Incentives in team contests

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Abstract

We study a contest between two teams that compete for multiple indivisible prizes. Team effort is determined as a function of Individual team members efforts. The allocation of prizes among teams is then determined through a Tullock contest function. The foucs of the paper is to study how intra-party prize allocation rules provide incentives to team mebers to exerrt effort. In particular, we study an egalitarian allocation rule that treats all memebers equally ex post by giving them an equal chance to receive a prixe, and a list allocation rule that deicdes ex ante which members will receive a prize. We show that the convexity of the cost function and complementarity of indivudual efforts determine which system leads to most of effort and thus would be chosen by the teams.

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1 Introduction

In many economic and political settings, we see competition between groups or organizations rather than between individuals. For instance, electoral competition between candidates involves strategic interaction between parties, which can be considered as teams of candidates.

In particular, in elections under proportional representation political parties propose to voters a pre-defined list of politicians who compete as a team to win as many seats as possible in the legislature. Each seat is a prize that is available to each of the candidates on the team. The number of seats won by each party depends on the aggregate effort of the candidates on the list. The system to allocate these seats between candidates on the list is de facto a way to provide incentives to candidates. For instance, it is common in countries such as Spain, Sweden or The Netherlands, to attribute won seats according to a pre-specified list that orders the candidates. The list system clearly treats team members in a discriminatory way. We can think of other systems in which all team members are treated the same way, with every team member getting the same chance to get a prize ex post. To understand how to best organize the allocation of prizes in teams, we need to study the interplay between intragroup incentives and intergroup competition.

The interplay between intragroup incentives and intergroup competition is also relevant in the business world. For example, imagine a firm that is made up of two main departments. The CEO of the firm, either to motivate his employees or to avoid creating unnecessary intrafirm imbalances, announces that the hiring and promotion policy is such that the number of hirings and promotions going to each department will be proportional to the department's relative performance. Within each department, depending on the announcement made by the CEO of the firm, the hirings and promotions will be allocated by the managers, either at their discretion or following rules set by the firm globally. Then, each employee of a department has an incentive to help their department win as many hirings or promotions as possible but also to have one of the prizes attributed to them, either because they want to be promoted or because being given the possibility of having a new employee working with them increases the relative importance of the area in which this employee works. Once again, to understand how to best organize the allocation of prizes within these teams, we need to study the interplay between incentives within each department and competition across the departments.

In this paper, we study contests between two teams of individuals who compete for multiple indivisible prizes. Teams of agents are engaged in a contest that determines the allocation of prizes. Teams win prizes on the basis of total effort provided by their members. We assume a constant elasticity of substitution technology that aggregates individual team member efforts into the team effort. This allows us to consider the case when efforts are substitute or complement. We also parametrize cost function of team members to allow for different degree of convexity.

We propose a novel inter-team allocation mechanism for the prizes. We extend Tullock contest success function to many prizes. Each prize is won with a probability equal to the ratio of the aggregate team efforts. The number of prizes won by a team follows a binomial distribution.

Teams choose the way to allocate prizes to their members as a function of the number of prizes won. We assume for most of the analysis that the allocation cannot depend on the efforts of individual members. This means that an allocation rule specifies the probability that a given team members wins a prize as a function of the total number of prizes won by his team.

When individual efforts are not observable (or contractible) by the team leadership, we compare a list allocation rule that allocates prizes according to a pre-specified list to an egalitarian allocation rule that treats all members equally by distributing randomly the prizes won. We show that which system dominates depends on two main characteristics of the environment, the complementarity (substitutability) of the individual efforts in the team production technology and the convexity of the individual cost function. In particular we show that the egalitarian system dominates when the marginal cost function is convex and when the individual efforts are complement. When the marginal cost of effort is concave and the efforts are substitutes, the list system dominates.

We than consider several extensions. First, we study the optimal mechanism. We show (for some examples), that the egalitarian rule is optimal among all possible rules, when effort is contractible. When effort is not-contractible, we show that under the conditions that lead to the egalitarian allocation rule to dominate the list allocation rule, the egalitarian allocation rule is indeed optimal. When the condition is not satisfied, we derive the optimal allocation rule. We show that the optimal rule is not always monotonic in the sense that the probability that a team member wins a prize can decrease with the number of prizes won by the team. We show that the list rule is optimal among the monotonic allocation rules.

We then show that when efforts are contractible and that the allocation of prizes can depend on the individual effort exerted by team members, the egalitarian rule always dominates. These results shed light on when we can expect teams to treat all their members symmetrically (fairly) and when discriminatory rules may be needed to provide better incentives.

We then show that our results are robust to the case with more than two teams, to the case of biased contests with one team having an advantage over the other team. We also consider settings in which teams are composed of more members than prizes available in the contest. We relate our findings to the group size paradox and the equal minority allocation rules that have been discussed in the literature.

1.1 Related Literature

Our analysis relates to two particular areas of research within the contest literature.[1] The analysis of collective rent seeking (contests between groups of individuals) has focused on situations where several teams compete in order to win one prize, which maybe of public or private nature, or a mix of both.[2] If the contested prize is a pure public good, it can be enjoyed in its totality by all group members (Baik (2008), Riaz et al. (1995), Ursprung (1990) and Katz et al. (1990), among others). To the contrary, if the contested prize has a private component, the latter has to be shared among the group members according to some sharing rule. Starting with Nitzan (1991), the literature has considered both exogenous (e.g. (e.g., Davis and Reilly (1999), Lee (1993), Esteban and Ray (2001)) and endogenous (e.g. Ueda (2002), Lee (1995), Baik and Lee (1997), Baik and Lee (2001), Balart et al. (2015b)) sharing rules, while it has assumed that the choice of such rules may occur under either public or private information (e.g., Baik and Lee (2007), Baik (2014), Nitzan and Ueda (2011)). In turn, group sharing rules affect the extent of total rent dissipation, the occurrence of the group size paradox, or group formation.[3] In this paper, we show how the equilibria resulting from egalitarian sharing rules over a multiple indivisible prizes compare with those resulting from asymmetric sharing rules.

The analysis of contests involving multiple prizes has focused on situations where contestants are individuals, and where each of these individuals can win at most one prize.[4] This litterature is mainly concerned about the way to generalize the standard Tullock contest success function (CSF henceforth) to a contest over multiple prizes, as there is no

[1] For surveys of the literature on rent seeking in general, see for instance Corchon (2007) and Konrad (2009).

[2] Kolmar (2013) offers a recent survey on collective rent seeking. See also Sheremata (2015) for a recent review of experimental evidence regarding behaviour in group contests.

[3] For a recent survey on prize-sharing rules in collective rent seeking, see Flamand and Troumpounis (2015).

[4] For a recent survey on contests with multiple prizes, see Sisak (2009). See also the references therein for sequential contests with multiple prizes.

unique and obvious way to pick up several winners. Berry (1993) is a first attempt in this direction.[5] Clark and Riis (1996) demonstrate that the mechanism proposed by Berry (1993) fact implies that only the first prize is awarded on the basis of contestants' efforts, while the remaining prizes are awarded on an egalitarian basis. The authors then propose an alternative winning probability for the multi-winner contest, using a nested probability function to construct a more convincing incentive structure.[6] Our contribution to the contest literature is threefold. First, we consider multiple prizes in the context of a contest among teams. Second, we set up an allocation mechanism such that each team can win several prizes. Third, we consider and analyze various intrateam allocation mechanisms of prizes. We thus bridge the two above-mentioned parts of the literature.

Our paper also contributes to the literature on incentives in teams, and in particular to the literature that links incentives and discrimination. Winter (2004) analyze whether agents that are identical in their qualifications should receive nonsymmetric rewards to improve incentives and efficiency. Our set-up is quite different, but we obtain a similar result with the list allocation rule being more efficient. Our model allows us to highlight the importance of the convexity of the cost function and of the complementarity of efforts as a necessary condition for equal rewards to be optimal. In Winter (2004), he uses O-Ring technology where all agents must succeed in their task for the team to be successful. We have a more continuous way to parametrize complementarity and we show that discirimination is optimal if individual efforts are sufficiently substitutes. Winter (2004) models effort as a binary choice and thus can not discuss the role of the convexity of the cost function.

Closer to our analysis is Ray, Baland and Dagnelie (2007). They show that when a group of individuals work on a joint project that creates some output as a function of individual efforts, unequal sharing rules are efficient when efforts are substitute. Our model differs on various dimensions. First, we have two teams competing while Ray Baland and Dagnelie (2007) only consider one group of individuals. Second, the team produces a conituous output that can be divided in any way, while in our model, the team output is measured by the number of prizes won, and individual rewards are binary (one prize or no prize). Finally, we also consider the convexity of the cost function as a parameter that explains unequal sharing rules. Our analysis goes further when equal hsaring. We derive the optimal

[5] The first models of contests with strict rankings of players come from the literature on tournaments (see for instance Nalebuff and Stiglitz (1983) and Green and Stokey (1983)).

[6] Chowdhury and Kim (2014) consider a variant of Clark and Riis (1996)'s mechanism in which losers are sequentially eliminated to attain the set of winners, and show that this is equivalent to the mechanism proposed by Berry (1993). Alternative allocation mechanisms for multi-winner contests have also been proposed recently by Fu et al. (2014) and Vesperoni (2013). The CSF for strict rankings in Clark and Riis (1996) and Fu et al. (2014) have been axiomatically characterized in Lu and Wang (2014).

allocation rule and show that the list allocation rule is optimal among monotonic sharing rules when efforts are sufficiently subsiuttute and the cost of effort not too convex. Bose Pal and Sappington (2010a) and (2010b) also study the unequal treatment o fidentical agents in teams. Their model is sequential and they show that unequal treatment is optimal when efforts are complements. Their technology like Winter is about the probability of success of a project and the outcome is binary.

2 Model

2.1 Individual and team efforts

Two teams are competing in a contest for n identical prizes. Each prize has value V. Each team is composed of n members who can at most win one prize. Member i of team j exerts costly effort $e_{ij} \geqslant 0$ to improve his team's chances of winning prizes. The individual effort cost function is increasing and convex:

$$c(e_{ij}) = e_{ij}^{\beta}/\beta, \text{ with } \beta > 1. \tag{1}$$

Team j aggregate effort is denoted by E_j. We assume that the production function aggregating individual efforts exhibits constant elasticity of substitution, we have:

$$E_j = \left(\sum_{i=1}^{n} (e_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{2}$$

2.2 Allocation of prizes between teams

The allocation of the n prizes between the teams depends on the aggregate effort of each team. We assume that the between-team allocation of the different prizes follows a Binomial-Tullock imperfectly discriminating contest success function. This assumption is a natural generalization of the Tullock contest success function to multiple prizes. As in a Tullock contest, the probability that team j wins a given prize is given by the ratio-form contest success function $p_j = \frac{E_j}{E_1+E_2}$. We assume that prizes are awarded to team j using independent draws for the Bernouilli distribution with parameter p_j. The probability that team j wins k prizes follows a binomial distribution and is given by:

$$P_j(k) = C_k^n \left(\frac{E_j}{E_1+E_2}\right)^k \left(1 - \frac{E_j}{E_1+E_2}\right)^{n-k}. \tag{3}$$

2.3 Allocation of prizes within teams

In this paper, we analyze how the interteam prize allocation rule affects individual team members' incentives to exert effort and as a consequence team success. We assume that individual efforts are not observable by the team leadership and that the allocation of prizes cannot depend on these efforts[7]. This means that the allocation of prizes can only depend on the number of prizes won by the team. An allocation rule specifies for each number of prizes won by the team the probability that a given team member wins a prize. When effort is not contractible, the probability of getting one of the prizes won by the team is independent of one's effort. Still, total team effort determines the number of prizes the team wins, so team members, by exerting effort, increase their chances of getting a prize.

We contrast two types of allocation rules, the egalitarian allocation rule that treats all team members the same way, and list allocation rule that gives priority to some team members in the allocation of prizes..

Under the **egalitarian** rule, all group members have the same probability getting one of the prizes won by the team. This allocation rule treats all team members the same. Team member i in team j chooses his level of effort to maximize:

$$Max_{e_{ij}} \left[\sum_{k=1}^{n} P_j(k) \frac{k}{n} V - \frac{e_{ij}^{\beta}}{\beta} \right]. \tag{4}$$

The egalitarian allocation rule can also be interpreted as a (fair and) random allocation procedure in which the prizes a team has won are allocated to its members through a fair and unbiased lottery.

Under the **list** allocation rule, the team orders the n members on a list which determines the allocation of prizes won by the team. The member in mth position on the list wins a prize if the team wins m prizes or more. The list order is *independent* of effort decisions but based on seniority, age, or any other effort-independent characteristics of the different team members. This allocation rule treats similar members in different ways and is thus biased and discriminatory.

Team member in mth position on the list in team j maximizes

$$Max_{e_{ij}} \left[\sum_{k=m}^{n} P_j(k) V - \frac{e_{mj}^{\beta}}{\beta} \right]. \tag{5}$$

[7] In section 4, we analyze the case where individual effort is perfectly contractible and the allocation of prizes can depend directly on individual efforts.

Notice that the summation goes from m to n nad not from 1 to n, as the team member in mth position on the list only gets a prize when his team wins at least m prizes.

3 Equilibrium efforts

Under the **egalitarian rule**, team member i in team j chooses his level of effort to solve Eq 4. Using the expectation of the binomial distribution, we can rewrite the objective function as:

$$\sum_{k=1}^{n} P_j(k)\frac{k}{n}V - \frac{e_{ij}^{\beta}}{\beta} = \frac{E_j}{E_1 + E_2}V - \frac{e_{ij}^{\beta}}{\beta}. \tag{6}$$

Proposition 1: Under the egalitarian allocation rule, in a symmetric Nash-Equilibrium:

Total effort E_E^* in each team is given by:

$$E_E^* = \left(\frac{V}{4}\right)^{\frac{1}{\beta}} n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.$$

Individual effort e_E^* is given by :

$$e_E^* = \left[\frac{1}{4E_E^*}Vn^{\frac{\sigma}{1-\sigma}}\right]^{\frac{1}{\beta-1}} = (\frac{1}{4})^{1/\beta}\left(\frac{V}{n}\right)^{\frac{\beta-1}{\beta}}.$$

Proof. See appendix. ■

We see that the individual efforts increase in the value of a prize V, decrease with the number of prizes n. Total effort increases in both V and n.

Under the **list allocation rule**, team member in position m on the list in team j maximizes

$$\sum_{k=m}^{n} P_j(k)V - \frac{e_{mj}^{\beta}}{\beta} = \sum_{k=m}^{n} C_k^n \left(\frac{E_j}{E_1 + E_2}\right)^k \left(1 - \frac{E_j}{E_1 + E_2}\right)^{n-k} V - \frac{e_{mj}^{\beta}}{\beta}.$$

Proposition 2: Under the list allocation rule, in a symmetric Nash-Equilibrium, total team effort E_L^* is given by:

$$E_L^* = \left\{\sum_{k=1}^{n}\left[kC_k^n\left(\frac{1}{2}\right)^{n-1}\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}\left(\frac{V}{4}\right)^{\frac{1}{\beta}}.$$

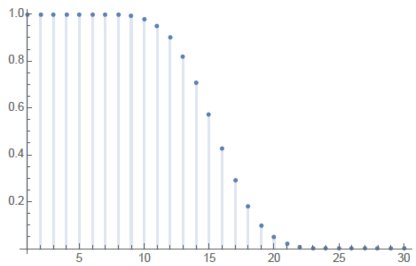

Figure 1:
Individual winning probability

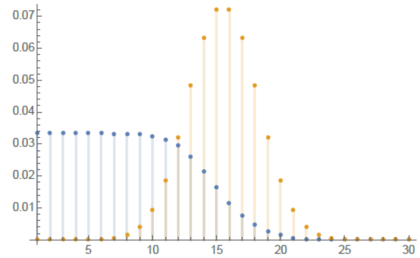

Figure 1bis:
Individual utility (blue) and effort (orange)

Individual effort e_m^* of the team member in the *mth* position on the list is given by :

$$e_m^* = \left[E_L^{*^{\sigma-1}} m C_m^n \left(\frac{1}{2}\right)^{n+1} V \right]^{\frac{1}{\beta+\sigma-1}}.$$

Proof. See appendix. ■

We see that individual efforts and total effort increase in V.

The main difference between the two allocation rules is that under the egalitarian rule all team members receive the same incentives and exert the same effort in equilibrium. Under the list allocation rule, the incentives and the individual equilibrium efforts vary with the position on the list. In particular, as we illustrate in the figure below, incentives are bell-shaped. Individuals who are in the first and last positions in the list have the list incentives to exert effort while individual in the middle of the list have stronger incentives. The next panel represents the winning probability , the equilibrium effort and the expected utility as a function of the position on the list for the following values of the parameters - $V = 1$, $\beta = 2$, $\sigma = 0$, $n = 30$. As we can see in the figure, individuals in the middle of the list exert more effort while individuals at the top and bottom of the list exert little effort.

3.1 Comparison of the allocation rules

We now compare team efforts and team success under the two allocation rules. We show that the convexity of the cost function parameter β and the degree of complementarity of the production function parameter σ play a central role. In particular, we show that

Condition 1: $(\beta - 1) \geq 1 - 2\sigma$.

Proposition 3: The egalitarian allocation rule leads to more effort when condition 1 is satisified, that is when the individual cost function is relatively convex and the team production technology exhibits enough complementarity.

Proof. See appendix. ■

The intuition behind this result comes from the individual incentives that the two allocation rule provide to team members. The egalitarian allocation rule gives all members the same incentives and individual efforts are equal within a team. With the list allocation rule, individual effort depends on the particular position on the list. The first and last members on the list have a very high or very low chance of seeing their team win enough prizes for them to receive one. As the slope of the binomial distribution function is flat in the tails, those members have a marginal benefit of exerting effort close to zero. The members in the middle of the list exert the highest levels of effort because they are in a position close to the expected number of prizes a team expects to win in a symmetric equilibrium. These positions correspond to the largest marginal benefit of effort, where the slope of the binomial distribution is the steepest.

When individual cost functions are very convex (in particular when the marginal cost is convex), asymmetric incentives are bad for team performance. Starting from equal marginal benefits of effort, increasing the marginal benefit of one team member and decreasing the benefit of another one will have a positive effect if the marginal cost increases more slowly for the individual with stronger incentives than for the one with weaker incentives. The complementarity of the team effort function also affects the impact of asymmetric incentives in similar fashion. When efforts are perfect substitutes, the total sum of efforts is what matters. When efforts are complementary, inducing differences in individual effort has drawbacks. The condition $2\sigma + \beta \geq 2$ reflects these two forces. When efforts are perfect substitutes ($\sigma = 0$) the list egalitarian rule dominates when the marginal cost of effort is convex ($\beta \geq 2$). When efforts are perfect substitute and the cost function is quadratic, the two systems lead to the same total team effort. When efforts become more complementary, the degree of convexity necessary for the egalitarian allocation rule to dominate decreases.

This result applies to a symmetric contests between teams that uses the same prize allocation rule. We can also show that, the comparison between allocation rules extends when we allow teams to choose between the two allocation rule beforehand. To do so, consider a two-stage game. In the first stage, each team leadership chooses an allocation rule to maximize team success. In the second stage, after observing the allocation rule chosen by both teams, individual team members choose their effort. The second stage may involve asymmetric contests in which teams have chosen different allocation rules. We show that the same condition on β and σ drives the choice of allocation rules in the Nash equilibrium in the first stage.

Proposition 4: In the Nash equilibrium of the two-stage game, the team leadership chooses the egalitarian rule when $(\beta - 1) \geq 1 - 2\sigma$.

Proof. See appendix. ■

This theorem shows that if the leadership of the team wants to maximize effort and thus the expected number of prizes won by the team, it chooses the allocation rule according to the condition 1. The intuition behind this result is that the incentives given by the two allocation rules do not depend on the effort exerted by the other team.

3.2 Discussion of the main result

Theorem 1 and 2 show that the convexity of the marginal cost and the complementarity of the team production function are the two main drivers behind the choice of allocation rule. When the marginal costs are convex giving very powerful incentives to a few individuals is not very productive. These individual are not going to produce a much higher effort level. With convex marginal costs, it is thus more efficient to give all team members the same incentives and treat them in a symmetric way. When marginal costs are concave, it is efficient to provide very powerful incentives to few individuals who will exert very high levels of effort. In that case, a rule that treats some members differently is optimal. The degree of complementarity plays a similar role. When efforts are substitute, there is no cost in getting very different effort levels within the team. When efforts are complementary, it is better to induce similar efforts and once again the egalitarian rule becomes more efficient in that context.

4 Extensions

In this section, we extend the basic model and show that condition (H) relating complementarity and convexity is robust. We first analyze the optimal allocation rule in the setting. We show that our result on treating team members equally or not extends. However, we show that the list allocation rule can be improved upon. In particular, we show that the optimal rule is a modified list system that uses a non-monotonic allocation of prizes. We then turn to the analysis of allocation rules that can depend directly on individual efforts. We show that in that context, the egalitarian rule always dominates the list allocation rule and is the optimal rule. Unequal treatment of team members is thus a consequence of the non-contractibility of individual efforts. We then go through a series of extensions. We analyze contests with more than two teams and contests in which one team has an advantage. We also consider the case of altruistic team members who value the result of the team as well as their own individual gains. In all cases, we show that condition 1 still drives the ranking between the egalitarian allocation rule and the list allocation rule. Finally, we look at the case of contests between teams in which there are more team members than prizes to be won. We relate our findings to the literature on the group size paradox and on equal minority allocation rules.

4.1 Optimal allocation rule

An allocation rule is $n * n$ matrix $\left[\lambda_i^j\right]$. Entry λ_i^j corresponds to the probability that team member i gets a prize when the team has won j prizes. Feasibility constraints lead to $0 \leq \lambda_i^j \leq 1$ and $\sum_{i=1}^n \lambda_i^j = j$. Probabilities need to be between 0 and 1, and the number of prizes distributed cannot be larger than the number of prizes won by the team.

The egalitarian allocation rule can be represented as a matrix in which each column has equal entries $\lambda_i^j = j/n$.

The list allocation rule can be represented as a matrix with $\lambda_i^j = 0$ if $i > j$ and $\lambda_i^j = 1$ if $i \leq j$.

Example: 3 prizes, 3 members per team, additive effort

An allocation rule is represented by the following matrix:

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ 1 & 0 & \lambda_1 & \mu_1 & 1 \\ 2 & 0 & \lambda_2 & \mu_2 & 1 \\ 3 & 0 & \lambda_3 & \mu_3 & 1 \end{array} \tag{7}$$

The constraints are $0 \leq \lambda_i \leq 1$, $\sum \lambda_i = 1$, $0 \leq \mu_i \leq 1, \sum \mu_i = 2$.

For instance, the egalitarian rule is represented by:

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ 1 & 0 & 1/3 & 2/3 & 1 \\ 2 & 0 & 1/3 & 2/3 & 1 \\ 3 & 0 & 1/3 & 2/3 & 1 \end{array} \tag{8}$$

and the list allocation rule is represented by:

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \end{array} \tag{9}$$

The first-order conditions of the effort choice by team member i are:

$$(e_i)^{\beta-1} = \frac{3V}{16E}\left(1+\mu_i-\lambda_i\right).$$

$$\begin{aligned} e_i &= \left(\frac{1}{E}\frac{3V}{16}\left(1+\mu_i-\lambda_i\right)\right)^{1/\beta-1} \\ E &= \sum\left(\frac{1}{E}\frac{3V}{16}\left(1+\mu_i-\lambda_i\right)\right)^{1/\beta-1} \\ E^{\beta/\beta-1} &= \sum\left(\frac{3V}{16}\left(1+\mu_i-\lambda_i\right)\right)^{1/\beta-1} \end{aligned}$$

Effort is thus maximized when $\sum\left(1+\mu_i-\lambda_i\right)^{1/\beta-1}$ is maximized

The optimal allocation rule can then be easily derived as a function β.

When $\beta=2$, we need to maximize $\sum\left(\mu_i-\lambda_i\right)$.

The optimal allocation rule is given by:

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 \end{array}$$

We see that the optimal allocation rule is not monotonic in the sense that team member 1 wins a prize if the team wins one prize but does not win a prize if the team wins 2 prizes.

If we insist on monotonic allocation rules, we conjecture that:

When $\beta>2$, the egalitarian allocation rule is the optimal monotonic allocation rule. When $\beta<2$, the list allocation rule is the optimal monotonic allocation rule.

4.2 Contractible efforts

We now turn to the study of allocation rules that can use the individual observed efforts to allocate prizes. We still only allow prizes as rewards. When individual efforts are observable and contractible, the team leader will ask each member for a given effort and make an allocation rule that depends on the number of prizes won. However, it can allocate zero prizes to any team member who does not exert the specified effort. This means that the team leader only needs to take into account the participation constraint of a team member.

Under an **egalitarian** rule, each team member who exerts effort $e \geq e^*$ gets an equal chance of getting one of the prizes won by the team. Under a **list** rule, each team member is assigned a rank in the list and an effort associated with the rank. If he exerts the specified effort, he will get a prize if the team wins more prizes than his rank. If he exerts less effort or if the team wins less prizes than his ranks, he gets nothing.

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Proposition 5: When efforts are contractible, the egalitarian allocation rule leads to more effort than the list allocation rule for all values of the parameters β and σ.

Proof. See appendix. ■

When efforts are observable, the level of effort is determined by the participation constraint of team members. The team leader can impose an effort level that drives the memeber utility to his outside option. In that respect, the cost of effort enters in the participation constraint while the marginal cost of effort was appearing in the maximization problem of the agent (incentive constraint) in the case where effort was not observable. If we consider the equivalent of condition 1 for the cost function (instead of the marginal cost function), we get $\beta \geq 1 - 2\sigma$ which is always satisfied.

When effort is contractible, it is thus always optimal to treat every team member in the same way. Unfair allocation rules are also inefficient allocation rules.

4.3 More than two teams

We now extend the model to the case in which $K > 2$ teams are competing for the prizes. This extension is important in the context of the application of the model to political parties competing for seats in elections with proportional representation in whcih it is common to see several parties competing. We show that the main results extend to this set-up.

The first interesting effect of having more than two teams happens under the list allocation rule that leads to an asymmetric distribution of efforts. With 2 teams ($K = 2$), the distribution of equilibrium individual efforts is symmetric and bell-shaped around the mean team list member(s). With more than 2 teams ($K > 2$), the distribution of equilibrium individual efforts is right-skewed: the first members of the list exert more effort than the last members on the list

In any symmetric equilibrium, each team member expects his team to win a share $1/K$ of the available prizes. Thus, depending on how many teams there are in the competition, the relevant 'intermediate' members we were referring to above are the ones that are around position n/K on their team list. Also, as we increase the number of teams in the competition, the position on the list for which the marginal benefit of exerting effort becomes sensitive to effort moves closer and closer to the first team member on the list. This explains intuitively why when $K > 2$ the vector of effort choices is right-skewed. Figure 2 illustrates this for $K = 2$ and $K = 4$ (with $V = 1$ $\beta = 2$ $\sigma = 0$ $n = 30$).

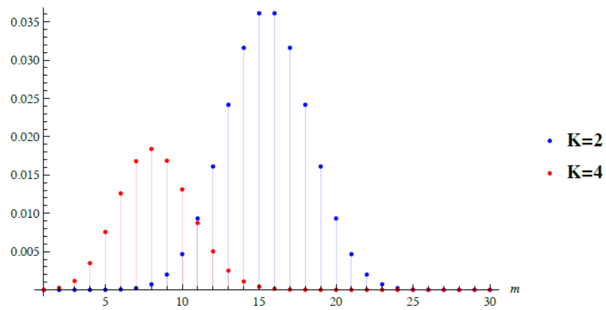

Figure 2: Individual effort under the non-competitive rule ($L = 30$)

Despite the fact that efforts are now distributed asymetrically, we can still show:

Proposition 6: With $K > 2$ teams, the egalitarian allocation rule leads to more effort than the list allocation rule when $2\sigma + \beta \geq 2$.

Proof. See appendix. ■

4.4 Asymmetric contests (team with advantage)

We can also show that when the team contest is biased, condition 1 still determines the raning of effort between allocation rules. Consider a contest betwen two teams biased in facor of team 1. We assume that the probability that team 1 wins a prize given efforts E_1 and E_2 is given by $\lambda E_1 / (\lambda E_1 + E_2)$ with $\lambda > 1$.

The distribution of the number of prizes is now:

$$P_1(k) = C_k^n \left(\tfrac{\lambda E_1}{\lambda E_1 + E_2}\right)^k \left(1 - \tfrac{\lambda E_1}{\lambda E_1 + E_2}\right)^{n-k}.$$

We have

Proposition 7: In a biased contest ($\lambda > 1$), the egalitarian allocation rule leads to more effort than the list allocation ruel when $2\sigma + \beta \geq 2$.

Proof. See appendix. ■

4.5 Other extensions

We discuss here some possible extensions. We have assumed in the model that the number of members in each team is the same as the numbers of prizes that a team can win. We can easily adapt the model to analyze how the number of team members influence total effort under the two allocation rules of interest. When the team uses the egalitarian allocation rule, when condition 1 is satisfied, it is easy to show increasing the number of team members leads to higher aggregate effort while when condition 1 is not satisfied increasing the number of team members leads to lower aggregate effort. Of course, with a list allocation rule, increasing the number of team members has no benefitt. Additional team members have no chance to get a prize and would exert zero effort. One way to interpret these findings is to think on the optimal way to organize a team when the number of team members is higher than the number of potential prizes. We see that our results are in line with the group size paradox literature. We also see that in many environments, the equal minority allocation rule discuss in Ray, Baland and Dagnlie (2007) appears to be optimal.

We can also extend the model to analyze atlruistic team members who also care about the total number of prizes their team wins. The presence of this type of preferences seems especially relevant for the political applications: the typical political candidate does not care only about himself getting elected, but also about the overall performance of his party. There are several ways to model this type of altruism. For isntance we could add a component to the utility function of the individual that depends on the number of prizes won by the team. We could also add a benefit if the team wins a majority of the prizes (majority of seats in an election). The model becomes more difficult to solve and the comparisons between allocation rules less straightforward, but the same condition is still driving the comparison of efforts.

We can also modify the contest technology without changing the result. In the model we ust the Tullock contest success function $\frac{E_i}{E_1+E_2}$ to determine the allocation of prizes among teams, we can easily use $\frac{E_i^r}{E_1^r+E_2^r}$ or $\frac{\gamma}{2}+(1-\gamma)\frac{E_i}{E_1+E_2}$ where r and γ parametrizes the responsiveness of success to effort. It is easy to show that it is still true that the ranking between the egalitarian allocation rule and the list allocation rules follows condition 1.

5 Conclusion and discussion

In this paper, we study the impact of intrateam allocation rules in team contests. We show that the convexity of the marginal cost function and the degree of effort complementarity drive which type of allocation rule is good for incentives. When the marginal cost of effort is convex and when efforts are complement, then egalitarian allocation rules that treat all team memebrs the same way dominate. When the marginal cost of effort is concave and when efforts are substitute, we show that it is optimal to treat team members nonsymmetrically. In particular, we show that list allocation rules are optimal. Our results obtain in a context in which individual efforts are not contractible. The allocation of prizes only depend on the total number of prizes won by the team. In a companion paper, Crutzen and Sahuguet (2017) study a competitive allocation rule in which individual effort also inlfuences the intrateam allocation mechanism. In particular, we study a model in which teams organize a contest for the spots on the list. We show that, if possible, this mechanism leads to very good incentives. More generally, there are still lots of open question on how team contests and team organizations work, for instance, the role of team leaders and the organization of teams with heteregeneous members.

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7 APPENDIX

Proof of proposition 1

We start by proving a useful lemma.

Lemma 1: $\frac{\partial e_{ij}}{dE_j} = \left(\frac{E_j}{e_{ij}}\right)^{\sigma}$.

Proof of lemma 1

Using the definition of E_j, $E_j = \left(\sum_{i=1}^{n} (e_{ij})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, we get:

$$\begin{aligned}
\frac{\partial e_{ij}}{\partial E_j} &= \frac{1}{1-\sigma}(1-\sigma)(e_{ij})^{-\sigma}\left(\sum_{i=1}^{n}(e_{ij})^{1-\sigma}\right)^{\frac{1}{1-\sigma}-1} \\
&= (e_{ij})^{-\sigma}\left(\sum_{i=1}^{n}(e_{ij})^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \\
&= \left(\frac{E_j}{e_{ij}}\right)^{\sigma}.
\end{aligned}$$

■

The first-order condition is:

$$V\frac{\partial E_1}{\partial e_{i1}}\frac{E_2}{(E_1+E_2)^2} - e_{i1}^{\beta-1} = 0.$$

At a symmetric Nash equilibrium, $e_{ij} = e_{kj}$ for any i and k and j, thus $\left(\frac{E_j}{e_{ij}}\right)^{\sigma} = n^{\frac{\sigma}{1-\sigma}}$ and $\frac{E_2}{(E_1+E_2)^2} = 1/4E$. We thus get:

$$\frac{1}{4E}Vn^{\frac{\sigma}{1-\sigma}} - e^{\beta-1} = 0.$$

Thus:

$$e = \left[\frac{1}{4E}Vn^{\frac{\sigma}{1-\sigma}}\right]^{\frac{1}{\beta-1}}$$

Therefore

$$
\begin{aligned}
E &= \left\{ \sum_{k=1}^{n} \left[\left[\frac{1}{4E} V n^{\frac{\sigma}{1-\sigma}} \right]^{\frac{1}{\beta-1}} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
E &= n^{\frac{1}{1-\sigma}} \left(\frac{1}{4E} V \right)^{\frac{1}{\beta-1}} n^{\frac{\sigma}{(1-\sigma)(\beta-1)}}
\end{aligned}
$$

which implies

$$
\begin{aligned}
E &= \left[n^{\frac{1}{1-\sigma}} \left(\frac{V}{4} \right)^{\frac{1}{\beta-1}} n^{\frac{\sigma}{(1-\sigma)(\beta-1)}} \right]^{\frac{\beta-1}{\beta}} \\
E &= \left(\frac{V}{4} \right)^{\frac{1}{\beta}} \left[n^{\frac{1}{1-\sigma}} n^{\frac{\sigma}{(1-\sigma)(\beta-1)}} \right]^{\frac{\beta-1}{\beta}} \\
E &= \left(\frac{V}{4} \right)^{\frac{1}{\beta}} n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.
\end{aligned}
$$

■

Proof of proposition 2

We start by proving a useful lemma.

Lemma 2

$\sum_{k=m}^{n} C_k^n (kp^{k-1} (1-p)^{n-k} - (n-k) (1-p)^{n-k-1} p^k) = mC_m^n p^{m-1} (1-p)^{n-m}$

Proof:

We show that terms in the sum cancel. to cancel terms in the sum. Consider the second term within the summation sign: $(n-k) C_k^n (1-p)^{n-k-1} p^k$. Using the identity $(n-k) C_k^n = (k+1) C_{k+1}^n$ we can write it as $(n-k) C_k^n (1-p)^{n-k-1} p^k = (k+1) C_{k+1}^n (1-p)^{n-k-1} p^k$, which corresponds exactly to the first term within the summation sign for the index $k+1$. These two terms cancel leaving only the first and last term of the sum. The first term is $mC_m^n p^{m-1} (1-p)^{n-m}$. The last term is equal to zero. ■

Under the list system, the individual on the mth position on the list maximizes:

$$
\sum_{k=m}^{n} C_k^n \left(\frac{E_j}{E_1 + E_2} \right)^k \left(1 - \frac{E_j}{E_1 + E_2} \right)^{n-k} V - \frac{e_{mj}^{\beta}}{\beta}.
$$

The first-order condition is:

$$V\sum_{k=m}^{n}\frac{\partial e_{ij}}{\partial E_j}\frac{E_i}{(E_1+E_2)^2}C_k^n\left(k\left(\frac{E_j}{E_1+E_2}\right)^{k-1}\left(1-\frac{E_j}{E_1+E_2}\right)^{n-k}-(n-k)\left(\frac{E_j}{E_1+E_2}\right)^{k}\left(1-\frac{E_j}{E_1+E_2}\right)^{n-k-}\right.$$

At a symmetric equilibrium, using lemma 1 and lemma 2, the first-order condition simplifies to:

$$\left(\frac{E}{e_m}\right)^{\sigma}\frac{m}{E}C_m^n\left(\frac{1}{2}\right)^{n+1}V-e^{\beta-1}=0.$$

Thus

$$\begin{aligned}
& (e_m)^{\beta+\sigma-1}=(E)^{\sigma-1}mC_m^n\left(\frac{1}{2}\right)^{n+1}V\\
\rightarrow\quad & e_m=\left[E^{\sigma-1}mC_m^n\left(\frac{1}{2}\right)^{n+1}V\right]^{\frac{1}{\beta+\sigma-1}}\\
\rightarrow\quad & E=\left\{\sum_{k=1}^{n}\left[\left[E^{\sigma-1}kC_k^n\left(\frac{1}{2}\right)^{n+1}V\right]^{\frac{1}{\beta+\sigma-1}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}\\
\rightarrow\quad & E^{\frac{\beta}{\beta+\sigma-1}}=\left\{\sum_{k=1}^{n}\left[kC_k^n\left(\frac{1}{2}\right)^{n+1}V\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{1}{1-\sigma}}\\
\rightarrow\quad & E=\left\{\sum_{k=1}^{n}\left[kC_k^n\left(\frac{1}{2}\right)^{n-1}\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}\left(\frac{V}{4}\right)^{\frac{1}{\beta}}.
\end{aligned}$$

■

Proof of proposition 3:

Comparing the efforts under both allocation rule we see that the egalitarian rule dominates the list rule when:

$$n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}>\left\{\sum_{k=1}^{n}\left[k\binom{n}{k}\left(\frac{1}{2}\right)^{n-1}\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.$$

We can rewrite this inequality as:

$$1 > \left(\frac{\sum_{k=1}^{n} \left[k \binom{n}{k} \left(\frac{1}{2}\right)^{n-1} \right]^{\frac{1-\sigma}{\beta+\sigma-1}}}{n} \right)^{\frac{1-\sigma}{\beta+\sigma-1}}$$

which simplifies as:

$$\frac{\sum_{k=1}^{n} \left[k C_k^n \left(\frac{1}{2}\right)^{n-1} \right]^{\frac{1-\sigma}{\beta+\sigma-1}}}{n} < 1.$$

Note that $\sum_{k=1}^{n} k C_k^n \left(\frac{1}{2}\right)^{n-1} = n$. We can now use Jensen's inequality to show that whether the inequality is satisfied depends on the concavity or convexity of the function $g(x) = x^{\frac{1-\sigma}{\beta+\sigma-1}}$.

So $\sum_{k=1}^{n} \left[k C_k^n \left(\frac{1}{2}\right)^{n-1} \right]^{\frac{1-\sigma}{\beta+\sigma-1}} / n \leq 1$ when $\frac{1-\sigma}{\beta+\sigma-1} \leq 1$. This last inequality simplifies to $\sigma \geq \frac{2-\beta}{2}$ which completes the proof.

■

Proof of proposition 4

Given the choice of allocation rule by the other team, the leader of a team will choose the allocation rule that maximizes effort and thus the number of prizes won. We need to prove that the condition $\sigma \geq (2-\beta)/2$ also determines the ranking of effort between the egalitarian rule and the list rule when efforts across teams are not symmetric.

We first consider the egalitarian allocation rule.

The first order condition of team 1's ith member under the egalitarian allocation rule is:

$$V \left(\frac{E_1}{e_{i1}}\right)^{\sigma} \frac{E_2}{(E_1+E_2)^2} - (e_{i1})^{\beta-1} = 0.$$

This yields, denoting $p = \frac{E_1}{E_1+E_2}$:

$$V n^{\frac{\sigma}{1-\sigma}} \frac{p(1-p)}{E_1} = (e_{i1})^{\beta-1}.$$

Thus:

$$e_1 = \left[V n^{\frac{\sigma}{1-\sigma}} \frac{p(1-p)}{E_1} \right]^{\frac{1}{\beta-1}}$$

Therefore

$$
\begin{aligned}
E_1 &= \left\{ \sum_{k=1}^{n} \left[\left[V n^{\frac{\sigma}{1-\sigma}} \frac{p\,(1-p)}{E_1} \right]^{\frac{1}{\beta-1}} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
E_1 &= n^{\frac{1}{1-\sigma}} \left(\frac{p\,(1-p)}{E_1} V \right)^{\frac{1}{\beta-1}} n^{\frac{\sigma}{(1-\sigma)(\beta-1)}}
\end{aligned}
$$

which implies

$$E_1 = (p\,(1-p)\,V)^{\frac{1}{\beta}}\, n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}.$$

We now turn to the list allocation rule.

The first order condition of team $1's$ mth member on the list under the list allocation rule is:

$$V \left(\frac{E_1}{e_{i1}} \right)^{\sigma} \frac{E_2}{(E_1+E_2)^2} \sum_{k=m}^{n} C_k^n \left(k \left(\frac{E_1}{E_1+E_2} \right)^{k-1} \left(1 - \frac{E_1}{E_1+E_2} \right)^{n-k} - (n-k) \left(\frac{E_1}{E_1+E_2} \right)^{k} \left(1 - \frac{E_1}{E_1+E_2} \right)^{n}$$

Using lemma 2, this simplifies to

$$V \left(\frac{E_1}{e_{i1}} \right)^{\sigma} \frac{E_2}{(E_1+E_2)^2} m C_m^n \left(\frac{E_1}{E_1+E_2} \right)^{m-1} \left(1 - \frac{E_1}{E_1+E_2} \right)^{n-m} - e_{i1}^{\beta-1} = 0.$$

$$V \left(\frac{E_1}{e_{m1}} \right)^{\sigma} \frac{m}{E_1} C_m^n p^m (1-p)^{n-m+1} - e^{\beta-1} = 0.$$

Thus

$$
\begin{aligned}
\left(e_{m1}\right)^{\beta+\sigma-1} &= V\left(\frac{E_1}{e_m}\right)^{\sigma} C_m^n \frac{m}{E_1} p^m (1-p)^{n-m+1}, \\
e_{m_1} &= \left[V\left(E_1\right)^{\sigma} C_m^n \frac{m}{E_1} p^m (1-p)^{n-m+1}\right]^{\frac{1}{\beta+\sigma-1}}, \\
E_1 &= \left\{\sum_{k=1}^{n}\left[\left[V\left(E_1\right)^{\sigma} C_m^n \frac{m}{E_1} p^m (1-p)^{n-m+1}\right]^{\frac{1}{\beta+\sigma-1}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} \\
E_1^{\frac{\beta}{\beta+\sigma-1}} &= \left\{\sum_{k=1}^{n}\left[kC_k^n p^m (1-p)^{n-m+1} V\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{1}{1-\sigma}} \\
E_1 &= \left\{\sum_{k=1}^{n}\left[kC_k^n p^{m-1} (1-p)^{n-m}\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} \left(p(1-p)V\right)^{\frac{1}{\beta}}.
\end{aligned}
$$

We need to compare the expected number of prizes won under both allocation rules, with the other team's allocation rule fixed. We know that if both teams choose the same allocation rule, the efforts are symmetric and both parties win on average $n/2$ prizes. We therefore need to compare efforts when teams chooses different allocation rule.

From the calculation above, if team 1 uses the egalitarian rule and team 2 uses the list rule, denoting $p = E_1/(E_1+E_2)$, we get:

$$
\begin{aligned}
E_1 &= \left(p(1-p)V\right)^{\frac{1}{\beta}} n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}, \\
E_2 &= \left\{\sum_{k=1}^{n}\left[kC_k^n (1-p)^{k-1} p^{n-k}\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}} \left(p(1-p)V\right)^{\frac{1}{\beta}}.
\end{aligned}
$$

Dividing them, we get:

$$
E_2/E_1 = \frac{\left\{\sum_{k=1}^{n}\left[kC_k^n (1-p)^{k-1} p^{n-k}\right]^{\frac{1-\sigma}{\beta+\sigma-1}}\right\}^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}}{n^{\frac{\beta+\sigma-1}{\beta(1-\sigma)}}}.
$$

We have $\sum_{k=1}^{n}\left[kC_k^n (1-p)^k p^{n-k}\right] = 1-p$ and $\sum_{k=1}^{n}\left[kC_k^n (1-p)^{k-1} p^{n-k}\right] = 1$.

We use the same argument based on Jensen's inequality used in the proof of Theorem 1 to show that the condition $\sigma \geq \frac{2-\beta}{2}$ determines which teams exerts the most effort. ■

Proof of proposition 5

When both teams use the egalitarian rule, in a symmetric equilibrium, individual efforts are given by the participation constraint:

$$e^{\beta}/\beta = V/2.$$

Individual effort is given by:

$$e = (\beta V/2)^{1/\beta}.$$

This leads to an aggregate effort of

$$\begin{aligned} E &= \left(\sum e^{1-\sigma}\right)^{1/1-\sigma} \\ &= n^{1/1-\sigma} e \\ &= n^{1/1-\sigma} (\beta V/2)^{1/\beta}. \end{aligned}$$

Turning to the case where both teams use the list allocation rule, at a symmetric equilibrium, individual effort of the team member in m^{th} position is given by the participation constraint:

$$\begin{aligned} (e_m)^{\beta}/\beta &= \sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^n V \\ e_m &= \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^n \beta V\right)^{1/\beta}. \end{aligned}$$

Aggregate effort is thus:

$$\begin{aligned} E &= \left(\sum_{m=1}^{n} e_m^{1-\sigma}\right)^{1/1-\sigma}. \\ &= \left(\sum_{m=1}^{n} \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^n \beta V\right)^{(1-\sigma)/\beta}\right)^{1/1-\sigma}. \\ &= \left(\sum_{m=1}^{n} \left(\sum_{k=m}^{n} C_k^n \left(\frac{1}{2}\right)^{n-1}\right)^{(1-\sigma)/\beta}\right)^{1/1-\sigma} (\beta V/2)^{1/\beta} \end{aligned}$$

The argument of theorem 1 applies but now the condition for the egalitarian rule to lead to more effort is that $1-\sigma/\beta<1$ that is $\beta>1-\sigma$ which is always the case.$\blacksquare$

Proof of proposition 6

The proof of proposition 2 applies directly with $p=E_1/\sum_{j=1}^{K}E_j$.

$\blacksquare$

Proof of proposition 7

The proof of proposition 2 applies directly with $p=\lambda E_1/\left(\lambda E_1+E_2\right)$.

We have $\frac{\partial}{\partial e_{i1}}\lambda E_1/\left(\lambda E_1+E_2\right)=\frac{\lambda E_2}{\left(\lambda E_1+E_2\right)^2}=\frac{p(1-p)}{E_1}$.

$\blacksquare$

Proof of egalitarian with $l>n$ members

$$\begin{aligned}
E &= \left\{\sum_{k=1}^{l}\left[\left[\frac{1}{4E}Vn^{\frac{\sigma}{1-\sigma}}\frac{n}{l}\right]^{\frac{1}{\beta-1}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}\\
E &= l^{\frac{1}{1-\sigma}}\left(\frac{1}{4E}\frac{n}{l}V\right)^{\frac{1}{\beta-1}}n^{\frac{\sigma}{(1-\sigma)(\beta-1)}}
\end{aligned}$$

which implies

$$\begin{aligned}
E &= \left[l^{\frac{1}{1-\sigma}}\left(\frac{1}{4}\frac{n}{l}V\right)^{\frac{1}{\beta-1}}n^{\frac{\sigma}{(1-\sigma)(\beta-1)}}\right]^{\frac{\beta-1}{\beta}}\\
E &= l^{\frac{\beta-1}{\beta(1-\sigma)}}l^{-1/\beta}\left(\frac{V}{4}\right)^{\frac{1}{\beta}}\left[n^{\frac{1}{\beta-1}}n^{\frac{\sigma}{(1-\sigma)(\beta-1)}}\right]^{\frac{\beta-1}{\beta}}\\
E &= l^{\frac{\beta+\sigma-2}{\beta(1-\sigma)}}\left(\frac{V}{4}\right)^{\frac{1}{\beta}}n^{\frac{\sigma+1}{\beta(1-\sigma)}}.
\end{aligned}$$

We see that the sign of$\frac{\partial E}{\partial l}$ depends on the sign of $\frac{\partial}{\partial l}l^{\frac{\beta+\sigma-2}{\beta(1-\sigma)}}$, which depends on the sign of $\beta+\sigma-2$.