The Impact of Tax Frequency: Theoretical and Empirical Investigations*

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Abstract

The pioneering work of Vickrey (1939), (1944) on the income-tax frequency has not inspired a lot of further studies. We propose an investigation of this issue both from a theoretical and empirical viewpoint within an annual framework. We first show that increasing the income-tax frequency (from an annual tax basis to a monthly tax basis) is Pareto-improving for any convex tax scheme. This result is obtained for the same tax revenue. Welfare gains are all the larger as the infra-annual volatility of income is large and the propensity to save is low, implying that the benefits should be larger for the bottom of the income distribution. We submit an empirical illustration using French administrative data and simulations of the current tax-benefit system. Despite that this system is not convex over the whole income domain, we show that increasing the tax frequency can lead to substantial social welfare gains.

1 Introduction

The temporality of taxation is complex. For instance, in withholding income tax systems, tax collection is based on monthly income while tax schedules are designed over yearly income, and filing at the end of the year allows using tax allowances to obtain rebates.
Benefits also have specific timing of income assessment (for instance in France, earnings of the three past months form the basis of means-tests for social assistance and in-work benefits). Effective taxation then possibly treats various time periods and, hence, different streams of income differently. In this paper, we question the welfare implications of varying the period of observation of the tax base.

Arguably, our question would be irrelevant if people had constant income flows over time. Yet it is clear that many households do face important income variation during the year, either expected (pay rise, seasonal work) or unexpected (income shocks due to business failure or unemployment), voluntary (chosen retirement) or involuntary (low-skill workers forced into temporary work). This is

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a major motivation for studying the question of within-year tax frequency and its impact on tax-payer utility. Note also that our question would be meaningless if effective taxation was flat. However, this never happens. Effective taxation, i.e. the impact of taxes and benefits, is usually nonlinear. Even in flat-tax countries, the presence of many other instruments, e.g. means-tested benefits, impose some regressivity or progressivity upon the effective tax schedule.

Interestingly, and potentially important for our welfare analysis, the question of tax frequency may be more relevant at the bottom of the distribution. Indeed, for high income households, large income variations are usually required to change tax brackets (especially given the global trend that has consisted in reducing the number of brackets, Peter et al. (2010, Fig. 3, p.468)). For them, this is as if they faced a marginal flat tax around the pre-shock income level. In the lower part of the distribution, however, effective brackets are narrower because of the numerous (and often overlapping) redistributive schemes at low and middle income levels (e.g. social assistance, tax credits, means-tested family benefits, etc.). Thus, even small variations of income may lead to change in effective marginal tax rates (MTR). There is also empirical evidence that this population experience more income fluctuations, and that income variation is more often involuntary, e.g. income shocks due to unemployment (Hills et al., 2006). These aspects are likely to contribute much to the welfare implications of tax frequency that we shall study. [1]

Strangely enough, the question of tax temporality has received little attention in the economic literature. Very few studies make the link between tax-payer utility and tax temporality. The seminal work of Vickrey (1939) focuses on yearly versus lifetime taxation. Vickrey develops the horizontal equity argument that over the lifecycle, people earning the same income should pay the same tax. This argument, closely linked to ours, leads to the recommandation in favor of a decrease in tax frequency over the lifecycle, i.e. a shift from an annual to a lifetime income basis. In contrast, we look at the infra-annual: we study the impact of an increase in tax frequency within the year, empirically illustrated with a move from annual to monthly taxation. [2]

To the best of our knowledge, we are the first to study the welfare impact of tax frequency within the year, in particular the implications of a shift from annual to monthly taxation. We first suggest a simple theoretical framework in which Jensen's inequality implies that a taxpayer with irregular income streams will pay more with a monthly tax scheme. We derive compensation schemes for the increase in tax liability due to income variation, i.e. compensations equating the amount of tax paid in both annual and monthly systems. We show that the reform with compensation is Pareto improving if utility is increasing,

1. Note also that poorer people are less able to smooth income because lower resources limit their saving possibilities (Dynan et al. (2004) finds a median saving rate of 1 percent for households in the bottom quintile for US households aged 30-59). In addition, they are often more credit constrained and possibly more myopic (cf. Hastings and Washington, 2010).

2. Vickrey's contribution has triggered important advances in capital taxation (Auerbach, CITE) and dynamic optimal taxation (CITE). An interesting extension of Vickrey's setting is Liebman (2002). He studies a distributionally-neutral Vickrey tax system that would equalize MTRs both over-time for the same individual and across individuals with the same lifetime earnings level, incorporating various motives for efficiency and equity gains (income shocks, myopic agents). Finally, note that the temporality of taxation has also received some attention in law studies following Vickrey, for instance Batchelder (2003) and Fennell and Stark (2005).

concave and intertemporally additive.[3] We characterize those most concerned by welfare gains, namely those whose incomes vary much over time and who face borrowing/saving constraints.

We then suggest an empirical illustration using French administrative data (tax filing data combined with work sequences from labor force surveys). Using simulations of the current income tax, we implement the higher-frequency tax reform and characterize winners and losers. We extend this empirical exercise to effective taxation, i.e. we additionally account for the main redistributive scheme in France. While our Pareto improving results require the income tax to be convex, real-world effective taxation is rarely so. Moreover, non-convex parts of the effective schedule are typically located in the first half of the income distribution, with the cumulative effect of different instruments (tax credit and benefit withdrawals, social contributions and income tax, etc). Despite this feature, which is particularly present in the case of French taxation, we show that increased tax frequency with compensation leads to substantial welfare gains.

The article is organized as follows. Section 2 presents our theoretical framework and results. Section 3 shows descriptive statistics on France taxpayers earnings. Section 4 suggests simulation of a tax frequency reform on French data while section 5 concludes.

2 Theory

The assessment of the tax determines the amount one is entitled to pay based on its situation (income streams, family situation, etc). The determined amount known as tax liability leads to a payment of the tax either directly by the taxpayer, or by a third party institution (employer, banks, etc). Appart from the frequency of the assessment of the tax, it can leads to different schedule of payments. In practice in countries with Pay-as-you-earn (PAYE) systems, an amount is withholded at each payroll period which can be viewed as sub-periods of taxation, eventhough in most countries final tax liability is computed over whole year incomes. Meaning that tax liability is meant to be based on yearly income, but tax payments are made before the realisation of incomes that constitutes the tax base.

There thus exists a *reference period* for the tax base in order to determine tax liability, but in cases the government wants to recover taxes before full realisation of income, it may wants to applies some rules on sub-periods to compute *sub-period tax liability* based on income, concomitant with the curent sub-period tax liability, or based on past income streams.

Those payments may lead to some differences with respect to tax liability and may implies some formal lending or borrowings : in the US fiscal household may have to fill a tax return in April for last year income to pay (or get refunded) the difference between the withholded amount and the tax one has to pay. France in 2016 has a delay of two years between the realisation of income

3. Not related to tax frequency but at the center of our theoretical results, Varian (1980) emphasizes the insurance property of an income tax and shows in a two-period model that this insurance component leads to a convex optimal tax scheme.

and the payment of the tax liability, which also ask the question of contemporaneity between income realisation and tax payment.

The assessment of the tax and the payment of the tax can be orthogonal in some sense.

2.1 Framework

Let's consider a reference period of $T \in \mathbb{N}$ and sub-periods t of integers ranging from 1 to T. If we want to take year length as the reference period and months as sub-periods, then T=12, and each month is labeled from 1 to 12. [4] We consider y_t income [5] at period $t = 1, ..., T$ with $y_t \geq 0$.
We define G_t the sub-period equivalent tax function where $G_t = \frac{1}{T}G(Ty_t)$;
Sub-period equivalent tax function depends only of the sub-period income, we should note that for equal income at each sub-periods the reference income tax is equal to the sum of the sub-periods income tax, formally :

$$\sum_{1}^{T} \frac{1}{T} G(Ty_t) = G(\sum_{1}^{T} y_t), \forall y_t = k, k \in \mathbb{R} \tag{1}$$

We refer to the passage from $G(\sum_1^T (y_t))$ to $\sum_1^T \frac{1}{T}G(Ty_t)$ tax scheme as the natural increase in the frequency of the tax. In a more general way, an increase in the frequency of the tax would either mean that the tax liability is computed over a tax base with span in a lower length in size, or that the computation of a tax liability evolves at a higher rate, the following analysis will not really disentangle those two aspects.

2.2 Existing systems

There exist several way to make a taxpayer pay for its income tax. We are going to see systems for US and France. These systems are described in a core way, and many aspects will not be described : there exist different tax payment options for the taxpayer, modifications in the amount to pay based on the taxpayers claim are possible, family situation plays a role, etc.
For the following section t represents the months of a given year (either fiscal or calendar), where 1 is the first month of the year and 12 the last. $G()$ represents the income tax [6] of a given country. g_t represents the payment made by the taxpayer for a given month t.

United States A simplified vision would be that income is withholded by the employer based solely on monthly income and monthly situation (family structure), at the end of the year a tax adjustment is proceeded. [7]

4. T = 4 if we consider trimestrial sub-periods relative to annual, 2 for five-year relative to decadal, etc.
5. y_t could be extended to a vector of characteristics such as disabilities, tax-unit composition, etc.
6. But the analysis could be extended to the effective tax function
7. https://www.irs.gov/publications/p505/index.html
https://www.irs.gov/pub/irs-pdf/p505.pdf

$$g_t = \begin{cases} \frac{G(Ty_t)}{T}, & \text{if } t \in [1, 11] \\ G(\sum_1^{12} y_t) - \sum_1^{11} \frac{G(Ty_t)}{T}, & \text{if } x = 12. \end{cases}$$

In fact the tax adjustment is made in April after the taxpayer has filled its tax returns. Thus the final income streams will be different in April since the taxpayer has to pay its withholded tax, plus the difference between last year income tax and the last year actual withholded amount.

$$g_t = \begin{cases} \frac{G(Ty_t)}{T}, & \text{if } t \in [1, 3] \cup [5, 12] \\ G(\sum_1^{12} y_t) - \sum_1^{12} \frac{G(Ty_t)}{T} + \frac{G(Ty_t)}{T}, & \text{if } x = 5. \end{cases}$$

France Current system France current system is based on year - 2 income tax amount, which is paid directly by the fiscal household on a monthly basis. [8] It consist of 10 payments from January to September that will match year - 2 income tax, with

$$g_t = \frac{G(\sum_{-23}^{-12} y_t)}{10}, \text{for } t \in [1, 10] \tag{2}$$

In the case that the amount of year - 1 income tax are higher or lower than last year a tax adjustment is done in November.

$$g_t = \begin{cases} \frac{G(\sum_{-23}^{-12} y_t)}{10}, & \text{for } t \in [1, 10] \\ G(\sum_{-11}^{0} y_t) - \sum_1^{10} \frac{G(\sum_{-23}^{-12} y_t)}{10}, & \text{for } x = 11 \\ 0, & \text{for } x = 12. \end{cases} \tag{3}$$

As a matter of fact there is other fiscal rule concerning the payment of the tax: if the amount of November tax payment is higher than preceding month tax payment, then the amount paid in November is equal to preceding payments, and the balance is paid in December.
For a matter of readability let's define year - 2 tax as a, and year - 1 tax as b.

$$a = G(\sum_{-23}^{-12} y_t)$$

$$b = G(\sum_{-11}^{-0} y_t)$$

$$g_t = \begin{cases} \frac{a}{10}, & \text{for } t \in [1, 10] \\ \begin{cases} b - a, & \text{if } b - a < \frac{a}{10}, \\ \frac{a}{10}, & \text{if } b - a \geq \frac{a}{10}, \end{cases} & \text{for } x = 11 \\ \begin{cases} 0, & \text{if } b - a < 10 \\ b - \left(11 \frac{a}{10}\right), & \text{if } b - a \geq 10 \end{cases} & \text{for } x = 12. \end{cases} \tag{4}$$

8. There also exist a second system called "paiement par tiers provisionnel" that consist of three payment during the year. The taxpayer can choose freely between the two system, but a majority of taxpayers take the one with monthly payment.

Forthcoming French tax withholding reform A tax withholding reform will takes place in France in 2017-2018 and will switch from precedent tax system to one where the employer will pay wages based on an average tax base (as in the UK) derived on year -2 income tax, with a tax adjustment in December on year - 1 tax.

$$g_t = \begin{cases} y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right), & \text{for } t \in [1, 11] \\ G\left(\sum_{-11}^{0} y_t\right) - \sum_1^{11} y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right), & \text{for } t = 12 \end{cases} \tag{5}$$

Actually the scheme is a bit mot complicated since its is year - 2 average rate that is applied from January to July. In august if the amount paid is higher than year-1 income tax, the taxpayer receive a rebate directly on its bank account. If the amount paid is lower, then the taxpayer will have to pay directly to the tax institution one fourth of the difference between year - 2 and year - 1 income tax for next remaining month.

$$g_t = \begin{cases} y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right), & \text{for } t \in [1, 7] \\ \begin{cases} G(\sum_{-11}^{0} y_t) - \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right), & \text{if } \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right) \geq G(\sum_{-11}^{0} y_t) \\ \frac{G(\sum_{-11}^{0} y_t) - \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right)}{4}, & \text{if } \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right) < G(\sum_{-11}^{0} y_t), \end{cases} & \text{for } t \in [8] \\ \begin{cases} \frac{G(\sum_{-11}^{0} y_t) - \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right)}{4}, & \text{if } \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right) < G(\sum_{-11}^{0} y_t) \\ 0, & \text{if } \sum_1^8 y_t \left(\frac{G(\sum_{t=-23}^{-12} y_t)}{\sum_{t=-23}^{-12} y_t} \right) \geq G(\sum_{-11}^{0} y_t), \end{cases}, & \text{for } t \in [9, 12] \end{cases} \tag{6}$$

On those 4 different tax schedules, we see some differences. France has a delay of at most 34 months and at least 10 month between the realisation of income, tax liability and the payment of the taxe, US has a delay of at most 1 year and two month between final tax liability and realisation of income, and at most one month between the tax base and tax payment. Forthcoming French tax system will be based on year minus two incomes to determine the tax rate.

2.3 Tax Frequency Proposals

We will compare different tax systems, to do so we consider a general income tax function $G()$ that is increasing and convex.

We consider that tax-payers have additive/separable utility functions :

$$U = \sum_1^T u_t \, , \tag{7}$$

where U is the reference period global utility function and u_t is the sub-period utility function.

We consider that tax-payers utility functions are increasing and concave in disposable income [9] – here income minus tax.
We do not include time discounting or interest rate in this framework. The reasons of that choice are discussed later.

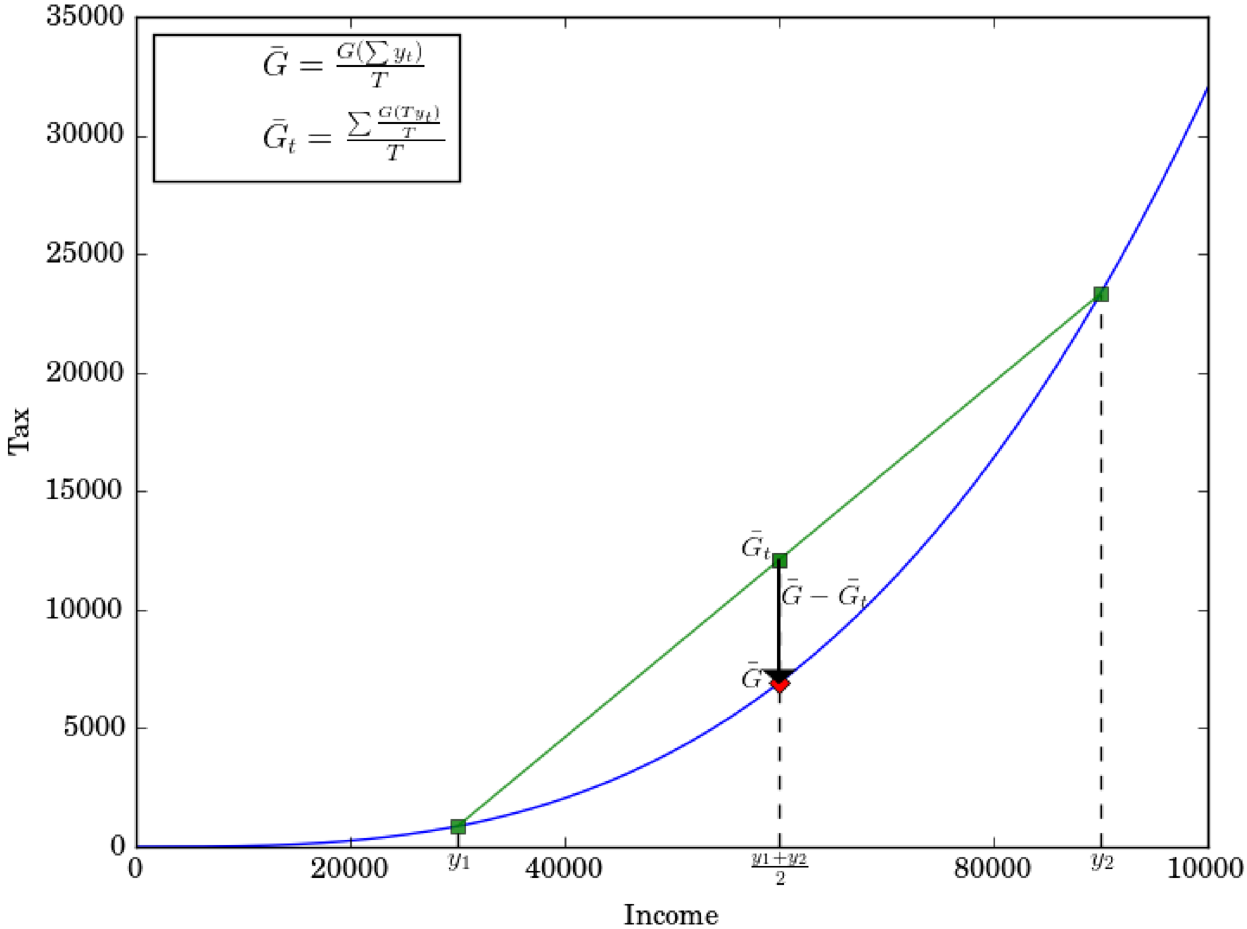
Equal Payment Tax In this system the amount of the tax is based over income of the reference period, $G(\sum_1^T y_t)$. The tax is split equally between all the sub-periods, each split being paid at each sub-periods. If we denote ${}^{RPT}g_t$ the payment made at period t, then :

$$ {}^{RPT}g_t = \frac{G(\sum_1^T y_t)}{T} \quad \text{, for the RPT scheme.} \tag{8} $$

Sub-period Tax Sub-period tax is the tax with the natural increase in frequency $\frac{1}{T}G(Ty_t)$ it leads to a sub-period tax payment ${}^{ST}g_t = \frac{G(Ty_t)}{T}$
From now on let's consider G() is a tax increasing and convex [10] income tax function $G(\sum_1^T(y_t)) \geq 0$, it depends only of the sum of income over the reference period.
A first remark is that, due to convexity, $\sum_1^T \frac{1}{T}G(Ty_t) \geq G(\sum_1^T y_t))$. With a strictly convex tax function those with varying income would have to pay more tax over the reference period with the sub-period equivalent tax than with the tax on the reference period. This fact is illustrated in Figure 1.

9. $\frac{\partial u_t(\tilde{y}_t)}{\partial \tilde{y}_t} \geq 0 \wedge \frac{\partial^2 u_t(\tilde{y}_t)}{\partial \tilde{y}_t^2} \leq 0$
10. $G'() > 0, G''() > 0$

Figure 1: More Tax Paid when Taxed over Subperiods



This graph illustrate the fact known as Jensen's Inequality which states that convex transformation of a mean is less than or equal to the mean applied after convex transformation. [11]
If we take a tax-payer income y_1 and y_2 respective to sub-periods $t = 1$ and $t = 2$, with $T = 2$. We apply a convex tax $\frac{G(Ty_t)}{T}$ to those two incomes, the amount of the tax is represented by the green squares of easting y_1 and y_2 (here respectively 30 k€and 90k€), we see that the average amount of tax with the sub-period tax $\bar{G}_t$ is higher than the reference period tax divided by the number of sub-periods $\bar{G}$, with an average gain for the tax-payer of $\bar{G} - \bar{G}_t$.

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11. The link with Jensen's Inequality is quite straightforward. Jensen's inequality theorem says : If f a convex function, $(x_1, .., x_n)$ a tuple of real numbers belonging to the defined interval of f, and $(\lambda_1, ..., \lambda_1)$ a tuple of positive reals numbers such that :

$$\sum_{i=1}^{n} \lambda_i = 1$$

Concerning the utility of the tax-payer, it's sub-period utility might be lower or higher with the sub-period tax compared to the equal payment tax. Indeed without savings or borrowing between periods, disposable income is $y_t - \frac{G(Ty_t)}{T}$ with the sub-period tax.
Two opposite effects co-exists : the first one is that more tax is paid with the sub-period tax, which reduces overall income, and thus overall utility. But on the other hand the fact that the tax is directly link to the sub-period income is smoothing the disposable income, one pay a small tax when earning little and a big tax when earning a lot. The fact that the utility is concave leads to a higher utility. This mixed effect depends thus of the shape of the tax and utility function, but also on the different income streams and their dispersion over the mean.

Compensated Sub-period Tax In order to equalize payment with the equal payment tax and the sub-period tax, we can compensate a tax-payer by adding to the sub-period tax denominator a parameter λ s.t .

$$\sum_1^T \frac{1}{T+\lambda} G(Ty_t) = G\left(\sum_1^T y_t\right) . \tag{9}$$

We thus have :

$$\lambda = \frac{\sum_1^T G(Ty_t)}{G\left(\sum_1^T y_t\right)} - T.$$

We now have three type of tax schemes : one at a reference period level, one at the sub-period level without compensation, and one at sub-period level with compensation ; which we now on will call respectively : the reference period tax (RPT), the sub-period tax (ST), and sub-period compensated tax (SCT).

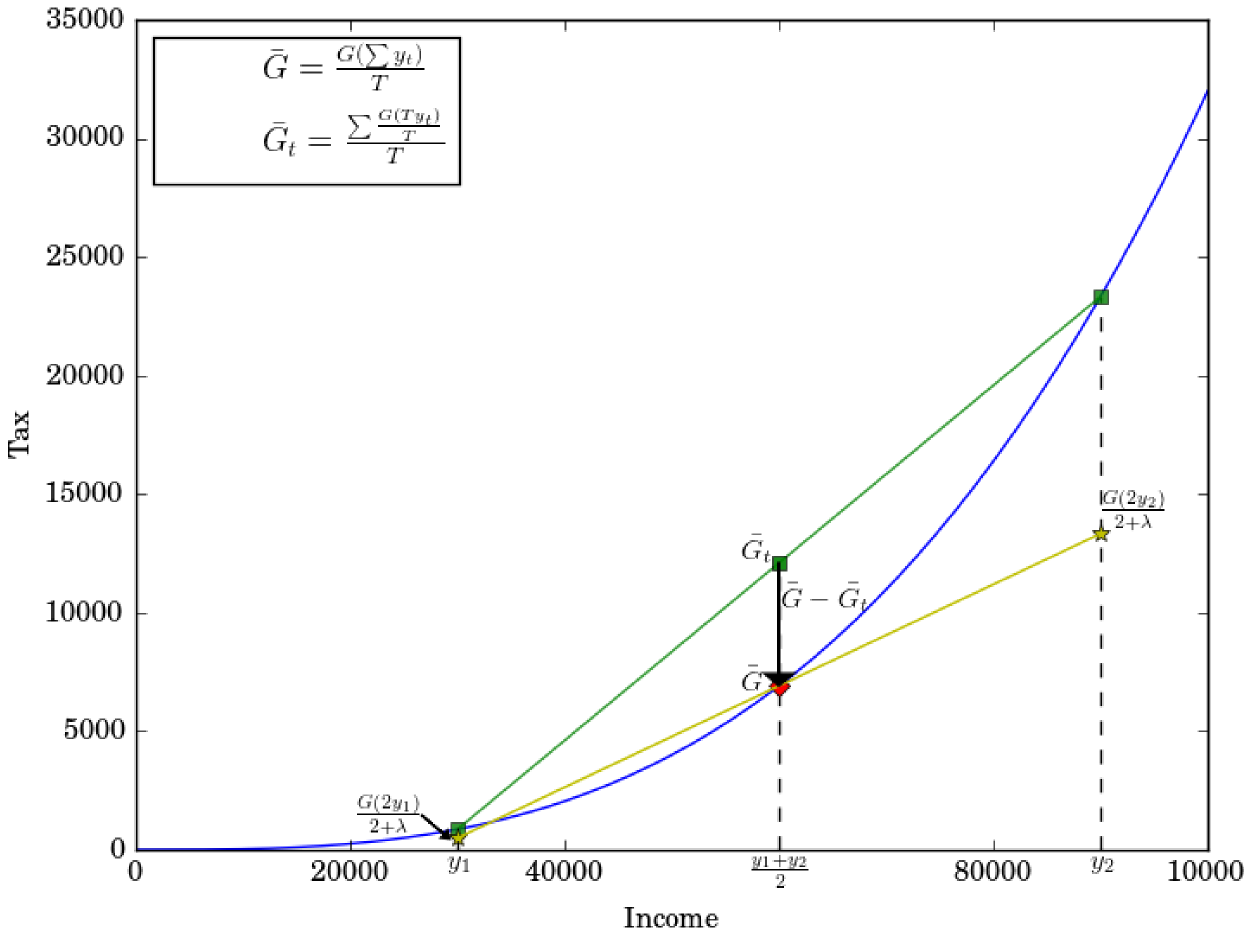
Then :

$$f(\sum_{i=1}^n (\lambda_i x_i)) \leq \sum_{i=1}^n \lambda_i f(x_i)$$

We just need to replace λ_i by $\frac{1}{T}$ and x_i by Ty_t to get back to our case :

$$G(\sum_1^T \frac{1}{T} Ty_t)) \leq \sum_1^T \frac{1}{T} G(Ty_t)$$

Figure 2: Compensated tax



We see on that figure that the compensated income tax represented by (yellow) stars leads to small absolute reduction w.r.t sub-period income tax for the low income sub-period, and a big absolute reduction for the hight income sub-period.

Such a compensated scheme is to some extent equivalent to Vickrey's proposal with no interest rate. Vickrey's proposal in our terminology would be to switch from ST scheme, where sub-periods would be years, to a compensated sub-period tax (even though the compensation is not the same as ours), where the RPT would be the largest possible reference period T (as the lifetime of a tax-payer).
The tax is paid at each sub-period : ${}^{SCT}g_t = \frac{G(Ty_t)}{T+\lambda}$
The following result is concerning rather individuals with no or very small saving, it is thus a result that can also be useful in development economics. It is

also our opinion that it matters much for poor household in rich and developed country. It is common for poor household that the beginning of the month is easier than the end because they have spent most of their wage or their social welfare payments during the month (see Hastings and Washington (2010)).[12]

Theorem. *If we value sub-period utility of disposable income equally over each sub-period and the overall utility is additive (7). That we make the hypothesis that sub-periods incomes are independent of the agents behavior. That we do an income tax reform from a sub-period payment over a reference period income tax (8) to a sub-period income tax with compensation (**??**) (as in (9)) for their overall lost in income, then they will have a higher utility with a SCT than with RPT :*

$$\sum_1^T U_t\left(y_t - \frac{G\left(\sum y_t\right)}{T}\right) \leq \sum_1^T U_t\left(y_t - \frac{G(Ty_t)}{T+\lambda}\right) \tag{10}$$

Proof. In order to prove the theorem we are going to use Karamata's inequality (Karamata, 1932) that generalizes the discrete form of Jensen's inequality. We recall the theorem as it figures on Wikipedia[13] :
Let I be an interval of the real line and let f denote a real-valued convex function defined on I.
If $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ are numbers in I such that $(x_1, ..., x_n)$ majorizes $(y_1, ..., y_n)$, then
$f(x_1) + \cdots + f(x_n) \geq f(y_1) + \cdots + f(y_n)$.
Here majorization means that :
$x_1 + \cdots + x_n = y_1 + \cdots + y_n$,
and, after relabeling the numbers $x_1, ..., x_n$ and $y_1, ..., y_n$, respectively, in decreasing order, i.e.,
$x_1 \geq x_2 \geq \cdots \geq x_n$ and $y_1 \geq y_2 \geq \cdots \geq y_n$
we have
$x_1 + \cdots + x_i \geq y_1 + \cdots + y_i$ for all $i \in \{1, ..., n-1\}$.

Let's get back to our case, let's say $\sum_1^T y_t = T\bar{y}$, if all sub-period incomes are not equal then there exist y_t such that :

$$\begin{aligned} & y_t \leq \bar{y} \\ \iff \quad & Ty_t \leq T\bar{y} \end{aligned}$$

G being a monotonically increasing function, $G(Ty_t) \leq G(T\bar{y})$ and it is easily shown[14] that $y_t - \frac{G(Ty_t)}{T+\lambda} \geq y_t - \frac{G(T\bar{y})}{T}$.

This being true $\forall y_t \leq \bar{y}$ and if $\exists y_{t'} \neq \bar{y} \Rightarrow \exists y_t \leq \bar{y}$. Then if we order y_t in a increasing order with the highest y_{tT} and smallest y_t being labeled y_{t1} we thus

12. it might be arguable, e.g. individuals might have higher utility from money in December than in March.
13. https://en.wikipedia.org/wiki/Karamata%27s_inequality
14. $G(Ty_t) \leq G(T\bar{y}) \iff \frac{G(Ty_t)}{T} \leq \frac{G(T\bar{y})}{T} \iff \frac{G(Ty_t)}{T+\lambda} \leq \frac{G(T\bar{y})}{T}$
$\iff -y_t + \frac{G(Ty_t)}{T+\lambda} \leq -y_t + \frac{G(T\bar{y})}{T} \iff y_t - \frac{G(Ty_t)}{T+\lambda} \geq y_t - \frac{G(T\bar{y})}{T}$

have: [15]

$$y_{tT} - \frac{G(Ty_{tT})}{T+\lambda} \geq ... \geq y_{t1} - \frac{G(Ty_{t1})}{T+\lambda} \text{ and } y_{tT} - \frac{G(T\bar{y})}{T} \geq ... \geq y_{t1} - \frac{G(T\bar{y})}{T} \quad (11)$$

We have [16] :

$$y_{tT} - \frac{G(Ty_{tT})}{T+\lambda} + ... + y_{ti} - \frac{G(Ty_{ti})}{T+\lambda} \geq y_{tT} - \frac{G(\sum_1^T y_t)}{T} + ... + y_{ti} - \frac{G(\sum_1^T y_t)}{T} \qquad \forall i \in \{1, ..., n-1\}$$

Thus $((y_1 - \frac{G(Ty_{t1})}{T+\lambda}), ..., ((y_{12} - \frac{G(Ty_{t12})}{T+\lambda}))$ majorizes $((y_{t1} - \frac{G(\sum_1^T y_t)}{T}), ..., (y_{t12} - \frac{G(\sum_1^T y_t)}{T}))$

The function U being concave thus we have :

$$\sum_1^T U(y_t - \frac{G(\sum y_t)}{T}) \leq \sum_1^T U(y_t - \frac{G(Ty_t)}{T+\lambda})$$

□

The general idea illustrated by Figure 3 is that an increase in the frequency smooth disposable income, the convexity of the tax is leading to higher payment which has a negative impact on utility, but the compensation take out that potential negative effect and lead to only have the benefit of the income smoothing effect.

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15. To the condition that the MTR $G'() \leq \frac{\lambda+T}{T}, since \lambda \geq 0$ then we need at least $G' \geq 1$ for (11) to be false . Thus the theorem is at least valid for non confiscatory tax.

Proof. Let's say that $y_t \geq y_{t'}$ and $y_t - \frac{G(Ty_t)}{T+\lambda} \leq y_{t'} - \frac{G(Ty_{t'})}{T+\lambda}$, namely that disposable income with SCT could be non increasing with income, we would have :

$$\begin{aligned} & y_t - \frac{G(Ty_t)}{T+\lambda} \leq y_{t'} - \frac{G(Ty_{t'})}{T+\lambda} \\ \iff \quad & T + \lambda \leq \frac{G(Ty_t) - G(Ty_{t'})}{y_t - y_{t'}} \end{aligned}$$

Thus by the mean value theorem there must exist a $y \in [y_{t'}, y_t]$ s.t. $\frac{G(Ty_t) - G(Ty_{t'})}{(y_t - y_{t'})} = \frac{\partial G(Ty)}{\partial y} = T\frac{\partial G(y)}{\partial y} \equiv TG'(y)$.

$$\begin{aligned} \iff \quad & T + \lambda \leq TG'(y) \\ \iff \quad & \frac{T+\lambda}{T} \leq G'(y) \end{aligned}$$

Thus for $y_t \geq y_{t'}$ but that $y_t - \frac{G(y_t)}{T+\lambda} \leq y_{t'} - \frac{G(Ty_{t'})}{T+\lambda}$ to be true we need the MTR to be higher than 100 % since $\frac{T+\lambda}{T} \geq 1$ since $\lambda \geq 0$. □

16. Keeping in mind that $\sum_1^T \frac{1}{T+\lambda} G(Ty_t) = G(\sum_1^T \frac{1}{T} Ty_t))$

Figure 3: Utility side

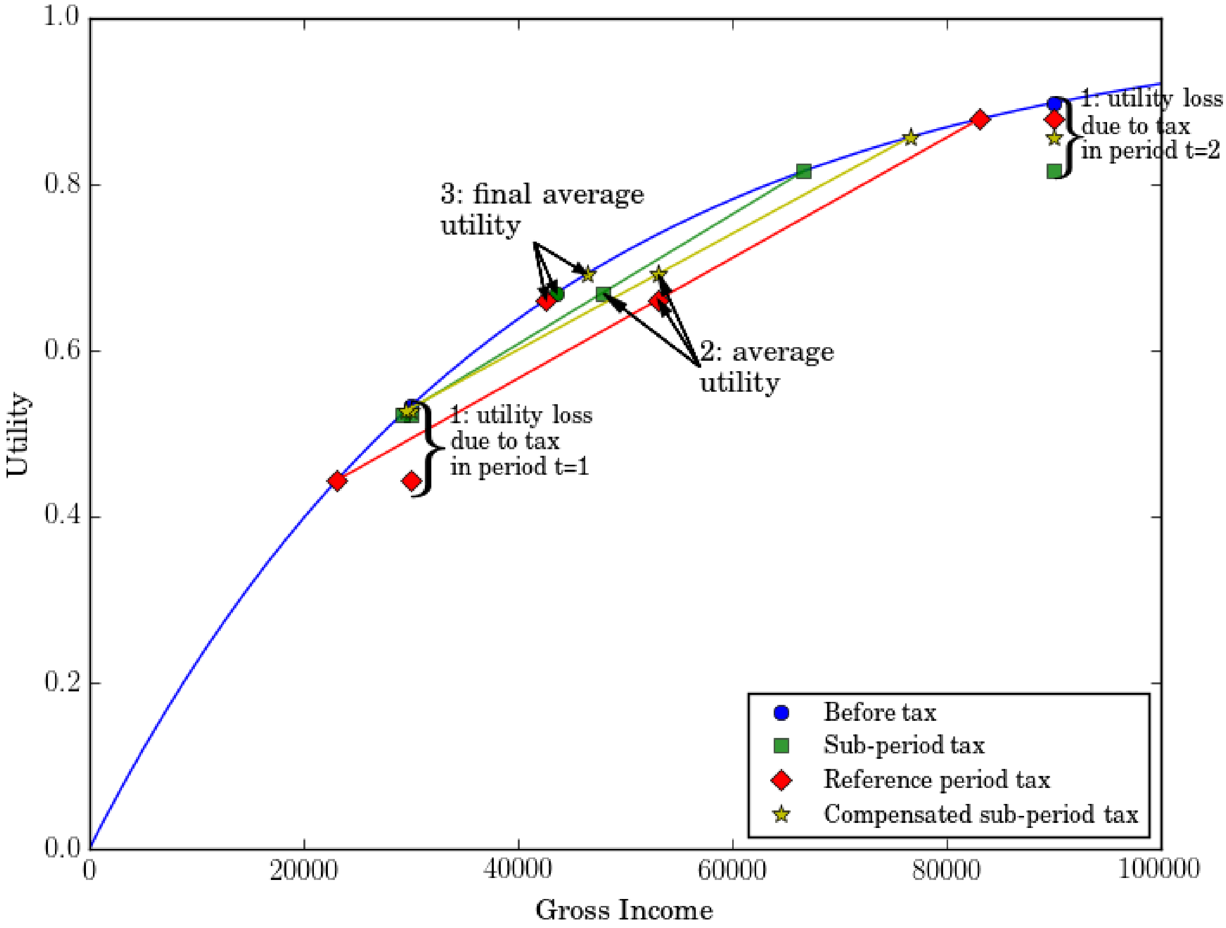


Figure 3 shows the tax-payers utility. The (blue) dots represent the before tax sub-period utility. Let's look at the first step **1.** which represent the utility loss due to tax for each tax system. We see that in period 1 when the tax-payer earns 30k€, the loss in utility for the ST (green squares) and CST (yellow stars) are very minor, while the RPT is imposing a big loss on utility. This is due to the fact that ST will be very low at t=1 (here 860 €), and even lower for the MCT (500 €). For the RPT even if the amount is of 6900, it entails (since the utility is concave, it is steep for low income) the utility. On the opposite, at t=2 while the RPT is still of 6900, the impact of the tax is small because we are on a flatter part of the utility function, however the ST is going to be quite large and have a big impact on utility since the amount of the tax is going to be big for that month (25'000 € of tax), the amount of the SCT is a bit lower (12'000 €) and have still a significant impact but lower than ST.
When we projects step **1.** markers on the graph of the utility function we obtain the disposable income utility, we then take the average of t=1 and t=2 for the three tax systems (step **2.**), then project it on the graph of the utility function. We see the final average utility function (step **3.**). Each of those points gives on the x axis the monthly disposable income equivalent for each tax-system, namely the amount sub-period disposable income equal for period t=1 and period t=2 for the tax-payer to be as well as with that amount over the two sub-periods than with y_1 and y_2 with the corresponding income.

For that specific numeric exemple average utility is of 0.66 with the RPT and it correspond to an individual having disposable income $y_1^d = y_2^d = 42600$ €, while ST leads to an average utility of à.67 and correspond to an disposable income of 43600 € for each sub-period, while the SCT correspond to a 46400 € of constant income for an average utility of 0.69.
In that exemple ST leads to a higher utility than RPT even-though they are quite close. More generally RPT could lead to a higher or lower utility than ST depending of how much "convex" is the tax, and how much "concave" is the utility function : a very curved convex tax will lead to pay a lot more with the ST than with the RPT and would thus leads to have a lower utility with the ST, conversely a very curved concave utility is going to benefit from the income-smoothing property of the ST and thus would lead to have an higher utility with ST. But as the theorem state, the CST will always lead to an higher utility than ST and RPT independently of the curvature of the functions or the sub-period incomes.

Sub-period Cumulated Tax Vickrey's proposal was that each sub-period payment should be equal to the amount one would have paid under the sub-period tax schedule if the total income had been earned in equal amounts. Without interest rate it leads to :

$$^{V}g_t = G\left(T\frac{\sum_{p=0}^{t} y_p}{t}\right) * \frac{t}{T} - \sum_{p=0}^{t-1} {}^{V}g_p \tag{12}$$

Where $^{V}g_t$ is the tax paid for a given sub-period t with the Vickrey's tax scheme. [17] This solution is equivalent it term of global amount over the period T to the SCT or RPT scheme. [18] We can thus implement a Vickrey's tax scheme on a yearly span –instead of a lifetime span–. This is indeed the solutions that seems to have been chosen by the Irish PAYE system [19].
This system might lead to positive transfers to the taxpayer in the case that the overall tax liability is lower than on the previous period. Thus such a cumulated tax could lead to higher utility than the one with the sub-period compensated tax. Indeed if we take a utility function that tends towards infinity in 0, if for a given sub-period income $y_t = 0$ then with SCT the tax will be $\frac{G(0)}{T+\lambda}$, if $G(0) = 0$ then utility will be $-\infty$, while with the Sub-period Cumulated Tax could be $g_t > 0$ which would lead to a higher overall utility. Such gains are conditional to a loss in income, meaning that one that will have a loss in income may be a utility winner w.r.t the SCT scheme, but one that has increasing income over time will be better off with SCT.

General Comments On those 4 tax schemes, the equal amount, the sub-period tax, the sub-period compensated tax-scheme, and the sub-period cumulated tax, only the sub-period tax and the sub-period cumulated tax are implementable. The equal amount and the sub-periods compensated tax needs for the government to foresee incomes before their realisation, which is not possible. The sub-period compensated tax-scheme is possible, but in case of a convex

17. We assume that there exist a period $t = 0$ where $y_0 = 0$

18. $\sum_1^T {}^{v}g_t = G(\sum_1^T y_t) \iff {}^{v}g_T = G\left(T\frac{\sum_{p=1}^{T} y_p}{T}\right)\frac{T}{T} - \sum_{p=1}^{T-1} {}^{v}g_p$
$\iff g_T = G(\sum_1^T y_t) - \sum_1^{T-1} {}^{v}g_t \iff {}^{v}g_T + \sum_1^{T-1} {}^{v}g_t = G(\sum_1^T y_t)$

19. http://www.revenue.ie/en/business/paye/guide/employers-guide-paye-calculation.html

tax scheme, it would penalise those with varying incomes. The cumulated sub-period tax scheme is easily implementable in real life since it only needs to know past income to derive the tax of a given month, but it tends to advantages in utility term those whom has low income at the end of the reference period T over those whom has increasing income during the year ; while all the other tax systems are neutral concerning the ordering of the sub-period streams of income.

Such a framework also ask the question of the length of the sub-period and of the reference period. [20]

Concerning the sub-period tax, if the saving ability of the taxpayer is low, if it is facing a borrowing contraint, and or is not able to foresee it's variation in income, the highest sub-period frequency of the tax seams desirable. The drawbacks of such an high frequency of taxation would be the potential administrative cost of such frequent transfert of information, computation, and financial transfert, but such costs are with computer automation negligible. It also implies a constant stream of income for the government whom would not have to borrowing between larger payment period. Thus we find no counter argument for the smallest sub-period possible, which is probably the payroll period (monthly in most country).

There is also the question of the length of the reference period T, considering only tax schemes where the amount of tax over a reference period T is the same. The length of the reference period is subject to opposite normative views. Let's state that the optimal tax scheme is a convex one (i.e. weakly increasing MTRs). We showed clearly that the length of the reference period is only subject to change in amount to the condition that income are varying. [21] But for those whom their income are varying we saw that they would be looser over multiple reference periods. [22].

Vickrey argued that one should pay the same tax for same income over life cycle ; many countries aimed at taxing more citizen with a higher ability to pay. If we define one's ability to pay based on it's average ability to pay over life cycle, then Vickrey's view should be followed, and thus reference period T should be the life cycle. But other could claim that one's ability to pay may vary over life, for reasons that one may or may not be held responsible. [23] Top sports players (or superstars) have extraordinary performance and thus a high productivity for a fraction of their life, seasonal workers (such as ski instructors) may have high a low and high period of productivity every year, writers may take a decade to write a book but income from royalties are highly concentrated on the following month of the publication of the book. Productivity could also change due to exogenous cause, increase in competition, technological change, market demand, and overall all variations related to skill premium. Unemployment, is also a strong element in income variation.

20. clearly fort the sub-period tax scheme, the reference period is not at stake, since the tax can be defined only relatively to the sub-period, and that each sub-period can bee viewed as the a reference period.

21. Such variation should be thought in real terms with the nominal sub-period tax being adapted at each sub-period in real terms.

22. c.f ST

23. French constitutional corpus for instance states explicitly that the tax should be paid on the basis of one's ability to contribute : "Art. 13. DDHC Pour l'entretien de la force publique, et pour les dépenses d'administration, une contribution commune est indispensable : elle doit être également répartie entre tous les citoyens, en raison de leurs facultés."

We question weather those variations in income should penalise the taxpayer or not. Vickrey's point of view is that *"same income, same tax over life cycle"*, which advocates for a T that is as large as possible. This embodies a *time liberal* view, meaning that an individual can choose freely to allocate its effort over life cycle without being penalised. A taxpayer could choose to work for 30 years, 60 hours a week ; or 45 years 40 hours a week without having a different tax amount to pay. It allows seasonal workers, or people with irregular incomes to be treated equally as regular employee. Vickrey's tax scheme with a T being the life cycle of an individual, is providing a solution for such a goal, it also gives incentives to work since tax-payer face lower marginal tax rate.
On the other hand we can take a view à la Derek Parfit : *the difference between different persons at the same time is more like the difference between the same person at different times.*, and that difference in income over time should be taken as difference in ability to contribute over each period of time. A sport superstar who is able to earn a lot of money over a period of 10 year should pay more taxes when it's ability to contribute is high, and less when it's ability to contribute is low. It is very in line with the static Mirrlees optimal tax model with the efficiency equity tradeoff. Vickrey's tax scheme looks at equity over life cycle, but if Derek Parfit view is embodied, it sacrifices equity over efficiency.

Behavior Such different tax schemes applied could imply some behaviour such as saving, modification in effort, which can be both impacted by time discounting, and or interest rate.
We choose to do not take into account of savings in our analysis. Indeed if agents foresee their future stream of incomes, and has no saving or borrowing contraint, the only salient notion for them is the amount of tax, which is a problem of social preferences. Application to empirics might not suffer that much from that choice as it will be discussed later. Time discounting is also not included : time discounting preferences is the expression that over a whole life cycle people prefers to consume goods when they are young more than when they are old, or that there is a risk of death from a period to an other, etc. Appart from that it can be the result of a cognitive bias that makes prefer nearer consumption in time w.r.t. further one. The increase in frequency of the tax would partly solve those issue, and could be seen as some kind of nudging. Moreover time discounting would weaken the Pareto improving result as the gains would now depend also of that parameter and of the order of the realisation of income (low income at the beginning of the reference period for high later would imply makes one better off, and vice versa). It can also easily be included in a simulation of the reform.

3 Data & Tax function

3.1 Statistics

France doesn't have to our knowledge available database that allows for a tax household's monitoring of monthly income. We dodge that difficulty by inferring monthly income from French labor survey matched with tax returns. [24]
We have individuals declared income by categories over the year. Work related categories are labor income, unemployment income, and retirement income. We have for each individual its monthly situation (employed, unemployed, retired, student, inactive).
In order to infer a monthly income for each individual, we split equally tax returns income into months for which an individual was at corresponding category. Employment income are attributed to months where one was declared employed, unemployment income for months where one was unemployed, and retirement income for the months where one was retired. If an individual declares for a given month that she was a student or inactive, the inferred income is of zero euro.
Other incomes (capital incomes, etc) are split equally over the year. Those attempt to deal between monthly income and annual income can be to some extent compared to those practice in British Household Panel Survey (see (Jenkins et al., 2000, p. 4-5)).
We should note that it is a very weak way of inferring monthly income and it is not likely to represent accurately households income streams. Moreover the micro-simulation has not been calibrated accurately. For those reasons, statistics and result of the simulations should be taken as a very rough estimate of what could be an increase in the frequency of the tax than a precise and

24. The official name of the survey is *enquête sur les revenus fiscaux et sociaux* (ERFS)

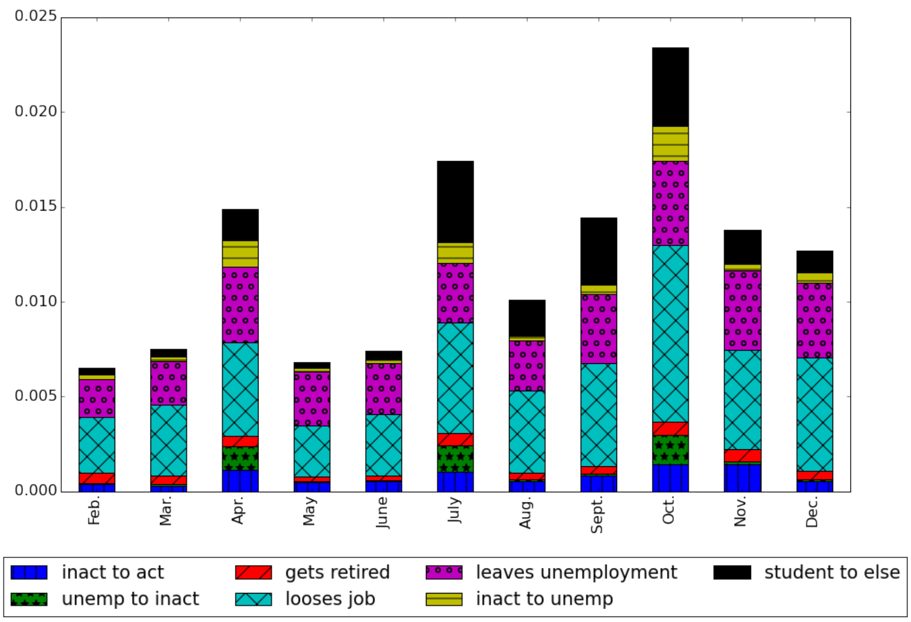
Table 1: Transitions by Months

value	sample size	student to else	inactive to active	unemployed to inactive	inactive to unemployed	get retired	loose job	unemployed to employed	sum
variable									
month2	48175	16	18	4	12	26	142	96	314
month3	48128	19	15	5	11	20	179	112	361
month4	47778	78	54	59	68	27	235	190	711
month5	48160	15	23	4	9	11	129	138	329
month6	48132	22	27	1	10	12	156	129	357
month7	47658	205	49	68	52	29	279	149	831
month8	48003	92	26	4	13	17	208	126	486
month9	47800	169	41	4	22	20	258	175	689
month10	47381	196	69	72	87	34	440	210	1108
month11	47830	85	68	9	16	30	250	201	659
month12	47882	54	25	6	27	20	286	189	607

This table show the non weighted transitions we obtained for each month.

realistic simulation of such a reform for France. However, the very precise description of the tax-benefit system in *OpenFisca* rise some insightful effects (such as thresholds effects, effects due to tax-breaks instruments) that are likely to be present in most tax-benefit systems but that economist may not notice in simple modeling. Datas estimations have some flaws, but gives, we believe, good order of magnitude of the different estimations and statistics we provide. In following sections, except where specified the observations are weighted such that it represents french population in 2009.
Technical details concerning the method are in the appendix.

Figure 4: Aggregate Monthly Transitions over the Year



Income Distribution and Variations Over Months Comment : a bit fuzzy due to declaration bias, figure will be corrected in the future to avoid this.

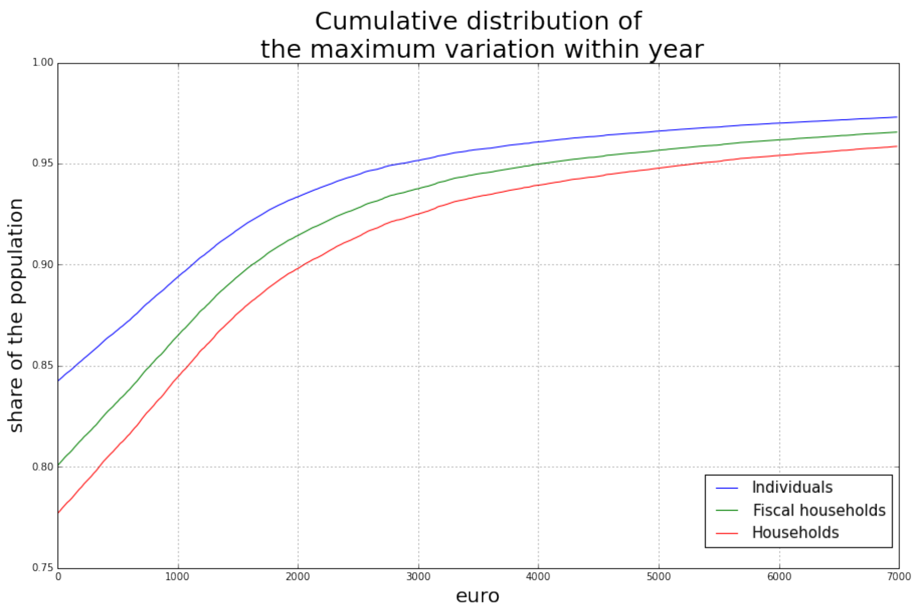
Figure 4 shows the declared monthly transitions by individuals over the year 2009, we see that approximately 1% of the sample changes its activity every month. From those 1%, 52% loses their job [25], 44% goes from no job to a job [26], 4% gets retired.

Table 2: Share of the Type of Transitions in the Sample

Job loss	Job taking	Retirement
52%	44%	4%

Figure 5 represent the cumulative distribution function of individuals maximum variation of income within the year (the difference between lowest and highest income. With the assumptions we made 89 % have a constant income over the year, and 11 % of individuals have a variation of income within the year. [27]

Figure 5: Infra-annual Variation of Income.

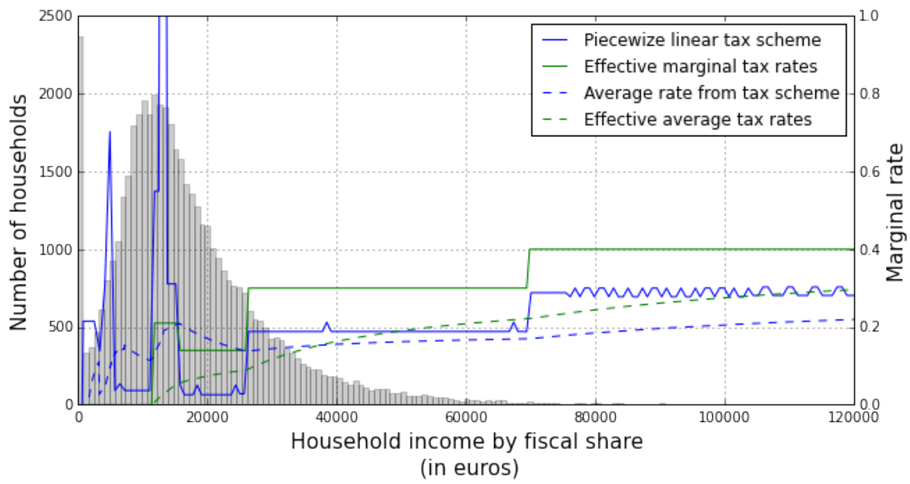


We can see that a significative share of individuals face a variation in income during the year. The French tax system being piecewise linear, a change in income doesn't necessarily mean a change in the tax amount between a monthly and annual frequency since a fiscal household would have to change of tax bracket

25. Retirement excluded
26. (sum a leaving unemployment, leaving inactivity for a job, and leaving student status for a job).
27. Those statements do not takes into account rise in salary or other type of variations except from those due to a change within unemployment, inactivity, retirement and employment.

to face a difference in the tax amount. Joint taxation also takes its part on that dimension since it is the average of incomes that are applied to the tax scheme.[28] Figure 7 shows that most population concerned by income variation during the month is at the bottom of the distribution, where the income tax scheme is concave. Those populations are on average negative savers as showed by Figure 8, thus marginal propensity to consume is quite high for those populations. We thus have most variation and poor savings that concerns the same population. This population also faces the highest number of potential tax backets, and thus potential effects of income variation (that are present only if one change of tax backet). As we can see on Figure 12, on the 10'000 euros to 30'000 euros there is 4 tax brackets where there is only 2 from 30'000 to 120'000 euros. The high number of means-tested benefits, the introduction of benefits to this analysis would make the number of tax-brackets to soar at the very bottom of the distribution as we can see on Figure 12 effective MTRs (which is for single individuals), the inclusion of family support benefits, local taxation, "social tariffs" for telephone and electricity utility services, Universal Health Care Cover, etc would increase much those MTR at the bottom of the distribution.[29]

Figure 6: 2009 French Income Tax, and Individuals Income Distribution.



28. More precision on the joint taxation system in France in appendix
29. See Anne and L'Horty (2012) for precision about diffent staking of benefits (monetary or in kind) depending of administrative level

Figure 7: Share of more than 20% variation of income per decile.

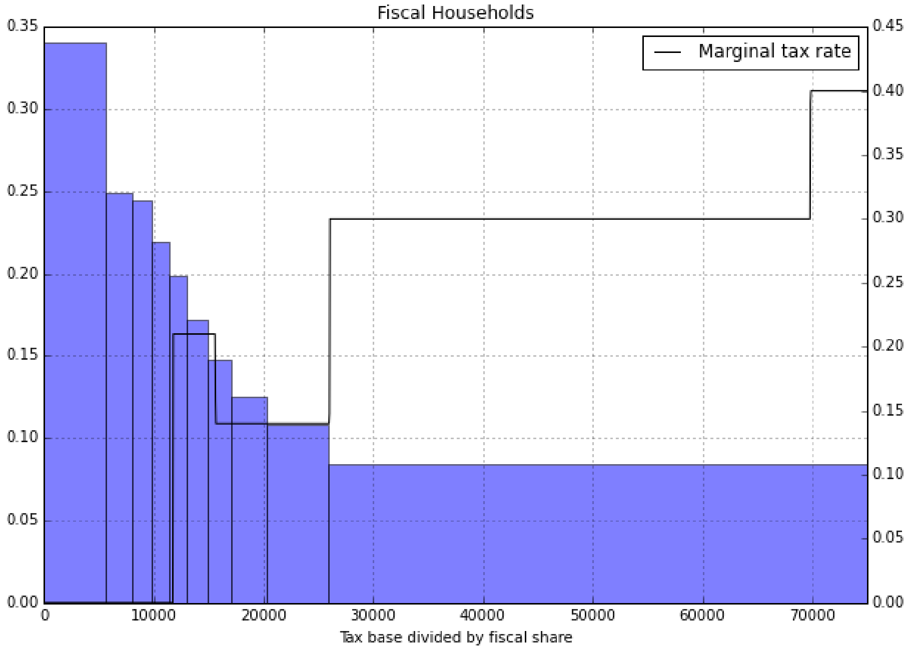


Figure 7 shows the share of each decile of normalized tax base that have more than 20% of income variation during the year. [30] The MTR represents the MTR of a tax-unit composed by one individual. The 22.5 % MTR is due to tax break called *"la décote"* and that –similarly to EITC– creates diminishing MTRs at the beginning of the tax-scheme.

We see that 90 % of the population is below the 30% marginal tax bracket. If we assume that those tax-units doesn't move drastically their month-to-month income (from one tax bracket to an other), we see that most tax-payers in the last decile will likely face a flat-tax scheme. But the other 90% of the population doesn't face a convex tax scheme: they might pay less tax with a monthly payment

30. Here by normalized tax base we mean the amount that pass through the piecewise linear tax function. Due to joint taxation, it doesn't correspond to the amount earned by a fiscal entity, but reflects the MTR a fiscal household is subject to. Joint taxation in France consist of a fiscal shares that is attributed to a fiscal household depending of its composition, the the tax is calculated such tax $G(income, fiscal\ shares) = P(\frac{income}{fiscal\ shares}) \times fiscal\ shares$ where G is the final income tax, and P the piecewise linear tax scheme. Thus $\frac{income}{fiscal\ shares}$ gives the MTR the tax unit is concerned by. Fiscal gain of fiscal shares due to children are nevertheless ceiled to an absolute amount – of 4602 euros per fiscal share in 2009 –, thus some of the household may face a higher MTR than the one corresponding to the $\frac{income}{fiscal\ shares}$ tax bracket, but since those ceilings can be attained only by last decile earners, it is not likely to change much the graph.

Figure 8: Mean saving rate by decile.

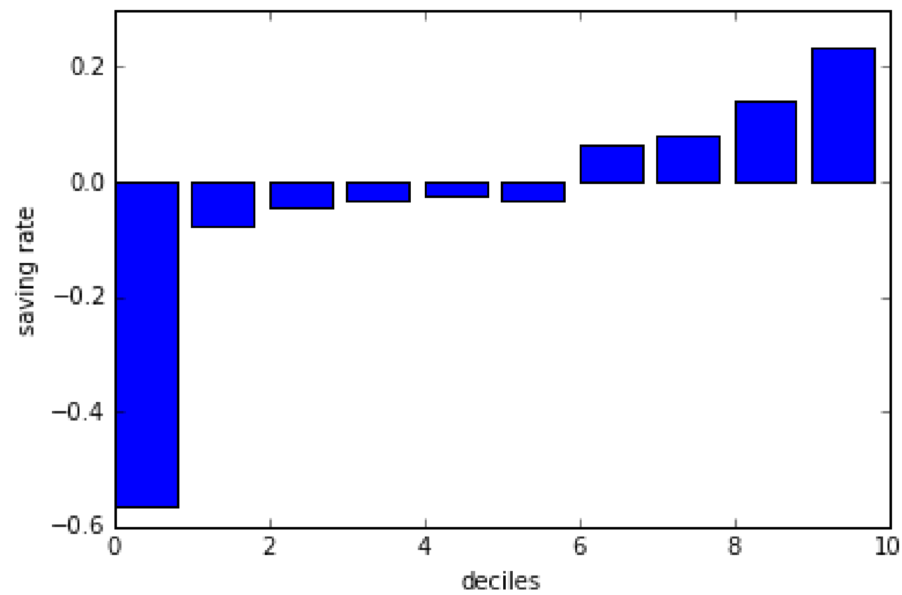
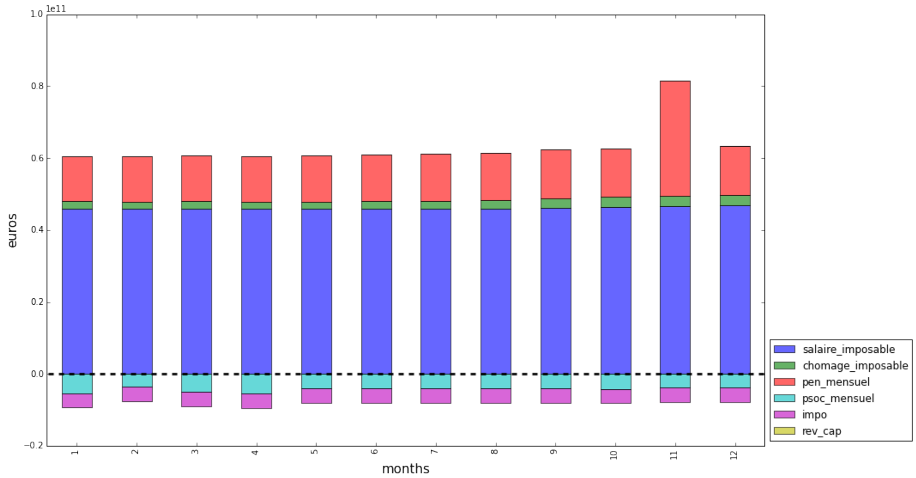


Figure 9: Aggregate Disposable Income by Months.



[Figure 9](#) shows aggregate income for each month, that is of a bit more than 60 billions euros for each month, to the notable exception of November that is due to a higher number of declare retired persons in the month of November in the 2009 french labor survey.

3.2 Tax functions and frequency

In the theoretical section, the general shape of G was very general and stated to be convex for the welfare gains results. Obviously the precise shape of the tax function would be of primary importance for the matter discussed, weather in terms of gain or loss, and in a more significant way the size of the gains. G could be seen as a specific tax such as income tax or as the effective tax (i.e the sum of all taxes and benefit one is entitled). Eventough effective taxation is the salient point for public economist to work on, since some taxes are not based on the same temporality it could lead to perverse effects w.r.t variation in income. [31]

Flat tax A flat tax would imply a constant MTR over the whole income distribution. We an easily show that change in frequency would not impact the global amount of tax :

$$\sum_{1}^{T} \frac{1}{T} G\left(T y_t\right) = G\left(\sum_{1}^{T} y_t\right)$$

Moreover we can state that varying income and a linear tax function are the only two cases where the amount of tax doesn't vary (for any income streams) between reference period tax and a sub-period tax. Flat tax schemes are thus *the only possible class of tax schemes that are frequency neutral.*
An increase in frequency for a flat tax schemes would still benefit to those that have varying incomes since they will benefit from the smoothing effet of the tax without changes in overall disposable income.
We can also note that with a linear $G()$ sub-period tax, compensated period tax, and cumulative tax are strictly equivalent.

Shape of the tax and frequency A convex tax leads to higher payment of the tax when the frequency is increased. And it implies under our assumptions that there is some possibilities to increase one's welfare by increasing the frequency of the tax. But empirically we often observe concave parts in the tax scheme. Concave parts would lead to a higher disposable income for those who have varying income, it can thus be a practical solution to compensate the beggining of the distribution for the variations that they can't be hold responsable. But it would prevent from having a progressive tax scheme at the beginning of the distribution. There is a potential tradeoff for the public planner between equity concerns of taking more to those earning more, and making those subjects to variation to pay less tax even-tough it entails the insurance component of a convex tax as pointed out in Varian (1980).

If we compensate with the SCT the overall effect is not clear anymore since compensation will have a negative effect on disposable income (λ would be

31. As it is the case with housing benefits and RSA in France : housing benefit are conditional that yearly income two calendar year before the claim to the benefit to be low enough, if that condition is fullfiled, monthly income of the month preceding the demand must be low enough. RSA is based on three preceding month. We see clearly that the static effective MTR could lead to some false conclusions of the impact of its shape.

negative), thus the gain in increasing the frequency with a concave tax scheme would depend of the shape of the utility function, shape of the tax function, and incomes : a "very" concave tax would increase taxes in low earning sub-period which can lead to loss in utility if the utility function is very steep near those small earnings sub-periods net income ; conversely the smoothing effect would still be present and may lead to a higher utility.
Reflechir Vickrey.

Piecewise linear tax scheme Most countries (to the notable exception of Germany) has piecewise linear income tax scheme.
A piecewize linear tax scheme is defined as a sequence of (usually) increasing marginal tax rate, [32]:

$$0 < m_0 < m_1 < ... < m_p$$

and a sequence of thresholds specifying the bands of taxable income to which the respective rates apply.

$$0 = \beta_0 < \beta_1 < ... < \beta_p$$

If $y \in [\beta_k, \beta_{k+1}]$, then the tax liability is :

$$s(y) = \sum_{j \leq k-1} m_j(\beta_{j+1} - \beta_j) + m_k(y - \beta_j)$$

It is clear that if one's income stays between two tax brackets, the taxpayer is comparable to one facing a flat tax, and all the characteristics concerned by the above paragraph still holds. So with a piecewise linear tax scheme, one would need to have large enough variation in income to be concerned by a change in tax frequency concerning the amount paid.

France tax system France has an annual income tax that most taxpayers pays on a monthly basis (one tenth of the tax based on last year income for most fiscal household). [33] It consist of a tax applied to a fiscal household. A fiscal household is an entity based on family situation : married couples or under civil union [34] will constitute one unit, single constitute one fiscal household and couples without civil union will form two fiscal households.
France Income tax consist of a piecewise linear tax scheme composed in 2009 of 5 tax brackets.

32. as defined in P.J. Lambert *The distribution and redistribution of income, third edition* P.181

33. In fact, there is several system where the taxpayer can choose between a three times payment of the tax (in February, May, and September), or a monthly payment of the tax, or a monthly payment of the tax that consist of 10 payment from January to October that correspond to one tenth of last year due tax. If the due income tax is greater for one year than the preceding year, additional payment are done in November and December. More than two third of the households choose that later option.

34. Pacte civil de solidarité (Pacs)

Income tax schedule	
from 0 to 6011 euros	0%
from 6011 to 11991 euros	5,5%
from 11991 to 26631 euros	14%
from 26631 to 71397 euros	30%
from 71397 to 151200 euros	41%
beyond 151200 euros	45%

thus in the formalised version of the piecewise linear tax scheme
$\vec{m} = [6011, 11991, 26631, 71397, 151200]$ and $\vec{\beta} = [0, 5.5, 14, 30, 41, 45]$

To compute tax liability, for a single without kid we just have to apply taxable income to the tax scheme.

$$\text{Final tax} = G(\text{taxable income}) = \sum_{j \leq k-1} m_j(\beta_{j+1} - \beta_j) + m_k(y - \beta_j)$$

Joint taxation consist of a system called *fiscal shares*[35]. A certain number of fiscal shares are attributed to each fiscal household based on its composition : 1 share for a single, two shares for a couple under civil union. Then each dependent gives additional shares : 0.5 for each two first child, 1 share for each child after the second one. [36] Then we divide taxable income by the number of fiscal shares, we apply the piecewise scheme, then we multiply the amount by the number of fiscal share :

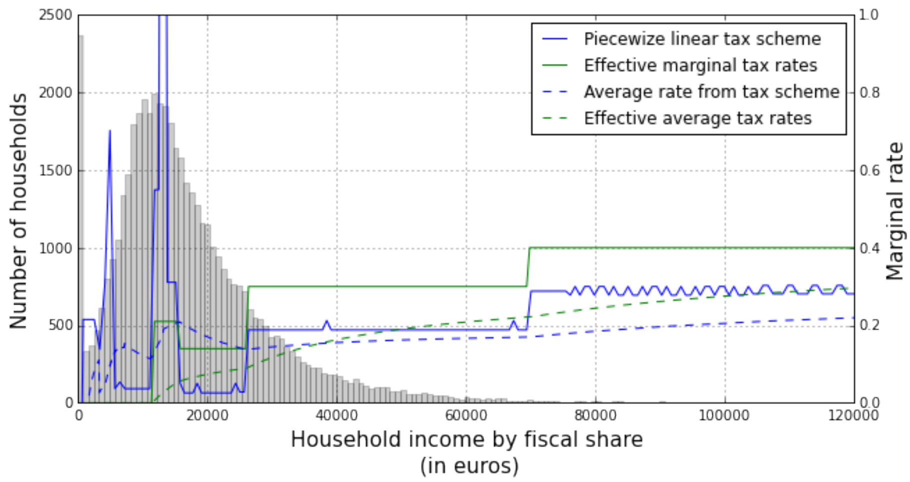
$$\begin{aligned} \text{Final tax} &= G\left(\frac{\text{taxable income}}{\text{fiscal shares}}\right) \times \text{fiscal shares} \\ &= \sum_{j \leq k-1} m_j(\beta_{j+1} - \beta_j) + m_k\left(\frac{y}{\text{fiscal shares}} - \beta_j\right) \times \text{fiscal shares} \end{aligned}$$

Their exist also a mechanism called *la décote* that is a tax break for household paying small amount of tax, that mechanism double the marginal tax rates in the two first brackets. It doesn't take family situation appart from being in couple or not into account.

35. *Parts fiscales* in French

36. There exists a certain number of exceptions and special case, single parents benefit from an additional half fiscal share, there exist specific rules for disable people, etc.

Figure 10: 2009 French Income Tax, and Individuals Income Distribution.



France has also many benefits that mainly concerns households not subject to income tax, and that have a not smooth and non convex shape of the effective MTR.

External Validity France has thus many benefits that imply non-convex effective MTR, and income tax that embodied a tax break increasing the MTR at the beginning of the income tax. It is the case for most OECD countries. For instance US has EITC that increase marginal tax rate at the beginning of the piecewise tax scheme. UK has also working tax credit and income support that gives decreasing marginal tax rate at the beginning of the distribution of income. Both UK, and US also have means tested benefits of which phase out implies high effective MTR, and diminishing MTR. Thus as in France, households in UK and US are likely to face both locally concave or convex effective tax schedule. We believe that the conclusions of our simulations for France could be extended to many EOCD countries.

Figure 11: UK scheme from Brewer et al. (2010) .

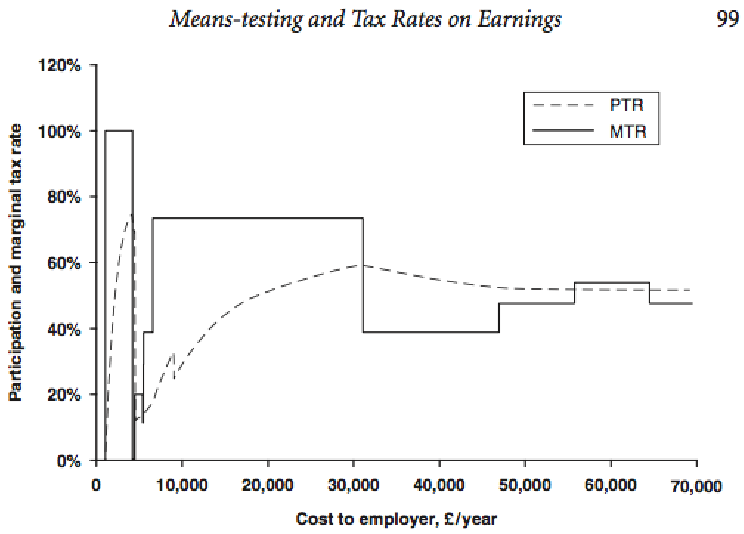
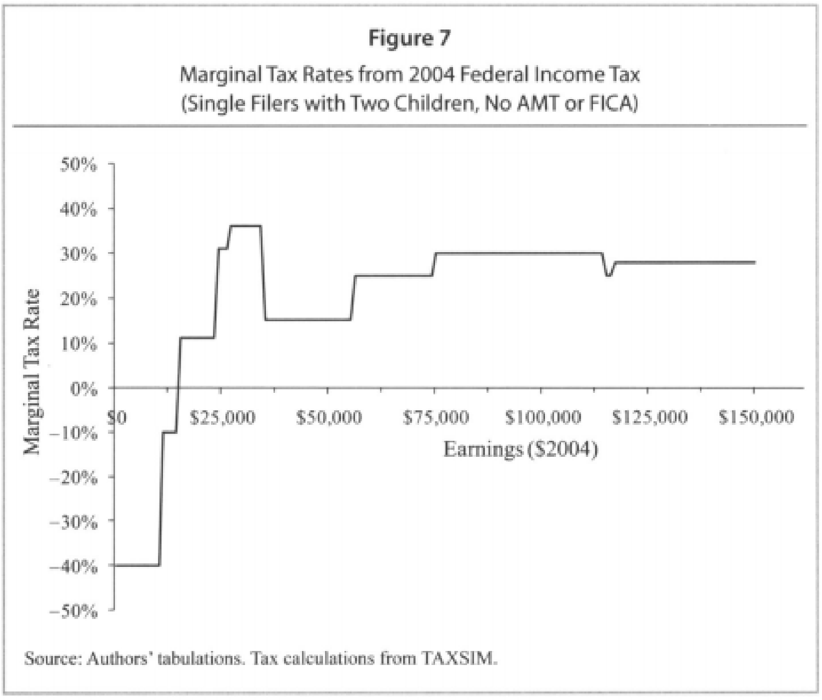


Figure 12: 2004 US tax scheme from Eissa and Hoynes (2008)



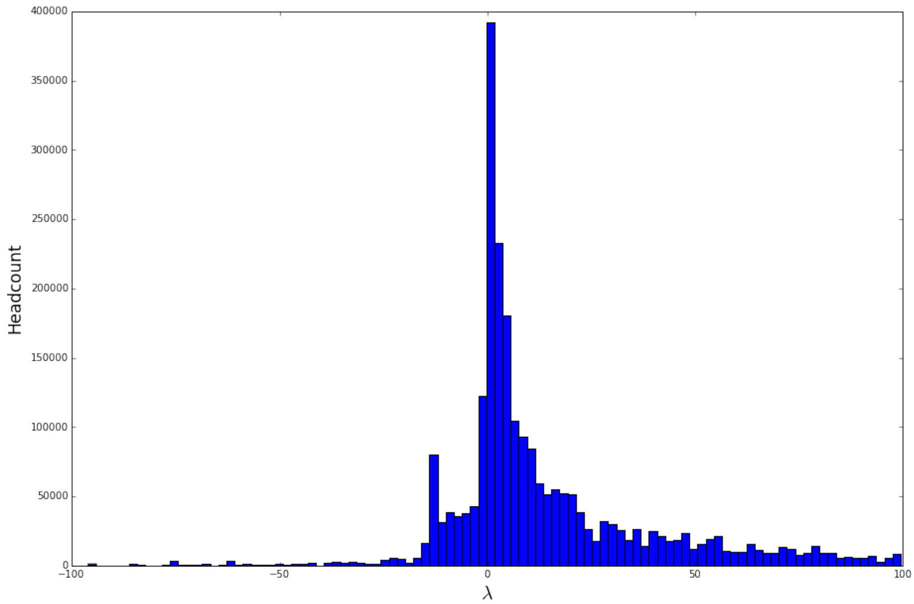
4 Simulations

Simulation Hypothesis We use the microsimulation software *OpenFisca* to simulate a increase in the frequency of the tax from an annual to a monthly one. We did that by changing all the tax formulas by making them working on a monthly period instead of an annual on, multiplying all the input by 12, and dividing the output by 12. Details and the code of the reform are available on GitHub.

Without compensation given the complete tax system (that has to some extent many complexities that will not be described here), the simulated income tax in of 48 billions euros for 2009, when put on a monthly basis, the amount goes to 54 billions. To put the income on a monthly basis would lead to an increase of 6 billion euros in the aggregate income tax.
We calculate the λ compensation term for each household. 50% of the sample has a λ equal to 0, 90 % is very close to zero ($abs(\lambda) < 0.001$). The income tax being not convex at the bottom of the distribution. We represent the distribution of λ on Figure 13, λs equal to zero are taken out for a matter of scale. We see on Figure 13 that a big part of the λ are negative. This is due to the non

convex part at the beginning of the tax scheme. [37]
We also see a mass at $\lambda = -12$, it is due to people that have an income tax on a monthly basis that is equal to 0 but non null on a monthly basis. This is mostly due to payment thresholds : if the income tax is below 62 euros on an annual basis (5 euros on a monthly basis), the due income tax is equal to 0. If for one month the due income tax is over 5 euros on a monthly basis, but below 62 with the annual tax scheme, then λ is equal to -12.

Figure 13: λ Distribution
(with $\lambda = 0$ taken out)



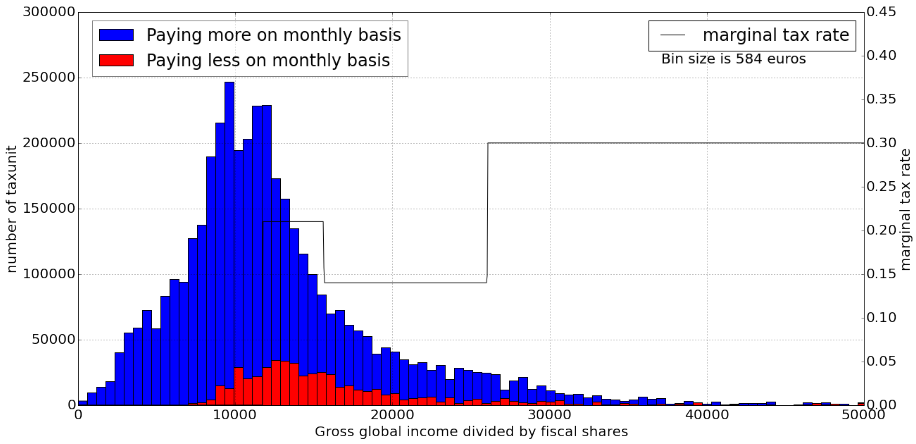
One should note that the French income tax being piecewise linear, having income that vary is not sufficient for a change in the income tax. One would need to change tax bracket in order to change the income tax with a monthly basis, otherwise it would stay on a linear tax bracket (which doesn't make any difference) . French tax schedule having quite large and few brackets, one would need big variation in income in order to observe a significant difference. It is even more true that France has a joint taxation (which double the length of the tax brackets).
To summarize that section, we could say that a significative part (10 %) of the population would face a variation of their income tax if it is put on a monthly basis, this is expressed by the fact that their $\lambda \neq 0$. Our tax frequency theorem state that λ should be greater than 0 for a convex tax scheme ($\equiv$ increasing marginal tax rate). If the tax is non convex, λ may be negative. [38]
In that case two effects of an increase in frequency take place :

37. From a mechanism called *la décote* in France but that could be –in terms of non convexity– compared to EITC for the US
38. and will be if varying income are present on a concave part of the tax function

1) one with varying income would pay less tax on a higher frequency [39] , and will thus have a positive effect on utility when not compensated.
2) its disposable income over periods will be smoothen, and will have a positive effect on utility.

Thus without compensation concavity of the tax will lead the increase in frequency to be pareto improving.
If compensated, the result is thus ambiguous between gains and loss, and will depend of *"how much concave"* is the utility function and *"how much concave is the tax"*[40].
[41]

Figure 14: Winner and Losers with Monthly Tax.



Winner/Losers. Figure 14 shows monetary winners and losers fiscal households. In order to make them comparable we divided the taxable income by the number of fiscal share (such that they are normalized to face the same tax schedule). We see that loosers (in blue – or light grey) are more numerous than losers (in red– or black). As expected we see that the taxpayers that pays less on a monthly basis are the one located near the concave part of the tax scheme: those whom has some monthly income in the 22.5% tax bracket.

Welfare Analysis. We are going to plug the monthly disposable income into an H.P Young's equal sacrifice utility function [42] based on yearly income

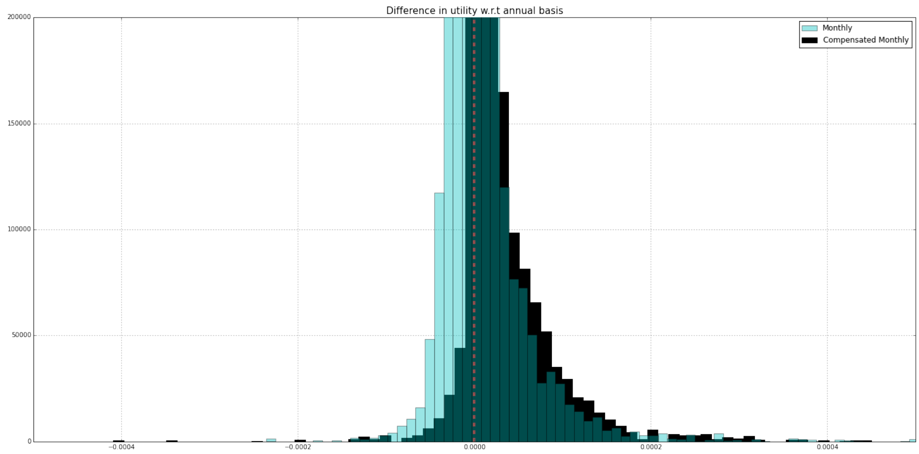
39. Jensen's inequality
40. the more concave the utility function, the bigger are the income smoothing gains. The more concave the tax the bigger the premium due to varying income, and thus the bigger the compensated tax negative impact on compensated income.
41. when talking about compensation each month rather than end of the year tax rebate, we can cite this: Increasing tax frequency could allow for intra-year change in the tax schedule, a decrease in the amount of the tax would boost demand. Such situation has been empirically studied by Shapiro and Slemrod (1993, 2003, 2009) that study the impact on aggregate demand of the US tax-refund.
42. Young (1990)

$U(\vec{y}) = -\left(\sum_1^{12} y_t\right)^{-0.89}$. [43]
But in order to assess monthly utility we are going to apply the function to a monthly basis :

$$u_t(y_t) = \frac{-\left(12y_t\right)^{-0.89}}{12} \tag{13}$$

Mean utility is of -0.00447867, -0.00447854, -0.00447669 for respectively the annual basis tax, the monthly tax, and the monthly basis compensated tax. Utilitarian gains of an increase in frequency is of $13 * 10^{-7}$, and gain from an increase with compensation is of $198 * 10^{-7}$.

Figure 15



43. Utility function is taken as given from an Alain Trannoy previous work on french income tax.

Figure 16

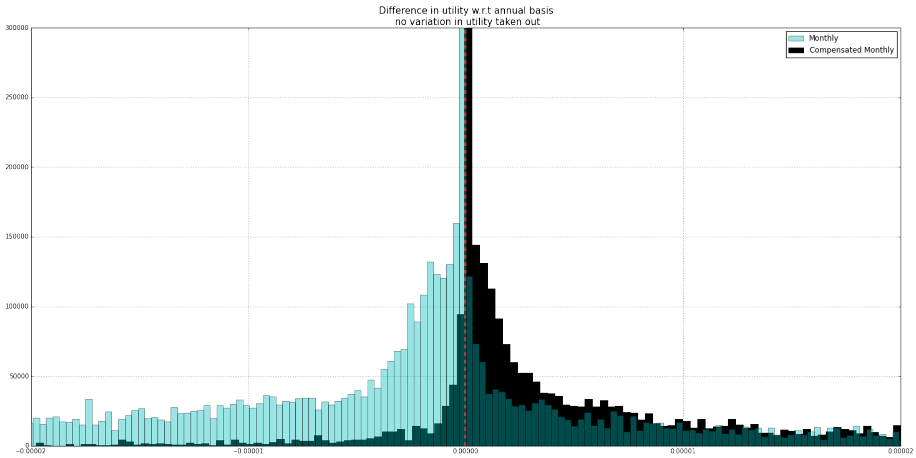
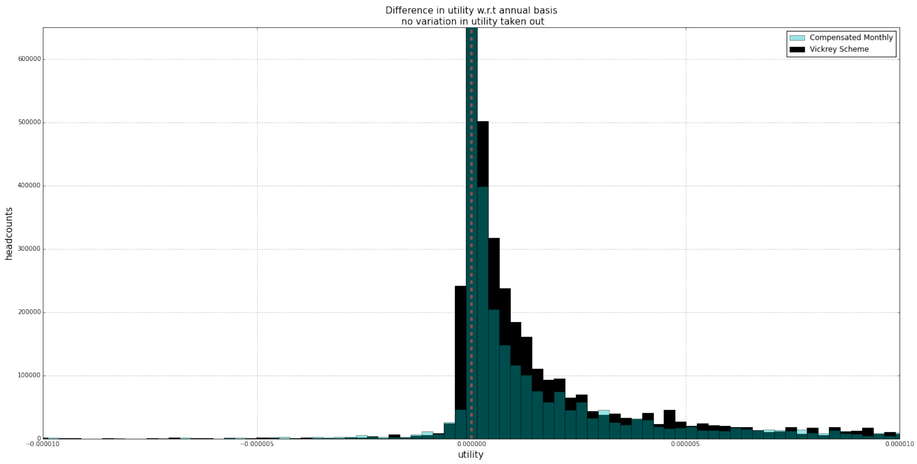


Figure 15 shows all losers and winners by utility points bins gains or loss. Cyan show utility gains with the monthly tax scheme, black shows the utility gains with the compensated monthly tax scheme, and dark green shows overlap.
Figure 16 take out outliers (less than 0.00002 utility points of variation, 25 % of the sample) and null utility variation taxunit. We see as expected that there is losers and winners with the monthly tax scheme, this mainly concerns taxpayers that have higher amount of tax to pay and benefit less from income smoothing than from monetary loss over the year. With compensated monthly tax scheme most people are net gainers from the reform, from those who are not 50% are because they have income on the concave part of the tax scheme, and 50% because they the face collection threshold of the tax.
In order to take into account the effect of joint taxation, we also implement OECD equivalent scale to compute utility.

Vickrey tax scheme: We have implemented Vickrey tax scheme as proposed in Equation 12 with the reference period being year 2009 and the sub-period span being the month. In Figure 17 we can see the difference in utility of the monthly compensated tax scheme, and the Vickrey tax scheme.

Figure 17



We see that most individuals a clear winners compared to the annual tax scheme. We see that on average Vickrey compensation scheme leads to higher utility gains than the compensated tax scheme, but a fraction of the population has a loss in utility with the Vickrey tax scheme compared to the annual one.

4.1 Utility Gains in Euros

Based on taxpayer utility function, we can apply the invert utility transform U^{-1} in order to estimate a money-metric utility function. U^{-1} is the function that gives for a given yearly utility, the monetary equivalent as if one has earned the same income over each month. U^{-1} is such that :

$$U^{-1}(U_t(\vec{y}_t)) = U^{-1}\left(\sum_{1}^{T} u_t\right).$$

We then sum the monthly utility gains over the year, the over all household to get the aggregate utility equivalent monetary gain. We can now compare our different tax systems to estimate the difference in mean yearly utility money metric.

We have compared the annual tax system with the monthly compensated tax system , this lead to utility monetary equivalent gains 1.19 billions euros for the households.
Such calculus consider each household being equal independently of its composition. We apply OECD-modified equivalence scale (as in Hagenaars et al. (1994)[44]) in order to make individuals as the relevant entity for the social planner.
This leads to u_t being :

44. Assigns a value of 1 to the household head, of 0.5 to each additional member aged 15 and above and of 0.3 to each member below 15.

Table 3: Utility Monetary Equivalent Gain Due to Reforms.

value	Monthly basis IR	Compensated MB IR	compensated MB IR + RSA	compensated MB IR + RSA	Overall compensation
Monthly aggregate	-203	1,246	4,859	TODO	
Monthly average		1,246	4,859	TODO	

$$u_t = -\left(\frac{y_t}{\text{equivalent scale}}\right)^{-0.89} \times \text{equivalent scale}$$

$$\iff y_t = exp\left(\frac{1,87 \times ln(\text{equivalent scale}) - ln(-u_t)}{0,89}\right)$$

5 Conclusion

In this article we have shown some mechanism implied by the frequency of the tax. We have proven given some standard hypothesis that it can be Pareto improving to increase the frequency of the tax. Then we have tried to confront our theoretical framework to reality by implementing our prescriptive reform to France.

We have shown that there is a mismatch between our framework where tax are convex and reality in developed countries : taxes on income are not convex and it has implication in terms of disposable income in a dynamic setting. Such shape of a global tax-benefit scheme has normative implications : convex tax scheme promote steady incomes and tax varying income, while concave tax schemes subsidize varying income. The dataset we used shows that most varying incomes are located at the bottom of the distribution, where precisely the income tax-benefit system is not convex.

We have then run with the microsimulation model *OpenFisca* the result of increase in frequency of the income tax and quantified monetary gains and loss for taxpayers and identified losers and winners. We then have conduced a welfare analysis based on the equal sacrifice principle to determine losers and winners in terms of utility of changing the frequency of the tax, both with and without compensation for monetary loss or gains. We have then estimated an utility equivalent monetary loss that is on average of 7.32 € for the monthly tax scheme and of 33 € per month with the monthly compensated tax scheme. [45]

Those results are driven by within year income variability. Most income variability studies in developed countries look at year to year incomes, to the notable exception of Hills et al. (2006) who looks at week-by-week effective disposable incomes of 93 poor families over a year in the UK. [46] Hills et al. (2006) emphasize that over nine tenth of those families has monthly income that vary over 10% of their average income [47] and half of the families in the sample has nothing left

45. Preliminary results problems with data that must be cleaned before relying on this result
46. We can also note Jenkins et al. (2000) who studied the difference between snapshot of current income vs. synthesize annual income on the observation of the distribution of income, their findings is that it has nearly no impact.
47.

> Only seven of the 93 families had incomes fitting our "highly stable" pattern, that is, varying less than 10 per cent either way from their annual average. Only a third had income in at least eleven periods within 15 per cent of their mean, and within 25 per cent of it in any other periods.

over for savings, and a quarter for which outgoings exceeded income. Literature on saving behavior –always yearly based– underline that bottom of the distribution has very high marginal propensity to consume.[48] Those facts suggest that a large parts of the bottom of the distribution cope with whatever is their monthly disposable income and gives credit to our "no-saving" framework. Poor savings and income variability can have huge impact on citizen well being, this suggest that the tax-benefit scheme should be studied in a non purely static models with those effects in mind.

Similarly to Brewer et al. (2010)[49] our public prescription is that bottom of the distribution tax-benefits should be based on high-frequency in order to smooth disposable income. If the public planner wants its tax-benefit system to be progressive, it should set the whole scheme convex. Such a decision would lead to deal with higher payment of the tax for economically unstable tax-units. If the public planner wants to respect to some extent luck egalitarianism, the bottom of the distribution facing more highly unemployment then the rest of the distribution, it should compensate for past-variation by taking large period history. Concerning variation for which individuals should be held responsible for, then it is a normative concerns to determine the length (or the amount) of the compensation period : time liberals would argue for a compensation as large as possible while peoples who lean more on ability to pay would argue for no compensation. Circumstances (chosen or unchosen) that changes the tax-benefit scheme, such as disability, children, etc, should be treated with no compensation.

Those effects problems open new research fields that should be investigated. Optimal taxation is for most contributions in a static framework, studying the incentives and its impact on the optimal tax scheme might lead to different public policy prescriptions as the one prescribed by actual models.

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48. Carrolls, etc, choisir les bonnes références.

49. " The assessment period and timing of taxes and transfers In most countries, individual income taxes are assessed on annual income, and transfers often assessed on a monthly basis (in the UK, weekly). Standard economic models predict that families should budget over long time periods, by borrowing (or using credit) and saving. If families have fluctuating incomes but are able to smooth consumption over time by borrowing and saving, then income assessed over a longer period of time is a better measure of economic welfare or well-being than income assessed over a short period of time. In reality, costs of using financial services and other credit market failures, low levels of literacy, numeracy and financial education, and self-control problems with savings all create significant departures from the standard model. These departures are likely to be more prevalent amongst low-income families, and tend to lead to such families budgeting consumption over short periods of time, such as a month or a fortnight. It therefore seems desirable to operate transfers for low-income families on a **high-frequency basis**, and operate taxes on higher incomes on a lower frequency, such as annual."

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