

Cartel Trigger Price Strategies and Time Inconsistency

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Abstract

This paper explores the impact of time inconsistency on the optimal trigger pricing strategies of a cartel. Time inconsistency on the part of firms is modelled by using hyperbolic discounting and we focus on $\beta - \delta$ preferences, where $\beta < 1$ in order to mimic the effect of the hyperbolic discount parameter. The case where $\beta = 1$ is well known and conforms to firms using exponential discounting. Green and Porter (1984) and Porter (1983) have shown for this case that trigger price strategies lead to a cartel in which cartel pricing is separated by periodic price wars between its members. Overall we find that there is a critical threshold value $\beta(\delta)$, which is decreasing in δ . For $\beta \geq \beta(\delta)$, there exists an optimal trigger strategy for the cartel featuring lower prices, higher output and periodic price wars. The duration of the price war increases as β decreases until the cartel collapses at a point where $\beta < \beta(\delta)$. Surprisingly, the trigger price decreases as β decreases.

Keywords: Time Inconsistency; Hyperbolic Discounting; Optimal Trigger Price Strategies; Cartels; Oligopoly.

JEL Classification: C73; D43; L13; L41

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1 INTRODUCTION

The objective of this paper is to provide a first step towards a behavioural theory of cartelization by focusing on the situation where participating firms' time preferences are quasi-hyperbolic (i.e. $\beta - \delta$ preferences). Firms using quasi-hyperbolic discounting reveal a tendency towards making choices that are inconsistent over time. They make choices today that their future self would prefer not to have made, despite using the same reasoning. To paraphrase Laibson (1997), this dynamic inconsistency happens because the value of future rewards for the firm is much lower under quasi-hyperbolic discounting than under exponential discounting. The setting for this paper is the market environment of Porter (1983) and Green and Porter (1984) in which there is imperfect public monitoring and firms can only observe a noisy price signal. Within this market environment, this paper provides a characterization of the optimality properties of the cartel. Hence, we calculate the values for the trigger price and punishment period length that maximize the expected industry discounted value, subject to the requirement that firms have no incentive to deviate from the cooperative output level. The implications for the setting, the optimal trigger price and punishment period length at their optimal values are also assessed.

Within the literature on collusion in industrial organization, the major problem with early formulations of the cartel problem (e.g., Friedman (1971); Osborne (1976)) is that incentives are perfect and the deterrent mechanisms are never observed. Green and Porter (1984) and Porter (1983) consider imperfect monitoring and find the trigger price strategies lead to a cartel in which cartel pricing is separated by periodic price wars between its members.¹ A key contribution of Green and Porter (1984) is to show that it was possible for periodic price wars to exist within the collusive equilibrium. In particular Porter (1985) empirically predicts that price wars are more likely under recession. Although this result has been challenged by Rotemberg and Saloner (1986), the body of empirical evidence (see, Aiginger (1993) for a discussion of this literature) has supported Green and Porter (1984). Recent anecdotal and empirical evidence suggests that cartels are either tolerant of persistent cheating by their members, or at least robust to persistent price shocks that would provide evidence of cheating (Genesove and Mullin

¹Important contributions in this literature include Green and Porter (1984), Rotemberg and Saloner (1986), Abreu et al. (1986, 1990), Abreu (1988), Bernheim and Whinston (1990), Athey and Bagwell (2001, 2008), Athey et al. (2004) and Harrington and Skrzypacz (2011).

(1998, 2001); Marshall et al. (2013)) and motivates the recent theoretical contribution of Bernheim and Madsen (2017) that studies the mechanism that supports cheating with cartels.

With regards to the role played by the discount parameters in this model, Porter (1983) showed that the exponential discount factor δ would not affect the optimal collusive output (although it does impact the duration of punishment phase). However, when we consider hyperbolic discounting, we find that the collusive output depends on both β and δ . We show that when β or δ are sufficiently small, the optimal cartel output increases to the Cournot output and the cartel collapses. At this point we find a threshold value for β ; if the hyperbolic discount parameter β falls below this threshold, then prices and output will converge on the Cournot levels and cartel collapses.

One would expect that present bias causes firms to place more weight on their immediate payoff, thus the cartel would require stricter punishment by increasing either the punishment period length or trigger price. Our results only partially agree with this intuition. We find it is better for the cartel to increase the punishment period length when $\beta < 1$, so that the length of the optimal punishment period is always larger than under exponential discounting. However, somewhat surprisingly, we find that the optimal trigger price will always be lower than under exponential discounting. This occurs because the optimal trigger price relative to the optimal price of the cartel is a fixed ratio. Hence, the optimal trigger price is proportional to the price (and output) set by the cartel. And, as mentioned above, because the output chosen by the cartel depends negatively on β , this implies that the both the cartel price level and trigger price will increase as $\beta \rightarrow 1$, so as to converge on the optimal levels determined in Porter (1983).

Within the repeated game literature, there are three related papers. The closest to our paper is Chade et al. (2008), which focuses on quasi-hyperbolic discounting in a repeated game setting. They adapt the approach of Abreu et al. (1986, 1990) to show the existence of a Strotz-Pollack equilibrium, as characterized in Peleg and Yaari (1973), by using an effective discount rate to give a recursive structure. They provide a characterization that identifies when the Strotz-Pollack equilibrium coincides with the sub-game perfect equilibrium. Obara and Park (2014) is slightly more general than Chade et al. (2008), as they examine generalized time preferences in a repeated game setting (with quasi-hyperbolic discounting as a special case). They show for quasi-hyperbolic discounting that cooperative equilibria can be supported by a stick-and-carrot strategy.

Our approach is different from these two papers in both setting and scope. Chade et al. (2008) and Obara and Park (2014) both focus on repeated games in an abstract setting. They focus exclusively on the perfect monitoring case, but ours examines an environment where there is imperfect public monitoring similar to Porter (1983) and Green and Porter (1984). The third paper is by Peeters (2004) and examines time inconsistency and cooperation within a stochastic game. The approach is abstract and is focused on formalizing and extending the analysis contained in O'Donoghue and Rabin (1999), where there are agents that exhibit naive and sophisticated optimizing behavior; its approach is therefore unrelated to this paper.

By contrast, our paper focuses on the optimal cartel problem under Nash reversion for quantity setting firms in an oligopoly. Within this context we are able to determine optimal output and punishment length. Because Porter (1983) and Green and Porter (1984) use exponential discounting and are a special case of our model, we are also able to benchmark off their results and we can show the impact of present bias on the optimal cartel strategy. Our equilibrium conforms with the definition of a Strotz-Pollack equilibrium, as contained in Chade et al. (2008). The underlying idea with the Strotz-Pollack equilibrium, due to Strotz (1956) and Pollack (1968), is that agents pick a plan q^* with the property that, for each time period t , q_t^* is the best action in that time period. The assumption is that all future actions chosen shall also be optimal. Thus, the plan q^* can, in principle, be determined by means of a backwards recursion. This idea, which is taken from the literature on intertemporal savings behavior was extended by Chade et al. (2008) to analyse the effect of time inconsistency in repeated games.

The paper is structured as follows. Section two sets up the model and provides preliminary results concerning the existence of cartel equilibria. Section three provides first order conditions determining the optimal Nash reversion strategy for the cartel. Section four provides supporting propositions and a numerical example that demonstrates the impact of a change in present bias.

2 THE MODEL

The market is composed of N oligopolistic firms producing a single homogenous commodity. The industry output vector in the t th time period is denoted by $\bar{q}_t = (q_{1,t}, \dots, q_{N,t})$, where $q_{i,t}$ denotes the t th period output of firm i and the bar denotes that \bar{q}_t is a vector.

Total industry output in period t is denoted by $Q_t = \sum_{i=1}^N q_{i,t}$. As there is no product differentiation firms face a common market price, which is determined by the linear inverse demand function

$$p(Q_t) = a - bQ_t, \quad a, b > 0.$$

The observed market price \hat{p}_t is determined by the inverse demand function and a multiplicative shock θ_t :

$$\hat{p}_t = p(Q_t)\theta_t,$$

The multiplicative shock is assumed to be an independent and identically distributed stochastic process $\{\theta_t\}_{t=1}^{\infty}$, with mean μ , density function f and cumulative distribution F . It is assumed that f and F are continuously differentiable and $F(0) = 0$ and $F(\infty) = 1$ and that F has positive support on its domain.

Firms are assumed to be symmetric with respect to cost structure, with each firm in each period facing the cost function

$$c(q_{i,t}) = c_0 + c_1 q_{i,t}, \quad c_0, c_1 > 0,$$

where c_0 is the fixed cost and c_1 is a constant marginal cost. Hence, the single-period expected profit function for the i th firm is given by

$$\pi_{i,t}(\bar{q}) = [A - B(Q_{-i,t} + q_{i,t})]q_{i,t} - c_0, \quad i = 1, \dots, N,$$

where $Q_{-i,t} = \sum_{j \neq i} q_{j,t}$, $A = a\mu - c_1$ and $B = b\mu$. It is assumed that $0 < c_1 < a\mu$. This assumption and the assumption that $b > 0$, implies that the constants A and B are positive.

Let $\bar{s}_t = (s_{1,t}, \dots, s_{N,t})$ denote the t th period Cournot equilibrium output vector. Then given $Q_{-i} = \sum_{j \neq i} s_j$ (and suppressing t), s_i maximizes $\pi_i(\bar{q})$ for firm i . For the i th firm, the single period Cournot equilibrium output and expected profit is then given by

$$s_i = \frac{A}{B(N+1)}, \quad i = 1, \dots, N$$

and

$$\pi_i(\bar{s}) = \frac{A^2}{B(N+1)^2} - c_0, \quad i = 1, \dots, N$$

It can be seen that the expected profit will be positive if and only if

$$0 < c_0 < \frac{(a\mu - c_1)^2}{b\mu(N + 1)^2}.$$

We let $\bar{q} = (q_1, \dots, q_N)$ denote the single period cooperative output vector. The output that maximizes the expected joint net return for a single time period is denoted by $\bar{r} = (r_1, \dots, r_N)$ (suppressing the t). It can be shown that this output level is given by

$$r_i = \frac{A}{2BN}, \quad i = 1, \dots, N.$$

The expected profit for the i th firm is given by

$$\pi_i(\bar{r}) = \frac{A^2}{4BN} - c_0, \quad i = 1, \dots, N.$$

When $N \geq 2$, the single period cooperative expected profit $\pi_i(\bar{r})$ for each firm i is always higher than the single period expected profit $\pi_i(\bar{s})$ at the Cournot equilibrium.

We will follow Porter (1983) and Green and Porter (1984) and assume that the Cartel behaves as follows:

1. Initially, at time period $t = 0$, firms have the choice of forming an industry-wide cartel or earning a non-collusive profit. Then in subsequent periods:
2. If at time period t the cartel is still active and the observed price $\hat{p}_t \geq \tilde{p}$, where \tilde{p} is the pre-determined trigger price, then at $t + 1$ all firms assume that the cartel is still active and set their current period output in line with Cartel policy.
3. If at time period t the cartel is still active and the observed price $\hat{p}_t < \tilde{p}$, then the market will revert to Cournot output levels for $T - 1$ periods, with the cartel resuming in the T th period.

The main departure from Porter (1983) is that firms' time preferences are hyperbolic, so that discount rates are much greater in the short-run than in the long-run. This implies that when $\beta < 1$, hyperbolic discounting captures the qualitative property that discount rates decline (weakly) as the time horizon length increases. In other words the short-run discount rate, $-\ln(\beta\delta)$, is greater than the long-run discount factor δ , where $-\ln(\delta)$. The other aspect of hyperbolic preferences is that they are non-recursive; Saez-Marti

and Weibull (2002) provide a decomposition that demonstrates this point. To provide a recursive structure to the firm's optimization problem time inconsistency requires us to distinguish between two different value functions, the expected value function and the long-run expected value function. Under the assumption of common long run discount rate δ and $\beta = 1$, they are exactly equal to each other. However, when $\beta < 1$, and firms have a hyperbolic discount rate, they are different.

We will begin by assuming that all firms in the industry face the same long-run discount factor δ , where $0 < \delta < 1$. The discounted present value of the long-run expected payoff by firm i is then given as follows

$$V_i(\bar{q}) = \pi_i(\bar{q}) + \Pr\{\tilde{p} \leq \theta p(Q)\} \delta V_i(\bar{q}) + \Pr\{\tilde{p} > \theta p(Q)\} \left[\sum_{\tau=1}^{T-1} \delta^\tau \pi_i(\bar{s}) + \delta^T V_i(\bar{q}) \right], \quad (1)$$

where \bar{q} denotes the long-run industry output under the cartel. This long-run value function is the discounted expected payoff given in Porter (1983), which can be rewritten as

$$V_i(\bar{q}) = \frac{\pi_i(\bar{q}) + F(\tilde{p}/p(Q)) [(\delta - \delta^T)/(1 - \delta)] \pi_i(\bar{s})}{(1 - \delta) + (\delta - \delta^T) F(\tilde{p}/p(Q))} \quad (2)$$

However, under the hyperbolic time preference, firm i 's expected value function $W_i(\bar{q})$ will solve the equation

$$W_i(\bar{q}) = \pi_i(\bar{q}) + \Pr\{\tilde{p} \leq \theta p(Q)\} \beta \delta V_i(\bar{q}) + \Pr\{\tilde{p} > \theta p(Q)\} \beta \left[\sum_{\tau=1}^{T-1} \delta^\tau \pi_i(\bar{s}) + \delta^T V_i(\bar{q}) \right]. \quad (3)$$

Upon substitution of Eq. (1) into Eq. (3) we arrive at

$$W_i(\bar{q}) = (1 - \beta) \pi_i(\bar{q}) + \beta \frac{\pi_i(\bar{q}) + F(\tilde{p}/p(Q)) [(\delta - \delta^T)/(1 - \delta)] \pi_i(\bar{s})}{1 - \delta + (\delta - \delta^T) F(\tilde{p}/p(Q))}. \quad (4)$$

The optimal cartel equilibrium is characterised by the trigger price \tilde{p} , the length of the punishment phase T and an output vector \bar{q}^* . To get the optimal cartel quantity, we

take the first order conditions with respect to q_i ; we have

$$[\beta C + (1 - \beta)C^2][A - (N + 1)Bq^*] = \beta(\delta - \delta^T)f[\tilde{p}b/(a - Nbq^*)^2]\Delta \quad (5)$$

where $C \equiv 1 - \delta + (\delta - \delta^T)F(\tilde{p}/p(Q))$ and $\Delta \equiv q^*(A - NBq^*) - A^2/B(N + 1)^2$.

Most of the preliminary propositions in Porter (1983) hold under hyperbolic discounting except for Lemma 4, which requires a more stringent convexity assumption. We state them as lemmas below:

Lemma 1 *For any strictly concave profit function, the single period Cournot output vector \bar{s} is an equilibrium quantity vector in cooperative periods for any values of \tilde{p} and T .*

Lemma 2 *Given the symmetric linear specification of the demand and cost functions, all firms which have positive equilibrium output levels will produce exactly the same quantity in cooperative periods.*

Lemma 3 *In the symmetric linear structure of this section, a noncooperative equilibrium is characterized by q^* , \tilde{p} , and T satisfying Eq. (5), where q^* lies within $(s/N, s]$. We further conclude that q^* lies within $(r, s]$.²*

Lemma 4 *If $F(\theta)$ is sufficiently convex, then $W_i(\bar{q})$ is concave in q_i , given the symmetric linear structure of this section.³*

3 OPTIMAL CARTEL STRATEGIES

We now derive the optimal cartel quantity by the first order condition, Eq. (5), which is a function of trigger price and punishment periods, $\bar{q}^* = \bar{q}^*(\tilde{p}, T)$. For simplicity, as in Porter (1983), we assume $\mu = 1$ when we derive the equilibrium, so that $b = \beta$. With

²Porter (1983) shows that the optimal cartel output is larger than r , the output that maximizes the joint profit of cartel members. This is because, under exponential discounting, for output levels below r the incentive to deviate increases and hence the likelihood of invoking the trigger price also increases without any payoff benefit. In the case of hyperbolic discounting, this logic is maintained.

³Porter (1983) shows that the convexity of F is sufficient to induce the concavity of Eq. (1) via the linearity of the first order condition given in his paper. Eq. (4) requires an additional restriction on the convexity of F .

respect to this locally optimal quantity, the expected value function can be expressed as follows $W_i(\bar{q}^*(\tilde{p}, T); \tilde{p}, T) = W_i(\tilde{p}, T)$. To get global optimal cartel strategy, we choose the trigger price \tilde{p} and punishment period T to maximize $W_i^*(\tilde{p}, T)$. The optimal pair (\tilde{p}, T) , if they are interior solutions, will satisfy the first order conditions

$$\frac{dW_i^*}{d\tilde{p}} = \sum_{j=1}^N \frac{\partial W_i}{\partial q_j} \frac{\partial q_j^*}{\partial \tilde{p}} + \frac{\partial W_i}{\partial \tilde{p}} = 0 \quad (6)$$

and

$$\frac{dW_i^*}{dT} = \sum_{j=1}^n \frac{\partial W_i}{\partial q_j} \frac{\partial q_j^*}{\partial T} + \frac{\partial W_i}{\partial T} = 0, \quad (7)$$

respectively. Adopting the symmetric conditions, these first order conditions can be rewritten as

$$(N-1) \frac{\partial W_i}{\partial q_j} \frac{\partial q_j^*}{\partial \tilde{p}} + \frac{\partial W_i}{\partial \tilde{p}} = 0 \quad \text{for } j \neq i \quad (8)$$

and

$$(N-1) \frac{\partial W_i}{\partial q_j} \frac{\partial q_j^*}{\partial T} + \frac{\partial W_i}{\partial T} = 0 \quad \text{for } j \neq i. \quad (9)$$

To solve Eqs. (8) and (9), we require the following:

$$\frac{\partial W_i(\bar{q}^*)}{\partial q_j} = \frac{-[\beta + (1-\beta)C](A - NBq^*)}{C} \quad \text{for } j \neq i, \quad (10)$$

$$\frac{\partial W_i(\bar{q}^*)}{\partial T} = \frac{p^{*2}[\beta + (1-\beta)C][A - (N+1)Bq^*]\delta^T \ln \delta F}{C(\delta - \delta^T)f\tilde{p}b}, \quad (11)$$

and

$$\frac{\partial W_i(\bar{q}^*)}{\partial \tilde{p}} = \frac{-p^*[\beta + (1-\beta)C][A - (N+1)Bq^*]}{C\tilde{p}b} \quad (12)$$

Totally differentiating the first order condition, Eq. (5), with respect to \tilde{p} and T , we get

$$\frac{\partial q^*}{\partial \tilde{p}} = \frac{p^* \{[\beta + 2(1-\beta)C][A - (N+1)Bq^*]p^* - \beta b \Delta \eta\}}{\tilde{p}bK'} \quad (13)$$

and

$$\frac{\partial q^*}{\partial T} = \frac{\delta^T \ln \delta p^* \{ \beta \tilde{p}b \Delta f - [\beta + 2(1-\beta)C][A - (N+1)Bq^*]Fp^{*2} \}}{\tilde{p}bf(\delta - \delta^T)K'} \quad (14)$$

where $\Delta = \pi_i(\bar{q}^*) - \pi_i(\bar{s})$,

$$\eta = 1 + \frac{f'}{f} \frac{\tilde{p}}{p^*},$$

and

$$K' = \beta \left[\frac{B\Delta(N+1)p^*}{A - (N+1)Bq^*} + bN\Delta(\eta+1) + p^*(A - 2NBq^*) \right] - [\beta + 2(1-\beta)C] [A - (N+1)Bq^*] Np^*$$

Eqs. (10)-(14) can now be compared to those derived in Porter (1983).

Plugging Eqs. (10), (12) and (13) into (8), we get the first order condition for the cartel trigger price \tilde{p} ,

$$0 = (N-1)(A - NBq^*) \{ [\beta + 2(1-\beta)C] [A - (N+1)Bq^*] p^* - \beta b\Delta\eta \} + [A - (N+1)Bq^*] K' \quad (15)$$

When $\beta = 1$ (i.e., Porter (1983)), Eq. (15) becomes

$$\eta(A - 2NBq^*) + \{ (N+1)p^* + N[A - (N+1)Bq^*] \} = 0,$$

which is independent of the punishment period T . Thus we can derive the the optimal cartel quantity for the case where $\beta = 1$

$$q^*|_{\beta=1} = \frac{A}{2NB} \left[\frac{N + \eta + (N+1)a/A}{N+1+\eta} \right] = r \left[\frac{N + \eta + (N+1)a/A}{N+1+\eta} \right].$$

However, when $\beta < 1$, the optimality condition given in Eq. (15) depends on T . Hence, we also require the first order condition for punishment period (Eq. (11)) in order to derive the optimal output.

The first order condition for the optimal punishment period is derived by plugging Eqs. (10), (11) and (14) into Eq. (9). This leads to

$$0 = (N-1)(A - NBq^*) \left\{ [\beta + 2(1-\beta)C] [A - (N+1)Bq^*] p^* - \beta b\Delta \frac{f\tilde{p}}{Fp^*} \right\} + [A - (N+1)Bq^*] K' \quad (16)$$

Comparing Eq. (16) to Eq. (15), it can be seen that $\eta^* = \frac{f(\theta^*)}{F(\theta^*)}\theta^*$, where $\theta^* = \tilde{p}^*/p^*$. By definition,

$$\eta^* \equiv 1 + \frac{f'(\theta^*)}{f(\theta^*)}\theta^* \Rightarrow \frac{f(\theta^*)}{F(\theta^*)}\theta^* - \frac{f'(\theta^*)}{f(\theta^*)}\theta^* = 1. \quad (17)$$

Observing the last function, we can derive the optimal relation between \tilde{p}^* and p^* from cumulative distribution function F . This relationship, between the optimal cartel price and the optimal trigger price, also holds in Porter (1983), and thus we know the discount rate has no effect on it. Therefore hyperbolic discounting has no impact on the size of optimal trigger price relative to the optimal price for the cartel. Hence, the probability for the optimal cartel switching into a price war remains constant even though the firms are present biased. Nevertheless, we know that finding the θ^* satisfying Eq. (17) is equivalent to finding the extreme point of $f(\theta^*)\theta^*/F(\theta^*)$. Therefore θ^* , as well as η^* , are derived from the cumulative distribution function and are independent of the cartel output and punishment period length. The implication of this can now be stated in the following proposition, which is analogous to Proposition 3.2 in Porter (1983).

Proposition 1 *The optimal trigger price \tilde{p}^* will adjust in exactly the same proportion as $p^* = p(Nq^*)$.*

Eqs. (5), (15) and (17) are required to derive q^* , \tilde{p}^* and T^* . Unfortunately, when the firms are hyperbolic discounting, we can not get a brief and intuitive analytical solution like Porter (1983). However, given a specific cumulative distribution F , we can provide a comparative static analysis, showing the relationship between the hyperbolic discount parameter and the optimal output and punishment length for the cartel. After we know the relationship between optimal cartel price and trigger price, we need only solve for the optimal cartel quantity and punishment periods. These are solved in turn in the next section.

4 IMPACT OF PRESENT BIAS

4.1 OPTIMAL CARTEL QUANTITY AND TRIGGER PRICE

In this section we derive the relationship between the optimal cartel quantity q^* and the hyperbolic discounting parameter β . According to Lemma 3, the optimal cartel quantity q^* must lie within $(r, s]$. For an interior solution, we find there is negative correlation

between them. That is, if a firm weights future profit less than its current profit, then the optimal cartel quantity must increase to maintain cooperation. The reason is intuitive: we substitute Eq. (17) into Eqs. (5) and (15) and rewrite them as

$$L(q, T, \beta) = [\beta C + (1 - \beta)C^2][A - (N + 1)Bq^*] - \beta(\delta - \delta^T)f[\theta^*b/p^*]\Delta = 0 \quad (18)$$

and

$$G(q^*(\tilde{p}, T), T, \beta) = (N - 1)(A - NBq^*) \{[\beta + 2(1 - \beta)C][A - (N + 1)Bq^*]p^* - \beta b\Delta\eta^*\} + [A - (N + 1)Bq^*]K' = 0 \quad (19)$$

Differentiating $L(q, T, \beta)$ and $G(q^*(\tilde{p}, T), T, \beta)$ with respect to β we get

$$L_\beta + L_q \frac{\partial q^*}{\partial \beta} + L_T \frac{\partial T^*}{\partial \beta} = 0 \quad (20)$$

$$G_\beta + G_q \frac{\partial q^*}{\partial \beta} + \left(G_q \frac{\partial q^*}{\partial T} + G_T \right) \frac{\partial T^*}{\partial \beta} = 0, \quad (21)$$

where the subscript denotes the partial derivative. Comparing with Eq. (18), $L_\beta = [C - C^2][A - (N + 1)Bq^*] - (\delta - \delta^T)f[\theta^*b/p^*]\Delta < 0$. Similarly, we know $G_\beta < 0, G_T > 0$. Using the implicit function theorem, we derive

$$\frac{\partial q^*}{\partial \beta} = - \frac{\begin{vmatrix} L_\beta & L_T \\ G_\beta & G_q \frac{\partial q^*}{\partial T} + G_T \end{vmatrix}}{\begin{vmatrix} L_q & L_T \\ G_q & G_q \frac{\partial q^*}{\partial T} + G_T \end{vmatrix}} \quad (22)$$

The second-order condition of the maximization problem requires the matrix of the denominator in Eq. (22) is negative definite, and thus we have $L_q < 0, G_q \frac{\partial q^*}{\partial T} + G_T < 0$ and

$$\begin{vmatrix} L_q & L_T \\ G_q & G_q \frac{\partial q^*}{\partial T} + G_T \end{vmatrix} > 0.$$

Thus, the sign of the numerator determines the sign of Eq. (22). It is easy to see that when $L_\beta (G_q \frac{\partial q^*}{\partial T} + G_T) - L_T G_\beta > 0$, then $\frac{\partial q^*}{\partial \beta} < 0$. This can be seen in the numerical simulation shown in Figure 1. A sufficient condition for this to occur is for $L_T =$

$\frac{-\delta^T \ln \delta F}{p^*} ([\beta + 2(1 - \beta)C] [A - (N + 1)Bq^*] p^* - \beta b \Delta \eta^*) > 0$. This result is now stated formally in the following proposition:

Proposition 2 *When $\beta > \beta(\delta)$ the optimal cartel quantity q^* is decreasing in hyperbolic discounting parameter β . When $\beta \leq \beta(\delta)$, the cartel collapses and $q^* = s$, where s is the Cournot equilibrium output.*

In Porter (1983), the optimal cartel quantity q^* is independent of discount factor δ which is counterintuitive since there is a widely acknowledged negative relationship between the optimal cartel quantity and the discount factor. When the firms are more patient (so that δ increases), they can achieve more collusive output (q^* decreases). Our results with hyperbolic discounting restore this intuition.

We use a numerical example to show this relation by adopting the same cumulative distribution used by Porter (1983). Assume $F(\theta) = [\alpha\theta/(\alpha + 1)]^\alpha$ for $0 \leq \theta \leq (\alpha + 1)/\alpha$ and $\alpha > 0$, then $f(\theta) = \alpha F(\theta)/\theta$, $f'(\theta) = (\alpha - 1)f(\theta)/\theta$ with $E(\theta) = 1$ and $Var(\theta) = 1/\alpha(\alpha + 2)$. In order to get an interior solution, we set $\alpha = 6$. We first derive the optimal θ^* , which we know must satisfy Eq. (17). This equation is satisfied for any θ^* belonging to the domain $[0, 7/6]$. Hence, without loss of generality, we will assume that $\theta^* = \tilde{p}^*/p^* = 1$. That is, the optimal trigger strategy \tilde{p}^* is equal to the optimal cartel price without market fluctuation p^* . When the stochastic market fluctuation $\theta \geq 1$, the market price $\hat{p} \geq \tilde{p}^*$ and firms continue to produce cartel quantities. When $\theta < 1$, $\hat{p} < \tilde{p}^*$ and firms revert to Cournot quantities. We calculate this possibility is $F(1) = 0.3966$ under our assumptions. Meanwhile, we can also derive the parameter $\eta^* = \frac{f(\theta^*)}{F(\theta^*)}\theta^* = \alpha$.

Figure 1 shows how the optimal cartel quantity changes with the hyperbolic discounting parameter β and the long run discount rate δ . Here we are setting $A = a = 2$, $B = b = 1$, and $N = 2$ and focus on the relationship between the two discount rates and optimal cartel quantity. We can see from Figure 1 that when δ is held constant q^* is decreasing in β .

To understand why this relationship holds, we set $\delta = 0.93$ to focus on the hyperbolic discounting parameter β . This relationship between q^* and β is shown in Figure 2, where s denotes the static Cournot output. It can be seen that as the hyperbolic discount parameter $\beta \rightarrow 1$, the optimal output $q^*(\beta) \rightarrow q^*$, where q^* is the optimal output for the cartel under exponential discounting (abusing notation). There is a threshold level $\beta(\delta)$

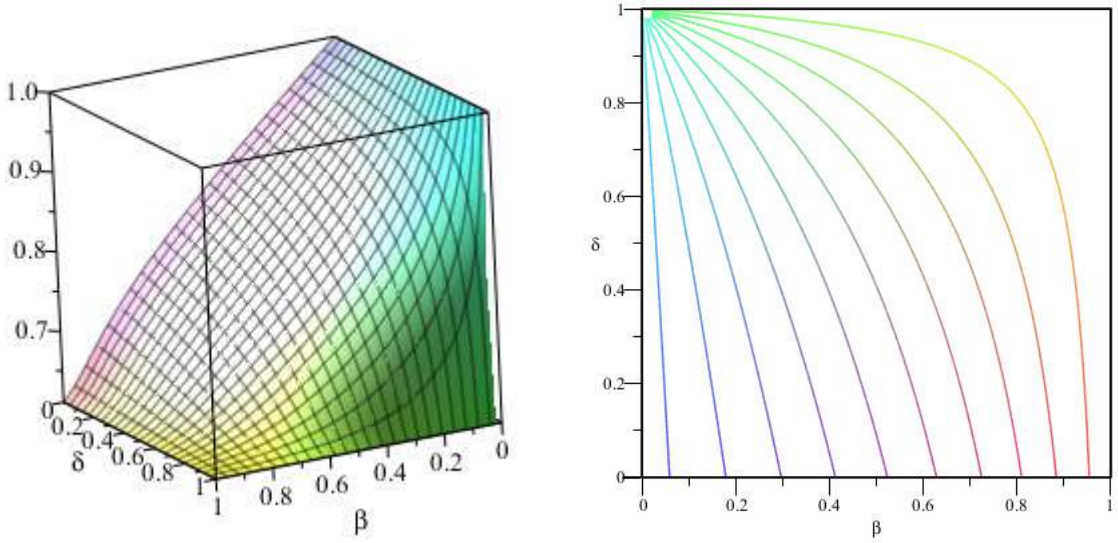


Figure 1: Optimal cartel quantity under different discount rate: $q^*(\beta, \delta)$

(at $\delta = 0.93$, the threshold $\beta(\delta) = 0.359$); for $\beta \leq \beta(\delta)$ the cartel can not support any output that is 'more collusive' than the Cournot equilibrium output and thus it collapses.

There is one further implication from Proposition 1 relating to the trigger price that needs to be discussed. Recall from Proposition 1 that the optimal trigger price \tilde{p} will adjust in exactly the same proportion as $p^* = p(Nq^*)$. This implies that \tilde{p} will adjust in response to changes in every parameter (including possibly β and δ). Proposition 2 states that there is a positive relationship between q^* and β . Hence, combining these two propositions we can see that there will also be a positive relationship \tilde{p} and β . This is expressed formally in the following corollary.

Corollary 1 *The optimal trigger price \tilde{p} increases in both β and δ when $\beta > \beta(\delta)$.*

As we know, unilaterally increasing trigger price would increase the chance of punishment. When firms' preferences are present biased, one would expect a larger \tilde{p}^* to strengthen the punishment. However, our results show that the trigger price decreases when β decreases. The reason is that the optimal cartel output q^* increases and the cartel price p^* decreases when β decreases. This implies that the cartel must lower \tilde{p}^* in order to maintain the likelihood of punishment at a constant level, a restriction implied by Proposition 1.

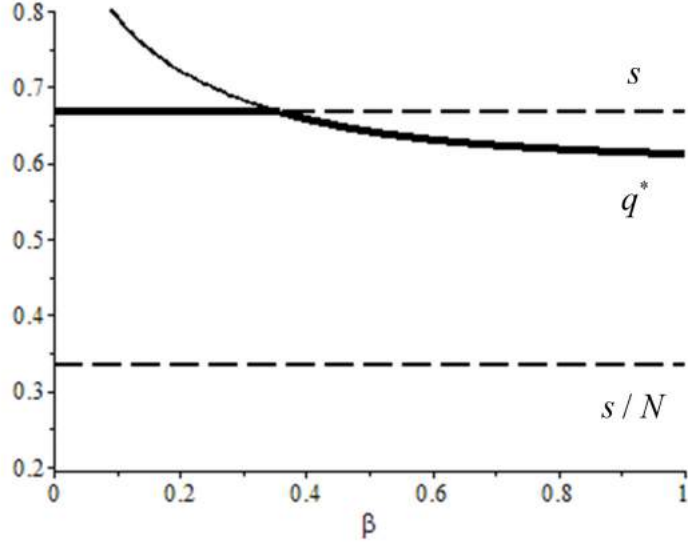


Figure 2: Optimal cartel quantity under different hyperbolic discounting rate: $q^*|_{\delta=0.93}(\beta)$

4.2 OPTIMAL CARTEL PUNISHMENT LENGTH

In this subsection, we derive the relationship between the optimal cartel punishment duration T^* and β . We know an interior solution q^* only appears when $\beta > \beta(\delta)$. Thus the interior optimal punishment period T^* is realized only when $\beta > \beta(\delta)$. Adopting the same method in Section 4.1, we derive

$$\frac{\partial T^*}{\partial \beta} = - \frac{\begin{vmatrix} L_q & L_\beta \\ G_q & G_\beta \end{vmatrix}}{\begin{vmatrix} L_q & L_T \\ G_q & G_q \frac{\partial q^*}{\partial T} + G_T \end{vmatrix}} \quad (23)$$

The second-order condition for maximization requires that the matrix contained in the denominator of Eq. (23) is negative definite. This implies that $G_q \frac{\partial q^*}{\partial T} + G_T < 0$ and since $G_T > 0$, we have $G_q \frac{\partial q^*}{\partial T} < 0$. From Eq. (9) we have $\frac{\partial q^*}{\partial T} < 0$ and thus we know $G_q > 0$. As shown above, $L_q < 0$, $L_\beta < 0$ and $G_\beta < 0$, this implies that $L_q G_\beta - L_\beta G_q > 0$, which implies that $\frac{\partial T^*}{\partial \beta} < 0$. Therefore, the optimal punishment length T^* is decreasing in the hyperbolic discount parameter β . We now formally state this as a proposition.

Proposition 3 *When $\beta > \beta(\delta)$, the optimal punishment period length T^* is decreasing in hyperbolic discounting parameter β .*

From Eqs. (5) and (15) we can plot T^* under different β and δ and adopt the same parameter values that were employed in Section 4.1. This plot is shown in Figure 3. The dotted line on the contour plot is $\beta(\delta)$. This is the critical value that demarcates interior and corner solutions. If $\beta \geq \beta(\delta)$, then there is an interior solution where the cooperative equilibrium is more profitable than the Cournot equilibrium. In this region, for fixed δ , it can be seen that T^* decreases as $\beta \rightarrow 1$. The region $\beta < \beta(\delta)$ is where the corner solution $T^* = 0$ is realized. In this region, punishment is not feasible since cooperative profits are always less than those generated under the Cournot equilibrium.

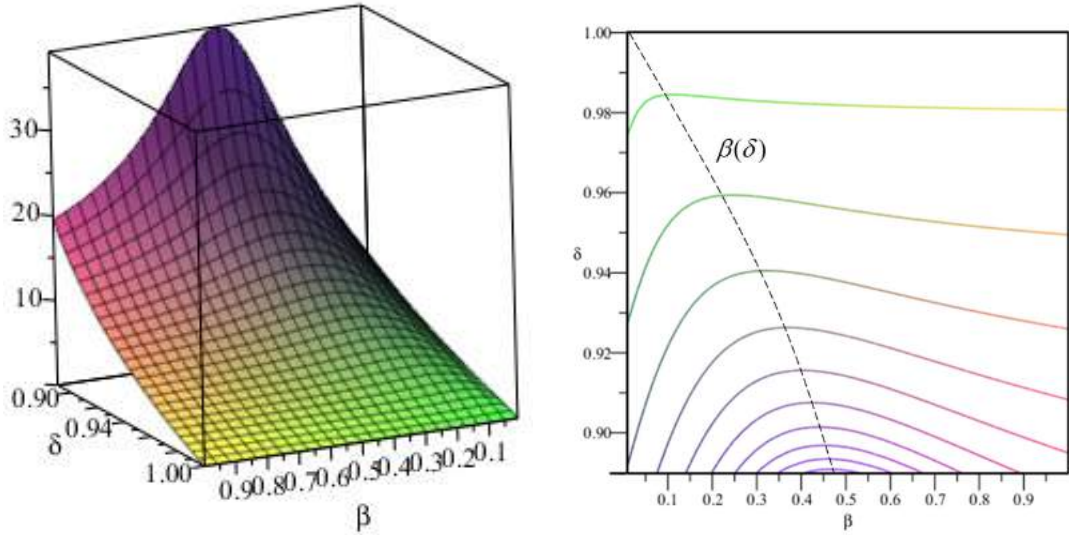


Figure 3: Optimal punishment period under different discount rate: $T^*(\beta, \delta)$

The reason why T^* is increasing in β when $\beta < 0.359$ in Figure 3 is that the optimal cartel quantity exceeds Cournot quantity which leads to lower profit relative to punishment periods. That is, $\Delta = \pi_i(\bar{q}^*) - \pi_i(\bar{s}) < 0$. For the first-order condition, $\frac{\partial W_i}{\partial T}$ turns to be positive since Cournot punishment is more profitable. Therefore the relation between T^* and β converts from negative to positive. According to Lemma 3, $q^* > s$ could be abandoned since its profit is even less than Cournot competition. When $q^* > s$, firms would rather take $q^* = s$ as the corner solution.

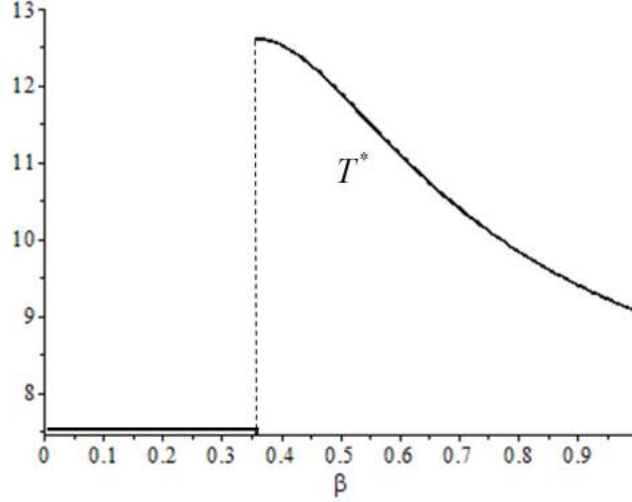


Figure 4: Optimal punishment periods under different hyperbolic discounting rate: $T^*|_{\delta=0.93}(\beta)$

We will now discuss the corner solution when $\beta < 0.359$. From Section 4.1, we know no cartel quantity which is less than the Cournot quantity s can be supported in the equilibrium when $\beta < 0.359$. This implies that when hyperbolic discounting degree is large enough, the optimal cartel quantity will be the Cournot quantity. Thus for Eq. (9),

$$(N - 1) \frac{\partial W_i}{\partial q_j} \frac{\partial q_j^*}{\partial T} + \frac{\partial W_i}{\partial T} = 0,$$

where $\frac{\partial W_i}{\partial T} = 0$, since punishment profit is equal to the cooperative profit; $\frac{\partial q_j^*}{\partial T} = 0$ since the cooperative quantity is equal to the Cournot quantity, which is independent of T . Therefore, Eq.(16) holds for any T . Hence, there is no point in choosing punishment periods T since punishment stage is equivalent to cooperative stage in this case. The optimal cartel strategy degenerates to be static Cournot strategy.

5 Conclusion

This paper explores the impact of time inconsistency on the trigger pricing strategies of cartels under imperfect public monitoring. Dynamic time inconsistency on the part of firms is modelled by using hyperbolic discounting and we focus on the case where

firms are present biased, so that $\beta < 1$. The case where $\beta = 1$ is well known and conforms to firms using exponential discounting. Green and Porter (1984) and Porter (1983) have shown for this case that trigger price strategies lead to a cartel in which cartel pricing is separated by periodic price wars between its members. They show that the cartel has to balance two effects: the increase in marginal profit from restricting output and the increase in the marginal loss brought on from increasing the possibility of causing Cournot punishment in subsequent periods. Porter (1983) shows that the optimal punishment period length is decreasing in discount factor δ . For the interior solution, both the optimal cartel output q^* and the optimal trigger price \tilde{p} are shown to be independent of δ .

In our paper we show that this result depends on β . When β decreases, the marginal profit from restricting output is dominated by the marginal loss effect. The optimal cartel quantity q^* does increase to rebalance these two effects. This restores the widely acknowledged negative relationship between the optimal cartel quantity and the discount factor. We also show that there is a threshold value $\beta(\delta)$, which is decreasing in δ . For $\beta > \beta(\delta)$, there exists an optimal trigger strategy for the cartel featuring lower prices, higher output and periodic price wars. We find that as β increases, the duration of the price war T^* also decreases and converges on the T^* predicted under exponential discounting. However, when $\beta \leq \beta(\delta)$, the cartel will collapse.

Consistent with Porter (1983), we find that neither β nor δ have any effect on the probability of detecting deviations from cartel pricing. However, we show that the optimal trigger price which depends on the price and output decreases as β or δ decrease. This is different from Porter (1983) and occurs because the output chosen by the cartel depends negatively on β . This implies that the price level and trigger price will increase as $\beta \rightarrow 1$ so as to converge on the optimal levels determined in Porter (1983).

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