Working paper

"*WHEN OLSON MEETS DAHL*": FROM INEFFICIENT GROUP FORMATION TO INEFFICIENT POLICY-MAKING[1]

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Abstract. Two conflicting interest groups buy favors from a decision-maker. Influence is modeled as a common agency game with lobbyists proposing monetary contributions contingent on decisions. When preferences are common knowledge, groups form efficiently and lobbying competition perfectly aggregates preferences. When preferences are private information, free riding in collective action arises *within groups*. Free riding implies that groups choose lobbyists with moderate preferences and that lobbying competition imperfectly aggregates preferences. By softening lobbying competition, asymmetric information might also increase groups' payoffs, although it always hurts the decision-maker. Importantly, informational frictions within each groups are best responses to each other and thus jointly determined at equilibrium. We draw from these facts a number of implications for the organization of interest groups, stressing that strong coalitions are better able to buy influence and showing the impact that entry costs in lobbying might have on influence.

Keywords. Lobbying, Collective Action, Free Riding, Asymmetric Information, Common Agency, Mechanism Design.

JEL codes. D72; D82.

1. INTRODUCTION

Motivation. The formation of interest groups, their competition in the political arena and, more generally, their influence on policy-making are key concerns for students of modern democracies. The role of lobbying as a vehicle for the representation of diverse interests, and its impact on the democratic process, although unanimously recognized, has nevertheless raised conflicting views among both political scientists and economists.

Following Dahl (1961)'s seminal work, the so-called *pluralistic approach* to politics views competition between interest groups as a healthy way to aggregate preferences. This view of politics argues with much optimism that policy-making reaches the right balance between various interests involved. In the economics literature, this approach, which certainly found its roots in the earlier works of Peltzman (1976) and Becker (1983), is nowadays best exemplified by the so-called *common agency model* of lobbying competition proposed earlier on by Bernheim and Whinston (1986) and then pursued by Grossman and Helpman (1994) and others over a broad range of applications.[1] Within this

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[1]Noticeable contributions include Persson and Tabellini (1994), Dixit, Grossman and Helpman (1997), Rama and Tabellini (1998), and Yu (2005) among others.

realm, interest groups influence a decision-maker through monetary payments (for instance campaign contributions) whose levels depend on the the decision-maker's choices. Importantly, such a multilateral bargaining protocol has always efficient equilibria where groups offer *truthful schedules* that reflect their preferences over alternatives. When utility is transferable, the decision-maker chooses a policy that maximizes the sum of his own payoff and those of active interest groups.[2] An immediate corollary of this approach is that the free riding problem both within and across groups can be solved at no cost at truthful equilibria; a somewhat unpalatable conclusion.[3]

In contrast, Olson (1965) has taken a less optimistic stance when viewing lobbying essentially as a rent-seeking activity. Strongly organized groups buy favors while unorganized rivals are unable to exert any influence. The major thrust of *The Logic of Collective Action* is that *intra-group* free riding might prevent groups from promoting their interests. Free riding being more of a curse as the size of the group increases, large latent groups might be dominated by small and better organized groups; the so called *Olson Paradox*.[4]

These two approaches thus deliver very different conceptions of the organization of interest groups. On the one hand, Olson (1965) argues that free riding hinders representation of interests. Yet, this argument fails to recognize that inefficiencies in collective action are to a large extent endogenous. Indeed, the stakes for forming as an active group depend on whether other groups might have already influenced decision-making or not. On the other hand, most models of lobbying competition do not even discuss why groups may face difficulties in forming at the outset leaving that as a choice of the modeler.[5] Reconciling the messages of these literatures still stands as a major challenge for our understanding of collective action.[6]

This paper makes progresses on this front. To address the free riding problem in collective action, we introduce asymmetric information on preferences. Individuals shade their willingnesses to pay for a policy shift to reduce their own contribution while still benefitting from their group's action. The stakes for groups formation are endogenously determined through lobbying competition. The endogenous costs and benefits of forming determine whether there is enough net gain from group formation to pay for the informational cost needed to induce preferences revelation. We ask whether the political process still efficiently aggregates preferences *within* and *across* groups.

[2]Throughout the paper, an efficient allocation is defined as being on the Pareto frontier of the set of payoffs for the groups and the decision-maker. This definition which is standard in the literature thus assumes that the decision-maker perfectly represents other interests in the polity.

[3]Another corollary is that games of voluntary contributions (Bergstrom et al. (1986)) only generate such inefficient free riding by restricting strategies.

[4]To illustrate, both the *Chicago School* (Stigler (1971), Posner (1974)) and the more recent *New Regulatory Economics* (Laffont and Tirole (1991)) have taken as granted inefficiencies in collective action and analyzed their impact on policy-making.

[5]See for instance Aidt (1996) and Siqueira (2001).

[6]Baumgartner and Leech (1998, pp. 88) wrote: *"One sees the interest-group system as hopelessly biased in favor of powerful economic interests and narrow special pleaders; another sees a greater diversity of interest in the Washington policy community and a positive role for groups in the creation of better citizens. The relative emphasis that scholars have placed on each of these views has changed from decade to decade, but neither has been shown to be completely accurate: a complete view must recognize elements of both views."*

In this endeavor, we rely on two important bodies of works; namely *mechanism design* and *common agency*. When taken separately, those two workhorses models have been extensively used to understand groups formation and lobbying competition. Yet, those two paradigms have evolved independently and, as such, have not been able to offer the more comprehensive framework needed. Before being active, groups must solve their collective action problem: an issue that calls for importing the tools of mechanism design. Within each group, mechanisms for collective actions are designed to ensure that members reveal their preferences. At the last stage of the game, lobbyists acting on behalf of interest groups compete for the decision-maker's influence: a standard common agency game.

MODEL AND MAIN RESULTS. Two interest groups with conflicting preferences over a policy decision buy favors from a policy-maker. Within each group, individuals have private information on their preferences. To be active, a group appoints a lobbyist and gives him a budget so as to exert political pressure on the decision-maker. The group must also determine how members share their overall contribution.

Aggregating preferences across groups. Because they represent interest groups with conflicting interests, each lobbyist is thus ready to pay up to the impact that his own influence has on the rest of society. Contributions, endogenously determined at the equilibrium of the common agency game, are thus *Vickrey-Clarke-Groves* (thereafter VCG) payments.[7] Because such payments are non-manipulable, groups thus have no incentives to choose the preferences of their own lobbyist for strategic purposes. The sole source of distortions, if any, may thus come from solving the intra-group free riding problem.

Aggregating preferences within groups. Had preferences been common knowledge, the free riding problem within a group could be solved by having each individual contribute up to his willingness to pay for a policy shift. Preferences would be perfectly aggregated within groups. Efficiency in solving the free riding problem together with the conditional efficiency of the common agency stage imply overall efficiency of policy-making. The basic take-away of this complete information scenario is that, if Olson is wrong and free riding in collective action is not an issue, then Dahl is also right and lobbying competition efficiently aggregates preferences.

Asymmetric information on preferences changes the picture. Group members may now free ride by shading their willingness to pay for a policy shift. As a result, information can only be revealed if individuals get enough *information rents*. A group now forms when the net gains from influencing the policy-maker cover those rents: A key *incentive-feasibility condition*. Inducing a large policy shift requires greater contributions from the group but it also exacerbates individual incentives to free ride.

Inefficient group formation. To limit information rents, two sorts of distortions arise. First, free riding is less of a concern when the group's overall contribution is reduced. The decision-maker is now more inclined to preserve the *status quo* policy he chooses under the sole influence of competing interests. Asymmetric information within groups thus softens lobbying competition. It also hurts the decision-maker who can no longer extract as much by playing one group against the other.

[7] Green and Laffont (1977).

Second, the group chooses a lobbyist with moderate preferences. Following insights from the mechanism design literature (Myerson (1981)) willingnesses to pay for a policy change are indeed replaced by *virtual willingnesses to pay* of lower magnitude. Aggregating *virtual willingnesses to pay* across group members is akin to having a group with less pronounced preferences for the policy shift. When free riding is too important, this group might even fail to get organized. Under those circumstances, *Olson's Paradox* finds strong informational foundations: An interest group is no longer active if its internal informational problems are too costly to solve.

The role of the types distribution. When the support of the types distribution includes a type with no strict preferences on policy, inefficiencies are pervasive and (under another weak extra condition) might not even depend on other groups' strategies. This result is best exemplified with large groups. Incentives to free ride are there at their worst. Group members would like to pretend having no preferences whatsoever for the policy so as to pay nothing for a policy shift while still enjoying its benefits.[8] Overall, zero contribution can be collected and such large groups cannot be active. *A contrario*, even large groups might be active when the lower bound of possible valuations is strictly positive. This is not smallness *per se* that facilitates group formation but the existence of a minimal stake. Our informational perspective on group formation thus challenges Olson-Stigler's view that groups are more likely to be successful when more heterogenous.

Joint determination of informational frictions. Studying the formation of coalitions among already active lobbyists, Holyoke (2009) shows that lobbyists with slightly divergent interests will be more likely to overcome their divergence and get organized into a coalition when they face a more intensive competition. Gray and Lowery (1996) also argue that the ability of an interest group to mobilize depends on its environment. Our model echoes those findings. In our framework, inefficiencies in collective action are fully captured by the shadow cost of an incentive-feasibility condition that pertains to that group. Since the net benefits of coalition formation depend on the *status quo* policy that would be chosen by a decision-maker under the sole influence of the competing group, this shadow cost might depend on the competing group's strategy. Inefficiencies in groups formation are thus *jointly* determined at equilibrium. In a *dual representation* of the game, shadow costs of the incentive-feasibility conditions for both groups are found as best responses to each other. In sharp contrast with the complete information scenario, inefficiencies within groups now percolate as inefficiencies in the lobbying game. To summarize, Olson's view is incomplete and free riding within a given group, when it matters, also depends on how competing groups solve their own organizational problem. Under asymmetric information, Dahl is wrong and lobbying competition cannot efficiently aggregate preferences.

Towards an I.O. perspective on the formation of interest groups. That informational frictions for competing groups are jointly determined at equilibrium suggests that those groups might also try to use various commitment devices to increase informational frictions for their competitor. This idea is well-known from the IO literature (Fudenberg and Tirole (1985), Bulow, Geanakoplos and Klemperer (1985)) and is here revisited in a political economy setting. Of course, whether a commitment device to ease internal frictions

[8]There also exists a tiny literature that analyzes incentives to free ride within groups in moral hazard settings (Lohmann (1998), Anesi (2009)).

gives a competitive edge on other groups depends on which of the *"policy-shifting"* or the *"status quo"* effect dominates. We analyze under which circumstances, *strong coalitions*, which are able to credibly share information, may worsen the free riding problems for competing groups, which in practice may limit the entry of those competitors in the political arena. More generally, we also unveil how a fixed cost of entry in lobbying has not only an impact at the extensive margin, precluding some groups to enter the political arena, but also at the intensive margin; inducing those entering to adopt more moderate stances as covering entry costs exacerbates free-riding.

Organization of the paper. Section 2 describes the model. Section 3 analyzes group formation under complete information. Section 4 presents a simple incentive-feasibility condition that summarizes all difficulties that a group may face to overcome the free riding problem. Section 5 provides conditions ensuring that groups form efficiently even under asymmetric information. In contrast, Section 6 investigates conditions for free riding to arise for large groups. Section 7 tackles the more complex scenario of groups of finite sizes. Section 8 shows how inefficiencies are jointly determined across groups. Section 9 then discusses various commitment devices to strategically affect frictions faced by competing groups and thus impact on lobbying competition. Section 10 assesses the welfare consequences of asymmetric information. Section 11 discusses the robustness of our findings. Proofs are relegated to an Appendix.

2. THE MODEL

2.1. *Interest Groups*

Preferences. Agents in the economy are divided into two groups.[9] Group l (for $l \in \{1,2\}$) has size $N_l \geq 1$. Agent i (for $i \in \{1,...N_l\}$) in group l has quasi-linear preferences over a policy x chosen by a decision-maker (sometimes referred to as *she* in the sequel) within an interval $\mathcal{X} = [-x_{max}, x_{max}]$ (x_{max} being large enough) and the monetary contribution t_l^i that he pays to influence the decision-maker:

$$\frac{\alpha_l^i}{N_l}u_l(x) - t_l^i.$$

The parameter α_l^i captures the intensity of agent i's preferences while $u_l(x)$ stems for a payoff function which is specific to group l. Members of a given group rank all policies in the same way although the intensity of preferences vary across individuals. Individuals of the same group are thus vertically differentiated. Individual preferences are scaled up by the size of the group N_l to normalize the group's overall influence.[10] For tractability, each function u_l is supposed to be linear in x although we often keep a more general expression to show how our results would apply more broadly.To capture the idea that groups are horizontally differentiated, we posit:

$$u_1(x) = -u_2(x) = -x \quad \forall x \in \mathcal{X}.^{11}$$

[9]The interpretation of agents as individuals is in line with Olson (1965). Yet, our analysis below has a broader scope and one may also interpret those agents themselves as interest groups or organizations. In U.S. legislative politics, coalitions of interest groups abound in various fields going from education (the *Committee for Education Funding* which involves more than one hundred organizations) to transportation (for instance, trade associations such as the *American Bus Association* or the *Air Transport Association*).

[10]This assumption does not give any positive role for size *per se*. Instead and in the spirit of McLean and Postlewaite (2002), this is the relative measure of individual information compared to size that matters to determine the impact of each agent on the group's strategy.

To illustrate, the decision may be the price of a regulated good or services in the spirit of Peltzman (1976); group 2 being then composed of producers while group 1 represents consumers. The decision x could also be the level of an import tariff for some intermediate good as in Gawande, Krishna and Olarreaga (2012). Group 2 might then stand for domestic producers. This group asks for protection from foreign competitors and thus lobby for an import tariff. Group 1 is made of final domestic users who instead call for a low protection to reduce their expenditures.

The decision-maker represents other (unorganized) groups in society or a median voter who might have more neutral stances on the policy at stake. The decision-maker's quasi-linear utility function is also defined over the decision x and the overall monetary payments she receives z as:

$$u_0(x) + z.$$

The function u_0 is twice continuously differentiable, strictly concave, single-peaked and symmetric around a bliss point at $x_0 = 0$. Let denote $\varphi = u_0'^{-1}$ with $\varphi(0) = 0$ and $\varphi' < 0$ (which follows from $u_0'' < 0$). Some of our results below depend on the curvature of φ.

Running example. Below, we will sometimes rely on a quadratic specification of the decision-maker's preferences; a familiar workhorse of the Political Science literature:

$$u_0(x) = -\frac{\beta_0}{2}x^2 \Rightarrow \varphi(y) = -\frac{y}{\beta_0}. \tag{2.1}$$

Information. While the degree of horizontal differentiation and the composition of the groups are common knowledge, preferences are not so even within a given group. Each individual i in group l has private information on his own preference parameter α_l^i.[12] For each group, these values are drawn from a common knowledge group-specific (atomless) distribution function F_l whose support is $\Omega_l = [\underline{\alpha}_l, \overline{\alpha}_l]$. Let f_l be the corresponding (positive) density. In the sequel, the non-negative lower bound $\underline{\alpha}_l$ of the distribution sometimes plays an important role. A specific case is obtained when preferences are diffuse enough so that individuals with no specific preferences for the policy might be part of group l, i.e., $\underline{\alpha}_l = 0$.[13] The mean value of the preference parameter for group l is $\alpha_l^e = \mathbb{E}_{\alpha_l^i}(\alpha_l^i)$.[14] We denote by $\boldsymbol{\alpha}_l = (\alpha_l^i)_{i\in\{1,\dots N_l\}}$ any arbitrary vector of preference parameters for group l and by $\alpha_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i$ its sample mean. Let Φ_{N_l} (resp. ϕ_{N_l}) be the cumulative distribution (resp. density) of this sample mean whose support is still Ω_l. Finally, we use the standard notations $\boldsymbol{\alpha}_l = (\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ when needed.

[11] Asymmetry in the strength of the groups could also be easily introduced. Suppose for instance that the two groups would like to push policies in their respective directions although with different intensities. Formally, say $u_1(x) = -kx$ while $u_2(x) = x$. Up to changing the support of types distributions, any such linear specification with opposite preferences could be transformed into our formulation.

[12] Esteban and Ray (2001) view competition between groups as a contest. Contrary to us, groups are homogenous and the cost of lobbying is not derived from informational constraints.

[13] The fact that some individuals might have no specific preferences for the policy under scrutiny is in particular justified when the group is a long-term venture banding agents on several related issues. A given individual may have found a positive value in belonging to that group in the past and still belong to that group nowadays even though he has no strict preferences for the decision at stake.

[14] In the sequel, we shall denote expectation operator with respect to x as $\mathbb{E}_x(\cdot)$.

Adopting the parlance of the mechanism design literature (Myerson (1981)) the *virtual preference parameter* of agent i in group l is defined as:

$$h_l(\alpha_l^i) = \alpha_l^i - \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)}, \quad \forall \alpha_l^i \in \Omega_l.$$

Following a standard requirement (Bagnoli and Bergstrom (2005)), the *monotone hazard rate property* holds, i.e., $\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}$ is non-increasing. This assumption ensures that $h_l(\alpha_l^i)$ is a non-decreasing transform of α_l^i. Following previous convention, the sample mean of virtual preference parameters for group l is defined as $h_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i)$.

2.2. *Lobbying Process*

The lobbying game determines contributions and policies as endogenous objects that impact in turn the costs and benefits of collective action. To model such interactions, we rely on a simple two-stage model that can be viewed as a metaphor for how groups form and act in the political arena.

Appointing Lobbyists. Each group delegates to a lobbyist the task of influencing the decision-maker. Following the literature on strategic delegation in legislative bargaining,[15] the group is free to choose the preferences of this delegate by specify how this lobbyist should trade-off political contributions against changes in decision. Formally, the lobbyist is endowed with the following objective:

$$\beta_l u_l(x) - T_l.$$

The payment T_l stands for the money that the lobbyist uses to buy the decision-maker's favors. Of course, such political contribution must be covered by individual contributions of the group's members. The weight β_l reflects the lobbyist's marginal rate of substitution between policy and contribution. This parameter is a convenient way of capturing how efficient the process of group formation is. When preferences are common knowledge, we will show that each group has a dominant strategy which is to endow its lobbyist with preferences that reflect the mean preference parameter of that group, i.e., $\beta_l \equiv \alpha_l^*(\boldsymbol{\alpha}_l)$. When preferences are private information and free riding is instead a concern, the wedge between β_l and $\alpha_l^*(\boldsymbol{\alpha}_l)$ captures how informational frictions undermine collective action.

The Common Agency Game. Endowed with such objectives, lobbyists compete for the decision-maker's favors. This stage is modeled following the standard *common agency* methodology (Bernheim and Whinston (1986), Grossman and Helpman (1994)). This framework is by now well admitted as being an adequate representation of how competing forces of lobbying groups impact policy-making. We thus adopt the corresponding informational assumptions, timing, and equilibrium refinements of that modeling. First, the lobbyists' objectives are common knowledge at this last stage of the game. In particular, each lobbyist perfectly knows his competitor's objective. Second, lobbyists offer to the decision-maker non-negative contributions $\tilde{T}_l(x)$. Such contributions are commitments to pay the decision-maker in response to her policy choice. The decision-maker is

[15] Christiansen (2013), Besley and Coate (2003), Dur and Roelfsema (2005), Persson and Tabellini (2002).

free to accept or refuse each of those contracts.[16] Third, we restrict attention to *truthful* equilibria obtained when lobbyists use *truthful contribution schedules* of the form:[17]

$$\tilde{T}_l(x) = \max\{\beta_l u_l(x) - V_l, 0\} \quad \forall x \in \mathcal{X}.^{18} \tag{2.2}$$

In this formula, V_l stems for the payoff that lobbyist l can always secure whatever the decision-maker's choice x. Truthful schedules perfectly reflect the lobbyist's preferences over alternatives. Hence, the equilibrium policy always maximizes the aggregate payoff of the grand-coalition made of those lobbyists and the decision-maker. Our goal is to investigate under which circumstances those induced preferences might no longer reflect the true preferences of an interest group as a result of informational frictions.

For future references, let $\beta = (\beta_1, \beta_2)$ be the lobbyists' preferences with $\Delta\beta = \beta_1 - \beta_2$ being a measure of polarization. Accordingly, the policy $x(\beta_1, \beta_2)$ that maximizes the payoff of the grand-coalition made of the decision-maker and the two lobbyists satisfies:

$$u_0'(x(\beta_1, \beta_2)) = \Delta\beta \Leftrightarrow x(\beta_1, \beta_2) = \varphi(\Delta\beta). \tag{2.3}$$

The optimal policy is tilted towards the group whose lobbyist has the strongest preferences. Of course, the definition of $x(\beta_1, \beta_2)$ given above applies equally well to characterize the optimal policy taken when only one group or none is active provided that we use the convention $\beta_l = 0$ for an inactive group. For instance, $x(0, \beta_2) = \varphi(-\beta_2)$ is the decision implemented if group 1 is not organized.

2.3. *Group Formation: Mechanism Design*

We envision the formation of group formation as a decentralized bargaining process among its potential members. Through this process, parties find out a way to share their overall contribution and to define their lobbyist's objective function. Of course, specifying extensive forms and communication procedures, and comparing their performances would be a daunting task. Following the methodology initiated by Myerson and Sattherwaite (1982) and pursued by Mailath and Postelwaite (1990) and Laffont and Martimort (1997) in other contexts, we take a more normative stance and model the choice of such bargaining between informed parties as a mechanism design problem. From the Revelation Principle (Myerson (1982)), any bargaining procedure would indeed lead to an allocation rule that could as well have been proposed by an uninformed mediator. The mediator has no physical existence *per se* but it is instead a metaphor for how the bargaining process unfolds. With that perspective in mind, a mechanism for the formation for group l determines, for all possible vector of reports $\hat{\boldsymbol{\alpha}}_l$ of its members, an objective function for his delegate and the set of individuals' contributions; $\left(\beta_l(\hat{\boldsymbol{\alpha}}_l), (t_i(\hat{\boldsymbol{\alpha}}_l))_{i=1}^{N_l}\right)$.

No-Veto Constraints. An important issue is to specify what happens if one potential member refuses its play. Following the existing mechanism design literature,[19] we assume

[16]The restriction to non-negative contributions is without any loss of generality in a model of delegated common agency. Indeed, the decision-maker being free to refuse any contract would never choose a policy corresponding to a negative payment.

[17]Because each lobbyist has a truthful schedule in his best-response correspondence, insisting on *truthfulness* is akin to imposing an equilibrium refinement (Bernheim and Whinston (1986)). Inefficient equilibria may nevertheless arise without the *"truthfullness"* refinement (Kirshteiger and Prat (2001)). Bernheim and Whinston (1986) demonstrate that *"truthfull"* equilibria are also coalition-proof.

[19]Laffont and Maskin (1982), Mailath and Postlewaite (1990).

individual veto power. If anyone refuses the mechanism, no lobbyist is appointed for group l and the decision-maker chooses $x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ so as to reflect only group $-l$'s own influence (if any). There are two ways of justifying this assumption. First, coalitions may themselves be made of interest groups that band together on some specific issues; a scenario that certainly echoes practices in nowadays U.S. Legislative Politics (Hula (1999)). In that case, it becomes again quite legitimate to give each of those groups equal veto power. Second, giving veto power to all individuals can be viewed as a metaphor for stressing the difficulties in gathering information on individual preferences and, as such, it showcases an upper bound on informational frictions that the group may face.[20]

To express veto constraints, it is useful to define the net gain of group formation for an agent with type α_l^i when the preferences profile within group l is $\boldsymbol{\alpha}_l = (\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ as:

$$\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) - t_i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) \text{ with } \Delta u_l(\beta_l, \beta_{-l}) \equiv u_l(x(\beta_l, \beta_{-l})) - u_l(x(0, \beta_{-l})).$$

Type α_l^i's expected net payoff from joining group l can thus be defined as:

$$\mathcal{U}_l(\alpha_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) - t_i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i})\right). \tag{2.4}$$

The no-veto constraint ensures that the net gains of participating to the group are non-negative for any individual in group l, whatever his type α_l^i:

$$\mathcal{U}_l(\alpha_l^i) \geq 0, \quad \forall \alpha_l^i \in \Omega_l. \tag{2.5}$$

Incentive Compatibility. From the Revelation Principle (Myerson (1982)), there is no loss of generality in considering direct mechanisms as above provided that each individual truthfully reports his type α_l^i at a Bayesian equilibrium.[21] The following incentive compatibility constraints must thus hold:

$$\mathcal{U}_l(\alpha_l^i) = \arg\max_{\hat{\alpha}_l^i \in \Omega_l} \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\Delta u_l(\beta_l(\hat{\alpha}_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) - t_i(\hat{\alpha}_l^i, \boldsymbol{\alpha}_l^{-i})\right), \quad \forall \alpha_l^i \in \Omega_l. \tag{2.6}$$

[20] Section 11.1 below shows that restricting veto powers to a few key members is enough to induce free riding and inefficiency. When no player has veto power, it is well known from the work of d'Aspremont and Gerard-Varet (1978) that the group can design efficient Bayesian incentive compatible mechanisms. *A contrario*, Section 11.2 also demonstrates how the free-riding problem can exacerbated if a non-ratifying agent can still enjoy the influence of those who actively act.

[21] In contrast with standard mechanism design problem, our context is one of competing mechanisms where multiple groups rely on their own mechanism. We must thus be somewhat careful in using this Revelation Principle. Indeed, each mechanism of group formation is now a best response to the mechanism designed by the competing group; a feature that has been studied in the general model of competing hierarchies by Myerson (1982) and in more specific contexts with secret contracts by Martimort (1996). For a given mechanism $\mathcal{G}_{-l}$ that determines a deterministic allocation β_{-l} for group l, there is no loss of generality in using the Revelation Principle to characterize group l's best response in the pure-strategy equilibria of this game that will be our focus below.

Budget Balance. Individual contributions must cover the payment needed to influence the decision-maker. [22] Taking into account that $\boldsymbol{\alpha}_{-l}$ is a random variable, the budget constraint for group l can be written in expected terms as follows:

$$\sum_{i=1}^{N_l} t_i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) - \mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0 \quad \forall(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) \in \Omega_l^{N_l}. \tag{2.7}$$ [23]

A mechanism $\mathcal{G}_l$ is *incentive-feasible* if and only if it satisfies no-veto (2.5), incentive compatibility (2.6), and budget balance (2.7).

2.4. *Timing and Equilibrium Concept*

The overall game of group formation and influence unfolds as follows. First, agents privately learn their preferences. Second, groups simultaneously (and secretly) propose mechanisms to their members. Third, within each group, each individual may accept or veto the proposed mechanism. If the mechanism is ratified, each agent reports his own preference parameter $\hat{\alpha}_l^i$. The lobbyist's induced preferences correspond to a weight $\beta_l(\hat{\boldsymbol{\alpha}}_l)$. Fourth, the common agency game between lobbyists unfolds with group l's equilibrium payment to the decision-maker being $T_l(\beta_l(\hat{\boldsymbol{\alpha}}_l), \beta_{-l}(\hat{\boldsymbol{\alpha}}_{-l}))$.

The equilibrium concept for the overall game is perfect Bayesian equilibrium with the addition of two refinements. First, we impose *passive beliefs* so that members of group l still believe that group $-l$ is ruled with the equilibrium mechanism if they receive an unexpected offer for their own group.[24] Second, equilibria of the common agency stage of the game are truthful; another standard refinement.

For any given incentive-feasible mechanism $\mathcal{G}_{-l}$ ruling group $-l$ (i.e., a mechanism which itself satisfies incentive compatibility, no veto and budget balance for that group), the mechanism $\mathcal{G}_l$ chosen by group l maximizes the sum of its members' expected payoffs subject to no-veto (2.5), incentive (2.6) and budget balance constraints (2.7). Of course, these payoffs take into account how competition between the groups unfolds.

3. GROUP FORMATION UNDER COMPLETE INFORMATION

Let us first consider the case where groups form under complete information. Focusing on the common agency stage, we may follow Laussel and Le Breton (2001) and show that, in the case of conflicting interest groups under scrutiny,[25] there is a unique truthful equilibrium in which, the decision-maker gets a positive payoff as a result of the lobbyists' *"head-to-head"* competition. The equilibrium payment from lobbyist l, say $T_l(\beta_l, \beta_{-l}) = \tilde{T}_l(x(\beta_l, \beta_{-l}))$, is defined as:

$$T_l(\beta_l, \beta_{-l}) = [\beta_{-l}u_{-l}(x) + u_0(x)]_{x(\beta_l,\beta_{-l})}^{x(0,\beta_{-l})}. \tag{3.1}$$ [26]

[22] The payment $T_l(\beta_l, \beta_{-l})$ is a random variable. It depends on both the realization of the whole vector of valuations $\boldsymbol{\alpha}_l$ for members of group l through the impact on β_l but also on the vector of preference parameters $\boldsymbol{\alpha}_{-l}$ of members of the competing group through its impact on β_{-l}. All incentive, budget balance and no-veto constraints that apply to the mechanism for group l take into account the fact that the vector of valuations $\boldsymbol{\alpha}_{-l}$ is viewed as being random from group l's viewpoint.

[24] This refinement is standard in the competing mechanisms literature (Martimort (1996)).

[25] See the Appendix for details.

This expression is remarkable. Contributions endogenously determined at the truthful equilibrium of the last stage of the game are in fact VCG payments. Each lobbyist pays for the externality that a change in policy he induces exerts both on the other lobbyist and the decision-maker. This fact has important implications on the groups' incentives to choose the preferences of their lobbyist. The logic of VCG mechanisms bites: There is no point in strategically choosing those preferences to affect subsequent lobbying competition. The objectives of a lobbyist always perfectly reflect the aggregate preferences of the group he represents. Then, competition between those lobbyists gives an efficient decision.

Proposition 1 Efficient group formation under complete information. *When group l forms under complete information, preferences are efficiently aggregated:*

$$\beta_l^*(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l), \quad \forall \boldsymbol{\alpha}_l \in \Omega_l^{N_l}. \tag{3.2}$$

From Proposition 1, the *pluralistic approach* of politics is valid under complete information. Preferences are perfectly aggregated both within and also across groups.

4. CHARACTERIZING INCENTIVE-FEASIBILITY

Our first task is to get a compact characterization of the set of incentive-feasible allocations. We follow a large body of the mechanism design literature[27] and characterize incentive compatibility conditions before aggregating (2.5), (2.6) and (2.7) into a single constraint which is both necessary and sufficient for incentive-feasibility.

Incentive compatibility. Incentive compatibility can be expressed in terms of properties of the mapping $\beta_l(\boldsymbol{\alpha}_l)$ and the payoffs profile $\mathcal{U}_l(\alpha_l^i)$ that such mapping induces. This is the purpose of next lemma whose proof is standard.

Lemma 1 *An allocation $(\mathcal{U}_l(\alpha_l^i), \beta_l(\boldsymbol{\alpha}_l))$ is incentive compatible if and only if $\mathcal{U}_l(\alpha_l^i)$ is convex, and admits the integral representation:*

$$\mathcal{U}_l(\alpha_l^i) = \mathcal{U}_l(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_l^i} \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l}} \left(\frac{1}{N_l} \Delta u_l(\beta_l(\tilde{\alpha}_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l})) \right) d\tilde{\alpha}_l^i; \tag{4.1}$$

Consider a member of group l with preferences α_l^i. To fix ideas, suppose also that $\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ is non-decreasing in α_l^i for any $(\boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l})$.[28] By pretending to have a slightly lower valuation $\alpha_l^i - d\alpha_l^i$, this type α_l^i can modify the decision chosen by the decision-maker which becomes $x(\beta_l(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$. At the same time, type α_l^i also reduces his own contribution $t_l^i(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_l^{-i})$ thereby letting other members of his own group contribute much of what is needed to influence the decision-maker. This modification of the individual payment is thus at the core of the free riding problem.

[26] To simplify notations, we denote in the sequel $[g(x)]_{x_0}^{x_1} = g(x_1) - g(x_0)$.

[27] Laffont and Maskin (1982), Mailath and Postlewaite (1990), Ledyard and Palfrey (1999), Hellwig (2003).

[28] We will see below, especially in Section 5, that such *ex post* monotonicity conditions hold quite naturally under some circumstances so that the weaker interim monotonicity conditions in Item 2. of Lemma 1 is satisfied.

More generally, type α_l^i's net gain from manipulating his own preferences is worth $\frac{1}{N_l}\Delta u_l(x(\beta_l(\alpha_l^i - d\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))d\alpha_l^i \approx \Delta u_l(x(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))d\alpha_l^i$. To induce information revelation, that type must thus pocket an informational rent $\mathcal{U}_l(\alpha_l^i) - \mathcal{U}_l(\alpha_l^i - d\alpha_l^i)$ which, at any point of differentiability, is approximatively worth $\dot{\mathcal{U}}_l(\alpha_l^i)d\alpha_l^i$ where $\dot{\mathcal{U}}_l(\alpha_l^i)$ is obtained from differentiating (4.1) as:

$$\dot{\mathcal{U}}_l(\alpha_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right). \tag{4.2}$$

From (4.2), $\mathcal{U}_l(\alpha_l^i)$ is necessarily non-decreasing. The no-veto constraint (2.5) is thus harder to satisfy for those individuals with type $\underline{\alpha}_l$ who are the most eager to veto. Ratification of the agreement by any type thus requires:

$$\mathcal{U}_l(\underline{\alpha}_l) \geq 0. \tag{4.3}$$

Aggregate feasibility condition. Equipped with the characterization of incentive compatibility (4.1) and no-veto (4.3), we now derive a feasibility condition that aggregates no-veto, incentive compatibility and budget balanced constraints into a single condition.

Proposition 2 Incentive-feasibility. *A mechanism $\mathcal{G}_l$ is incentive-feasible if and only if:*

1. *The virtual net gain from group l formation is non-negative:*

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left([u_0(x) + h_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right) \geq 0. \tag{4.4}$$

2. *$\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})))$ is non-decreasing in α_l^i.*

Proposition 2 is a fundamental step on our way to simplify the design problem. Condition (4.4) indeed summarizes all the difficulties that asymmetric information might bring to the collective action problem. When computing the net benefit of forming under asymmetric information, preference parameters are replaced by virtual parameters while, on the cost side, the overall contribution remains unchanged. Those virtual valuations are lower so that the overall group incentives to contribute diminish.

5. Conditions for an efficient equilibrium under asymmetric information

We now wonder whether the outcome of the game may remain efficient even under asymmetric information. We will refer to *an efficient equilibrium* as an equilibrium (if any) such that each group costlessly solves its own internal informational problem; lobbyists are endowed with the aggregate preferences of their respective group; and the decision-maker chooses an efficient decision at the last stage of the game.

Suppose that such an equilibrium exists. Given that group $-l$ efficiently solves its own informational problem, group l must also do so. Lobbyists are thus endowed with objectives that perfectly reflect the preferences of the interest groups they respectively represent, i.e., $\beta_l^*(\boldsymbol{\alpha}_l) \equiv \alpha_l^*(\boldsymbol{\alpha}_l)$, for all $\boldsymbol{\alpha}_l \in \Omega_l^{N_l}$.

PROPOSITION 3 EXISTENCE OF AN EFFICIENT EQUILIBRIUM. *An efficient equilibrium exists if and only if:*

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+\alpha_l^*(\boldsymbol{\alpha}_l)u_l(x)+\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}\right)$$
$$\geq \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\Delta u_l(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))\right)\quad \forall l\in\{1,2\}. \tag{5.1}$$

The feasibility condition (5.1) has a simple interpretation. The l.h.s. is a measure of the welfare gain obtained from influencing the decision-maker so as to shift his decision from $x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ which is chosen when group l is not active to $x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ at an efficient equilibrium. Since $x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ maximizes overall welfare, this gain is necessarily positive. If collective action were to take place under symmetric information, this l.h.s. difference would be the payoff that group l could capture.

More novel is the r.h.s. of (5.1). This term stems for the overall informational cost that such change of decision induces. Under complete information within group l, it would disappear. Condition (5.1) is thus a fundamental equation to understand how groups solve their collective action problem. Since the l.h.s. is always non-negative, there would always exist an efficient equilibrium under complete information. We retrieve here the standard efficiency result that backs up the *pluralistic approach.* Asymmetric information introduces a cost of coalition formation that may preclude such an efficient equilibrium and call for less optimistic conclusions about the efficiency of lobbying competition.

RUNNING EXAMPLE. Let us consider the case where u_0 is quadratic and suppose that types are drawn from a uniform distribution on $[\underline{\alpha}_l,\overline{\alpha}_l]$. Condition (5.1) becomes:

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{3}{2}\alpha_l^*-\overline{\alpha}_l\right)\alpha_l^*\right)\geq 0\quad \forall l\in\{1,2\}. \tag{5.2}$$

This condition is independent of group $-l$'s preferences; a specific feature on which we will come back below. In the quadratic example, inefficiencies in group formation are indeed fully determined by the group's own composition and not by the composition of its rival. Condition (5.2) also holds when $\underline{\alpha}_l$ is close enough to $\overline{\alpha}_l$ and more specifically when $\underline{\alpha}_l\geq\frac{2}{3}\overline{\alpha}_l$. In other words, enough homogeneity of the group (defined as $\underline{\alpha}_l$ and $\overline{\alpha}_l$ being close enough) ensures existence of an efficient equilibrium under those circumstances. We will come back on the generalization of this result below. ■

6. FREE RIDING AND EFFICIENCY IN LARGE GROUPS

Following a tradition that goes back to Bowen (1943), we now investigate how stringent Condition (5.1) is when groups become large. Of course, this is in such settings that intra-group free riding is the most difficult to solve.

Taking the limit $N_l\to+\infty$, the *Strong Law of Large Numbers* tells us that the empirical mean of samples made of true (resp. virtual) preference parameters converges with probability one towards the mean (resp. the lowest value) of α_l^i:

(6.1)

$$\alpha_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i \underset{N_l\to+\infty}{\overset{a.s.}{\to}} \mathbb{E}_{\alpha_l^i}(\alpha_l^i) = \alpha_l^e \quad \left(\text{resp. } h_l^*(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} \mathbb{E}_{\alpha_l^i}(h(\alpha_l^i)) = \underline{\alpha}_l\right).$$

Running example. Inserting (6.1) into (5.2), the condition for an efficient equilibrium and a uniform distribution certainly holds in the limit of a large population when $3\underline{\alpha}_l > \overline{\alpha}_l$ for all $l \in \{1,2\}$. This example suggests that efficiency can only be obtained if types have enough willingness to pay but that their distribution is not too disperse. ■

Repeatedly throughout the paper, we refer to the following assumption that ranks those limits and requires that the lowest possible type has no taste for the policy.

Assumption 1 $\quad \underline{\alpha}_l = 0 < \alpha_l^e \quad \forall l.$

We can now easily prove the following important result.

Proposition 4 Free Riding in large heterogenous groups. *Suppose that Assumption 1 holds. An efficient equilibrium never exists for N_l large enough.*

With a large group, inefficiencies arise whenever the types distribution contains individuals with no strict preferences for shifting the policy. To understand this property, one has to come back on the forces that lie behind such formation. On the one hand, an efficient equilibrium requires each lobbyist to be endowed with the average preference parameter of his own group. When group l is influential, the policy has thus to move away from the decision $x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ that would be taken if only group $-l$ was active towards the efficient decision $x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$. The welfare gain that accrues to group l from such a move has to be compared with the overall information rent that has to be distributed internally to members of that group to induce information revelation. When group l is large, each individual member only cares about minimizing his own contribution to such a policy shift. Each agent has thus an incentive to behave as having the lowest possible valuation within the group, namely $\underline{\alpha}_l = 0$ from Assumption 1. So overall, the group behaves as being made only of agents with no preferences for moving the policy away from the *status quo*. The overall contribution of group l is thus zero as a whole and it becomes impossible to move towards the efficient policy $x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$.

Proposition 4 has its counterpart: Efficiency is achieved when the distribution of types is not too diffuse and the lowest valuation types has *some* willingness to change the policy.

Proposition 5 Efficient equilibria in large homogenous groups. *Suppose that $\alpha_l^e - \underline{\alpha}_l$ (for $l = 1,2$) is small enough but that $\underline{\alpha}_l > 0$. An efficient equilibrium always exists when N_l (for $l = 1,2$) is large enough.*

When α_l^e is close to $\underline{\alpha}_l$ and $\underline{\alpha}_l > 0$, types are rather close to each other and they are all ready to pay at least some positive amount to induce a policy shift away from $x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ towards $x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$. Aggregating over a large group, even tiny individual contributions may suffice to change the outcome. Even under asymmetric information and in the limit of a large size, the group may remain influential under those circumstances.[29] Proposition 5 is certainly a strong qualifier of the *Olson Paradox*. In an asymmetric information context, this is not size *per se* that undermines group formation but instead the addition of size and enough heterogeneity in the types distribution.

7. GROUPS OF FINITE SIZE: THE CHOICE OF MODERATE LOBBYISTS

Proposition 4 is reminiscent of Mailath and Postlewaite (1990) who stress the existing free riding for large groups in a public good context. Beyond differences in functional forms, contexts and the nature of the decision (discrete in Mailath and Postlewaite (1990) while it is continuous here), the main difference comes from the fact that the net gains from forming are here equilibrium objects. Although this difference has less bite for large groups, it significantly matters with groups of finite sizes. With large groups, there is almost no remaining uncertainty about both aggregate preferences and aggregate contributions. Instead, with groups of finite sizes, there is still some remaining uncertainty not only about the distribution of aggregate valuations within the group, but also about the competing group's own preferences. This makes the analysis at finite sizes certainly more complex, but it also introduces some new strategic features that unveil the true determinants of inefficiencies in group formation.

7.1. *Inefficient Best Responses*

This Section analyzes how lobbyists might no longer inherit of the true preferences of the group they represent and how lobbying competition might in turn be modified. We look for a best response to any arbitrary appointment rule $\beta_{-l}(\cdot)$ chosen by group $-l$.

When group l forms under asymmetric information, $\beta_l(\boldsymbol{\alpha}_l)$ is chosen so as to satisfy the incentive-feasibility condition (4.4). When this condition is indeed a binding constraint, group l faces a trade-off. On the one hand, choosing $\beta_l(\boldsymbol{\alpha}_l)$ close to the efficient rule $\alpha_l^*(\boldsymbol{\alpha}_l)$ is good for group l since this choice induces a large policy shift from $x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))$ to $x(\alpha_l^*(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))$. On the other hand, such shift also requires making a large payment to the decision-maker. This means increasing individual contributions which in turn exacerbates free riding within the group. To moderate this rent-efficiency trade-off, group l chooses a preference parameter for his lobbyist $\beta_l^{sb}(\boldsymbol{\alpha}_l)$ which now only partially reflects the group's own preferences. So doing, policy shifts are of a lower magnitude, contributions diminish and free riding is less of a concern.

PROPOSITION 6 INEFFICIENT BEST RESPONSES. *At a best response, group l endows his lobbyist with a moderate preference parameter $\beta_l^{sb}(\boldsymbol{\alpha}_l) \leq \alpha_l^*(\boldsymbol{\alpha}_l)$:*

$$\beta_l^{sb}(\boldsymbol{\alpha}_l) = \max\left\{0, \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right\} \tag{7.1}$$

where λ_l is the non-negative Lagrange multiplier of the incentive-feasibility constraint (4.4).

When $\lambda_l > 0$, the decision-maker chooses a policy that no longer perfectly reflects the preferences of group l. Indeed, the overall contribution of that group is reduced to weaken

[29] Hellwig (2003) criticizes Mailath and Postlewaite (1990)'s findings and argues that it depends a lot about the cost function for producing the public good and that efficiency may sometimes be reached. Our argument for efficiency relies instead on the distribution of preferences.

intra-group free riding. The decision-maker is thus biased towards the outcome he would have chosen when selling his favors to the competing group only. Informational frictions also weaken competition for the decision-maker's favors. The welfare consequences of this insight are studied in Section 10.

A priori, the parameter $\beta_l^{sb}(\boldsymbol{\alpha}_l)$ might now depend on group $-l$'s own choice of induced preferences $\beta_{-l}(\boldsymbol{\alpha}_{-l})$ through the impact that this variable has on the incentive-feasibility constraint (4.4) and thus on λ_l. Section 8 analyzes the equilibrium joint determination of frictions by means of a pair of Lagrange multipliers that pertain to each group.

7.2. *A Sufficient Condition Ensuring Inefficiency*

This section further unveils the nature of informational frictions that hinder group formation. To this end, we state the following assumption.

ASSUMPTION 2

$$\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \Big| \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i = \alpha_l^*\right) \textit{ is decreasing in } \alpha_l^* \textit{ for } N_l > 1.$$

Assumption 2 requires the strict monotonicity of the difference between the hazard rate of the sample mean and the conditional sample mean of hazard rates. This condition thus only depends on properties of the types distribution.[30] It will ensure inefficient group formation for any appointment rule chosen by the competing group. That efficiency is not possible means that (4.4) fails for $\beta_l^*(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l)$, i.e.

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x) + h_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right) < 0. \tag{7.2}$$

Proposition 4 already showed that Condition (7.2) holds when N_l is large enough and group $-l$ already implements the efficient appointment rule, i.e., $\beta_{-l}^*(\boldsymbol{\alpha}_{-l}) = \boldsymbol{\alpha}_{-l}^*(\boldsymbol{\alpha}_{-l})$. Assumptions 1 and 2 jointly ensure that this result is already true for finite sizes.

PROPOSITION 7 INEFFICIENT GROUP FORMATION OF FINITE SIZES. *Suppose that Assumptions 1 and 2 both hold. Group l never forms efficiently: $\lambda_l > 0$.*

It should be no surprise that inefficiencies are obtained when making assumptions on the types distribution. It is so throughout many contributions of the mechanism design literature including the seminal works by Myerson and Satterthwaite (1983) and Mailath and Postlewaite (1990). The important aspect of Assumption 2 is that this condition is universal and sufficient for inefficiencies in all possible competitive environments. The sole role of lobbyist $-l$'s objective is to affect the value of the positive Lagrange multiplier of the incentive-feasibility constraint (4.4) but Condition (7.2) is already enough to ensure that group l's formation suffers from asymmetric information. This was certainly the case

[30] When valuations are uniformly distributed on $[0,\overline{\alpha}_l]$, we have $\alpha^e = \frac{\overline{\alpha}_l}{2}$ and $\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} = \overline{\alpha}_l - \alpha_l^i$. Assumption 1 is trivially true. Assumption 2 holds since it amounts to checking that $\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \overline{\alpha}_l + \alpha_l^*$ is increasing in α_l^*. Proof available upon request.

for a large group as shown in Proposition 4 but Assumptions 1 and 2 taken in tandem ensure that inefficiencies also arise at finite sizes.[31]

RUNNING EXAMPLE (CONTINUED). With u_0 quadratic, the optimal policy is given by $x(\beta_1,\beta_2)=\frac{\beta_2-\beta_1}{\beta_0}$. To illustrate the input of Assumption 2, observe that Condition (7.2) can then be transformed as:[32]

$$\text{(7.3)}\qquad \mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\middle|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i=\alpha_l^*\right)\right)\frac{\alpha_l^*}{\beta_0}\right)<0.$$

This condition is independent of the appointment rule $\beta_{-l}(\boldsymbol{\alpha}_{-l})$ chosen by group $-l$. From Assumption 2, the first factor in the expectation is decreasing. This term has also zero mean. The second factor $\frac{\alpha_l^*}{\beta_0}$ is the efficient rule which is monotonically increasing in α_l^*. A simple integration by parts then shows that the l.h.s. of (7.3) is negative as requested. ■

Proposition 7 summarizes how asymmetric information offers a drastic departure from the pluralistic view of politics. Under asymmetric information, the lobbying process fails to adequately aggregate the groups' interests. This failure can even be extreme, with a group being with no influence with positive probability. To illustrate, consider the case where all members of group l have preference parameters α_l^i such that $\alpha_l^i<\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}$; a possibility that arises when α_l^i is close enough to $\underline{\alpha}_l$ and $\underline{\alpha}_l<\frac{1}{f_l(\underline{\alpha}_l)}$. Under such configurations, $h_l^*(\boldsymbol{\alpha}_l)<0$ and thus $\beta_l^{sb}(\boldsymbol{\alpha}_l)=0$. Everything happens as if group l was no longer active under asymmetric information while it would have been so under complete information. Too many members of group l are tempted to shade their preferences.[33] *A contrario*, even if inefficiencies still arise and the lobbyist's objectives remain moderated by informational frictions, some representation may always be guaranteed provided that types expresses enough preferences for a policy shift. For instance, whenever $\underline{\alpha}_l\geq\frac{1}{f_l(\underline{\alpha}_l)}$, group l always forms (i.e., $\beta_l^{sb}(\boldsymbol{\alpha}_l)>0$) although it is almost always inefficiently. Indeed, $\beta_l^{sb}(\boldsymbol{\alpha}_l)=\beta^*(\boldsymbol{\alpha}_l)$ only arises when $\boldsymbol{\alpha}_l=(\overline{\alpha}_l,....,\overline{\alpha}_l)$.

7.3. *From Finite to Large Groups*

This section fills the gap between the finite sizes scenario and the limiting case of a large group. We are interested in asymptotic properties, making now explicit the dependence of the optimal appointment rule $\beta_l^{sb}(\boldsymbol{\alpha}_l,N_l)$ on N_l.

PROPOSITION 8 TOWARDS LARGE GROUPS. *Suppose that Assumptions 1 and 2 both hold. The appointment rule converges in probability towards no influence as N_l gets large:*

[31] Similar conditions for the impossibility of implementing the first-best allocation under asymmetric information have flourished throughout the whole mechanism design literature both in public and private good contexts (Laffont and Maskin (1982), Myerson and Sattherwaite (1983), Cramton, Gibbons and Klemperer (1987), Mailath and Postelwaite (1990), Hellwig (2003)).

[32] The proof in the Appendix relies on several rounds of integration by parts.

[33] This missing influence is reminiscent of the analysis in Le Breton and Salanié (2003), Martimort and Semenov (2008) and Martimort and Stole (2015) although, in those papers, asymmetric information is not on the demand side of the market for favors but rather on its supply side: Private information pertains to the preferences of the decision-maker. When the group's preferences are *"too far away"* from those of the decision-maker, agency costs might be so large that they prevent some groups from being active as in Martimort and Stole (2015).

$$(7.4)\qquad \beta_l^{sb}(\boldsymbol{\alpha}_l, N_l) \underset{N_l \to +\infty}{\overset{p}{\to}} 0.$$

Proposition 4 already showed the inexistence of an efficient equilibrium for large groups. Proposition 8 is stronger since it applies whatever group $-l$'s own appointment rule. As group size increases, the appointment rule is entirely determined by the incentive-feasibility Condition (4.4). This latter condition is trivially satisfied when group l exerts no influence. This is precisely what (7.4) shows: A large group leaves the decision-maker under the sole influence of its rival.

8. DUAL REPRESENTATION OF EQUILIBRIA

A priori, the Lagrange multiplier λ_l that appears on the r.h.s. of formula (7.1) depends on the appointment rule $\beta_{-l}(\cdot)$ chosen by group $-l$. Indeed, those preferences affect how much money is paid by group l to buy the policy-maker and thus the magnitude of the intra-group free riding. Since the preferences parameter $\beta_l^{sb}(\boldsymbol{\alpha}_l)$ for group l's lobbyist is fully characterized by a single non-negative parameter λ_l, it becomes quite natural to summarize an equilibrium by a pair of non-negative numbers $(\lambda_l, \lambda_{-l}) \in \overline{\mathbb{R}}_+^2$ that determine appointment rules $(\beta_l^{sb}(\boldsymbol{\alpha}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))$ which are best responses to each other. With this *dual representation*, an equilibrium amounts to a pair (λ_1, λ_2) satisfying:

$$(8.1)\qquad \lambda_l = \Lambda_l^*(\lambda_{-l}) \quad \forall l \in \{1, 2\}$$

where the Λ_l^* are *"best-response mappings"* defining the Lagrange multiplier characterizing group l's formation in terms of the Lagrange multiplier pertaining to group $-l$.

Under complete information, each group perfectly passes its own aggregate preferences to its lobbyist; contributions to the decision-maker are always *truthful*. In other words, the lobbyists' preferences are not interdependent. Whether the opposite group easily raises money to influence the decision-maker or not has no impact on group l's choice of objective function for its own lobbyist. It changes the level of group l's contributions but not how preferences among alternatives are finally passed on the decision-maker. This stands in sharp contrast with asymmetric information. Frictions now depend on how much money is needed to influence the decision-maker. This provides a channel by which the strength of the opposite group impacts on group l's choice of an objective for its own lobbyist. Frictions within each group are now jointly determined at equilibrium.

Thanks to the simple dual representation that views an equilibrium as a pair of Lagrange multipliers and provided that the set of relevant Lagrange multipliers is conveniently compactified, a simple fixed-point argument ensures existence of an equilibrium.

PROPOSITION 9 EXISTENCE OF EQUILIBRIA. *Suppose that Assumptions 1 and 2 both hold. There always exists a (pure-strategy) equilibrium, i.e., a pair (λ_1, λ_2) that solves (8.1) with $\lambda_l > 0$ for $l \in \{1, 2\}$.*

Existence of a pure strategy equilibrium is interesting in its own sake. First, it means that we should not expect much instability in lobbying competition as would be the case if only mixed-strategy equilibria could arise. Beyond existence, more interesting comparative statics follow from carefully looking at the the properties of best-response mappings. Slightly abusing language and using the well-known parlance of the I.O. literature, Proposition 10 shows that the game between competing groups might exhibit either *strategic complementarity* with both mappings Λ_l^* (for $l \in \{1,2\}$) being everywhere non-decreasing or *strategic substitutability* when those mappings are instead everywhere non-increasing. Those monotonicity properties depend in fine details of the decision-maker's preferences.

Proposition 10 Monotonicity of best-response mappings. *Λ_l^* ($l \in \{1,2\}$) is everywhere non-decreasing (resp. non-increasing) if and only if $u_0''' \geq 0$ (resp. $u_0''' \leq 0$).*

The intuition for those different patterns comes from understanding how asymmetric information impacts on lobbying competition. Suppose that group $-l$ finds it more difficult to organize itself, in other words that λ_{-l} increases. The first consequence is that this group has less impact on the decision. It becomes easier for group l to shift the *status quo* towards its own preferred direction: a *"policy-shifting" effect*. Lower contributions from group l are needed and thus free riding within that group is less of a curse. On the other hand, that group $-l$ does not influence so much the decision also means that the *status quo* might already please group l. Not organizing efficiently is thus less costly for that group. Free riding is exacerbated: a *"status quo" effect*.

Which effect dominates depends on the sign of u_0'''. When $u_0''' \leq 0$, the *"policy-shifting" effect* dominates and distortions within group l are less significant as competing interests find it more difficult to organize. *A contrario*, when $u_0''' \geq 0$, the *"status quo" effect* prevails and distortions are more pronounced. A key lesson is thus that group formation generally depends on its environment. Informational frictions within a group impacts on how easily its competitor organizes.

Running example (continued). For quadratic preferences, $u_0''' \equiv 0$, the *"status quo"* and the *"policy-shifting"* effects cancel out. The best-response mappings Λ_l^* are flat and the Lagrange multipliers (λ_1, λ_2) are determined separately. Whether group l faces a strong opponent or not does not affect its own difficulties in solving the free riding problem. The preferences of its lobbyist are independent of the surrounding environment. ∎

9. TOWARDS AN "INDUSTRIAL ORGANIZATION" THEORY OF GROUPS FORMATION

That informational frictions are determined altogether suggests that groups may enter into various strategies to undermine the ability of their rivals to organize themselves.

Information Sharing. Suppose that members of group $-l$ can credibly share information. It may be so because the group is small in size and peer monitoring is possible. In some related contexts, Ostrom (1990) has argued that agents may invest resources to monitor each other and avoid free riding. Free riders could also be punished by means of group stigma, or through repeated interactions that overcome informational problems. In this regard, Hula (1999, p.24)'s analysis of coalition formation by interest groups in U.S. legislative politics is worth noticing. This author first reports that *"The presence or absence of a given group is likely to be noticed in a small universe of organizations, and the potential exists for the applications of coercive sanctions by other members of* *the coalition to discourage free riding. At least, there must be a strong element of peer pressure."* He also points out that *"the increasing use of long-term, recurrent, and institutionalized coalitions in many policy arenas"* build *strong coalitions* of interest groups which circumvent the free riding problem. Finally, even when information cannot be credibly shared, private information might sometimes have a limited impact, especially when group members have limited veto power, a point that will be discussed in Section 11.1.

Information sharing within group $-l$ can be viewed as a commitment device to fix $\lambda_{-l} = 0$. The strategic impact of such choice on the frictions faced by group l can be easily deduced from Proposition 10.

Proposition 11 Information Sharing. *Let $(\lambda_l, \lambda_{-l})$ be an equilibrium of the game when both groups form under asymmetric information. With information sharing within group $-l$, the new equilibrium $(\tilde{\lambda}_l, 0)$ is such that $\tilde{\lambda}_l \leq \lambda_l$ (resp. $\geq$) if $u_0''' \geq 0$ (resp. $\leq$).*

With information sharing, group $-l$ certainly finds it easier to buy influence. It then becomes more difficult for group l to buy the decision-maker's favors. Group l has to raise its own contribution which worsens its own free riding problem. The *"policy-shifting" effect* exacerbates frictions within group l, while the *"status quo" effect* does the reverse. When the first effect dominates (i.e., $u_0''' \leq 0$), a group which has solved its own free riding problem can not only buy influence more easily but it also weakens its competitor's representation. Coalitions bound by strong ties might thus exclude rivals more easily than what less well-organized groups would do.

Illustration 1. The U.S. sugar industry might illustrate these findings. This industry is indeed thought as being one of the strongest lobbying groups in Washington, with the specificity that it is organized around four major producers; the Fanjul brothers. Between 2007 and 2014, sugar growers donated up to \$18.5 million, according to the *Center for Responsive Politics* although sugar represents only a small fraction U.S. farm output. Because of their strong ties, sugar producers have thus been very successful in resisting repeated attempts by sugar-users (like Pepsi Co Inc., Hershey Co., the *National Foreign Trade Council*, a pro-trade group that includes Coca-Cola Co., WalMart Stores Inc., and the *Sweetener Users Association,* a collection of sugar-buyers) and foreign countries like Australia to open the market. As a result, a quota system dating back to the Great Depression has persisted with prices remaining much higher than elsewhere.

Illustration 2. The strong political influence that senescent industries exert has been subject of much interests especially by comparison with the less successful lobbying exerted by growing sectors (Hillman (1982), Brainard and Verdier (1997), Baldwin and Robert-Nicoud (2007)). Standard arguments point out that entrants might not generate enough profits to pay for the fixed cost of entry in lobbying activities while senescent industries may still be able to hold their influence for a while with the possible cost of a delayed transition towards novel technologies. Our model suggests another, maybe complementary line of arguments. Through repeated interactions over time, firms in senescent industries may have knitted tight bounds, maybe up to the point of credibly sharing information and alleviating the free riding problem. As these industries become more efficient at lobbying, entrants find it harder to overpass their own free riding problem.

Entry Costs/Coercive Stigma. To be active, an interest group might also have to incur further organization costs beyond asymmetric information *per se*. For instance, hiring a lobbyist might require to incur search costs, to pay contingent fees, sometimes to give up extra rent if those lobbyists have to gather expert information to access key Congressmen. Entry costs harden the incentive-feasibility condition (4.4) and exacerbate frictions. To see how, let denote by K_l a fixed cost of group formation. The incentive-feasibility condition (4.4) becomes:

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left([u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right)\geq K_l>0. \tag{9.1}$$

Group l's contribution must increase to cover this extra fixed cost. In response, informational frictions are more pronounced. Formally, the whole best-response mapping Λ_l^* is shifted upwards, modifying accordingly the set of possible equilibria of the game.

Proposition 12 Fixed costs. *Let (λ_l,λ_{-l}) be an equilibrium of the game when both groups form under asymmetric information and there are no fixed costs. If group l incurs some small fixed cost of formation (i.e., K_l small enough), there exists a new equilibrium $(\tilde{\lambda}_l,\tilde{\lambda}_{-l})$ such that $\tilde{\lambda}_l\geq\lambda_l$ (resp. $\geq$) and $\tilde{\lambda}_{-l}\geq\lambda_{-l}$ (resp. $\leq$) if $u_0'''\geq 0$ (resp. $\leq$).*

When the *"policy shifting" effect* dominates, (i.e., $u_0'''\geq 0$), group l gets a strategic advantage when its own fixed cost of organization decreases since it also worsens free riding problem for his rival. The opposite would happen had the *"status quo" effect* dominated (i.e., $u_0'''\leq 0$).

Illustration 3. To be politically active, each individual might have to pay a fixed cost of participation, say $\frac{K_l}{N}$. Such fixed cost may stem for the cost of learning his own willingness to pay. Individual participation constraints then require that, for each type, the net gains of participating exceeds that individual share of the overall entry cost. Aggregating those participation constraints with other budget-balance and incentive constraints again leads to the incentive-feasibility condition (4.4).

Proposition 12 has a counterpart. Suppose instead that each agent suffers from some reputation loss, or stigma, for not participating to the agreement. The term $\frac{K_l}{N}$ becomes a negative number and the incentive-feasibility condition (4.4) is eased. More coercive ways of enforcing the agreement would thus shift group l's best response downwards, leading to changes in equilibria that mirror those of Proposition 12. Of course, there exists a value of those stigmas beyond which efficiency can be restored. From (9.2), the aggregate stigma that ensures that an efficient equilibrium exists is given by:

$$\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+h_l^*(\boldsymbol{\alpha}_l)u_l(x)+\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}\right)=K_l^*. \tag{9.2}$$

Illustration 4. In its usual complete information version, the common agency model of lobbying is unable to explain group formation unless a fixed cost of group formation is introduced. Without fixed costs, this is the modeler's choice to let some groups organize while others remain unorganized. With fixed costs, only those groups whose expected benefits cover the fixed cost may then be active. Following Mitra (1999), many authors have taken this path to endogenize the set of active groups (Baldwin and Robert-Nicoud

(2007), Bombardini (2008), Martimort and Semenov (2007)). In those models, fixed costs have thus only an impact at the extensive margin but no consequence on the intensive margin since all active groups always use truthful strategies. Illustrating this approach in a trade context, Bombardini (2008) provides some empirical analysis showing that, when the distribution of firm sizes has a fatter tail, lobbying for protection is more effective.

Although attractive this conclusion suffers from a caveat. Indeed, those firms whose gain exceeds by much the fixed cost of entry should subsidize those whose profits are insufficient. So doing, firms could collectively be more powerful in buying influence. In other words, unless there are hidden frictions that make it impossible for heterogenous groups to cross-subsidize participation of their members (may be by means of setting up a trade association, a frequent vehicle), the requirement for being active is akin to the complete information version of condition (9.2). Because that condition is taken on aggregate, a change in the distribution of firms has a lesser impact on lobbying than could have been expected. Instead, if firms have private information as in our setting, the incentive-feasibility condition (9.2) directly depends on the tail of the distribution through its impact on $h_l^*(\boldsymbol{\alpha}_l)$. As the distribution puts more weights on higher types, $h^*(\boldsymbol{\alpha}_l)$ comes "closer" to $\alpha_l^*(\boldsymbol{\alpha}_l)$ and the condition becomes easier to satisfy.

10. WELFARE ANALYSIS

To understand how informational frictions within groups impact on welfare, we must remind a logic which is now familiar from Section 9. Inefficiencies in group formation have two effects on groups' payoffs. First, for a given strategy followed by his rival, each group would be better off if informational constraints *within this group* could be circumvented so as to endow its lobbyist with an objective that would perfectly reflect the group's aggregate preferences. Second, informational frictions *within the competing group* also contribute to soften competition. Each group benefits from facing a weaker competitor who has less influence on the decision-maker. The overall impact of those competing effects on the groups' payoffs can thus go either way. A group might take advantage of informational frictions while, at the same time, its rival could be hurt by the very same frictions (this is for instance the case if the lower bound $\underline{\alpha}_l$ is sufficiently positive so that group l always manages to get organized while group $-l$ fails being so). However, it is also possible that both groups benefit from softening competition.

Getting unambiguous welfare results is thus difficult in general. We thus content ourselves with pointing out a few effects that arise in specific contexts. The first one is that frictions in coalition formation may soften competition between symmetric groups.

PROPOSITION 13 GROUPS' PAYOFFS. *Suppose that Assumption 1 holds, groups have the same size $N_1 = N_2 = N$, valuations are drawn from the same distribution on $\Omega_1 = \Omega_2 = [0, \overline{\alpha}]$, and the decision-maker's objective is quadratic as defined in (2.1). For N large enough, interest groups are ex ante better off under asymmetric information.*

We already know that inefficiencies are pervasive in large groups when preferences are sufficiently dispersed. Those groups have no influence on decision-making. When both groups face such a huge free riding problem, the decision-maker still goes for the *status quo* which reflects only her own preferences. Under complete information, groups of equal force would have competed *"head-to-head"* for influence, and the same decision would also have been chosen.The difference is that each group would pay a lot for maintaining the *status quo* just to avoid that the other group tilts the decision in his own direction. From an *ex ante* viewpoint, the groups' expected gains remain the same under both informational scenarios. Yet, under complete information, groups waste money in *"head-to-head"* competition for the decision-maker's services while they refrain from doing so under asymmetric information. Asymmetric information moderates lobbying competition.

Illustration 5. The complete information common agency model suggests that conflicting groups may use large contributions to preserve the *status quo*. This conclusion seems at odds with evidence. Why there is so little money used by interest groups, the so called *"Tullock Paradox"* (Tullock (1972)), is a puzzle that has indeed retained much attention. Ansolabehere et al. (2003) have found explanations on the demand side of the market for influence, arguing that interest groups' political campaign contributions only account for a small fraction of the overall contributions that a legislator may gather. Focusing on the supply side, Helpman and Persson (2001) offer a model where lobbying takes place prior to a legislative bargaining stage. Competition between legislators for being in the billing majority leads to small equilibrium contributions. Our model offers another demand side-driven explanation for low contributions. Groups reduce their own contributions to ease the free riding problem and, at equilibrium, very little contributions are used. *A contrario*, the size of donations in *Illustration 1* above suggests that strong coalitions are instead better able to channel contributions.

Informational frictions also have a non-ambiguous impact on the decision-maker.

PROPOSITION 14 DECISION-MAKER'S PAYOFF. *The decision-maker's payoff is always lower under asymmetric information than under full information.*

By making lobbyists' objective less sensitive to the decision, asymmetric information softens lobbying competition. It thus reduces the rent that the decision-maker extracts from playing one group against the other.

11. DISCUSSIONS AND EXTENSIONS

This section discusses several extensions of our basic framework.

11.1. *Veto Power For Key Players Only*

Following the mechanism design literature, we have assumed that unanimous agreement was required to enforce a mechanism within each group. Beyond, we may ask what would happen if only a few players had veto power. Frictions are certainly of a lesser magnitude in that scenario. More specifically, suppose that agents indexed by $i \in \{1, ..., \hat{N}_l\}$ have no veto right while those indexed by $i \in \{\hat{N}_l + 1, ...N_l\}$ have such right. Importing an important insight due to d'Aspremont and Gerard-Varet (1979) into our specific context, we know that incentive compatibility for agents with no veto power comes for free. Everything thus happens as if their preferences were common knowledge within the group. This immediately leads to redefine the lobbyist's preferences for that group as:

$$(11.1) \quad \beta_l^{sb}(\boldsymbol{\alpha}_l) = \frac{1}{N_l} \max\left(\sum_{i=1}^{\hat{N}_l} \alpha_l^i + \left\{ \sum_{i=\hat{N}_l+1}^{N_l} \alpha_l^i - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l} \frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right\} ; 0 \right).$$

$\hat{\lambda}_l$ is the Lagrange multiplier for the incentive-feasibility condition that now takes into account that no information rent is left to individuals with no veto power, namely:

$$(11.2)\quad \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x)+\tilde{h}_l^*(\boldsymbol{\alpha}_l)u_l(x)+\beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right)\geq 0$$

where $\tilde{h}_l^*(\boldsymbol{\alpha}_l)=\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\alpha_l^i-\sum_{i=\hat{N}_l+1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)$.

There is nothing specific to the analysis of such environments with limited veto power. The only difference is the lesser inefficiency coming from the ability of the group to implement more allocations since free riding is less of a concern.

11.2. *Free-Riding on Participation*

Following a standard assumption of the mechanism design literature, we have posited that the whole coalition breaks down and thus can no longer influence the decision-maker whenever an agent vetoes. Another interpretation is that the group can commit to stop lobbying without the agreement of all members. Such commitment ability is of course a strong requirement that allows us to give its full force to the conflict between incentive compatibility and participation to the agreement, assuming a very weak outside option following non-ratification. Alternatively, we could assume that the mechanism can still be somewhat enforced, and some coordination continues, even if an agent fails to ratify.[34] Such scenario creates the possibility of free riding on participation. A non-ratifying agent now free rides on the influence of the rest of the group that remains organized and active.

There is a plethora of possible scenarios to describe such limited commitment environments. One of the simplest one is to assume that, had agent i not ratified the mechanism, that mechanism still enforces the allocation $\beta_l(0,\boldsymbol{\alpha}_l^{-i})$ (which is a possible offer in the menu if $\underline{\alpha}_l=0$, an assumption that is made from now on). The mechanism thus relies on the sole contributions of ratifying agents to influence the decision-maker. Such free riding on participation is prevented when:

$$(11.3)\quad \mathcal{U}_l(\alpha_l^i)\geq\mathcal{U}_l^r(\alpha_l^i)=\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(0,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

A priori, this seems a more stringent participation constraint than (2.5) because the right-hand side is non-negative. Frictions in group formation might now be more significant. Nevertheless, observe that $\mathcal{U}_l^r(0)=0$ and

$$\dot{\mathcal{U}}_l(\alpha_l^i)-\dot{\mathcal{U}}_l^r(\alpha_l^i)=\frac{1}{N_l}\left(\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(\alpha_l^i,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))-\Delta u(\beta_l(0,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l})))\right)\geq 0$$

where the last inequality follows from the fact that $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u(\beta_l(\alpha_l^i,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))$ is non-decreasing in α_l^i. Those conditions, taken in tandem, thus ensure that (11.3) is only binding at $\underline{\alpha}_l=0$ if it is so. Hence, (11.3) is no more stringent than the standard veto constraint (2.5). When $\underline{\alpha}_l=0$, our results are thus all robust to the possibility of free riding in participation.

[34] See Martimort and Sand-Zantman (2016) for another mechanism design example of such scenario.

11.3. *Congruent Groups*

Our assumption of considering only two groups with conflicting preferences is also consistent with casual evidence reported by Hula (1999). Interest groups with similar interests tend to coalesce to push their own collective interests. Keeping two conflicting groups is thus a way to short-cut the full-fledged modelig of such cooperation of coalition of interest groups with *congruent* interests.

Nevertheless, the case of two congruent groups is interesting as such, and Appendix B is devoted to briefly develop the corresponding analysis. A first important feature that distinguishes this analysis from the case of conflicting interests is that, even under complete information on preferences, *inter-group* free riding now arises. Indeed, the equilibrium contributions that are determined at the common agency stage of the game are no longer VCG payments as with conflicting groups. Congruent groups design contributions so as to leave the decision-maker indifferent between taking both contributions and his next best option which is now to refuse all contributions and choose his most preferred status quo policy free of any influence.[35] Because contributions are no longer VCG payments, groups no longer pass sincerely their aggregate preferences to their delegates. Each group shades its preferences to reduce its own share of the cost of moving the decision-maker away from the status quo.[36] Congruent groups thus enjoy efficiency gains when merging.

Under asymmetric information, those gains should be compared with the information rents that accrue to group members. When groups remain split apart, vetoing the mechanism is not very costly for any individual. Provided that the other group forms, the decision is indeed already tilted in the right direction. There are not too much rent to grasp under this scenario. Instead, when congruent groups are merged, each individual by vetoing the mechanism triggers the choice of the status quo by the decision-maker and this decision is further apart. Much more rent can now be obtained under this scenario. Overall the informational cost of a merger is quite large. Whether asymmetric information is more of a blessing with split congruent groups or with a merger is thus generally difficult to assess beyond specific examples. Appendix B nevertheless provides an example showing that efficiency gains may dominate even under asymmetric information. Congruent groups may then prefer to merge to solve their collective action problems. This again provides some justifications for our focus on two groups with opposite preferences.

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[35] In the parlance of Laussel and Le Breton (2001), the *"no-rent property"* holds in this setting.

[36] Similar results are found in Dixit and Olson (2000) and Furusawa and Konishi (2011). These authors consider games where lobbies have first to decide wether or not to become active. Free riding occurs with some groups remaining outside the policy process.

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APPENDIX A: PROOFS OF THE MAIN RESULTS

PROOF OF PROPOSITION 1: Laussel and Lebreton (2001) identify the properties of the lobbyists' payoffs at truthful equilibria of the common agency game with those of a cooperative game whose characteristic function is the payoff $W(\beta_1, \beta_2)$ of a coalition including a subset of lobbyists and the decision-maker. These payoffs can be defined in terms of the vector of induced preferences (β_1, β_2) as:

$$W(\beta_1, \beta_2) = \max_{x \in \mathcal{X}} u_0(x) - \Delta\beta x = u_0(\varphi(\Delta\beta)) - \Delta\beta\varphi(\Delta\beta).$$

To illustrate, $W(0, \beta_2)$ stands for the payoff when group 1 fails to get organized and a similar convention applies to $W(\beta_1, 0)$ when group 2 is not organized. In our context with conflicting interest groups, this cooperative game turns out to be *sub-additive* since:

$$W(\beta_1, \beta_2) + W(0,0) \leq W(\beta_1, 0) + W(0, \beta_2) \quad \forall(\beta_1, \beta_2) \in \mathbb{R}^2_+.$$

Laussel and Le Breton (2001) then demonstrate that there is a unique truthful equilibrium with each lobbyist's payoff being his *incremental value* to the grand-coalition's surplus, namely:

$$V_l(\beta_l, \beta_{-l}) = W(\beta_1, \beta_2) - W(0, \beta_{-l}). \tag{A.1}$$

As a result, the decision-maker gets a positive share of the value of the grand-coalition:

$$W(\beta_1, 0) + W(0, \beta_2) - W(\beta_1, \beta_2) \geq W(0,0) = 0$$

(with a strict inequality whenever $\beta_l > 0$ for all l). The decision-maker pit one lobbyist against the other to extract some positive surplus. Using (2.2) and the expression of the lobbyist's payoff in (A.1) yields the expression of lobbyist l's contribution, $T_l(\beta_l, \beta_{-l}) = \tilde{T}_l(x(\beta_1, \beta_2))$, as (3.1).

Under complete information, group l's net gains from forming can thus be written as:

$$\sum_{i=1}^{N_l} \mathbb{E}_{\boldsymbol{\alpha}_l}(\mathcal{U}_l(\alpha_l^i)) = \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\alpha_l^*(\boldsymbol{\alpha}_l)\Delta u_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right).$$

Using (3.1), $\sum_{i=1}^{N_l} \mathbb{E}_{\boldsymbol{\alpha}_l}(\mathcal{U}_l(\alpha_l^i))$ can be simplified as being:

$$\text{(A.2)} \qquad \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(\left[u_0(x) + \alpha_l^*(\boldsymbol{\alpha}_l)u_l(x) + \beta_{-l}(\boldsymbol{\alpha}_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}^{x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))}\right).$$

Optimizing with respect to $\beta_l(\boldsymbol{\alpha}_l)$ the latter expression pointwise (i.e., for all realizations of $(\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l})$) and taking into account that $x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) = \varphi((-1)^l(\beta_{-l}(\boldsymbol{\alpha}_{-l}) - \beta_l(\boldsymbol{\alpha}_l)))$ also solves (2.3), we obtain the following first-order condition:

$$\text{(A.3)} \qquad \left(u_0'(x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) + (-1)^l(\alpha_l^*(\boldsymbol{\alpha}_l) - \beta_{-l}(\boldsymbol{\alpha}_{-l}))\right)\frac{\partial x}{\partial \beta_l}(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) = 0.$$

Using that $x(\beta_l,\beta_{-l}) = \varphi((-1)^l(\beta_{-l} - \beta_l))$ to simplify (A.3) yields (3.2). Finally, denoting the l.h.s. of (A.3) as function of the optimizing variable β_l as $\psi_l(\beta_l,\beta_{-l}(\boldsymbol{\alpha}_{-l}))$, we have:

$$\frac{\psi_l(\beta_l,\beta_{-l}(\boldsymbol{\alpha}_{-l}))}{(-1)^l\frac{\partial x}{\partial \beta_l}(\beta_l,\beta_{-l}(\boldsymbol{\alpha}_{-l}))} = \alpha_l^*(\boldsymbol{\alpha}_l) - \beta_l.$$

It follows that the objective function of group l is quasi-concave in β_l so that (3.2) is both necessary and sufficient for optimality. *Q.E.D.*

Proof of Lemma 1: Necessity. Fix $\beta_{-l}(\cdot)$, and consider any incentive-feasible mechanism $\mathcal{G}_l$. To keep notations simple, we define the expected utility gains and expected payments for agent i who reports having type $\hat{\alpha}_l^i$ respectively as:

$$G_l(\hat{\alpha}_l^i) = \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}\left(\Delta u_l(\beta_l(\hat{\alpha}_l^i,\boldsymbol{\alpha}_l^{-i}),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) \text{ and } \mathcal{T}(\hat{\alpha}_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}\left(t_l^i(\hat{\alpha}_l^i,\boldsymbol{\alpha}_l^{-i})\right).$$

With those notations, incentive compatibility constraints (2.6) can be written as:

$$\text{(A.4)} \qquad \mathcal{U}_l(\alpha_l^i) = \max_{\hat{\alpha}_l^i\in\Omega_l} \alpha_l^i G_l(\hat{\alpha}_l^i) - \mathcal{T}_l^i(\hat{\alpha}_l^i).$$

$\mathcal{U}_l$ is convex as the maximum of linear functions. Since $\mathcal{X}$ is bounded above, $\mathcal{U}_l$ is also Lipschitz continuous. It is thus a.e. differentiable and admits the integral representation:

$$\text{(A.5)} \qquad \mathcal{U}_l(\alpha_l^i) = \mathcal{U}_l(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_l^i} G_l(\tilde{\alpha}_l^i)d\tilde{\alpha}_l^i.$$

This condition rewrites as (4.1). Finally, convexity of $\mathcal{U}_l$ implies that G_l is non-decreasing.

Sufficiency. Suppose that the allocation $(\mathcal{U}_l, G_l)$ satisfies (4.1) with $\mathcal{U}_l$ convex (i.e., G_l non-decreasing). By convexity, $\mathcal{U}_l$ has always a subdifferential that contains $G_l(\hat{\alpha}_l^i)$ at any $\hat{\alpha}_l^i$. Incentive compatibility then follows since:

$$\alpha_l^i G_l(\alpha_l^i) - \mathcal{T}_l^i(\alpha_l^i) = \mathcal{U}_l(\alpha_l^i) \geq \mathcal{U}_l(\hat{\alpha}_l^i) + G_l(\hat{\alpha}_l^i)(\alpha_l^i - \hat{\alpha}_l^i) = \alpha_l^i G_l(\hat{\alpha}_l^i) - \mathcal{T}_l^i(\hat{\alpha}_l^i).$$

Q.E.D.

PROOF OF LEMMA 2: NECESSITY. Taking expectations of (2.7) with respect to $\boldsymbol{\alpha}_l$ yields:

$$\text{(A.6)}\qquad \sum_{i=1}^{N_l}\mathbb{E}_{\alpha_l^i}\left(\alpha_l^i G_l(\alpha_l^i) - \mathcal{U}_l(\alpha_l^i)\right) - \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0.$$

Using (4.1) and integrating by parts, we obtain:

$$\mathbb{E}_{\alpha_l^i}\left(\mathcal{U}_l(\alpha_l^i)\right) = \mathcal{U}_l(\underline{\alpha}_l) + \mathbb{E}_{\alpha_l^i}\left(\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}G_l(\alpha_l^i)\right).$$

Inserting into (A.6) and simplifying yields:

$$\sum_{i=1}^{N_l}\mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)G_l(\alpha_l^i)\right) - \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq N_l\mathcal{U}_l(\underline{\alpha}_l) \geq 0$$

where the last inequality follows from (2.5). This condition can be written as:

$$\text{(A.7)}\qquad \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(h_l^*(\boldsymbol{\alpha}_l)\Delta u_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right) \geq 0.$$

Using the expression of payments from (3.1) yields (4.4).

SUFFICIENCY. Consider an allocation that satisfies (4.4), or (A.7) once (3.1) has been used, and such that G_l is non-decreasing. Define now a rent profile such that (A.5) hold and

$$\text{(A.8)}\qquad \mathcal{U}_l(\underline{\alpha}_l) = \mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)G_l(\alpha_l^i)\right) - \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) \geq 0.$$

From Lemma 1, such allocation is incentive compatible. From the fact that $\mathcal{U}_l$ is non-increasing, (A.8) also implies that (2.5) holds everywhere. Moreover, the expected payment $\mathcal{T}_l^i$ satisfies:

(A.9)

$$\mathcal{T}_l^i(\alpha_l^i) = \alpha_l^i G_l(\alpha_l^i) - \int_{\underline{\alpha}_l}^{\alpha_l^i} G_l(\tilde{\alpha}_l^i)d\tilde{\alpha}_l^i - \mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)G_l(\alpha_l^i)\right) + \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

Taking expectations and integrating by parts, we get:

$$\text{(A.10)}\qquad \mathbb{E}_{\alpha_l^i}(\mathcal{T}_l^i(\alpha_l^i)) = \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))).$$

From the expression of $\mathcal{T}_l^i(\alpha_l^i)$ in (A.9), we reconstruct payments t_l^i as follows:

$$t_l^i(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \varphi(\alpha_l^i) - \frac{1}{N_l-1}\left(\sum_{j\neq i}\varphi_j(\alpha_j) - \mathbb{E}_{\alpha_j}(\varphi_j(\alpha_j))\right).$$

where $\varphi(\alpha_l^i) = \mathcal{T}_l^i(\alpha_l^i) - \frac{1}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})))$. Observe that $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i}}(t_l^i(\boldsymbol{\alpha}_l)) = \mathcal{T}_l^i(\alpha_l^i)$ and that (2.7) is satisfied when (A.10) holds. *Q.E.D.*

PROOF OF PROPOSITION 3: Expressing the incentive-feasibility condition (4.4) for $\beta_l(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l)$ ($l \in \{1,2\}$) and developing yields (5.1). Turning to Item 2. of Lemma 2, observe that:

$$\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(\Delta u_l(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))) = (-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l^{-i},\boldsymbol{\alpha}_{-l}}(x(\alpha_l^*(\boldsymbol{\alpha}_l),\alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0,\alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))).$$

Since $x(\beta_l,\beta_{-l}) = \varphi((-1)^l(\beta_{-l}-\beta_l))$ and $\varphi' < 0$, the r.h.s. above is non-decreasing in α_l^i as requested. *Q.E.D.*

PROOF OF PROPOSITION 4: Taking limits as $N_l \to +\infty$ and using (6.1), the policy chosen by the decision-maker converges $a.s.$ towards:

$$x(\alpha_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) = \arg\max_{x \in \mathcal{X}} u_0(x) + \alpha_l^e u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}) u_{-l}(x) \quad \forall \boldsymbol{\alpha}_{-l}. \tag{A.11}$$

Taking limits as $N_l \to +\infty$, and again using (6.1), (5.1) certainly fails when:

$$\mathbb{E}_{\boldsymbol{\alpha}_{-l}} \left(\left[u_0(x) + \underline{\alpha}_l u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}) u_{-l}(x) \right]_{x(0,\boldsymbol{\alpha}_{-l})} x(\alpha_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) \right) < 0. \tag{A.12}$$

Notice that $x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) = \arg\max_{x \in \mathcal{X}} u_0(x) + \underline{\alpha}_l u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}) u_{-l}(x)$ when $\underline{\alpha}_l = 0$ and observe that $x(\boldsymbol{\alpha}_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) \neq x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$ when $\alpha_l^e > 0$. We thus have:

$$\left[u_0(x) + \underline{\alpha}_l u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}) u_{-l}(x) \right]_{x(\alpha_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(0,\boldsymbol{\alpha}_{-l})} > 0 \quad \forall \boldsymbol{\alpha}_{-l}. \tag{A.13}$$

Taking expectations expectations over $\boldsymbol{\alpha}_{-l}$ yields (A.13) as requested. *Q.E.D.*

PROOF OF PROPOSITION 5: To show that an efficient equilibrium exists when N_l and N_{-l} are both large enough, suppose that group $-l$ has chosen an efficient appointment rule, i.e., $\beta_{-l}(\boldsymbol{\alpha}_{-l}) = \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})$. Condition (5.1) certainly holds for N_l large enough when:

$$\left[u_0(x) + \underline{\alpha}_l u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}) u_{-l}(x) \right]_{x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))} > 0 \quad \forall \boldsymbol{\alpha}_{-l}. \tag{A.14}$$

By (A.11), the following strict inequality holds when $x(\alpha_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) \neq x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))$:

$$\left[u_0(x) + \alpha_l^e u_l(x) + \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}) u_{-l}(x) \right]_{x(0, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))}^{x(\alpha_l^e, \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))} > 0 \quad \forall \boldsymbol{\alpha}_{-l}.$$

Thus, (A.14) holds if $\alpha_l^e - \underline{\alpha}_l$ is small enough and $\underline{\alpha}_l > 0$. Taking expectations over $\boldsymbol{\alpha}_{-l}$ yields (5.1). *Q.E.D.*

PROOF OF PROPOSITION 6: The mechanism design problem for group l can be written as:

$$(\mathcal{GF}): \quad \max_{\mathcal{U}_l, \mathcal{G}_l} \sum_{i=1}^{N_l} \mathbb{E}_{\boldsymbol{\alpha}_l}(\mathcal{U}_l(\alpha_l^i)) \text{ subject to (2.4), (2.5), (2.6) and (2.7).}$$

We shall neglect the monotonicity condition $\mathbb{E}_{\boldsymbol{\alpha}_l^{-i}, \boldsymbol{\alpha}_{-l}}(\Delta u_l(\beta_l(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}), \beta_{-l}(\boldsymbol{\alpha}_{-l})))$ non-decreasing. This condition will be checked *ex post* on the solution of the so relaxed problem. Taking into account (A.2) and the fact that (2.5), (2.6) and (2.7) can be aggregated into a single incentive-feasibility constraint (4.4), we rewrite $(\mathcal{GF})$ as:

$$\max_{\beta_l \geq 0} \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(u_0(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \alpha_l^*(\boldsymbol{\alpha}_l) u_l(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \beta_{-l}(\boldsymbol{\alpha}_{-l}) u_{-l}(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) \right)$$

subject to (4.4). Let denote by λ_l the non-negative Lagrange multiplier for this constraint. The Lagrangean $\mathcal{L}_l(\beta_l, \alpha, \lambda_l)$ satisfies (up to terms independent of $\beta_l(\boldsymbol{\alpha}_l)$):

$$\frac{\mathcal{L}_l(\beta_l, \alpha, \lambda_l)}{1 + \lambda_l} = u_0(x(\beta_l, \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) u_l(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + \beta_{-l}(\boldsymbol{\alpha}_{-l}) u_{-l}(x(\beta_l(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})))$$

where we define $\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l)$ as

$$\text{(A.15)}\quad \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}.$$

For each possible realization of $(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l})$, the optimality condition in β_l under the constraint $\beta_l \geq 0$ writes as follows:

$$\text{(A.16)}\quad \Big(u_0'(x(\beta_l^{sb}(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + (-1)^l(\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) - \beta_{-l}(\boldsymbol{\alpha}_{-l}))\Big)\frac{\partial x}{\partial \beta}(\beta_l^{sb}(\boldsymbol{\alpha}_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})) \leq 0.$$

We distinguish two cases.

1. $\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) \geq 0$. Observe that, by definition, $x(\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l), \beta_{-l}(\boldsymbol{\alpha}_{-l})) = \varphi((-1)^l(\beta_{-l}(\boldsymbol{\alpha}_{-l}) - \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l)))$. Thus, we deduce that

   $$\text{(A.17)}\quad \beta_l^{sb}(\boldsymbol{\alpha}_l) = \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) \geq 0$$

   satisfies condition (A.16).

2. $\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) < 0$. Becase $x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l})) = \varphi((-1)^l\beta_{-l}(\boldsymbol{\alpha}_{-l})$, we deduce that:

   (A.18)
   $$u_0'(x(0, \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + (-1)^l(\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) - \beta_{-l}(\boldsymbol{\alpha}_{-l})) = (-1)^l\tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l)\begin{cases} < 0 & \text{if } l = 2, \\ > 0 & \text{if } l = 1. \end{cases}$$

   Observe that:

   $$\text{(A.19)}\quad \frac{\partial x}{\partial \beta_1}(\beta_1, \beta_2) < 0 < \frac{\partial x}{\partial \beta_2}(\beta_1, \beta_2).$$

   Putting together (A.18) with (A.19) and using (A.16) yields:

   $$\text{(A.20)}\quad \beta_l^{sb}(\boldsymbol{\alpha}_l) = 0.$$

Gathering (A.17) and (A.20) finally gives us (7.1).

Moreover, because of the monotonicity condition of the hazard rate, β_l^{sb} is non-decreasing in α_l^i. As a result, $\mathcal{U}_l$ is convex as requested by Lemma 1. *Q.E.D.*

PROOF OF PROPOSITION 7: We rewrite Condition (7.2) by means of integrals inside the expectation as:

(A.21)
$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\Big(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l}))\Big(u_0'(x(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l}))) + (-1)^l(\alpha_l^*(\boldsymbol{\alpha}_l) - \beta_{-l}(\boldsymbol{\alpha}_{-l}))\Big)d\beta\Big)$$
$$< (-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}\frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right).$$

Using the fact that $x(\beta_l, \beta_{-l}) = \varphi((-1)^l(\beta_{-l} - \beta_l))$, (A.21) becomes

$$\text{(A.22)}\quad (-1)^l\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)}(\alpha_l^*(\boldsymbol{\alpha}_l) - \beta)\frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l}))d\beta\right)$$

$$< (-1)^l \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)} \left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right).$$

Using the *Law of Iterated Expectations*, we can rewrite:

$$\mathbb{E}_{\boldsymbol{\alpha}_l} \left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)} (\alpha_l^*(\boldsymbol{\alpha}_l) - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right)$$

$$= \mathbb{E}_{\alpha_l^*} \left(\mathbb{E}_{\boldsymbol{\alpha}_l} \left(\int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)} (\alpha_l^*(\boldsymbol{\alpha}_l) - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta | \alpha_l^*(\boldsymbol{\alpha}_l) = \alpha_l^* \right) \right)$$

$$= \int_0^{\overline{\alpha}_l} \left(\int_0^{\alpha_l^*} (\alpha_l^* - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right) \phi_{N_l}(\alpha_l^*) d\alpha_l^*.$$

Integrating by parts, this last term becomes:

$$\left(\left[-(1 - \Phi_{N_l}(\alpha_l^*)) \int_0^{\alpha_l^*} (\alpha_l^* - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right]_0^{\overline{\alpha}_l} \right) \tag{A.23}$$

$$+ \int_0^{\overline{\alpha}_l} (1 - \Phi_{N_l}(\alpha_l^*)) \left((\alpha_l^* - \alpha_l^*) \frac{\partial x}{\partial \beta_l}(\alpha_l^*, \beta_{-l}(\boldsymbol{\alpha}_{-l})) + \int_0^{\alpha_l^*} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right) d\alpha_l^*.$$

Or, simplifying

$$\mathbb{E}_{\alpha_l^*} \left(\frac{1 - \Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} \int_0^{\alpha_l^*} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right).$$

Using again the *Law of Iterated Expectations*, we also get:

$$\mathbb{E}_{\boldsymbol{\alpha}_l} \left(\left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right) \int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right) \tag{A.24}$$

$$= \mathbb{E}_{\alpha_l^*} \left(\mathbb{E}_{\boldsymbol{\alpha}_l} \left(\left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right) \int_0^{\alpha_l^*(\boldsymbol{\alpha}_l)} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta | \alpha_l^*(\boldsymbol{\alpha}_l) = \alpha_l^* \right) \right)$$

$$= \mathbb{E}_{\alpha_l^*} \left(\mathbb{E}_{\boldsymbol{\alpha}_l} \left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} | \alpha_l^*(\boldsymbol{\alpha}_l) = \alpha_l^* \right) \int_0^{\alpha_l^*} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right).$$

Using (A.23) and (A.24) and taking expectations over $\boldsymbol{\alpha}_{-l}$, Condition (A.22) becomes:

$$\mathbb{E}_{\alpha_l^*, \boldsymbol{\alpha}_{-l}} \left(\left(\frac{1 - \Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \mathbb{E}_{\boldsymbol{\alpha}_l} \left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} | \alpha_l^*(\boldsymbol{\alpha}_l) = \alpha_l^* \right) \right) \int_0^{\alpha_l^*} (-1)^l \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) d\beta \right) < 0. \tag{A.25}$$

Observe that this r.h.s. inequality cannot be strict for $N_l = 1$ since the first bracketed term is identically null. Suppose thus that $N_l > 1$. Observe then that $\kappa(\alpha_l^*) = \int_0^{\alpha_l^*} (-1)^l \mathbb{E}_{\boldsymbol{\alpha}_{-l}} \left(\frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\boldsymbol{\alpha}_{-l})) \right) d\beta$ is increasing in α_l^* while $\frac{\zeta(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} = \frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \mathbb{E}_{\boldsymbol{\alpha}_l} \left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)} | \alpha_l^*(\boldsymbol{\alpha}_l) = \alpha_l^* \right)$ is decreasing in α_l^* from Assumption 2. By definition, we have:

$$\int_0^{\overline{\alpha}_l} \zeta(\alpha_l^*) \kappa(\alpha_l^*) d\alpha_l^* =$$

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha(\alpha_l^*)=\alpha_l^*\right)\right)\int_0^{\alpha_l^*}(-1)^l\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right)d\beta\right)$$

Integrating by parts, we thus get:

$$\int_0^{\overline{\alpha}_l}\zeta(\alpha_l^*)\kappa(\alpha_l^*)d\alpha_l^*=\left[\left(\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma\right)\kappa(\alpha_l^*)\right]_0^{\overline{\alpha}_l}-\int_0^{\overline{\alpha}_l}\left(\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma\right)\kappa'(\alpha_l^*)d\alpha_l^*. \tag{A.26}$$

Since $\underline{\alpha}_l=0$ from Assumption 1 and α_l^* has mean α^e, it can be checked that:

$$\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\right)=\alpha^e-\underline{\alpha}_l=\alpha^e.$$

Using the *Law of Iterated Expectations*, we also get:

$$\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}|\alpha_l^*(\boldsymbol{\alpha}_l)=\alpha_l^*\right)\right)=\mathbb{E}_{\boldsymbol{\alpha}_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)$$

$$=\mathbb{E}_{\alpha_l^i}\left(\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right)=\alpha^e-\underline{\alpha}_l=\alpha^e.$$

Therefore, we obtain:

$$\mathbb{E}_{\alpha_l^*}\left(\frac{\zeta(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\right)=\int_0^{\overline{\alpha}_l}\zeta(\gamma)d\gamma=0.$$

Inserting into (A.26), we get:

$$\int_0^{\overline{\alpha}_l}\zeta(\alpha_l^*)\kappa(\alpha_l^*)d\alpha_l^*=-\int_0^{\overline{\alpha}_l}\left(\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma\right)\kappa'(\alpha_l^*)d\alpha_l^*. \tag{A.27}$$

From Assumption 2, $\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma$ is strictly quasi-concave in α_l^* and zero only at $\alpha_l^*=0$ and $\alpha_l^*=\overline{\alpha}_l$. Hence, $\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma<0$ for $\alpha_l^*\in(0,\overline{\alpha}_l)$. Since $\kappa'(\alpha_l^*)>0$, $\int_0^{\overline{\alpha}_l}\zeta(\alpha_l^*)\kappa(\alpha_l^*)d\alpha_l^*<0$. Therefore, Condition (7.2) holds. The feasibility condition (4.4) does not hold for $\beta_l(\boldsymbol{\alpha}_l)=\alpha_l^*(\boldsymbol{\alpha}_l)$ and necessarily $\lambda>0$. *Q.E.D.*

Proof of Proposition 8: First, remember that the feasibility condition for group l is given by (4.4). Second, we prove a preliminary Lemma.

Lemma A.1

$$\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(T_l(\beta_l,\beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l})))>0\quad\forall\beta_l>0. \tag{A.28}$$

Proof of Lemma A.1: From (3.1) and the definition of $x(\beta_l,\beta_{-l})$, we can write:

$$T_l(\beta_l,\beta_{-l})=\int_{x(0,\beta_{-l})}^{x(\beta_l,\beta_{-l})}(u_0'(x(\beta,\beta_{-l}))-\beta_{-l}(-1)^l)\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l})d\beta$$

$$=\int_{x(0,\beta_{-l})}^{x(\beta_l,\beta_{-l})}\beta_l(-1)^{l+1}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l})d\beta=\beta_l(-1)^l(x(\beta_l,\beta_{-l})-x(0,\beta_{-l})).$$

Thus, $T_l(\beta_l,\beta_{-l})>0$ if $\beta_l>0$. Therefore, $\min_{\beta_{-l}\in[0,\overline{\alpha}]}T_l(\beta_l,\beta_{-l})>0$. Condition (A.28) immediately follows by taking expectations. *Q.E.D.*

Assumption 1 implies that $\mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)\right) = \underline{\alpha}_l = 0$. Let us make explicit the dependence of the sample mean $h^*_{N_l}(\boldsymbol{\alpha}_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_l^i)$ on N_l. For a given $\hat{\lambda}_l \in \mathbb{R}_+$, we now define

$$\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l) \equiv \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_l^i - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}.$$

Since $|\Delta u_l(\hat{\beta}_l(\cdot,\cdot,\hat{\lambda}_l), \beta_{-l}^{sb})|$ is uniformly bounded in N_l on $[0,\overline{\alpha}_l]\times\mathbb{N}$, (6.1) implies that

$$B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l) \equiv h^*_{N_l}(\boldsymbol{\alpha}_l)\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} 0. \tag{A.29}$$

Since convergence almost surely implies convergence in probability, Condition (A.29) also implies that, for all $\gamma>0$ and $\epsilon>0$, there exists N^* such that, for all $N_l \geq N^*$, $\mathbb{P}\left\{|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)| \geq \gamma\right\} \leq \epsilon$. Because $h^*_{N_l}(\boldsymbol{\alpha}_l)$ is uniformly bounded in N_l, $|B(\cdot,\cdot,\hat{\lambda}_l)|$ is also bounded on $[0,\overline{\alpha}_l]\times\mathbb{N}$ by some constant M. Fixing such γ, ϵ, and N^*, we can write, for all $N_l \geq N^*$,

$$\begin{aligned}
&|\mathbb{E}_{\boldsymbol{\alpha}_l}\left(B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)\right)| \leq \mathbb{E}_{\boldsymbol{\alpha}_l}\left(|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l|)\right)\\
&= \mathbb{P}\left\{|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)| \geq \gamma\right\}\mathbb{E}_{\boldsymbol{\alpha}_l}\left[|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)|\,|\,|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)| \geq \gamma\right]\\
&+\mathbb{P}\left\{|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)| < \gamma\right\}\mathbb{E}_{\boldsymbol{\alpha}_l}\left[|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)|\,|\,|B(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l)| < \gamma\right]\\
&\leq \epsilon M + \gamma
\end{aligned}$$

Choosing $\gamma = \epsilon = \epsilon'/(M+1)$ shows that for any $\epsilon' > 0$, there exists N^{**} such that, for all $N_l \geq N^{**}$,

$$|\mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(h^*_{N_l}(\boldsymbol{\alpha}_l)\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right)| \leq \epsilon' \tag{A.30}$$

which proves

$$\lim_{N_l\to\infty}\ \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left(h^*_{N_l}(\boldsymbol{\alpha}_l)\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right) = 0. \tag{A.31}$$

In addition, we have

$$\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} \alpha_l^e - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}(\alpha_l^e - \underline{\alpha}_l) = \frac{1}{1+\hat{\lambda}_l}\alpha_l^e > 0 \tag{A.32}$$

where the last equality follows from Assumption 1. Therefore, for all $\hat{\lambda}_l < +\infty$, (A.28) implies:

$$\lim_{N_l\to\infty}\ \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left[T_l(\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right] > 0. \tag{A.33}$$

It follows from (A.31) and (A.33) that:

$$\lim_{N_l\to\infty}\ \mathbb{E}_{\boldsymbol{\alpha}_l,\boldsymbol{\alpha}_{-l}}\left[h^*_{N_l}(\boldsymbol{\alpha}_l)\Delta u_l(\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l})) - T_l(\hat{\beta}_l(\boldsymbol{\alpha}_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))\right] < 0.$$

Therefore, for any $\hat{\lambda}_l > 0$, there exists N^e such that, for $N_l \geq N^e$, $\lambda_l(N_l) \geq \hat{\lambda}_l$. Hence,

$$\lim_{N_l\to\infty}\ \lambda_l(N_l) = +\infty. \tag{A.34}$$

Condition (A.32) also implies convergence in probability and thus

$$\beta_l^{sb}(\boldsymbol{\alpha}_l, N_l) = \frac{1}{N_l}\left(\sum_{i=1}^{N_l}\alpha_l^i - \frac{\lambda_l(N_l)}{1+\lambda_l(N_l)}\frac{1-F_l(\alpha_l^i)}{f_l(\alpha_l^i)}\right) \underset{N_l\to+\infty}{\overset{p}{\to}} \mathbb{E}_{\alpha_l^i}\left(h_l(\alpha_l^i)\right) = \underline{\alpha}_l = 0$$

where again the last equality follows from Assumption 1. Hence, (7.4) holds. *Q.E.D.*

PROOFS OF PROPOSITIONS 11 AND 12: We start with Proposition 11. Assume that $u_0''' \geq 0$. From Proposition 10, the mapping Λ_l^* is everywhere non-decreasing. Since $0 < \lambda_{-l}$, we thus get $\tilde{\lambda}_l = \Lambda_l^*(0) \leq \lambda_l$. Finally, the opposite condition holds if $u_0''' \leq 0$ which ends the proof. The proof of Proposition 11 is similar and omitted. *Q.E.D.*

PROOF OF PROPOSITION 13: Let consider an equilibrium of the game obtained when the common size of the groups is $N_1 = N_2 = N$. It corresponds to the appointment rules $(\beta_l^{sb}(\cdot, N), \beta_{-l}^{sb}(\cdot, N))$ where we now make the dependence on N explicit. It is easy to check that the limiting behaviors described in Proposition 8 still apply when both groups have variable sizes:

$$\lim_{N\to\infty} \lambda_l(N) = +\infty \text{ and } \beta_l^{sb}(\boldsymbol{\alpha}_l, N) \underset{N\to+\infty}{\overset{p}{\to}} = 0.$$

In addition, the function $x(\beta_l^{sb}(\boldsymbol{\alpha}_l, N), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}, N))$ is bounded because $\mathcal{X}$ itself is. Reminding that the ideal point of the decision-maker is 0, we can deduce (with the method we used to establish (A.30)) that

$$\lim_{N\to\infty} \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_l^{sb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N)) = 0 \tag{A.35}$$

where group l's payoff writes as

$$\mathcal{U}_l^{sb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N) = \alpha_N^*(\boldsymbol{\alpha}_l) u_l(x(\beta_l^{sb}(\boldsymbol{\alpha}_l, N), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}, N))) - T_l(\beta_l^{sb}(\boldsymbol{\alpha}_l, N), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}, N))$$

and $\alpha_N^*(\boldsymbol{\alpha}_l) = \frac{1}{N}\sum_{i=1}^N \alpha_l^i$.

Consider now the case when both groups form under complete information. When the profile of preferences is $(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l})$, group l's payoff writes as:

$$\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}, N) = W(\alpha_N^*(\boldsymbol{\alpha}_l), \alpha_N^*(\boldsymbol{\alpha}_{-l})) - W(0, \alpha_N^*(\boldsymbol{\alpha}_{-l})).$$

Taking expectations yields:

$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l})\right) = \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(W(\alpha_N^*(\boldsymbol{\alpha}_l), \alpha_N^*(\boldsymbol{\alpha}_{-l})) - W(0, \alpha_N^*(\boldsymbol{\alpha}_{-l}))\right).$$

Taking into account that the decision-maker's objective function is quadratic, we obtain:

$$W(\alpha_N^*(\boldsymbol{\alpha}_l), \alpha_N^*(\boldsymbol{\alpha}_{-l})) - W(0, \alpha_N^*(\boldsymbol{\alpha}_{-l})) = \frac{1}{2\beta_0}\left(\alpha_N^{*2}(\boldsymbol{\alpha}_l) - 2\alpha_N^*(\boldsymbol{\alpha}_l)\alpha_N^*(\boldsymbol{\alpha}_{-l})\right).$$

That all $\alpha_l^i s$ are independently distributed on $[0, \Delta\alpha]$ within and across groups implies that $\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\alpha_N^*(\boldsymbol{\alpha}_l)\alpha_N^*(\boldsymbol{\alpha}_{-l})\right) = (\alpha^e)^2$ and $\lim_{N\to+\infty} \mathbb{E}_{\boldsymbol{\alpha}_l}\left((\alpha_N^*(\boldsymbol{\alpha}_l))^2\right) = (\alpha^e)^2$. It follows that:

$$\lim_{N\to\infty} \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\mathcal{U}_l^{fb}(\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l})\right) = -\frac{(\alpha^e)^2}{2\beta_0} < 0 \tag{A.36}$$

The result directly follows by comparing the r.h.s. of (A.35) and (A.36). *Q.E.D.*

PROOF OF PROPOSITION 14: We show that the decision-maker is always worse off under incomplete information in the *ex post* sense. The *ex ante* result will directly follow by taking expectations. Let denote by T_0 the total contribution received by the decision-maker. From our earlier findings, we get:

$$T_0(\beta_l, \beta_{-l}) = [\beta_{-l}u_{-l}(x) + u_0(x)]_{x(\beta_l, \beta_{-l})}^{x(0, \beta_{-l})} + [\beta_l u_l(x) + u_0(x)]_{x(\beta_l, \beta_{-l})}^{x(\beta_l, 0)}$$

For any such configuration (β_l, β_{-l}), the decision-maker's payoff can thus be written as:

$$\mathcal{U}_0(\beta_l, \beta_{-l}) = u_0(x(\beta_l, \beta_{-l})) + T_0(\beta_l, \beta_{-l}).$$

Differentiating with respect to β_l, we get:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) = u_l(x(\beta_l, 0)) - u_l(x(\beta_l, \beta_{-l})) + \frac{\partial x}{\partial \beta_l}(\beta_l, 0)\left(u_0'(x(\beta_l, 0)) + \beta_l u_l'(x(\beta_l, 0))\right)$$

$$-\frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l})\left(u_0'(x(\beta_l, \beta_{-l})) + \beta_l u_l'(x(\beta_l, \beta_{-l})) + \beta_{-l} u_{-l}'(x(\beta_l, \beta_{-l}))\right).$$

Using the definitions of $x(\beta_l, \beta_{-l})$ and $x(0, \beta_{-l})$, the latter expression becomes:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) = u_l(x(\beta_l, 0)) - u_l(x(\beta_l, \beta_{-l})) = (-1)^l(x(\beta_l, 0) - x(\beta_l, \beta_{-l})).$$

It follows that:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) > 0 \quad \forall \beta_l > 0.$$

Therefore, for all $\beta_l < \alpha_l^*$, $\beta_{-l} < \alpha_{-l}^*$, the following string of inequalities holds:

$$\mathcal{U}_0(\beta_l, \beta_{-l}) < \mathcal{U}_0(\alpha_l^*, \beta_{-l}) < \mathcal{U}_0(\alpha_l^*, \alpha_{-l}^*).$$

In particular, we may take $\beta_l = \beta_l^{sb}(\boldsymbol{\alpha}_l)$ and $\beta_{-l} = \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l})$. Taking expectations, and taking into account that those inequalities are strict on a set of positive measure, we finally obtain:

$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_0(\beta_l^{sb}(\boldsymbol{\alpha}_{-l}), \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))) < \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_0(\alpha_l^*, \beta_{-l}^{sb}(\boldsymbol{\alpha}_{-l}))) < \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}(\mathcal{U}_0(\alpha_l^*, \alpha_{-l}^*))$$

which ends the proof. *Q.E.D.*

APPENDIX B: CONGRUENT GROUPS (SUPPLEMENTARY MATERIAL)

Consider now the case where groups have *congruent preferences*. To mirror our previous analysis, we suppose that $u_1(x) = u_2(x) = x$ for all $x \in \mathcal{X}$. We first need to come back on the specification of payoffs in the common agency stage of the game. With congruent interest groups, the cooperative game constructed by Laussel and Le Breton (2001) turns out to be *super-additive*. Indeed, for any profile of lobbyists' preferences $(\beta_1, \beta_2) \in \mathbb{R}_+^2$, we now have:

$$W(\beta_1, \beta_2) + W(0, 0) \geq W(\beta_1, 0) + W(0, \beta_2) \quad \forall (\beta_1, \beta_2) \in \mathbb{R}_+^2$$

where again $W(0, 0) = 0$. Laussel and Le Breton (2001) demonstrated that the associated common agency game has the so-called *no rent property*, i.e., in all truthful continuation equilibria, the surplus of the decision-maker is always fully extracted by lobbyists. The lobbyists' payoffs lie in an interval with non-empty interior which is fully determined by the following constraints:

$$\text{(B.1)} \qquad V_l(\beta_l, \beta_{-l}) \leq W(\beta_1, \beta_2) - W(0, \beta_{-l}) \quad \forall l \in \{1, 2\},$$

$$\text{(B.2)} \qquad V_1(\beta_1, \beta_2) + V_2(\beta_2, \beta_1) = W(\beta_1, \beta_2).$$

The choice of the optimal appointment rules, either under complete or asymmetric information, of course depends on how the lobbyists' payoffs are precisely determined. To highlight new phenomena that might arise with congruent groups, we shall assume that those payoffs are given by the lobbyists' *Shapley Values* since this allocation satisfies both (B.1) and (B.2), namely:

$$V_l(\beta_l,\beta_{-l}) = \frac{1}{2}\left(W(\beta_1,\beta_2) - W(0,\beta_{-l}) + W(\beta_l,0)\right). \tag{B.3}$$

Using (2.2), we retrieve the expression of the equilibrium payment made by lobbyist l:

$$T_l(\beta_l,\beta_{-l}) = \beta_l x(\beta_l,\beta_{-l}) - \frac{1}{2}\left(W(\beta_1,\beta_2) - W(0,\beta_{-l}) + W(\beta_l,0)\right). \tag{B.4}$$

These payments are not VCG.[37] It is no longer a sincere (i.e., dominant) strategy for each group to pass to its lobbyist the aggregate preferences of the group. Each group manipulates these preferences even under complete information. There is now inter-group free riding.

Running Example (Continued). To illustrate, consider the case of quadratic preferences. It is immediate to derive the policy chosen by the decision-maker at the last stage of the game as $x(\beta_1,\beta_2) = \frac{\beta_1+\beta_2}{\beta_0}$ while coalitional payoffs are given by $W(\beta_1,\beta_2) = \frac{(\beta_1+\beta_2)^2}{2\beta_0}$. Under complete information, each group l endows its lobbyist with an objective β_l that maximizes the net surplus of the group, taking as given its conjectures on similar choices made by group $-l$ and taking into account that preferences in that competing group remain unknown. Because mechanisms for group formation are secret, members of group $-l$ conjecture the formation of group l even if it may be vetoed off equilibrium. Following veto, the policy chosen remains $x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))$ where $\beta_{-l}(\boldsymbol{\alpha}_{-l})$ represents the preferences given to lobbyist $-l$. The net utility of an individual with type α_l^i who belongs to group l and knows the preferences $\boldsymbol{\alpha}_l^{-i}$ of other members is thus:

$$\frac{\alpha_l^i}{N_l}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))) - t_i(\boldsymbol{\alpha}_l) \quad \forall(,\boldsymbol{\alpha}_l)\in\Omega_l^{N_l}. \tag{B.5}$$

Aggregating those expressions over the whole group l, $\beta_l(\boldsymbol{\alpha}_l)$ should be chosen to maximize:

$$\mathbb{E}_{\boldsymbol{\alpha}_{-l}}\left(\alpha_l^*(\boldsymbol{\alpha}_l)(x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - x(0,\beta_{-l}(\boldsymbol{\alpha}_{-l}))) - T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l}))\right).$$

Using (B.4) yields the expression of group l's overall contribution:

$$T_l(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) = \beta_l(\boldsymbol{\alpha}_l)x(\beta_l(\boldsymbol{\alpha}_l),\beta_{-l}(\boldsymbol{\alpha}_{-l})) - \frac{1}{2}\left(\frac{(\beta_l(\boldsymbol{\alpha}_l)+\beta_{-l}(\boldsymbol{\alpha}_{-l}))^2}{2\beta_0} - \frac{\beta_{-l}^2(\boldsymbol{\alpha}_{-l})}{2\beta_0} + \frac{\beta_l^2(\boldsymbol{\alpha}_l)}{2\beta_0}\right).$$

The (necessary and sufficient) first-order condition that characterizes the equilibrium choices of lobbyists' preferences yields:

$$\beta_l^*(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l) - \frac{1}{2}\mathbb{E}_{\boldsymbol{\alpha}_{-l}}(\beta_{-l}^*(\boldsymbol{\alpha}_{-l})).$$

For a symmetric equilibrium, assuming symmetric distributions, we thus obtain:

$$\beta_l^*(\boldsymbol{\alpha}_l) = \alpha_l^*(\boldsymbol{\alpha}_l) - \frac{\alpha^e}{3} \tag{B.6}$$

where we assume $\underline{\alpha} \geq \frac{\alpha^e}{3}$ to ensure that $\beta_l^*(\boldsymbol{\alpha}_l)$ remains non-negative and avoid a corner solution.

[37] Even if we were to choose within the range of allocations defined by (B.1) and (B.2) another allocation than that defined with Shapley values, manipulations would still arise. It is indeed well-known that, in those contexts, there is no sincere (i.e., dominant strategy) mechanism that implements the first-best allocation and extracts all surplus from the decision-maker. Furosawa and Konishi (2011) find a similar result in a more specific game.

From (B.6), each group chooses a lobbyist with moderate preferences. There is now inter-group free riding. The policy that ends up being chosen by the decision-maker is obviously lower than the first-best policy that would have been chosen had groups merged into a single entity so as to perfectly pass their preferences on the decision-maker:

$$x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) = x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) - \frac{2\alpha^e}{3\beta_0} < x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})).$$

■

Under asymmetric information within group l, the net utility of an individual with type α_l^i when the rule $\beta_l^*(\boldsymbol{\alpha}_l)$ is still adopted within group l is thus:

(B.7)
$$\mathcal{U}_l(\alpha_l^i) = \mathbb{E}_{\boldsymbol{\alpha}_l^{-i}} \left(\frac{\alpha_l^i}{N_l} \mathbb{E}_{\boldsymbol{\alpha}_{-l}}(x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0, \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))) - t_i(\alpha_l^i, \boldsymbol{\alpha}_l^{-i}) \right) \quad \forall \alpha_l^i \in \Omega_l.$$

We do not expect that an efficient equilibrium exists under asymmetric information. Indeed, free riding already bites across groups even under complete information. Yet, we might still be interested in determining conditions such that intra-group free riding does not add inefficiencies on top of those already brought by inter-group free riding. Proceeding as in the case of conflicting interests (Condition (5.1)), we may obtain a condition ensuring that the decision $x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))$ remains implementable even under asymmetric information as:

(B.8)
$$\begin{aligned} &\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\alpha_l^*(\boldsymbol{\alpha}_l)(x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0, \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))) - T(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) \right) \\ &\geq \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\frac{1}{N_l} \left(\sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right) (x(\beta_l^*(\boldsymbol{\alpha}_l), \beta_{-l}^*(\boldsymbol{\alpha}_{-l})) - x(0, \beta_{-l}^*(\boldsymbol{\alpha}_{-l}))) \right). \end{aligned}$$

The r.h.s. above is by now familiar. It represents the expected information rent left to all members of group l. The l.h.s. is the expected net gain from group formation given the continuation equilibrium and payments.

Had groups cooperated when dealing with the decision-maker, inter-group free riding would disappear. The efficient decision $x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) = \frac{1}{\beta_0}(\alpha_l^*(\boldsymbol{\alpha}_l) + \boldsymbol{\alpha}_{-l}^*(\boldsymbol{\alpha}_{-l}))$ would be implemented while the decision-maker would choose his ideal point, namely 0, when the merged group does not organize. The incentive-feasibility condition would now become:

(B.9)
$$\begin{aligned} &\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(u_0(x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))) + (\alpha_l^*(\boldsymbol{\alpha}_l) + \boldsymbol{\alpha}_{-l}^*(\boldsymbol{\alpha}_{-l})) x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l})) \right) \\ &\geq \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\left(\frac{1}{N_l} \left(\sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_l^i)}{f_l(\alpha_l^i)} \right) + \frac{1}{N_{-l}} \left(\sum_{j=1}^{N_{-l}} \frac{1 - F_{-l}(\alpha_j)}{f_{-l}(\alpha_j)} \right) \right) x(\alpha_l^*(\boldsymbol{\alpha}_l), \alpha_{-l}^*(\boldsymbol{\alpha}_{-l}))) \right) \quad \forall l \in \{1, 2\}. \end{aligned}$$

The comparison of the incentive-feasibility conditions (B.8) and (B.9) may already highlight two important driving forces. On the one hand, the net gains of forming is certainly greater in the case of a merger of the two groups since the *status quo* entails no production at all while its formation induces an efficient policy. Instead, with two groups, each of them can free ride and benefit from the policy induced by the sole contribution of its rival. Overall the gains from forming for those two groups are certainly lower. On the other hand, with a merger, the *status quo* if that merged group does not form is the null policy that is chosen by the decision-maker on her own. This means that, with groups merging, the overall information rents that must be distributed are also quite large.

RUNNING EXAMPLE (CONTINUED). We assume that, for both groups, types are symmetrically and uniformly distributed on the same interval $\Theta = [\underline{\alpha}, \overline{\alpha}]$ with mean $\alpha^e = \frac{\underline{\alpha}+\overline{\alpha}}{2}$ and variance $\frac{\Delta\alpha^2}{12}$ (where also $\Delta\alpha = \overline{\alpha} - \underline{\alpha}$ and where $\alpha^e > \frac{3}{4}\Delta\alpha$ to avoid corner solutions). Groups have also the same size $N = N_1 = N_2$. Condition (B.8) then amounts to:

$$\frac{7}{3}(\alpha^e)^2 + \frac{9}{4}\frac{(\Delta\alpha)^2}{12N} \geq \overline{\alpha}\alpha^e.$$

Instead, Condition (B.9) writes as:

$$3(\alpha^e)^2 + \frac{3}{2}\frac{(\Delta\alpha)^2}{12N} \geq \overline{\alpha}\alpha^e.$$

It can be checked that, when N is large, this second condition is easier to achieve. Inter-group free riding reduces the overall surplus and makes it more difficult to implement for each delegate the same objectives (B.6) as under complete information (even though this objective is distorted by inter-group free-riding). Merging congruent groups helps solving the collective action problem.■

APPENDIX C: PROOFS OF SECTION 9

PROOF OF PROPOSITION 9: Let us first define for any non-negative value λ, $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda)$ as:

(C.1) $$\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda) = \max\left\{0, \tilde{\beta}_l(\boldsymbol{\alpha}_l, \lambda)\right\}.$$

Observe that $\hat{\beta}_l(\boldsymbol{\alpha}_l, 0) = \alpha_l^*$ and $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) = \beta^{sb}(\boldsymbol{\alpha}_l)$ when λ_l is the equilibrium value of the Lagrange multiplier for group l. Define the function $\tilde{S}_l$ from $\mathbb{R}_+ \times \mathbb{R}_+$ on $\mathbb{R}$ as:

(C.2) $$\tilde{S}_l(\lambda, \lambda_{-l}) = \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\Delta W_l(h_l^*(\boldsymbol{\alpha}_l), \hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l}))\right)$$

where

(C.3) $$\Delta W_l(\tilde{\beta}_l, \beta_l, \beta_{-l}) = \left[u_0(x) + \tilde{\beta}_l u_l(x) + \beta_{-l}u_{-l}(x)\right]_{x(0,\beta_{-l})}^{x(\beta_l,\beta_{-l})}.$$

LEMMA C.1 *The mapping $\Lambda_l^*(\lambda_{-l})$ such that $\tilde{S}_l(\Lambda_l^*(\lambda_{-l}), \lambda_{-l}) = 0$ is defined on $\mathbb{R}_+$, single-valued and continuous.*

PROOF OF LEMMA C.1: We start with three preliminary results.

RESULT C.1

(C.4) $$\frac{\partial \tilde{S}_l}{\partial \lambda}(\lambda, \lambda_{-l}) > 0.$$

PROOF OF RESULT C.1: Differentiating (C.2) with respect to λ, we find:

(C.5)
$$\frac{\partial \tilde{S}_l}{\partial \lambda}(\lambda, \lambda_{-l}) = (-1)^l \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\left(h_l^*(\boldsymbol{\alpha}_l) - \hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda)\right)\frac{\partial x}{\partial \beta_l}(\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l}))\frac{\partial \hat{\beta}_l}{\partial \lambda}(\boldsymbol{\alpha}_l, \lambda)\right).$$

From (C.1), we get $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda) \geq h_l^*(\boldsymbol{\alpha}_l)$ with a strict inequality for $\lambda > 0$. From (C.1), we also get: $\frac{\partial \hat{\beta}_l}{\partial \lambda}(\boldsymbol{\alpha}_l, \lambda) \leq 0$. Gathering those facts with (A.19) and inserting into (C.5) yields (C.4). *Q.E.D.*

Result C.2

$$\lim_{\lambda \to +\infty} \tilde{S}_l(\lambda, \lambda_{-l}) > 0. \tag{C.6}$$

Proof of Result C.2: Define now $\beta_l^\infty(\boldsymbol{\alpha}_l) = \max\{0, h_l^*(\boldsymbol{\alpha}_l)\}$. We can compute:

$$\lim_{\lambda \to +\infty} S_l(\lambda, \lambda_{-l}) = \mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}} \left(\Delta W_l(h_l^*(\boldsymbol{\alpha}_l), \beta_l^\infty(\boldsymbol{\alpha}_l), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l})) \right)$$

When $\beta_l^\infty(\boldsymbol{\alpha}_l) = h_l^*(\boldsymbol{\alpha}_l) > 0$, $x(\beta_l^\infty(\boldsymbol{\alpha}_l), \beta_{-l})$ is the unique maximizer of $u_0(x) + \beta_l^\infty(\boldsymbol{\alpha}_l) u_l(x) + \beta_{-l} u_{-l}(x)$. From which it follows that $\Delta W_l(h_l^*(\boldsymbol{\alpha}_l), \beta_l^\infty(\boldsymbol{\alpha}_l), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l})) > 0$. Since $\beta_l^\infty(\boldsymbol{\alpha}_l) > 0$ on a set of positive measure, it follows that (C.6) holds. *Q.E.D.*

Result C.3 *Suppose that Assumptions 1 and 2 both hold, then we have:*

$$\tilde{S}_l(0, \lambda_{-l}) < 0. \tag{C.7}$$

Proof of Result C.3: Taking into account that $\beta_l(\boldsymbol{\alpha}_l, 0) = \alpha_l^*(\boldsymbol{\alpha}_l) > 0$, (C.7) amounts to: Condition (7.2) which holds, from Proposition 7, when Assumptions 1 and 2 are both satisfied. *Q.E.D.*

Putting together Results C.1, C.2 and C.3, there always exists a unique solution $\lambda_l > 0$ to $\tilde{S}_l(\lambda_l, \lambda_{-l}) = 0$. Λ_l^* is thus single-valued. The mappings Λ_l^* is also continuous on $[0, +\infty)$. Indeed, consider any converging sequence λ_{-l}^n, and denote $\overline{\lambda}_{-l} = \lim_{n \to +\infty} \lambda_{-l}^n$. We want to show that

$$\lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^n) = \Lambda_l^*(\overline{\lambda}_{-l}). \tag{C.8}$$

Observe first that $\Lambda_l^*(\lambda_{-l}^n)$ is bounded. Indeed, if it was not the case, there would exist a subsequence $\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$, where φ is an increasing function from $\mathbb{N}$ into $\mathbb{N}$, such that $\lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^{\varphi(n)}) = +\infty$. From the fact that $\tilde{S}_l(\Lambda_l^*(\lambda_{-l}^{\varphi(n)}), \lambda_{-l}^{\varphi(n)}) = 0$ for all n, it would follow that $\tilde{S}_l(+\infty, \overline{\lambda}_{-l}) = 0$. This is a contradiction since $\tilde{S}_l(+\infty, \overline{\lambda}_{-l}) > 0$.

Second, consider any converging subsequence $\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$ (the Bolzano-Weirstrass theorem guarantees existence of such a subsequence), and define $\lambda_l^s = \lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^{\varphi(n)})$. It is again the case that $\tilde{S}_l(\lambda_l^s, \overline{\lambda}_{-l}) = 0$. This implies that $\lambda_l^s = \Lambda_l^*(\overline{\lambda}_{-l})$.

We thus have shown that $\Lambda_l^*(\lambda_{-l}^n)$ is a bounded sequence, such that all converging subsequences have the same limit $\Lambda_l^*(\overline{\lambda}_{-l})$. Therefore, $\lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^n)$ exists and thus (C.8) holds.

Q.E.D.

Since $\beta_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l}) = \frac{1}{N_{-l}} \sum_{i=1}^{N_{-l}} \alpha_l^i - \frac{\lambda_{-l}}{1+\lambda_{-l}} \frac{1-F_{-l}(\alpha_l^i)}{f(\alpha_l^i)}$, and $\lim_{\lambda_{-l} \to +\infty} \frac{\lambda_{-l}}{1+\lambda_{-l}} = 1$, $\lim_{\lambda_{-l} \to +\infty} \Lambda_l^*(\lambda_{-l})$ exists and takes a finite value. It follows that Λ_l^* is bounded over $[0, +\infty)$. There exists $A_l > 0$ such that for all $\lambda_{-l} \geq 0$, $\Lambda_l^*(\lambda_{-l}) \leq A_l$. We now define the function ζ on the domain $[0, A_l] \times [0, A_{-l}]$ as $\zeta(\lambda_l, \lambda_{-l}) = (\Lambda_l^*(\lambda_{-l}), \Lambda_{-l}^*(\lambda_l))$. The function ζ is continuous on a compact set and onto. From Brouwer's Fixed Point Theorem, it has a fixed point (λ_1, λ_2) which gives us a dual representation of the equilibrium of the game. Moreover, Proposition 7 that holds when Assumptions 1 and 2 are satisfied implies that $\lambda_l > 0$. *Q.E.D.*

PROOF OF PROPOSITION 10: From the Theorem of Implicit Functions, we have:

$$\text{(C.9)}\qquad \frac{\partial \Lambda_l^*}{\partial \lambda_{-l}}(\lambda_{-l}) = -\frac{\frac{\partial \tilde{S}_l}{\partial \lambda_{-l}}(\Lambda_l^*(\lambda_{-l}), \lambda_{-l})}{\frac{\partial \tilde{S}_l}{\partial \lambda_l}(\Lambda_l^*(\lambda_{-l}), \lambda_{-l})}$$

where the denominator is positive from Result C.2 and the numerator is given by

$$\frac{\partial \tilde{S}_l}{\partial \lambda_{-l}}(\lambda, \lambda_{-l}) =$$

$$\mathbb{E}_{\boldsymbol{\alpha}_l, \boldsymbol{\alpha}_{-l}}\left(\frac{\partial x}{\partial \beta_{-l}}(\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l})))\frac{\partial \hat{\beta}_{-l}}{\partial \lambda_{-l}}(\boldsymbol{\alpha}_{-l}, \lambda_{-l})\frac{\partial \Delta W_l}{\partial \beta_{-l}}(h_l^*(\boldsymbol{\alpha}_l), \hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda), \hat{\beta}_{-l}(\boldsymbol{\alpha}_{-l}, \lambda_{-l})))\right).$$

Differentiating (C.3) with respect to β_{-l}, and simplifying using (2.3), we find:

(C.10)
$$\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l}) = (-1)^{l+1}\left((\beta_l - \tilde{\beta}_l)\frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) + \tilde{\beta}_l\frac{\partial x}{\partial \beta_{-l}}(0, \beta_{-l}) + x(\beta_l, \beta_{-l}) - x(0, \beta_{-l})\right).$$

From (2.3), we also get:

$$\text{(C.11)}\qquad \frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l}) + \frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) = 0 \quad \forall(\beta_l, \beta_{-l}).$$

Differentiating (C.10) with respect to β_l, and using (C.11) to simplify the expression, we find:

$$\frac{\partial^2 \Delta W_l}{\partial \beta_l \partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l}) = (-1)^{l+1}(\beta_l - \tilde{\beta}_l)\frac{\partial^2 x}{\partial \beta_l \partial \beta_{-l}}(\beta_l, \beta_{-l}).$$

From (2.3), we know that

$$\frac{\partial^2 x}{\partial \beta_l \partial \beta_{-l}}(\beta_l, \beta_{-l}) \leq 0 \text{ (resp. } \geq 0) \;\Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

When $\beta_l \geq \tilde{\beta}_l$, we thus find:

$$\frac{\partial^2 \Delta W_l}{\partial \beta_l \partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l}) \leq 0 \text{ (resp. } \geq 0) \;\Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

Since $\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, 0, \beta_{-l}) = 0$, we finally get:

$$u_0''' \geq 0 \text{ (resp. } \leq 0) \Leftrightarrow \frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, \beta_l, \beta_{-l})\begin{cases} \leq 0 & \forall \beta_l \geq \max\{0, \tilde{\beta}_l\} \text{ (resp. } \geq 0) & \text{if } l = 1\,, \\ \geq 0 & \forall \beta_l \geq \max\{0, \tilde{\beta}_l\} \text{ (resp. } \leq 0) & \text{if } l = 2 \end{cases}.$$

From (A.19) and the fact that $\hat{\beta}_l(\boldsymbol{\alpha}_l, \lambda_l) \geq h_l^*(\boldsymbol{\alpha}_l)$, we finally obtain:

$$\text{(C.12)}\qquad \frac{\partial \tilde{S}_l}{\partial \lambda_{-l}}(\lambda, \lambda_{-l}) \geq 0 \text{ (resp. } \geq 0) \;\Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

Inserting into (C.9) gives the result.

Q.E.D.