Segmentation versus Agglomeration: Competition between Platforms with Competitive Sellers[*]

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May 15, 2017

Abstract

Two competing non-differentiated platforms bring together sellers and buyers who face the discrete choice problem which platform to visit. Platforms charge listing fees to sellers for their service. If competition between sellers is soft, only agglomeration equilibria exist, i.e. all sellers and buyers locate on one platform. By contrast, if competition between sellers is fierce, in the unique equilibrium, buyers and sellers segment, and sellers enjoy a monopoly position vis-a-vis buyers. This allows platforms to obtain strictly positive profits in equilibrium. If competition between sellers is moderate, in the unique equilibrium, buyers and sellers segment with positive probability. This equilibrium features dispersion of listing fees. We characterize the equilibrium and extend the analysis to allow for multi-homing sellers and buyers.

keywords: intermediation, two-sided markets, price competition, imperfect and perfect competition

JEL-classification: L13, D43

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1 Introduction

In many industries, platforms play the essential role to enable transactions between buyers and sellers. As trade migrates from physical venues to the Internet, platforms increasingly serve as intermediaries for purchase decisions. For example, in the rental market, the main bulk of matching of landlords and tenants is done via Internet platforms such as Craigslist in the US, Rightmove and Zoopla in the UK, or Immobilienscout24 and Immowelt in German speaking countries. Another example is the used car market, where a large fraction of transactions is initiated via portals.

However, the market structure differs considerably across industries and space. For example, in the US, Craigslist dominates the market in several cities, foremost in the bay area. Buyers and sellers almost exclusively choose this portal, leaving other platforms only specialized market niches. The dominance of a single platform may not come as a surprise, as Craigslist is non-profit and charges low fees. By contrast, in the UK and Germany, the market is segmented and two (or more) platforms have non-negligible market shares.

Due to positive cross-group external effects between buyers and sellers, it has been recognized that platforms have the tendency to tip (as shown by Caillaud and Jullien, 2001, 2003). This explains the phenomenon of market agglomeration, where all buyers and sellers choose one platform over the other. However, as the examples above indicate, in several industries two or more platforms have non-negligible market shares. A possible explanation is that platforms offer differentiated matching services and, therefore, are active in the market (e.g., Rochet and Tirole, 2003, and Armstrong, 2006). However, in the above examples (and more broadly for many Internet platforms) there appears to be little room for differentiation, that is, platforms offer homogeneous services to their customers. In this case, it is unclear how multiple platforms can survive, and it is puzzling that they compete with each other for several years without tipping taking place. In this paper, we resolve the puzzle how multiple homogeneous platforms can survive in an industry that exhibits strong positive network externalities.

Our explanation is based on endogenous differentiation of competitive sellers via their platform choice. We present a theoretical model in which sellers and buyers decide on which platform to be active.[1] If sellers locate on the same platform, it is optimal for buyers to do the same. An agglomeration equilibrium arises. Buyers are then informed about all offers, implying that sellers are in competition with each other. However, if competition between sellers in the same industry is sufficiently intense, they prefer to be active on different platforms. Then, consumers also prefer to be active on different

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[1]In the basic model, agents on both side single-home. We explain below that our results carry over to the case of multi-homing buyers and sellers.

platforms, which implies that single-homing consumers do not become informed about all offers on the market. This relaxes seller competition on each platform. Hence, platforms allow for segmentation of the product market and obtain positive profits for playing this role.

We show that using the natural selection criterion of profit dominance of sellers in addition to Strong Nash Equilibrium yields a unique equilibrium.This allows us to make predictions under which conditions an agglomeration and under which conditions a segmentation equilibrium exists. First, we obtain that if the degree of competition between sellers is low, segmentation cannot occur and tipping prevails. Sellers obtain a higher demand from consumers in the agglomeration equilibrium because all consumers are exposed to all offers by sellers. This increased-demand effect dominates the increased-competition effect. Platforms compete fiercely to win the market. This leads to a Bertrand-style competition between platforms, and their listing fees are driven down to zero.

By contrast, if the degree of competition between sellers is high, segmentation occurs. Sellers use the platforms to avoid competition with their rivals in the product market. Platforms serve the role of segmenting the market and receive positive margins for providing this service. Thus, depending on the degree of competition between sellers, very different market structures can emerge and our paper provides clear predictions how the competitive environment between sellers drives the market structure.

If the degree of competition between sellers is moderate, we show that a mixed-strategy equilibrium in listing fees occur. This equilibrium might consist of two disjoint segments of fees. The upper segment of fees are charged if a platform intends to segment the market. By contrast, a listing fee in the lower segment is charged if the platform intends to agglomerate agents on its platform. In this mixed-strategy equilibrium, platforms segment the market with positive probability. We demonstrate that the probability for segmentation taking place increases if the degree of competition between sellers gets larger. Our result therefore contributes to the explanation of why different market structures emerge in industries with similar competitive conditions.

Interestingly, the mixed-strategy equilibrium contains one or two mass points. The logic behind the best-response dynamic in our model is similar to that of Bertrand-Edgeworth cycles. In our case, if a platform sets a high listing fee, the rival's optimal response is to set a fee which is lower by a discrete amount to induce agglomeration. The best response of the platform is then to lower its fee slightly to induce segmentation. The rival's optimal response to lower its fee slightly to induce agglomeration again, and so on. This tendency goes on until the fee of the platform with the lower fee is so low that it prefers to set a discretely higher fee than the other platform in order to induce segmentation instead of lowering its fee further. In contrast to Bertrand-Edgeworth

cycles, there is no marginal undercutting of the rival's fee but a discrete one. Because there is a continuum of fees between the best responses, mass points occur.

To illustrate segmentation in the real world, consider a search on the platforms Immobilienscout24 and Immowelt. We searched for apartments to rent in the city of Frankfurt am Main, Germany. Our search criteria were "at least 3 rooms", "at least 100 m^2", and "distance no less than 1 kilometer to the centre". A search on November 23, 2015 gave 12 matches on each portal. We report the matches in Table 1 in ascending order of the rental price by stating the square meters of the apartment and the rental price in Euros. Out of these 12 offers, only 2 could be found on both platforms.[2]

	Immobilienscout24		Immowelt	
1.	m^2:103.78;	Rent:1.350	m^2:111.00;	Rent:1.285
2.	m^2:110.00;	Rent:1.450	m^2:104.00;	Rent:1.290
3.	m^2:100.00;	Rent:1.450	m^2:117.00;	Rent:1.350
4.	m^2:105.90;	Rent:1.450	m^2:103.56;	Rent:1.490
5.	m^2:129.02;	Rent:1.548	m^2:114.00;	Rent:1.550
6.	m^2:124.74;	Rent:1.597	m^2:145.00;	Rent:1.650
7.	m^2:142.00;	Rent:1.700	m^2:100.00;	Rent:1.800
8.	m^2:136.00;	Rent:1.890	m^2:140.00;	Rent:1.970
9.	m^2:137.48;	Rent:2.007	m^2:140.00;	Rent:2.450
10.	m^2:173.00;	Rent:2.290	m^2:160.00;	Rent:2.800
11.	m^2:140.00;	Rent:2.450	m^2:152.00;	Rent:2.830
12.	m^2:152.00;	Rent:2.830	m^2:200.00;	Rent:3.200

Table 1: Apartment offers in ascending order of the rental price

Although some consumers may multi-home on both platforms, we expect that many of them single-home, as it is time-consuming to conduct searches on various platforms. As a consequence, the listing behavior of sellers gives rise to segmentation of sellers, which dampens competition. Such segmentation is inefficient because network effects are not fully exploited and, with price-sensitive demand, the deadweight loss is higher than under agglomeration due to to higher product market prices.

While our base model features single-homing on both sides of the market, we allow for multi-homing sellers and buyers in our extensions and show that our solution to the puzzle that multiple platforms are active (and profitable) carries over to those settings. First, we consider the case in which some (but not all) buyers multi-home. The intuition for the existence of the segmentation equilibrium remains: the remaining single-homing consumers do not observe all offers, which dampens competition on the product market. Hence, if the degree of competition is fierce, firms prefer segmentation over agglomeration and platforms can demand positive listing fees in equilibrium. The sellers' and

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[2]Offer 11 on Immobilienscout24 is the same apartment as offer 9 on Immowelt, and offer 12 on Immobilienscout24 is the same apartment as offer 11 on Immowelt.

platform's profit from multi-homing consumers depends on the trade-off between increased competition brought about by multi-homing and a higher demand because more consumers are informed. We show profits may change non-monotonically in the mass of multi-homing consumers. A few multi-homers lead to increased profit whereas profits fall if the number of multi-homers is above a certain threshold.

Finally, we analyze multi-homing of sellers. We find that platforms can be hurt by the possibility that sellers multi-home. This result is in contrast to the existing literature which shows that multi-homing benefits platforms because platforms do no longer compete for multi-homing agents. Our result shows, however, that this is not true if agents (here sellers) compete against each other. The intuition is as follows: consider listing fees that lead to a segmentation equilibrium with single-homing sellers. If sellers can multi-home, a profitable deviation from the segmentation equilibrium may exits for each seller. This deviation is to multi-home and to serve buyers on both platforms. This possibly breaks the segmentation equilibrium, in which platforms can earn positive profits. Then, only an agglomeration equilibrium exists, in which buyers observe all offers, and platforms end up competing à la Bertrand and receive zero profits.

Our paper contributes to the literature on competition in two-sided markets, pioneered by Caillaud and Jullien (2001), Rochet and Tirole (2003, 2006), and Armstrong (2006). These papers focus on the cross-group externalities between agents on both sides and did not consider competition between agents on the same side (as the sellers do in our model).

Several papers in the two-sided markets literature analyze competition between sellers. Nocke, Peitz and Stahl (2007) and Galeotti and Moraga-González (2009) consider the effect of platform ownership on prices and of search for sellers' products, respectively, but focus on a monopoly platform. Hagiu (2006) is primarily interested in price commitment by platforms when agents of the two sides make their decision in a sequential order. However, he shows that if commitment is not possible and agents single-home, an agglomeration equilibrium with zero profits emerges. Belleflamme and Toulemonde (2009) show how a fee-setting platform can gain market shares from a platform with zero fees by applying a divide-conquer-strategy (i.e., negative prices on one side and positive prices on the other). Hagiu (2009) considers the effects of product variety on platform prices in a model with differentiated platforms, implying that both platforms are active in equilibrium.[3] None of these papers has considered the effect identified in our paper and analyzed how the market structure depends on seller competition.

In an early paper, Gehrig (1998), considering Hotelling competition between platforms and competition on the circle (Salop, 1979) between sellers, analyzes the effect

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[3]Belleflamme and Peitz (2010) consider congestion externalities between sellers, which lead to similar effects as competition. They focus on how investment incentives are influenced by the platform prices.

of transportation costs on entry and location of platforms. In contrast to our analysis, he is mainly interested in agglomeration equilibria.[4] Armstrong and Wright (2007) endogenize the decision of agents to single-home or to multi-homes (thereby endogenizing the market structure in a different way than we do) and determine how differentiation between platforms affects this choice.

Ellison, Fudenberg, and Möbius (2004) consider competition between two auction sides and analyze market thickness of the platforms. They show that concentration tends to be optimal but under some conditions sellers may prefer different platforms because this lowers the seller-buyer ratio on each platform and leads to (higher) expected prices. Hence, platforms can co-exist in equilibrium. In contrast to our paper, they do not consider how platforms can influence the market structure via their fees; instead they determine how the homing decision of agents affects the equilibrium. Lee (2014) considers a model with non-atomistic sellers and determines how bilateral contracting between platforms and sellers affects the market structure. He shows that even without competition between sellers, platforms may achieve segmentation, which leads to co-existence of platforms.

Our paper also contributes to the literature on competition in the Internet, in particular, on price comparison websites. For example, Baye an Morgan (2001) show how sellers can obtain positive profits, even if a website informs buyers about their prices. The intuition is that sellers still sell on their local market where buyers are not informed about all prices. This leads to price dispersion in equilibrium. Ronayne (2015) uses this framework and demonstrates that due to the website's fee, all prices increase in expectation, leading to lower surplus for buyers. Instead, our paper analyzes competition between websites and we obtain that price dispersion can occur not for sellers' prices but for the platforms.

Finally, our paper is connected to the literature obtaining mixed-strategy equilibria in price competition, as is often the case in the seach literature (Varian, 1980; Janssen and Moraga-Gozález, 2004). If firms are symmetric, the prices in the mixing domain are usually atomless as the best response is to slightly undercut the rival's price. If firms are asymmetric, this is no longer true. The distribution of the firm with lower quality or higher cost often entails a mass point at the price where its profit equals zero (see e.g., Narasimhan, 1988, or De Corniére and Taylor, 2014). In our equilibrium, mass points also exist with symmetric firms because the best responses involve a discrete increase or decrease in the price relative to the one of the rival.

The rest of the paper is organized as follows: The next Section sets out the model. Section 3 determines the equilibrium. Section provides an extension to multi-homing

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[4]See also Gal-Or and Dukes (2003) for an analysis how competition between sellers affects platforms' location decision.

firms and Section 5 analyzes the effects of multi-homing sellers. Section 6 concludes. All proofs are in the Appendix.

2   The Model

There are three types of agents in our model: platforms, firms, and consumers. We describe the agents in turn.

Platforms

Two homogeneous platforms A and B offer listing services. The platforms bring together sellers and buyers of products. To be active on platform i, a firm has to pay a listing fee f_i, $i \in \{A, B\}$. Such listing or posting fees are prevalent in markets in which the platform cannot or does not monitor the sale of a product. For example, in the housing or the rental market, platforms posting ads usually charge listing fees. Also, the portal craigslist.org charges listing fees for posting ads for cars/trucks or therapeutic services. Consumers can access platforms for free.

Firms

Firms (or sellers) have to decide which, if any, platform to join. In the base model, they cannot be active on both platforms (i.e., firms single-home) – in Section 5, we show that our main results carry over to the case with multi-homing firms. The product of each firm belongs to a product category. There is a mass 1 of such categories, indexed by $k \in [0, 1]$. There are two sellers per product category and a platform can accommodate up to two sellers per product category.

For simplicity, we assume that the two sellers in each category are symmetric. Firms set uniform prices to consumers. We denote the symmetric equilibrium duopoly price by p^d and the monopoly price by p^m. Equilibrium profits per consumer in duopoly are denoted by π^d. If consumers can buy from only one of the firms because only one firm is listed on the platform that consumers are patronizing, this firm makes monopoly profits π^m per consumer. Our formulation implies that per-consumer profit in duopoly and monopoly are independent of the number of consumers. At the end of this section, we provide two illustrations that generate π^d and π^m from two widely-used demand functions. However, as will become clear later, our qualitative results do not depend on π^d and π^m being independent of the number of consumers, but hold more generally.

Consumers

Each consumer (or buyer) single-homes, that is, she decides to be active on (up to) one platform.[5] On the chosen platform, each consumer makes a choice among the products

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[5]In Section 4, we provide the analysis with multi-homing consumers and demonstrate that our main insights remain valid.

encountered on the platform; this includes the option not to buy. She is interested in a single product category and derives a positive gross utility from products in this category; products in all other segments give zero utility.[6] There is mass 1 of consumers per product category. When visiting a platform, a consumer becomes informed about the product category and the price of all products listed on the platform. If a platform lists all products from a fraction $\alpha \in [0, 1]$ of all product categories, then consumers expect to find a product from the liked category with probability α.

A consumer obtains a different (indirect) utility if one or two sellers are listed in her preferred category. Consider a consumer who has found at least one listed seller in the category she is interested in. Prior to observing the idiosyncratic taste realization within this category, this consumer obtainsan expected utility of V^d if two products are listed in the category. If only one product is listed, the consumer obtains an expected utility of $V^m < V^d$. The reason for this inequality is twofold: First, if two sellers are listed, there is competition between them, implying that they charge the duopoly price p^d in equilibrium, which is lower than the monopoly price p^m, which is the price a single listed seller charges. Second, if sellers are differentiated, a consumer will find a product closer to her preferences or benefits due to a taste for variety if two sellers are listed instead of only one.

Timing

The timing is as follows:

1. Platforms A and B set listing fees f_A and f_B, respectively.

2. Firms and consumers make a discrete choice between platform A and B, and the outside option (normalized to zero).

3. Firms in each category set product prices.

4. Consumers observe all offers on the platform they are visiting and make their buying decisions.

We make two observations regarding our setup: First, according to our timing firms decide where to list before setting their prices on the product market. This is the relevant timing in most applications because the decision on which platform to list is typically more long term than the pricing decision. In our setting, firms set prices after learning about the number of competitors in the product market.[7] Second, listing fees do not enter the pricing decisions of the firms in the third stage because they are "fixed" costs for firms (which are, in addition, sunk when firms set prices). Hence, the market for listing services is in fact two-sided in the sense of Rochet and Tirole (2006).

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[6] See, e.g., Burguet, Caminal, and Ellman (2014) for a similar structure.

[7] In addition, listing fees are often paid on a subscription basis. This makes them lumpy. By contrast, prices charged by the firms are flexible.

Payoffs

For simplicity, we assume that all platform costs are zero. The profit of platform i is then the number of sellers active on platform i multiplied by the listing fee f_i. The profit of a firm which is listed on platform i is $\beta_i \pi - f_i$, where β_i is the fraction of consumers in the firm's category, who are active on platform i, and π is either π^m if the rival seller is not listed on platform i or π^d if the rival also lists on platform i. As mentioned above, the utility of a consumer is V^d or V^m, dependent on the number of sellers listed in the consumer's preferred category; the utility is 0 if none of those sellers is listed on the platform.

Solution Concept

Our solution concept is subgame perfect equilibrium. In the second stage, firms and consumers face a coordination game on which platform(s) to be active, which leads to a multiplicity of equilibria. To deal with this well-known problem in two-sided markets, we impose the refinement of Strong Nash Equilibrium (see e.g., Aumann, 1959, or Bernheim et al., 1987), that is, we select only Nash equilibria that are stable against deviations by coalitions of consumers and firms. In addition, if this concept is not sufficient to select a unique equilibrium, we select the equilibrium that is the profit dominant one for firms at this stage. We will show that the joint application of these refinements leads to a unique equilibrium at this stage and also at the full game.

A justification of the refinement is that the outcome is equivalent to the outcome of a sequential game in which sellers decide which platform to join before consumers do, as considered in the models by Hagiu (2006) and Lee (2014), and sellers select the Strong Nash equilibrium. In Section 6, we analyze the mirror case, in which the Pareto dominant equilibrium for consumers is selected. The main insights of or analysis will be unchanged.

Finally, if consumers expects one firm in each category to list on platform A and the other firm on platform B, consumers are indifferent between the two platforms. We assume that in this case half of the consumers in each category join platform A and the other half platform B. A natural interpretation is that each consumer mixes with equal probability to be active either on platform A or B. Since there is a continuum of consumers, both platforms will in fact be patronized by 1/2 of the consumers.[8] As we point out below, this assumption is not crucial for the results and can be relaxed

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[8] Another interpretation is that platforms are differentiated by different platform designs (with half of the consumers in each category preferring platform A and the other half platform B) but that this differentiation is negligibly small. For example, platforms are differentiated along a Hotelling line and the transport cost parameter t goes to zero. This means that consumers ex ante have lexicographic preferences in the sense that they prefer the platform that has a higher probability to list a product in the consumer's preferred category. Only if consumers expect these probabilities to be the same across platforms, they decide according to their preference for different platform designs.

allowing for unequal distribution of consumers in case of indifference.

Examples on buyer-seller interaction

We provide two examples of widely-used demand functions in oligpoly (i.e., Hotelling and linear demand as in Singh and Vives, 1984), to provide explicit expression for π^d and π^m. As an alternative to differentiated product oligopoly, one may use models in which firms after entry obtain private information about their marginal cost or in which there are differentially informed consumers within a product segment or consumers incur search costs.

Example 1: Hotelling.

Consider Hotelling competition in each product category. Each firm is located at one of the extreme points of the unit interval in a particular category; i.e., a firm j is characterized by its category k_j and its location l_j on the unit interval, $(k_j, l_j) \in [0,1] \times \{0,1\}$. The consumers' valuation of a product at the ideal location in the preferred category equals v. The utility of a consumer who likes category m and is located at x_k, $(k, x_k) \in [0,1] \times [0,1]$ when buying a product which belongs to category k and is located at l_j is $= v - t|x_k - l_j| - p_{l_j}$, where the parameter t captures the degree of product differentiation. Instead, her utility is zero for products in all categories which are not equal to k. Price competition among Hotelling duopolists leads to equilibrium prices $c + t$ and equilibrium profits $\pi^d = t/2$ per unit mass of consumers. The monopoly seller sets price $(v + c)/2$ and its profit is $\pi^m = (v - c)^2/(4t)$ per unit mass of consumers, if the market is not fully covered. This is the case if $2t \geq v - c$. If the reverse inequality $2t < v - c$ holds, there is full coverage under monopoly and the monopolist sets $p^m = v - t$. Its profit is $\pi^m = v - t - c$.

Example 2: Linear demand for differentiated products by representative consumer.

Consider that consumers with the same preferred category have utility function $v = q_1 + q_2 - 1/2(q_1^2 + q_2^2) - \gamma q_1 q_2 - p_1 q_1 - p_2 q_2$, with $\gamma \in [0,1]$, expressing the degree of substitutabiliy between products. This is a representative consumer setting, where each consumer obtains utility from positive quantities of each product in her preferred category. Maximizing this utility function with respect to q_1 and q_2, we obtain the indirect demand functions $p_i = 1 - q_i - \gamma q_{-i}$, $i = 1, 2$. Inverting this demand system yields the direct demand functions

$$q_i = \frac{\beta - \gamma - p_i + \gamma p_{-i}}{1 - \gamma^2}, \quad i = 1, 2.$$

Duopoly equilibrium profits are

$$\pi^d = \frac{(1 - \gamma)(1 - c)^2}{(1 + \gamma)(2 - \gamma)^2}$$

per consumer.

For a monopolist, the direct demand is

$$q_i = 1 - p_i.$$

and monopoly profits are

$$\pi^m = \frac{(1-c)^2}{4}.$$

3   Equilibrium Analysis

We solve the model by backward induction. Consumers' choices in stage 4 and firms' pricing decisions at stage 3 are straightforward: in the fourth stage, consumers buy a product in their preferred product category given that there is one according to their demand function. In the third stage, firms listed on a platform know whether they face a competitor in their product category or not. They therefore set price p^d in case of competition and p^m in case of monopoly.

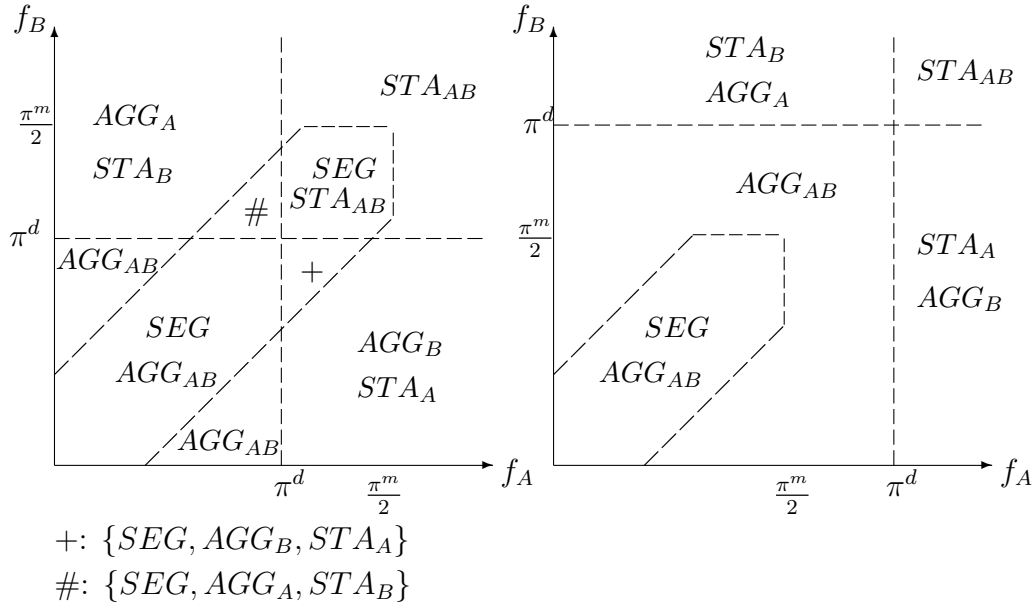+: $\{SEG, AGG_B, STA_A\}$
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Figure 1: Possible equilibrium configurations at stage 2 : $\pi^d \leq \pi^m/2$ on the left-hand side and $\pi^d > \pi^m/2$ on the right-hand side

In stage 2, multiple Nash equilibria exist given the listing fees chosen by platforms in the first stage. Our refinement selects a unique one of those. We next characterize the set of Nash equilibria in stage 2 and demonstrate our equilibrium selection. A detailed analysis is provided in Appendix A.

As an example, suppose listing fees f_i and f_{-i} are close to zero. Then, it is a Nash equilibrium that all sellers and all consumers are active on only one platform, i.e. on

either platform i or $-i$. In such an agglomeration equilibrium, for instance, on platform i, a seller's profit is $\pi^d - f_i$ and a consumer's utility is V^d. In addition to two such agglomeration equilibria, there is also a segmentation equilibrium, in which half of the consumers are active on platform i and the other half on platform $-i$ as firms in each category also locate on different platforms. Therefore, a firm's profit when being active on platform i is $\pi^m/2 - f_i$, whereas the utility of a consumer is V^m. If instead, for example, both platforms' listing fees are higher than π^d, sellers would make losses when competing on the same platform. In this case, there also exist two equilibria in which in each category only one seller is active, either on platform i or on platform $-i$ and all consumers use this platform (stand-alone equilibria).

The set of Nash equilibria is visualized in Figure 1. The left-hand side of the figure displays the case $\pi^d \leq \pi^m/2$, whereas the right-hand side is the opposite case. In the figure, the agglomeration equilibrium on platform i, in which all sellers are active is abbreviated by AGG_i (and by AGG_{AB} if an agglomeration equilibrium on either platform exists), the stand-alone equilibrium with only half of the sellers being active is denoted by STA_i, and the segmentation equilibrium is denoted by SEG. As it can be seen on the left panel, there are regions in which all three equilibrium configurations exist.
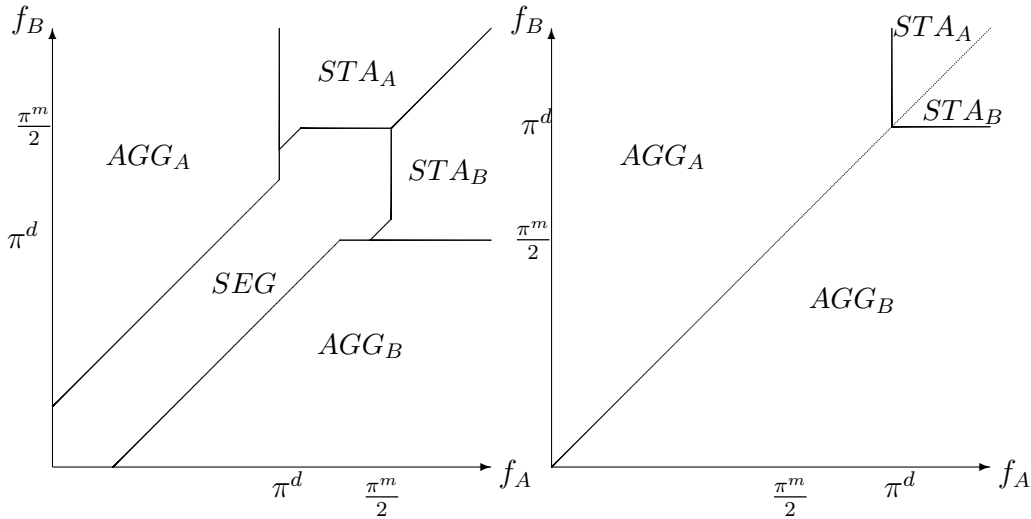

Figure 2: Selected equilibrium configurations at stage 2: $\pi^d \leq \pi^m/2$ on the left-hand side and $\pi^d > \pi^m/2$ on the right-hand side

We now turn to the equilibrium selection accomplished through our refinement, as illustrated Figure 2. First, applying the concept of Strong Nash equilibrium eliminates the multiplicity of agglomeration equilibria. The reason is that a coalition of sellers and consumers will always choose to be active on the platform with the lower fee which makes the coalition members better off. The same reasoning holds if there is a multiplicity of stand-alone equilibria and if an agglomeration and a stand-alone equilibrium co-exist.

Second, the joint use of Strong Nash and profit-dominance of sellers also singles out a unique equilibrium for the regions in which the segmentation equilibrium exists together with another equilibrium. To see this, consider the case above, in which listing fees of both platforms are close to zero. Then, if $\pi^d > \pi^m/2$, the segmentation equilibrium is not stable to the deviation of a coalition of sellers and consumers who are active on the platform with the higher price. If this coalition switches to the rival platform, consumers are better off because $V^d > V^m$ and firms are also better off because they now serve all consumers instead of only half of them. This is profitable because $\pi^d > \pi^m/2$. Hence, in this region, all segmentation equilibria are eliminated, as can be seen on the right panel in Figure 2. By contrast, if $\pi^d \leq \pi^m/2$, the Strong Nash refinement has no bite as the deviation is no longer profitable for sellers. Using seller dominance, however, now singles out a unique equilibrium. In particular, if $f_i \leq \pi^m/2 - \pi^d + f_{-i}$, the segmentation is more profitable for sellers than agglomeration. This is displayed in the left panel of Figure 2, which shows that segmentation is an equilibrium if f_i and f_{-i} are relatively close to each other.

Our equilibrium concept is also different to the one imposed by Caillaud and Jullien (2003), who consider favorable expectations for one platform (the incumbent) in case of a price deviation by the rival platform (the entrant). This implies that in equilibrium agents agglomerate on the incumbent platform. By contrast, in our model, agents form expectations after observing prices and expectations are symmetric.

We now turn to the first stage. As is evident from the discussion above, the agglomeration and the segmentation equilibrium have different benefits for the sellers. The segmentation equilibrium has the advantage that each seller is in a monopoly position vis-a-vis consumers but, in contrast to the agglomeration equilibrium, a seller can reach only half of the consumers. If the first effect dominates, market tipping may not occur. This effect may break the Bertrand logic that platforms undercut each other until price equals marginal costs. We will now show under which conditions platforms can exploit this and obtain positive profits. The next four propositions describe the equilibrium listing fees and the associated (expected) profits for all parameter ranges.

Proposition 1. *Agglomeration. If $\pi^d \geq 1/2\pi^m$, in the unique equilibrium, the equilibrium listing fees are $f_A^\star = f_B^\star = 0$, platforms' profits are $\Pi_A^\star = \Pi_B^\star = 0$.*

If duopoly profits are relatively large (i.e, $\pi^d \geq 1/2\pi^m$), the agglomeration regime applies. From a firm's point of view, the effect that agglomeration reduces profits due to firm competition is dominated by the demand expansion effect that all consumers instead of half of them consider the firm's offer. Since each platform can obtain all demand by setting a lower fee than its rival, platforms end up in Bertrand competition and equilibrium fees of zero. Thus, in this region, the classic Bertrand argument applies

and competing homogeneous platforms obtain zero profits in equilibrium.

Proposition 2. *Segmentation with deterministic listing fees. If $\pi^d < 1/4\pi^m$, the equilibrium listing fess are $f_A^\star = f_B^\star = \pi^m/2$, platforms profits' are $\Pi_A^\star = \Pi_B^\star = \pi^m/2$.*

If duopoly profits are particularly low (i.e., $\pi^d < 1/4\pi^m$), the segmentation regime applies. Firms avoid competition by choosing segmentation. This can be exploited by platforms. To see this, suppose that both platforms charge a fee of zero. If π^d is lower than $\pi^m/2$, firms choose to segment. But then a platform can raise its fee slightly without loosing any firms. Thus, the platform with the higher fee remains active and raises strictly positive profits.

In the segmentation regime, platforms can extract the full surplus from firms. The argument is as follows. When a platform deviates from the equilibrium listing fee $f_i^\star = \pi^m/2$ to a listing fee slightly below π^d, this induces firms and buyers to agglomerate on this deviating platform. The deviant platform then makes profit $2\pi^d$. The profit in equilibrium is instead equal to $\pi^m/2$ which is greater than $2\pi^d$ if $\pi^d < \pi^m/4$. Hence, in the segmentation regime, no platform has an incentive to deviate from the subscription fee $\pi^m/2$. To sum up, if competition between firms is sufficiently intense, platforms obtain positive profits by inducing firms to segment the market. This result obtains despite the fact that platforms offer the same matching service. Interestingly, fierce competition among firms enables platforms to sustain high profits in equilibrium.

We now turn to the range $1/4\pi^m \leq \pi^d < 1/2\pi^m$. As we will show, in this range platforms randomize over subscription fees. The intuition for the non-existence of a pure-strategy equilibrium in this range is as follows: For any fee set by platform i, the competing platform $-i$'s best response is to either set a lower fee to induce agglomeration on its platform or set a higher fee, which leads to firm segmentation, where platform $-i$ receives higher profits than platform i. Suppose that platform i sets a relatively high fee. The competing platform $-i$ then optimally sets a fee that is discontinuously lower, so as to just induce agglomeration. The optimal response of platform i is to lower its fee slightly and induce segmentation again. This sequence of best responses continues until the fee of platform i reaches a level that further adjustments by platform $-i$ to induce agglomeration is no longer its best response, but instead platform $-i$ prefers to set a fee that is discontinuously higher than the one of platform i, so as to just induce segmentation. In turn, it is then the best response of platform i to reduce its fee slightly to induce agglomeration. Therefore, the sequence continues and does not converge.

From the argument above, it is evident that the range of subscription fees over which platforms mix can be divided into two intervals, a lower and an upper interval. In the lower interval, fees are set with the intention to induce agglomeration. In the upper interval, fees are set with the intention to induce segmentation.

Proposition 3. *Segmentation or agglomeration with listing fees chosen from a convex set. If $3/8\pi^m \le \pi^d < 1/2\pi^m$, there is a unique symmetric mixed-strategy equilibrium, in which platforms set fees in the domain $f_i \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$. The mixing probability is characterized by a cdf of*

$$G(f) = \begin{cases} \frac{f - (\pi^m - 2\pi^d)}{f + 1/2(\pi^m - 2\pi^d)}, & \text{if } f \in [\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d); \\ \frac{2f - 5/2(\pi^m - 2\pi^d)}{f - 1/2(\pi^m - 2\pi^d)}, & \text{if } f \in [3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d]. \end{cases}$$

There is a mass point at $f = 3/2\pi^m - 3\pi^d$ with point mass $1/4$. The corresponding generalized density is given by[9]

$$g(f) = G'(f) + \frac{1}{4}\delta^D(f - (3/2\pi^m - 3\pi^d)),$$

where $\delta^D(f - f_0)$ denotes Dirac's delta function which is 0 everywhere except for f_0 where it is ∞. Furthermore, $\int \delta^D(f - f_0)df = 1$. The expected profit is $\Pi_A^\star = \Pi_B^\star = 3\pi^m/2 - 3\pi^d$.

In the region of $3/8\pi^m \le \pi^d < 1/2\pi^m$, the upper bound of the lower interval coincides with the lower bound of the upper interval and platforms randomize over the interval $[\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$. The lower interval is $[\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d)$ and the upper interval is $[3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d]$. Setting a fee of $3/2\pi^m - 3\pi^d$ therefore induces segmentation with probability 1. There is a mass point on this fee $3/2\pi^m - 3\pi^d$. Since the event that both firms choose this fee occurs with strictly positive probability, the expected equilibrium profit in this regime must equal $3/2\pi^m - 3\pi^d$.

The cdf and the corresponding generalized density in the region of $3/8\pi^m \le \pi^d < 1/2\pi^m$ are illustrated in Figures 1 and 2, respectively. There is a mass point at $3/2\pi^m - 3\pi^d$.[10] The mass point is at the fee which separates the interval of fees that are intended to induce agglomeration from those that are intended to induce segmentation. As explained above, the fee always leads to segmentation, and the platform's profit when charging this fee is certain.

The question arises why there is no mass point in the best response to $3/2\pi^m - 3\pi^d$. In Varian's (1980) seminal model of sales indeed mass points can be excluded because of such a type of best response. The answer is that the best response is a downward jump in the fee (given that firms play the agglomeration equilibrium in case they are indifferent)

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[9]Because the distribution is not absolutely continuous with respect to the Lebesgue measure, it fails to have a density. Nevertheless, we define a generalized density, which is a generalized function (since it will be comprised of a dirac delta function) such that integration against this generalized function yields the correct cumulative distribution function.

[10]Mixed-strategy equilibria in symmetric oligopoly models typically do not feature mass points (e.g., Varian, 1980, or Moraga-Gonzales and Janssen, 2004). A number of contributions find mass points in asymmetric oligopoly models in which one firm is disadvantaged and therefore obtains zero profits in equilibrium; see, among others, Narasimhan (1988) and de Corniére and Taylor (2015).
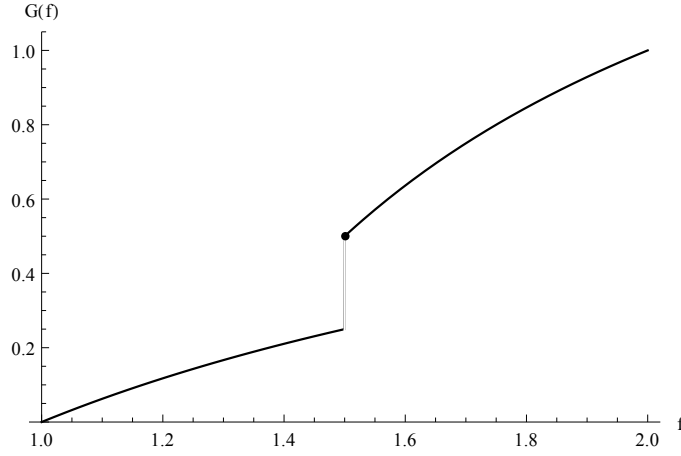
Parameters are $\delta = 1/2$ which leads to $f \in [1, 2]$.

Figure 1: First Mixed-Strategy Equilibrium: Cumulative Distribution Function
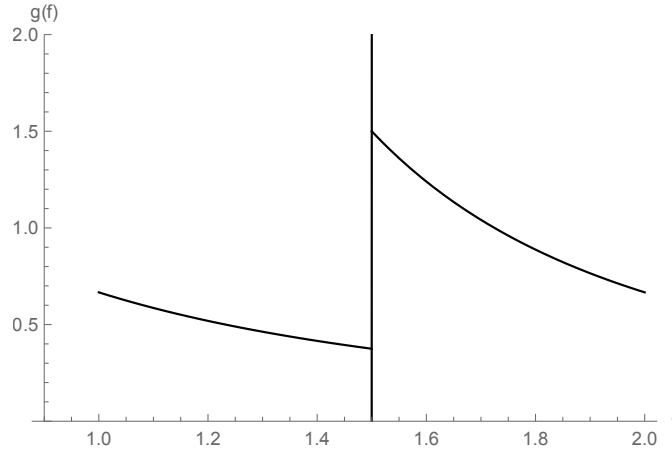
Parameters are $\delta = 1/2$ which leads to $f \in [1, 2]$.

Figure 2: First Mixed-Strategy Equilibrium: Generalized Density

and not just a marginal undercutting. But this implies that there is a continuum of fees between $3/2\pi^m - 3\pi^d$ and the best response $\pi^m - 2\pi^d$, to which $\pi^m - 2\pi^d$ does not constitute a best response. Because this continuum of fees has positive probability, there is no mass point at $\pi^m - 2\pi^d$.

The maximal fee that platforms can possibly charge to obtain positive demand is $f_i = \pi^m/2$. In the region of $3/8\pi^m \le \pi^d < 1/2\pi^m$, as π^d decreases and reaches $3/8\pi^m$, the upper bound in the upper interval of the first mixed regime, $2\pi^m - 4\pi^d$, reaches this level. For lower π^d, i.e., in the region of $1/4\pi^m \le \pi^d < 3/8\pi^m$, the two intervals that form the support of the fee distribution then become separate. This implies that the support becomes non-convex and we enter a second mixed regime. Here, platforms set listing fees in the set $[\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. As in the first mixed region, the fee

$3\pi^m/4 - \pi^d$ induces segmentation with probability 1, and this fee is chosen with strictly positive probability (i.e., $3\pi^m/4 - \pi^d$ is a mass point in the distribution over subscription fees). Therefore, the expected equilibrium profit in the second mixed regime must equal $3\pi^m/4 - \pi^d$.

Proposition 4. *Segmentation or agglomeration with listing fees chosen from a non-convex set. If $1/4\pi^m \leq \pi^d < 3/8\pi^m$, there is a unique symmetric mixed-strategy equilibrium, in which platforms set fees in the domain $f_i \in [\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. The mixing probability is characterized by a cdf of*

$$G(f) = \begin{cases} \frac{f - 1/4\pi^m}{f + 1/2(\pi^m - 2\pi^d)}, & \text{if } f \in [\pi^m/4, \pi^d); \\ \frac{2f - 1/4\pi^m - 3/2(\pi^m - 2\pi^d)}{f - 1/2(\pi^m - 2\pi^d)}, & \text{if } f \in [3\pi^m/4 - \pi^d, \pi^m/2); \\ 1, & \text{if } f = \pi^m/2. \end{cases}$$

There are mass points at $f = 3\pi^m/4 - \pi^d$ and $f = \pi^m/2$. The respective point masses are $(3/4\pi^m - 2\pi^d)/\pi^d$ and $(2\pi^d - 1/2\pi^m)/\pi^m$. The corresponding generalized density is given by

$$g(f) = G'(f) + \frac{3/4\pi^m - 2\pi^d}{\pi^d}\delta^D(f - (3\pi^m/4 - \pi^d)) + \frac{2\pi^d - 1/2\pi^m}{\pi^m}\delta^D(f - \pi^m/2).$$

*The expected profit is $\Pi_A^\star = \Pi_B^\star = 3\pi^m/4 - \pi^d$.*

Figures 3 and 4 illustrate $G(f)$ and $g(f)$ of the second mixed regime. It is evident that in this regime the equilibrium strategy features two mass points, one at the highest fee in the support and the other at the lower bond of the upper interval. The intuition for the latter mass point (at $f = 3\pi^m/4 - \pi^d$) is the same as the one given in the first mixed regime. This fee induces segmentation with probability 1 and a platform sets this fee with positive probability.

The intuition for the mass point at $f = \pi^m/2$ is different. It is rooted in the non-convexity of the mixing region: Charging a fee equal to the upper bound of the lower interval (i.e., infinitesimally below π^d) to induce agglomeration is only a best response to the rival platform charging $\pi^m/2$. For all other fees in the upper interval, charging a fee infinitesimally below π^d does not lead to agglomeration, implying that a lower or a higher fee from the non-convex mixing region does strictly better. As a consequence, to render the fee infinitesimally below π^d optimal, the rival platform must set the highest fee with a strictly positive probability. Otherwise, a fee infinitesimally below π^d can never be part of the mixing domain, and an equilibrium would fail to exist. Again, reacting to a fee of $\pi^m/2$ by also playing the best response (i.e. the fee infinitesimally below π^d) with strictly positive probability cannot be optimal because this is not the optimal reaction to all fees in the upper interval.
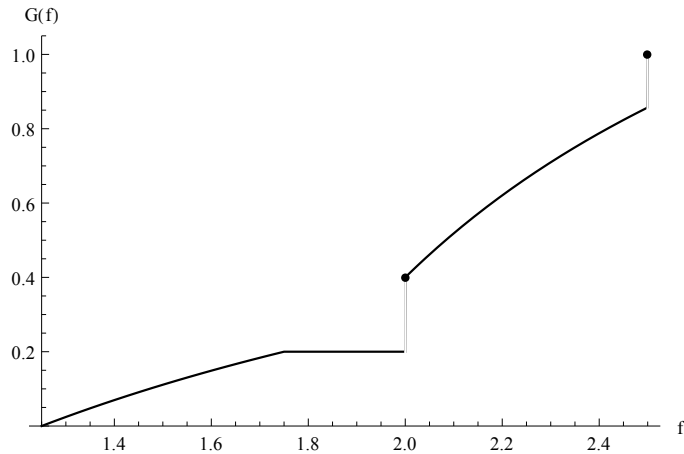
Parameters are $\delta = 1/2$ and $\eta = 3/4$ which leads to $f \in [5/4, 7/4) \cup [2, 5/2]$.

Figure 3: Second Mixed-Strategy Equilibrium: Cumulative Distribution Function
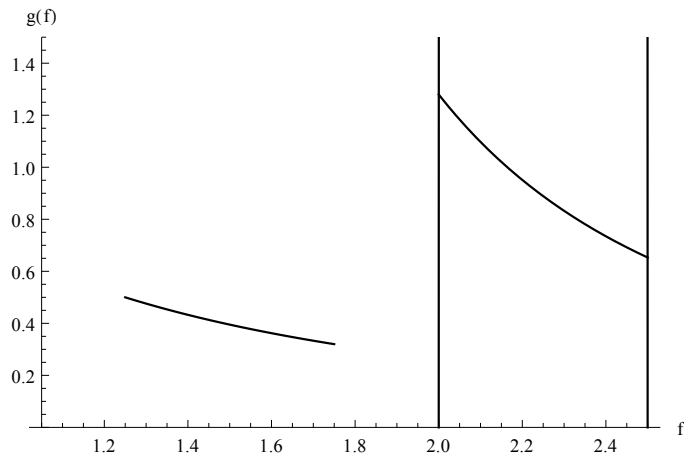
Parameters are $\delta = 1/2$ and $\eta = 3/4$ which leads to $f \in [5/4, 7/4) \cup [2, 5/2]$.

Figure 4: Second Mixed-Strategy Equilibrium: Generalized Density

The argument why only a mixed-strategy equilibrium exists in the first and second mixing region is reminiscent of Bertrand-Edgeworth-cycles. However, different from these cycles, in our equilibrium, platforms charge different subscriptions fees because one of them intends to induce agglomeration whereas the other intends to induce segmentation. This can lead to two disjoint intervals from which subscription fees are chosen, something which cannot happen in a Bertrand-Edgeworth cycle.

In the two examples given in the previous section, the degree of competition determines π^d relative to π^m. Therefore, the boundaries of the regions can be expressed with the parameter indicating how differentiated firms are.

In the Hotelling example, this degree is given by t, that is, a higher t represents more differentiation. We obtain that the agglomeration region occurs for $t \geq (v - c)/2$,

the first mixing region if $3(v-c)/7 \leq t < (v-c)/2$, the second mixing region if $(v-c)/3 \leq t < 3(v-c)/7$, and the segmentation region if $t < (v-c)/3$.

In example 2, the boundaries of the regions can be determined with the help of $\gamma \in [0, \beta]$. A higher γ means fiercer competition. The agglomeration region occurs approximately for $\gamma \leq 0.62\beta$, the first mixing region if $0.62\beta < \gamma \leq 0.74\beta$, the second mixing region if $0.74\beta < \gamma \leq 0.85\beta$, and the segmentation region if $\gamma \geq 0.85\beta$.

We note that the (expected) equilibrium platform profit as a function of π^d is continuous and has three kinks. The profit is 0 for $\pi^d \geq 1/2\pi^m$. For $3/8\pi^m \leq \pi^d < 1/2\pi^m$, the profit is $3\pi^m/2 - 3\pi^d$. At the boundaries, this implies that the profit is 0 as π^d approaches $\pi^m/2$. At $\pi^d = 3/8\pi^m$, the profit is $3\pi^m/8$. For $1/4\pi^m < \pi^d < 3/8\pi^m$, the profit is $3\pi^m/4 - \pi^d$, which goes to $3\pi^m/8$ as π^d approaches $3\pi^m/8$, and is $\pi^m/2$ for $\pi^d \to 1\pi^m/4$. Finally, if $\pi^d \leq 1/4\pi^m$, the profit is $\pi^m/2$.

The mixed-strategy equilibrium implies that the market outcome in industries with similar conditions might be very different. Specifically, in some markets agglomeration takes place and one platform has the lion's share of demand. By contrast, as pointed out in the introduction, there are other markets which look very different. In many European countries two (or more) platforms are active in the rental market for flats and compete on almost on equal grounds. For example, in Germany Immobilienscout 24 and Immowelt are widely used by buyers and firms and often have exclusive offers (and, thus, induce segmentation).

Regarding welfare properties, we find that the market equilibrium may not be welfare maximizing in a second-best sense. Suppose that the social planner cannot control firm prices but the market structure. This brings out the trade-off between the agglomeration and the segmentation equilibrium. Welfare is the sum of platforms' and firms' profits and consumer welfare. Since platforms charge listing fees, the fees are just transfers from firms to platforms. They therefore do not enter the welfare function directly, but affect welfare as they determine whether agglomeration or segmentation prevails. Welfare in the agglomeration equilibrium is then given by $V^d + 2\pi^d$ and welfare in segmentation equilibrium is $V^m + \pi^m$. There are two inefficiencies in the segmentation equilibrium. First, because consumers are not informed about all prices, there is mismatch between a consumer's preference and the firm's offer. Ann agglomeration equilibrium avoids such mismatch. Second, because $p^m < p^d$, the quantity bought by consumers in an agglomeration equilibrium is (weakly) higher. Both effects imply that welfare in the agglomeration equilibrium is higher than in the segmentation equilibrium. However, platforms induce segmentation if competition between firms is fierce because their incentives are driven by p^m versus p^d. Instead, welfare considerations are driven by V^m versus V^d.

4   Multi-Homing of Consumers

So far, we focused on the case in which consumers are single-homing. In this section, we show that our qualitative results extend to multi-homing consumers, as long as not all consumers multi-home.

To this end, assume that a fraction $\alpha \in [0, 1]$ of consumers join both platforms. An interpretation is that consumers incur some time cost to be active on the second platform. Consumers are heterogeneous with respect to these time costs, implying that only those consumers with low enough time costs are active on both platforms. A higher α can then be interpreted as a reduction in time costs.[11]

The profits of the firms need then to be modified. In fact, a firm will never obtain the full monopoly profit because there are always some consumers who have seen the offers of both firms in each category. In particular, in the segmentation equilibrium where one firm lists on platform A and the other on platform B, half of the single-homing consumers are active on platform A and the other half on platform B. Because there is a mass $1 - \alpha$ of single-homing consumers, each firm has a mass of $(1 - \alpha)/2$ of single-homing consumers and a mass α of multi-homing consumers. The total consumer mass is then $(1 + \alpha)/2$.

Firms do not know which consumer is a single-homing and which one is a multi-homing one and set a single price in the product market. Since single-homers are less price-sensitive than the multi-homers, the equilibrium price with only single-homers is larger than that with only multi-homers. Hence, with a single price in the product market, the equilibrium price depends on α. We can therefore write the expected profit that a firm obtains from a consumer as $\pi(\alpha)$. In particular, $\pi(0) = \pi^m$ and $\pi(1) = \pi^d$. As α gets larger, we obtain $\pi'(\alpha) \leq 0$; hence, for all $\alpha \in [0, 1]$, $\pi(\alpha) \in [\pi^d, \pi^m]$. Below, we will show how a change in α plays out in the two examples of demand functions.[12]

Deriving the equilibrium with multi-homing consumers, we obtain the following proposition:

Proposition 5. *All results of Propositions 1 to 4 carry over to the case of consumer multi-homing with the exception that $\pi^m/2$ needs to be replaced by*

$$\pi(\alpha)\frac{1 + \alpha}{2}.$$

The proposition shows that the qualitative result of Propositions 1 to 4 remains valid

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[11]For example, if distribution S of time costs among consumers first-order stochastically dominates distribution S', then the latter distribution leads to a higher fraction α of multi-homing consumers.

[12]In any agglomeration equilibrium, a firm's profit is unchanged since all consumers see both offers. This leads to a profit of π^d for each firm.

if consumers can multi-home. With multi-homing consumers, segmentation does not give firms monopoly power over their consumers. Segmentation nevertheless lowers the competitive pressure because some consumers are still only informed about one firm's offer, and this will be exploited by platforms.

The question arises if platforms benefit from multi-homing of consumers. If we are in the range of the agglomeration equilibrium, nothing changes compared to single-homing consumers because platforms are in Bertrand competition. However, this is not true for the regions in which the segmentation equilibrium occurs with some or full probability. There are two countervailing forces. First, platforms have more consumers, which leads to a larger demand for firms. This allows platforms to charge higher listing fees and is therefore beneficial to platforms' profits. This effect can also be seen in the formulas: Instead of serving a consumer mass of $1/2$ (as with single-homing consumers), platforms now have a mass of $(1+\alpha)/2$ of consumers. However, the countervailing force is that firms make smaller profits in the product market because some consumers are now informed about both offers. As can be seen in the formulas, the profit is now $\pi(\alpha) < \pi^m$. It follows that platforms are hurt by the possibility of consumers to multi-home if the competition effect dominates the demand-enhancing effect.

We can illustrate this result with the example of the concrete demand functions. In the case of Hotelling demand, $\pi(\alpha)$ is given by

$$\frac{(2-\alpha)(\alpha t + 2(v-c)(1-\alpha))^2}{2t(4-3\alpha)^2}.$$

Comparing $\pi(\alpha)(1+\alpha)/2$ with $\pi^m/2$, we obtain a non-monotonic effect in α. Taking the derivative of $\pi(\alpha)(1+\alpha)/2 - \pi^m/2$ with respect to α at $\alpha = 0$ yields $(v-c)/8 > 0$. This implies that platforms benefit from a small fraction of multi-homing consumers. However, as α gets larger the difference between $\pi(\alpha)(1+\alpha)/2$ and $\pi^m/2$ falls in α. (In the limit, as $\alpha \to 1$, $\pi(\alpha)(1+\alpha)/2 \to \pi^d$, implying that for the whole parameter range, only the agglomeration equilibrium with zero profits for platforms exist.)

A similar picture arises with the linear demand example. Here,

$$\pi(\alpha) = \frac{(\beta-\gamma)(\beta-\gamma^2(1-\alpha)(\beta-\gamma(1-\alpha))^2(1-c)^2}{(\beta+\gamma)(2(\beta-\gamma^2)-\alpha\gamma(1-2\gamma))^2}.$$

Taking the derivative of $\pi(\alpha)(1+\alpha)/2 - \pi^m/2$ with respect to α at $\alpha = 0$ yields $(\alpha - c)^2/(8(\beta + \gamma) > 0$. Therefore, a small fraction of multi-homing consumers increases platforms' profits in this case as well.

5   Multi-Homing of Firms

In this section we consider the effects of multi-homing of firms. In contrast to consumers, firms need to pay for being active on the platforms. Therefore, even without any costs for using a second platform, firms do not necessarily find it profitable to join both platforms.

We therefore proceed in a different way than in the last section by assuming that firms decide to single-home or to multi-home (or not to participate at all). They do not incur any intrinsic costs from doing so but need to pay the listing fees. We are particularly interested if platforms benefit from the possibility that firms can multi-home. The literature on two-sided markets predicts that platforms can exploit the multi-homing side because they do not compete for agents on this side (see e.g., Armstrong, 2006, or Hagiu, 2006). The question is if this is also true in our framework in which agents compete against each other.

With multi-homing firms, new possibilities for the distribution of firms come into play. First, both firms in a segment may multi-home. In that case, all consumers are exposed to the offer of both firms, implying that each firm receives the duopoly profit π^d per consumer. But the profit per consumer is then equivalent to that in the situation where both firms agglomerate on one platform. In the latter case, however, firms only have to pay the listing fee of one platform. Therefore, the agglomeration equilibrium configuration is weakly preferred by firms. In fact, firms are indifferent only if both listing fees are zero. We assume that firms choose the agglomeration equilibrium then. This assumption is without loss of generality because both configurations give rise to the same surplus for all agents.

Second, a configuration is possible in which one firm in a category single-homes and the other one multi-homes.[13] Competition in the product market works then differently to the situation described above. In particular, denote the mass of consumers on the platform on which both firms are active by x and the mass on the platform in which only one firm is active by $1 - x$. Then, there is asymmetric competition in the product market. A mass $1 - x$ of consumers can only buy from the multi-homing firm whereas the remaining mass can buy from both firms. Let us denote the profit of the multi-homing firm by $\pi^{MH}(1 - x)$ and the profit of the single-homing firm by $\pi^{SH}(x)$. To save on notation we denote $\pi^{SH}(1/2)$ by π^{SH} and $\pi^{MH}(1/2)$ by π^{MH}.[14]

Finally, with multi-homing firms, the situation can occur in which there are multiple equilibria in the fee-setting game between platforms. If this occurs, we use as a selection

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[13]This situation can never occur in equilibrium because the best response of consumers is then to join the platform on which both firms are active. Multi-homing is then not a best response because the firm receives no consumers on one of the platforms where it is active. However, the configuration can occur as a potential deviation.

[14]We will provide the formulas in the examples with our demand functions below.

criterion that platforms coordinate on the profit-dominant equilibrium.

We can now establish the equilibrium with multi-homing firms.

Proposition 6.
- *In the agglomeration and segmentation region, the equilibrium is the same as that characterized in Propositions 1 and 2.*

- *In the first mixing region, the equilibrium is the same as that characterized in Proposition 3 if $\pi^{MH} < 3/2\pi^m - 2\pi^d$.*
  Similarly, in the second mixing region, the equilibrium is the same as that characterized in Proposition 4 if $\pi^{MH} < 3/4\pi^m$.

- *In the first and second mixing region, for $\pi^{MH} \geq 3/2\pi^m - 2\pi^d$ and $\pi^{MH} \geq 3/4\pi^m$, respectively, in equilibrium platforms set fees of $f_A^\star = f_B^\star = 0$ and firms play an agglomeration equilibrium.*

The proposition shows that for some parameter constellations, the equilibrium derived in Propositions 1 to 4 stays unchanged. Foremost, if competition between firms is relatively fierce, the segmentation equilibrium still exists. Although firms can multi-home, this reduces their profits by too large an amount and so they prefer segmentation. Again platforms exploit this by charging high listing fees. Therefore, our insight that segmentation leads to high platform profits although platforms are homogeneous, is robust to multi-homing of firms.

The proposition also shows that the equilibrium with segmentation occurs for a smaller parameter range than in case of single-homing firms. In the range in which segmentation no longer occurs with multi-homing firms, platforms charge zero listing fees and obtain zero profits. We therefore obtain the result that platforms can exploit agents less if agents multi-home—a result in contrast to the standard insight derived on two-sided markets.

The intuition behind the result is as follows: homogeneous platforms make positive profits because they allow firms to segment themselves and thereby reduce competition in the product market. If firms can multi-home, segmentation may break down because firms have an incentive to deviate from the segmentation equilibrium. As can be seen in the proposition, segmentation is more likely to break down if the profit of a multi-homing firm π^{MH} becomes large. As a result, platforms can no longer charge high fees and exploit the possibility that they grant monopoly power to firms. The homogeneity of the platforms then drives fees and profits down to zero and, due to the indirect network effects, firms choose agglomeration.

We can illustrate the result with the help of the two examples of Hotelling and linear

demand. Determining π^{MH} for Hotelling demand yields

$$\pi^{MH} = \frac{3(2(v-c)+t)^2}{100t}.$$

Comparing this with half of the monopoly profit $(v-c)^2/(8t)$ yields that $\pi^{MH} > \pi^m/2$ if and only if $t > (v-c)(5/\sqrt{6}-2) \approx 0.412(v-c)$. The threshold for t is then in the second mixing region, which is relevant for values of t between $(v-c)/3 \approx 0.333(v-c)$ and $(v-c)3/7 \approx 0.429(v-c)$. Therefore, for the whole first mixing region, the possibility of firms to multi-home destroys the segmentation equilibrium.

The result is even more extreme for the linear demand example. Here,

$$\pi^{MH} = \frac{(\beta-\gamma)(2\beta-\gamma^2)(2\beta+\gamma)(1-c)^2}{2(\beta+\gamma)(4\beta-\gamma-2\gamma^2)^2}.$$

Comparing this with $\pi^m/2 = (\alpha-c)^2/(8\beta)$ yields that $\pi^{MH} > \pi^m/2$ for all $\gamma < 0.89\beta$. Since the first and second mixing regions are relevant for γ between 0.62β and 0.85β, we obtain that the segmentation equilibrium does not exist in these regions.

6 Buyer-Preferred Equilibrium

In this subsection, we demonstrate how our equilibrium selection would change if we used the concept of Pareto dominance of consumers (instead of sellers) in addition to Strong Nash. Because $V^d > V^m$, consumers prefer an agglomeration equilibrium over a segmentation or stand-alone equilibrium. This implies that whenever the refinement of Strong Nash alone does not suffice to select a unique equilibrium, it is Pareto dominant for consumers to choose an agglomeration equilibrium whenever it exists. In the range $\pi^d \leq \pi^m/2$, an agglomeration equilibrium does not exist at stage 2 if both listing fees are larger than π^d. In the case where the segmentation and a stand-alone equilibrium co-exist, only the segmentation equilibrium is a Strong Nash one. Therefore, the segmentation equilibrium is selected whenever it exists in this range. For $\pi^d > \pi^m/2$, the selected equilibrium stays unchanged. The next figure represents the different equilibrium regions with this concept.

Turning to the first stage, if $\pi^d < 1/4\pi^m$, platforms set their fees in equilibrium (i.e., $f_A^\star = f_B^\star = \pi^m/2$) so that the segmentation configuration is still an equilibrium of the full game.[15]

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[15]There also exists an equilibrium in which platforms set their fees equal to zero, thereby inducing agglomeration. However, since platforms obtain zero profits in such an equilibrium, the one with fees of $\pi^m/2$ is Pareto dominant for platforms.
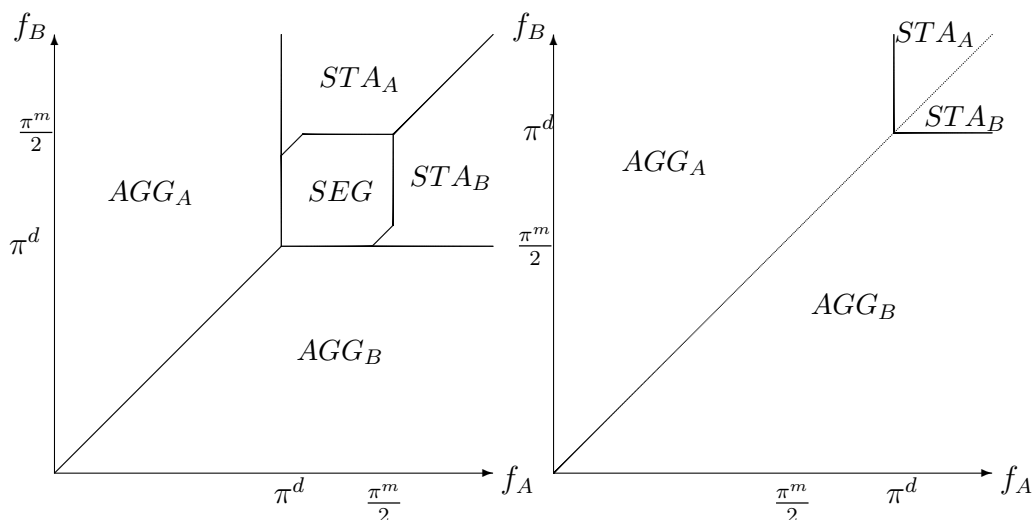
Figure 3: Selected equilibrium configurations with Pareto-dominance of consumers

7 Conclusion

We propose a model of competing platforms that bring together buyers and sellers. Platforms are homogeneous and set listing fees to sellers who compete and against each other in the product market. Generally speaking, we have analyzed how the competitive environment between agents on one side of the market affects the platform market structure. Based on the Bertrand logic adjusted to platform markets, one may expect that, due to positive network effects, only one platform will be active in the market. As argued in the introduction, this is not what we frequently observe in real life.

Can multiple intermediaries exist and make positive profits, given there is no differentiation between them? We show that the function of multiple intermediaries as a differentiation device for competitive sellers explains such an outcome. To obtain such an outcome, sellers must choose to be active on different platforms and thereby avoiding fierce competition with each other. Platforms can exploit this by demanding positive fees and thus obtain strictly positive equilibrium profits. Thus imperfect competition among sellers can explain that homogeneous platforms can survive in the market and make positive profits. Such a segmentation equilibrium exists if competition between sellers is sufficiently strong. If, by contrast, there is little competition between sellers the standard intuition is confirmed and the equilibrium features agglomeration; i.e., all buyers and sellers go to the same platform.

For moderate degrees of competition between sellers the equilibrium features mixing by platforms. In this equilibrium, agglomeration and segmentation occur with positive probability. The price distribution generically features at least one mass point and its support is, under some condition, non-convex. Overall this paper informs us whether multiple platforms can co-exist as a function of the intensity of competition among sellers.

We have also shown that the possibility of firms to multi-home may break the segmentation equilibrium. The agglomeration equilibrium then occurs on a larger parameter range. Since platforms obtain zero profits in the agglomeration equilibrium, multi-homing of sellers hurts platforms. This insight contrasts with results from two-sided markets with differentiated platforms, which finds that multi-homing agents can be exploited because platforms do not compete for them (e.g., Armstrong, 2006).

In our model, we assumed that platforms do not incur fixed costs independent of the number of participants the platform is catering to. If platforms had (arbitrarily small) fixed cost, instead of an agglomeration equilibrium with zero profits for both platforms,[16] with endogenous entry, only one platform would be present in the market and the market would be a natural monopoly. Thus, with fixed costs only one platform is present and obtains a large profit when there is little competition between sellers. In such a setting fierce competition between sellers is needed to avoid monopolization of the platform market.

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[16]Zero profits obtain, since, at the participation stage of buyers and sellers, we selected the equilibrium which is most favorable to sellers.

8   Appendix

8.1   Appendix A: Equilibrium Selection in Stage 2

We start by determining all Nash equilibria in stage 2, given $\{f_i, f_{-i}\}$. In a second step, we show how the refinement of Strong Nash plus profit dominance of sellers singles out a unique equilibrium.

First suppose that the mass of sellers is unequal on both platforms, i.e., α_i sellers are on platform i and $\alpha_{-i} < \alpha_i$ sellers are on platform $-i$. Then all consumers will join platform i. This implies that the sellers on platform $-i$ have a profitable deviation to either go the platform i or be inactive for any $f_{-i} > 0$. It follows that in equilibrium either one platform has no sellers and no consumers, or $\alpha_i = \alpha_{-i}$, which makes consumers indifferent and induces them to split equally between the two platforms.

We start with the first type of equilibrium, in which one platform is inactive. As sellers are homogeneous across categories, there cannot be an equilibrium in which sellers in different categories follow different strategies. The reason is that if it is profitable for one or both sellers in some categories to list on a platform, this must also be true for the remaining categories. Given this, there are two equilibrium confidurations in which only one platform is active.

The first configuration is an agglomeration equilibrium, in which all sellers and all buyers agglomerate on one platform. A seller's profit is then π^d. Hence, an equilibrium in which agglomeration on platform i takes place exists if $f_i \leq \pi^d$, independent of f_{-i}.[17] The second equilibrium configuration is stand-alone equilibrium, in which in each category only one seller is active on platform i and all consumers go to platform i. This configuration occurs if $\pi^m \geq f_i > \pi^d$, independent of f_{-i}. This equilibrium cannot occur with $f_i < \pi^d$, as then both sellers in each category prefer to be active on platform i.[18]

We now turn to the third Nash equilibrium type in which $\alpha_i = \alpha_{-i}$. There are three possibilities for $\alpha_i = \alpha_{-i}$ to arise.

(i) In each category, one seller lists on platform i and one seller lists on platform $-i$.

(ii) In $1/2$ of the categories, both sellers list on platform i and in the other half both sellers list on platform $-i$.

(iii) In $1/2$ of the categories, one seller lists on platform i and in the other half one seller lists on platform $-i$.

Note that it can never be an equilibrium that in fewer than $1/2$ of the categories both or one seller list on platform i and platform $-i$. The reason is that in the categories,

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[17]The configuration is referred to in Figure 1 as AGG_i.

[18]The configuration is referred to in Figure 1 as STA_i.

in which sellers are not active, they must have a profitable deviation to become active. This is because platform fees must be such that the resulting profits are are higher than the listing fees as otherwise there can be no categories in which sellers are willing to list.

We now show that possibilities (ii) and (iii) can never occur in equilibrium. Consider case (ii). Since in $1/2$ of the categories, both sellers are active on platform i, we must have $f_i \leq \pi^d/2$. If a seller active on platform $-i$ then deviates to platform i, its profit changes from $\pi^d/2 - f_{-i}$ to $\pi^m/2 - f_i$. By a similar argument, if a seller deviates from platform i to platform $-i$, it profit changes from $\pi^d/2 - f_i$ to $\pi^m/2 - f_{-i}$. This implies that case (ii) can only be an equilibrium if $\pi^d/2 - f_{-i} \geq \pi^m/2 - f_i$ and $\pi^d/2 - f_i \geq pi^m/2 - f_{-i}$. Since $\pi^m > \pi^d$, both conditions cannot hold jointly, implying that there must be profitable deviation. Similarly, in case (iii) platform fees must be smaller than $\pi^m/2$, which implies that non-active sellers have a profitable deviation to list on the platform in which the competitor is not active.

As a consequence, the configuration in which both platforms are active can only be such that each platform is host to one seller in each category. This equilibrium can only occur if platform fees are below $\pi^m/2$ and no seller has an incentive to switch to the other platform because it can achieve a higher profit there. The latter condition implies

$$\frac{\pi^m}{2} - f_i \geq \frac{\pi^d}{2} - f_{-i}$$

Rewriting this condition, we obtain that a segmentation equilibrium exists if and only if

$$f_i \leq \min\left\{\frac{\pi^m - \pi^d}{2} + f_{-i}, \frac{\pi^m}{2}\right\}. \tag{1}$$

As can be visualized in Figure 1, for any combination of listing fees $\{f_i \leq \pi^m, f_{-i} \leq \pi^m\}$, multiple equilibria exist in stage 2.

We now demonstrate how our selection rule singles out a unique equilibrium. We start with the cases in which only a single equilibrium configuration exists (i.e., agglomeration or stand-alone) but multiple equilibria occur because agents can coordinate on either platform. First, consider the case in which there are the two equilibria, where all sellers (and consumers) agglomerate on platform A or platform B. If both platforms charge a fee below π^d, profit-dominance of sellers implies that they coordinate on the platform with the lower fee, that is, they agglomerate on platform i if $f_i \leq f_{-i}$. Similarly, if both platform charge a fee larger than π^d and the two equilibria in which only half of the sellers are active on platform A or on platform B exist (i.e., the stand-alone equilibria), then sellers choose platform i if and only if $f_i \leq f_{-i}$.

Now we turn to cases, in which multiple equilibrium configurations exist. First, consider the case in which both an agglomeration equilibrium and a stand-alone equilibrium

exists. From the arguments above, this occurs if one platform, say platform $-i$, charges a fee below π^d whereas the other one charges a fee above π^d. However, the stand-alone equilibrium is then not a Strong Nash equilibrium because a coalition consisting of all consumers and all inactive sellers has a profitable deviation. If all these agents choose to be active on platform $-i$, then consumers are indifferent (as the same number of sellers is then active on each platform) but the profits of the formerly inactive sellers strictly increase from 0 to $\pi^m - f_i > 0$. By the same argument, if both a stand-alone equilibrium and a segmentation equilibrium exist, the stand-alone one is not a Strong Nash equilibrium. By contrast, the segmentation equilibrium is a Strong Nash one. The reason is that such a co-existence only occurs if both fees are larger than π^d and $\pi^d < \pi^m/2$. Then, no coalition of sellers has the incentive to deviate from a segmentation equilibrium.

Finally, we turn to the region, in which both a segmentation and an agglomeration equilibrium exist. The profit of each seller in an agglomeration equilibrium on platform i is $\pi^d - f_i$. By contrast, in a segmentation equilibrium, the profit of a sellers is either $\pi^m/2 - f_i$ or $\pi^m/2 - f_{-i}$ dependent on which platform the seller is active. Let us first look at the case $\pi^d > \pi^m/2$. It is evident that a coalition of all sellers active on the platform with the higher fee, say platform i (i.e., $f_i \geq f_{-i}$), and all consumers on this platform now have a profitable deviation to switch to platform $-i$. After such a deviation, the sellers are better off because $\pi^d - f_{-i} > \pi^m/2 - f_i$ due to the fact that $\pi^d > \pi^m/2$ and $f_{-i} \leq f_i$, and consumers are better off because $V^d > V^m$. It follows that for $\pi^d > \pi^m/2$, the segmentation equilibrium is eliminated.

We turn to the case $\pi^d \leq \pi^m/2$. We first show that a similar mechanism as the one in the previous paragraph does only partly work then. In particular, a seller on platform i (i.e., the platform with the higher fee) wants to deviate from the segmentation equilibrium if and only if $\pi^m/2 - f_i < \pi^d - f_{-i}$ or

$$f_i > \frac{\pi^m}{2} - \pi^d + f_{-i}.$$

If this inequality holds, the segmentation equilibrium is not a Strong Nash one. This shrinks the range for the segmentation equilibrium. In particular, for fees below π^d, the equilibrium was valid for $f_i \leq (\pi^m - \pi^d)/2 + f_{-i}$ without the refinement, whereas with the refinement it is valid only if $f_i \leq \pi^m/2 - \pi^d + f_{-i}$.[19] If instead $f_i \leq \pi^m/2 - \pi^d + f_{-i}$, the refinement of Strong Nash has no bite. However, the refinement of profit-dominance for sellers then selects the segmentation equilibrium as the unique equilibrium. In particular, the inequality ensures that sellers on platform i are better off in the segmentation equilibrium than in the agglomeration equilibrium, and the condition $\pi^d <$

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[19]If both fees are above π^d there is now restriction because the agglomeration equilibrium does not exist then.

$\pi^m/2$ guarantees that also sellers on platform $-i$ prefer segmentation over agglomeration because $\pi^m/2 - f_{-i} > \pi^d - f_{-i}$.

Therefore, our equilibrium refinement selects a unique equilibrium for all $\{f_i \leq \pi^m, f_{-i} \leq \pi^m\}$. This unique equilibrium can be written as follows:

(i) If $\pi^d > \pi^m/2$, then

- for $f_i, f_{-i} \geq \pi^d$, the equilibrium is STA_i if $f_i \leq f_{-i}$;

- for all other values, the equilibrium is AGG_i if $f_i \leq f_{-i}$.

(ii) If $\pi^d \leq \pi^m/2$, then

- for $f_i, f_{-i} \leq \pi^d$, the equilibrium is SEG if $f_{-i} \leq \pi^m/2 - \pi^d + f_i$, and AGG_i if $f_{-i} > \pi^m/2 - \pi^d + f_i$.

- for $f_i \leq \pi^d$ and $f_{-i} > \pi^d$, the equilibrium is AGG_i.

- for $f_i, f_{-i} \in (\pi^d, \pi^m/2]$, the equilibrium is SEG if $f_{-i} \leq (\pi^m - \pi^d)/2 - f_i$ and AGG_i if $f_{-i} > (\pi^m - \pi^d)/2 - f_i$.

- for $f_i > \pi^d$ and $f_{-i} > \pi^m/2$, the equilibrium is STA_i if $f_i \leq f_{-i}$.

8.2 Appendix B: Proof of Propositions

Proof of Proposition 1. Agglomeration equilibrium. Suppose that the agglomeration equilibrium is played in the second stage. A firm's profit in this equilibrium is $\pi^d - f_i$, if it is listed on platform i. It follows that a firm is willing to participate as long as $f_i \leq \pi^d$. Therefore, an agglomeration equilibrium can be obtained with fees $(f_A, f_B) \in [0, \pi^d] \times [0, \pi^d]$. We focus on equilibria, which are preferred by the firms at stage 2. Firms and consumers will therefore coordinate on the equilibrium at stage 2 such that they list on the platform with the lower fee. As a consequence, all agglomeration equilibria with strictly positive listing fees do not survive our selection criterion. It follows that there is a unique equilibrium within the set of all agglomeration equilibria that survives our selection criterion with listing fees $(f_A^*, f_B^*) = (0, 0)$.

We can now determine under which conditions the agglomeration equilibrium with listing fees $(f_A^*, f_B^*) = (0, 0)$ exists. If firms and consumers in the second stage play the agglomeration equilibrium, a firm's profit is π^d. Instead, if the segmentation equilibrium is played, a firm's profit equals $\pi^m/2$. Hence, given listing fees $(f_A^*, f_B^*) = (0, 0)$, the agglomeration equilibrium is preferred from the firm's perspective if

$$\pi^d \geq \frac{\pi^m}{2}.$$

Now consider listing fees $(f_A, f_B) \neq (0, 0)$ but $f_A, f_B \leq \pi^d$ It is evident, that, as long as $\pi^d \geq \pi^m/2$, firms prefer the agglomeration equilibrium on the platform with the lower listing fee to the segmentation equilibrium. Therefore, in the region $\pi^d \geq \pi^m/2$, a segmentation equilibrium does not exist. It follows that in this region, the unique equilibrium is an agglomeration equilibrium with listing fees $(f_A^*, f_B^*) = (0, 0)$. $\qquad \square$

Proof of Proposition 2. Segmentation equilibrium. Consider the region of $\pi^d < \pi^m/2$. In a segmentation equilibrium, a firm active on platform i obtains profits of $\pi^m/2 - f_i$. Therefore, the highest possible fees that platform can charge equals $\pi^m/2$, leaving firms with zero profits. Let us first determine under which conditions an equilibrium with listing fees $\pi^m/2$ exist. If both platforms charge $f_i = \pi^m/2$, the only possible configuration in the second stage is the separating equilibrium. This follows because the profit that a firm obtains in the agglomeration configuration equals π^d, which is below the listing fee. Therefore, we can focus on deviations in the listing fees in the first stage.

Suppose that platform i deviates to induce an agglomeration equilibrium in the second stage such that all participate on platform i. To do so, it needs to charge a lower fee $f_i^d = \pi^d - \epsilon$, where $\epsilon > 0$ can be arbitrarily small. Since all consumers will agglomerate on platform i if all firms do, firms earn then a small positive profit when agglomerating on platform i but zero in the segmentation equilibrium. The deviation profit of platform i is then (letting $\epsilon \to 0$) $\Pi_i^d = 2\pi^d$. A deviation is therefore not profitable if $\pi^m/2 > 2\pi^d$ or

$$\pi^d < \frac{\pi^m}{4}.$$

It follows that in the region $\pi^d < 1/4\pi^m$, a segmentation equilibrium with listing fees $(f_A^\star, f_B^\star) = (\pi^m/2, \pi^m/2)$ is the unique equilibrium. Platforms' equilibrium profits are $\pi^m/2$. $\qquad \square$

Proof of Proposition 3. Convex randomization domain. We first show the *non-existence of a pure-strategy equilibrium.* Consider the region of $\pi^m/4 \leq \pi^d < \pi^m/2$. We know that, in this region, a pure-strategy segmentation equilibrium will be played in the second stage if both platforms charge the same listing fees (conditional on these fees being lower than $\pi^m/2$, which will always be fulfilled in equilibrium). However, platforms cannot extract the full profits from firms because then each platform will have an incentive to deviate and induce firms and consumers to play an agglomeration equilibrium. We proceed by first determining the highest fee that platforms can charge to make such a downward deviation unprofitable. Suppose that both platforms charge a fee of $\pi^m/2 - x$. The platforms' resulting profit is then $\pi^m/2 - x$, whereas the profit of a firm is x. If platform i deviates to attract all firms and consumers in the second stage, it must offer firms at least a profit of x. This implies that its fee must be such that $\pi^d - f_i^{dev} > x$. The

highest possible deviation listing fee is therefore $f_i^{dev} = \pi^d - x - \epsilon$, leading to a deviation profit of (letting $\epsilon \to 0$) $2\pi^d - 2x$. Such a deviation is unprofitable if $\pi^m/2 - x \geq 2\pi^d - 2x$ or $x \geq 2\pi^d - \pi^m/2$. Hence, with an x equal to $2\pi^d - \pi^m/2$, platforms prevent such a downward deviation. The resulting listing fee is then

$$f_i = \pi^m/2 - x = \pi^m - 2\pi^d$$

and the platforms profit is also $\pi^m - 2\pi^d$.

To determine if listing fees $f_i = f_j = \pi^m - 2\pi^d$ can constitute an equilibrium, we need to check if a platform has an incentive to deviate by charging a higher listing fee (upward deviation). Suppose that platform i charges $f_i = \pi^m - 2\pi^d$ and platform j charges a deviation fee $f_j^{dev} > f_i$ such that segmentation is still the continuation equilibrium in the second stage. To induce a segmentation equilibrium, we must have $\pi^m/2 - f_j^{dev} > 2\pi^d - 2x = 3\pi^d - \pi^m$. The right-hand side is the profit that firms obtain when agglomerating on platform i. The inequality therefore states that a firm's profit when listing on platform j in a segmentation equilibrium is higher than in an agglomeration equilibrium. The highest possible listing fee is therefore $f_j^{dev} = 3\pi^m/2 - 3\pi^d - \epsilon = 3(\pi^m/2 - \pi^d) - \epsilon$ which yields a larger platform profit than $f_i = 2(\pi^m/2 - \pi^d)$. As a consequence, a profitable upward deviation exists and both platforms charging listing fees of $\pi^m - 2\pi^d$ cannot constitute an equilibrium.

It follows that in the range of $\pi^m/4 \leq \pi^d < \pi^m/2$ no equilibrium in pure strategies exists. The only candidate equilibrium, which prevents downward deviations was $f_i = f_j = \pi^m - 2\pi^d$ but then an upward deviation is profitable. In turn, for all listing fees above $\pi^m - 2\pi^d$, a downward deviation is profitable. We will now characterize the mixed-strategy equilibrium.

Randomization domain. We now derive the randomization domain of the first mixed strategy equilibrium using platforms' best responses. Suppose platform j sets a fee f_j to induce segmentation. This implies that a firm on platform j receives a profit of $\pi^m/2 - f_j$ and the platform j receives a profit of f_j. f_j must not be higher than $\pi^m/2$. We know derive the maximum and the minimum of platform $i \neq j$'s best response to f_j. Given f_j, the best downward deviation $f_i^{dev^-}$ just triggers agglomeration on platform i, i.e., firms prefer agglomeration on i to segmentation on j: $\pi^d - f_i^{dev^-} > \pi^m/2 - f_j$. This yields platform i a deviation profit of $2f_i^{dev^-} = 2(\pi^d - \pi^m/2 + f_j)$. For the same f_j, the best upward deviation $f_i^{dev^+}$ induces segmentation with the highest possible fee on platform i, i.e., firms (weakly) prefer segmentation on i to agglomeration on j: $\pi^m/2 - f_i^{dev^-} \geq \pi^d - f_j$. This leads to a deviation profit of $f_i^{dev^+} = \pi^m/2 - \pi^d + f_j$.

The two deviation profits are equal at $f_j = 3/2\pi^m - 3\pi^d$. Platform i's best response to $f_j = 3/2\pi^m - 3\pi^d$ is to induce segmentation at $f_i^{dev^+} = \pi^m/2 - \pi^d + f_j = 2\pi^m - 4\pi^d$.

Since we consider the range where $\pi^d < \pi^m/2$, for $f_j > 3/2\pi^m - 3\pi^d$, platform i's deviation profit from its best downward deviation is higher than that from its best upward deviation, i.e., $2(\pi^d - \pi^m/2 + f_j) > \pi^m/2 - \pi^d + f_j$. This implies that its best response to $f_j > 3/2\pi^m - 3\pi^d$ is $f_i^{dev^-} = \pi^d - \pi^m/2 + f_j$. $f_i^{dev^-}$ is increasing in f_j. Vice versa, platform i's best response to $f_j \leq 3/2\pi^m - 3\pi^d$ is $f_i^{dev^+} = \pi^m/2 - \pi^d + f_j$, where $f_i^{dev^+}$ is also increasing in f_j. It follows that the maximum of platform i's best response to f_j is reached at $f_i^{dev^+} = 2\pi^m - 4\pi^d$ for $f_j = 3/2\pi^m - 3\pi^d$. By symmetry, this leads to an upper interval of the randomization domain equal to $f_j \in [3/2\pi^m - 3\pi^d, 2\pi^m - 4\pi^d]$. Analogously, the minimum of platform i's best response to f_j is reached at $f_i^{dev^-} = \pi^m - 2\pi^d$ for $f_j = 3/2\pi^m - 3\pi^d$, respectively). This leads to a lower interval of the randomization domain equal to $f_j \in [\pi^m - 2\pi^d, 3/2\pi^m - 3\pi^d)$.

As a consequence, there is mixed-strategy equilibrium in which $f_i, f_j \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$. The expected profit in this equilibrium is $3\pi^m/2 - 3\pi^d$. This is because when charging this fee, a platform induces the segmentation with probability 1.

In this mixed-strategy equilibrium, the highest listing fee is $2\pi^m - 4\pi^d$. To ensure participation of firms, the highest fee a platform can charge (in a segmentation equilibrium) is $\pi^m/2$. Therefore, the equilibrium characterized above is only valid if $2\pi^m - 4\pi^d \leq \pi^m/2$ or $\pi^d \geq 3/8\pi^m$.

Mixing probabilities. In the range $3/8\pi^m \leq \pi^d < 1/2\pi^m$, platforms set fees in the domain $f_i, f_j \in [\pi^m - 2\pi^d, 2\pi^m - 4\pi^d]$ and the expected profit is $\Pi_A^\star = \Pi_B^\star = 3\pi^m/2 - 3\pi^d$.

Let $\delta \equiv (\pi^m/2 - \pi^d)$ and $\epsilon > 0$ but infinitesimally small. Denote $\underline{f} \equiv 2\delta$, $\tilde{f} \equiv 3\delta$, and $\overline{f} \equiv 4\delta$ such that the domain of interest can be expressed as $f_i, f_j \in [\underline{f}, \overline{f}] = [2\delta, 4\delta]$. For $i \neq j$ and $i, j \in \{A, B\}$, the corresponding best response function is given by

$$\hat{f}_i(f_j) = \begin{cases} f_j - \delta - \epsilon, & \text{if } f_j \in (\tilde{f}, \overline{f}]; \\ f_j + \delta, & \text{if } f_j \in [\underline{f}, \tilde{f}]. \end{cases}$$

We know that all fees in the mixing domain should give an expected profit of 3δ because setting a fee of 3δ is triggering a segmentation equilibrium with probability 1 yielding a profit of 3δ. In this mixing domain we need to distinguish between two cases, a lower and an upper range. The lower range is $f_i \in [2\delta, 3\delta]$ and the upper range from $f_i \in [3\delta, 4\delta]$. The reason for this distinction is that in the lower range, firms may agglomerate on platform i (i.e., this happens if $f_j > f_i + \delta$) but will never agglomerate on platform j. That is, if f_i is in this lower range, platform i will always obtain a positive profit. By contrast, if f_i is an element of the upper range, with some probability firms will choose to agglomerate on platform j (i.e., this occurs if platform j charges

$f_j < f_i - \delta$) and platform i obtains no profit then. This can be expressed as follows

$$\Pi_i(f_i, f_j) = \begin{cases} 0, & \text{if } f_i \in (f_j + \delta, 4\delta] \wedge f_j \in [2\delta, 3\delta); \\ f_i, & \text{if } f_i \in [\max\{2\delta, f_j - \delta\}, \min\{f_j + \delta, 4\delta\}] \wedge f_j \in [2\delta, 4\delta]; \\ 2f_i, & \text{if } f_i \in [2\delta, f_j - \delta) \wedge f_j \in (3\delta, 4\delta]. \end{cases}$$

Let us start with the case in which platform i charges a fee in the lower range, that is, $f_i \in [2\delta, 3\delta]$. Denote the cumulative density function with which platform j mixes by $G(f_j)$. The profit of platform i when setting fees in this lower range is then given by (abbreviating f_i by f)

$$G(f + \delta)f + (1 - G(f + \delta))\, 2f.$$

In equilibrium, this must be equal to 3δ, yielding a first equation of

$$G(f + \delta)f + (1 - G(f + \delta))\, 2f = 3\delta. \tag{2}$$

This equation determines the mixing probabilities of platform j in its upper range. This is because only if platform j sets a fee above $f + \delta$ (which happens with probability $1 - G(f + \delta)$), firms will agglomerate on platform i. Such a fee must necessarily be in the upper range.

In case platform i charges a fee from the upper range, that is, $f_i \in [3\delta, 4\delta]$, the equation is

$$G(f - \delta)0 + (1 - G(f - \delta))\, f = 3\delta. \tag{3}$$

This equation determines the mixing probability in the lower range.

Let us first look at (2). We can substitute $h \equiv f + \delta$ to get

$$G(h)\,(h - \delta) + (1 - G(h))\, 2\,(h - \delta) = 3\delta. \tag{4}$$

Therefore, h is the fee in the upper range. Recall that (2) was relevant for f in the lower range and since $h = f + \delta$, these are exactly the fees in the upper range. Solving (4) for $G(h)$ gives

$$G(h) = \frac{2h - 5\delta}{h - \delta}. \tag{5}$$

It is easy to check that $G(4\delta) = 1$.

Now we turn to (3). Here we can substitute $h \equiv f - \delta$ representing that h is now in the lower range. We obtain

$$(1 - G(h))\,(h + \delta) = 3\delta. \tag{6}$$

Solving (6) for $G(h)$ gives

$$G(h) = \frac{h - 2\delta}{h + \delta}. \tag{7}$$

It is easy to check that $G(2\delta) = 0$. Using (5) and (7), we obtain $\lim\limits_{h \searrow 3\delta} = 1/2$ and $\lim\limits_{h \nearrow 3\delta} = 1/4$. This implies the existence of a mass point with mass $1/4$ at $h = 3\delta$. Intuitively, equation (3) requires a sufficiently low probability of $f - \delta$ being close to 3δ because otherwise setting f close to 4δ would lead to zero profit too often due to an agglomeration equilibrium in the lower range. A profitable deviation because of the mass point at 3δ is ruled out by equation (2) and (3).

The resulting mixing probability is characterized by a cdf of

$$G(f) = \begin{cases} \frac{f - 2\delta}{f + \delta}, & \text{if } f \in [2\delta, 3\delta); \\ \frac{2f - 5\delta}{f - \delta}, & \text{if } f \in [3\delta, 4\delta]. \end{cases}$$

Because the distribution is not absolutely continuous with respect to the Lebesgue measure, it fails to have a density. Nevertheless, we define a generalized density, which is a generalized function (since it will be comprised of a dirac delta function) such that integration against this generalized function yields the correct desired probabilities. The corresponding probability density function is given by

$$g(f) = G'(f) + \frac{1}{4} \delta^D (f - 3\delta),$$

where

$$G'(f) = \begin{cases} \frac{3\delta}{(f + \delta)^2}, & \text{if } f \in [2\delta, 3\delta); \\ \frac{3\delta}{(f - \delta)^2}, & \text{if } f \in [3\delta, 4\delta], \end{cases}$$

and $\delta^D (f - f_0)$ denotes Dirac's delta function which is 0 everywhere except for f_0 where it is ∞. Furthermore, $\int \delta^D (f - f_0) df = 1$. Inserting $\delta = \pi^m / 2 - \pi^d$ yields the result stated in the proposition. $\qquad \square$

Proof of Proposition 4. Non-convex randomization domain. As shown in the proof of the previous proposition, there exist no pure-strategy equilibria in the region of $\pi^m / 4 \leq \pi^d < \pi^m / 2$. Furthermore, for $3\pi^m / 8 \leq \pi^d < \pi^m / 2$, there exist a first mixed-strategy equilibrium whose upper bound of the randomization domain is always lower than $\pi^m / 2$ except for $\pi^d = 3\pi^m / 8$, where it is equal to $\pi^m / 2$. For $\pi^m / 4 \leq \pi^d < 3\pi^m / 8$, we next derive the randomization domain of the second mixed-strategy equilibrium whose upper bound is always equal to $\pi^m / 2$.

Randomization domain. For $\pi^d < 3\pi^m / 8$ the highest fee in any mixed strategy equilibrium is $\pi^m / 2$. Suppose platform j sets this fee. The best response of platform

i is then to set its fee such that it attracts all firms and consumers (i.e., induces an agglomeration equilibrium on its platform). To do so, it needs to set $f_i = \pi^d - \epsilon$. As a best response, platform j wants to marginally reduce its fee to $\pi^m/2 - \epsilon$ and induce a segmentation equilibrium, and so on.

Denote the lowest fee in the mixing domain (i.e., the fee at which a platform i prefers to raise its fee to the highest fee $\pi^m/2$ instead of marginally reducing it) by $f_i = \underline{f}$. We have that $\underline{f}$ is given by $2\underline{f} = \pi^m/2$. The resulting lowest fee is therefore $\underline{f} = \pi^m/4$. This fee makes firms exactly indifferent between agglomeration on platform i and segmentation on platform j if platform j charges a fee such that $\pi^d - \underline{f} = \pi^m/2 - f_j$. This leads to $f_j = 3\pi^m/4 - \pi^d$. Furthermore, it is easy to check that with fees $f_i = \pi^m/2$ and $f_j = 3\pi^m/4 - \pi^d$, we in fact have a segmentation equilibrium in the second stage.[20]

Finally, note that $\pi^d - \epsilon$ (i.e., the fee that induces agglomeration if the rival platform charges the highest fee) is strictly lower than $3\pi^m/4 - \pi^d$ (i.e., the fee which induces a platform to stop undercutting and instead raise the fee to the highest one) since we are in the range $\pi^d < 3\pi^m/8$. This implies that in the mixed-strategy equilibrium, there are two disjoint sets of mixing ranges. The upper one $[3\pi^m/4 - \pi^d, \pi^m/2]$ is a best response to a fee in the lower range $[\pi^m/4, \pi^d)$, that induces segmentation, whereas a fee in the lower range is intended to induce agglomeration.

Therefore, in the range $\pi^m/4 \leq \pi^d < 3\pi^m/8$, there is a symmetric mixed-strategy equilibrium with fees $f_i \in [\pi^m/4, \pi^d) \cup [3\pi^m/4 - \pi^d, \pi^m/2]$. The expected profit in this range is given by $3\pi^m/4 - \pi^d$. As above, this is because setting this fee induces segmentation with probability 1.

Mixing Probabilities. Let $\delta \equiv (\pi^d - \pi^m/4)$, $\eta \equiv (\pi^m/2 - \pi^d)$ and $\epsilon > 0$ but infinitesimally small. Denote $\underline{f} \equiv \pi^m/4$, $\overline{f} \equiv \pi^d$, $\underline{f'} \equiv 3/4\pi^m - \pi^d$, and $\overline{f'} \equiv \pi^m/2$ such that the domain of interest can be expressed as $f_i \in [\underline{f}, \overline{f}) \cup [\underline{f'}, \overline{f'}]$. In addition, it holds that $\overline{f} - \underline{f} = \overline{f'} - \underline{f'} = \delta$ and $\overline{f'} - \overline{f} = \underline{f'} - \underline{f} = \eta$. Using that $2\underline{f} = \overline{f'}$ and $\underline{f} + \delta + \eta = \overline{f'}$ yields $\underline{f} = \delta + \eta$. This implies that $f_i \in [\delta + \eta, 2\delta + \eta) \cup [\delta + 2\eta, 2\delta + 2\eta]$. For $i \neq j$ and $i, j \in \{A, B\}$, the corresponding best response function is given by

$$\hat{f}_i(f_j) = \begin{cases} f_j - \eta - \epsilon, & \text{if } f_j \in (\underline{f'}, \overline{f'}]; \\ f_j + \delta, & \text{if } f_j = \underline{f'}; \\ f_j + \eta, & \text{if } f_j \in [\underline{f}, \overline{f}). \end{cases}$$

We know that all fees in the mixing domain should give an expected profit of $3/4\pi^m - \pi^d = \underline{f'} = \delta + 2\eta$.

We now proceed analogously to above. Let us start with the case in which platform

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[20]This is because in an agglomeration equilibrium, firms profits are $\pi^d - f_j = 2\pi^d - 3\pi^m/4$, which is negative because $\pi^d < 3\pi^m/8$.

i charges a fee in the lower range, that is, $f_i \in [\underline{f}, \overline{f})$. Denote the cumulative density function with which platform j mixes by $G(f_j)$. The profit of platform i when setting fees in this lower range is then given by (again abbreviating f_i by f)

$$G(f + \eta)f + (1 - G(f + \eta))\, 2f.$$

In equilibrium, this must be equal to $\delta + 2\eta$, yielding a first equation of

$$G(f + \eta)f + (1 - G(f + \eta))\, 2f = \delta + 2\eta. \tag{8}$$

This equation determines the mixing probabilities of platform j in its upper range.

In case platform i charges a fee from the upper range, that is, $f_i \in [\underline{f'}, \overline{f'}]$, the equation is

$$G(f - \eta)0 + (1 - G(f - \eta))\, f = \delta + 2\eta. \tag{9}$$

This equation determines the mixing probability in the lower range.

Let us first look at (8). We can substitute $h \equiv f + \eta$ to get

$$G(h)\, (h - \eta) + (1 - G(h))\, 2\, (h - \eta) = \delta + 2\eta. \tag{10}$$

Therefore, h is the fee in the upper range. Recall that (8) was relevant for f in the lower range and since $h = f + \eta$, these are exactly the fees in the upper range. Solving (10) for $G(h)$ gives

$$G(h) = \frac{2h - 4\eta - d}{h - \eta}. \tag{11}$$

It is easy to check that $\lim_{f \searrow \underline{f'}} G(f) = \lim_{f \searrow \delta + 2\eta} G(f) = \delta/(\delta + \eta) < 1/2$ because $\eta > \delta$. Moreover, it holds that $\lim_{f \nearrow \overline{f'}} G(f) = \lim_{f \nearrow 2\delta + 2\eta} G(f) = 3\delta/(2\delta + \eta) < 1$, which implies the existence of a mass point with mass $1 - 3\delta/(2\delta + \eta) = (\eta - \delta)/(2\delta + \eta)$ at $h = 2\delta + 2\eta$. The intuition for this result is that equation (8) is barely satisfied for f close to $\overline{f} = 2\delta + \eta$ because the support of G is a non-convex set and $\overline{f} = 2\delta + \eta < \delta + 2\eta$, the expected profit. In order to satisfy this equation, there must be a positive probability of triggering an agglomeration equilibrium and receiving $2f$ in the lower range even for $f = \overline{f}$. This is achieved by a mass point at $h = 2\delta + 2\eta = \overline{f'}$. Note that in the lower range, a profitable deviation from the mass point at $2\delta + 2\eta$ is ruled out by equation (8). We show next that $2\delta + 2\eta$ also satisfies the equilibrium condition.

Consider (9). Here we can substitute $h \equiv f - \eta$ representing that h is now in the lower range. We obtain

$$(1 - G(h))\, (h + \eta) = \delta + 2\eta. \tag{12}$$

Solving (12) for $G(h)$ gives

$$G(h) = \frac{h - \delta - \eta}{h + \eta}. \tag{13}$$

It is easy to check that $G(\underline{f}) = G(\delta + \eta) = 0$, whereas $\lim_{f \nearrow \overline{f}} G(f) = \lim_{f \nearrow 2\delta + \eta} G(f) = \delta/(2(\delta + \eta))$. Note that $\lim_{f \nearrow \overline{f}} G(f) = \delta/(2(\delta + \eta)) < \lim_{f \searrow \underline{f'}} G(f) = \lim_{f \searrow \delta + 2\eta} G(f) = \delta/(\delta + \eta)$, which implies the existence of a second mass point with mass $\delta/(2(\delta + \eta))$ at $h = \delta + 2\eta$. Intuitively, equation (3) requires a sufficiently low probability of $f - \eta$ being close to $\overline{f} = 2\delta + \eta$ because otherwise setting f close to $\overline{f'} = 2\delta + 2\eta$ would lead to zero profit too often due to an agglomeration equilibrium in the lower range. A profitable deviation because of the mass point at $\delta + 2\eta$ is ruled out by equation (8) and (9).

The resulting mixing probability is characterized by a cdf of

$$G(f) = \begin{cases} \frac{f - \delta - \eta}{f + \eta}, & \text{if } f \in [\delta + \eta, 2\delta + \eta); \\ \frac{2f - \delta - 4\eta}{f - \eta}, & \text{if } f \in [\delta + 2\eta, 2\delta + 2\eta); \\ 1, & \text{if } f = \delta + 2\eta. \end{cases}$$

Because the distribution is not absolutely continuous with respect to the Lebesgue measure, we define a generalized density. The corresponding generalized density is given by

$$g(f) = G'(f) + \frac{\delta}{2(\delta + \eta)}\delta^D(f - (\delta + 2\eta)) + \frac{\eta - \delta}{(2\delta + \eta)}\delta^D(f - (2\delta + 2\eta)),$$

where

$$G'(f) = \begin{cases} \frac{\delta + 2\eta}{(f + \eta)^2}, & \text{if } f \in [\delta + \eta, 2\delta + \eta); \\ \frac{\delta + 2\eta}{(f - \eta)^2}, & \text{if } f \in [\delta + 2\eta, 2\delta + 2\eta), \end{cases}$$

and $\delta^D(f - f_0)$ denotes Dirac's delta function. Replacing δ and η by their respective definitions yields the result stated in the proposition. $\square$

Proof of Proposition 5. From the main text we know that if firms segment themselves, then half of the single-homing consumers are active on platform A and the other half on platform B. This implies that the total number of consumers of a firm is $(1 + \alpha)/2$. Instead, if firms agglomerate, all single-homing consumers also choose the platform where firms agglomerate and the total number of consumers per firm is 1.

In the latter case, the profit of a firm is π^d. If a firm in a category deviates and is active on the other platform, it offers its products only to the mass α of multi-homing consumers who have seen both offers. The firm's profit is then $\alpha\pi^d < \pi^d$. Therefore, a deviation is not profitable, implying that an agglomeration equilibrium exists, given that a platform's

fee is lower (or equal) than π^d. If firms segment (and single-homing consumers follow suit), the profit of each firm is $\pi(\alpha)(1+\alpha)/2$. If a firm in a category deviates, it obtains a profit of $\pi^d(1+\alpha)/2 < \pi(\alpha)(1+\alpha)/2$. As a consequence, an segmentation equilibrium exists, if platforms charge fees lower (or equal) to $\pi(\alpha)(1+\alpha)/2$.

Again, only these two types of equilibria can exist. There can never be an equilibrium in which firms in some categories choose segmentation whereas in others they choose agglomeration. The reason is that in this case all single-homing consumers are active on the platform with a larger number of firms (say, platform i). Therefore, the total number of consumers on the two platforms is $x_i = 1$ and $x_{-i} = \alpha$. But then in all categories in which firms segment, the firm on platform $-i$ wants to deviate and go to platform i because it obtains a profit of π^d per consumer on both platforms but platform i has a larger number of consumers.

Having established that there are only two types of equilibria in the second stage, we can now move to the first-stage choices of the platforms. Agglomeration is preferred from the firms' perspective if

$$\pi^d \geq \pi(\alpha)\frac{1+\alpha}{2}.$$

Following the same argument as in the proof of Proposition 1, we obtain that in this range an agglomeration equilibrium with fees $f_i = f_{-i} = 0$ is the unique equilibrium.

Similarly, if both platforms charge a fee of $\pi(\alpha)(1+\alpha)/2$, the only equilibrium is that firms segment, and a platform's profit equals $\pi(\alpha)(1+\alpha)/4$. A platform has no incentive to deviate from this fee combination, if

$$\pi^d < \pi(\alpha)\frac{1+\alpha}{4}.$$

Hence, in this range, the unique equilibrium involves $f_i = f_{-i} = \pi(\alpha)(1+\alpha)/2$ and firms segment.

It is evident, that the regions are the same as in case where $\alpha = 0$ but $\pi^m/2$ is replaced by $\pi(\alpha)(1+\alpha)/2$. The same logic applies for the region

$$\pi(\alpha)\frac{1+\alpha}{2} > \pi^d \geq \pi(\alpha)\frac{1+\alpha}{4}.$$

By following the same steps as in the proof of Propositions 1 to 4, we obtain the same results as in those propositions.   □

Proof of Proposition 6. As before, we start with the potential equilibrium configurations at stage 2. For any set of listing fees, there can be three potential configurations. First, an agglomeration equilibrium, in which in each category both firms and all consumers are active on only one platform, and firms obtain a gross profit of π^d. Second, a segmentation

equilibrium, in which in each category firms and consumers segment and firms obtain a gross profit of $\pi^m/2$. Third, all firms multi-home in equilibrium, consumers split equally on both platforms and firm earn π^d. There can never be an equilibrium with partial multi-homing, that is, in some categories only one firm multi-homes, because then all consumers would join the platform on which both firms are active. The multi-homing firm's best response is then to single-home on the platform where all consumers are active.

We now move to the first stage and examine the equilibrium in the fee-setting game of platforms. First, note that in the agglomeration and in the multi-homing equilibrium, firms' profits are equal to π^d. From the proof of Proposition 1, we know that the agglomeration equilibrium involves listing fees of zero. If one firm sets a positive listing fee, it is better for all firms to choose agglomeration on the other platform instead of coordinating on the multi-homing equilibrium. This is because the gross profit in both equilibria is π^d and firms can save listing fees by agglomerating. As a consequence, the multi-homing equilibrium can only exist if both platforms charge a listing fee of zero. The outcome of the multi-homing and the agglomeration equilibrium is then equivalent. As stated in the man text, without loss of generality, we assume that firms will play the agglomeration equilibrium in this case. We can therefore focus on the conditions under which the agglomeration and the segmentation equilibrium exist.

First, we know that in the agglomeration region, given that $f_A = f_B = 0$, firms obtain higher profits in the agglomeration than in the segmentation equilibrium. Therefore, in this region the agglomeration equilibrium is the unique equilibrium.

If $\pi^m/2 > \pi^d$, firms obtain higher profits when separating instead of agglomerating , given that $f_A = f_B = 0$. However, in contrast to the previous analysis, segmentation is not necessarily an equilibrium of the second stage even if $\pi^m/2 > \pi^d$ and $f_A = f_B = 0$. This is because a firm can multi-home. In particular, suppose that platforms set $f_A = f_B = 0$, and firms and consumers in the second stage play the segmentation equilibrium. The profit of each firm is then $\pi^m/2$. By deviating to multi-homing, a firms obtains a profit of π^{MH} – we recall that π^{MH} is the profit a firm receives when it multi-homes, the competitor single-homes, and consumers are split equally on the two platforms. Hence, if

$$\pi^{MH} > \frac{\pi^m}{2},$$

the separating equilibrium does not exist for $f_A = f_B = 0$.

We will now show that an equilibrium, in which both listing fees are equal to zero and firms play the agglomeration equilibrium exists for $\pi^{MH} > \pi^m/2$. First, note that neither a single firm nor a single consumer have an incentive to deviate from this equilibrium because no agent is active on the other platform. Second, since the segmentation

equilibrium fails to exist, there is no other equilibrium but agglomeration on which firms can coordinate on in the second stage. Finally, if platform $-i$ sets a listing fee of $f_{-i} = 0$ and $\pi^{MH} > \pi^m/2$, the best response of platform i is to set a listing fee of zero as well. This reasoning holds regardless of the exact parameter constellation (i.e., it also holds for $\pi^d < \pi^m/2$).[21]

It follows that platforms setting $f_A = f_B = 0$ and firms playing the agglomeration equilibrium, always constitutes an equilibrium of the full game. The question is if there are constellations such that another equilibrium exists, which is profit-dominant for platforms. As in the proof of Proposition 2, let us start with the case $\pi^d < \pi^m/4$ (i.e., the segmentation region). If both platforms charge a listing fee of $\pi^m/2$, no firm has an incentive to multi-home. The reason is that a multi-homing firm's payment is then equal to π^m, which is below the profit the firms earns on the product market. Firms will therefore play the segmentation equilibrium. In addition, a platform cannot profitably set a different listing fee, because its profit in an agglomeration equilibrium is lower. Therefore, setting $f_A = f_B = \pi^m/2$ and firms segmenting constitutes an equilibrium in the segmentation region. This equilibrium dominates the zero-profit equilibrium for the platforms and will therefore by played.

Finally, we turn to the range $\pi^m/2 > \pi^d \geq \pi^m/4$. We know from above that for $\pi^{MH} > \pi^m/2$ an agglomeration equilibrium with zero listing fees exist. From the proof of Propositions 1 to 4, we also know that if a segmentation equilibrium exist, it must be that described in those propositions. We will now check under which conditions the possibility to multi-home breaks the segmentation equilibrium. This equilibrium exist if the circle of best responses described in the proofs of Propositions 1 to 4 works in the same way if firms can multi-home. However, this circle does no longer exist if one of the fees in the mixing range is below $\pi^{MH} - \pi^m/2$. The reason is as follows: Suppose platform i sets a fee below $\pi^{MH} - \pi^m/2$. Platform $-i$'s best response in case of single-homing firms was to set a higher fee to induce segmentation. However, inducing segmentation is no longer possible with multi-homing firms. Specifically, if a firm decides to single-home on platform $-i$ it obtains a profit of $\pi^m/2 - f_{-i}$. Instead if it multi-homes, its profit is

$$\pi^{MH} - f_i - f_{-i} > \pi^{MH} - (\pi^{MH} - \pi^m/2) - f_{-i} = \pi^m/2 - f_{-i},$$

where the inequality follows from the fact that f_i is lower than $\pi^{MH} - \pi^m/2$. As a consequence, the best response of platform $-i$ to a listing fee of f_i below $\pi^{MH} - \pi^m/2$

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[21]The inequality $\pi^{MH} > \pi^d$ holds because if the multi-homing firm sets the duopoly price p^d, its rival firm will best-respond with p^d. The multi-homing firm then gets weakly higher profits than π^d because for half of the consumers it obtains π^d whereas for the other half it has set a price of π^d but faces no competitor. By a revealed-preference argument, if the firms sets a different price than p^d, it must earn even higher profits than π^d.

is to undercut this fee slightly to induce an agglomeration equilibrium on platform $-i$ in the second stage. The lowering of prices then leads to the agglomeration equilibrium with $f_A = f_B = 0$.

It remains to check, under which conditions the lowest fee in the mixing range is below $\pi^{MH} - \pi^m/2$. Starting with the first mixing region we obtain that this is true if $\pi^{MH} - \pi^m/2 > \pi^m - 2\pi^d$ or

$$\pi^{MH} > \frac{3\pi^m}{2} - 2\pi^d.$$

If this is fulfilled, then π^{MH} is also larger than $\pi^m/2$, implying that the unique equilibrium is $f_A = f_B = 0$ and agglomeration. Instead, if $\pi^{MH} \leq 3\pi^m/2 - 2\pi^d$, the mixed-strategy equilibrium derived above exists and gives higher profits to platforms.

Proceeding in the same way for the second mixing region, we obtain that for

$$\pi^{MH} > \frac{3\pi^m}{4}$$

the unique equilibrium involves $f_A = f_B = 0$ and agglomeration, whereas for $\pi^{MH} \leq 3\pi^m/4$, platforms coordinate on the mixed-strategy equilibrium. $\qquad \square$

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