Vertical Mergers in Platform Markets*

Jérôme Pouyet† Thomas Trégouët‡

May 16, 2017

Abstract

We consider the competitive analysis of vertical integration between a platform and a manufacturer when platforms provide operating systems for the devices sold by manufacturers to customers, and, customers care about applications developed for the operating systems.

First, two-sided network effects make manufacturers more collaborators than competitors. As a result, when it brings efficiency gains, vertical integration benefits the non-integrated manufacturer and increases consumer surplus.

Second, with direct network effects on one side of the market or when developers bear a cost to make their applications available on a platform, manufacturers boost their demands by coordinating on the same platform. This creates some market power for the integrated firm. Vertical integration then leads to foreclosure and harms consumers.

Third, developer fee enables the integrated firm to harness indirect network effects by implementing an asymmetric price structure. Foreclosure may then arise, but is only the consequence of the integrated firm's ability to internalize asymmetric indirect network effects.

Keywords: Vertical integration, two-sided markets, network effects, foreclosure.

JEL Code: L40, L10, D43.

*We sincerely thank Özlem Bedre-Defolie, Bruno Jullien, Daniel O'Brien, Markus Reisinger and Yaron Yehezkel for their detailed comments and suggestions on a previous draft. We gratefully acknowledge the comments of Paul Belleflamme, Alexandre de Cornière, Jacques Crémer, Vincent Lefrere, David Martimort, Régis Renault, Patrick Rey and Nicolas Schutz. We are also thankful to participants to ICT (Paris, 2015), IIOC (Philadelphia, 2016), EARIE (Lisbon, 2016), Tenth IDEI-TSE-IAST Conference on The Economics of Intellectual Property, Software and the Internet (Toulouse, 2017), 2nd workshop on the Economics of Platforms (Berlin, 2017) as well as to seminar participants at PSE, CREST, Université de Caen, Université de Cergy-Pontoise, Université de Paris-Dauphine, Toulouse (Digital Workshop), CORE (Université Catholique de Louvain), GAEL (Université de Grenoble), Université Paris Sud. This work was supported by a grant overseen by the French National Research Agency (ANR-12-BSH1-0009), by the Cepremap (Paris) and by the Labex MME-DII (ANR11-LBX-023-01). All remaining errors are ours.

†Paris School of Economics (CNRS & ENS). E-mail: jerome.pouyet@ens.fr. Address: PSE, 48 boulevard Jourdan, 75014 Paris, France.

‡Université de Cergy-Pontoise, THEMA. E-mail: thomas.tregouet@u-cergy.fr. Address: Thema – Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy Pontoise Cedex, France.

1. INTRODUCTION

MOTIVATION. Software platform industries have recently witnessed many sudden changes in the nature of the relationship between software and hardware producers. A prime example is Google's venture in the smartphone markets, which started six years ago. Google initially maintained arm's length relationships with several smartphone hardware producers to build the Nexus range, even after the acquisition of Motorola. Perhaps not surprisingly, that acquisition has had the side-effect of making relationship with the main licensees of its Android mobile operating system increasingly strained.[1] It certainly came as a greater surprise that Motorola was sold back only three years afterwards, in 2014, and many observers believed that more integration with hardware manufacturers was not such a smart strategy for software platforms. As of October 2016, praising the synergies associated with the so-called full-stack (that is, integrated) model, Google announced the launch of its Pixel, the first smartphone entirely conceptualized and engineered in-house, which will benefit from exclusive Google's technologies and be the first to boast the new Android operating system. Rumors are now rampant that Samsung, subjugated to Google for the use of its Android platform but delivering Google substantial money through services installed on its phones, may equip all of its devices with its own Tizen operating system, despite the problem of getting enough traction from application developers.

The main issues we are interested in this paper can be stated as follows: In platform markets, what are the competitive effects of vertical integration between a platform and an equipment manufacturer and the risk of foreclosure? And, how direct or indirect network effects, which are endemic in these industries, ought to be considered in the analysis? In short, we obtain three insights. First, indirect network effects make manufacturers more collaborators and less competitors, and, as a result, lessen foreclosure concerns. Second, and by contrast, direct network effects provide manufacturers with the incentives to coordinate on the same operating system. Vertical integration creates thus some market power and always leads to harmful foreclosure. Third, an integrated platform harnesses indirect network effects by implementing an asymmetric price structure. The impact on the non-integrated manufacturer's profit then depends on the structure of indirect network effects. In passing, we also discuss how network effects impact the profitability of vertical integration as well as its impact on welfare.

Our analysis starts with the following base model, which builds on the extant literature.[2] Platforms compete to offer operating systems to two downstream manufacturers, which then sell devices to final customers. A device gives buyers access to the applications developed for that platform. Accordingly consumers care not only about the devices' prices but also about the number of developers joining a platform. One assumption is

[1] At the time of Motorola's acquisition, some experts argued that Google's primary objective was to strengthen its patents portfolio; many now retrospectively think that this also was a test of the feasibility of a more integrated business model.

[2] We focus on the literature that determines circumstances under which vertical integration creates some market power, and, thus, may lead both to soften downstream competition at the expense of final customers and to harmful foreclosure of non-integrated competitors; see, e.g., Salop and Scheffman (1983), Ordover et al. (1990) or Chen (2001). Another strand, following Hart and Tirole (1990), shows that vertical integration may be used as a way not to create but, rather, to restore the upstream market power, which was eroded by a lack of commitment; see Rey and Tirole (2007) and Riordan (2008) for surveys.

made: there are neither developer fees nor additional platform-specific costs beyond the development cost so that application developers care only about the total number of customers; this is relaxed later on.

INDIRECT NETWORK EFFECTS. Our first contribution is to show that strong indirect network effects make manufacturers more collaborators and less competitors: a manufacturer's demand may either increase or decrease when the other manufacturer cuts its price. The reason is simple and illuminates the role of indirect network effects on the downstream interaction between manufacturers: a price reduction on one device increases the total number of buyers and, thus, makes platforms more attractive for multi-homing developers; this, in turn, increases buyers' willingness to pay for all the devices. When indirect network effects are strong (weak) relative to product market competition, manufacturers end up being collaborators (competitors): a price decrease by one manufacturer boosts (reduces) the rival manufacturer's demand and, formally, everything happens 'as if' manufacturers were producing demand complements (substitutes).[3] We say that there is a positive (negative) externality among manufacturers when the demand faced by a manufacturer increases (decreases) with the price set by its rival.

EFFICIENCY GAINS. Our analysis then proceeds by analyzing the competitive impact of vertical integration when it creates efficiency gains or when platforms are asymmetrically efficient. In equilibrium, whatever the externality between manufacturers, the non-integrated manufacturer buys from the integrated firm at a royalty strictly above the pre-merger level. The impact of vertical integration on foreclosure depends, however, on whether manufacturers are collaborators or competitors: with a positive externality, a merger which makes the integrated manufacturer more efficient (through the removal of a double marginalization or through synergies) benefits the non-integrated manufacturer; with a negative externality, the merger hurts the non-integrated manufacturer. Hence, although vertical integration always leads to a higher royalty, it benefits (hurts) the non-integrated manufacturer when indirect network effects are strong (weak) relative to the intensity of product market competition.

From an antitrust perspective, everything happens as if indirect network effects 'scale down' the intensity of product market competition at the retail level. This suggests to adopt a more lenient stance vis-à-vis vertical integration than in standard markets, and all the more so that indirect network effects are strong.

COST TO PORT APPLICATIONS AND DIRECT NETWORK EFFECTS. Once an application is developed, a developer has to bear some additional costs to make it available on a specific platform or on a specific operating system. These costs lead to the phenomenon of 'fragmentation,' according to which the sometimes-prohibitive costs to port an application on different operating systems lead to the scattering of developers across competing platforms.

Costly porting implies a first departure of our base model, for by coordinating on the same platform manufacturers reduce the developers' total cost and thus increase the

[3] In a broad sense, that the 'two-sidedness' nature of a market can change the nature of the interactions has appeared in other contexts. For instance, in media markets, Reisinger et al. (2009) show that advertising levels can be either strategic complements or substitutes. Amelio and Jullien (2012) make a similar observation in the context of tying in two-sided markets.

number of available applications, thereby making their devices more attractive to buyers.[4] A very similar phenomenon arises in the presence of direct network effects on one side of the market: if each buyer values the total number of smartphone buyers (through messaging apps or games), or if developers value the total number of application developers (because this facilitates solving coding issues), then manufacturers have incentives to adopt the same operating system.

These motives for coordination are pervasive in platform markets and we show that their impact on the competitive analysis of vertical integration does not depend on whether manufacturers are competitors or collaborators: they always reinforce concerns about foreclosure. Intuitively, motives for coordination between manufacturers provide platforms with some market power, for some gains are lost if manufacturers do not buy from the same platform. When platforms are symmetric and non-integrated, and thus compete fiercely to license their operating systems, these gains end up being fully pocketed by the manufacturers. Integration, however, forces coordination on the integrated platform, and empowers the integrated firm with the ability to raise its royalty so as to extract some of the non-integrated manufacturer's gain from coordination.

Welfare. Whether consumers gain or lose from vertical integration also depends on the source of the integrated platform's market power: vertical integration increases (decreases) consumer surplus with efficiency gains combined with indirect network effects (with motives for coordination between manufacturers).

The private profitability of vertical integration is ambiguous, even when it brings efficiency gains. On the one hand, when indirect network effects are strong and manufacturers are collaborators, the strategic response of the non-integrated manufacturer impacts negatively the integrated firm's profit. On the other hand, motives for coordination make vertical integration profitable. From a managerial perspective, our model suggests that in platform markets with strong indirect network externalities and weak direct network effects on both sides, platforms may well prefer to remain at arm's length relationships with device manufacturers.

Developer Fee and the Structure of Indirect Network Effects. Last, we introduce fees that developers have to pay to join the platforms and assume away any efficiency gains or coordination motives. Developer fee introduces two new aspects: first, the integrated platform has a monopoly market power over the buyers of its devices, which allows to charge application developers; second, the integrated platform can control the pricing on both sides of the market, the price for its device and the fee paid by developers. Whereas under separation a platform has no direct contact with the buyers of devices (this is mediated by manufacturers), an integrated platform can now harness the indirect network effects between both sides of the market by choosing the price structure appropriately. Much in the spirit of the two-sided market literature, the structure of the indirect network effects now determines which side ought to be 'subsidized' or 'taxed' by the integrated platform.

When application developers value strongly the number of buyers of devices, the integrated platform sets a low price for its own device, and, simultaneously, a high fee

[4] The base model neutralizes these motives for coordination since the choice of platform by a manufacturer did not change the participation of developers.

on developers. The non-integrated manufacturer's profit is thus negatively impacted, for its demand shrinks. This result is obtained in a setting where the externality between manufacturers is null, and there are neither efficiency gains nor motives for coordination: foreclosure does not emerge from the desire to soften competition, but, rather, simply because the integrated platform has the ability, and is sometimes willing, to implement an asymmetric price structure.

Not all asymmetric price structures lead to foreclosure, however: when buyers of devices value strongly the number of applications, the integrated platform sets a negative developer fee and a high price for its device, and the non-integrated manufacturer benefits from the vertical merger. Last, with developer fee, vertical integration is always profitable and its impact on consumer surplus is ambiguous. Roughly speaking, consumers gain when indirect network effects are sufficiently asymmetric, that is when there are large gains to internalize the network effects across both sides of the market: this can be reached under integration, although imperfectly.

Related literature. To the best of our knowledge, our paper is the first to link, on the one hand, the literature on two-sided markets, and, on the other hand, the literature on strategic vertical integration in the specific context of platforms-manufacturers relationships.

From the literature on two-sided markets, we borrow the general insight that indirect network effects are key to understanding the platform pricing and competition (Caillaud and Jullien, 2003; Armstrong, 2006; Rochet and Tirole, 2006; Weyl, 2010). That literature has considered more recently the effect of exclusive dealing between a platform and content providers (that is, developers in our model): Evans (2013) discusses the antitrust of such vertical relations in platform industries; Doganoglu and Wright (2010) and Hagiu and Lee (2011) provide a rationale for why platforms sign exclusive contract with content providers; Church and Gandal (2000) describe the incentives of a manufacturer that is integrated with a developer to make its applications compatible with the hardware of a rival manufacturer; Hagiu and Spulber (2013) show that investment in first-party content (that is, vertical integration with one side of the market) depends on whether a platform faces a 'chicken-and-egg' coordination problem; in the videogame industry, Lee (2013) finds that exclusivity tends to be pro-competitive, in that it benefits more to an entrant platform than to an incumbent platform. While we share with these papers the issue of the competitive impact of vertical restraints in a two-sided market, our work also differs substantially, for we are interested in the vertical relations between platforms/operating systems and manufacturers when devices are an essential link to connect buyers and developers.

Our analysis also belongs to the strategic approach of foreclosure initiated by Ordover et al. (1990). A message conveyed by that literature is that vertical integration can lead to input foreclosure and be detrimental to consumer surplus. Analyses that feature trade-offs between the pro- and the anti-competitive effects of vertical integration include: Ordover et al. (1990) and Reiffen (1992) when integration generates an extra commitment power; Riordan (2008) and Loertscher and Reisinger (2014) when the integrated firm is dominant; Chen (2001) when manufacturers have switching costs; Choi and Yi (2000) when upstream suppliers can choose the specification of their inputs; Chen and Riordan (2007) when exclusive dealing can be used in combination with integration;

Nocke and White (2007) and Normann (2009) when upstream suppliers tacitly collude; Hombert et al. (2016) when there are more manufacturers than upstream suppliers; and Hunold and Stahl (2016) when integration can be either controlling or passive.[5] None of these papers deal with multi-sided markets and our analysis bring several new insights. First, indirect network effects affect the nature of the interaction in the downstream market by making manufacturers behave as if they were producing demand complements; we thus contribute to this literature by considering the case of demand complements. Second, foreclosure emerges if manufacturers have incentives to coordinate on the same platform, a phenomenon which arises naturally in the context of platform markets, but, surprisingly, has not been considered in that literature. Third, the structure of indirect network effects plays a critical role when platforms compete both with royalties and with developer fees: here we fully exploit the two-sided aspect of our framework to understand how competition on the manufacturers' side and on the developers' side may lead to foreclosure.

Organization of the Paper. Section 2 gives a quick overview of the smartphone industry, putting the emphasis on the relationships between operating systems and manufacturers of devices. Section 3 describes the model. Section 4 characterizes the impact of vertical integration on manufacturers' profits. Section 5 shows that, in platform markets, several characteristic features provide manufacturers with the incentives to join the same platform and derives the implications of such a motive for coordination. Section 6 studies the impact of vertical integration on consumer surplus. Section 7 analyzes the impact of vertical merger when platforms charge fees to developers. Section 8 concludes. All proofs are relegated to the Appendix. Some results are illustrated in a more structured version of our model, which is available in the Online Appendix.

2. A Quick Background on the Smartphone Industry

As of 2016, the smartphone industry is dominated by two software platforms: market shares are 84% for Google Android, 15% for Apple iOS and 1% for the other platforms (Windows Phone, Blackberry, etc.). Google and Apple have different business models: Android is an open-source platform that can be installed by any manufacturers willing to do so; Apple is fully integrated and does not license its operating system. Google makes profits through ads displayed on Android phones and from its mobile applications store (Google Play). It also collects user data from a set of applications: Google Search, Google Maps, etc. Though Android is an open-source platform, manufacturers can use the Android brand and the core applications developed by Google (Google Play, Google Search, Google Maps, etc.) only if they sign a Mobile Application Distribution Agreement (MADA). A MADA requires that a manufacturer makes its device "compatible" with Android (that is, a device must satisfies minimum requirements established by Google) and that all Google applications are installed and placed prominently (that is, not far from the default home screen). This makes the manufacturers' applications and application stores less visible to the end users. Accordingly, a MADA determines how the stream of revenue from the purchasing of applications and the monetization of users data (through

[5]For empirical analyses, see, e.g., Lafontaine and Slade (2007) and Crawford et al. (2016) and the references therein.

advertising for instance) is shared between Google and the manufacturer.[6,7]

The top Android devices' manufacturers are, in order of market share, Samsung, Huawei, Oppo and LG. These manufacturers produce a number of new devices each year, ranging from high-end expensive 'flagship' smartphones to low-end cheap smartphones. Though they all use Android, manufacturers often add an in-house user interface (Touchwiz for Samsung, Emotion UI for Huawei, etc.). This allows a manufacturer to differentiate its products from its competitors and to promote its own services and applications. Until recently, Google did not engineered smartphones for Android.

It is worth noting that the major manufacturers have developed or are in the process of developing their own mobile operation systems: Samsung's Tizen is already installed on a handful of devices (TV, watches, etc.); rumors indicate that Huawei is currently working on its own operating system with former developers from Nokia; in 2013, LG bought a license of WebOS from HP; Amazon developed a non-compatible "fork" of Android named FireOS; etc. In addition, there are alternative licensable mobile operating systems: Microsoft sells licenses of Windows 10 Mobile; Firefox OS (the development of which has been abandoned recently); various forks of Android (Aliyun OS, Cyanogen, etc.). Manufacturers are however reluctant to launch smartphones with operating systems non compatible with Android, for they will lose the benefits of the thousands of applications available on the Google Play Store. In addition, manufacturers that offer devices on which Google applications are installed sign an anti-fragmentation agreement which prevents them for selling devices non compatible with Android.[8]

Concerns about foreclosure of Android manufacturers are currently investigated by the EU commission.[9] Similar concerns were also raised in two recent merger cases. In the Google-Motorola merger: "The Commission considered whether Google would be likely to prevent Motorola's competitors from using Google's Android operating system."[10] Similarly, in the Microsoft-Nokia merger, the Commission "investigated the vertical relationships between the merged entity's activities in the downstream market for smart mobile devices and Microsoft's upstream activities in mobile operating systems."[11]

3. Model and Preliminary Results

We consider a two-sided market where buyers and developers of applications may interact. These interactions require three types of decision: buyers must purchase a device from a manufacturer; developers must decide which platforms to develop for; manufacturers must equip their devices with an operating system licensed by a platform.

[6] Choi and Jeon (2016) study how a platform can leverage some market power through such tying agreement. See also Edelman (2015).

[7] The MADAs are confidential. However, the agreements between Google and HTC/Samsung have been made available to the public during a litigation in 2014. See "Secret Ties in Google's "Open" Android," http://www.benedelman.org/news/021314-1.html

[8] In 2012, Acer attempted to launch a smartphone with Aliyun OS, a mobile operating system developed by Alibaba. Aliyun was partially compatible with Android applications and, in particular, with Google applications. Despite the claims of Alibaba, Google argued that Aliyun OS was based on Android and, accordingly, had to have a license to use the Google applications. Acer promptly renounced to launch an Aliyun smartphone when Google threatened it to cancel its Android license.

[9] http://europa.eu/rapid/press-release_IP-16-1492_en.htm

[10] http://europa.eu/rapid/press-release_IP-12-129_en.htm

[11] http://europa.eu/rapid/press-release_IP-13-1210_en.htm

3.1. Main Assumptions

Platforms. There are three platforms, an incumbent I and two entrants E_1 and E_2. I has a competitive advantage over E_1 and E_2: I's marginal cost to provide its operating system is nil whereas it is equal to $\delta > 0$ for E_i, $i = 1, 2$, with δ not too large to ensure that platforms E_i put an effective competitive pressure on the more efficient platform I.[12,13] Platforms levy non-negative royalties from manufacturers.[14] Let w_k denote the royalty paid by manufacturer M_k ($k = 1, 2$) to the chosen platform for each device using that platform's operating system and sold to buyers. The royalty may also be interpreted as the share of the revenues generated from user data that accrues to the platform. Contracts between a manufacturer and a platform, such as the MADA discussed in Section 2, typically specify which party owns the data and, accordingly, who can monetize it, through advertising for instance.

For clarity of exposition, we assume that platforms levy no fees from developers; that assumption is relaxed later on in Section 7.

Application Developers. We assume there is no platform-specific costs beyond the development cost.[15] Then, since there is no developer fee, once an application is developed, it can be made available on all platforms so as to reach all buyers of devices. Let $Q_S(n_B)$ denote the quasi-demand of developers, that is, the number of developers who develop an application given that there are $n_B = n_{B1} + n_{B2}$ buyers of devices, where n_{Bk} stands for the number of buyers of device produced by manufacturer M_k. Assume that application developers value positively the number of buyers ($\partial Q_S/\partial n_B > 0$).

Manufacturers. Manufacturers are symmetric and produce at the same constant marginal cost normalized to nil. In our two-sided framework, the buyers' demand for manufacturer M_k's device depends on the number of applications developed, and on the non-negative prices charged by manufacturers to buyers, denoted by p_1 and p_2. Hence, manufacturer M_k's quasi-demand may be written as $Q_B(p_k, p_\ell, n_S)$. Assume that devices are demand substitutes for buyers ($\partial Q_B/\partial p_k < 0 < \partial Q_B/\partial p_\ell$), that the direct price effect is stronger than the indirect one ($|\partial Q_B/\partial p_k| > \partial Q_B/\partial p_\ell$), and, that buyers value positively the number of applications ($\partial Q_B/\partial n_S > 0$).[16]

Linear Example. We sometimes use the following specification of the model:

- Buyers' demand is given by[17]

$$Q_B(p_k, p_\ell, n_S) = v + u_B n_S - p_k - \gamma \left(p_k - \frac{p_k + p_\ell}{2} \right),$$

[12] The competitive advantage of platform I may, alternatively, come from an existing base of developers, a better software, etc. The assumption that I has a lower marginal cost is a convenient shortcut to capture these scenarios. Notice that the least efficient platform E can be seen as an alternative mobile operating system that may be developed in-house by a manufacturer (for instance, Tizen for Samsung) but at a higher cost.

[13] Having three platforms ensures intense competition between platforms both under separation and integration. As detailed in the analysis below, it prevents a non-integrated platform from exerting market power on non-integrated manufacturers.

[14] We consider two-part tariffs in Appendix A.5 and show that our results are qualitatively the same.

[15] That assumption is relaxed later on in Section 5.

[16] This specification of quasi-demands implies that buyers of devices and developers of applications have no intrinsic preferences for the operating systems, that is, platforms are not differentiated.

[17] This is Shubik and Levitan (1980)'s linear demands system, to which we append indirect network

where $\gamma \geq 0$ measures the degree of substitutability between products (or of product market competition) and u_B the strength of indirect network effects from the buyer's side.

- There is a unit mass of developers. A developer's gross profit from interacting with a mass n_B of buyers is given by $u_S n_B$, where u_S relates to the strength of indirect network effects from the developer's side. Developers are heterogeneous with respect to their development cost $\tilde{f}$, drawn according to the uniform distribution on $[0, 1]$. Hence $Q_S(n_B) = u_S n_B$.

- The overall strength of indirect network effects plays a key role in our analysis and is denoted by $\mu = u_B u_S$.

TIMING. In stage 1, platforms I, E_1 and E_2 set royalties for manufacturers M_1 and M_2. Then, in stage 2, M_1 and M_2 decide which platforms to buy from.[18] Once operating systems are chosen, manufacturers set the prices of their devices in stage 3. Last, in stage 4, buyers decide whether to buy a device, and, simultaneously, developers decide whether to develop an application. All prices and affiliation decisions are public. We look for subgame-perfect equilibria of the game.

3.2. Participation Decisions

At the last stage of the game, given devices prices p_1 and p_2, the number of buyers of each device and the number of developers must be consistent with each other and solve

$$\left\{ \begin{array}{rcl} n_{B1} & = & Q_B(p_1, p_2, n_S), \\ n_{B2} & = & Q_B(p_2, p_1, n_S), \\ n_S & = & Q_S(n_{B1} + n_{B2}). \end{array} \right. \tag{3.1}$$

Denote by $D_k(p_k, p_\ell)$,[19] $k \neq \ell \in \{1, 2\}$, and $D_S(p_1, p_2)$ the solution of (3.1) and assume that solution is interior for the relevant range of prices. Then, as expected, the developers demand is decreasing in both prices ($\partial D_S/\partial p_k < 0$), the demand for a device decreases with its own price ($\partial D_k/\partial p_k < 0$) and we further impose that it is more responsive to its own price than to the price of the other device ($|\partial D_k/\partial p_k| > |\partial D_k/\partial p_\ell|$).[20]

Perhaps more surprising is the fact that indirect network effects impact the nature of the interaction between manufacturers on the product market in the following sense: the demand for device faced by a manufacturer may either increase or decrease with the price

effects additively. The gross utility function of the representative buyer is then given by

$$q_0 + (v + u_B n_S) \sum_{k=1,2} q_k - \frac{1}{2} \frac{1}{2(1+\gamma)} (2 \sum_{k=1,2} q_k^2 + \gamma (\sum_{k=1,2} q_k)^2),$$

where q_0 is the numéraire and q_k is the quantity of device k bought.

[18] Since platforms are not differentiated, manufacturers have incentives to make their devices available on one platform only.

[19] Since manufacturers are symmetric, $D_1(p_1, p_2) = D_2(p_1, p_2)$, for all prices p_1 and p_2.

[20] In Appendix A.1, we show, first, that (3.1) has a unique solution if indirect network effects are not too strong, which amounts to $\mu < 1/2$ in the linear example, and, second, that $|\partial D_k/\partial p_k| > |\partial D_k/\partial p_\ell|$ for symmetric prices, and for any prices in the linear example.

of the rival manufacturer, depending on the strength of indirect network effects relative to the degree of product market competition. Indeed, using (3.1), it comes immediately

$$\frac{\partial D_k}{\partial p_\ell} = \frac{\partial Q_B}{\partial p_\ell} + \frac{\partial Q_B}{\partial n_S}\frac{\partial D_S}{\partial p_\ell},$$

which can be positive or negative. If, say, p_ℓ increases, then some consumers are diverted from M_ℓ and M_k's demand increases by $\partial Q_B/\partial p_\ell$. This is the standard rivalry effect created by product market competition between manufacturers. The increase of p_ℓ has, moreover, a negative impact on the total number of buyers, since the direct price effect on buyers of device ℓ is stronger than the indirect price effect on buyers of device k $(d(Q_B(p_\ell, p_k, n_S) + Q_B(p_k, p_\ell, n_S))/dp_\ell < 0)$. Since there are less buyers overall, there are fewer applications too, for developers find it less attractive to develop. This negatively affects M_k's demand by $\partial Q_B/\partial n_S$. We therefore expect that when indirect network effects are small (that is, when $\partial Q_B/\partial n_S\, \partial D_S/\partial p_\ell \approx 0$), the rivalry effect created by product market competition dominates and $\partial D_k/\partial p_\ell \leq 0$; by contrast, when product market competition is weak (that is, when $\partial Q_B/\partial p_\ell \approx 0$), then the interaction created by indirect network effects dominates and $\partial D_k/\partial p_\ell \geq 0$. This points toward introducing the following distinction.

DEFINITION 1. *A manufacturer exerts a globally positive (respectively, negative) externality on its rival if a decrease in its price increases the demand faced by the rival, that is, if $\partial D_k/\partial p_\ell \leq 0$ (respectively, $\partial D_k/\partial p_\ell \geq 0$), $k \neq \ell \in \{1, 2\}$, in the relevant range of prices.*

In the sequel, we say that that manufacturers are collaborators (competitors) when the externality is positive (negative). When collaborators (competitors), everything happens 'as if' manufacturers were producing demand complements (substitutes). In the linear example, the externality across manufacturers is positive if and only if $\gamma \leq 2\mu/(1 - 2\mu)$ or, equivalently, $\mu \geq \gamma/(2(1 + \gamma))$.

3.3. Competition Between Manufacturers

At stage 3, manufacturers compete on the product market. Given a royalty w_k that it pays, M_k's profit writes as $\pi_k(p_k, p_\ell) = (p_k - w_k)D_k(p_k, p_\ell)$, $k \neq \ell$. We need several standard assumptions which ensure that that price competition subgame is 'well-behaved.' First, the best response in price of manufacturer M_k, denoted by $R_k(p_\ell, w_k)$, is uniquely characterized by the first-order condition $\frac{\partial \pi_k}{\partial p_k}(R_k(p_\ell, w_k), p_\ell) = 0$.[21] Second, the slope of manufacturers' best responses has a constant sign and is smaller than 1 in absolute value. Accordingly, there exists a unique pair of prices $(p_1^*(w_1, w_2), p_2^*(w_2, w_1))$ which forms the Nash equilibrium of stage 3 of the game. Third, the equilibrium price of a manufacturer is increasing in its marginal cost, and, the cost pass-through is smaller than 1, that is, $0 < \frac{\partial p_k^*}{\partial w_k}(w_k, w_\ell) < 1,\ k \neq \ell \in \{1, 2\}$.[22,23]

[21]The dependence of a manufacturer's best response on its marginal cost is made explicit for future reference.

[22]That the pass-through is smaller than 1 is an assumption often made in industrial organization. Weyl and Fabinger (2013) study the link between the log-curvature of demand functions and pass-through rates; see also Ritz (2017).

[23]All these assumptions are satisfied in the linear example.

Slightly abusing notations, $\pi_k(w_k, w_\ell)$ denotes M_k's profit from the subgame starting at stage 3, that is, $\pi_k(w_k, w_\ell) = (p_k^*(w_k, w_\ell) - w_k)D_k(p_k^*(w_k, w_\ell), p_\ell^*(w_\ell, w_k))$. In the sequel, it will be sometimes useful to keep track of the operating systems chosen by manufacturers. To this end, subscript 'ij' refers to situations where manufacturers M_1 and M_2 buy from platforms i and j respectively, with $i, j \in \{I, E_1, E_2\}$. M_k's profit then writes $\pi_k^{ij}(w_k, w_\ell)$. Demands and prices are denoted accordingly.

A first and straightforward result is the following.

LEMMA 1. *When the externality between manufacturers is positive (respectively, negative) a manufacturer's profit increases (respectively, decreases) when its rival becomes more efficient, that is*

$$\frac{\partial D_k}{\partial p_\ell}(p_k, p_\ell) \leq 0 \Leftrightarrow \frac{\partial \pi_k}{\partial w_\ell}(w_k, w_\ell) \leq 0.$$

Proof. Using the envelope theorem, it comes immediately that

$$\frac{\partial \pi_k}{\partial w_\ell}(w_k, w_\ell) = (p_k^*(w_k, w_\ell) - w_k)\frac{\partial D_k}{\partial p_\ell}(p_k^*(w_k, w_\ell), p_\ell^*(w_\ell, w_k))\frac{\partial p_\ell^*}{\partial w_\ell}(w_\ell, w_k),$$

which is positive when $\frac{\partial D_k}{\partial p_\ell}(p_k, p_\ell) \geq 0$ and negative otherwise. □

As argued later on, this result will be instrumental to assess whether vertical integration leads to foreclosure of the non-integrated manufacturer.

4. INDIRECT NETWORK EFFECTS AND FORECLOSURE

We study in this section the consequences of vertical integration between the efficient platform I and one manufacturer.

SEPARATION BENCHMARK. These consequences are assessed relative to the benchmark situation of no integration (sometimes called separation). Absent integration, platforms compete in royalties to sell their operating systems to manufacturers. At equilibrium of this asymmetric Bertrand game, the most efficient platform sells to both manufacturers at a royalty equal to the marginal cost of the least efficient platforms, that is, $w_I = \delta$. Manufacturers' profits are denoted by $\hat{\pi}_k^{II}(\delta, \delta)$, $k = 1, 2$. For future reference, it it useful to write manufacturer M_k's best response in price, which is characterized by

$$(p_k - \delta)\frac{\partial D_k}{\partial p_k}(p_k, p_\ell) + D_k(p_k, p_\ell) = 0. \tag{4.1}$$

Under separation, manufacturers buy from the most efficient platform but at a price strictly above that platform's marginal cost ($\delta > 0$). There is thus a double marginalization between platform I and both manufacturers.

4.1. Equilibrium Market Structure

Consider now that platform I is vertically integrated with manufacturer M_1. Since E_1 and E_2 are symmetric, they will undercut each other to attract the non-integrated manufacturer M_2. To streamline the analysis, we say in the following that if M_2 buys from one of the non-integrated platform, it chooses platform E that offers a royalty $w_E = \delta$. The next proposition determines the market structure at equilibrium, as well as the royalty paid by the non-integrated manufacturer M_2.

PROPOSITION 1. *In equilibrium, the vertically integrated platform supplies the non-integrated manufacturer at a royalty* $w^*(\delta)$*, strictly above the pre-merger level* δ*, such that the non-integrated firm is indifferent with buying the less efficient platform's operating system at a royalty* δ*:* $\pi_2^{II}(w^*(\delta), 0) = \pi_2^{IE}(\delta, 0)$.

Proof. See Appendix A.2. □

Vertical integration has three effects: an efficiency effect, an accommodation effect, and, an upstream market power effect.

First, vertical integration allows to get rid of one double marginalization: the integrated manufacturer M_1 can now buy I's operating system at its marginal cost (0). All else equal, the integrated manufacturer M_1 becomes more efficient on the product market.

Second, suppose the non-integrated manufacturer M_2 adopts platform E's operating system. The integrated firm's profit is thus $p_1 D_1(p_1, p_2)$ and its best response in price solves

$$p_1 \frac{\partial D_1}{\partial p_1}(p_1, p_2) + D_1(p_1, p_2) = 0. \tag{4.2}$$

If, instead, the non-integrated manufacturer M_2 chooses I's operating system at some royalty w, then the integrated firm's profit is $p_1 D_1(p_1, p_2) + w D_2(p_2, p_1)$ and its best response now solves[24]

$$p_1 \frac{\partial D_1}{\partial p_1}(p_1, p_2) + D_1(p_1, p_2) + w \frac{\partial D_2}{\partial p_1}(p_2, p_1) = 0. \tag{4.3}$$

Comparing (4.2) and (4.3) shows that when M_2 buys from I rather than from E, the integrated platform becomes accommodating in that it sets its downstream price p_1 with an eye on how this impacts the revenue earned from licensing its operating system, or, equivalently, so as to boost the demand faced by the non-integrated manufacturer.

Third, as a consequence of the accommodation effect, if the integrated platform offers $w = \delta$, the non-integrated manufacturer strictly prefers buying from I rather than from E. The integrated firm is therefore empowered with some upstream market power, which allows to attract M_2 even though it sets a royalty above the pre-merger level ($w > \delta$).

Provided that δ is not too large, the integrated firm wants to license the non-integrated manufacturer at the highest royalty consistent with its participation. The royalty is used to shift part of the non-integrated manufacturer's profit into the integrated firm's pocket. At equilibrium, M_2's profit, $\pi_2^{II}(w^*(\delta), 0)$, coincides with that obtained if it were choosing E's operating system instead, namely $\pi_2^{IE}(0, \delta)$. Observe finally that these results do not depend on whether the externality between manufacturers is positive or negative.[25]

[24]For the subgame of price competition between the integrated manufacturer and the non-integrated one, we adopt assumptions similar to those made in Section 3.3.

[25]Charging $w > \delta$ implies that M_2's price increases: a positive (respectively, negative) effect when the externality is negative (respectively, positive). Hence, all else equal, the integrated firm has less incentives to raise the royalty when the externality is positive than when it is negative. However, for δ not too large, whatever the sign of the externality, the integrated platform wants to raise its royalty as much as possible.

4.2. Foreclosure

From Proposition 1, one could be tempted to conclude that vertical integration always leads to harmful foreclosure, for the royalty paid by the non-integrated manufacturer increases above the pre-merger level.

However, what matters is not the royalty, but, rather, the non-integrated manufacturer's profit after the merger, which is equal to $\pi_2^{IE}(\delta, 0)$ from Proposition 1. But when M_2 buys from E, there are neither an accommodation effect nor an upstream market power one, and that profit easily compares with M_2's profit before the merger: with respect to the separation benchmark, the only change is that M_2 faces a more efficient competitor on the product market. Applying Lemma 1 immediately yields the following result.[26]

PROPOSITION 2.

- *With a positive externality across manufacturers, the non-integrated manufacturer gains from the vertical merger:* $\pi_2^{II}(w^*(\delta), 0) = \pi_2^{IE}(\delta, 0) > \hat{\pi}_2^{II}(\delta, \delta)$.
- *With a negative externality across manufacturers, the non-integrated manufacturer loses from the vertical merger:* $\pi_2^{II}(w^*(\delta), 0) = \pi_2^{IE}(\delta, 0) < \hat{\pi}_2^{II}(\delta, \delta)$.

Proof. Immediate from the text. □

Strong indirect network effects relative to the intensity of product market competition make manufacturers more collaborators than competitors; as a result, since vertical integration makes the integrated manufacturer more efficient, the non-integrated manufacturer benefits from the merger. By contrast, a strong rivalry on the product market relative to the strength of indirect network effects make manufacturers more competitors than collaborators and vertical integration hurts the non-integrated manufacturer.

4.3. Profitability of Vertical Integration

To assess whether the merger is profitable, we compare the joint profit of platform I and manufacturer M_1 before and after the merger.

PROPOSITION 3. *Consider the linear example.*

- *With a positive externality across manufacturers, the merger is profitable if and only if indirect network effects are not too strong relative to product market competition: there exists* $\hat{\mu}(\gamma)$ *such that the merger is profitable if and only if* $\mu \leq \hat{\mu}(\gamma)$.
- *With a negative externality across manufacturers, the merger is always profitable.*

Proof. See the Online Appendix. □

Remind that when vertical integration brings some efficiency gains, it commits the integrated manufacturer to reduce its price. The accommodation and upstream market power effects both lead the integrated manufacturer to reduce (increase) its price when manufacturers are collaborators (competitors). Notice, finally, that in the linear example

[26]We show in Appendix A.5 that the same result obtains when we allow two-part tariffs.

prices are strategic substitutes when the externality is positive and strategic complements when it is negative.[27]

Hence, all three effects push the integrated manufacturer to lower its price when manufacturers are collaborators, which leads the non-integrated manufacturer to increase its own price when devices prices are strategic substitutes. That strategic reaction impacts negatively the integrated firm's profit, which explains the first part of the proposition.[28] By contrast, when manufacturers are competitors and prices are strategic complements, the integrated manufacturer's price may either increase or decrease, and so does the non-integrated manufacturer's price; in the linear example, the efficiency effect turns out to dominate and makes the merger always profitable.

5. Coordination Motives and Foreclosure

Our modeling has assumed so far that the total numbers of developers and of buyers do not depend on whether or not manufacturers choose the same operating system. While that assumption allowed to obtain clear-cut results, it may not always be satisfactory in the context of platform markets for the following reasons:

- *Cost to port applications.* The cost of porting an existing application to a new operating system, while usually less than the cost of writing it from scratch, is non-negligible.[29]

- *Direct network effects among users on one side of the market.* The larger the community of developers using the same programming langage is, the easier it becomes to find help to overcome coding issues. Similarly, platforms often promote in-house services that encourage interactions among its users (for instance, messaging apps or games).[30]

Costs to port applications and direct network effects provide manufacturers with incentives to coordinate on the same platform. This limits the fragmentation of developers across different systems and boosts the number of applications available; or, this increases the value for users benefiting from direct network effects.

This section analyzes how these coordination motives across manufacturers affect the competitive analysis of vertical integration. To emphasize the differences with the previous analysis, we assume from now on that platform I has no cost advantage, that is, $\delta = 0$. In the base model, this implies that a vertical merger has no impact with respect to the separation benchmark (since $w^*(0) = 0$).

[27] Observe that we made, so far, no assumption on the nature of the strategic interaction between manufacturers.

[28] A similar effect arises in the case of an horizontal mergers between Cournot competitors: because quantities are then strategic substitutes, the strategic reaction of outsiders hurts the merging firms, thereby reducing the profitability of the merger.

[29] Page and Lopatka (2009, pp. 98-99) discuss the importance of porting costs for developers, and their consequences for antitrust.

[30] Katz and Shapiro (1985) highlight another form of direct network effects: more users on a platform may lead to a higher quality of postpurchase services, a relevant dimension for electronic devices that require both hardware maintenance and software update on a regular basis.

5.1. Cost to Port Applications

Let us amend the base model as follows. Suppose developers are heterogeneous in the cost to make their application available on a platform: a share $\alpha \in [0, 1]$ bears the development cost each time they want to publish their application on a platform; the remaining $1-\alpha$ share of developers incurs only the development cost before making their application available on both platforms. Parameter α captures the idea that porting an application on several operating systems can be costly and is thus an inverse measure of scale economy in application development. When $\alpha = 1$, developers care only about the number of customers on the platform when deciding whether to develop for that platform; when $\alpha = 0$, developers do not care about which platform customers are affiliated with, but only about the total number of customers brought by the manufacturers.

SEPARATION BENCHMARK. When M_1 and M_2 buy from I and E respectively, developers' quasi-demands are given by[31]

$$\tag{5.1} \begin{cases} n_{SI} &= \alpha Q_S(n_{B1}) + (1-\alpha) Q_S(n_{B1} + n_{B2}), \\ n_{SE} &= \alpha Q_S(n_{B2}) + (1-\alpha) Q_S(n_{B1} + n_{B2}). \end{cases}$$

If, instead, manufacturers coordinate on, say, platform I, developers' quasi-demands are given by

$$\tag{5.2} \begin{cases} n_{SI} &= Q_S(n_{B1} + n_{B2}), \\ n_{SE} &= 0. \end{cases}$$

For $\alpha < 1$, $Q_S(n_{B1} + n_{B2}) > \alpha Q_S(n_{Bk}) + (1-\alpha) Q_S(n_{B1} + n_{B2})$, $\forall k \in \{1, 2\}$: when manufacturers coordinate their affiliation decisions and choose the same operating system so as to benefit from a larger number of applications, this, in turn, boosts the demands for their devices. Assume that this then leads to higher downstream profits.[32]

Since platforms are symmetric ($\delta = 0$), they compete fiercely to attract manufacturers, which, in equilibrium, pay a nil royalty and coordinate on one of the platform.[33] Intense competition on the market for operating systems prevents platforms from charging royalties above the marginal cost and manufacturers fully pocket the gains associated to their coordination. Given that developers bear no cost to port at equilibrium, the manufacturers' profit is given by $\hat{\pi}_k^{II}(0, 0)$, $k = 1, 2$.

IMPACT OF VERTICAL INTEGRATION. The next result describes how such coordination motives impact the equilibrium after vertical integration.

[31] n_{Si} stands for the number of developers that affiliate with platform i and n_{Bk} denotes the number of customers that buys M_k's device.

[32] In imperfectly competitive industries, it could be that a common positive shock on demand ends up decreasing the firms' profit. In a Cournot framework, Seade (1980) finds conditions under which a common increase in the firms' marginal cost increases or decreases equilibrium profits. Cowan (2004) extends the analysis to demand shocks. Given the focus of our analysis, we directly consider that (given royalties) a positive demand shock increases the manufacturers' profits.

[33] To be more precise, M_1 and M_2 play a coordination game in stage 2. Accordingly, for given royalties, there can be multiple equilibria in the choice of platforms by manufacturers. We ignore this potential coordination problem and assume that, for all relevant royalties, manufacturers coordinate on the cheapest platform. See our working paper (Pouyet and Trégouët, 2016) where we discuss coordination failures.

PROPOSITION 4. *Assume no efficiency gains (that is, $\delta = 0$) and a cost to port application (that is, $\alpha > 0$). Then:*

- *The non-integrated manufacturer affiliates with the integrated platform and pays a royalty $w^{**}(\alpha)$ strictly above the pre-merger level* 0.

- *The non-integrated manufacturer looses from the vertical merger and the extent of foreclosure increases with α (that is, $\pi_2^{II}(w^{**}(\alpha), 0) < \hat{\pi}_2^{II}(0, 0)$ and $(w^{**})'(\alpha) \geq 0$).*

- *Vertical integration is always profitable.*

Proof. See Appendix A.3. □

While the first part of Proposition 4 bears some resemblance with Proposition 1, the underlying logic is different.

Vertical integration still generates an accommodation effect when the non-integrated manufacturer buys the integrated platform's operating system. But since platforms are symmetric (because $\delta = 0$), the non-integrated platform can always undercut until the royalty is nil, at which point the accommodation and upstream market power effects discussed in Section 4 both disappear.

The presence of a cost to port application creates actually a different source of upstream market power, for vertical integration forces the coordination of manufacturers on the integrated platform. Indeed, if the non-integrated manufacturer were to buy the non-integrated platform's operating system, it would loose the gains associated to the coordination of manufacturers on the same operating system (the number of developers would be given by (5.1) instead of (5.2)). Put differently, porting costs reduce the non-integrated manufacturer's outside option in its bargaining with the integrated platform. That threat of loosing the benefits of coordination enables the integrated platform to extract a higher royalty from the non-integrated manufacturer. Part of the gains associated to the coordination of manufacturers are now pocketed by the integrated firm.

5.2. *Direct Network Effects*

Last, we briefly consider the possibility of intra-group network effects. To account for network effects among, say, buyers, the quasi-demand of our base model is modified as follows. If device k is equipped with platform i's operating system, then the number of buyers of device k may be written as follows

$$n_{Bk} = Q_B(p_k, p_\ell, n_S, n_{Bi}),$$

where n_{Bi} is the total number of users on platform i. Positive direct effects among buyers arise when $Q_B(\cdot)$ is increasing in n_{Bi}. If device k is the sole running platform i's operating system, then $n_{Bi} = n_{Bk}$. If, however, both devices run that operating system, then $n_{Bi} = n_{B1} + n_{B2}$. Demands derived from these quasi-demands are obviously larger when manufacturers coordinate on the same operating system than when they choose different operating systems. Hence, direct network effects lead to qualitatively similar effects than those obtained with a cost to port application.

6. WELFARE

To analyze how vertical integration impacts surplus, we must determine how integration affects the prices of devices on the product market, but also on how these price variations affect the participation of developers on the other side of the market.

THE TOTAL PRICE. We can combine the quasi-demands defined by (A.1) to express the participation of developers as a function of the devices prices

$$n_S = Q_S(Q_B(p_1, p_2, n_S) + Q_B(p_2, p_1, n_S)). \tag{6.1}$$

Consider now a symmetric situation in which manufacturers set the same price $p_1 = p_2 = \hat{p}$, and the number of applications is $\hat{n}_S$. Using symmetry of quasi-demands, a first-order Taylor expansion of (6.1) leads to

$$n_S - \hat{n}_S \approx \underbrace{\left. \frac{Q'_S(\frac{\partial Q_B}{\partial p_k} + \frac{\partial Q_B}{\partial p_\ell})}{1 - 2Q'_S \frac{\partial Q_B}{\partial n_S}} \right|_{(\hat{p}, \hat{p}, \hat{n}_S)}}_{<0} (dp_1 + dp_2)\,. \tag{6.2}$$

Equation (6.2) shows that the number of developers depends only how the total price $p_1 + p_2$ varies: if the total price increases (decreases), the number of applications decreases (increases), an intuitive result since the total number of buyers decreases (increases) and developers only care about the total number of buyers they can reach.[34]

Let $U(q_1, q_2, n_S)$ be the (gross) utility function from which the buyers' quasi-demands in (3.1) are derived (with $\partial U / \partial n_S > 0$). The buyers' net surplus is then given by $V(p_1, p_2, n_S) = \max_{(q_1 \geq 0, q_2 \geq 0)} U(q_1, q_2, n_S) - p_1 q_1 - p_2 q_2$. Using (6.2), a first-order Taylor expansion leads to

$$V(p_1, p_2, n_S) - V(\hat{p}, \hat{p}, \hat{n}_S) \approx \underbrace{\left(-Q_B + \frac{\partial U}{\partial n_S} \frac{Q'_S(\frac{\partial Q_B}{\partial p_k} + \frac{\partial Q_B}{\partial p_\ell})}{1 - 2Q'_S \frac{\partial Q_B}{\partial n_S}} \right)}_{<0} (dp_1 + dp_2)\,. \tag{6.3}$$

Accordingly, the total surplus of buyers increases (decreases), at the first order, when the total price $p_1 + p_2$ decreases (increases).

When quasi-demands are linear, as in our linear example, these first-order approximations are exact and we need to focus only on the total price to assess the impact of the merger on buyers and developers.

TOTAL PRICE VARIATION. To provide some intuitions on the impact of vertical integration on the total price, we analyze how the merger changes the manufacturers' best responses. Remind that, under separation, manufacturer M_k's best response is characterized by Equation (4.1). Assume that the integrated firm sets a royalty $w > \delta$.

Under integration, the best responses of the integrated manufacturer and of the non-

[34] We use the assumption ensuring the existence of an equilibrium at the participation subgame, that is, $1 - 2Q'_S(\partial Q_B / \partial n_S) > 0$ (see Appendix A.1).

integrated one are respectively characterized by

$$p_1 \frac{\partial D_1}{\partial p_1}(p_1, p_2) + D_1(p_1, p_2) + w \frac{\partial D_2}{\partial p_1}(p_2, p_1) = 0, \tag{6.4}$$

$$(p_2 - w)\frac{\partial D_2}{\partial p_2}(p_2, p_1) + D_2(p_2, p_1) = 0, \tag{6.5}$$

Comparing Equations (4.1) and (6.4) shows that the integrated firm's best response moves by

$$\underbrace{\delta \frac{\partial D_1}{\partial p_1}}_{\text{Efficiency}} + \underbrace{\delta \frac{\partial D_2}{\partial p_1}}_{\text{Accommodation}} + \underbrace{(w - \delta)\frac{\partial D_2}{\partial p_1}}_{\text{Upstream market power}}. \tag{6.6}$$

While the efficiency effect always shifts the integrated manufacturer's best response downward, the accommodation and upstream market power effects shift it upward when there is a negative externality across manufacturers and downward otherwise. All else equal, vertical integration leads the integrated manufacturer to set a lower price if there is a positive externality and has an ambiguous impact otherwise.

Comparing (4.1) and (6.5), the non-integrated manufacturer's best response always shifts upward by

$$- \underbrace{(w - \delta)\frac{\partial D_2}{\partial p_2}}_{\text{Upstream market power}} \tag{6.7}$$

because that manufacturer's marginal cost raises from 0 to δ. All else equal, the non-integrated manufacturer tends to increase its price under integration.

How much the total price varies following integration depends on the magnitude of these shifts in best responses. In the linear example, those shifts do not depend on price levels and are thus constant. This allows a clean and simple characterization of the total price variation, and, consequently, of consumer surplus and developer participation.

Proposition 5. *Consider the linear example.*

(i) With efficiency gains and no cost to port applications (that is, $\delta > 0$ and $\alpha = 0$), there exists a threshold $\tilde{\mu}(\gamma)$ such that the total price decreases if and only if $\mu > \tilde{\mu}(\gamma)$. Moreover, $\tilde{\mu}(\gamma) < \gamma/(2(1+\gamma))$: with a positive externality, developer participation and consumer surplus always increase.

(ii) With no efficiency gains and a cost to port applications (that is, $\delta = 0$ and $\alpha > 0$), the total price always increases, and developer participation and consumer surplus thus decrease.

Proof. See the Online Appendix. □

Absent positive motives for coordination but with efficiency gains (see Section 4), the extent to which the integrated firm increases the royalty depends on δ: in the linear example, it turns out that the total price decreases (increases) when indirect network effects are strong (weak) relative to product market competition, leading both to a higher (lower) consumer surplus and to more (fewer) applications developed.

With no efficiency gains and a cost to port applications, there is only an upstream market power effect at work (see Section 5). The non-integrated manufacturer increases its price, for its marginal cost increases, and that effect dominates any change in the price set by the integrated manufacturer. Overall, the total price unambiguously increases, and both developer participation and consumer surplus decrease.

7. Developer Fee: Structure of Indirect Network Effects and Foreclosure

Software platforms often charge developers on participation (for instance, Google charges developers $25 for each application published on the Play Store) or on transaction each time an application is sold on the platform (for instance, both Apple and Google charges a 30% royalty on each transaction on their respective applications stores). Fees paid by developers to platforms are now introduced in our base model. In the following, a_I and a_{E_i} denote the fees charged by platforms I and E_i, $i = 1, 2$. Developer fees are chosen together with royalties in stage 1 of our game.

The quasi-demand of developers writes now $n_S = Q_S(a, n_{B1} + n_{B2})$ where a stands for the total fee they paid to platforms (which depends on whether developers publish their applications on both platforms or one only one) and $\partial Q_S/\partial a < 0$. At the last stage of the game, consistency of participation decisions by buyers and developers leads to a number of developers, $D_S(a, p_1, p_2)$, and to demands for device k, $D_k(a, p_k, p_\ell)$, which all are decreasing with the developer fee a. The total fee developers pay to platforms acts as a shifter of the demands for devices and the number of applications.

MAIN ASSUMPTIONS. To emphasize the novel aspects introduced by developer fee, we neutralize the effects analyzed in Sections 4 and 5 with the following assumptions.

- First, platforms are symmetric and there are no costs to port applications, that is, $\delta = 0$ and $\alpha = 0$.

- Second, the externality across manufacturers is nil, that is, $\partial D_1/\partial p_2 = \partial D_2/\partial p_1 = 0$ for all (a, p_1, p_2) in the relevant range. In the linear example, this amounts to having $\gamma = 2\mu/(1-2\mu)$. If foreclosure emerges in equilibrium, it will therefore not be driven by the desire of the integrated firm to soften competition on the product market.

Under these assumptions, the demand for device k writes as $D_k(a, p_k)$. Since developers want to reach all buyers, the number of developers still depends on both prices and writes thus as $D_S(a, p_1, p_2)$.

SEPARATION. These assumptions lead to a clear separation benchmark, which is the natural generalization of the standard Bertrand when platforms compete on one side of the market only: All platforms offer a nil royalty and a nil developer fee.

Given that royalties are assumed to be non-negative, there are two cases to consider.[35]

First, it cannot be an equilibrium that a platform attracts manufacturers while charging a positive developer fee. Indeed, another platform can profitably undercut on both sides of the market.

Second, suppose that, say, platform I subsidizes developers ($a_I < 0$) and attracts both manufacturers with a positive royalty ($w_I > 0$). Platform E_i can then profitably attract manufacturers by slightly undercutting I's royalty ($w_{E_i} = w_I - \varepsilon$ with ε positive small) and setting a nil developer fee ($a_{E_i} = 0$): manufacturers gain, for their marginal costs decrease; developers can still benefit from the subsidy offered by I and pay no fee to access buyers since $a_{E_i} = 0$.

Differently put, a platform which boosts the participation of developers through a subsidy is exposed to a hold-up problem. Subsidizing developers is indeed akin to an investment that increases the manufacturers' profits. But the benefits of that investment in terms of a higher royalty can be expropriated by another competing platform.

The manufacturers' prices and profits in this separation benchmark are denoted by $\hat{p}_k$ and $\hat{\pi}_k$.[36] Platforms obviously make no profit.

Preliminary Result. Assume now that platform I is integrated with manufacturer M_1. From the perspective of the non-integrated platforms E_1 and E_2, the same argument as that made in the separation benchmark applies: they cannot do better than offering a nil royalty and a nil developer fee.

It turns out that the integrated platform cannot leverage its competitive advantage into a higher royalty. Indeed, if $w_I > 0$, then M_2 is better off choosing, say, E_i because, first, the accommodation effect does not compensate for the nil royalty offered by E_i, and, second, the affiliation with E_i has no adverse impact on developers' participation since $a_{E_i} = 0$. This directly echoes the analysis undertaken in Sections 4 and 5: absent efficiency gains (because $\delta = 0$) and without motives for coordination (because $a_{E_i} = 0$), there are no accommodation and upstream market power effects, and the integrated platform must license its operating system at a nil royalty. We consider from now on that $w_I = 0$ in any continuation.

The integrated platform has, however, some market power over its membership fee a_I, for developers always have to pay that fee to access the buyers of the integrated manufacturer's device. In other words, being integrated with a manufacturer provides a platform with a market power over that manufacturer's customers, which allows in turn to set a non-null developer fee. In stark contrast with the separation benchmark, the

[35]The assumption that royalties are non-negative allows us, besides, to bypass another issue. To understand, observe that, from the perspective of the platform-manufacturer vertical relationship, the royalty and developer fee act jointly as an 'imperfect' two-part tariff offered to the manufacturer (since the developer fee affects indirectly the manufacturer's profit). In the usual framework of upstream suppliers competing in two-part tariff to supply several downstream manufacturers, Schutz (2012) shows that a pure strategy equilibrium may fail to exist, and discusses issues related to its characterization. Restricting the fixed component of the two-part tariff to be positive allows to get rid of these issues. Our assumption on the royalties plays a similar role.

[36]We maintain the assumption made so far that the subgame of price competition between manufacturers is 'well-behaved'; see Section 3.3.

integrated platform is no longer entirely expropriated of its 'investment' on developers participation ($a_I < 0$). By the same token, the integrated platform is able to charge a positive developer fee ($a_I > 0$).

Notice that Sections 4 and 5 were primarily concerned with the choice of operating system by the non-integrated manufacturer and the corresponding royalty paid. This is now an irrelevant issue since M_2 pays no royalty in equilibrium. The introduction of developer fee has changed the focus of the analysis, as we now detail.

MANUFACTURERS' PRICES. At the last stage of the game, the profit of the integrated firm is $p_1 D_1(a_I, p_1) + a_I D_S(a_I, p_1, p_2)$ and its best response thus solves

$$D_1(a_I, p_1) + p_1 \frac{\partial D_1}{\partial p_1}(a_I, p_1) + a_I \frac{\partial D_S}{\partial p_1}(a_I, p_1, p_2) = 0. \tag{7.1}$$

The profit of the non-integrated manufacturer is $\pi_2^{II}(a_I, p_2) = p_2 D_2(a_I, p_2)$ and its best response is given by

$$D_2(a_I, p_2) + p_2 \frac{\partial D_2}{\partial p_2}(a_I, p_2) = 0. \tag{7.2}$$

Denote by $p_1^{II}(a_I)$ and $p_2^{II}(a_I)$ the prices which solve (7.1) and (7.2) simultaneously, and assume that $p_k^{II}(a_I)$, $k = 1, 2$ is decreasing in a_I.[37] The non-integrated manufacturer's profit is then given by $\pi_2^{II}(a_I, p_2^{II}(a_I))$.

FORECLOSURE AND PROFITABILITY OF VERTICAL INTEGRATION. The following result is then immediately obtained.

PROPOSITION 6. *Assume no externality across manufacturers, no efficiency gains, and no cost to port applications.*

- *The non-integrated manufacturer is foreclosed following vertical integration if and only if the developer fee increases beyond its pre-merger level, that is,*

$$\pi_2^{II}(a_I, p_2^{II}(a_I)) \leq \hat{\pi}_2 \Leftrightarrow a_I \geq 0.$$

- *Vertical integration is always profitable.*

Proof. See Appendix A.4. □

The first part is rather immediate: the integrated firm is now able to charge a strictly positive fee on developers because it has some market power over the buyers of its device; and when it is willing to do so, this contracts the non-integrated manufacturer's demand and harms its profits.

The second part is straightforward in light of the analysis undertaken in 4: a merger which creates no efficiency gains never triggers an adverse reaction of the non-integrated manufacturer; with developer fee, even if the integrated firm distorts the price of its device to accommodate the revenues earned from developers, it can always set a nil developer fee and gets the separation profit.

[37] As already noted, the developer fee acts as a negative shifter of the demands for devices. Though it is possible that equilibrium prices increase following a negative shift of demands, we rule out this possibility and assume that the manufacturers' prices are decreasing in the developer fee. A similar assumption was made in Section 5.

7.1. Asymmetric Price Structure

Charging a_I on developers has the following consequences:

- Consider first the non-integrated manufacturer. Under the nil externality assumption, the only effect effect of vertical integration for M_2 is to shift its demand, so that the price for device 2 is higher if $a_I < 0$ and lower otherwise. This situation is depicted in Figure 1, where M_2's best response shifts from $\hat{R}_2(0, p_1)$ to $R_2^{II}(a_I, p_1)$.

- Consider now the integrated manufacturer. Exactly as for the non-integrated manufacturer, a developer fee induces a demand shift. Much in the spirit of Section 4, the integrated firm chooses now its downstream price with an eye, not on the upstream profit since its royalty is nil, but rather on the impact on the revenues earned from the developers. Since $\partial D_S/\partial p_1 < 0$, the integrated firm's best response moves further downwards when $a_I > 0$ and further upwards when $a_I < 0$.[38]

Figure 1 illustrates these two effects in the linear example.

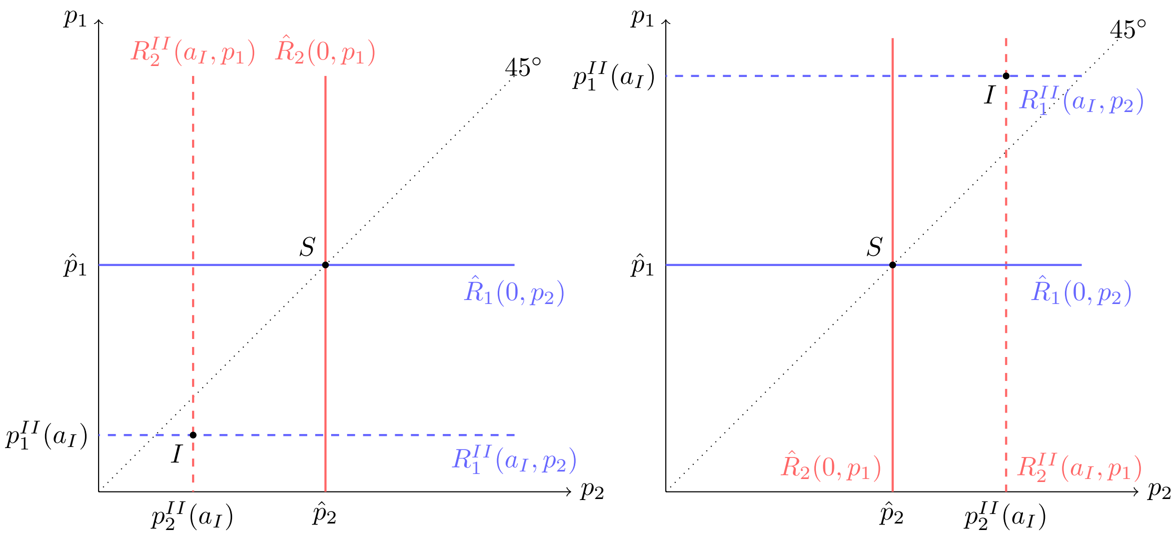

Figure 1 – Impact of vertical integration on best responses and downstream prices when the integrated platform charges $a_I > 0$ (left panel) or $a_I < 0$ (right panel). Best responses under separation are $\hat{R}_k(0, p_\ell)$; best responses under integration are $R_k^{II}(a_I, p_\ell)$. In the linear example, best responses are flat.

Considering an interior solution, the optimal developer fee, denoted by a_I^*, satisfies

[38] There is actually another implication, which has no consequences on the results that we derive below. From Equation (7.1), developer fee creates a new form of strategic interaction for the integrated firm only. Indeed, differentiating (7.1) with respect to p_2 shows that the sign of the slope of the integrated firm's best response depends on the sign of $a_I \partial^2 D_S/\partial p_1 \partial p_2$. $\partial^2 D_S/\partial p_1 \partial p_2$ itself has the sign of $\partial^2 Q_S/\partial n_S^2$. Therefore, the slope of the integrated manufacturer's best response depends both on whether developers are taxed or subsidized and on the concavity/convexity of network effects on the developer side. In the linear example, $\partial^2 Q_S/\partial n_S^2 = 0$, so that the integrated manufacturer's best response is flat, as under separation. If $\partial^2 Q_S/\partial n_S^2 \neq 0$, the integrated firm's best response is not flat, but it shifts more than the non-integrated manufacturer's, implying that p_1 varies more than does p_2 following the merger.

the following first-order condition (omitting some arguments)

$$(7.3)\quad \underbrace{p_1^{II}\frac{\partial D_1}{\partial a_I}(a_I^*, p_1^{II})}_{\text{Impact on buyers side}}$$
$$+ \underbrace{D_S(a_I^*, p_1^{II}, p_2) + a_I^*\left(\frac{\partial D_S}{\partial a_I}(a_I^*, p_1^{II}, p_2^{II}) + \frac{\partial D_S}{\partial p_2}(a_I^*, p_1^{II}, p_2^{II})\frac{dp_2^{II}}{da_I}\right)}_{\text{Impact on developers side}} = 0.$$

Charging a positive developer fee yields additional revenues, but reduces the demand for the integrated firm's devices and thus its retail profit. Moreover, this also negatively affects the non-integrated manufacturer's demand, thereby further lowering the number of applications developed.

A two-sided logic is now at work. When buyers of devices benefit weakly from the number of applications, the integrated firm should tax developers and 'subsidize its buyers' through a low price of its device. By contrast, when application developers value weakly the number of buyers, the integrated firm chooses to 'tax buyers' through a high price for its devices and subsidize developers. While the competitive pressure exerted by the non-integrated platforms does not allow the integrated firm to raise the royalty, it can still internalize the externalities across both sides of the market by playing simultaneously on the developer fee and on the price of its device. This intuition is illustrated in the case of our linear example.

PROPOSITION 7. *Consider the linear example with no externality across manufacturers, no efficiency gains and no motives for coordination.*

- *The integrated platform taxes developers when $u_B \leq 2u_S$ and subsidizes them otherwise, that is, $a_I^* \geq 0 \Leftrightarrow u_B \leq 2u_S$.*
- *Vertical integration increases consumer surplus when either $u_B \geq 2u_S$ or when $u_B \leq 1/2\, u_S$, that is, when indirect network effects are sufficiently asymmetric.*

Proof. See the Online Appendix. □

The first part follows a two-sided logic: when indirect network effects are asymmetric, the integrated firm implements an asymmetric price structure, in which the side which benefits the most (least) from the participation of the other side is taxed (subsidized). From Proposition 6, foreclosure arises when indirect network effects are stronger for developers than for buyers.

When indirect network effects are stronger for developers than for buyers, the integrated firm charges a positive developer fee and lowers the price of its device. As indirect network effects become more asymmetric, so does the price structure and the price of the integrated manufacturer's device falls below its cost. Selling its devices at a loss is a profitable strategy for the integrated firm because these losses are recouped from the fee charged on developers. But as the developer fee increases, the demand faced by the non-integrated manufacturer contracts. At some point, the non-integrated manufacturer is forced to exit the market.[39]

[39] See the Online Appendix for the details.

Such a foreclosure is, however, not driven by any anti-competitive motive, for we assumed the externality across manufacturers is nil; rather, it is the mere consequence of the internalization of the indirect network effects through the adequate design of the integrated firm's price structure.

The internalization of indirect network effects by the integrated platform may well increase consumer surplus, as shown in the second part of Proposition 7. The intuition relates to that of the two-sided markets literature. The price structure implemented by a monopoly platform is similar to that chosen by a benevolent planner, although the price levels obviously differ, as put forward by Rochet and Tirole (2006) or Weyl (2010). In our context, under separation, platforms cannot internalize indirect network effects because competition leads to nil royalties and manufacturers capture the retail profits. By contrast, the vertically integrated firm can use both the developer fee and the price of its device to harness the indirect network effects, thereby bringing prices closer to an efficient structure and enhancing consumer surplus.

Observe that total consumer surplus and the non-integrated manufacturer's profit no longer vary in the same direction following the merger. Section 6 has shown that, with efficiency gains, vertical integration leads to higher post-merger consumer surplus and non-integrated manufacturer's profit when there is a positive externality across manufacturers, whereas both are lower with motives for coordination. The logic at work with developer fee is thus different, and is linked to the structure, rather than the level, of indirect network effects.

7.2. Positive and Negative Externalities between Manufacturers

We now briefly discuss the role of the externality between manufacturers. When the externality between manufacturers is non nil, the demand for device k depends on both prices, $D_k(a_I, p_k, p_\ell)$. Following similar steps as previously, we obtain (omitting some arguments)

$$\frac{d\pi_2^{II}}{da_I}(a_I, p_2^{II}, p_1^{II}) = p_2^{II}\left(\frac{\partial D_2}{\partial a_I}(a_I, p_2^{II}, p_1^{II}) + \underbrace{\frac{\partial D_2}{\partial p_1}(a_I, p_2^{II}, p_1^{II})}_{\text{Externality}}\frac{dp_1^{II}}{da_I}\right)$$

Since $dp_1^{II}/da_I < 0$, when the externality between manufacturers is negative, $d\pi_2^{II}/da_I < 0$ and the non-integrated manufacturer is foreclosed as soon as the integrated platform sets a developer fee above the pre-merger level. When the externality is positive, this might no longer be the case. This echoes the analysis undertaken in Section 4, which showed that foreclosure becomes less a concern when manufacturers are more collaborators than competitors. In the Online Appendix, we use the linear example to confirm the intuition that vertical integration, first, leads to foreclosure when developers value sufficiently more the number of buyers than buyers value the number of applications, and, second, increase consumer surplus when indirect network are sufficiently asymmetric in one direction or another.

8. Conclusion

We develop a model of a platform market, in which platforms interact with device manufacturers and there are indirect network effects between buyers of devices and application developers. While our prime example is the smartphone market, our analysis is relevant, more generally, to the market of connected devices also called 'the Internet of Things.'

The main messages conveyed by our analysis may be summarized as follows: indirect network externalties change the nature of the downstream interaction between manufacturers, and, therefore, the competitive assessment of a vertical merger; direct and indirect network effects have different impacts on foreclosure; the presence of developer fee adds another layer of complexity in the competitive assessment of vertical integration. Overall, the sources of upstream market power, and their consequences in terms of foreclosure or consumer surplus, are different from those unveiled in the extant literature. Our analysis therefore warns policy-makers against a blind application of the traditional view about foreclosure when dealing with platform markets, and offers some guidance as to where competition authorities should focus their investigation.

As in standard markets, antitrust authorities may want to limit the anti-competitive effects of vertical integration by constraining the pricing of the royalty. Our analysis somewhat supports that idea: with coordination motives giving rise to harmful foreclosure, limiting the royalty paid by manufacturers prevents the non-integrated manufacturer from being hurt by the merger. In the context of platform markets, this remedy raises, however, several issues. First, with strong indirect network effects, capping the royalty reduces the price decrease of the integrated manufacturer. Second, a cap on the royalty is likely to impact the pricing on the developer side of the market. A more complete assessment of such a behavioral remedy is left for future research.

While we have voluntary laid down our analysis in a setting as close as possible to the literature on strategic vertical integration, it could be readily applied to the motivating example given in introduction. One platform has a dominant position on the market for operating systems and decides whether to launch its own smartphone, which will compete directly with the device of the incumbent manufacturer; the manufacturer has a costly possibility to bypass the platform's operating system. The difference is that the introduction of the new product is also a source of demand creation. Our working paper (Pouyet and Trégouët, 2016) shows that the impact of such a downstream expansion by the dominant platform on foreclosure depends, again, on the nature of the externalities between manufacturers. As of today, Google Pixel's low sales seem unlikely to put a strong competitive pressure at the retail level on other smartphone producers. Our analysis suggests therefore that its introduction may well benefit the industry as well as consumers.

Finally, whether manufacturers end up being collaborators or competitors depends on a combination of indirect network effects and product differentiation. Roughly speaking, investment by platforms are more likely to affect the intensity of (direct or indirect) network effects whereas manufacturers can change the design and characteristics of their products. Hence, our analysis could be extended to endogenize the degree of collaboration/competition between manufacturers, given that both manufacturers and platforms may jointly contribute to that variable.

REFERENCES

AMELIO, A. AND B. JULLIEN (2012): "Tying and freebies in two-sided markets," *International Journal of Industrial Organization*, 30, 436–446.

ARMSTRONG, M. (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37, 668–691.

CAILLAUD, B. AND B. JULLIEN (2003): "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34, 309–28.

CHEN, Y. (2001): "On Vertical Mergers and Their Competitive Effects," *RAND Journal of Economics*, 32, 667–85.

CHEN, Y. AND M. H. RIORDAN (2007): "Vertical Integration, Exclusive Dealing, and Expost Cartelization," *RAND Journal of Economics*, 38, 1–21.

CHOI, J. P. AND D.-S. JEON (2016): "A Leverage Theory of Tying in Two-Sided Markets," TSE Working Paper 16-689, TSE.

CHOI, J. P. AND S.-S. YI (2000): "Vertical Foreclosure with the Choice of Input Specifications," *RAND Journal of Economics*, 31, 717–743.

CHURCH, J. AND N. GANDAL (2000): "Systems Competition, Vertical Merger, and Foreclosure," *Journal of Economics & Management Strategy*, 9, 25–51.

COWAN, S. (2004): "Demand Shifts and Imperfect Competition," Tech. Rep. 188, Discussion paper, University of Oxford.

CRAWFORD, G. S., R. S. LEE, M. WHINSTON, AND A. YURUKOGLU (2016): "The Welfare Effects of Vertical Integration in Multichannel Television Markets," CEPR Discussion Papers 11202, C.E.P.R. Discussion Papers.

DOGANOGLU, T. AND J. WRIGHT (2010): "Exclusive Dealing with Network Effects," *International Journal of Industrial Organization*, 28, 145–154.

EDELMAN, B. (2015): "Does Google Leverage Market Power Through Tying and Bundling?" *Journal of Competition Law & Economics*, 11, 365–400.

EVANS, D. (2013): "Economics of Vertical Restraints for Multi-Sided Platforms," *CPI Journal*, 9.

HAGIU, A. AND R. S. LEE (2011): "Exclusivity and Control," *Journal of Economics & Management Strategy*, 20, 679–708.

HAGIU, A. AND D. SPULBER (2013): "First-Party Content and Coordination in Two-Sided Markets," *Management Science*, 59, 933–949.

HART, O. AND J. TIROLE (1990): "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity: Microeconomics*, special issue, 205–276.

Hombert, J., J. Pouyet, and N. Schutz (2016): "Anticompetitive Vertical Merger Waves," *PSE Working Paper*.

Hunold, M. and K. Stahl (2016): "Passive Vertical Integration and Strategic Delegation," *RAND Journal of Economics*, 47, 891–913.

Katz, M. L. and C. Shapiro (1985): "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75, 424–40.

Lafontaine, F. and M. Slade (2007): "Vertical Integration and Firm Boundaries: The Evidence," *Journal of Economic Literature*, 45, 629–685.

Lee, R. S. (2013): "Vertical Integration and Exclusivity in Platform and Two-Sided Markets," *American Economic Review*, 103, 2960–3000.

Loertscher, S. and M. Reisinger (2014): "Market Structure and the Competitive Effects of Vertical Integration," *RAND Journal of Economics*, 45, 471–494.

Nocke, V. and L. White (2007): "Do Vertical Mergers Facilitate Upstream Collusion?" *American Economic Review*, 97, 1321–1339.

Normann, H.-T. (2009): "Vertical integration, raising rivals' costs and upstream collusion," *European Economic Review*, 53, 461–480.

Ordover, J. A., G. Saloner, and S. C. Salop (1990): "Equilibrium Vertical Foreclosure," *American Economic Review*, 80, 127–42.

Page, W. H. and J. E. Lopatka (2009): *The Microsoft Case: Antitrust, High Technology, and Consumer Welfare*, Chicago, IL: University of Chicago Press.

Pouyet, J. and T. Trégouët (2016): "Vertical Mergers in Platform Markets," *PSE Working Paper*.

Reiffen, D. (1992): "Equilibrium Vertical Foreclosure: Comment," *American Economic Review*, 82, 694–97.

Reisinger, M., L. Ressner, and R. Schmidtke (2009): "Two-sided markets with pecuniary and participation externalities," *Journal of Industrial Economics*, 57, 32–57.

Rey, P. and J. Tirole (2007): *A Primer on Foreclosure*, Elsevier, vol. 3 of *Handbook of Industrial Organization*, chap. 33, 2145–2220.

Riordan, M. (2008): *Competitive Effects of Vertical Integration*, MIT Press, 145–182, Handbook of Antitrust Economics.

Ritz, R. (2017): *Oligopolistic competition and welfare*, Edward Elgar, forthcoming, Handbook of Game Theory and Industrial Organization.

Rochet, J.-C. and J. Tirole (2006): "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37, 645–667.

Salop, S. C. and D. T. Scheffman (1983): "Raising Rivals' Costs," *American Economic Review*, 73, 267–71.

Schutz, N. (2012): "Competition with Exclusive Contracts in Vertically Related Markets: An Equilibrium Non-Existence Result," Tech. rep.

Seade, J. (1980): "The Stability of Cournot Revisited," *Journal of Economic Theory*, 23, 15–27.

Shubik, M. and R. Levitan (1980): *Market Structure and Behavior*, Cambridge, MA: Harvard University Press.

Weyl, E. G. (2010): "A Price Theory of Multi-sided Platforms," *American Economic Review*, 100, 1642–72.

Weyl, G. E. and M. Fabinger (2013): "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition," *Journal of Political Economy*, 121, 528–583.

A. APPENDIX

A.1. Demand Functions and Indirect Network Effects

At the last stage of the game, given devices prices p_1 and p_2, the number of buyers of each device and the number of developers must be consistent and solve

$$\left\{\begin{array}{rcl} n_{B1} & = & Q_B(p_1, p_2, n_S), \\ n_{B2} & = & Q_B(p_2, p_1, n_S), \\ n_S & = & Q_S(n_{B1} + n_{B2}). \end{array}\right. \tag{A.1}$$

In order to avoid 'cornered-market' solutions, where all consumers or all developers participate in equilibrium, we make the usual assumption that indirect network effects are not too strong so that, in the relevant range, each manufacturer faces a demand that is locally elastic with respect to prices.

ASSUMPTION A.1 (Indirect Network Effects Are Not Too Strong). *For the relevant range of prices (p_1, p_2), the total number of buyers $n_{B1} + n_{B2}$ and number of developers n_S,*

$$Q'_S(n_{B1} + n_{B2}) \left(\frac{\partial Q_B}{\partial n_S}(p_1, p_2, n_S) + \frac{\partial Q_B}{\partial n_S}(p_2, p_1, n_S) \right) < 1.$$

We can then show the following result.

LEMMA A.1. *For the relevant range of prices (p_1, p_2), system (A.1) has a unique solution.*

Proof. Let $D(p_1, p_2) = D_1(p_1, p_2) + D_2(p_2, p_1)$. From system (A.1), we have

$$D(p_1, p_2) = Q_B(p_1, p_2, Q_S(D(p_1, p_2)) + Q_B(p_2, p_1, Q_S(D(p_1, p_2)). \tag{A.2}$$

Put differently, for a given pair (p_1, p_2), $D(p_1, p_2)$ is a fixed point of $\psi(x) = Q_B(p_1, p_2, Q_S(x)) + Q_B(p_1, p_2, Q_S(x))$. Notice then that

$$\psi'(x) = Q'_S(x) \left(\frac{\partial Q_B}{\partial n_S}(p_1, p_2, Q_S(x)) + \frac{\partial Q_B}{\partial n_S}(p_2, p_1, Q_S(x)) \right).$$

Assumption A.1 then implies that $|\psi'(\cdot)| < 1$: $\psi(\cdot)$ is a contraction mapping and Equation (A.2) has a unique solution. □

We can then show the following result.

LEMMA A.2. *The following properties hold: $\partial D_k / \partial p_k(p_k, p_\ell) < 0$, $\partial D_S / \partial p_k(p_k, p_\ell) < 0$ and $|\partial D_k / \partial p_k(p, p)| > |\partial D_k / \partial p_\ell(p, p)|$.*

Proof. To avoid any ambiguity, let $Q_{B1}(p_1, p_2, n_S) = Q_B(p_1, p_2, n_S)$ and $Q_{B2}(p_2, p_1, n_S) = Q_B(p_2, p_1, n_S)$. By the implicit function theorem, $D(p_1, p_2)$ is continuously differentiable. Differentiating wrt p_1 Equation (A.2) and rearranging terms, we find

$$\frac{\partial D}{\partial p_1}(p_1, p_2) \left\{ 1 - Q'_S(D(p_1, p_2)) \left(\frac{\partial Q_{B1}}{\partial n_S}(p_1, p_2, D(p_1, p_2)) + \frac{\partial Q_{B2}}{\partial n_S}(p_2, p_1, D(p_1, p_2)) \right) \right\}$$
$$= \frac{\partial Q_{B1}}{\partial p_1}(p_1, p_2, D(p_1, p_2)) + \frac{\partial Q_{B2}}{\partial p_1}(p_2, p_1, D(p_1, p_2)).$$

By Assumption A.1 the term in curly brackets is positive, and, therefore, $\partial D / \partial p_1 \, (p_1, p_2)$ is negative. Similarly, $\partial D / \partial p_2 \, (p_1, p_2) < 0$. Since $Q'_S(\cdot) > 0$, it is then immediate that

$D_S(p_1, p_2) = Q_S(D(p_1, p_2))$ is decreasing in both p_1 and p_2. Then (omitting some notations) $\partial D_1/\partial p_1 = \partial Q_{B1}/\partial p_1 + (\partial D/\partial p_1)(\partial Q_{B1}/\partial n_S)Q'_S(D)$, which shows that $\partial D_1/\partial p_1 < 0$. Similarly, $\partial D_1/\partial p_2 = \partial Q_{B1}/\partial p_2 + (\partial D/\partial p_2)(\partial Q_{B1}/\partial n_S)Q'_S(D)$. For symmetric prices, $\partial D/\partial p_1 = \partial D/\partial p_2$, and therefore $|\partial D_1/\partial p_1| - |\partial D_1/\partial p_2| = |\partial Q_{B1}/\partial p_1| - |\partial Q_{B1}/\partial p_2| < 0$, which is negative from our assumptions. $\square$

A.2. Proof of Proposition 1

The proof proceeds in several steps.[40] First, we prove that if I and E offer the same positive royalty, then M_2 strictly prefers to buy from I. Second, we show that I is better off selling to manufacturer M_2. Last, we show that I supplies M_2 at a royalty strictly above its marginal cost, that is, $w^*(\delta) > \delta$ for δ small enough.

LEMMA A.3 (Accommodation Effect). *Suppose that both platforms offer the same royalty $w > 0$. With a positive (negative) externality across manufacturers, the integrated manufacturer sets a lower (higher) price when the non-integrated manufacturer buys from the integrated platform, that is, for all $w > 0$, $p_1^{II}(0, w) < p_1^{IE}(0, w) \Leftrightarrow \frac{\partial D_2}{\partial p_1}(p_2, p_1) < 0$.*

Accordingly, the non-integrated manufacturer is strictly better off buying from the integrated platform, that is, $\pi_2^{II}(w, 0) > \pi_2^{IE}(w, 0)$.

Proof. By definition, $(p_1^{IE}(0, w), p_2^{IE}(w, 0))$ and $(p_1^{II}(0, w), p_2^{II}(w, 0))$ solve respectively

$$\text{(A.3)} \qquad \begin{cases} D_1\left(p_1^{IE}(0,w), p_2^{IE}(w,0)\right) + p_1^{IE}(0,w)\dfrac{\partial D_1}{\partial p_1}\left(p_1^{IE}(0,w), p_2^{IE}(w,0)\right) = 0 \\ D_2\left(p_2^{IE}(w,0), p_1^{IE}(0,w)\right) + \left(p_2^{IE}(w,0) - w\right)\dfrac{\partial D_2}{\partial p_2}\left(p_2^{IE}(w,0), p_1^{IE}(0,w)\right) = 0 \end{cases}$$

and

$$\text{(A.4)} \quad \begin{cases} D_1\left(p_1^{II}(0,w), p_2^{II}(w,0)\right) + p_1^{II}(0,w)\dfrac{\partial D_1}{\partial p_1}\left(p_1^{II}(0,w), p_2^{II}(w,0)\right) + w\dfrac{\partial D_2}{\partial p_1}\left(p_2^{II}(w,0), p_1^{II}(0,w)\right) = 0 \\ D_2\left(p_2^{II}(w,0), p_1^{II}(0,w)\right) + \left(p_2^{II}(w,0) - w\right)\dfrac{\partial D_2}{\partial p_2}\left(p_2^{II}(w,0), p_1^{II}(0,w)\right) = 0 \end{cases}$$

We first prove that, when there is a positive (negative) externality across manufacturers, M_1's best response shifts downward (upward) when w increases. To this end, let $R_k^{Ij}(p_\ell, w)$ denote manufacturer M_k's best response to manufacturer M_ℓ ($\ell \neq k$) when M_2 buys from platform $j \in \{I, E\}$ at some royalty w. From the systems (A.3) and (A.4), $R_2^{IE}(p_1, w) = R_2^{II}(p_1, w)$ and $R_1^{IE}(p_2, w) = R_1^{II}(p_2, 0)$. By definition $\frac{\partial}{\partial p_1}\pi_1^{II}(R_1^{II}(p_2, w), p_2) = 0$. Therefore (omitting some notations to ease the exposition)

$$\frac{dR_1^{II}}{dw} = -\frac{\frac{\partial^2 \pi_1^{II}}{\partial p_1 \partial w}}{\frac{\partial^2 \pi_1^{II}}{\partial p_1^2}} = -\frac{\frac{\partial D_2}{\partial p_1}}{\frac{\partial^2 \pi_1^{II}}{\partial p_1^2}},$$

which is strictly negative (positive) when there is a positive (negative) externality across manufacturers since the denominator is negative from the second-order condition ($\partial^2 \pi_1^{II}/\partial p_1^2 < 0$). Therefore, M_1's best response shifts downward (upward) when w increases.

We are now in position to prove that the integrated platform becomes accommodating when M_2 buys from I rather than E.

[40] The proof extends that in Chen (2001) to the case of demand complements. It is however more general in that it does not hinge on the fact that the manufacturers' prices are strategic complements or substitutes.

To begin with, assume that there is a positive externality across manufacturers. Since M_1's best response shifts downward when w increases, we have, for all $w > 0$, $R_1^{II}(p_2, w) < R_1^{II}(p_2, 0) = R_1^{IE}(p_2, w)$. It follows that for all $w > 0$

$$p_1^{II}(0,w) = R_1^{II}\left(R_2^{II}(p_1^{II}(0,w),w),w\right) < R_1^{IE}\left(R_2^{IE}(p_1^{II}(0,w),w),w\right). \tag{A.5}$$

Define now $\Phi(p) = R_1^{IE}(R_2^{IE}(p,w),w) - p$, and notice that $\Phi(p_1^{IE}(0,w)) = 0$. $\Phi(\cdot)$ is continuously differentiable and strictly decreasing, since the slopes of the best responses are strictly smaller than 1. Therefore, $\Phi(p) > 0$ if and only if $p < p_1^{IE}(0,w)$. Together with inequality (A.5), this implies that $p_1^{II}(0,w) < p_1^{IE}(0,w)$.

It is immediate to see that the opposite result obtains when there is a negative externality across manufacturers, that is, p_1 is higher when M_2 buys from I. To summarize, for all $w > 0$, $p_1^{II}(0,w) < p_1^{IE}(0,w)$ if and only if there is a positive externality across manufacturers.

To conclude, it remains to show that M_2 is better off buying from I than from E when I and E offer the same royalty $w > 0$. We have

$$\begin{aligned}
\pi_2^{II}(w,0) &= (p_2^{II}(w,0) - w)D_2\left(p_2^{II}(w,0), p_1^{II}(0,w)\right) \\
&> (p_2^{IE}(w,0) - w)D_2\left(p_2^{IE}(w,0), p_1^{II}(0,w)\right) \qquad \text{(by revealed preferences)} \\
&> (p_2^{IE}(w,0) - w)D_2\left(p_2^{IE}(w,0), p_1^{IE}(0,w)\right) = \pi_2^{IE}(0,w),
\end{aligned}$$

where the last inequality comes from the fact that, with a positive (negative) externality across manufacturers, $p_1^{II}(0,w) < p_1^{IE}(0,w)$ ($p_1^{II}(0,w) > p_1^{IE}(0,w)$) and $D_2(p_2,p_1)$ is decreasing (increasing) in p_1. □

We prove now that the integrated platform is better off serving M_2.

LEMMA A.4 (Incentives to Sell). *The integrated platform is strictly better off serving the non-integrated manufacturer.*

Proof. Suppose first that there is a positive externality across manufacturers. We have

$$\begin{aligned}
\pi_1^{II}(0,0) &= p_1^{II}(0,0)D_1\left(p_1^{II}(0,0), p_2^{II}(0,0)\right) \\
&> p_1^{IE}(0,\delta)D_1\left(p_1^{IE}(0,w), p_2^{II}(0,0)\right) \qquad \text{(by revealed preferences)} \\
&> p_1^{IE}(0,\delta)D_1(p_1^{IE}(0,\delta), p_2^{IE}(\delta,0)) = \pi_1^{IE}(0,\delta)
\end{aligned}$$

where the last inequality comes from the fact that $p_2^{II}(0,0) = p_2^{IE}(0,0) < p_2^{IE}(w,0)$ and $D_1(p_1,p_2)$ is decreasing in p_2.

Suppose then that there is a negative externality across manufacturers. With a positive externality across manufacturers, the proof relied on the fact that making M_2 more efficient through a nil royalty benefits the integrated platform. This argument no longer holds with a negative externality, and, accordingly, we proceed differently. We prove that if δ is small enough, then $\pi_1^{II}(0,\delta) > \pi_1^{IE}(0,\delta)$.

Notice first that, since $\pi_1^{II}(0,0) = \pi_1^{IE}(0,0)$, it is sufficient to show that $\delta \mapsto \Delta\pi_1(\delta) = \pi_1^{II}(0,\delta) - \pi_1^{IE}(0,\delta)$ is strictly increasing in a (right) neighborhood of 0. We have (omitting some notations)

$$(\Delta\pi_1)'(\delta) = \left(p_1^{II}(0,\delta)\frac{\partial p_2^{II}}{\partial w_2}(\delta,0)\frac{\partial D_1}{\partial p_2} + D_2 + \delta\frac{\partial p_2^{II}}{\partial w_2}(\delta,0)\frac{\partial D_2}{\partial p_2}\right) - \left(p_1^{IE}(0,\delta)\frac{\partial p_2^{IE}}{\partial w_2}(\delta,0)\frac{\partial D_1}{\partial p_2}\right)$$

And, since $p_k^{II}(0,0) = p_k^{IE}(0,0)$ for all $k \in \{1,2\}$, we obtain (still omitting some notations)

$$\begin{aligned}
(\Delta\pi_1)'(0) &= p_1^{II}(0,0)\frac{\partial D_1}{\partial p_2}\left(\frac{\partial p_2^{II}}{\partial w_2}(0,0) - \frac{\partial p_2^{IE}}{\partial w_2}(0,0)\right) + D_2 \\
&= p_1^{II}(0,0)\frac{\partial D_1}{\partial p_2}\left(\frac{\partial p_2^{II}}{\partial w_2}(0,0) - \frac{\partial p_2^{IE}}{\partial w_2}(0,0)\right) - p_2^{II}(0,0)\frac{\partial D_2}{\partial p_2} \\
&= p_1^{II}(0,0)\left(\frac{\partial D_2}{\partial p_1}\left(\frac{\partial p_2^{II}}{\partial w_2}(0,0) - \frac{\partial p_2^{IE}}{\partial w_2}(0,0)\right) - \frac{\partial D_2}{\partial p_2}\right) \\
&> p_1^{II}(0,0)\left(-\frac{\partial D_2}{\partial p_1} - \frac{\partial D_2}{\partial p_2}\right) \\
&> 0
\end{aligned}$$

where the second equality comes from the first-order condition on p_2; the third equality from the fact that $p_1^{II}(0,0) = p_2^{II}(0,0)$ and $\frac{\partial D_1}{\partial p_2}(p_1^{II}(0,0), p_2^{II}(0,0)) = \frac{\partial D_2}{\partial p_1}(p_2^{II}(0,0), p_1^{II}(0,0))$; the first inequality from the fact that $0 < \partial p_k/\partial w_k < 1$; and the last inequality from the fact that $-\partial D_2/\partial p_2 > |\partial D_2/\partial p_1| > 0$. This concludes the proof. □

We can now prove the announced result. The problem of the integrated platform writes as

$$\max_w \pi_1^{II}(0,w) \text{ s.t. } \pi_2^{II}(w,0) \geq \pi_2^{IE}(\delta,0) \tag{A.6}$$

Let us show first that $\pi_2^{II}(w,0)$ is decreasing in w. This will show that the inequality in problem (A.6) rewrites as $w \leq w^*(\delta)$, where $w^*(\delta)$ is given by $\pi_2^{II}(w^*(\delta),0) = \pi_2^{IE}(\delta,0)$.

$$\begin{aligned}
\frac{d\pi_2^{II}}{dw}(w,0) &= -D_2 + (p_2^{II} - w)\tfrac{\partial D_2}{\partial p_1}\tfrac{\partial p_1^{II}}{\partial w} && \text{(by the Envelope Theorem)} \\
&= (p_2^{II} - w)\left(\tfrac{\partial D_2}{\partial p_2} + \tfrac{\partial D_2}{\partial p_1}(R_1^{II})'(p_2^{II}, w)\tfrac{\partial p_2^{II}}{\partial w}\right) && \text{(using } M_2\text{'s first-order condition)} \\
&< 0,
\end{aligned}$$

where the last inequality stems from the facts that pass-throughs are smaller than 1, the slopes of the best-response functions are smaller than 1, and $|\partial D_2/\partial p_2| > |\partial D_2/\partial p_1|$. Then, since $\pi_2^{II}(\delta,0) > \pi_2^{IE}(\delta,0) > 0$ by Lemma A.3 and $\pi_2^{II}(w,0) = 0$ when w is large enough, there exists a unique royalty $w^*(\delta) > \delta$ such that $\pi_2^{II}(w,0) > \pi_2^{IE}(\delta,0)$ iff $w < w^*(\delta)$. Therefore, M_2 buys from platform I iff $w \leq w^*(\delta)$.

Notice then that $w^*(\delta)$ goes to 0 when δ goes to 0. Accordingly, if δ is small enough, $w^*(\delta)$ is in a neighborhood of 0 and the solution of problem (A.6) is $w^*(\delta)$ if $\pi_1^{II}(0,w)$ is increasing in w in this neighborhood. Let us prove that it is indeed the case. We have

$$\frac{d\pi_1^{II}}{dw}(0,0) = \frac{\partial p_2^{II}}{\partial w_2}(0,0)\frac{\partial D_1}{\partial p_2}\left(p_1^{II}(0,0), p_2^{II}(0,0)\right)p_1^{II}(0,0) + D_2\left(p_2^{II}(0,0), p_1^{II}(0,0)\right)$$

When there is a negative externality across manufacturers, the two terms in the right-hand side of the previous equation are positive and we immediately conclude that $(d\pi_{I1}^{II}/dw)(0,0) > 0$.

Suppose then that there is a positive externality across manufacturers. We have

$$\begin{aligned}
\frac{d\pi_1^{II}}{dw}(0,0) &> \frac{\partial D_1}{\partial p_2}\left(p_1^{II}(0,0), p_2^{II}(0,0)\right)p_1^{II}(0,0) + D_2\left(p_2^{II}(0,0), p_1^{II}(0,0)\right) \\
&> \frac{\partial D_2}{\partial p_2}\left(p_2^{II}(0,0), p_1^{II}(0,0)\right)p_2^{II}(0,0) + D_2\left(p_2^{II}(0,0), p_1^{II}(0,0)\right) = 0
\end{aligned}$$

where the first inequality stems from the fact that pass-throughs are positive and smaller than 1, $0 < \frac{\partial p_2^{II}}{\partial w_2}(0) < 1$, and $\frac{\partial D_1}{\partial p_2} \leq 0$; the second inequality stems from the fact that $p_1^{II}(0,0) = p_2^{II}(0,0)$

and $|\frac{\partial D_2}{\partial p_2}(p_2, p_1)| > |\frac{\partial D_1}{\partial p_2}(p_1, p_2)|$; the last equality stems from the first-order condition associated to $p_2^{II}(0,0)$. This concludes the proof.

A.3. *Proof of Proposition 4*

Let $I1$ denotes the integrated firm composed of platform I and manufacturer M_1. To prove the first item in Proposition 4, we follow the same steps as in the proof of Proposition 1.

First, the non-integrated manufacturer M_2 is better off affiliating with the integrated platform when both platforms set the same royalty since, by so doing, it receives a strictly higher demand and benefits from an accommodation effect (see the proof of Proposition 1): for all w, $\pi_2^{II}(w,0) > \pi_2^{IE}(w,0)$.

Second, $I1$ is better off supplying the non-integrated manufacturer. In equilibrium, E sets a nil royalty: M_2's profit is $\pi_2^{IE}(0,0)$ if it affiliates with E. By setting a nil royalty, we already know that $I1$ attracts M_2. It is also strictly profitable for, by so doing, $I1$'s upstream profits are unaffected (they are nil irrespective of whether M_2 affiliates with I or E) and downstream profits are strictly higher (M_1's downstream demand is strictly higher when M_2 affiliates with I).

Finally, the problem of the integrated platform writes as

$$\max_{w} \pi_1^{II}(0,w) \text{ s.t. } \pi_2^{II}(w,0) \geq \pi_2^{IE}(0,0) \tag{A.7}$$

By the same argument than in the proof of Proposition 1, there exists $\overline{w} > 0$ such that the constraint in equation (A.7) rewrites $w \leq \overline{w}$. Put differently, the optimal royalty $w^{**}(\alpha)$ is in the interval $[0, \overline{w}]$. To prove that $w^{**}(\alpha)$ is positive, it is then sufficient to show that the function $w \mapsto \pi_1^{II}(0,w)$ is strictly increasing in a neighborhood of $w = 0$, and this has already been shown in the proof of Proposition 1.

The second item in Proposition 4 comes from the following observation: since $w \mapsto \pi_2^{II}(w,0)$ is strictly decreasing in w, we have $\pi_2^{II}(w^{**},0) < \pi_2^{II}(0,0) = \hat{\pi}_2^{II}(0,0)$.

Let us then prove that $(w^{**})'(\alpha) \geq 0$. If α is large, the gains from coordination are large and, therefore, it is possible that M_2's participation constraint is not binding, in which case $w^{**}(\alpha) = \arg\max_w \pi_1^{II}(0,w)$, which does not depend on α. Suppose then that M_2's participation constraint is binding for smaller values of α. Note that there always exists an interval of values of α for which it is the case since $\pi_2^{II} = \pi_2^{IE}$ when $\alpha = 0$. $w^{**}(\alpha)$ is now given by $\pi_2^{II}(w^{**}(\alpha),0) = \pi_2^{IE}(0,0)$ and (omitting notations)

$$(w^{**})'(\alpha) = \frac{d\pi_2^{IE}/d\alpha}{\partial\pi_2^{II}/\partial w_2},$$

which is positive since, by assumption, $d\pi_2^{IE}/d\alpha < 0$ and $\partial\pi_2^{II}/\partial w_2 < 0$.

The third item is immediate since, by revealed preferences, $\pi_1^{II}(0,w^{**}) > \pi_1^{II}(0,0) = \hat{\pi}_1(0,0) + \hat{\pi}_I(0,0)$.

A.4. *Proof of Proposition 6*

For the first part, observe first that $\pi_2^{II}(0, p_2^{II}(0)) = \hat{\pi}_2$ because, first, the integrated firm charges a nil royalty, and, second, $p_2^{II}(0) = \hat{p}_2$. Then, using an envelope argument, we have

$$\frac{d\pi_2^{II}}{da_I}(a_I, p_2^{II}(a_I)) = p_2^{II}(a_I)\frac{\partial D_2}{\partial a_I}(a_I, p_2^{II}(a_I)).$$

The result is then obtained using the facts that $\partial D_2/\partial a_I < 0$ and that the non-integrated manufacturer is never willing to sell its device at a negative price.

The second part is obtained with a revealed preferences argument. The integrated firm can always set $a_I = 0$, in which case its profit is equal to the joint profit of I and M_1 under separation. Since setting $a_I = 0$ is non-generic, vertical integration is generically strictly profitable.

A.5. Two-Part Tariffs

We now extend the model to allow two-part tariffs. Suppose hereafter that least-efficient platforms E_1 and E_2 are non-strategic and offer $(\delta, 0)$. That assumption is needed to ensure that, in the separation benchmark, an equilibrium exists and can be characterized; see Schutz (2012). In the following, the most-efficient platform I's two-part tariff is denoted by (w, F). Then, we show the following proposition.

Proposition A.1. *Suppose that manufacturers' prices are strategic complements (substitutes) when there is a negative (positive) externality across manufacturers.*[41] *Then, the non-integrated manufacturer loses from the vertical merger iff there is a negative externality across manufacturers.*

Proof. Consider first the separation benchmark. In equilibrium, I's offer (w_k, F_k) to M_k is such that M_k is indifferent with buying from one of the least-efficient platforms at $(\delta, 0)$, that is,

$$(\hat{p}_k(w_k, w_\ell) - w_k)D_k(\hat{p}_k(w_k, w_\ell), \hat{p}_\ell(w_\ell, w_k)) - F_k = (\hat{p}_k(\delta, w_\ell) - \delta)D_k(\hat{p}_k(\delta, w_\ell), \hat{p}_\ell(w_\ell, \delta)),$$

I's equilibrium royalties are thus given by

$$(w_1^*, w_2^*) = \arg\max_{(w_1,w_2)} \sum_{k\neq\ell} \Big(\hat{p}_k(w_k, w_\ell)D_k(\hat{p}_k(w_k, w_\ell), \hat{p}_\ell(w_\ell, w_k)) - (\hat{p}_k(\delta, w_\ell) - \delta)D_k(\hat{p}_k(\delta, w_\ell), \hat{p}_\ell(w_\ell, \delta))\Big),$$

or $w_1^* = w_2^* = w^*$ with

$$w^* = \arg\max_w \; \Pi(w) \equiv \hat{p}_k(w, w)D_k(\hat{p}_k(w, w), \hat{p}_\ell(w, w)) - (\hat{p}_k(\delta, w) - \delta)D_k(\hat{p}_k(\delta, w), \hat{p}_\ell(w, \delta)). \tag{A.8}$$

M_k's equilibrium profit under separation is therefore equal to its outside option $\hat{\pi}_k(\delta, w^*)$.

Consider now the integration case. I's offer is such that M_2 is indifferent between buying from I, and buying from one of the least-efficient platforms at $(\delta, 0)$ and earning a profit equal to $\hat{\pi}_2(\delta, 0)$ (since M_1's marginal cost is nil). M_2 therefore benefits from the vertical merger iff $\hat{\pi}_2(\delta, 0) > \hat{\pi}_2(\delta, w^*)$. Notice that, if $w^* > 0$, M_2 benefits from the vertical merger iff there is a negative positive externality across manufacturers.

Let us then prove that w^* is indeed strictly positive. To do so, it is sufficient to prove that $\Pi'(0) > 0$. Taking the derivative wrt to w in equation (A.8), we obtain

$$\Pi'(w) = w\frac{d\hat{p}_k}{dw}(w, w)\frac{\partial D_k}{\partial p_k}(w, w) + \hat{p}_k(w, w)\frac{d\hat{p}_\ell}{dw}(w, w)\frac{\partial D_k}{\partial p_\ell}(w, w) - \left(\delta\frac{\partial \hat{p}_k}{\partial w_\ell}(\delta, w)\frac{\partial D_k}{\partial p_k}(\delta, w) + \hat{p}_k(\delta, w)\frac{\partial \hat{p}_\ell}{\partial w_\ell}(\delta, w)\frac{\partial D_k}{\partial p_\ell}(\delta, w)\right), \tag{A.9}$$

[41] This is always the case in the linear example.

where, with a slight abuse of notations, $\partial D_k/\partial p_\ell\,(w_k, w_\ell) = \partial D_k/\partial p_\ell\,(\hat{p}_k(w_k, w_\ell), \hat{p}_\ell(w_\ell, w_k))$. Evaluating equation (A.9) at $w = 0$, we obtain

$$\begin{aligned}(\text{A.10}) \quad \Pi'(0) = \hat{p}_k(0,0)\left(1 + R_k'(\hat{p}_\ell(0,0),0)\right)\frac{\partial \hat{p}_\ell}{\partial w_\ell}(0,0)\frac{\partial D_k}{\partial p_\ell}(0,0) \\ - \left(\delta R_k'(\hat{p}_\ell(\delta,0),0)\frac{\partial \hat{p}_\ell}{\partial w_\ell}(\delta,0)\frac{\partial D_k}{\partial p_k}(\delta,0) + \hat{p}_k(\delta,0)\frac{\partial \hat{p}_\ell}{\partial w_\ell}(\delta,0)\frac{\partial D_k}{\partial p_\ell}(\delta,0)\right),\end{aligned}$$

where $R_k(p_\ell, w)$ is M_k's best response to price p_ℓ, $\ell \neq k$, when its marginal cost is w. Evaluating equation (A.10) at $\delta = 0$, we finally obtain

$$\Pi'(0)\big|_{\delta=0} = \hat{p}_k(0,0)R_k'\left(\hat{p}_\ell(0,0),0\right)\frac{\partial \hat{p}_\ell}{\partial w_\ell}(0,0)\frac{\partial D_k}{\partial p_\ell}(0,0).$$

Since $\hat{p}_\ell$ is increasing in M_ℓ's marginal cost, $\Pi'(0)|_{\delta=0} > 0$ has the sign of $R_k'(\hat{p}_\ell, 0)\partial D_k/\partial p_\ell\,(0,0)$. Then, since prices are strategic complements (substitutes) when there is a negative (positive) externality across manufacturers, $R_k'(\hat{p}_\ell, 0)$ and $\partial D_k/\partial p_\ell$ have the same sign, and, therefore, $\Pi'(0)|_{\delta=0} > 0$. By continuity, this shows that, when δ is small enough, $\Pi(w)$ is strictly increasing in the neighborhood of 0, and, accordingly, $w^* > 0$. □