Working paper

"WHEN OLSON MEETS DAHL": FROM INEFFICIENT GROUP FORMATION TO INEFFICIENT POLICY-MAKING[1]

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Abstract. Two conflicting interest groups buy favors from a decision-maker. Influence is modeled as a common agency game with lobbyists proposing monetary contributions contingent on decisions. With common knowledge preferences, groups form efficiently and lobbying competition perfectly aggregates preferences. When preferences are private information, free riding in collective action arises *within groups*. This free riding implies that groups choose lobbyists with moderate preferences tilting final decisions towards their competitors. Intra-group inefficiencies are jointly determined at equilibrium. Lobbying competition imperfectly aggregates preferences. Asymmetric information, by softening lobbying competition, might increase groups' payoffs, although it always hurts the decision-maker.

Keywords. Lobbying, Collective Action, Free Riding, Asymmetric Information, Common Agency, Mechanism Design.

JEL codes. D72; D82.

1. INTRODUCTION

Motivation. The formation of interest groups, their competition in the political arena and, more generally, their influence on policy-making are key concerns for students of modern democracies. The role of lobbying as a vehicle for the representation of diverse interests, and its impact on the democratic process, although unanimously recognized, has nevertheless raised conflicting views among both political scientists and economists.

Following Dahl (1961)'s seminal work, the so-called *pluralistic approach* to politics views competition between interest groups as a healthy way to aggregate conflicting interests. This view of politics argues with much optimism that diverse interests always get represented in policy-making. As a result, efficient decisions which correctly balance conflicting preferences are reached. From a theoretical viewpoint, this approach, which certainly found its roots in the earlier works of Peltzman (1976) and Becker (1983), is nowadays best exemplified by the so-called *common agency model* of lobbying competition proposed earlier on by Bernheim and Whinston (1986) and then pushed forward by Grossman and Helpman (1994) and Dixit, Grossman and Helpman (1997) over a broad range of applications.[1] Within this realm, interest groups influence a decision-maker through monetary payments (for instance campaign contributions) whose levels depend on the the decision-maker's choices. Importantly, the multilateral bargaining protocol so constructed has

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[1]See also Goldberg and Maggi (1999) for an empirical assessment.

always efficient equilibria. To illustrate, the decision-maker chooses policies maximizing the sum of his own payoff and those of active interest groups when utility is transferable.[2]

In sharp contrast, less optimistic stances view lobbying essentially as a rent-seeking activity. Strongly organized and well-connected groups buy favors while unorganized rivals are unable to exert influence on policy-making. This second approach follows the steps of Olson (1965) whose pathbreaking work has resonated throughout all social sciences. The major thrust of *The Logic of Collective Action* is that *intra-group* free riding might prevent groups from correctly promoting their interests. Free riding being more of a curse as the size of the group increases, large latent groups might be dominated by small groups which are better organized. This so called *Olson Paradox* has paved the way for a vast research agenda about groups organization and their impact on policy-making both through Political Science and Economics. To illustrate, both the *Chicago School* (Stigler (1971), Posner (1974)) and the *New Regulatory Economics* (Laffont and Tirole (1991)) testify of various trends of the Political Economy literature that have taken as granted inefficiencies in collective action but have analyzed their impact on policy-making.

Reconciling the messages conveyed by those two approaches of politics stands as a major challenge for our understanding of collective action. On the one hand, Olson (1965) argues that free riding strongly hinders representation of interests. Yet, this line of research fails to recognize that inefficiencies in collective action are to a large extent endogenous. Indeed, the stakes for forming as an active group depend on whether other groups might have already influenced decision-making or not. On the other hand, models of lobbying competition do not even discuss why groups may face difficulties in forming at the outset. By focusing on different aspects of groups behavior, these two approaches thus deliver very different conceptions of the organization of interest groups. An important item on the research agenda is thus to reconcile those two approaches so as to get a more complete and convincing view of the representation of interests in modern democracies. Unfortunately, this step has been awaited by political science scholars for decades, without any substantial progress being made as recognized by Baumgartner and Leech (1998, pp. 88).[3]

This paper makes progresses on this front. In our framework, groups formation responds to stakes which are endogenously determined by the outcome of lobbying competition. To address the free riding problem in collective action in a modern manner, we introduce asymmetric information on preferences. We then ask whether and when the political process efficiently aggregates preferences both within and across groups.

THE MODEL. Two interest groups with conflicting preferences over a policy decision buy favors from a policy-maker. Within each group, individuals have private information on

[2] Throughout the paper, an efficient allocation is defined as being on the Pareto frontier of the set of payoffs for the groups and the decision-maker. This definition which is standard in the literature thus assumes that the decision-maker perfectly represents other interests in the polity.

[3] They wrote: *"One sees the interest-group system as hopelessly biased in favor of powerful economic interests and narrow special pleaders; another sees a greater diversity of interest in the Washington policy community and a positive role for groups in the creation of better citizens. The relative emphasis that scholars have placed on each of these views has changed from decade to decade, but neither has been shown to be completely accurate: a complete view must recognize elements of both views."*

their own preferences. To become politically active, a group must lobby the decision-maker. It first means appointing a lobbyist and endowing this lobbyist with an objective and a budget so as to exert political pressure over the decision-maker through monetary contributions. It also requires to determine how the overall contribution of the group is shared. Because of private information, individuals might shade their willingnesses to pay for a policy shift to reduce their own contribution while still benefitting from their group's action. Free riding in collective action matters *within* each group.

An important aspect of our modeling is that the difficulties faced by an interest group in solving its own free riding problem are *endogenously* derived as the equilibrium outcome of lobbying competition. Indeed, the costs and benefits of forming as an active group depend on the exact influence that other competing interest groups might have on the decision-maker since that influence determines both the *status quo* payoff had group formation failed but also its *equilibrium* value had it succeeded. In turn, those costs and benefits determine whether intra-group free riding can ever be solved.

To tackle this double-sided problem, our model builds on two important bodies of theoretical works; namely *mechanism design* and *common agency*. When taken separately, those two workhorses models have been extensively used to understand groups formation on the one hand and their competition on the other hand. Yet, those two paradigms have evolved independently and, as such, have not been able to offer a more comprehensive framework. Before being active, groups must solve their collective action problem: an issue that calls for importing the tools of mechanism design. Within each group, mechanisms for collective actions are designed to ensure that members reveal their preferences. At the last stage of the game, lobbyists acting on behalf of interest groups compete for the decision-maker's influence: a standard common agency game. Compounding the endogenity of payoffs inherited from the lobbying game with the mechanism design stage gives us a fresh view on how interest groups form and interact.

MAIN RESULTS. To understand how preferences end up being represented through this two-stage political process, we must carefully analyze how preferences are aggregated *within* and *across* groups.

Aggregating preferences across groups. Because they represent interest groups with conflicting interests, lobbyists compete *"head-to-head"* for the decision-maker's favors. To prevent the competing group from buying the decision-maker, each group is thus ready to bid up to his incremental contribution to the overall payoff of the grand-coalition made of those groups and the decision-maker. Those contributions, which are endogenously determined at the equilibrium of the common agency stage of the game, are thus akin to *Vickrey-Clarke-Groves* (thereafter VCG) payments.[4] Absent any possible internal free riding, this property implies that the group would sincerely choose these preferences with no strategic view on how lobbying competition could be distorted. Indeed, the sole purpose of distorting the lobbyist's preferences is thus to solve the internal free riding problem. Conditionally on such incentive distortions, the groups' preferences are thus sincerely passed onto their delegates.

[4]Green and Laffont (1977).

Aggregating preferences within groups. Had the preferences of members been common knowledge within a group, the free riding problem could be easily solved by having each member contributing up to his willingness to pay for a policy shift. Under those circumstances, preferences are perfectly aggregated within the group and the lobbyist's preferences reflect those of the group as a whole. Efficiency in solving the free riding problem *within* groups together with the conditional efficiency of the common agency stage implies overall efficiency of policy-making. The basic take-away of this complete information scenario is that, if Olson is wrong and free riding in collective action is not an issue, then Dahl is also right and lobbying competition efficiently aggregates preferences.

Asymmetric information on preferences *within groups* radically changes the picture. Group members may now free ride by shading their willingness to pay for a policy shift. As a result, information can only be shared within a group if individuals get *information rents* that compensate for such strategic possibilities. A group now forms whenever the net gains from influencing the policy-maker also cover those rents: A key *incentive-feasibility condition*. An important *rent-efficiency trade-off* in collective action now arises. Inducing a large policy shift requires greater contributions from the group but it also exacerbates individual incentives to free ride and hardens the incentive-feasibility condition.

Inefficient group formation. To limit information rents within a group, the optimal mechanism for group formation is distorted along two dimensions. First, free riding is less of a concern when the group's overall contribution is reduced. The decision-maker is now more inclined to preserve the *status quo* policy he chooses under the sole influence of competing interests. Second, the group chooses a lobbyist with moderate preferences. Following insights from the mechanism design literature (Myerson (1981)) the willingness to pay for a policy change of any individual member is indeed replaced by a *virtual willingness to pay*. This parameter, which encapsulates the cost of information constraints, is of a lower magnitude than the true willingness to pay. Aggregating *virtual willingnesses to pay* across group members is akin to having a group with less pronounced preferences for the policy shift. When the free riding problem is of enough significance, this group may even fail to get organized. Under those circumstances, *Olson's Paradox* finds strong informational foundations: An interest group might no longer be represented in lobbying competition if its internal informational problems are too costly to solve.

From a more technical viewpoint, inefficiencies in collective action are fully captured by the shadow cost of the incentive-feasibility condition that pertains to that group. Since the net benefits of coalition formation depend on the *status quo* policy that would be chosen by a decision-maker under the sole influence of the competing group, this shadow cost depends on the competing group's strategy. Inefficiencies in groups formation are thus *jointly* determined at equilibrium.[5] In sharp contrast with the complete information scenario, inefficiencies within groups now percolate as inefficiencies in the lobbying game. To summarize, Olson's view is incomplete and free riding in collective action within a given group, when it matters, also depends on how competing groups solve their own

[5]Studying the formation of coalitions among already active lobbyists, Holyoke (2009) shows that lobbyists with slightly divergent interests will be more likely to overcome their divergence and get organized into a coalition when they face a more intensive competition by a rival coalition. Gray and Lowery (1996) also argue that the ability of an interest group to mobilize depends on its environment.

organizational problem. Under asymmetric information within groups, Dahl is wrong and lobbying competition cannot efficiently aggregate preferences.

Inefficiencies and types distributions. When the support of this distribution is broad enough to include types with no strict preferences on policy, inefficiencies are pervasive. This result is best exemplified with large groups. Incentives to free ride are there at their worst. Group members would like to pretend having no preferences whatsoever for the policy so as to pay nothing for a policy shift while still enjoying its benefits. Overall, zero contribution can be collected and such large groups cannot be active. *A contrario*, even large groups are active when the lower bound of possible valuations is strictly positive. This is not smallness *per se* that facilitates group formation but the existence of a minimal stake for individuals. Our informational perspective on group formation thus challenges Olson-Stigler's view that groups are more likely to be successful when more heterogenous.

Welfare. Intra-group free riding has important welfare consequences. Because it reduces contributions, asymmetric information within groups softens lobbying competition. It also hurts the decision-maker who can no longer extract as much by playing one group against the other. These results might explain the puzzling observation that lobbying contributions generally appear too small.[6]

Organization of the paper. Section 2 reviews the relevant literature. Section 3 describes our two buildings blocks: the common agency model of lobbying competition and the mechanism design model of groups formation. Section 4 characterizes the set of incentive-feasible mechanisms for groups by means of a simple incentive-feasibility condition. Section 5 provides conditions ensuring that groups form efficiently even under asymmetric information. In contrast, Section 6 investigates conditions for free riding to arise within large groups. Section 7 tackles the more complex scenario where groups are of finite size. Free riding in collective action leads to inefficiencies that are jointly determined across competing groups at equilibrium. Taking an *IO* perspective, Section 8 discusses how organizational choices may act as commitment devices to affect lobbying competition. Section 9 assesses the welfare impact of asymmetric information. Section 10 proposes some possible extensions. Proofs are relegated to an Appendix.

2. Literature Review

Our paper blends together two important trends of the economic literature: the common agency model of lobbying competition, and the mechanism design approach that is used to model intra-group agreements.

Common agency. Bernheim and Whinston (1986) have laid down the theoretical foundations for common agency models of lobbying competition. They demonstrate that those games have efficient equilibria implemented by means of the so-called *"truthful"* contribution schedules that perfectly reflect the groups' preferences.[7] On the theoretical side, this research was pursued by Laussel and Le Breton (2001) who provided a careful analysis of the set of payoffs that arise at truthful equilibria with transferable utility (the most common setting in the more applied literature). We borrow from this work the structure

[6] See Tullock (1972) and Helpman and Persson (2001) among others.

[7] Inefficient equilibria may nevertheless arise when the *"truthfullness"* refinement is given up.

of payoffs that come out of the lobbying game. Those payoffs determine endogenously the costs and benefits of group formation.

On the more applied side, a series of important contributions by Grossman and Helpman (1994, 2001) have paved the way for applying complete information common agency models in regulation, trade and public economics. (See for instance Persson and Tabellini (1994), Dixit, Grossman and Helpman (1997), Rama and Tabellini (1998), Yu (2005).) Those models take as given how efficient a group is in channelling influence on the policy-maker. Instead, we will derive these frictions from the solution of an intra-group mechanism design problem. In the existing literature, whether some groups form or not is given at the outset (Aidt (1996), Siqueira (2001)) while it results from informational frictions in our framework. Attempts to reconcile models of common agency with Olson's analysis have essentially followed two trends. The first one (Mitra (1999), Martimort and Semenov (2007)) endogenizes the number and the identity of active lobbyists by considering an exogenous cost of entry into the lobbying process. Entry costs may distort groups' representation and thus equilibrium policies. A second direction (Dixit and Olson (2000), Furusawa and Konishi (2011)) considers a game where lobbies have first to decide wether or not to become active. Free riding occurs with some groups remaining outside the policy process. Even under complete information, the *Coase Theorem* does not apply when participation is a strategic decision.

Another trend of the common agency literature has introduced asymmetric information with regard to the decision-maker's preferences. The agency costs paid by interest groups to influence a privately informed decision-maker might then make competition among groups inefficient (Le Breton and Salanié (2003), Martimort and Semenov (2008)). Agency costs might be so large that they prevent some groups from being active as in Martimort and Stole (2015). We borrow from this latter paper the structure of conflicting preferences for the competing groups although the loci of private information differ. Asymmetric information is no longer on the supply side of the market for favors but rather on its demand side: Private information pertains to the preferences of members of a given group.

Mechanism design. Introducing private information requires an explicit modeling of group formation under the threat of free riding. To do so, we borrow techniques from the mechanism design literature on public good provision. (See Laffont and Maskin (1982), Mailath and Postlewaite (1990), Ledyard and Palfrey (1999) and Hellwig (2003) among others.) When agents have private information on their preferences, a conflict between incentives, budget balance and participation might prevent efficiency.[8] A major departure away from this literature is that stakes are here endogenous: The costs and benefits of group formation are derived from equilibrium behavior in the lobbying game.

Contests. Our model bears some resemblance with Esteban and Ray (2001). Those authors view competition between groups as a contest. Contrary to us, groups are homogenous and the cost of lobbying is not derived from informational constraints. Detailed properties of lobbying costs determine whether *Olson's Paradox* holds or not.

[8]There also exists a tiny literature that analyzes incentives to free ride within groups in moral hazard settings (Lohmann (1998), Anesi (2009)).

3. THE MODEL

3.1. *Interest Groups*

PREFERENCES. Agents in the economy are divided into two groups. Group l ($l \in \{1, 2\}$) has size $N_l \geq 1$. Those groups have distinct preferences over a policy x chosen by a decision-maker (sometimes referred to as *she* in the sequel) within an interval $\mathcal{X} = [-x_{max}, x_{max}]$ (x_{max} being large enough). Agent i who belongs to group l has a quasi-linear utility function which is defined over the policy x and the monetary contribution t_i that he pays to influence the decision-maker:

$$\frac{\alpha_i}{N_l} u_l(x) - t_i.$$

The parameter α_i captures the intensity of agent i's preferences while $u_l(x)$ stems for a payoff function which is specific to group l. Members of a given group rank all policies in the same way although the intensity of preferences vary across individuals. Individuals of the same group are thus vertically differentiated. Individual preferences are scaled up by the size of the group N_l to normalize the group's overall influence.[9]

For simplicity, each function u_l is supposed to be linear in x although we often keep a more general expression to show how our results would apply more broadly. To capture the idea that groups are horizontally differentiated, we posit:

$$u_1(x) = -u_2(x) = -x \quad \forall x \in \mathcal{X}.$$[10]

To illustrate, the decision x might be the level of an import tariff for some intermediate good. Group 2 might stand for domestic producers. This group asks for protection from foreign competitors and thus lobby for an import tariff. Group 1 are final users of this input who instead call for a low protection to reduce expenditures.

The decision-maker represents other (unorganized) groups in society or a median voter who might have more neutral stances on the policy at stake. We assume that the decision-maker has her own quasi-linear utility function which is also defined over x and overall monetary payments he receives z as:

$$u_0(x) + z.$$

The function u_0 is twice continuously differentiable, strictly concave, single-peaked and symmetric around a bliss point at $x_0 = 0$. Let denote $\varphi = u_0'^{-1}$ with $\varphi(0) = 0$ and $\varphi' < 0$ (which follows from $u_0'' < 0$). Some of our results below depend on the curvature of φ.

[9]This assumption does not give any positive role for size *per se*. Instead and, in the spirit of McLean and Postelwaite (2002), this is the relative measure of individual information compared to size that matters to determine the impact of each agent on the group's strategy.

[10]Asymmetry in the strength of the groups could also be easily introduced. Suppose for instance that the two groups would like to push policies in their respective directions although with different intensities. Formally, say $u_1(x) = -kx$ while $u_2(x) = x$. Up to changing the support of types distributions, any such linear specification with opposite preferences could be transformed into our formulation.

Running example. To illustrate our findings, we will repeatedly rely on a quadratic specification of preferences which is a workhorse of the Political Science literature:

$$u_0(x) = -\frac{\beta_0}{2}x^2 \Rightarrow \varphi(y) = -\frac{y}{\beta_0}. \quad (3.1)$$

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Information. While the degree of horizontal differentiation and the composition of the groups are common knowledge, preferences are not so even within a given group. Each individual has private information on his own preference parameter α_i. For each group, these values are drawn from a group-specific cumulative distribution function F_l whose support is $\Omega_l = [\underline{\alpha}_l, \overline{\alpha}_l]$. We denote the corresponding (atomless and positive) density by f_l. In the sequel, the lower bound $\underline{\alpha}_l \geq 0$ of the distribution sometimes plays an important role. A specific case is obtained when preferences are diffuse enough so that $\underline{\alpha}_l = 0$.[11] We denote also the average preference parameter for group l as $\alpha_l^e = \int_{\underline{\alpha}_l}^{\overline{\alpha}_l} \alpha dF_l(\alpha)$.

We denote by $\alpha_l = (\alpha_i)_{i\in l}$ any arbitrary vector of preference parameters for group l and by $\alpha_l^*(\alpha_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_i$ its sample mean. We refer to Φ_{N_l} (resp. ϕ_{N_l}) as the cumulative distribution (resp. density) of this sample mean whose support is still Ω_l.

Adopting the parlance of the mechanism design literature (Myerson (1981)) the *virtual preference parameter* of agent i in group l is defined as:

$$h_l(\alpha_i) = \alpha_i - \frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)}, \quad \forall \alpha_i \in \Omega_l.$$

Following a standard requirement (Bagnoli and Bergstrom (2005)), the *monotone hazard rate property* holds:

$$\frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)} \text{ is non-increasing on } \Omega_l.$$

This assumption ensures that $h_l(\alpha_i)$ is a non-decreasing transform of α_i. Following previous convention, the average virtual preference parameter for group l is defined as $h_l^*(\alpha_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_i)$.

3.2. *Lobbyists*

For the purpose of our analysis, it is key to have a detailed model the lobbying process. Indeed, the lobbying game determines contributions and policies as endogenous objects that impact in turn on the costs and benefits of collective action. In this respect, we rely on a simple model that pictures the lobbying process in American Politics. Each group appoints a lobbyist and instructs this agent on how to influence the decision-maker. Once appointed, lobbyists compete *"head-to-head"* for the favors of the decision-maker. This

[11] The fact that some individuals might have no specific preferences for the policy under scrutiny is justified when the group is a long-term venture banding agents on several related issues. A given individual may have found a positive value in belonging to that group in the past and still belong to that group nowadays even though he has no strict preferences for the decision at stake.

process can be seen as *"down-top"*, with group members deciding to appoint lobbyists. Alternatively and much in lines with Salisbury (1969), it could instead be considered as more *"top-down"* in nature, with lobbyists behaving as political entrepreneurs who initiate collective action. An interest group thus endows his appointed lobbyist both with an objective $\beta_l u_l(x)$ and with enough money T_l to buy the decision-maker's favors.The lobbyist's induced preferences can be written as:

$$\beta_l u_l(x) - T_l. \quad (3.2)$$ [12]

The weight β_l is a simple and convenient way of capturing how efficient is the process of group formation. When preferences are common knowledge, the lobbyist is endowed with the average preferences of the group, namely $\beta_l = \alpha_l^*(\alpha_l)$. When preferences are private information and free riding in collective action is a concern, the wedge between β_l and $\alpha_l^*(\alpha_l)$ captures how informational frictions undermine collective action.

3.3. *Lobbying the Decision-Maker: The Common Agency Game*

Lobbyists compete for influence in a standard *common agency game*. We adopt Bernheim and Whinston (1986) and Grossman and Helpman (1994)'s framework both in terms of informational assumptions and timing. First, the lobbyists' objectives are common knowledge at this last stage of the game. In particular, each lobbyist perfectly knows his competitor's objective.[13] Second, lobbyists offer to the decision-maker non-negative contributions $T_l(x)$ that stipulate a payment which depends on the chosen policy. The decision-maker is free to accept or refuse each contract.[14] Third, we restrict attention to *truthful* (continuation) equilibria of that common agency game. Those equilibria are obtained when lobbyists use *truthful contribution schedules* of the form:[15]

$$T_l(x) = \max\{\beta_l u_l(x) - V_l, 0\} \quad \forall x \in \mathcal{X}. \quad (3.3)$$ [16]

Truthful schedules perfectly reflect the lobbyist's preferences over alternatives; an attractive requirement. An important consequence of truthfulness is that the equilibrium policy always maximizes the aggregate payoff of the grand-coalition made of those lobbyists and the decision-maker herself. Our goal is to investigate circumstances such that those induced preferences no longer reflect the true average preferences of an interest group as a result of informational frictions.

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[12] We could account for the possibility that the lobbyist has some intrinsic preferences on policy, in which case his objective writes as $(\hat{\beta}_l + \beta_l)u_l(x) - T_l$ where $\hat{\beta}_l$ reflects intrinsic preferences. Everything then happens as if the lobbyist was a member of the group himself with the only modeling specificity (simplifying analysis) being that his own preferences would be common knowledge, a justification being that this lobbyist finds value to disclose his preferences so as to act as a leader for the group.

[13] This is by no means saying that asymmetric information between individuals disappears since, as we will see below, the lobbyist's objective is only a rough statistics for individual's preferences.

[14] The restriction to non-negative contribution schedules is without any loss of generality in a model of delegated common agency. Indeed, the decision-maker being free to refuse any contract would never choose a policy corresponding to a negative payment.

[15] Bernheim and Whinston (1986) offer a justification for truthful strategies. Because each lobbyist has a truthful schedule in his best-response correspondence, insisting on *truthfulness* is akin to imposing an equilibrium refinement. Bernheim and Whinston (1986) then demonstrate that these equilibria are also coalition-proof Nash equilibria. The same refinement could be applied to select the truthful allocations at the last stage of our game.

[16] Observe that V_l is the payoff that lobbyist l secures irrespectively of the decision-maker's choice x.

For future references, let $\beta = (\beta_1, \beta_2)$ be the lobbyists' preferences with $\Delta\beta = \beta_1 - \beta_2$ measuring polarization. Accordingly and thanks to the strict concavity of u_0 and the fact that x_{max} is large enough, the policy $x(\beta_1, \beta_2)$ that maximizes the payoff of the grand-coalition made of the decision-maker and the two lobbyists is interior and satisfies:

$$u_0'(x(\beta_1, \beta_2)) = \Delta\beta \Leftrightarrow x(\beta_1, \beta_2) = \varphi(\Delta\beta). \tag{3.4}$$

Intuitively, the optimal policy is tilted towards the group whose lobbyist has the strongest preferences. Of course, the definition of $x(\beta_1, \beta_2)$ given above applies equally well to characterize the optimal policy taken by a coalition made of the decision-maker and any subset $A \subset \{\emptyset, 1, 2\}$ of lobbyists provided that we use the convention $\beta_l = 0$ when $l \notin A$. For instance, $x(0, \beta_2) = \varphi(-\beta_2)$ is the decision implemented if group 1 is not organized. With such convention, the payoff of any such coalition A can also be defined in terms of the vector of induced preferences (β_1, β_2) as:

$$W(\beta_1, \beta_2) = \max_{x \in \mathcal{X}} u_0(x) - \Delta\beta x = u_0(\varphi(\Delta\beta)) - \Delta\beta\varphi(\Delta\beta).$$

To illustrate, $W(0, \beta_2)$ stands for the payoff when group 1 fails to get organized and a similar convention applies to $W(\beta_1, 0)$ when group 2 is not organized.

That the common agency game takes place under complete information on the lobbyists' preferences allows us to import *mutatis mutandis* the general characterization of truthful equilibrium payoffs found in Laussel and Le Breton (2001). A key step of their analysis is to identify the properties of payoffs in the common agency game with those of a cooperative game among coalitions of principals, whose characteristic function is $W(\beta_1, \beta_2)$. In our context with conflicting interest groups, this cooperative game turns out to be *strongly sub-additive* since:

$$W(\beta_1, \beta_2) + W(0, 0) < W(\beta_1, 0) + W(0, \beta_2) \quad \forall(\beta_1, \beta_2) \in \mathbb{R}_+^2.$$

With strong sub-additivity, Laussel and Le Breton (2001) demonstrate that, at a truthful equilibrium, each lobbyist's payoff is uniquely determined as his *incremental value* to the grand-coalition's surplus, namely:

$$V_l(\beta_l, \beta_{-l}) = W(\beta_1, \beta_2) - W(0, \beta_{-l}). \tag{3.5}$$

As a result, the decision-maker gets a positive share of the value of the grand-coalition:

$$W(\beta_1, 0) + W(0, \beta_2) - W(\beta_1, \beta_2) > W(0, 0) = 0.$$

Intuitively, the decision-maker can pit one lobbyist against the other to extract some positive surplus from their *"head-to-head"* competition.

Using (3.3), we retrieve the expression of the equilibrium payment from lobbyist l as:

$$T_l(\beta_l, \beta_{-l}) = \beta_{-l}\left(u_{-l}(x(0, \beta_{-l})) - u_{-l}(x(\beta_l, \beta_{-l})) + u_0(x(0, \beta_{-l})) - u_0(x(\beta_l, \beta_{-l})\right). \tag{3.6}$$

This expression is remarkable. Contributions that are endogenously determined at the common agency stage of the game are in fact VCG payments. Each lobbyist pays for the externality that a change in policy he induces exerts both on his rival lobbyist and on the decision-maker (representing the rest of society). This fact has important implications on the groups' incentives to choose the preferences of their lobbyist. The logic of VCG mechanisms bites: Groups choose sincerely the preferences of their delegates, up to the frictions induced by intra-group free riding. There is no point in strategically choosing these preferences to affect subsequent lobbying competition.

3.4. *Group Formation: Mechanism Design*

When appointing a lobbyist, a group must not only choose the preferences of his delegate but also specify how members share the contribution paid to the decision-maker. Because group members have private information on their preferences, we model group formation as a mechanism design problem. In response to group $-l$'s own formation mechanism, a mediator proposes to all members of group l some induced preferences β_l *cum* a rule to share the expected payment $\mathbb{E}_{\alpha_{-l}}(T_l(\beta_l, \beta_{-l}(\alpha_{-l})))$ left to the decision-maker.[17] This mediator could be a broker organizing collusion on behalf of group members as pointed out by Loomis (1986) in his analysis of lobbying practices. Summarizing, a mechanism for group l's formation determines for all possible vector of reports $\hat{\alpha}_l$ of its members an objective function for his delegate and individual contributions; $\left(\beta_l(\hat{\alpha}_l), (t_i(\hat{\alpha}_l))_{i=1}^{N_l}\right)$.

3.4.1. *No-Veto Constraints*

An important issue for the design of any such mechanism is to specify what happens if one potential member refuses the mechanism, maybe because he fears that he will be asked to pay too high a contribution. Following the standard mechanism design literature,[18] we assume individual veto power. If anyone refuses the mechanism, no lobbyist is appointed for group l and the decision-maker chooses $x(0, \beta_{-l}(\alpha_{-l}))$ so as to reflect only group $-l$'s own influence (if any). The net gain of group formation for an agent with type α_i when the preferences profile within his own group l is $\alpha_l = (\alpha_i, \alpha_{-i})$ is thus:

$$\frac{\alpha_i}{N_l}\mathbb{E}_{\alpha_{-l}}(\Delta u_l(\beta_l(\alpha_i, \alpha_{-i}), \beta_{-l}(\alpha_{-l}))) - t_i(\alpha_i, \alpha_{-i})$$

where

$$\Delta u_l(\beta_l, \beta_{-l}) \equiv u_l(x(\beta_l, \beta_{-l})) - u_l(x(0, \beta_{-l})).$$

Type α_i's expected net payoff from joining group l can thus be defined as:

$$\mathcal{U}_l(\alpha_i) = \mathbb{E}_{\alpha_{-i}}\left(\frac{\alpha_i}{N_l}\mathbb{E}_{\alpha_{-l}}(\Delta u_l(\beta_l(\alpha_i, \alpha_{-i}), \beta_{-l}(\alpha_{-l}))) - t_i(\alpha_i, \alpha_{-i})\right) \quad \forall \alpha_i \in \Omega_l. \tag{3.7}$$

The no-veto constraint for an individual with type α_i in group l then writes as:

$$\mathcal{U}_l(\alpha_i) \geq 0, \quad \forall \alpha_i \in \Omega_l. \tag{3.8}$$

[17] The payment $T_l(\beta_l, \beta_{-l})$ is a random variable. It depends on both the realization of the whole vector of valuations α_l for members of group l through the impact on β_l but also on the vector of preference parameters α_{-l} of members of the competing group through its impact on β_{-l}. All incentive, budget balance and no-veto constraints that apply to the mechanism for group l take into account the fact that the vector of valuations α_{-l} is viewed as being random from group l's viewpoint.

[18] See Laffont and Maskin (1982) and Mailath and Postlewaite (1990) among others.

3.4.2. *Incentive Compatibility*

From the Revelation Principle, there is no loss of generality in considering direct mechanisms as above provided that each individual truthfully reports his type α_i at a Bayesian equilibrium.[19] The following incentive compatibility constraints must thus hold:

$$\mathcal{U}_l(\alpha_i) \geq \mathbb{E}_{\alpha_{-i}}\left(\frac{\alpha_i}{N_l}\mathbb{E}_{\alpha_{-l}}\left(\Delta u_l(\beta_l(\hat{\alpha}_i, \alpha_{-i}), \beta_{-l}(\alpha_{-l}))\right) - t_i(\hat{\alpha}_i, \alpha_{-i})\right), \quad \forall(\alpha_i, \hat{\alpha}_i) \in \Omega_l^2. \tag{3.9}$$

3.4.3. *Budget Balance*

Taking into account that α_{-l} is a random variable, the budget constraint for group l can be written in expected terms as follows:

$$\sum_{i=1}^{N_l} t_i(\alpha_i, \alpha_{-i}) - \mathbb{E}_{\alpha_{-l}}(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) \geq 0 \quad \forall(\alpha_i, \alpha_{-i}) \in \Omega_l^2. \tag{3.10}$$ [20]

A mechanism $\mathcal{G}_l$ is *incentive-feasible* if and only if it satisfies no-veto (3.8), incentive compatibility (3.9), and budget balance (3.10).

3.5. *Timing*

To summarize, the overall game of group formation and influence unfolds as follows. First, agents privately learn their preferences. Second, groups simultaneously (and secretly) propose mechanisms to their members. Third, within each group, each individual may accept or veto the proposed mechanism. If the mechanism is ratified by everyone, each member reports his own preference parameter $\hat{\alpha}_i$. The lobbyist's induced preferences correspond to a weight $\beta_l(\hat{\alpha}_l)$ and an objective function as defined in (3.2). Fourth, the common agency game between lobbyists unfolds with group l's equilibrium payment to the decision-maker being $T_l(\beta_l(\hat{\alpha}_l), \beta_{-l}(\hat{\alpha}_{-l}))$.

The equilibrium concept for the overall game of group formation *cum* lobbying is Bayesian-Nash equilibrium with the addition of two refinements. First, we impose *passive beliefs* so that members of group l still believe that group $-l$ is ruled with the equilibrium mechanism if they get an unexpected offer for their own group.[21] Second, equilibria of the common agency stage of the game are truthful; another standard refinement.

[19] In contrast with standard mechanism design problem, our context is one of competing mechanisms where multiple groups rely on their own mechanism. We must thus be somewhat careful in using this Revelation Principle. Indeed, each mechanism of group formation is now a best response to the mechanism designed by the competing group; a feature that has been studied in the general model of competing hierarchies by Myerson (1982) and in more specific contexts with secret contracts by Martimort (1996). For a given mechanism $\mathcal{G}_{-l}$ that determines a deterministic allocation β_{-l} for group l, there is no loss of generality in using the Revelation Principle to characterize group l's best response in the pure-strategy equilibria of this game that will be our focus below.

[20] The budget-balance requirement could be thought to be more demanding if those constraints had to hold for all possible realizations of group $-l$'s preferences α_{-l}. However, such complications would not change our results.

[21] This refinement is standard in the competing mechanisms literature (Martimort (1996)).

4. CHARACTERIZING INCENTIVE-FEASIBILITY

For a given incentive-feasible mechanism $\mathcal{G}_{-l}$ ruling group $-l$ (i.e., a mechanism which satisfies incentive compatibility, no veto and budget balance) that defines the mapping $\beta_{-l}(\alpha_{-l})$, the optimal mechanism $\mathcal{G}_l$ chosen by group l maximizes the sum of its members' expected payoffs subject to no-veto (3.8), incentive (3.9) and budget balance constraints (3.10). The first step consists in getting a compact characterization of the set of incentive-feasible allocations. We follow a large body of the mechanism design literature[22] and characterize incentive compatibility conditions before aggregating (3.8), (3.9) and (3.10) into a single constraint which is both necessary and sufficient for incentive-feasibility.

Incentive compatibility. Incentive compatibility can be expressed in terms of properties of the mapping $\beta_l(\alpha_l)$ that determines the delegate's preferences and of the payoffs profile $\mathcal{U}_l(\alpha_i)$ that such mapping induces. This is the purpose of next lemma.

Lemma 1 *An allocation $(\mathcal{U}_l(\alpha_i), \beta_l(\alpha_l))$ is incentive compatible if and only if:*

1. *$\mathcal{U}_l(\alpha_i)$ is absolutely continuous, almost everywhere differentiable and satisfies the integral representation*

$$\mathcal{U}_l(\alpha_i) = \mathcal{U}_l(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_i} \mathbb{E}_{\alpha_{-i},\alpha_{-l}}\left(\frac{1}{N_l}\Delta u_l(\beta_l(\tilde{\alpha}_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l}))\right) d\tilde{\alpha}_i; \tag{4.1}$$

2. *$\mathbb{E}_{\alpha_{-i},\alpha_{-l}}(\Delta u_l(\beta_l(\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l})))$ is non-decreasing in α_i.*

Consider a member of group l with preferences α_i. To fix ideas, suppose also that $\Delta u_l(\beta_l(\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l}))$ is non-decreasing in α_i for all $(\alpha_{-i},\alpha_{-l})$.[23] By pretending to have a slightly lower valuation $\alpha_i - d\alpha_i$, this type α_i can modify the decision chosen by the decision-maker which becomes $x(\beta_l(\alpha_i - d\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l}))$. At the same time, type α_i also reduces his own contribution $t_i(\alpha_i - d\alpha_i,\alpha_{-i})$ thereby letting other members of his own group contribute much of what is needed to influence the decision-maker. This modification of the individual payment is thus at the core of the free riding problem in group formation. To illustrate, when group l is large enough, shading his own preferences might have little impact on the decision but it drastically reduces individual contribution.

More generally, type α_i's net gain from manipulating preferences ends up being worth $\frac{1}{N_l}\Delta u_l(x(\beta_l(\alpha_i - d\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l}))d\alpha_i \approx \Delta u_l(x(\beta_l(\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l}))d\alpha_i$. To induce information revelation, type α_i must thus pocket an extra informational rent $\mathcal{U}_l(\alpha_i) - \mathcal{U}_l(\alpha_i - d\alpha_i)$ which, at any point of differentiability, is approximatively worth $\dot{\mathcal{U}}_l(\alpha_i)d\alpha_i$ where $\dot{\mathcal{U}}_l(\alpha_i)$ is obtained by differentiating (4.1) as:

$$\dot{\mathcal{U}}_l(\alpha_i) = \mathbb{E}_{\alpha_{-i},\alpha_{-l}}\left(\frac{1}{N_l}\Delta u_l(\beta_l(\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l}))\right). \tag{4.2}$$

[22] Laffont and Maskin (1982), Mailath and Postlewaite (1990), Ledyard and Palfrey (1999) and Hellwig (2003) among others.

[23] We will see below, especially in Section 5, that such *ex post* monotonicity arises quite naturally under some circumstances and that Item 2. of Lemma 1 can sometimes follow from a set of more stringent conditions that apply *ex post*, namely:

$$\frac{1}{N_l}\Delta u_l(\beta_l(\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l})) \text{ being non-decreasing in } \alpha_i \text{ for all } (\alpha_{-i},\alpha_{-l})\ .$$

From (4.2), the payoff profile $\mathcal{U}_l(\alpha_i)$ is necessarily non-decreasing. The no-veto constraint (3.8) is thus harder to satisfy for those individuals with type $\underline{\alpha}_l$ who are the most eager to veto. Unanimous agreement on the formation of group l arises when:

$$\mathcal{U}_l(\underline{\alpha}_l) \geq 0. \tag{4.3}$$

Aggregate feasibility condition. Equipped with the characterization of incentive compatibility (4.1) and no-veto (4.3), we now derive a feasibility condition that aggregates no-veto, incentive compatibility and budget balanced constraints. To this end, we define group l *virtual net gain* of forming as:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h_l^*(\alpha_l)\Delta u_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l})) - T_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))\right).$$

This virtual gain is obtained when each member's preference parameter is replaced by its own virtual parameter while, on the cost side, the overall contribution remains unchanged.

Lemma 2 Incentive-feasibility. *A mechanism $\mathcal{G}_l$ is incentive-feasible if and only if:*

1. *The virtual net gain is non-negative:*

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h_l^*(\alpha_l)\Delta u_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l})) - T_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))\right) \geq 0. \tag{4.4}$$

2. $\mathbb{E}_{\alpha_{-i},\alpha_{-l}}(\Delta u_l(\beta_l(\alpha_i,\alpha_{-i}),\beta_{-l}(\alpha_{-l})))$ *is non-decreasing in* α_i.

Lemma 2 is a fundamental step on our way to simplify the design problem. Condition (4.4) indeed summarizes all the difficulties that asymmetric information might bring to the collective action problem. Valuations are replaced by virtual valuations which are lower so that the overall incentives of the group to contribute diminish.

5. EFFICIENT EQUILIBRIUM

We now assess whether lobbying competition can still be efficient under asymmetric information. We thus ask whether the *pluralistic approach* of politics that predicts such efficiency is still valid in this context. We will refer to *an efficient equilibrium* as an equilibrium (if any) such that each group solves its own internal informational problem at no cost, lobbyists are endowed with the aggregate preferences of their respective group, and the decision-maker chooses an efficient decision at the last stage of the game.

Let us thus suppose such existence of such equilibrium. Given that group $-l$ efficiently solves its own informational problem, group l must also do so. For such an equilibrium to exist, lobbyists must be endowed with an objective that perfectly reflects the preferences of the interest group they represent, i.e., $\beta_l^*(\alpha_l) = \alpha_l^*(\alpha_l)$. Slightly abusing notations, we may define the efficient decision that is then taken by the decision-maker at the last stage of the game in terms of the overall vector of preferences as:

$$x^*(\alpha_l,\alpha_{-l}) = \arg\max_{x\in\mathcal{X}} u_0(x)+\alpha_l^*(\alpha_l)u_l(x)+\alpha_{-l}^*(\alpha_{-l})u_{-l}(x) \equiv \varphi((-1)^l(\alpha_{-l}^*(\alpha_{-l})-\alpha_l^*(\alpha_l))). \tag{5.1}$$

We are now ready to unveil conditions for the existence of an efficient equilibrium.[24]

[24]To simplify notations, we denote in the sequel $[g(x)]_{x_0}^{x_1} = g(x_0) - g(x_1)$.

PROPOSITION 1 EXISTENCE OF AN EFFICIENT EQUILIBRIUM. *An efficient equilibrium exists if and only if:*

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\left[u_0(x)+\alpha_l^*(\alpha_l)u_l(x)+\alpha_{-l}^*(\alpha_{-l})u_{-l}(x)\right]_{x^*(0,\alpha_{-l})}^{x^*(\alpha_l,\alpha_{-l})}\right) \tag{5.2}$$

$$\geq \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)\left[u_l(x)\right]_{x^*(0,\alpha_{-l})}^{x^*(\alpha_l,\alpha_{-l})}\right) \quad \forall l\in\{1,2\}.$$

The feasibility condition (5.2) has a simple interpretation. The l.h.s. is the overall welfare gain from influencing the decision-maker so as to shift his decision from $x^*(0,\alpha_{-l})$ which is chosen when group l is not active to $x^*(\alpha_l,\alpha_{-l})$ which is instead chosen at an efficient equilibrium. Since $x^*(\alpha_l,\alpha_{-l})$ maximizes overall welfare, this gain is necessarily positive. If collective action were to take place under symmetric information, this l.h.s. difference would be the payoff that group l could capture.

The r.h.s. stems for the overall informational cost that such change of decision induces. Under complete information within group l this second term would disappear. Condition (5.2) is thus a fundamental equation to understand how groups solve their collective action problem. Since the l.h.s. is always non-negative, there would always exist an efficient equilibrium under complete information. We retrieve here the standard efficiency result that backs up the *pluralistic approach*. Asymmetric information introduces a cost of coalition formation that may preclude such efficient equilibrium and call for less optimistic conclusions about the efficiency of lobbying competition.

RUNNING EXAMPLE. To illustrate, let us consider the quadratic example and suppose that valuations are drawn from a uniform distribution on $[\underline{\alpha}_l,\overline{\alpha}_l]$. Condition (5.2) becomes:

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{3}{2}\alpha_l^*-\overline{\alpha}_l\right)\alpha_l^*\right)\geq 0 \quad \forall l\in\{1,2\}. \tag{5.3}$$

This feasibility condition is independent of group $-l$'s preferences. We will see below that, in the quadratic example, inefficiencies in group formation are fully determined by the group's own composition. Observe also that Condition (5.3) holds when $\underline{\alpha}_l$ is close enough to $\overline{\alpha}_l$ and more specifically when $\underline{\alpha}_l \geq \frac{2}{3}\overline{\alpha}_l$. In other words, homogeneity of the group is a key factor to ensure existence of an efficient equilibrium. ■

6. FREE RIDING AND EFFICIENCY IN LARGE GROUPS

Following a tradition that goes back to Bowen (1943), we now investigate how stringent Condition (5.2) is when groups become large. Of course, this is in such settings that intra-group free riding is the most difficult to solve.

There are two consequences of taking the limit $N_l \to +\infty$ while keeping N_{-l} fixed. First, the *Strong Law of Large Numbers* tells us that the empirical mean of samples made of true preference parameters converges with probability one towards the mean of α_i:

$$\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i \underset{N_l\to+\infty}{\overset{a.s.}{\to}} \int_{\underline{\alpha}_l}^{\overline{\alpha}_l}\alpha_i f_l(\alpha_i)d\alpha_i=\alpha_l^e.$$

Second, this same law also implies that the empirical mean of samples made of virtual preference parameters converges with probability one towards the mean of $h_l(\alpha_i)$, i.e., the lowest possible valuation within group l:

$$\frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_i) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} \int_{\underline{\alpha}_l}^{\overline{\alpha}_l} h_l(\alpha_i) f_l(\alpha_i) d\alpha_i = \underline{\alpha}_l.$$

Repeatedly throughout the paper, we refer to the following assumption that ranks those limits and requires that the distribution of tastes is diffuse enough.

Assumption 1 $\underline{\alpha}_l = 0 < \alpha_l^e \quad \forall l.$

We can now easily prove the following important result.

Proposition 2 Free Riding in large heterogenous groups. *Suppose that Assumption 1 holds. An efficient equilibrium never exists for N_l large enough.*

Inefficiencies in the formation of a large group are pervasive whenever the types distribution contains individuals with no strict preferences for the policy. To understand this property, one has to come back on the forces that lie behind such formation. On the one hand, an efficient equilibrium requires that the lobbyist is endowed with the average preference parameter of the group he represents. When group l is influential, the policy has thus to move away from the decision $x^*(0, \alpha_{-l})$ that would be taken if only group $-l$ was intervening towards the efficient decision $x^*(\alpha_l, \alpha_{-l})$. The welfare gain that accrues to group l from such a move has to be compared with the overall information rent that has to be distributed to members of that group to induce information revelation. When group l is large, each individual member only cares about minimizing his own contribution to such a policy shift. Each member has thus an incentive to behave as having the lowest possible valuation within the group, namely $\underline{\alpha}_l = 0$ from Assumption 1. So overall, the group behaves as being made only of those agents with no preferences for the policy. But if it was so, those types would just be collectively indifferent between being influential and letting the decision $x^*(0, \alpha_{-l})$ be enforced. They are not ready to contribute anything to change that outcome. The overall contribution of group l is thus zero as a whole and it becomes impossible to move towards the efficient policy $x^*(\alpha_l^*, \alpha_{-l})$.

Proposition 2 has its counterpart which states that efficiency is achieved when both groups are sufficiently homogenous.

Proposition 3 Efficient equilibria in large homogenous groups. *Suppose that $\alpha_l^e - \underline{\alpha}_l$ (for $l = 1, 2$) is small enough. An efficient equilibrium always exists when N_l (for $l = 1, 2$) is large enough.*

When α_l^e is close to $\underline{\alpha}_l$ and even if all agents behave as being the worst type within group l, they are still ready to pay some positive amount to induce a policy shift away from $x^*(0, \alpha_{-l})$. Aggregating over a large group, even tiny individual contributions may be enough to change the outcome. Enough homogeneity suffices to ensure that the group is influential even in the limit of a large size. To conclude, this is not size *per se* that undermines group formation but the addition of size and heterogeneity.

7. GROUPS OF FINITE SIZE AND INEFFICIENT EQUILIBRIA

When groups are large, the *Strong Law of Large Numbers* shows that there is almost no remaining uncertainty about both aggregate preferences and aggregate contributions. Such arguments explained our findings in Propositions 2 and 3. However, with groups of finite size, there is still some remaining uncertainty not only about the distribution of aggregate valuations within the group, but also about the competitor's own preferences. This makes the analysis *at finite distances* certainly more complex than what we just saw in Section 6, but it also introduces some new strategic features of much interest if one wants to unveil the true determinants of inefficiencies in group formation.

7.1. *Benchmark: Group Formation under Complete Information*

As a benchmark, let us consider the hypothetical case where group l forms under complete information. We already alluded to the fact that the feasibility condition (4.4) is now replaced with the simpler requirement:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\alpha_l^*(\alpha_l)\Delta u_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))-T_l(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))\right)\geq 0. \tag{7.1}$$

This condition just expresses the fact that, under complete information, the group l's overall net gains from forming must be non-negative.

PROPOSITION 4 EFFICIENT GROUP FORMATION UNDER COMPLETE INFORMATION. *When group l forms under complete information, preferences are efficiently aggregated, irrespectively of the other group's characteristics:*

$$\beta_l^*(\alpha_l)=\alpha_l^*(\alpha_l). \tag{7.2}$$

Under complete information, the objectives of lobbyists always perfectly reflect the aggregate preferences of the group they represent. This result holds whatever the preferences of the competing group since the efficient decision rule (7.2) is independent of the decision rule $\beta_{-l}(\alpha_{-l})$ that pertains to group $-l$. Under complete information, competition has thus no impact on group formation.

Although apparently intuitive and much in lines with Bernheim and Whinston (1986), this result is far from being trivial especially because the game of coalition formation adds a preliminary stage where the preferences of delegates could be strategically chosen. To fully understand the strength of Proposition 4, remember that contributions at the last stage of the game are VCG payments. Henceforth, each lobbyist ends up getting the incremental value (3.5) that he brings to the grand-coalition made of the decision-maker and the competing lobbyist. At the margin, changing the objective of his lobbyist thus brings to group l the full marginal social value of such a change. There is no reason to manipulate those preferences because VCG mechanisms are non-manipulable. The lobbyist's preferences sincerely reflect those of the group he represents.

Finally, it follows from Proposition 4 and from the fact that the common agency stage of the game perfectly reflects the preferences of the lobbyists that, under complete information within groups, the lobbying process leads to an efficient decision. In other words, the *pluralistic approach* of politics would be valid under complete information.

7.2. *Inefficient Group Formation as a Best Response*

Under asymmetric information, groups may have to incur informational costs to solve their free riding problems. As a result, an efficient equilibrium of the overall game may not exist. This Section analyzes how our results must then be amended.

7.2.1. *Second-Best Appointment Rule*

We look for a best response to any arbitrary appointment rule $\beta_{-l}(\alpha_{-l})$ that may have been chosen by group $-l$. For a given appointment rule $\beta_l(\alpha_l)$ chosen now by group l, the contribution $T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))$ that this group offers to the decision-maker can be expressed from (3.6) as:

$$T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l})) = [\beta_{-l}(\alpha_{-l})u_{-l}(x) + u_0(x)]_{x(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))}^{x(0,\beta_{-l}(\alpha_{-l}))}. \tag{7.3}$$

Taking into account this expression, the incentive-feasibility condition (4.4) becomes:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left([u_0(x) + h_l^*(\alpha_l)u_l(x) + \beta_{-l}(\alpha_{-l})u_{-l}(x)]_{x(0,\beta_{-l}(\alpha_{-l}))}^{x(\beta_l(\alpha_l),\beta_{-l}(\alpha_{-l}))}\right) \geq 0. \tag{7.4}$$

When group l forms under asymmetric information, the induced preferences parameter $\beta_l(\alpha_l)$ given to its lobbyist is chosen with an eye on the incentive-feasibility condition (7.4). When this condition is indeed a relevant binding constraint, group l faces a trade-off. On the one hand, choosing $\beta_l(\alpha_l)$ close to the efficient rule (7.2) is good from an efficiency point of view within group l since this choice induces a large policy shift from $x(0, \beta_{-l}(\alpha_{-l}))$ to $x(\alpha_l^*(\alpha_l), \beta_{-l}(\alpha_{-l}))$. On the other hand, such policy shift also requires a large compensation payment for the decision-maker. This in turn exacerbates the intra-group free riding problem and hardens the incentive-feasibility condition (7.4).

To moderate this rent-efficiency trade-off, the lobbyist's preference parameter $\beta_l^{sb}(\alpha_l)$ now only partially reflects group l's preferences. So doing, policy shifts are of a lower magnitude, contributions diminish and the intra-group free riding problem is weakened.

Proposition 5 Inefficient best responses. *At a best response, group l endows his own lobbyist with a preference parameter $\beta_l^{sb}(\alpha_l)$ such that:*

$$\beta_l^{sb}(\alpha_l) = \max\left\{0, \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right\} \tag{7.5}$$

where λ_l is the Lagrange multiplier of the incentive-feasibility constraint (7.4).

Formula (7.5) shows how informational frictions force group l to choose a lobbyist with more moderate preferences than its own. Formally, $\beta_l^{sb}(\alpha_l) \leq \beta_l^*(\alpha_l) = \alpha_l^*(\alpha_l)$ when the multiplier λ_l of the incentive-feasibility constraint is positive.

A priori, the preferences of group l's lobbyist $\beta_l^{sb}(\alpha_l)$ might now depend on group $-l$'s own choice of induced preferences $\beta_{-l}(\alpha_{-l})$ through the impact that this variable has on the incentive-feasibility constraint (7.4) and thus on λ_l. We will come back later on this joint determination of informational frictions by means of Lagrange multipliers.

Choosing moderate preferences for the lobbyist might sometimes mean giving up any influence at all. Consider the case where group l's aggregate preferences $\alpha_l^*(\alpha_l) = \frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i$ are small enough so as not to cover the overall information distortion $\frac{1}{N_l}\frac{\lambda_l}{1+\lambda_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}$. This arises for instance when all members of group l have preference parameters α_i close enough to $\underline{\alpha}_l = 0$. Under such configurations, group l prefers to endow its lobbyist with *no strict preferences on policy*: $\beta_l^{sb}(\alpha_l) = 0$. As a result, the lobbyist receives no money from the interest group to influence the decision-maker. Group l is no longer active under asymmetric information while it would have been so under complete information. The intuition is straightforward. Too many members of group l are tempted to shade their preferences and, as a result, the group has no influence.

This result is reminiscent of the analysis in Martimort and Stole (2015) although the loci of asymmetric information in the two papers differ. Martimort and Stole (2015) model private information on the decision-maker side instead. They demonstrate that interest groups eschew intervention as soon as their preferences are *"too far away"* from those of the decision-maker to be willing to incur the corresponding agency costs. Here instead, this is intra-group asymmetric information that creates enough agency costs so that influencing the decision-maker is no longer attractive.

More generally, with asymmetric information, the policy that is chosen by the decision-maker does not perfectly reflect the preferences of this group. Contributions are smaller to weaken intra-group free riding. The decision-maker is thus systematically biased towards the outcome he would have chosen when selling his favors to the competing group only. In other words, informational frictions also weaken competition for the decision-maker's favors. The welfare consequences of this insight will be studied in more details below.

7.2.2. *Inefficiencies*

This section further unveils the nature of informational frictions that hinder group formation. To this end, we state the following assumption.

Assumption 2

$$\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)} - \mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\Big|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i = \alpha_l^*\right) \text{ is non-increasing in } \alpha_l^*.$$

Assumption 2 requires the monotonicity of the difference between the hazard rate of the sample mean and the conditional sample mean of hazard rates. Although apparently complex, this property ensures inefficient group formation irrespectively of what happens for the competing group.This condition depends only on properties of the types distribution and not on the appointment rule $\beta_{-l}(\alpha_{-l})$ that has been chosen by group $-l$.

Proposition 6 Inefficient Group formation at finite size. *Suppose that Assumptions 1 and 2 both hold and $N_l > 1$. Group l never forms efficiently whatever the appointment rule β_{-l} implemented by group $-l$.*

That efficiency is not possible means that (7.4) fails for $\beta_l^*(\alpha_l) = \alpha_l^*(\alpha_l)$, i.e.

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\left[u_0(x) + h_l^*(\alpha_l)u_l(x) + \beta_{-l}(\alpha_{-l})u_{-l}(x)\right]_{x(0,\beta_{-l}(\alpha_{-l}))}^{x(\alpha_l^*(\alpha_l),\beta_{-l}(\alpha_{-l}))}\right) < 0. \tag{7.6}$$

Proposition 2 already showed that Condition (7.6) always holds when N_l is large enough. Assumptions 1 and 2 jointly ensure that this result is already true at finite sizes. Indeed, when N_l is large enough, the *Strong Law of Large Numbers* shows that both $\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}$ and $\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i=\alpha_l^*\right)$ converge almost surely towards $\alpha_l^e=\mathbb{E}_{\alpha_i}\left(\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)$. Thus, Assumption 2 trivially holds for N_l large enough and Assumption 1 alone was already enough to obtain the inefficiency result in Proposition 2.

Condition (7.6) highlights an important inefficiency result. Similar conditions for the impossibility of implementing the first-best allocation under asymmetric information have flourished throughout the whole mechanism design literature both in public and private good contexts (Laffont and Maskin (1982), Myerson and Sattherwaite (1983), Cramton, Gibbons and Klemperer (1987), Mailath and Postelwaite (1990), Hellwig (2003)). In those models, each agent has veto power and veto yields zero payoff to all players. In our context, any potential group member may veto group formation. Yet, reservation payoffs remain non-zero and, more precisely, those payoffs are given by the policy chosen by the decision-maker under the sole influence of the rival group.

Running example (continued). With quadratic preferences example, the optimal policy is given by $x^*(\beta_1,\beta_2)=\frac{\beta_2-\beta_1}{\beta_0}$. To illustrate the value of Assumption 2, observe that Condition (7.6) can then be transformed as:[25]

$$(7.7)\qquad \mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i=\alpha_l^*\right)\right)\frac{\alpha_l^*}{\beta_0}\right)<0.$$

This condition is independent of the appointment rule $\beta_{-l}(\alpha_{-l})$ that is chosen by group $-l$. Assumption 2 ensures that the first factor in the expectation is non-decreasing. It can be easily checked that this term has also zero mean. The second factor $\frac{\alpha_l^*}{\beta_0}$ is monotonically increasing; a property that holds beyond the quadratic case. A simple integration by parts then shows that the l.h.s. of (7.7) is negative as requested.

Lastly, when valuations are uniformly distributed on $[0,\overline{\alpha}_l]$, we have $\alpha^e=\frac{\overline{\alpha}_l}{2}$ and $\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}=\overline{\alpha}_l-\alpha_i$. Assumption 1 is trivially true. Assumption 2 holds since it amounts to checking that $\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\overline{\alpha}_l+\alpha_l^*$ is non-decreasing in α_l^*.[26] ■

We now turn to a more detailed description of the inefficiencies in group formation.

Corollary 1 Strong inefficiency. *Suppose that Assumptions 1 and 2 both hold.*

1. *Group l's best response is always inefficient:*

$$\lambda_l>0.$$

2. *The probability that group l has no influence is always strictly positive:*

$$\mathbb{P}\{\beta_l^{sb}(\alpha_l)=0\}>0.$$[27]

[25] The proof in the Appendix relies on several rounds of integration by parts.
[26] Proof available upon request.
[27] The notation $\mathbb{P}\{\Gamma\}$ stands for the probability of the event Γ.

Item 1. points at a very strong form of inefficiency that arises independently of frictions in group $-l$ if any. The sole role of lobbyist $-l$'s objective is to affect the value of the positive Lagrange multiplier of the incentive-feasibility constraint (7.4) but Condition (7.6) is already enough to ensure that group l's formation suffers from asymmetric information. This was certainly the case for a large group as shown in Proposition 2 but Assumptions 1 and 2 ensure that inefficiencies also arise at finite sizes.

Finally, Corollary 1 nicely summarizes how asymmetric information offers a drastic departure from the pluralistic view of politics. According to this view, lobbying competition should lead to an efficient representation of diverse interests. Under asymmetric information, not only the lobbying process generically fails to adequately aggregate the groups' interests, but this failure can even be extreme, with a group being simply absent from this process with some positive probability. This arises provided that the types distribution is sufficiently diffuse to include types with no strict preferences for the policy.

A contrario, even if inefficiencies still arise and the lobbyist's objectives remain moderated by informational frictions, some representation in the political process may always be guaranteed provided that preferences are not too dispersed. Next proposition provides a lower bound on how diffuse the types distribution should be to do so.

Corollary 2 Weak inefficiency. *Suppose that* $\underline{\alpha}_l \geq \frac{1}{f_l(\underline{\alpha}_l)}$. *Group* l *always forms, i.e.,* $\beta_l^{sb}(\alpha_l) > 0$ *although it is almost always inefficiently so:*

$$\mathbb{P}\{\beta_l^{sb}(\alpha_l) = \beta^*(\alpha_l)\} = 0.$$

7.2.3. *From Finite to Large Groups*

Corollaries 1 and 2 echo Propositions 2 and 3 respectively, stressing the role of heterogeneity as a key factor behind frictions in group formation. This Section further fills the gap between the finite size scenario and the limiting case of a large group. In particular, we are interested in asymptotic properties, making now explicit the dependence of the Lagrange multiplier $\lambda_l(N_l)$ and the optimal appointment rule $\beta_l^{sb}(\alpha_l, N_l)$ on N_l.

Proposition 7 Towards Large Groups. *Suppose that Assumptions 1 and 2 both hold. The following limiting behaviors arise as* N_l *becomes large.*

1. *Informational frictions become arbitrarily large:*

$$\lim_{N_l \to \infty} \lambda_l(N_l) = +\infty, \tag{7.8}$$

2. *The appointment rule converges in probability towards no influence:*

$$\beta_l^{sb}(\alpha_l, N_l) \underset{N_l \to +\infty}{\overset{p}{\to}} 0. \tag{7.9}$$

Condition (7.8) implies that, as size increases, the appointment rule is entirely determined by the incentive-feasibility Condition (4.4). Proposition 2 already highlighted the inexistence of an efficient equilibrium under those conditions. Condition (7.9) is a stronger qualifier: A large group leaves the decision-maker under the sole influence of its rival.

7.3. *Dual Representation of Equilibria*

A priori, the Lagrange multiplier λ_l that appears on the r.h.s. of formula (7.5) depends on the induced preferences β_{-l} of the lobbyist acting on behalf of group $-l$. Indeed, those preferences affect how much money must be paid by group l to buy the policy-maker and thus the magnitude of the intra-group free riding. Since the preferences parameter $\beta_l^{sb}(\alpha_l)$ for group l's lobbyist is fully characterized by a single non-negative parameter λ_l, it becomes quite natural to summarize an equilibrium by a pair of non-negative numbers $(\lambda_l, \lambda_{-l}) \in \overline{\mathbb{R}}_+^2$ that determine appointment rules $(\beta_l^{sb}(\alpha_l), \beta_{-l}^{sb}(\alpha_{-l}))$ which are best responses to each other. With this *dual representation*, an equilibrium amounts to a pair (λ_1, λ_2) satisfying:

$$\lambda_l = \Lambda_l^*(\lambda_{-l}) \quad \forall l \in \{1, 2\} \tag{7.10}$$

where the Λ_l^* are *"best-response mappings"* defining the Lagrange multiplier characterizing group l's formation in terms of the Lagrange multiplier pertaining to group $-l$.

In the frictionless case of complete information that was summarized in Proposition 4, each group perfectly passes its own aggregate preferences to its lobbyist; contributions to the decision-maker are *truthful* and this is so independently on what the other groups is offering. In other words, induced preferences are not interdependent. Whether the opposite group easily raises money to influence the decision-maker or not has no impact on group l's own choice of an objective function for its lobbyist. It changes the level of group l's contributions but not how preferences among alternatives are finally passed on the decision-maker. This stands in sharp contrast with what arises under asymmetric information. Frictions in group formation now depend on how much money is needed to influence the decision-maker. This provides a channel by which the strength of the opposite group impacts on group l's choice of an objective for its own lobbyist. Frictions within each group are now determined simultaneously at equilibrium.

Thanks to the simple dual representation that views an equilibrium as a pair of Lagrange multipliers and provided that the set of relevant Lagrange multipliers is conveniently compactified, a simple fixed-point argument ensures existence of an equilibrium.

PROPOSITION 8 EXISTENCE OF EQUILIBRIA. *There always exists a (pure-strategy) equilibrium, i.e., a pair $(\lambda_l, \lambda_{-l})$ that solves (7.10) and that corresponds to induced preferences $(\beta_l^{sb}(\alpha_l), \beta_{-l}^{sb}(\alpha_{-l}))$ given by formula (7.5).*

Existence of a pure strategy equilibrium is interesting in its own sake. First, it means that we should not expect much instability in lobbying competition as would be the case if only mixed-strategy equilibria could arise. Beyond existence, more interesting comparative statics follow from carefully looking at the the properties of best-response mappings. This is the purpose of next subsection.

7.4. *Monotonicity Properties of the Best-Response Mappings*

LEMMA 3 MONOTONICITY PROPERTIES OF BEST-RESPONSE MAPPINGS. *Λ_l^* ($l \in \{1, 2\}$) is everywhere non-decreasing (resp. non-increasing) if and only if $u_0''' \geq 0$ (resp. $u_0''' \leq 0$).*

Slightly abusing language, Lemma 3 shows that the game between competing groups might exhibit either *strategic complementarity* with both mappings Λ_l^* (for $l \in \{1,2\}$) being everywhere non-decreasing or *strategic substitutability* when those mappings are instead everywhere non-increasing. Those monotonicity properties depend in fine details of the decision-maker's preferences.

The intuition for those different patterns comes from understanding how asymmetric information impacts on lobbying competition. Suppose that group $-l$ finds it more difficult to organize itself, in other words that λ_{-l} increases. The first consequence is that this group has less impact on the decision. It becomes easier for group l to shift the *status quo* towards its own preferred direction: a *"policy-shifting" effect*. Lower contributions from group l are needed and thus free riding within that group is less of a curse. On the other hand, that group $-l$ does not influence so much the decision also means that this *status quo* might already please group l. Not organizing efficiently is thus less costly for that group. This in turn exacerbates free riding: a *"status quo" effect*.

Which effect dominates depends on the sign of u_0'''. When $u_0''' \leq 0$, the *"policy-shifting" effect* dominates and distortions within group l are less significant as competing interests find it more difficult to organize. *A contrario*, when $u_0''' \geq 0$, the *"status quo" effect* prevails and distortions are more pronounced. A key lesson is thus that the formation of a group generally depends on its environment. The existence of informational frictions within a group impacts on how easily its competitor organizes.

Running example (continued). With quadratic preferences, $u_0''' \equiv 0$. The *"status quo"* and the *"policy-shifting"* effects just compensate each other. The best-response mappings Λ_l^* are then constant and the equilibrium values of the Lagrange multipliers (λ_1, λ_2) are determined independently. Whether group l faces a strong opponent or not does not affect its own difficulties in solving the internal free riding problem and the preferences left to its lobbyist are independent of the surrounding environment. ■

8. TOWARDS AN "INDUSTRIAL ORGANIZATION" THEORY OF GROUPS FORMATION

Referring to the case of increasing (resp. decreasing) best responses as featuring strategic complementarity (resp. substitutability) reminds of the well-known parlance of the IO literature. This reference also suggests that groups might try to use various commitment devices to move the equilibrium along best-response mappings so as to favor their own interests. Following important insights by Fudenberg and Tirole (1985) and Bulow, Geanakoplos and Klemperer (1985), this idea is well-known in the IO literature. This section now revisits those insights in the case of group formation. Of course, whether a commitment device to worsen internal frictions gives a competitive edge on a competing group depends on whether the *"policy-shifting"* or the *"status quo"* effect dominates.

8.1. *Information Sharing*

Suppose that members of group $-l$ can credibly share information, maybe because this group has a small size and peer monitoring is readily available,[28] or because free riders can

[28] Ostrom (1990) argued that agents invest resources to monitor each other and reduce the occurrence of the free riding problem.

be sufficiently punished by means of group stigma, or repeated interactions that overcome informational problems. In this respect, considering how coalitions of interest groups themselves form, Hula (1999) argued that *"the increasing use of long-term, recurrent, and institutionalized coalitions in many policy arenas"* build what he coins as being *strong coalitions* of interest groups. Finally, even if information is not shared, it might not have much of an impact on how a group designs preferences for its lobbyist. This is so when individuals have no veto power and we will comment on this point further in Section 10.2.

In those contexts, complete information eliminates frictions in group formation. Formally, credibly sharing information within say group $-l$ can be viewed as a commitment device to fixing $\lambda_{-l} = 0$. Its effect on the frictions faced by group l when itself forming can be easily deduced from Lemma 3.

PROPOSITION 9 INTRA-GROUP INFORMATION SHARING. *Let* $(\lambda_l, \lambda_{-l})$ *be an equilibrium of the game when both groups form under asymmetric information. If group* $-l$ *now forms under complete information, the new equilibrium* $(\tilde{\lambda}_l, 0)$ *is such that* $\tilde{\lambda}_l \leq \lambda_l$ *(resp.* $\geq$*) if* $u_0''' \geq 0$ *(resp.* $\leq$*).*

This proposition nicely illustrates how the *"policy-shifting"* and *"status quo"* effects are now modified. When it is efficiently organized, group $-l$ buys influence on the decision-maker more easily. In turn, it becomes more difficult for group l to buy such influence. Group l has to raise its own contribution which worsens its own free riding problem. The *"policy-shifting" effect* exacerbates frictions within group l, while the *"status quo" effect* instead attenuates those frictions. When the first effect dominates (i.e., $u_0''' \leq 0$), a group which has solved its own free riding problem can not only affect decision more easily but it also benefits in addition from weakening its competitor's representation. This indirect effect increases the group's payoff and decreases that of the weaker competitor. Strong interest groups might thus exclude rivals from the political arena more often than what less well-organized groups would do. In contrast, when the *"status quo" effect* dominates (i.e., $u_0''' \geq 0$), a strong group facilitates the formation of its rival and faces a tougher competition in the political arena.

8.2. *Transaction Costs*

Groups may also incur further organization costs beyond asymmetric information *per se*. For instance, hiring a lobbyist might require to incur search costs, to pay contingent fees, sometimes to give up extra rent if lobbyists have market power or private information on the market for their services. Following on this idea, Mitra (1999) and Martimort and Semenov (2007) have proposed symmetric information common agency models that also rely on the existence of fixed costs of group formation to model entry and endogenize the composition of active groups. Here, the effect of transaction costs is to harden the incentive-feasibility condition (4.4) and exacerbate frictions. To see how, let denote by K_l a positive fixed cost of group formation. The incentive-feasibility condition (4.4) becomes:

$$\text{(8.1)} \qquad \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h_l^*(\alpha_l)\Delta u_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))\right) - \mathbb{E}_{\alpha_{-l}}\left(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))\right) \geq K_l > 0.$$

The overall contribution of group l must increase to cover this extra fixed cost. In response, frictions in group formation are more pronounced. Formally, the Lagrange multiplier λ_l increases, shifting upwards the whole best-response mapping Λ_l^*, and modifying accordingly the set of possible equilibria of the game.

Proposition 10 Transaction costs. *Let* $(\lambda_l, \lambda_{-l})$ *be an equilibrium of the game when both groups form under asymmetric information and there are no transaction costs. If group* l *incurs some small transaction costs of formation (i.e.,* K_l *small enough), there exists a new equilibrium* $(\tilde{\lambda}_l, \tilde{\lambda}_{-l})$ *such that* $\tilde{\lambda}_l \geq \lambda_l$ *(resp.* $\geq$*) and* $\tilde{\lambda}_{-l} \geq \lambda_{-l}$ *(resp.* $\leq$*) if* $u_0''' \geq 0$ *(resp.* $\leq$*).*

When the *"status quo" effect* dominates, (i.e., $u_0''' \geq 0$), group l gets a strategic advantage by worsening its own informational problem since it also hardens the organization of his rival. The opposite happens when the *"policy shifting" effect* dominates (i.e., $u_0''' \leq 0$).

9. WELFARE ANALYSIS

This section investigates some welfare consequences of having group formation taking place asymmetric information. To understand how those frictions impact on welfare, we must remind a logic which is now familiar from Section 8. Inefficiencies in group formation have two effects on groups' payoffs. First, for a given strategy followed by his rival, each group would be better off if informational constraints *within this group* could be circumvented so as to endow its lobbyist with an objective that would perfectly reflect the group's aggregate preferences. Second, informational frictions *within the competing group* also contribute to soften competition. Each group benefits from facing a weaker competitor who has less influence on the decision-maker. The overall impact of those competing effects on the groups' payoffs can thus go either way. A group might take advantage of informational frictions while, at the same time, its rival could be hurt by the very same frictions (this is for instance the case if the lower bound $\underline{\alpha}_l$ is sufficiently positive so that group l always manages to get organized while group $-l$ fails being so). However, it is also possible that both groups benefit from softening competition.

Getting unambiguous welfare results is thus difficult in general. We thus content ourselves with pointing out a few effects that arise in specific contexts. The first one is that frictions in coalition formation may soften competition between symmetric groups.

Proposition 11 Groups' payoffs. *Suppose that Assumption 1 holds, groups have the same size* $N_1 = N_2 = N$*, valuations are drawn from the same distribution on* $\Omega_1 = \Omega_2 = [0, \overline{\alpha}]$*, and the decision-maker's objective is quadratic as defined in (3.1). For* N *large enough, interest groups are ex ante better off under asymmetric information.*

We already know that inefficiencies are pervasive in large groups when preferences are sufficiently dispersed. Those groups have no influence on decision-making. When both groups face such a huge free riding problem, the decision-maker still goes for the *status quo* which reflects only her own preferences. Under complete information, groups of equal force would have competed *"head-to-head"* for influence, and the same decision would also have been chosen.The difference is that each group would pay a lot for maintaining the *status quo* just to avoid that the other group tilts the decision in his own direction. From an *ex ante* viewpoint, the groups' expected gains remain the same under both informational scenarios. Yet, under complete information, groups waste money in *"head-to-head"* competition for the decision-maker's services while they refrain from doing so under asymmetric information. Asymmetric information moderates lobbying competition.

By the same token, informational frictions have also a non-ambiguous impact on the decision maker's payoff.

PROPOSITION 12 DECISION-MAKER'S PAYOFFS. *The decision-maker's ex ante and ex post payoffs are always lower under asymmetric information than under full information.*

By making lobbyists' objective less sensitive to the decision, asymmetric information softens lobbying competition. It thus reduces the rent that the decision-maker extracts from playing one group against the other. As a result, the decision-maker might want to favor group formation in order to boost competition and increase her own rent.

10. DISCUSSIONS AND EXTENSIONS

This short section discusses several extensions of our basic framework.

10.1. *Towards an Informational Theory of Transaction Costs*

The literatures on lobbying and regulatory capture have repeatedly referred to the idea that groups incur transaction costs when targeting a decision-maker. In a nutshell, if a group's stakes for changing a policy is $\alpha\Delta u$ and the group exerts real influence on the decision-maker by paying him T dollars, then its net payoff writes as

$$\alpha\Delta u - (1+\mu)T \tag{10.1}$$

where the quantity μT can be interpreted as the transaction costs of transferring those T dollars to the decision-maker.

In Bernheim and Whinston (1986)'s model of lobbying competition or in Laffont and Tirole (1991)'s model of regulatory capture, such deadweight loss is group specific. Policies then reflect the distribution of transaction costs across groups. An important albeit hidden assumption is that those frictions are independent of the stakes induced by the chosen policy. The argument is that side-transfers entail a deadweight loss that reflects the imperfect enforceability of side-contracts, the group's organizational costs of collective action or the shadow cost of raising contributions on financial markets. Even when deadweight losses are nonlinear as in Esteban and Ray (2001), a partial equilibrium perspective is taken, fixing frictions at the outset.[29] This is certainly a valid first step but modelers should be aware of possible limits in using such partial approach. Modelers might instead be concerned with the feed-back effect that policies have on the endogenous stakes of group formation. If they do, they should also realize that the simple formula (10.1) above is only an approximation for the informationally-founded approach that is advocated in this paper. To showcase the possible modifications of those transaction costs, we now interpret our previous results in terms of *informational transaction costs* that characterize inefficiency in group formation. Consider an equilibrium $(\lambda_l, \lambda_{-l})$ of the game and define some *informational transaction costs* of group l's payments, say $\mu_l(\alpha_l, \lambda_l)$, as follows:

$$\mu_l(\alpha_l, \lambda_l) \equiv \frac{\lambda_l}{1+\lambda_l}\frac{\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}}{\max\left\{\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}, 0\right\}} \tag{10.2}$$

[29] Martimort (1999) and Faure-Grimaud, Laffont and Martimort (2003) have proposed various models that endogenize transaction costs when side-contracts are impeded by either asymmetric information or imperfect enforceability. Even there, the analysis of frictions has been decoupled from the study of the political game that endogenously determines the stakes for collective action.

with the convention that $\mu_l(\alpha_l, \lambda_l) = +\infty$ when the denominator is zero. We can easily check that for all (α_l, α_{-l}) the equilibrium weights (β_l, β_{-l}), and the subsequent decision $x(\beta_l, \beta_{-l})$ are the same had groups formed under complete information, but with a deadweight loss of transfers being $\mu_i(\alpha_l, \lambda_l)$ so that each group now maximizes:

$$\alpha_l^*(\alpha_l)\mathbb{E}_{\alpha_{-l}}(\Delta u(\beta_l, \beta_{-l}(\alpha_{-l}))) - (1+\mu(\alpha_l, \lambda_l))\mathbb{E}_{\alpha_{-l}}(T_l(\beta_l, \beta_{-l}(\alpha_{-l}))).$$

This formula is very close to the reduced form (10.1) above. Yet, it shows that modeling frictions in reduced form models should make transaction costs dependent on various parameters. The profile of preferences within the group, the strength of rivals and the stakes are all important ingredients. This suggests that interesting and important comparative statics could be obtained if modelers were to take the short-cut of specifying stake-dependent transaction costs μ that would increase with group size, strength and preferences profiles both within and across groups as shown in Section 8.

10.2. *Limited Veto Power*

Following the mechanism design literature, we have assumed that unanimous agreement was required to enforce a mechanism within each group. There are two ways of justifying this assumption. First, individuals may also themselves be groups that band together on some specific issues; a scenario that certainly echoes practices in nowadays U.S. Legislative Politics as it has been forcefully stressed by Hula (1999). In that case, it becomes again quite legitimate to give each of those groups equal veto power on an agreement for their coalition. Second, giving veto power to all individuals within a group can be viewed as a metaphor for stressing the difficulties in gathering information on individual preferences and, as such, it showcases an upper bound on informational frictions that a group faces.

Beyond, we may ask what would happen if the assumption of unanimous agreement to a mechanism were relaxed. Frictions are certainly of a lesser magnitude when only a subgroup of agents is entitled with veto power while others are just bound on whatever mechanism is proposed for their group. More specifically, suppose that all agents indexed by $i \in \{1, ..., \hat{N}_l\}$ have veto right on an agreement while those indexed by $i \in \{\hat{N}_l + 1, ...N_l\}$ have no such right. Importing an important insight due D'Aspremont and Gerard-Varet (1979) into our specific context; incentive compatibility for those agents with no veto power comes for free. Therefore, everything happens as if the preferences of those agents where common knowledge within the group. This immediately leads to redefine the objective of the lobbyist for that group as

$$\beta_l^{sb}(\alpha_l) = \frac{1}{N_l}\max\left(\sum_{i=1}^{\hat{N}_l}\alpha_i + \left\{\sum_{i=\hat{N}_l+1}^{N_l}\alpha_i - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right\};0\right). \tag{10.3}$$

$\hat{\lambda}_l$ is now the Lagrange multiplier for an aggregate feasibility constraint that takes into account that no information rent is left to the first $\hat{N}_l$ individuals and thus writes as:

(10.4)
$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{\hat{N}_l}\alpha_i + \sum_{i=\hat{N}_l+1}^{N_l}\alpha_i - \frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)\Delta u_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l})) - T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))\right) \geq 0.$$

There is nothing specific to the analysis of such environments with limited veto power. We could derive the same insights as above with the only difference being the lesser inefficiency coming from the ability of the group to implement more allocations when asymmetric information is less of a concern. The consequences on the equilibrium outcome of the overall game are thus very similar to those highlighted in Section 8.1.

10.3. *Groups Preferences*

We have developed our analysis in what could be seen by some readers as a specific environments: two groups with conflicting (linear) preferences of the same magnitude. Our analysis could be easily extended to more complex scenarios although with very little additional insights.

Nonlinear payoffs; Number of groups. First, the choice of linearity for the group's preferences plays little role beyond giving us a simple and tractable representation of informational frictions that might be lost with more complex specifications of preferences. As far as the number of active groups is concerned, having only two groups with conflicting preferences is also consistent with casual evidence reported by Hula (1999). In U.S. Legislative Politics, interest groups with similar interests tend to coalesce to push their own collective interests. Keeping two conflicting groups is thus a way to short-cut the full-fledged model of such cooperation of coalition of interest groups with *congruent* interests; i.e., groups that would like to push the policy decision in the same direction.

Congruent groups. This case of congruent groups is interesting as such, and Appendix B is devoted to briefly develop the corresponding analysis. A first important feature that distinguishes this analysis from the case of conflicting interests is that, even under complete information on preferences, *inter-group* free riding now arises. Indeed, the equilibrium contributions that are determined at the common agency stage of the game are no longer VCG payments as in the case of conflicting groups. Congruent groups design contributions so as to leave the decision-maker indifferent between taking both contributions and his next best option which is now to refuse all contributions and choose his most preferred status quo policy free of any influence.[30] Because contributions are no longer VCG payments, groups no longer pass sincerely their aggregate preferences to their delegates. Each group shades its preferences to reduce its own share of the cost of moving the decision-maker away from the status quo.[31] There are thus strong efficiency gains for congruent groups from merging so as to avoid such wasteful competition.

Under asymmetric information, those gains should be compared with the information rents that accrue to group members. When groups remain split apart, vetoing the mechanism is not very costly for any individual. Provided that the other group forms, the decision is indeed already tilted in the right direction. There are not too much rent to grasp under this scenario. Instead, when congruent groups are merged, each individual by vetoing the mechanism triggers the choice of the status quo by the decision-maker and this decision is further apart. Much more rent can now be obtained under this scenario.

[30] In the parlance of Laussel and Le Breton (2001), the *"no-rent property"* holds in this setting.

[31] Strategic delegation to a representative in the context of legislative bargaining has also been studied in Christiansen (2013), Besley and Coate (2003) and Dur and Roelfsema (2005), among others.

Overall the informational cost of a merger is quite large. Whether asymmetric information is more of a blessing with split congruent groups or with a merger is thus generally difficult to assess beyond specific examples. Appendix B nevertheless provides an example showing that efficiency gains may dominate even under asymmetric information. Congruent groups may then prefer to merge to solve their collective action problems. This again provides some justifications for our focus on two groups with opposite preferences.

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APPENDIX A: PROOFS OF THE MAIN RESULTS

PROOF OF LEMMA 1: NECESSITY. Fix the function $\beta_{-l}(\alpha_{-l})$, and consider an incentive mechanism $\mathcal{G}_l$. We shall use in the sequel the standard notations $\alpha_l = (\alpha_i, \alpha_{-i})$ and sometimes express functions of α_l accordingly. To keep notations simple, we also define the expected utility gains and expected payments for an individual in group l who reports having type α_i respectively as:

$$G_l(\hat{\alpha}_i) = \frac{1}{N_l}\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\Delta u_l(\beta_l(\hat{\alpha}_i, \alpha_{-i}), \beta_{-l}(\alpha_{-l}))\right) \text{ and } \mathcal{T}_i(\alpha_i) = \mathbb{E}_{\alpha_l}\left(t_i(\alpha_i, \alpha_{-i})\right).$$

With those notations, incentive compatibility constraints can be written as:

$$\text{(A.1)} \qquad \mathcal{U}_l(\alpha_i) = \alpha_i G_l(\alpha_i) - \mathcal{T}_i(\alpha_i) \geq \alpha_i G_l(\hat{\alpha}_i) - \mathcal{T}_i(\hat{\alpha}_i) = \mathcal{U}_l(\hat{\alpha}_i) + G_l(\hat{\alpha}_i)(\alpha_i - \hat{\alpha}_i), \quad \forall(\hat{\alpha}_i, \alpha_i).$$

Summing incentive constraints for types α_i who could mimic $\hat{\alpha}_i$ and $\hat{\alpha}_i$ who could mimic α_i respectively, we easily find:

$$(\alpha_i - \hat{\alpha}_i)(G_l(\alpha_i) - G_l(\hat{\alpha}_i)) \geq 0.$$

The function $G_l(\alpha_i)$ is thus non-decreasing which proves Item 2.

From (A.1) and the fact that $\mathcal{X}$ is bounded above, it follows that $\mathcal{U}_l$ is Lipschitz continuous. Thus, $\mathcal{U}_l$ is also absolutely continuous and a.e. differentiable, with the integral representation:

$$\text{(A.2)} \qquad \mathcal{U}_l(\alpha_i) = \mathcal{U}_l(\underline{\alpha}_l) + \int_{\underline{\alpha}_l}^{\alpha_i} G_l(\tilde{\alpha}_i)d\tilde{\alpha}_i.$$

This latter condition can finally be rewritten as (4.1).

SUFFICIENCY. Suppose that the allocation $(\mathcal{U}_l, G_l)$ satisfies (4.1) where G_l is non-decreasing. We show that this allocation is incentive compatible, i.e., satisfies:

$$\alpha_i G_l(\hat{\alpha}_i) - \mathcal{T}_i(\hat{\alpha}_i) = \mathcal{U}_l(\hat{\alpha}_i) + G_l(\hat{\alpha}_i)(\alpha_i - \hat{\alpha}_i) \leq \mathcal{U}_l(\alpha_i) = \mathcal{U}_l(\hat{\alpha}_i) + \int_{\hat{\alpha}_i}^{\alpha_i} G_l(\tilde{\alpha}_i) d\tilde{\alpha}_i \quad \forall(\hat{\alpha}_i \neq \alpha_i),$$

Simplifying, we indeed obtain:

$$G_l(\hat{\alpha}_i)(\alpha_i - \hat{\alpha}_i) \leq \int_{\hat{\alpha}_i}^{\alpha_i} G_l(\tilde{\alpha}_i) d\tilde{\alpha}_i \quad \forall \hat{\alpha}_i \neq \alpha_i,$$

which immediately follows from the fact that G_l is non-decreasing. *Q.E.D.*

PROOF OF LEMMA 2: NECESSITY. Taking expectations of (3.10) with respect to α_l yields:

$$\text{(A.3)} \qquad \sum_{i=1}^{N_l} \mathbb{E}_{\alpha_i} \left(\alpha_i G_l(\alpha_i) - \mathcal{U}_l(\alpha_i)\right) - \mathbb{E}_{\alpha_l, \alpha_{-l}}(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) \geq 0.$$

Using (4.1) and integrating by parts, we obtain:

$$\mathbb{E}_{\alpha_i} \left(\mathcal{U}_l(\alpha_i)\right) = \mathcal{U}_l(\underline{\alpha}_l) + \mathbb{E}_{\alpha_i} \left(\frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)} G_l(\alpha_i)\right).$$

Inserting into (A.3) and simplifying yields:

$$\sum_{i=1}^{N_l} \mathbb{E}_{\alpha_i} \left(h_l(\alpha_i) G_l(\alpha_i)\right) - \mathbb{E}_{\alpha_l, \alpha_{-l}}(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) \geq N_l \mathcal{U}_l(\underline{\alpha}_l) \geq 0$$

where the last inequality follows from (3.8). Rearranging yields (4.4).

SUFFICIENCY. Consider an allocation that satisfies (4.4) and such that G_l is non-decreasing as requested by Item 2. Define now a rent profile such that :

$$\text{(A.4)} \qquad \mathcal{U}_l(\underline{\alpha}_l) = \mathbb{E}_{\alpha_i} \left(h_l(\alpha_i) G_l(\alpha_i)\right) - \frac{1}{N_l} \mathbb{E}_{\alpha_l, \alpha_{-l}}(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) \geq 0$$

and (A.1) both hold. From Lemma 1, such allocation is incentive compatible. From the fact that $\mathcal{U}_l$ so constructed is non-increasing, (A.4) ensures that (3.8) holds everywhere. Moreover, the expected payment $\mathcal{T}_i$ satisfies:

$$\text{(A.5)} \quad \mathcal{T}_i(\alpha_i) = \alpha_i G_l(\alpha_i) - \int_{\underline{\alpha}_l}^{\alpha_i} G_l(\tilde{\alpha}_i) d\tilde{\alpha}_i - \mathbb{E}_{\alpha_i} \left(h_l(\alpha_i) G_l(\alpha_i)\right) + \frac{1}{N_l} \mathbb{E}_{\alpha_l, \alpha_{-l}}(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))).$$

Taking expectations, we get:

$$\text{(A.6)} \qquad \mathbb{E}_{\alpha_i}(\mathcal{T}_i(\alpha_i)) = \frac{1}{N_l} \mathbb{E}_{\alpha_l, \alpha_{-l}}(T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))).$$

From the expression of $\mathcal{T}_i(\alpha_i)$ in (A.5), we can reconstruct payments t_i that satisfy (3.10) as:

$$\text{(A.7)} \qquad t_i(\alpha_i, \alpha_{-i}) = \mathcal{T}_i(\alpha_i) - \frac{1}{N_l - 1} \left(\sum_{j \neq i} (\mathcal{T}_j(\alpha_j) - \mathbb{E}_{\alpha_i}(\mathcal{T}_i(\alpha_i)))\right).$$

It is straightforward to check that (3.10) holds with those transfers. *Q.E.D.*

Proof of Proposition 1: From (3.6), we may define group l's payment to the decision-maker at an efficient equilibrium as:

$$T_l^*(\alpha_l,\alpha_{-l}) = \alpha_{-l}^*(\alpha_{-l})\,(u_{-l}(x^*(0,\alpha_{-l})) - u_{-l}(x^*(\alpha_l,\alpha_{-l}))) + u_0(x^*(0,\alpha_{-l})) - u_0(x^*(\alpha_l,\alpha_{-l})).$$

From (4.4), the non-negativity condition on group l's virtual net gain from forming becomes:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h_l^*(\alpha_l)\Delta u_l(\beta_l^*(\alpha_l),\beta_{-l}^*(\alpha_{-l})) - T_l^*(\alpha_l,\alpha_{-l})\right) \geq 0. \tag{A.8}$$

Developing (A.8) yields:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x^*(\alpha_l,\alpha_{-l})) + h_l^*(\alpha_l)u_i(x^*(\alpha_l,\alpha_{-l})) + \alpha_{-l}^*(\alpha_{-l})u_{-l}(x^*(\alpha_l,\alpha_{-l}))\right)$$
$$\geq \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x^*(0,\alpha_{-l})) + h_l^*(\alpha_l)u_i(x^*(0,\alpha_{-l})) + \alpha_{-l}^*(\alpha_{-l})u_{-l}(x^*(0,\alpha_{-l}))\right)$$

which can finally be expressed as (5.2).

Turning to Item 2. of Lemma 2, observe that:

$$\mathbb{E}_{\alpha_{-i},\alpha_{-l}}(\Delta u_l(\beta_l^*(\alpha_i,\alpha_{-i}),\beta_{-l}^*(\alpha_{-l}))) = (-1)^l\mathbb{E}_{\alpha_{-i},\alpha_{-l}}(x^*(\alpha_i,\alpha_{-i},\alpha_{-l}) - x^*(0,\alpha_{-l})).$$

Differentiating with respect to α_i, we get:

$$\frac{\partial}{\partial\alpha_i}(\mathbb{E}_{\alpha_{-i},\alpha_{-l}}(\Delta u_l(\beta_l^*(\alpha_i,\alpha_{-i}),\beta_{-l}^*(\alpha_{-l})))) = (-1)^l\mathbb{E}_{\alpha_{-i},\alpha_{-l}}\left(\frac{\partial x^*}{\partial\alpha_i}(\alpha_i,\alpha_{-i},\alpha_{-l})\right). \tag{A.9}$$

Differentiating (5.1) with respect to α_i, we also get:

$$\frac{\partial x^*}{\partial\alpha_i}(\alpha_i,\alpha_{-i},\alpha_{-l}) = \frac{(-1)^{l+1}}{N_l}\varphi'((-1)^l(\alpha_{-l}^*(\alpha_{-l}) - \alpha_l^*(\alpha_l))).$$

Inserting into (A.9), and taking into account that $\varphi' < 0$ finally yields that $\mathbb{E}_{\alpha_{-i},\alpha_{-l}}(\Delta u_l(\beta_l^*(\alpha_i,\alpha_{-i}),\beta_{-l}^*(\alpha_{-l})))$ is non-decreasing in α_i. *Q.E.D.*

Proof of Proposition 2: Taking limits as $N_l \to +\infty$ and using the *Strong Law of Large Numbers*, (5.1) gives us the policy chosen by the decision-maker with probability one as:

$$x^*(\alpha_l^e,\alpha_{-l}) = \arg\max_{x\in\mathcal{X}} u_0(x) + \alpha_l^e u_l(x) + \alpha_{-l}u_{-l}(x). \tag{A.10}$$

Using again the *Strong Law of Large Numbers*, Condition (5.2) certainly does not hold for N_l large enough when:

$$u_0(x^*(\alpha_l^e,\alpha_{-l})) + \underline{\alpha}_l u_l(x^*(\alpha_l^e,\alpha_{-l})) + \alpha_{-l}u_{-l}(x^*(\alpha_l^e,\alpha_{-l})) \tag{A.11}$$
$$< u_0(x^*(0,\alpha_{-l})) + \underline{\alpha}_l u_l(x^*(0,\alpha_{-l})) + \alpha_{-l}u(x^*(0,\alpha_{-l})) \quad \forall\alpha_{-l}.$$

Taking $\underline{\alpha}_l = 0$, noticing that $x^*(0,\alpha_{-l})$ satisfies

$$x^*(0,\alpha_{-l}) = \arg\max_{x\in\mathcal{X}} u_0(x) + \alpha_{-l}u_{-l}(x).$$

and observing that $x^*(\alpha_l^e,\alpha_{-l}) \neq x^*(0,\alpha_{-l})$ when $\alpha_l^e > 0$, we immediately get that (A.11) always holds. This gives the inefficiency result that we are looking for. *Q.E.D.*

Proof of Proposition 3: We show that an efficient equilibrium exists when N_l and N_{-l} are both large enough. To this end, suppose that group $-l$ has chosen an efficient appointment rule $\beta_{-l}(\alpha_{-l}) = \alpha^*_{-l}(\alpha_{-l})$. Condition (5.2) certainly holds for N_l large enough when:

$$\text{(A.12)} \quad u_0(x^*(\alpha_l^e, \alpha_{-l})) + \underline{\alpha}_l u_l(x^*(\alpha_l^e, \alpha_{-l})) + \alpha_{-l}u_{-l}(x^*(\alpha_l^e, \alpha_{-l})) > u_0(x^*(0, \alpha_{-l})) + \underline{\alpha}_l u_l(x^*(0, \alpha_{-l})) + \alpha_{-l}u_{-l}(x^*(0, \alpha_{-l})) \quad \forall \alpha_{-l}.$$

By (A.10), the following strict inequality holds when $x^*(\alpha_l^e, \alpha_{-l}) \neq x^*(0, \alpha_{-l})$:

$$u_0(x^*(\alpha_l^e, \alpha_{-l})) + \alpha_l^e u_l(x^*(\alpha_l^e, \alpha_{-l})) + \alpha_{-l}u_{-l}(x^*(\alpha_l^e, \alpha_{-l})) > u_0(x^*(0, \alpha_{-l})) + \alpha_l^e u_l(x^*(0, \alpha_{-l})) + \alpha_{-l}u_{-l}(x^*(0, \alpha_{-l})) \quad \forall \alpha_{-l}.$$

Hence, (A.12) also holds for $\alpha_l^e - \underline{\alpha}_l$ small enough. Taking expectations over α_{-l} yields (5.2). *Q.E.D.*

Proof of Proposition 5: The mechanism design problem for group l can be written as:

$$(\mathcal{GF}): \quad \max_{\mathcal{U}_l, \mathcal{G}_l} \sum_{i=1}^{N_l} \mathbb{E}_{\alpha_l}(\mathcal{U}_l(\alpha_i)) \text{ subject to (3.7), (3.8), (3.9) and (3.10).}$$

Taking into account that

$$\text{(A.13)} \quad \sum_{i=1}^{N_l} \mathbb{E}_{\alpha_l}(\mathcal{U}_l(\alpha_i)) =$$

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \alpha_l^*(\alpha_l)u_l(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l})))\right)$$
$$-\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(0, \beta_{-l}(\alpha_{-l}))) + \alpha_l^*(\alpha_l)u_l(x(0, \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(0, \beta_{-l}(\alpha_{-l})))\right)$$

and the fact that (3.8), (3.9) and (3.10) can be aggregated into a single incentive-feasibility constraint (7.4), we rewrite $(\mathcal{GF})$ as:

$$\max_{\beta_l \geq 0} \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \alpha_l^*(\alpha_l)u_l(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l})))\right)$$

subject to (7.4). Let denote by λ_l the non-negative Lagrange multiplier for (7.4). The Lagrangean $\mathcal{L}_l(\beta_l, \alpha, \lambda_l)$ satisfies (up to terms independent of $\beta_l(\alpha_l)$):

$$\frac{\mathcal{L}_l(\beta_l, \alpha, \lambda_l)}{1+\lambda_l} = u_0(x(\beta_l, \beta_{-l}(\alpha_{-l}))) + \tilde{\beta}_l(\alpha_l, \lambda_l)u_l(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))$$

where we define $\tilde{\beta}_l(\alpha_l, \lambda_l)$ as

$$\text{(A.14)} \quad \tilde{\beta}_l(\alpha_l, \lambda_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_i - \frac{\lambda_l}{1+\lambda_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}.$$

For each possible realization of (α_l, α_{-l}), the optimality condition in β_l under the constraint $\beta_l \geq 0$ writes as follows:

$$\text{(A.15)} \quad \left(u_0'(x(\beta_l^{sb}(\alpha_l), \beta_{-l}(\alpha_{-l}))) + (-1)^l(\tilde{\beta}_l(\alpha_l, \lambda_l) - \beta_{-l}(\alpha_{-l}))\right)\frac{\partial x}{\partial \beta}(\beta_l^{sb}(\alpha_l), \beta_{-l}(\alpha_{-l})) \leq 0.$$

We distinguish two cases.

1. *Case 1:* $\tilde{\beta}_l(\alpha_l, \lambda_l) \geq 0$. Observe that, by definition,

$$x(\tilde{\beta}_l(\alpha_l, \lambda_l), \beta_{-l}(\alpha_{-l})) = \varphi((-1)^l(\beta_{-l}(\alpha_{-l}) - \tilde{\beta}_l(\alpha_l, \lambda_l))).$$

   Thus, we deduce that

$$\beta_l^{sb}(\alpha_l) = \tilde{\beta}_l(\alpha_l, \lambda_l) \geq 0 \tag{A.16}$$

   satisfies condition (A.15).

2. *Case 2:* $\tilde{\beta}_l(\alpha_l, \lambda_l) < 0$. From the definition of $x(0, \beta_{-l}(\alpha_{-l}))$ through a first-order condition, we deduce that:

$$u_0'(x(0, \beta_{-l}(\alpha_{-l}))) + (-1)^l(\tilde{\beta}_l(\alpha_l, \lambda_l) - \beta_{-l}(\alpha_{-l})) = (-1)^l \tilde{\beta}_l(\alpha_l, \lambda_l) \begin{cases} < 0 & \text{if } l = 2, \\ > 0 & \text{if } l = 1. \end{cases} \tag{A.17}$$

   Observe that:

$$\frac{\partial x}{\partial \beta_1}(\beta_1, \beta_2) < 0 < \frac{\partial x}{\partial \beta_2}(\beta_1, \beta_2). \tag{A.18}$$

   Putting together (A.17) with (A.18) and using (A.15) yields:

$$\beta_l^{sb}(\alpha_l) = 0. \tag{A.19}$$

Putting together (A.16) and (A.19) finally gives us (7.5). *Q.E.D.*

Proof of Proposition 6: We rewrite Condition (7.6), or equivalently (C.12), as:

$$\mathbb{E}_{\alpha_l, \alpha_{-l}}\left(\int_0^{\alpha_l^*(\alpha_l)} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) \left(u_0'(x(\beta, \beta_{-l}(\alpha_{-l}))) + (-1)^l(\alpha_l^*(\alpha_l) - \beta_{-l}(\alpha_{-l})) \right) d\beta \right) \tag{A.20}$$
$$< (-1)^l \mathbb{E}_{\alpha_l, \alpha_{-l}} \left(\left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)} \right) \int_0^{\alpha_l^*(\alpha_l)} \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) d\beta \right).$$

Using the fact that $x(\beta_l, \beta_{-l}) = \varphi((-1)^l(\beta_{-l} - \beta_l))$, (A.20) becomes

$$(-1)^l \mathbb{E}_{\alpha_l, \alpha_{-l}} \left(\int_0^{\alpha_l^*(\alpha_l)} (\alpha_l^*(\alpha_l) - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) d\beta \right) \tag{A.21}$$
$$< (-1)^l \mathbb{E}_{\alpha_l, \alpha_{-l}} \left(\int_0^{\alpha_l^*(\alpha_l)} \left(\frac{1}{N_l} \sum_{i=1}^{N_l} \frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)} \right) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) d\beta \right).$$

Using the Law of Iterated Expectations, we can rewrite:

$$\mathbb{E}_{\alpha_l} \left(\int_0^{\alpha_l^*(\alpha_l)} (\alpha_l^*(\alpha_l) - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) d\beta \right)$$
$$= \mathbb{E}_{\alpha_l^*} \left(\mathbb{E}_{\alpha_l} \left(\int_0^{\alpha_l^*(\alpha_l)} (\alpha_l^*(\alpha_l) - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) d\beta | \alpha_l^*(\alpha_l) = \alpha_l^* \right) \right)$$
$$= \int_0^{\overline{\alpha}_l} \left(\int_0^{\alpha_l^*} (\alpha_l^* - \beta) \frac{\partial x}{\partial \beta_l}(\beta, \beta_{-l}(\alpha_{-l})) d\beta \right) \phi_{N_l}(\alpha_l^*) d\alpha_l^*.$$

Integrating by parts, this last term becomes:

$$\text{(A.22)}\quad \left(\left[-(1-\Phi_{N_l}(\alpha_l^*))\int_0^{\alpha_l^*}(\alpha_l^*-\beta)\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta\right]_0^{\overline{\alpha}_l}\right)$$

$$+\int_0^{\overline{\alpha}_l}(1-\Phi_{N_l}(\alpha_l^*))\left((\alpha_l^*-\alpha_l^*)\frac{\partial x}{\partial\beta_l}(\alpha_l^*,\beta_{-l}(\alpha_{-l}))+\int_0^{\alpha_l^*}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta\right)d\alpha_l^*$$

$$=\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\int_0^{\alpha_l^*}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta\right).$$

Using again the Law of Iterated Expectations, we also get:

$$\text{(A.23)}\quad \mathbb{E}_{\alpha_l}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)\int_0^{\alpha_l^*(\alpha_l)}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta\right)$$

$$=\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\alpha_l}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)\int_0^{\alpha_l^*(\alpha_l)}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta|\alpha_l^*(\alpha_l)=\alpha_l^*\right)\right)$$

$$=\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\alpha_l^*(\alpha_l)=\alpha_l\right)\int_0^{\alpha_l^*}\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta\right).$$

Using (A.22) and (A.23) and taking expectations over α_{-l}, Condition (A.21) becomes:

(A.24)

$$\mathbb{E}_{\alpha_l^*,\alpha_{-l}}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\alpha_l^*(\alpha_l)=\alpha_l^*\right)\right)\int_0^{\alpha_l^*}(-1)^l\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))d\beta\right)<0.$$

Observe that this r.h.s. inequality cannot be strict for $N_l=1$ since the first bracketed term is identically null. Suppose thus that $N_l>1$. Observe then that $\kappa(\alpha_l^*)=\int_0^{\alpha_l^*}(-1)^l\mathbb{E}_{\alpha_l^*}\left(\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))\right)d\beta$ is non-decreasing in α_l^* while $\frac{\zeta(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}=\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\alpha(\alpha_l^*)=\alpha_l^*\right)$ is non-increasing in α_l^* from Assumption 2. By definition, we have:

$$\int_0^{\overline{\alpha}_l}\zeta(\alpha_l^*)\kappa(\alpha_l^*)d\alpha_l^*=$$

$$\mathbb{E}_{\alpha_l^*}\left(\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}-\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\alpha(\alpha_l^*)=\alpha_l^*\right)\right)\int_0^{\alpha_l^*}(-1)^l\mathbb{E}_{\alpha_{-l}}\left(\frac{\partial x}{\partial\beta_l}(\beta,\beta_{-l}(\alpha_{-l}))\right)d\beta\right)$$

Integrating by parts, we thus get:

$$\text{(A.25)}\quad \int_0^{\overline{\alpha}_l}\zeta(\alpha_l^*)\kappa(\alpha_l^*)d\alpha_l^*=\left[\left(\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma\right)\kappa(\alpha_l^*)\right]_0^{\overline{\alpha}_l}-\int_0^{\overline{\alpha}_l}\left(\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma\right)\kappa'(\alpha_l^*)d\alpha_l^*.$$

Since $\underline{\alpha}_l=0$ from Assumption 1 and α_l^* has mean α^e, it can be checked that:

$$\mathbb{E}_{\alpha_l^*}\left(\frac{1-\Phi_{N_l}(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\right)=\alpha^e-\underline{\alpha}_l=\alpha^e.$$

Using the Law of Iterated Expectations, we also get:

$$\mathbb{E}_{\alpha_l^*}\left(\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}|\alpha_l^*(\alpha_l)=\alpha_l^*\right)\right)=\mathbb{E}_{\alpha_l}\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)$$

$$= \mathbb{E}_{\alpha_i}\left(\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right) = \alpha^e - \underline{\alpha}_l = \alpha^e.$$

Therefore, we obtain:

$$\mathbb{E}_{\alpha_l^*}\left(\frac{\zeta(\alpha_l^*)}{\phi_{N_l}(\alpha_l^*)}\right) = \int_0^{\overline{\alpha}_l} \zeta(\gamma)d\gamma = 0.$$

Inserting into (A.25), we get:

$$\text{(A.26)} \quad \int_0^{\overline{\alpha}_l} \zeta(\alpha_l^*)\kappa(\alpha_l^*)d\alpha_l^* = -\int_0^{\overline{\alpha}_l}\left(\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma\right)\kappa'(\alpha_l^*)d\alpha_l^*.$$

From Assumption 2, $\int_0^{\alpha_l^*}\zeta(\gamma)d\gamma$ is quasi-concave in α_l^* and zero at both $\alpha_l^* = 0$ and $\alpha_l^* = \overline{\alpha}_l$. Hence, it is non-negative. Since $\kappa'(\alpha_l^*) \geq 0$, the r.h.s. of (A.26) is non-positive which ends the proof. *Q.E.D.*

Proof of Proposition 7: First, remember that the feasibility condition for group l is given by (4.4). Second, we prove a preliminary Lemma.

Lemma A.1

$$\text{(A.27)} \quad \mathbb{E}_{\alpha_{-l}}(T_l(\beta_l, \beta_{-l}^{sb}(\alpha_{-l}))) > 0 \quad \forall \beta_l > 0.$$

Proof of Lemma A.1: From (3.6) and the definition of $x(\beta_l, \beta_{-l})$, we can write:

$$T_l(\beta_l, \beta_{-l}) = \int_{x(0,\beta_{-l})}^{x(\beta_l,\beta_{-l})} (u_0'(x(\beta,\beta_{-l})) - \beta_{-l}(-1)^l)\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l})d\beta$$

$$= \int_{x(0,\beta_{-l})}^{x(\beta_l,\beta_{-l})} \beta_l(-1)^{l+1}\frac{\partial x}{\partial \beta_l}(\beta,\beta_{-l})d\beta = \beta_l(-1)^l(x(\beta_l,\beta_{-l}) - x(0,\beta_{-l})).$$

Thus, $T_l(\beta_l, \beta_{-l}) > 0$ if $\beta_l > 0$. Therefore, $\min_{\beta_{-l}\in[0,\overline{\alpha}]} T_l(\beta_l, \beta_{-l}) > 0$. Condition (A.27) immediately follows by taking expectations. *Q.E.D.*

Assumption 1 implies that $\mathbb{E}_{\alpha_i}(h_l(\alpha_i)) = \underline{\alpha}_l = 0$. Introducing the subscript N_l to make explicit the dependence of the sample mean $h_{N_l}^*(\alpha_l) = \frac{1}{N_l}\sum_{i=1}^{N_l} h_l(\alpha_i)$ on N_l, and using the *Strong Law of Large Numbers*, we thus get

$$\text{(A.28)} \quad h_{N_l}^*(\alpha_l) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} 0.$$

For a given $\hat{\lambda}_l \in \mathbb{R}_+$, we now define

$$\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l) \equiv \frac{1}{N_l}\sum_{i=1}^{N_l} \alpha_i - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}\frac{1-F(\alpha_i)}{f(\alpha_i)}.$$

Since $|\Delta u_l(\hat{\beta}_l(\cdot,\cdot,\hat{\lambda}_l), \beta_{-l}^{sb})|$ is uniformly bounded in N_l on $[0,\overline{\alpha}] \times \mathbb{N}$, (A.28) implies that

$$\text{(A.29)} \quad B(\alpha_l, N_l, \hat{\lambda}_l) \equiv h_{N_l}^*(\alpha_l)\mathbb{E}_{\alpha_{-l}}\left(\Delta u_l(\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l), \beta_{-l}^{sb}(\alpha_{-l}))\right) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} 0.$$

Since convergence almost surely implies convergence in probability, Condition (A.29) also implies that, for all $\gamma > 0$, for all $\epsilon > 0$, there exists N^* such that, for all $N_l \geq N^*$:

$$\mathbb{P}\left\{|B(\alpha_l, N_l, \hat{\lambda}_l)| \geq \gamma\right\} \leq \epsilon.$$

Because $h^*_{N_l}(\alpha_l)$ is uniformly bounded in N_l, $|B(\cdot,\cdot,\hat{\lambda}_l)|$ is also bounded on $[0,\overline{\alpha}] \times \mathbb{N}$ by some constant M. Fixing such γ, ϵ, and N^*, we can write, for all $N_l \geq N^*$,

$$\begin{aligned}
&|\mathbb{E}_{\alpha_l}\left(B(\alpha_l, N_l, \hat{\lambda}_l)\right)| \leq \mathbb{E}_{\alpha_l}\left(|B(\alpha_l, N_l, \hat{\lambda}_l|)\right)\\
&= \mathbb{P}\left\{|B(\alpha_l, N_l, \hat{\lambda}_l)| \geq \gamma\right\}\mathbb{E}_{\alpha_l}\left[|B(\alpha_l, N_l, \hat{\lambda}_l)|\,|\,|B(\alpha_l, N_l, \hat{\lambda}_l)| \geq \gamma\right]\\
&+\mathbb{P}\left\{|B(\alpha_l, N_l, \hat{\lambda}_l)| < \gamma\right\}\mathbb{E}_{\alpha_l}\left[|B(\alpha_l, N_l, \hat{\lambda}_l)|\,|\,|B(\alpha_l, N_l, \hat{\lambda}_l)| < \gamma\right]\\
&\leq \epsilon M + \gamma
\end{aligned}$$

Choosing $\gamma = \epsilon = \epsilon'/(M+1)$ shows that for any $\epsilon' > 0$, there exists N^{**} such that, for all $N_l \geq N^{**}$,

$$|\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h^*_{N_l}(\alpha_l)\Delta u_l(\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l), \beta^{sb}_{-l}(\alpha_{-l}))\right)| \leq \epsilon' \tag{A.30}$$

which proves

$$\lim_{N_l\to\infty} \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(h^*_{N_l}(\alpha_l)\Delta u_l(\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l), \beta^{sb}_{-l}(\alpha_{-l}))\right) = 0. \tag{A.31}$$

In addition, we have

$$\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l) \underset{N_l\to+\infty}{\overset{a.s.}{\to}} \alpha^e_l - \frac{\hat{\lambda}_l}{1+\hat{\lambda}_l}(\alpha^e_l - \underline{\alpha}_l) = \frac{1}{1+\hat{\lambda}_l}\alpha^e_l > 0 \tag{A.32}$$

where the last equality follows from Assumption 1. Therefore, for all $\hat{\lambda}_l < +\infty$, (A.27) implies:

$$\lim_{N_l\to\infty} \mathbb{E}_{\alpha_l,\alpha_{-l}}\left[T_l(\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l), \beta^{sb}_{-l}(\alpha_{-l}))\right] > 0. \tag{A.33}$$

It follows from (A.31) and (A.33) that:

$$\lim_{N_l\to\infty} \mathbb{E}_{\alpha_l,\alpha_{-l}}\left[h^*_{N_l}(\alpha_l)\Delta u_l(\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l), \beta^{sb}_{-l}(\alpha_{-l})) - T_l(\hat{\beta}_l(\alpha_l, N_l, \hat{\lambda}_l), \beta^{sb}_{-l}(\alpha_{-l}))\right] < 0.$$

Therefore, for any $\hat{\lambda}_l > 0$, there exists N^e such that, for $N_l \geq N^e$, $\lambda_l(N_l) \geq \hat{\lambda}_l$. Hence, (7.8) holds.

Condition (A.32) also implies convergence in probability and thus

$$\beta^{sb}_l(\alpha_l, N_l) = \frac{1}{N_l}\left(\sum_{i=1}^{N_l}\alpha_i - \frac{\lambda_l(N_l)}{1+\lambda_l(N_l)}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right) \underset{N_l\to+\infty}{\overset{p}{\to}} \mathbb{E}_{\alpha_i}\left(h_l(\alpha_i)\right) = \underline{\alpha}_l = 0$$

where again the last equality follows from Assumption 1. Hence, (7.9) holds. *Q.E.D.*

Proof of Proposition 9: Assume that $u'''_0 \geq 0$. From Lemma 3, the mapping Λ^*_l is everywhere non-decreasing. Therefore, since $0 < \lambda_{-l}$, we get:

$$\tilde{\lambda}_l = \Lambda^*_l(0) \leq \lambda_l.$$

Finally, the reverse condition holds if $u'''_0 \leq 0$ which ends the proof. *Q.E.D.*

Proof of Proposition 11: Let consider an equilibrium of the game obtained when the common size of the groups is N. It corresponds to the appointment rules $(\beta_l^{sb}(\cdot,N),\beta_{-l}^{sb}(\cdot,N))$ where we now make the dependence on N explicit. It is easy to check that the limiting behaviors described in Proposition 7 still apply when both groups have variable sizes:

$$\lim_{N\to\infty} \lambda_l(N)=+\infty \text{ and } \beta_l^{sb}(\alpha_l,N) \underset{N\to+\infty}{\overset{p}{\to}} = 0.$$

In addition, the function $x(\beta_l^{sb}(\alpha_l,N),\beta_{-l}^{sb}(\alpha_{-l},N))$ is bounded because $\mathcal{X}$ itself is. Reminding that the ideal point of the decision-maker is 0, we can deduce (with the method we used to establish (A.30)) that

$$\lim_{N\to\infty} \mathbb{E}_{\alpha_l,\alpha_{-l}}(\mathcal{U}_l^{sb}(\alpha_l,\alpha_{-l},N))=0 \tag{A.34}$$

where group l's payoff writes as

$$\mathcal{U}_l^{sb}(\alpha_l,\alpha_{-l},N)=\alpha_N^*(\alpha_l)u_l(x(\beta_l^{sb}(\alpha_l,N),\beta_{-l}^{sb}(\alpha_{-l},N)))-T_l(\beta_l^{sb}(\alpha_l,N),\beta_{-l}^{sb}(\alpha_{-l},N))$$

and $\alpha_N^*(\alpha_l)=\frac{1}{N}\sum_{i=1}^N\alpha_i$.

Consider now the case when both groups form under complete information. When the profile of preferences is (α_l,α_{-l}), group l's payoff writes as:

$$\mathcal{U}_l^{fb}(\alpha_l,\alpha_{-l},N)=W(\alpha_N^*(\alpha_l),\alpha_N^*(\alpha_{-l}))-W(0,\alpha_N^*(\alpha_{-l})).$$

Taking expectations yields:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\mathcal{U}_l^{fb}(\alpha_l,\alpha_{-l})\right)=\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(W(\alpha_N^*(\alpha_l),\alpha_N^*(\alpha_{-l}))-W(0,\alpha_N^*(\alpha_{-l}))\right).$$

Taking into account that the decision-maker's objective function is quadratic, we obtain:

$$W(\alpha_N^*(\alpha_l),\alpha_N^*(\alpha_{-l}))-W(0,\alpha_N^*(\alpha_{-l}))=\frac{1}{2\beta_0}\left(\alpha_N^{*2}(\alpha_l)-2\alpha_N^*(\alpha_l)\alpha_N^*(\alpha_{-l})\right).$$

That all α_i are independently distributed on $[0,\Delta\alpha]$ within and across groups implies that $\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\alpha_N^*(\alpha_l)\alpha_N^*(\alpha_{-l})\right)=(\alpha^e)^2$ and $\lim_{N\to+\infty}\mathbb{E}_{\alpha_l}\left((\alpha_N^*(\alpha_l))^2\right)=(\alpha^e)^2$. It follows that:

$$\lim_{N\to\infty} \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\mathcal{U}_l^{fb}(\alpha_l,\alpha_{-l})\right)=-\frac{(\alpha^e)^2}{2\beta_0}<0 \tag{A.35}$$

The result directly follows by comparing the r.h.s.s of (A.34) and (A.35). *Q.E.D.*

Proof of Proposition 12: We show that the decision-maker is always worse off under incomplete information in the *ex post* sense. The *ex ante* result will directly follow by taking expectations. Let denote by T_0 the total contribution received by the decision-maker. From our earlier findings, we get:

$$T_0(\beta_l,\beta_{-l})=\beta_{-l}(u_{-l}(x(0,\beta_{-l}))-u_{-l}(x(\beta_l,\beta_{-l})))+u_0(x(0,\beta_{-l}))-u_0(x(\beta_l,\beta_{-l}))$$

$$+\beta_l(u_l(x(\beta_l,0))-u_l(x(\beta_l,\beta_{-l})))+u_0(x(\beta_l,0))-u_0(x(\beta_l,\beta_{-l}))\quad\forall(\beta_l,\beta_{-l}).$$

For any such configuration (β_l,β_{-l}), the decision-maker's payoff can thus be written as:

$$\mathcal{U}_0(\beta_l,\beta_{-l})=u_0(x(\beta_l,\beta_{-l}))+T_0(\beta_l,\beta_{-l}).$$

Differentiating with respect to β_l, we get:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) = u_l(x(\beta_l, 0)) - u_l(x(\beta_l, \beta_{-l})) + \frac{\partial x}{\partial \beta_l}(\beta_l, 0)\left(u_0'(x(\beta_l, 0)) + \beta_l u_l'(x(\beta_l, 0))\right)$$

$$-\frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l})\left(u_0'(x(\beta_l, \beta_{-l})) + \beta_l u_l'(x(\beta_l, \beta_{-l})) + \beta_{-l} u_{-l}'(x(\beta_l, \beta_{-l}))\right).$$

Using the definitions of $x(\beta_l, \beta_{-l})$ and $x(0, \beta_{-l})$, the latter expression becomes:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) = u_l(x(\beta_l, 0)) - u_l(x(\beta_l, \beta_{-l})) = (-1)^l(x(\beta_l, 0) - x(\beta_l, \beta_{-l})).$$

It follows that:

$$\frac{\partial \mathcal{U}_0}{\partial \beta_l}(\beta_l, \beta_{-l}) > 0 \quad \forall \beta_l > 0.$$

Therefore, for all $\beta_l < \alpha_l^*$, $\beta_{-l} < \alpha_{-l}^*$, the following string of inequalities holds:

$$\mathcal{U}_0(\beta_l, \beta_{-l}) < \mathcal{U}_0(\alpha_l^*, \beta_{-l}) < \mathcal{U}_0(\alpha_l^*, \alpha_{-l}^*).$$

In particular, we may take $\beta_l = \beta_l^{sb}(\alpha_l)$ and $\beta_{-l} = \beta_{-l}^{sb}(\alpha_{-l})$. Taking expectations, and taking into account that those inequalities are strict on a set of positive measure, we finally obtain:

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}(\mathcal{U}_0(\beta_l^{sb}(\alpha_{-l}), \beta_{-l}^{sb}(\alpha_{-l}))) < \mathbb{E}_{\alpha_l,\alpha_{-l}}(\mathcal{U}_0(\alpha_l^*, \beta_{-l}^{sb}(\alpha_{-l}))) < \mathbb{E}_{\alpha_l,\alpha_{-l}}(\mathcal{U}_0(\alpha_l^*, \alpha_{-l}^*))$$

which ends the proof. *Q.E.D.*

APPENDIX B: CONGRUENT GROUPS (SUPPLEMENTARY MATERIAL)

Considered now the case where groups have *congruent preferences*. To mirror our previous analysis, we suppose that $u_1(x) = u_2(x) = x$ for all $x \in \mathcal{X}$. We first need to come back on the specification of payoffs in the common agency stage of the game. With congruent interest groups, the cooperative game constructed by Laussel and Le Breton (2001) turns out to be *super-additive*. Indeed, for any profile of preferences (β_1, β_2) for the lobbyists, the following property holds:

$$W(\beta_1, \beta_2) + W(0, 0) > W(\beta_1, 0) + W(0, \beta_2) \quad \forall (\beta_1, \beta_2) \in \mathbb{R}_+^2$$

where again $W(0, 0) = 0$. Laussel and Le Breton (2001) demonstrated that the associated common agency game has the so-called *no rent property*, i.e., in all truthful continuation equilibria, the surplus of the decision-maker is always fully extracted by lobbyists. The lobbyists' payoffs lie in an interval with non-empty interior which is fully determined by the following constraints:

$$\text{(B.1)} \qquad V_l(\beta_l, \beta_{-l}) \leq W(\beta_1, \beta_2) - W(0, \beta_{-l}) \quad \forall l \in \{1, 2\},$$

$$\text{(B.2)} \qquad V_1(\beta_1, \beta_2) + V_2(\beta_2, \beta_1) = W(\beta_1, \beta_2).$$

The choice of the optimal appointment rules, either under complete or asymmetric information, of course depends on how the lobbyists' payoffs are precisely determined. To highlight new phenomena that might arise with congruent groups, we shall assume that those payoffs are given by the lobbyists' *Shapley Values* since this allocation satisfies both (B.1) and (B.2), namely:

$$\text{(B.3)} \qquad V_l(\beta_l, \beta_{-l}) = \frac{1}{2}\left(W(\beta_1, \beta_2) - W(0, \beta_{-l}) + W(\beta_l, 0)\right).$$

Using (3.3), we retrieve the expression of the equilibrium payment made by lobbyist l:

$$T_l(\beta_l, \beta_{-l}) = \beta_l x(\beta_l, \beta_{-l}) - \frac{1}{2}\left(W(\beta_1, \beta_2) - W(0, \beta_{-l}) + W(\beta_l, 0)\right). \tag{B.4}$$

These payments are not VCG. It is no longer a sincere (i.e., dominant) strategy for each group to pass to its lobbyist the aggregate preferences of the group. Each group manipulates these preferences even under complete information. There is now inter-group free riding.[32]

Running Example (Continued). To illustrate, consider the case of quadratic preferences. It is immediate to derive the policy chosen by the decision-maker at the last stage of the game as $x(\beta_1, \beta_2) = \frac{\beta_1+\beta_2}{\beta_0}$ while coalitional payoffs are given by $W(\beta_1, \beta_2) = \frac{(\beta_1+\beta_2)^2}{2\beta_0}$. Under complete information, each group l endows its lobbyist with an objective β_l that maximizes the net surplus of the group, taking as given its conjectures on similar choices made by group $-l$ and taking into account that preferences in that competing group remain unknown. Because mechanisms for group formation are secret, members of group $-l$ conjecture the formation of group l even if it may be vetoed off equilibrium. Following veto, the policy chosen remains $x(0, \beta^*_{-l}(\alpha_{-l}))$ where $\beta^*_{-l}(\alpha_{-l})$ represents the preferences given to lobbyist $-l$. The net utility of an individual with type α_i who belongs to group l and knows the preferences α_{-i} of other members is thus:

$$\frac{\alpha_i}{N_l}\mathbb{E}_{\alpha_{-l}}(x(\beta_l, \beta^*_{-l}(\alpha_{-l})) - x(0, \beta^*_{-l}(\alpha_{-l}))) - t_i(\alpha_i, \alpha_{-i}) \quad \forall(\alpha_i, \alpha_{-i}) \in \Omega_l^{N_l}. \tag{B.5}$$

Aggregating those expressions over the whole group l, β_l should be chosen to maximize over β_l:

$$\mathbb{E}_{\alpha_{-l}}\left(\alpha^*_l(\alpha_l)(x(\beta_l, \beta^*_{-l}(\alpha_{-l})) - x(0, \beta^*_{-l}(\alpha_{-l}))) - T_l(\beta_l, \beta^*_{-l}(\alpha_{-l}))\right).$$

Using (B.4) yields the expression of group l's overall contribution:

$$T_l(\beta_l, \beta_{-l}(\alpha_{-l})) = \beta_l x(\beta_l, \beta_{-l}(\alpha_{-l})) - \frac{1}{2}\left(\frac{(\beta_l + \beta_{-l}(\alpha_{-l}))^2}{2\beta_0} - \frac{\beta^2_{-l}(\alpha_{-l})}{2\beta_0} + \frac{\beta_l^2}{2\beta_0}\right).$$

The (necessary and sufficient) first-order condition that characterizes the equilibrium choices of lobbyists' preferences yields:

$$\beta^*_l(\alpha_l) = \alpha^*_l(\alpha_l) - \frac{1}{2}\mathbb{E}_{\alpha_{-l}}(\beta^*_{-l}(\alpha_{-l})).$$

For a symmetric equilibrium, we thus obtain:

$$\beta^*_l(\alpha_l) = \alpha^*_l(\alpha_l) - \frac{\alpha^e}{3} \tag{B.6}$$

where we assume $\underline{\alpha} \geq \frac{\alpha^e}{3}$ to ensure that $\beta^*_l(\alpha_l)$ is non-negative so as to avoid a corner solution.

From (B.6), each group chooses a lobbyist with moderate preferences. There is now inter-group free riding. The policy that ends up being chosen by the decision-maker is obviously lower than the first-best policy that would have been chosen had groups merged into a single entity so as to perfectly pass their preferences on the decision-maker:

$$x(\beta^*_l(\alpha_l), \beta^*_{-l}(\alpha_{-l})) = x^*(\alpha_l, \alpha_{-l}) - \frac{2\alpha^e}{3\beta_0} < x^*(\alpha_l, \alpha_{-l}).$$

[32] Even if we were to choose within the range of allocations defined by (B.1) and (B.2) another allocation than that defined with Shapley values, manipulations would still arise. It is indeed well-known that, in those contexts, there is no sincere (i.e., dominant strategy) mechanism that implements the first-best allocation and extracts all surplus from the decision-maker. Furosawa and Konishi (2011) find a similar result in a more specific game.

■

Under asymmetric information within group l, the net utility of an individual with type α_i when the rule $\beta_l^*(\alpha_l)$ is still adopted within group l is thus:

$$\text{(B.7)}\quad \mathcal{U}_l(\alpha_i) = \mathbb{E}_{\alpha_{-i}}\left(\frac{\alpha_i}{N_l}\mathbb{E}_{\alpha_{-l}}(x(\beta_l^*(\alpha_l),\beta_{-l}^*(\alpha_{-l})) - x(0,\beta_{-l}^*(\alpha_{-l}))) - t_i(\alpha_i,\alpha_{-i})\right) \quad \forall \alpha_i \in \Omega_l.$$

We do not expect that an efficient equilibrium exists under asymmetric information. Indeed, free riding already bites across groups even under complete information. Yet, we might still be interested in determining conditions such that intra-group free riding does not add inefficiencies on top of those already brought by inter-group free riding. Proceeding as in the case of conflicting interests (Condition (5.2)), we may obtain a condition ensuring that the decision $x(\beta_l^*(\alpha_l),\beta_{-l}^*(\alpha_{-l}))$ remains implementable even under asymmetric information as:

$$\text{(B.8)}\quad \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\alpha_l^*(\alpha_l)(x(\beta_l^*(\alpha_l),\beta_{-l}^*(\alpha_{-l})) - x(0,\beta_{-l}^*(\alpha_{-l}))) - T(\beta_l^*(\alpha_l),\beta_{-l}^*(\alpha_{-l}))\right)$$
$$\geq \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right)(x(\beta_l^*(\alpha_l),\beta_{-l}^*(\alpha_{-l})) - x(0,\beta_{-l}^*(\alpha_{-l})))\right).$$

The r.h.s. above is by now familiar. It represents the expected information rent left to all members of group l. The l.h.s. is the expected net gain from group formation given the continuation equilibrium and payments.

Had groups cooperated when dealing with the decision-maker, inter-group free riding would disappear. The efficient decision $x^*(\alpha_l,\alpha_{-l}) = \frac{1}{\beta_0}(\alpha_l^*(\alpha_l) + \alpha_{-l}^*(\alpha_{-l}))$ would be implemented while the decision-maker would choose his ideal point, namely 0, when the merged group does not organize. The incentive-feasibility condition would now become:

$$\text{(B.9)}\quad \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x^*(\alpha_l,\alpha_{-l})) + (\alpha_l^*(\alpha_l) + \alpha_{-l}^*(\alpha_{-l}))x^*(\alpha_l,\alpha_{-l}))\right)$$
$$\geq \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\left(\frac{1}{N_l}\left(\sum_{i=1}^{N_l}\frac{1-F_l(\alpha_i)}{f_l(\alpha_i)}\right) + \frac{1}{N_{-l}}\left(\sum_{j=1}^{N_{-l}}\frac{1-F_{-l}(\alpha_j)}{f_{-l}(\alpha_j)}\right)\right)x^*(\alpha_l,\alpha_{-l}))\right) \quad \forall l \in \{1,2\}.$$

The comparison of the incentive-feasibility conditions (B.8) and (B.9) may already highlight two important driving forces. On the one hand, the net gains of forming is certainly greater in the case of a merger of the two groups since the *status quo* entails no production at all while its formation induces an efficient policy. Instead, with two groups, each of them can free ride and benefit from the policy induced by the sole contribution of its rival. Overall the gains from forming for those two groups are certainly lower. On the other hand, with a merger, the *status quo* if that merged group does not form is the null policy that is chosen by the decision-maker on her own. This means that, with groups merging, the overall information rents that must be distributed are also quite large.

Running Example (Continued). We assume that, for both groups, types are symmetrically and uniformly distributed on the same interval $\Theta = [\underline{\alpha},\overline{\alpha}]$ with mean $\alpha^e = \frac{\underline{\alpha}+\overline{\alpha}}{2}$ and variance $\frac{\Delta\alpha^2}{12}$ (where also $\Delta\alpha = \overline{\alpha} - \underline{\alpha}$ and where $\alpha^e > \frac{3}{4}\Delta\alpha$ to avoid corner solutions). Groups have also the same size $N = N_1 = N_2$. Condition (B.8) then amounts to:

$$\frac{7}{3}(\alpha^e)^2 + \frac{9}{4}\frac{(\Delta\alpha)^2}{12N} \geq \overline{\alpha}\alpha^e.$$

Instead, Condition (B.9) writes as:

$$3(\alpha^e)^2 + \frac{3}{2}\frac{(\Delta\alpha)^2}{12N} \geq \overline{\alpha}\alpha^e.$$

It can be checked that, when N is large, this second condition is easier to achieve. Inter-group free riding reduces the overall surplus and makes it more difficult to implement for each delegate the same objectives (B.6) as under complete information (even though this objective is distorted by inter-group free-riding). Merging congruent groups helps solving the collective action problem.■

APPENDIX C: PROOFS OF OTHER RESULTS (SUPPLEMENTARY MATERIAL)

PROOF OF PROPOSITION 4: Under complete information, group l's gains from forming is:

$$\sum_{i=1}^{N_l} \mathbb{E}_{\alpha_l}(\mathcal{U}_l(\alpha_i)) = \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\alpha_l^*(\alpha_l)\Delta u_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l})) - T_l(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))\right).$$

Using (7.3), this expression can be simplified as (A.13). Optimizing the latter expression pointwise (i.e., for all realizations of (α_l, α_{-l})) with respect to $\beta_l(\alpha_l)$ and taking into account that $x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))$ which is by definition equal to $\varphi((-1)^l(\beta_{-l} - \beta_l))$ also solves (3.4) yields the following first-order condition:

$$\text{(C.1)} \qquad \left(u_0'(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + (-1)^l(\alpha_l^*(\alpha_l) - \beta_{-l}(\alpha_{-l}))\right)\frac{\partial x}{\partial \beta_l}(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l})) = 0.$$

Inserting (A.18) into (C.1) then gives us:

$$u_0'(x(\beta_l(\alpha_l), \beta_{-l}(\alpha_{-l}))) + (-1)^l(\alpha_l^*(\alpha_l) - \beta_{-l}(\alpha_{-l})) = 0.$$

Using that $x(\beta_l, \beta_{-l}) = \varphi((-1)^l(\beta_{-l} - \beta_l))$ then yields:

$$\text{(C.2)} \qquad \beta_l(\alpha_l) = \alpha_l^*(\alpha_l).$$

Finally, denoting the l.h.s. of (C.1) as function of the optimizing variable β_l as $\psi_l(\beta_l, \beta_{-l}(\alpha_{-l}))$, we have:

$$\text{(C.3)} \qquad \frac{\psi_l(\beta_l, \beta_{-l}(\alpha_{-l}))}{(-1)^l\frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l}(\alpha_{-l}))} = \alpha_l^*(\alpha_l) - \beta_l.$$

It follows from this simple condition that the objective function of group l is quasi-concave in β_l so that (C.2) is both necessary and sufficient for optimality. *Q.E.D.*

PROOF OF COROLLARY 1: We start by proving that the Lagrange multiplier is positive when $\underline{\alpha}_l = 0$. To this end, let define for any non-negative value λ, $\beta_l(\alpha_l, \lambda)$ as:

$$\text{(C.4)} \qquad \beta_l(\alpha_l, \lambda) = \max\left\{0, \tilde{\beta}_l(\alpha_l, \lambda)\right\}.$$

Observe that $\beta_l(\alpha_l, 0) = \alpha_l^*$ and $\beta_l(\alpha_l, \lambda_l) = \beta^{sb}(\alpha_l)$. Let also define the virtual surplus for group l from forming as:

$$\text{(C.5)} \qquad S_l(\lambda, \beta_{-l}) =$$

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(\beta_l(\alpha_l, \lambda), \beta_{-l}(\alpha_{-l}))) + h_l^*(\alpha_l)u_l(x(\beta_l(\alpha_l, \lambda), \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(\beta_l(\alpha_l, \lambda), \beta_{-l}(\alpha_{-l})))\right)$$

$$-\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(0,\beta_{-l}(\alpha_{-l}))) + h_l^*(\alpha_l)u_l(x(0,\beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(0,\beta_{-l}(\alpha_{-l})))\right).$$

Using now the definition (C.14), we can rewrite

$$\tilde{S}_l(\lambda,\lambda_{-l}) = \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\Delta W_l(h_l^*(\alpha_l), x(\beta_l(\alpha_l,\lambda),\beta_{-l}(\alpha_{-l},\lambda_{-l})))\right)$$

and thus

$$\frac{\partial \tilde{S}_l}{\partial \lambda_{-l}}(\lambda,\lambda_{-l}) = \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\frac{\partial x}{\partial \beta_{-l}}(\beta_l(\alpha_l,\lambda),\beta_{-l}(\alpha_{-l},\lambda_{-l}))\frac{\partial \beta_{-l}}{\partial \lambda_{-l}}(\alpha_{-l},\lambda_{-l})\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\beta_l(\alpha_l,\lambda),\beta_{-l}(\alpha_{-l},\lambda_{-l}))\right).$$

We now prove three lemmas that helps us to get our result.

Lemma C.1

$$\frac{\partial S_l}{\partial \lambda}(\lambda,\beta_{-l}) > 0. \tag{C.6}$$

Proof of Lemma C.1: Differentiating with respect to λ, we find:

$$\frac{\partial S_l}{\partial \lambda}(\lambda,\beta_{-l}) = (-1)^l\mathbb{E}_{\alpha_l,\alpha_{-l}}\left((h_l^*(\alpha_l) - \beta_l(\alpha_l,\lambda))\frac{\partial x}{\partial \beta_l}(\beta_l(\alpha_l,\lambda),\beta_{-l}(\alpha_{-l}))\frac{\partial \beta_l}{\partial \lambda}(\alpha_l,\lambda)\right). \tag{C.7}$$

From (C.4), we get:

$$\beta_l(\alpha_l,\lambda) \geq h_l^*(\alpha_l) \tag{C.8}$$

with a strict inequality for $\lambda > 0$. From (C.4), we also get that:

$$\frac{\partial \beta_l}{\partial \lambda}(\alpha_l,\lambda) \leq 0. \tag{C.9}$$

Gathering (A.18), (C.8) and (C.9) and inserting into (C.7), we obtain (C.6). *Q.E.D.*

Lemma C.2

$$\lim_{\lambda\to+\infty} S_l(\lambda,\beta_{-l}) > 0. \tag{C.10}$$

Proof of Lemma C.2: Define now:

$$\beta_l^\infty(\alpha_l) = \max\{0, h_l^*(\alpha_l)\}.$$

We can compute:

$$\lim_{\lambda\to+\infty} S_l(\lambda,\beta_{-l}) =$$

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\Big(u_0(x(\beta_l^\infty(\alpha_l),\beta_{-l}(\alpha_{-l})))+\beta_l^\infty(\alpha_l)u_l(x(\beta_l^\infty(\alpha_l),\beta_{-l}(\alpha_{-l})))+\beta_{-l}(\alpha_{-l})u_{-l}(x(\beta_l^\infty(\alpha_l),\beta_{-l}(\alpha_{-l})))\Big)$$

$$-\mathbb{E}_{\alpha_l,\alpha_{-l}}\Big(u_0(x(0,\beta_{-l}(\alpha_{-l}))) + \beta_l^\infty(\alpha_l)u_l(x(0,\beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l})u_{-l}(x(0,\beta_{-l}(\alpha_{-l})))\Big).$$

When $\beta_l^\infty(\alpha_l) = 0$, the following equality holds for all β_{-l}:

$$u_0(x(\beta_l^\infty(\alpha_l),\beta_{-l})) + \beta_l^\infty(\alpha_l)u_l(x(\beta_l^\infty(\alpha_l),\beta_{-l})) + \beta_{-l}u_{-l}(x(\beta_l^\infty(\alpha_l),\beta_{-l}))$$

$$= u_0(x(0, \beta_{-l})) + \beta_l^\infty(\alpha_l) u_l(x(0, \beta_{-l})) + \beta_{-l} u_{-l}(x(0, \beta_{-l})).$$

Instead, when $\beta_l^\infty(\alpha_l) > 0$, for all β_{-l}, $x(\beta_l^\infty(\alpha_l), \beta_{-l})$ is the unique maximizer of

$$u_0(x) + \beta_l^\infty(\alpha_l) u_l(x) + \beta_{-l} u_{-l}(x).$$

From which it follows that:

$$u_0(x(\beta_l^\infty(\alpha_l), \beta_{-l})) + \beta_l^\infty(\alpha_l) u_l(x(\beta_l^\infty(\alpha_l), \beta_{-l})) + \beta_{-l} u_{-l}(x(\beta_l^\infty(\alpha_l), \beta_{-l}))$$

$$> u_0(x(0, \beta_{-l})) + \beta_l^\infty(\alpha_l) u_l(x(0, \beta_{-l})) + \beta_{-l} u_{-l}(x(0, \beta_{-l})).$$

Since $\mathbb{P}\{\beta_l^\infty(\alpha_l) > 0\} > 0$, it follows that (C.10) holds. *Q.E.D.*

LEMMA C.3 *When Condition (7.6) holds, we have:*

(C.11) $S_l(0, \beta_{-l}) < 0.$

PROOF OF LEMMA C.3: Taking into account that $\beta_l(\alpha_l, 0) = \alpha_l^*(\alpha_l) > 0$, (C.11) amounts to:

(C.12)

$$\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(\alpha_l^*(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \alpha_l^*(\alpha_l) u_l(x(\alpha_l^*(\alpha_l), \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l}) u_{-l}(x(\alpha_l^*(\alpha_l), \beta_{-l}(\alpha_{-l})))\right)$$

$$-\mathbb{E}_{\alpha_l,\alpha_{-l}}\left(u_0(x(0, \beta_{-l}(\alpha_{-l}))) + \alpha_l^*(\alpha_l) u_l(x(0, \beta_{-l}(\alpha_{-l}))) + \beta_{-l}(\alpha_{-l}) u_{-l}(x(0, \beta_{-l}(\alpha_{-l})))\right)$$

$$< \mathbb{E}_{\alpha_l,\alpha_{-l}}\left(\left(\frac{1}{N_l}\sum_{i=1}^{N_l}\frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)}\right)(u_i(x(\alpha_l^*(\alpha_l), \beta_{-l}(\alpha_{-l}))) - u_i(x(0, \beta_{-l}(\alpha_{-l}))))\right).$$

This can be rewritten as Condition (7.6) which ends the proof. *Q.E.D.*

Putting together Lemmas C.1, C.2 and C.3, there exists a unique solution $\lambda_l > 0$ to:

$$S_l(\lambda_l, \beta_{-l}) = 0.$$

Finally, Item 1. follows from the text. Item 2. follows from observing that, $\underline{\alpha}_l = 0$ also implies:

$$\mathbb{P}\{\beta^{sb}(\alpha_l) = 0\} = \mathbb{P}\left\{\frac{1}{N_l}\sum_{i=1}^{N_l}\alpha_i - \frac{\lambda_l}{1 + \lambda_l}\frac{1 - F_l(\alpha_i)}{f_l(\alpha_i)} < 0\right\} > 0.$$

Q.E.D.

PROOF OF COROLLARY 2: It follows from (7.5) and $\lambda_l \geq 0$ that

$$\beta_l^{sb}(\alpha_l) \geq h_l^*(\alpha_l) > h(\underline{\alpha}_l) > 0$$

which ends the proof. *Q.E.D.*

Proof of Proposition 8: Let us define $\tilde{S}_l$ as:

$$\tilde{S}_l(\lambda_l, \lambda_{-l}) = S_l(\lambda_l, \beta_{-l}(\cdot, \lambda_{-l})).$$

Define accordingly the mappings $\Lambda_l^*(\lambda_{-l})$ such that:

$$\tilde{S}_l(\Lambda_l^*(\lambda_{-l}), \lambda_{-l}) = 0.$$

From Corollary 1, these mappings are well-defined. Observe also that $\tilde{S}_l(\lambda_l, \lambda_{-l})$ is continuously differentiable in each variable, with $\frac{\partial \tilde{S}_l}{\partial \lambda_l} > 0$ (from Lemma C.1), $\tilde{S}_l(+\infty, \lambda_{-l}) > 0$ (from Lemma C.2) and $\tilde{S}_l(0, \lambda_{-l}) < 0$ (from Lemma C.3). It follows that the mappings Λ_l^* for $l \in \{1, 2\}$ are single-valued and continuous on $[0, +\infty)$. Consider any converging sequence λ_{-l}^n, and denote $\overline{\lambda}_{-l} = \lim_{n \to +\infty} \lambda_{-l}^n$. We want to show that

$$\lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^n) = \Lambda_l^*(\overline{\lambda}_{-l}). \tag{C.13}$$

Observe first that $\Lambda_l^*(\lambda_{-l}^n)$ is bounded. Indeed, if it was not the case, there would exist a subsequence $\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$, where φ is an increasing function from $\mathbb{N}$ into $\mathbb{N}$, such that $\lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^{\varphi(n)}) = +\infty$. From the fact that $\tilde{S}_l(\Lambda_l^*(\lambda_{-l}^{\varphi(n)}), \lambda_{-l}^{\varphi(n)}) = 0$ for all n, it would follow that $\tilde{S}_l(+\infty, \overline{\lambda}_{-l}) = 0$. This is a contradiction since $\tilde{S}_l(+\infty, \overline{\lambda}_{-l}) > 0$.

Second, consider any converging subsequence $\Lambda_l^*(\lambda_{-l}^{\varphi(n)})$ (the Bolzano-Weirstrass theorem guarantees existence of such a subsequence), and define $\lambda_l^s = \lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^{\varphi(n)})$. It is again the case that $\tilde{S}_l(\lambda_l^s, \overline{\lambda}_{-l}) = 0$. This implies that $\lambda_l^s = \Lambda_l^*(\overline{\lambda}_{-l})$.

We thus have shown that $\Lambda_l^*(\lambda_{-l}^n)$ is a bounded sequence, such that all converging subsequences have the same limit $\Lambda_l^*(\overline{\lambda}_{-l})$. Therefore, $\lim_{n \to +\infty} \Lambda_l^*(\lambda_{-l}^n)$ exists and thus (C.13) holds.

Since $\beta_{-l}(\alpha_{-l}, \lambda_{-l}) = \frac{1}{N_{-l}} \sum_{i=1}^{N_{-l}} \alpha_i - \frac{\lambda_{-l}}{1+\lambda_{-l}} \frac{1-F_{-l}(\alpha_i)}{f(\alpha_i)}$, and $\lim_{\lambda_{-l} \to +\infty} \frac{\lambda_{-l}}{1+\lambda_{-l}} = 1$, $\lim_{\lambda_{-l} \to +\infty} \Lambda_l^*(\lambda_{-l})$ exists and takes a finite value. It follows that Λ_l^* is bounded over $[0, +\infty)$. There exists $A_l > 0$ such that for all $\lambda_{-l} \geq 0$, $\Lambda_l^*(\lambda_{-l}) \leq A_l$.

We now define the function ζ as:

$$\begin{aligned} \zeta: \quad [0, A_l] \times [0, A_{-l}] &\to [0, A_l] \times [0, A_{-l}] \\ (\lambda_l, \lambda_{-l}) &\mapsto (\Lambda_l^*(\lambda_{-l}), \Lambda_{-l}^*(\lambda_l)). \end{aligned}$$

The function ζ is continuous on a compact set and onto. From Brouwer's Theorem, it has a fixed point which gives us a dual representation of the equilibrium of the game. *Q.E.D.*

Proof of Lemma 3: Let define the *incremental virtual surplus* for group l when its virtual (aggregate) preference parameter is $\tilde{\beta}_l$ and the decision $x(\beta_l, \beta_{-l})$ as:

$$\begin{aligned} \Delta W_l(\tilde{\beta}_l, x(\beta_l, \beta_{-l})) &= u_0(x(\beta_l, \beta_{-l})) + \tilde{\beta}_l u_l(x(\beta_l, \beta_{-l})) + \beta_{-l} u_{-l}(x(\beta_l, \beta_{-l})) \\ &- \left(u_0(x(0, \beta_{-l})) + \tilde{\beta}_l u_l(x(0, \beta_{-l})) + \beta_{-l} u_{-l}(x(0, \beta_{-l})) \right). \end{aligned} \tag{C.14}$$

Differentiating this expression with respect to β_{-l}, and simplifying using (3.4), we find:

$$\frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, x(\beta_l, \beta_{-l})) = (-1)^{l+1} \left((\beta_l - \tilde{\beta}_l) \frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) + \tilde{\beta}_l \frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) + x(\beta_l, \beta_{-l}) - x(0, \beta_{-l}) \right). \tag{C.15}$$

From (3.4), we also get:

$$\text{(C.16)}\quad \frac{\partial x}{\partial \beta_l}(\beta_l, \beta_{-l}) + \frac{\partial x}{\partial \beta_{-l}}(\beta_l, \beta_{-l}) = 0 \quad \forall(\beta_l, \beta_{-l}).$$

Differentiating (C.15) with respect to β_l, and using (C.16) to simplify the expression, we find:

$$\frac{\partial^2 \Delta W_l}{\partial \beta_l \partial \beta_{-l}}(\tilde{\beta}_l, x(\beta_l, \beta_{-l})) = (-1)^{l+1}(\beta_l - \tilde{\beta}_l)\frac{\partial^2 x}{\partial \beta_l \partial \beta_{-l}}(\beta_l, \beta_{-l}).$$

From (3.4), we know that

$$\frac{\partial^2 x}{\partial \beta_l \partial \beta_{-l}}(\beta_l, \beta_{-l}) \leq 0 \text{ (resp. } \geq 0) \ \Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

When $\beta_l \geq \tilde{\beta}_l$, we thus find:

$$\frac{\partial^2 \Delta W_l}{\partial \beta_l \partial \beta_{-l}}(\tilde{\beta}_l, x(\beta_l, \beta_{-l})) \leq 0 \text{ (resp. } \geq 0) \ \Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

Since $\frac{\partial \Delta W_l}{\partial \beta_{-l}}(0, \beta_{-l}) = 0$, we finally get:

$$u_0''' \geq 0 \text{ (resp. } \leq 0) \Leftrightarrow \frac{\partial \Delta W_l}{\partial \beta_{-l}}(\tilde{\beta}_l, x(\beta_l, \beta_{-l})) \begin{cases} \leq 0 & \forall \beta_l \geq \max\{0, \tilde{\beta}_l\} \text{ (resp. } \geq 0) & \text{if } l = 1 \ , \\ \geq 0 & \forall \beta_l \geq \max\{0, \tilde{\beta}_l\} \text{ (resp. } \leq 0) & \text{if } l = 2 \end{cases}.$$

From (A.18), we finally obtain:

$$\text{(C.17)}\quad \frac{\partial \tilde{S}_l}{\partial \lambda_{-l}}(\lambda, \lambda_{-l}) \geq 0 \text{ (resp. } \geq 0) \ \Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

From the Theorem of Implicit Functions, we finally deduce that

$$\frac{\partial \Lambda_l^*}{\partial \lambda_{-l}}(\lambda_{-l}) = -\frac{\frac{\partial \tilde{S}_l}{\partial \lambda_{-l}}(\Lambda_l^*(\lambda_{-l}), \lambda_{-l})}{\frac{\partial \tilde{S}_l}{\partial \lambda_l}(\Lambda_l^*(\lambda_{-l}), \lambda_{-l})} \geq 0 \text{ (resp. } \geq 0) \ \Leftrightarrow u_0''' \geq 0 \text{ (resp. } \leq 0).$$

where the last inequality follows from and (C.6) and (C.17). *Q.E.D.*

Proof of Proposition 10: The proof is similar to the Proof of Proposition 9 and is thus omitted. *Q.E.D.*