

Contract Law with Multi-Unit Sales

Preliminary and Incomplete

Tracy R. Lewis

Fuqua School of Business

Alan Schwartz

Yale Law School

March 10, 2017

Abstract

Private contract enforcement often depends on the collective actions of individual agents. For instance high rates of credit default obstruct enforcement of duties to perform, and individuals indifference to infringing on the property rights of owners can render private contracts ineffective. We develop a theory of contracting enabling private parties to internalize the effects of their behavior on others and of enforcing private provisions that yield positive spillovers

K22: Business and Securities Law

1 Introduction

For students of the law and contract theory the 2007-2008 collapse of the residential mortgage-backed securities (RMBS) market was notable for the failure of presumably well designed securities contracts. Securitization has had and continues to have an important role in the U.S Economy with over 11 trillion of outstanding securitized assets, which substantially exceeds the size of of all outstanding marketable US treasury securities and related assets.¹. But to avoid or at least reduce the likelihood of a future financial crisis, one needs to know why, where and how the securitization process went awry.

¹See Gorton and Metrick (2011)

The conventional wisdom of leading financial economists is that the crisis revealed a number of serious problems with our financial system in general and securitization of assets in particular. A generic cause of the crisis were rampant conflicts of interest, better known to lawyers and economists and as agency problems. Agency problems arise in securitization contracts when a principal, i.e. an intermediary requests an agent, i.e. a bank, to supply prime grade assets backed by residential mortgages. Potential gains from trade between the principal and the agent exist. The parties rely on contract provisions to realize these gains and on the courts to enforce the provisions when one party fails to perform. In this paper we demonstrate why and how securitization agreements failed despite efforts to enforce them. The creation of mortgage backed securities withered under the weight of agency problems, including moral hazard, hidden information and financial regulations that overlooked the social costs of failed loans.

In section 2 we review the string of events that led to the failure of individualized security contracts for intermediation. Like most financial transactions future payoffs were difficult to predict and the parties in this case, the borrowers, banks and intermediaries had disparate information and expectations about their relative payoffs. The large variations in payoffs of the operators made it difficult to attribute unexpected payoff gains or losses to one party's action or to market swings beyond the agents' control. Despite this, had there been vigilant regulators, parties would have regarded this as a risk of "doing business" and insured themselves against payoff variations with capital and diversified portfolios.

However, because this indeterminacy in payoffs went unchecked the operators had incentives to shirk their duty to invest to avoid harming each other. Under normal conditions, the courts overseeing a bi-lateral contract between a single bank and an investor have a decent chance of determining which of the parties has breached her duty to perform, and to fix that breach with remedies of specific performance or expectation damages. However in the midst of the financial crisis that followed, there were widespread failures of residential loans and a dire need for repair and replacement of large numbers of loan agreements. It was impossible for the courts to identify and rectify every loan failure in real time.

To understand why contract individuation failed in this financial crisis, is to appreciate that each contract dispute generates negative externalities for the regulation of financial

markets, just as each ton of emission generates negative externalities for environmental regulation. As is well known it is normally impossible to write a complete financial contract for the initiation and sale of a single loan that addresses contingencies in each possible state. Matters worsen in a financial crisis. One failed loan precipitates the failure of other loans. Wide spread loan defaults delay repair of individual mortgages allowing delinquent borrowers to evade eviction from their homes, and so on. Each failed loan imposes social costs on the RMBS market; creditors wait longer to be repaid; investors' returns from asset backed securities decline, forcing housing prices to fall and confidence in financial markets to wane.

Section 3 provides a simple formal model of the intermediation process. The individuation of bi-lateral contracts for the origination and sale of mortgage backed bank loans to intermediaries is compared with two intermediation alternatives. One alternative relies on government regulation of mortgage backed securities in place of open market securitization. We show this alternative is preferable to securitization of contracts in preventing financial collapse and reducing the social costs loan defaults when regulation is depoliticized.

The second alternative is a return to old style intermediation where small local banks retained sufficient capital to fix any loan defaults. We show this alternative trumps the others if the social costs of systemic failures are sufficiently high and the cost of private capital is relative low.

Section 4 concludes with observations on the role of individuated contracts and court enforcement of private investor performance in markets subject to agency costs and collective action issues.

2 The failure of individuated security contracts for intermediation

INSERT CONTRACT LAW WITH MULTI-UNIT SALES

3 A Simple Setting: Individuation Contracts

We begin with a simple formal model to highlight the central effects of intermediation on loan quality in the context of individuated contracts. For concreteness we focus on the benchmark case of one time origination with eventual sale of mortgage backed securities to intermediaters. Let B be a representative local bank who originates collateral backed loans on real property. There is a continuum of loans $l_B \in [0, 1]$ of mass 1 expressed in (millions of dollars). The loan is intended (but not guaranteed) to yield a risk free return of 1. The loan type is g (good) or b (bad). Type g loans are repaid while type b loans are defaulted. The quality $q_B(e_B) \in [0, 1]$ of the loan portfolio is the probability a given loan l_B is good. Quality $q_B(e_B)$ is an increasing and concave function of the bank's effort e_B to find credit worthy borrowers and to monitor their actions. We assume loan quality is observed and the bank is liable for bad loans.

Once B originates loans of quality q_B they are sold to intermediary I by contract. The contract stipulates price p_B for the loans which are guaranteed by B to perform. Should a loan fail and B breaches her responsibility to repair, the court enforces a specific performance or expectation damages remedy. In the formal analysis to come we refer to these arrangements as *individuated contracts*. Because B originated the loan she is uniquely qualified to repair or replace the non-performing loan at minimal cost, denoted c_R . The contract provisions are efficient if they induce B to exert efficient effort, $e_B := \arg \max_e p_B - c_R(1 - q_B(e)) - c_B e$, that minimizes the cost of guaranteeing performance. B acts efficiently because she is the residual claimant to net surplus created by guaranteeing risk free returns of the loan portfolio.

An important complication arises when there is an additional stage of intermediation following loan origination. To illustrate, I contracts *separately* with banks B_i and B_j for identical and independently distributed loan portfolios with the same quality $q_B(e_B)$. As before, each bank guarantees the performance of her loans and should one fail and she breaches her responsibility to repair, the court enforces the appropriate remedy. However, now I combines the portfolios from the different banks to form a mixture of loans, called a synthetic asset. I constructs a continuum of assets, $l_I \in [0, 1]$ consisting of equal parts

of loans originated by B_i and by B_j . Furthermore I may invest effort e_I to increase the synthetic asset quality $q_I(e_{B_i}, e_{B_j}, e_I)$. Presumably $q_I(e_B, e_B, e_I)$ exceeds $q_B(e_B)$, because I 's asset portfolio consists of mixtures of independently distributed loans of quality $q_B(e_B)$ that reduces the variation in returns and decreases the likelihood of failure.

If the operators I , B_i and B_j shared knowledge of the make-up of the synthetic asset, I could presumably ensure her ideal outcome by relying on separate individuated contracts described above, that delegate repairs of failed mortgages to the individual originators. However, after B_i sells a loan to I for intermediation she does not control the monitoring of the loan, or how her loan is combined with other bank loans. As a practical matter, when loans from different originators are combined to form an investor grade security, accurate determination of individual liability for repair of a failed asset is impossible. Under individuation the court must use best efforts to negotiate a liability assignment to minimize the total cost, including legal fees and court costs, of repairing the failed asset. For modeling purposes it seems safe to assume $c_R(m)$ the unit cost of restoring a failed assets under individuation is strictly increasing in m the number of operators involved. And, without knowing their liability under the contract, originators are not incented to make efficient investments in loan quality to reduce default. Thus, a lack of common information can preclude seamless intermediation of financial assets, limit the tailoring of remedies to prevent breach, thus resulting in unnecessary financial distress.

Now imagine separate contracts for origination and intermediation still exist but that the sellers of financial products provide express warranties to ensure performance. The originator agrees to repair or replace defective mortgages provided the intermediary has otherwise maintained the loans in working order. Suppose that it is common knowledge that the quality of the loans that originators B_i and B_j originate and sell to I for intermediation is an increasing and concave function $q(e_{B_i}, e_{B_j}, e_I)$ of the observable efforts $\langle e_{B_i}, e_{B_j}, e_I \rangle$ the parties expend. Provided the failure of a loan is publicly observed, the operators are responsible for repairing or replacing the loan if collectively or individually they have been negligent in maintaining the loan quality. Assuming there are competitive markets for origination and intermediation, it is readily shown that enforceable contracts exist that induce the parties to exert efficient efforts $\langle e_{B_i}, e_{B_j}, e_I \rangle := \arg \max_{\langle e_i, e_j, e_k \rangle} 1 - c_R(1)(1 - q(e_i, e_j, e_k)) - (c_{B_i}e_i + c_{B_j}e_j + c_Ie_k)$ that

minimize the cost of risk free loans. Courts that enforce express warranties allow the originator to signal the intermediary that she also is complicit in loan failures. The costs of maintaining loan quality is borne by all of the parties.

This is precisely the strategy for the courts to use in general. When intermediation involves combining loans from several suppliers it affords intermediaries a choice of originators to pair with. For example I creates a synthetic security combining the loan portfolios of B_i and B_j . The resulting security has quality $q(e_{B_i}, e_{B_j}, e_I)$ based on the parties' observable investments in loan quality. Should a synthetic security fail, each of the parties contributes a predetermined amount to its repair. It is not necessary to determine which of the parties is at fault, because the parties have agreed to compensate public sector investors for their out-of-pocket losses. This arrangement could be implemented by a pair of two-way contracts between (i) I and B_i and (ii) I and B_j , however one three-way contract between all the parties would probably be less costly to write and enforce.

These findings may help to explain why intermediation of asset-backed securities is at once so appealing but sometimes susceptible to failure. When there is good information about the quality of loans and the composition of mortgage backed securities is transparent, warranties guaranteeing loan performance are possible to enforce, provided operators are liable. Intermediation affords the originators a choice of retaining their low quality loans or selling their high quality loans to the market, when loan quality or investment in loan quality is verifiable. At the same time, intermediators can increase the volume of minimum risk investor grade securities by combining high and moderate quality mortgages given the investments in loan quality of the originators and intermediators are observed. However intermediation may fail, sometimes on a grand scale, when loan quality and investment are not transparent or monitored.

4 Intermediation with Private Investment and Multiple Operators

We now demonstrate that these qualitative conclusions persist more generally, albeit with some qualifications about the access to capital that originators are assumed to have. For

concreteness we focus on contracts where investment is private and subject to moral hazard.

Assume there is one intermediary I who contracts with $n \geq 1$ independent, identical originators B to buy a mass of its loans each worth \$1mil. I combines the loans from each B in equal amounts to form a continuum of synthetic assets $l_I \in [0, 1]^2$. It is easily shown because banks are assumed to be identical the optimal contract treats banks symmetrically and is characterized by the terms offered to the *representative* originator B . I instructs B to invest e_B in loan quality at unit cost of c_B . As in Section 1 I simultaneously invests coordination capital θ_I to package and coordinate loans from multiple banks at one time to attain asset quality equal to the probability that each of the continuum of synthetic assets l_I is a *good* type and yields a riskless return equal to 1. Quality $\theta_I q(ne_B,)$ is an increasing and concave function of the investments.³ For convenience the elasticity of the probability of success with respect to bank effort (γ) is assumed constant and the same for all originators. Formally we assume⁴

$$(A1) \quad \theta_I q(ne_B) = \theta_I (ne_B)^\gamma \quad \text{where } n\gamma \leq 1$$

The expected cost $c_R(m)(1 - \theta_I (ne_B)^\gamma)$ of replacing the fraction of failed assets is a linear function of $c_R(m)$ the unit replacement cost depending on the type of contract mediation m employed. Operator disputes over repairs are said to be settled by *unilateral mediation* ($m = 1$) when I contracts simultaneously with n originators who bear collective liability for fixing any non performing loan, independent of the bank that originated the loan. The unit cost of repair for unilateral mediation, $c_R(1)$ is minimal because it is not necessary to establish which originating bank is at fault, which can be time consuming and costly. Also it may be less costly to compensate the harmed parties for payment shortfalls rather than repair or replace the failed mortgage. In contrast when *multilateral mediation* or individuated contracts are used to resolve liability disputes, the part(ies) responsible for non performing

² I mixes loans from independent banks in order to form minimal risk securities, although we don't model this feature explicitly.

³Because the originators are symmetric, it is understood that e_B is the investment made by each of the N originators in the contract equilibrium, with asset quality $q(e_B, e_I)$.

⁴Each synthetic loan $l_I \in [0, 1]$ is a equal mixture of N independent bank loans each with of same quality e_B^γ . I invests e_I^γ in each loan $l_I \in [0, 1]$. Hence the quality of loan portfolio is $e_B^\gamma e_I^\gamma$. By assuming $\gamma < 1/2$ we ensure that $q(e_B^\gamma e_I^\gamma)$ is a concave function.

assets are determined by individuated contracts, where the originator of each loan is determined to be liable or not liable for repair of her individual loan. In this case the unit cost of repairing a shortfall in payments is $c_R(n)$ which exceeds $c_R(1)$ because I must establish which of the n originators is liable for the repair of a failed synthetic asset. This is a time consuming process, that adds to the unit cost of repair as we have shown in section 1.

This section compares the construction of synthetic risk-free assets under different forms and degrees of contract individuation. By now it should be clear individuation would not be the contract of choice, were it not for the possibility that originator banks are sometimes capital constrained, and unable to guarantee the quality of their loans. To analyze the effect of capital constraints on contracting for bank loans we begin by analyzing optimal intermediation for the unconstrained capital setting.

A. OPTIMAL CONTRACTS WHEN BANKS HAVE UNLIMITED CAPITAL

In the setting where bank capital is unconstrained, I maximizes the net social surplus from the representative risk free security by choosing investments to solve problem $[P_U]$,

$$\begin{aligned}
 (1) \quad S(1) &\equiv \underset{e_B, \lambda_B, \lambda_I, \tau_B, \tau_I}{\text{maximize}} \quad [1 - c_R(1)(1 - \theta_I(ne_B)^\gamma) - \theta_I - c_B e_B] \\
 &\quad \text{subject to} \\
 (2) \quad &\quad \pi_B, \pi_I \geq 0 \\
 (3) \quad &\quad \pi_B = \tau_B - \lambda_B c_R(1)(1 - \theta_I(ne_B)^\gamma) - c_B e_B \\
 (4) \quad &\quad \pi_I = 1 + \tau_I - \lambda_I c_R(1)(1 - \theta_I(ne_B)^\gamma) - \theta_I \\
 (5) \quad &\quad N\tau_B + \tau_I = 0
 \end{aligned}$$

Expression (1) implies the net social surplus of a synthetic security is the difference between the total expected surplus of 1 and failed security replacement and investment costs, $c_R(1)(1 - \theta_I(ne_B)^\gamma) - \theta_I - c_B e_B$. Expression (2) ensures the operators participate because each expects to receive non negative profits. Expressions (3) and (4) imply profits equal operator's share of the synthetic assets, minus her share of replacement and investment costs plus transfer payments⁵. There is an implicit assumption the operators have enough capital

⁵We assume the intermediary receives the proceeds from the sale of the synthetic asset, but this not necessary for the analyses to follow.

to pay the restoration cost of the synthetic asset in the profit expression for originators. Required modifications in the contract when operators are capital constrained, are discussed below in Section B. Expression (5) requires operators' transfers sum to zero, ensuring the synthetic asset is created by a self supporting joint venture⁶.

Lemma 1 characterizes efficient investment in the synthetic asset when the operator's investment in quality is not observed but operators have sufficient capital available for any loan failures.

LEMMA 1 *Suppose the banks are liable for failed loans, bank investments e_B are unverifiable and the failure and repair of the risk free asset at unit cost $c_R(1)$ is known. Then a sufficient condition for efficient origination and intermediation is the marginal share of repair costs is $\lambda_B = 1$ for all banks B .*

Proposition 1 reports that unilateral mediation, where I contracts simultaneously with all originators to provide high quality loans is cost minimizing under the conditions of Lemma 1. The Proposition refers to $S(1)$ and $e_B(1)$, which are the expected surplus and investments at the solution $[P_U]$ when mediation is unilateral

PROPOSITION 1 *Suppose the operators have unlimited liability, operator investments are unverifiable and the failure and repair of the risk free asset is observed. Then the solution $[P_U]$ is efficiently implemented by a unilateral mediation agreement with these properties:*

- (i) *The unit cost of repairing the failed assets is minimized at $c_R(1)$,*
- (ii) *Minimal regulation or oversight by the courts is required,*
- (iii) *Dispersed control, i.e. the construction and maintenance of the risk-free asset is operator controlled to minimize the total cost of ensuring risk-free assets*

$$e_B(1) := \arg \max_{e_B} -c_R(1)(1 - \theta_I(ne_B)^\gamma) - \theta_I - c_B e_B,$$

⁶In practice the Federal government is the insurer of last resort for synthetic assets and securities, but it cannot guarantee to make all investors whole when there is large scale default. cite

(iv) *Relative profit shares guarantee each operator is the residual claimant to her investment surplus*⁷,

(v) *Total surplus is increasing in the number of independent originators, n .*

The conclusions of Proposition 1 stem from the following considerations: The operators induce each other to undertake the efficient investments at minimal or no costs of monitoring. Assuming operators have sufficient capital and are liable for paying the marginal costs of repairing a failed loan, it is unnecessary to identify the operator(s) at fault under the contract conditions. Barring an unforeseen change in conditions, the contract stipulates each operator is individually liable for loan failures, implying each has incentives to invest efficiently when each is the residual claimant of the rewards from its investment. These terms induce efficient investment without unnecessary expenditures to check for specific operator liability. Finally Proposition 1 reports there are (weakly) increasing returns to scale in the number of independent originators employed to construct the synthetic asset⁸. The import of this finding is to illustrate that when banks are well capitalized, it is efficient to delegate the origination of loans to numerous banks from diverse sectors of the economy *to* form low risk securities at least cost.

B. INDIVIDUATED CONTRACTS AND CAPITAL CONSTRAINED BANKS

We now extend the analysis in two directions. First we consider *a limited liability setting* where loan originators are partially capital constrained; that is banks have sufficient capital to fund mortgages but not enough capital to pay expectation damages for a failed loan in timely fashion. This means it is impossible to obtain the economies of scale in the oversight of risky loans, afforded by unilateral mediation. Instead I purchases loans from n different banks on an individuated basis. Because the banks are capital constrained the courts enforce a specific performance remedy. This together with the difficulty of attributing a loan default to a specific bank increases the unit cost of repairs to $c_R(n)$

⁷The appendix provides a proof of Lemma 1, and all other formal results.

⁸The returns to scale are strictly increasing provided $n < 1/\gamma$.

In the partial liability setting I maximizes the net social surplus by choosing investments to solve problem $[P_L]$

$$(6) \quad S(n) \equiv \underset{e_B, \tau_B, \tau_I}{\text{maximize}} [1 - c_R(n) (1 - \theta_I (ne_B)^\gamma) - \theta_I - c_B e_B]$$

subject to

$$(7) \quad \pi_B, \pi_I \geq 0$$

$$(8) \quad \pi_B = \tau_B - c_R(n) (1 - \theta_I (ne_B)^\gamma) - c_B e_B$$

$$(9) \quad \pi_I = 1 - n\tau_B - \theta_I$$

PROPOSITION 2 *Suppose I contracts separately with each of n representative banks for the delivery of minimal risk loans. All originators have limited liability and operator investments are unverifiable. Then at the solution to $[P_L]$*

(i) *When bank loans are integrated, the determination of liability \in individuated contracts results in unit repair cost of $c_R(n)$.*

(ii) *I induces efficient investment in quality*

$$e_B(n) = \arg \max_{e_B} -c_R(n)(1 - \theta (ne_B)^\gamma) - c_B e_B$$

Originators exert effort to avoid large repair costs caused by individuation.

COROLLARY 1: *Efficient intermediations in an unlimited liability setting yields greater surplus per origination of bank loans than efficient intermediation in a limited liability setting.*

The conclusions of Proposition 2 and Corollary 1 stem from the following considerations. When originators are liquidity constrained, repair of non performing loans is tedious, slow and costly. The lack of liquidity precludes operators from the use of the streamlined full liability remedy reported in Proposition 1, which provides for efficient repair of failing loans and investment in quality. Proposition 2 reports that liquidity constrained operators exert more effort to ensure loans perform, because a failed security is more costly and time consuming to replace. As a result the originator's asset backed securities fail less often. When the high cost

of repairs under individuated contracting is borne by the operators, as we have assumed, the inefficient remedy for non performance has punitive effects by forcing originators to search for safer loans. However, this distortion leaves the operator with less capital to fix non performing loans if they fail. Corollary 1 concludes that although bank loans may be safer under individuated contracts, they are more costly to fund and more costly to repair if they fail. In view of the fact that this comparison is driven by liability and access to capital, it confirms what we already know: free capital is a valuable and scarce resource in financial intermediation.

Our second extension considers the setting most applicable to the process often employed currently in intermediation. The bank originates loans by finding credit worthy borrowers and funding mortgages. Following origination she sells the cash flows from her loans to an intermediary. In theory she retains responsibility for ensuring the loans perform. In practice, the bank has no capital or incentive to fix failing loans. Whatever new capital the bank receives goes towards originating new loans for eventual sale. Because the bank's liability for fixing loans is essentially judgement proof, we call this the judgement proof setting.

In this setting we assume I relies on the government to certify the quality of the loans it purchases. Assuming competitive markets and government certification of quality the price of a certified quality loan is μc_B , where $\mu > 1$ is the social cost of funds when financial markets are government regulated. The intermediary's selection of loan quality solves problem [P_J],

$$(10) \quad \pi_I(n) \equiv \underset{e_B}{\text{maximize}} [1 - c_R(1)(1 - \theta_I(ne_B)^\gamma) - \theta_I - \mu c_B e_B]$$

Expression (10) implies that I is acting to maximize her profit which is the difference between the total expected surplus of 1 and the failed security replacement and investment costs, $[1 - c_R(1)(1 - \theta_I(ne_B)^\gamma) - \theta_I - \mu c_B e_B]$. In this setting I is responsible for compensating investors for any shortfall in security returns. Assuming markets are competitive and the certification of loan quality is accurate, $\pi_I(n)$ coincides with net social surplus which includes the social cost of government regulation.

Proposition 3 characterizes the efficient construction of the synthetic asset, when banks are judgement proof and the intermediary guarantees the returns on the synthetic asset.

PROPOSITION 3 *Suppose I contracts separately with each of n judgement proof representative banks for the delivery of bank loans of certified quality. Then at the solution to $[P_J]$*

(i) *Unit repair cost of failed assets is minimized at $c_R(1)$.*

(ii) *I induces efficient investment in certifiable quality*

$$e_B(n) := \arg \max_{e_B} -c_R(1) (1 - \theta (ne_B)^\gamma) - \mu c_B e_B$$

COROLLARY 2: *Efficient intermediation in an unlimited liability setting yields greater surplus per origination of bank loans than efficient intermediation in a regulated judgement proof setting*

Discussion to Follow

5 Appendix

5.1 A. Sketch of the Proof of Proposition 1 and Lemma 1

Substituting conditions (2) – (5) into expression (1) for the maximand we can rewrite $[P_U]$ as,

$$\begin{aligned} \underset{\tau_I, \tau_B, e_B}{\text{maximize}} J = & [1 - c_R(1) [1 - \theta_I (ne_B)^\gamma] - c_B e_B] + \\ & \alpha_B [\pi_B - \tau_B + \lambda_{BCR}(m) (1 - \theta_I (ne_B)^\gamma) + c_B e_B] + \\ & \alpha_I [\pi_I - \tau_I - 1 + \lambda_{ICR}(m) (1 - \theta_I (ne_B)^\gamma)] + \\ & \varepsilon (\tau_B + \tau_I) + \mu_B \pi_B + \mu_I \pi_I \end{aligned}$$

with the following first order conditions:

$$\begin{aligned} (a1) \quad J_{e_B} &= \lambda_{BCR}(1) \gamma \left(n^\gamma e_B^{\gamma-1} \theta_I \right) - c_B &= 0 \\ (a2) \quad J_{\alpha_B} &= [\pi_B - \tau_B + \lambda_{BCR}(1) (1 - \theta_I (ne_B)^\gamma) + c_B e_B] &= 0 \\ (a3) \quad J_{\alpha_I} &= [\pi_I - \tau_I - 1 + \lambda_{ICR}(1) (1 - \theta_I (ne_B)^\gamma)] &= 0 \\ (a4) \quad J_\varepsilon &= \tau_B + \tau_I &= 0 \end{aligned}$$

These conditions can be shown to be necessary and sufficient for a solution to $[P_U]$. Condition (a1) implies that $\lambda_B = 1$ is necessary and sufficient for efficient investment in origination and mediation thus proving Lemma 1. Properties (i) – (v) of Proposition 1 follow immediately from conditions (a1) – (a4).