Duopolies with social propensity

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Abstract

In duopolies where in the first stage firms engage in a Bertrand competition, we study the effect of introducing a generic form of social influence in the utility of a finite set of consumers, who choose strategically taking into account the consumption choice of other consumers. We characterize local market equilibria through the notion of subgame-perfect Nash equilibrium with first stage local pure price equilibrium for firms.

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1 Introduction

A decisive main characteristic of a market is how demand changes, in particular how it reacts to price. In the duopoly case under consideration, this essencially relies on the social profile and on the properties of the social externality function, as they reveal how consumers interact and the interdependence of their choices, which ultimately will be inherited by demand. The choice and characteristics of the social externality function are thus decisive to determine the type of duopoly and consequent results. Two main lines can be identified as crucial in this choice: the degree of social heterogeneity and its functional form. Both play a role on the existence of equilibria, but while social heterogeneity will essencially play a role on the symmetries of the equilibrium set (different from heterogeneity of consumers personal preferences), the smoothness properties of demand rely mainly on the functional form. Nevertheless, to understand demand changes, the crucial aspect is not the externality function itself, but rather how changes in some consumer strategy affect the rest of consumers. As an example think of a social network like facebook. There may be a large network of conections between users, which naturally provoke externalities, but the decision itself depend on how users look and interpret this conections, how they are influenced by them, which need not be by the whole network. In our context this superstructure within the actual consumer network structure is what we call the *local influence network*. The nodes in the network are consumers and the edges represent the influence two consumers have on eachother, which is dependent on the context created by the consumers choice. The consumer network is thus directed, weighted and state-dependent. Hence, consumers may have diferent influence on eachother, and that influence need not be symmetric nor have the same value throughout the network. Furthermore, it is state-dependent in the sense that the weight will depend on the consumers choice. A natural way to represent the network is through its weighted adjacency matrix. In order to use the standard notation and to provide a more intuitive representation, an weighted directed edge from i to j should represent the influence i has on j, which should be the value of the entry ij. The adjacency matrix of the influence network is defined as the transpose of the Jacobian matrix.[1] The influence network reveals changes in social differentiation provoked by a change in consumers strategy. This in turn provokes changes in the consumers utility differential. Note however that for loyal consumers this may not result in a strategy change, unless the Nash equilibrium condition is strict, a change needs to be sufficiently high to result in a change of their best response. As such, when the best reply of loyal consumers is constant we say that loyal consumers have *lower sensitivity*. This means that the crucial aspect to capture local changes in demand is the non-loyal consumers influence network. The idea that loyal consumers may have lower sensibility and not contribute to social propensity is rather natural, and intuitive to the very notion of brand loyalty. Note however that loyalty differs from installed base, since being loyal is a strategical behavior (those who opt for pure strategies) and not an exogeneously imposed choice, or a choice deriving from some switch cost or other stabilizing variable. We are interested in duopolies with some social propensity.

Social propensity is a measure of how changes are captured by the social component of a market. The interpretation is that it reveals how consumers may change their strategy in response to local changes in the overall consumer profile. We prove the existence of local market equilibria with shared demand and positive profits, but also characterize completely prices and show that the equilibria reveals consumer personal preferences. The conditions are rather general and rely exclusively on the properties of the social profile through the social propensity index. Socially prone duopolies thus disrupt the Bertrand paradox and provide pure price solutions These solutions do not rely on heterogeneity to exist or to be assymetrical.

2 The duopoly setup

We consider the duopoly as a two stage game. In the first stage, the *firms subgame*, two firms independently and simultaneously set a price for the service

[1] We hope this comment avoids more confusion than it creates. This is just a clarification, as it will only be used in the graphical representation.

they provide: p_1 for the service provided by firm 1, p_2 for the service provided by firm 2, defining the price profile $\mathbf{p} \equiv (p_1, p_2)$. We assume firms have no costs in providing the service (neither variable nor fixed costs). In the concluding section we show that introducing a cost structure does not change the results as it leads to an isomorphic set of equilibria through a change of parameters. In the second stage, the *consumers subgame*, a finite set of consumers (individuals) $\mathcal{I} \equiv \{1, \ldots, n\}$ observe the prices set in the first stage and each consumer $i \in \mathcal{I}$ independently decides the probability σ_1^i and σ_2^i of using each one of the two services $S \equiv \{s_1, s_2\}$, provided, respectively, by each one of the two firms. The choice is mandatory i.e. $\sigma_1^i + \sigma_2^i = 1$ (there is no reservation price set exogeneously), which means that given a pair of prices from the first stage, the choice of a consumer is a probability distribution over the set of services S provided by firms, i.e. over the space of pure strategies. Consumers are thus assumed to use standard mixed strategies in the space $\mathcal{S} \equiv S^n$. For simplicity of notation we will identify the distribution by a single parameter $(\sigma^i, 1 - \sigma^i) \equiv (\sigma_1^i, \sigma_2^i)$ and as such the space of mixed strategies can be identified with $[0, 1]^n$. The consumers choice is summarized in the profile of consumer (mixed) strategies denoted by $\boldsymbol{\sigma} \equiv (\sigma^1, \ldots, \sigma^n) \in [0, 1]^n$. An outcome of the game is a pair $(\mathbf{p}, \boldsymbol{\sigma}) \in (\mathbb{R}_0^+)^2 \times [0, 1]^n$ formed by a pair of prices $\mathbf{p}$ and a consumers choice $\boldsymbol{\sigma}$. The characterization of outcomes that can arise in a market equilibrium will be done according to the notion of subgame perfect equilibrium, hence by characterizing the Nash equilibria of both stages.

2.1 Firms

The demand for each firm stems from the profiles of consumer choices that maximize their utility, and it is therefore contained in the set of Nash equilibrium of the consumers subgame. For a given pair of prices $\mathbf{p}$ from the first stage, the *Nash domain* $\mathcal{N}(\mathbf{p})$ for the consumers subgame is the set of consumers choices $\boldsymbol{\sigma}$ that are a Nash equilibrium of the consumers subgame. We say that an outcome $(\mathbf{p}, \boldsymbol{\sigma})$ is credible if $\boldsymbol{\sigma} \in \mathcal{N}(\mathbf{p})$. In characterizing demand it is useful to use the partition of the set of individuals for a given consumers choice $\boldsymbol{\sigma}$ according to whether they use a pure strategy or a strictly mixed strategy. Let us call *loyal* consumers to those consumers who choose firm 1 or 2 with probability one, and *non-loyal* consumers to those using a strict mixed strategy. The partition is given by $\mathcal{L}_1(\boldsymbol{\sigma}) \cup \mathcal{L}_2(\boldsymbol{\sigma}) \cup \mathcal{M}(\boldsymbol{\sigma})$, where $\mathcal{L}_1(\boldsymbol{\sigma}) \equiv \{i \in \mathcal{I} : \sigma_1^i = 1\}$ and $\mathcal{L}_2(\boldsymbol{\sigma}) \equiv \{i \in \mathcal{I} : \sigma_2^i = 1\}$ are the sets of consumers loyal to each firm and $\mathcal{M}(\boldsymbol{\sigma}) \equiv \{i \in \mathcal{I} : 0 < \sigma_1^i < 1\}$ is the subset of non-loyal consumers (those that play with strictly mixed strategies, i.e. non-integer probabilities). The number of loyal consumers are respectively given by the cardinalities $l_1(\boldsymbol{\sigma}) \equiv \#\mathcal{L}_1(\boldsymbol{\sigma})$, $l_2(\boldsymbol{\sigma}) \equiv \#\mathcal{L}_2(\boldsymbol{\sigma})$, and $m(\boldsymbol{\sigma}) = n - l_1(\boldsymbol{\sigma}) - l_2(\boldsymbol{\sigma})$. We call (l_1, l_2) the *loyalty characterization* of the outcome $(\mathbf{p}, \boldsymbol{\sigma})$, omitting the dependence when it is clear what outcome we are refereing to. The demand for each firm is, respectively, given by

$$D_1(\boldsymbol{\sigma}) \equiv l_1(\boldsymbol{\sigma}) + \sum_{i \in \mathcal{M}(\boldsymbol{\sigma})} \sigma^i, \quad D_2(\boldsymbol{\sigma}) \equiv l_2(\boldsymbol{\sigma}) + \sum_{i \in \mathcal{M}(\boldsymbol{\sigma})} (1 - \sigma^i).$$

Note that, as the choice is mandatory it always leads to full market coverage $D_1(\boldsymbol{\sigma}) + D_2(\boldsymbol{\sigma}) = n$. The *profit* $\Pi : \mathcal{R} \times (\mathbb{R}_0^+)^2 \to \mathbb{R}$ is, respectively, given by

$$\Pi_1(p_1, \boldsymbol{\sigma}) = p_1 D_1(\boldsymbol{\sigma}), \quad \Pi_2(p_2, \boldsymbol{\sigma}) = p_2 D_2(\boldsymbol{\sigma}).$$

Local deviation beliefs. For the characterization of price equilibria it is necessary to understand the dependence of consumer behavior on prices. In particular, to figure out if a given price is a best response, each firm needs to know how consumers would react to a price change. In this regard, we consider that firms have *local deviation beliefs*: given an outcome $(\mathbf{p}^*, \boldsymbol{\sigma}^*)$ and a neighbourhood $P_1(p_1^*) \times P_2(p_2^*) \subset (\mathbb{R}_0^+)^2$ of the outcome prices $\mathbf{p}^* = (p_1^*, p_2^*)$, local deviation beliefs are maps $\phi_1 : P_1 \to \mathcal{N}(P_1, p_2^*)$ and $\phi_2 : P_2 \to \mathcal{N}(p_1^*, P_2)$ that represent how firms believe consumers will respond to small price deviations. That is, firm 1 believes that a deviation from charging price p_1^* to charging price $p \in P_1$ will lead consumers to respond with a change from the given consumer choice $\boldsymbol{\sigma}^*$ to a consumer choice $\phi_1(p) \in \mathcal{N}(p, p_2^*)$, producing demand $D_1(\phi_1(p))$. Analogously for ϕ_2, the deviation belief of firm 2. By definition $\phi_1(p_1^*) = \phi_2(p_2^*) = \boldsymbol{\sigma}^*$.[2] As deviation beliefs are a way for firms to evaluate if a deviation is profitable, and we will define market equilibrium through the notion of subgame-perfect equilibrium, it is natural to consider only credible beliefs by restricting beliefs to credible outcomes. We say that a local deviation belief ϕ *preserves loyalty* if we have $\mathcal{L}_1(\phi) = \mathcal{L}_1(\boldsymbol{\sigma}^*)$ and $\mathcal{L}_2(\phi) = \mathcal{L}_2(\boldsymbol{\sigma}^*)$; and we say that firms have a *common local belief* if $\phi_1(p_1^* + \varepsilon) = \phi_2(p_2^* - \varepsilon)$. The profit expected from a small price deviation, taking into account local deviation beliefs, is $\Pi_1^*(p_1, \phi_1) = p_1 D_1(\phi_1(p_1))$ for firm 1, and $\Pi_2^*(p_2, \phi_2) = p_2 D_2(\phi_2(p_2))$ for firm 2.

2.2 Consumers

Given an outcome $(\mathbf{p}, \boldsymbol{\sigma})$ the payoff of a consumer is built on the utility derived from the use of each service which depends on three components: (i) the price of each service; (ii) the personal benefit derived from the use of each service; and (iii) the externality arising from the social influence exerted at each service by the choice of the other consumers. We assume that the utility has the property that personal and social components are commensurable with money. Therefore, we can characterize the payoff a consumer i derives from the use of a service s through: (i) the personal component $\omega(i; s) = -p_s + b_s$, which is additively separable in price and personal benefit $b_s^i \in \mathbb{R}$; and (ii) the social component measured by some social externality function $e : \mathcal{I} \times S \times [0,1]^{(n-1)} \to \mathbb{R}$. With this, the use of each service respectively induces the following payoffs

$$u_1^i(p_1; \boldsymbol{\sigma}_{-i}) = -p_1 + b_1^i + e_1^i\left(\boldsymbol{\sigma}_{-i}\right), \qquad u_2^i(p_2; \boldsymbol{\sigma}_{-i}) = -p_2 + b_2^i + e_2^i\left(\boldsymbol{\sigma}_{-i}\right).$$

The utility function $u : \mathcal{I} \times (\mathbb{R}_0^+)^2 \times [0,1]^n \to \mathbb{R}$ is given by

$$u^i(\mathbf{p}, \boldsymbol{\sigma}) = \sigma_1^i u_1^i(p_1; \boldsymbol{\sigma}_{-i}) + \sigma_2^i u_2^i(p_2; \boldsymbol{\sigma}_{-i}).$$

Product differentiation. The type of duopoly in consideration and consequent results are naturally heavily dependent on the choice of the consumers

[2] This is not interely a new concept, just a reinterpretation of mixed strategy in the context of multistage games (see for example [?] p103). In our case we only want the local part of deviation beliefs.

utility function as the personal and social parameters and its relation with prices will determine the Nash equilibria of the consumers subgame, and ultimately market equilibria. Nevertheless, whether a consumers choice is a Nash equilibrium or not is invariant to changes of parameters that do not affect the utility differentials $\Delta u^i = u_2^i - u_1^i$. Consequently, the characterization of equilibria can be done up to isomorphism through the differentials induced by Δu, namely, the price differential

$\Delta p \equiv p_2 - p_1,$ (*price difference*).

and the differentials of personal benefit and social externalities, which characterize product differentiation and are given by

$\Delta b^i \equiv b_2^i - b_1^i$ (*standard product differentiation*);

$\Delta e^i(\boldsymbol{\sigma}_{-i}) \equiv e_2^i(\boldsymbol{\sigma}_{-i}) - e_1^i(\boldsymbol{\sigma}_{-i})$ (*social product differentiation*).

Observe that while standard product differentiation is 'intrinsic' to a consumer, social product differentiation has a contextual nature, in the sense of representing how consumers differentiate the product taking into account its momentaneous consumption profile. The duopoly is thus characterized by two maps: the *personal profile* $\Delta\mathbf{b} : \mathcal{I} \to \mathbb{R}^n$ and the *social profile* $\Delta\mathbf{e} : [0,1]^{n_\mathcal{I}} \to \mathbb{R}^n$ which form the *consumers profile* $(\Delta\mathbf{b}, \Delta\mathbf{e})$.

Local influence network. Based on the social profile we can build a local network of influences that reveals how small changes in the consumers strategy change social differentiation, and thus payoffs. Note however that for loyal consumers this may not result in a strategy change. When the best-reply contains a pure strategy, two situations can occur: either the best-reply is constant in the neighborhood of the outcome $(\mathbf{p}, \boldsymbol{\sigma})$, i.e. $\mathrm{br}_i(\mathbf{p}, \boldsymbol{\sigma}_{-i} + \varepsilon) = \mathrm{br}_i(\mathbf{p}, \boldsymbol{\sigma}_{-i})$ for some $\varepsilon > 0$, or it is not constant and thus susceptible to small changes. What we mean is that for these consumers, unless the Nash equilibrium condition is strict, a change needs to be sufficiently high to result in a change of their best response. As such, when the best reply of loyal consumers is constant we say that loyal consumers have *lower sensitivity.* The crucial aspect to capture local changes in demand is the non-loyal consumers strategy. Let us define the network: the nodes are non-loyal consumers and the edges represent the influence two consumers have on eachother, which is captured by the partial derivatives $\partial\Delta e^i/\partial\sigma^j(\boldsymbol{\sigma})$. The consumer network is completely characterized by the non-loyal Jacobian matrix

$$J_{\Delta e}(\boldsymbol{\sigma}; \mathcal{M}) \equiv \left[(\partial\Delta e^i/\partial\sigma^j)(\boldsymbol{\sigma}), i, j \in \mathcal{M}\right].$$

Note that the network is directed, weighted and state-dependent. Hence, consumers may have diferent influence on eachother, and that influence need not be symmetric nor have the same value throughout the network. Furthermore, it is state-dependent in the sense that the weight will depend on the consumers choice.

Let now $J_{\Delta e}^{(i)}$ be the matrix obtained by replacing column i with -1 in $J_{\Delta e}$ (from Cramer's rule).

Definition 1 (Social propensity index)**.** The *social propensity index* κ of a consumers choice $\boldsymbol{\sigma}$ is

$$\kappa(\boldsymbol{\sigma}) \equiv \frac{\det\left[J_{\Delta e}(\boldsymbol{\sigma}; \mathcal{M})\right]}{\sum_i \det\left[J_{\Delta e}^{(i)}(\boldsymbol{\sigma}; \mathcal{M})\right]}.$$

Social propensity is a measure of how changes are captured by the social component of a market. The interpretation is that it reveals how consumers may change their strategy in response to local changes in the overall consumer profile. When $\kappa > 0$ changes in demand are amplified by social differentiation (similar to a conformity, herd or bandwagon effect). When $\kappa < 0$ changes in demand are mitigated by social differentiation (similar to a congestion, snob or Veblen effect). Hence, the response of demand to prices is amplified or mitigated by social differentiation, acording to the social propensity of non-loyal consumers. When $\#\mathcal{M}(\boldsymbol{\sigma}) = 1$ (there is at most one non-loyal consumer), we have $\kappa_\alpha = 0$, since social externality is by definition the effect of others, and the diagonal entries of A are zero. When $\mathcal{M}(\boldsymbol{\sigma}) = \emptyset$, if loyal consumers have lower sensitivity we assume $\kappa = -\infty$, else we assume $\kappa = 0$. We are interested in duopolies with some social propensity.

Socially prone duopolies. We say that a duopoly is *socially prone* if there is a set of consumer choices $\mathsf{Sp} \subset [0,1]^n$ with the following properties: for every $\boldsymbol{\sigma} \in \mathsf{Sp}$

1. *Loyal consumers have lower sensitivity*;
2. *Non-loyal consumers are socially prone:* $\kappa(\boldsymbol{\sigma}) \neq 0$.

Socially prone outcomes are non-monopolistic. Property (i) means that in a neighborhood of the outcome the best-reply of a loyal individual is constant, i.e. for $i \in L(\boldsymbol{\sigma})$ $\mathrm{br}_i(\mathbf{p} + \varepsilon, \boldsymbol{\sigma}_{-i} + \delta) = \mathrm{br}_i(\mathbf{p}, \boldsymbol{\sigma}_{-i})$. Note that, by the results of the first chapter, there is an open set of personal preferences $\boldsymbol{\Delta}\mathbf{b}$ such that (i) holds. Furthermore, the set of consumer choices such that (ii) holds is dense in $[0,1]^n$. So Sp is a well behaved set, and in general $\mathsf{Sp} \neq \emptyset$.

There are some natural restricions that non-zero social propensity imposes on the network of non-loyal consumers. Namely, the strong components of the network cannot be singletons, i.e. there there are no sinks or sources. The idea that loyal consumers may have lower sensibility and not contribute to social propensity is rather natural, and intuitive to the very notion of brand loyalty. Note however that loyalty differs from installed base, since being loyal is a strategical behavior (those who opt for pure strategies) and not an exogeneously imposed choice, or a choice deriving from some switch cost or other stabilizing variable.

3 Local Market Equilibria

An outcome $(\mathbf{p}^*, \boldsymbol{\sigma}^*)$ with associated local deviation beliefs ϕ_1, ϕ_2 forms a *local market equilibrium* if it is a subgame-perfect equilibrium for an open set containg $\mathbf{p}^*$, that is, if $\boldsymbol{\sigma}^* \in \mathcal{N}(\mathbf{p}^*)$ and $\mathbf{p}^*$ is a local Nash equilibrium for the firms subgame taking into account their deviation beliefs. More formally, the prices $\mathbf{p}^*$ are a *local pure price equilibrium* for firms if there is a neighbourhood $P_1 \times P_2$ of prices, in which, for $j = 1, 2$, and for all $p_j \in P_j(p_j^*)$, we have $\Pi_j(p_j, \phi_j) \leq \Pi_j(p_j^*, \boldsymbol{\sigma}^*)$. Although we are using standard definitions, let us define the notion of local market equilibrium formally to emphasize the notion that it is local in prices and subgame-perfect.

Definition 2 (Local market equilibrium)**.** An outcome $(\mathbf{p}, \boldsymbol{\sigma})$ with deviation beliefs ϕ_1, ϕ_2 is a *local market equilibrium* if (i) $\boldsymbol{\sigma}$ is a Nash equilibrium of the consumers subgame, i.e. $\boldsymbol{\sigma} \in \mathcal{N}(\mathbf{p})$; and (ii) $\mathbf{p}$ is a local pure price equilibrium for the firms subgame.

Recall that both demand and beliefs must come from strategies contained in the Nash equilibria of the consumers subgame, hence as price is the unique strategic variable for firms, the characterization of local market equilibrium is essentially dependent on the local structure of $\mathcal{N}(\mathbf{p})$. Namely, through characterization of admissable deviation beliefs, which rely on the existence or non-existence of multiple equilibria, the relation between loyal and non-loyal consumers and the price regions where they hold.

Main result

Suppose there is no product differentiation, meaning that $\Delta b^i = 0$ and $\Delta e^i = 0$ for all $i \in \mathcal{I}$. The game becomes essencially the original Bertrand framework. The paradox arises since $\mathcal{N}(\mathbf{p})$ is a singleton except when $\Delta p = 0$, which induces the following unique demand beliefs: $D_1(\phi_1^*) = n$ if $\Delta p < 0$; $D_1(\phi_1^*) = 0$ if $\Delta p > 0$. [3] This means the only credible non-monopolistic outcomes have associated discontinuous beliefs, which leads to the paradox. Since $\mathcal{I}$ is finite, introducing standard product differentiation $\Delta b^i \neq 0$ for some subset of consumers $I \subseteq \mathcal{I}$, but no social differentiation $\Delta e^i = 0$ for all $i \in \mathcal{I}$, may lead to a shift to a monopoly equilibrium, but the behavior of consumers is identical. The set $\mathcal{N}(\mathbf{p})$ still a singleton except for a finite set of prices where $\Delta b^i = \Delta p$ for at least some consumer $i \in \mathcal{I}$. For all other prices, consumers best response is unique and consumers will use pure strategies, which will again lead to beliefs that are either discontinuous or constant in a neighbourhood of the outcomes candidate for equilibria, thus creating an incentive for firms to deviate. The introduction of a social component, will give rise to conected price regions where, not only there are multiple Nash equilibria for the consumers subgame, but these include strictly mixed strategy equilibria with larger domains. This means there are price regions where the loyalty characterization is constant and small price changes may be captured by non-loyal consumers, leading to smooth changes in demand. Nevertheless, although $\mathcal{N}(\mathbf{p})$ is no longer a singleton almost everywhere, if beliefs are contained in the pure strategy choices of consumers, a market equilibrium where both firms have positive profits will still fail to exist.

Lemma 1. If firms beliefs are that consumers use only pure strategies, or use pure strategies for almost every price, i.e. $\mathcal{M}(\phi) = \emptyset$ a.s., then in a market equilibrium at least one firm has zero profit.

Naturally an equilibrium may not exist, but for any outcome, a deviation, if admissable, is profitable for firms. When we allow consumers to use mixed strategies, the effect of a price deviation may be captured by smooth changes in the non-loyal consumers probability through the externality function. The properties of the externality function will be inherited by demand and allow the existence of continuous deviation beliefs that stabilize prices and create market equilibria where both firms earn positive profits. The drawback with these new equilibium for consumers is the coordination posed by the multiplicity of

[3] There are multiple equilibria only when $\Delta p = 0$, and in fact as any choice is a Nash equilibrium of the consumers subgame $\mathcal{N}(p, p) = [0, 1]^n$.

Nash equilibria of the consumers subgame. Firms will thus have a coordination problem, since local deviation beliefs are in general not unique.

Lemma 2 (Demand responsiveness)**.** Consider a socially prone duopoly and a credible outcome $(\mathbf{p}^*, \boldsymbol{\sigma}^*)$ with a socially prone consumer choice $\boldsymbol{\sigma}^* \in \mathsf{Sp}$. There is a unique continuous local deviation belief ϕ^*. Furthermore, this belief preserves loyalty, is common for both firms and in equilibrium

$$\frac{\partial D_s}{\partial p_s}(\phi^*(\mathbf{p})) = \frac{1}{\kappa(\boldsymbol{\sigma}^*)},\ s \in S.$$

Although discontinuous beliefs are credible alternatives, since they are contained in the set of Nash equilibria of the consumers subgame, they are hard to justify from an economic perspective. It's hard to envision a situation where firms believe that small price deviations provoke a disruptive behavior in consumers, when there is a credible smooth alternative. The second part of lemma 2 shows why outcomes with positive social propensity will in general not allow for equilibria with positive profits, while negative social propensity will create the effect of slowing the demand response to price changes, opening the possibility of a shared market equilibrium. When social propensity gets close to zero it leads towards jumps, meaning consumers will be essentially reacting to price, which may prevent firms from finding an equilibrium. On the other hand if social propensity tends to infinity, consumers will essencially not be reacting to price changes at all, which leads to the opposite effect, also preventing firms from finding non-monopolistic equilibria.

Theorem 1 (Local Market Equilibrium)**.** *Consider a socially prone duopoly and let $\boldsymbol{\sigma} \in \mathsf{Sp}$. The outcome $(\mathbf{p}, \boldsymbol{\sigma})$ is a local market equilibrium with continuous deviation beliefs and positive profits for both firms if, and only if, $\kappa(\boldsymbol{\sigma}) < 0$, prices are given by*

$$p_1 = -\kappa(\boldsymbol{\sigma})D_1(\boldsymbol{\sigma})$$

$$p_2 = -\kappa(\boldsymbol{\sigma})D_2(\boldsymbol{\sigma})$$

and personal preferences for $i \in \mathcal{M}(\boldsymbol{\sigma})$ are

$$\Delta b^i = -\kappa(\boldsymbol{\sigma})\Delta D(\boldsymbol{\sigma}) - \Delta e^i(\boldsymbol{\sigma}_{-i}).$$

Furthermore, there is a unique continuous local deviation belief, which is common for both firms and preserves loyalty.

Note that if $\boldsymbol{\sigma} \in \mathsf{Sp}$ then $0 < D_1(\boldsymbol{\sigma}), D_2(\boldsymbol{\sigma}) < n$. The theorem, which is the main result of this work not only proves the existence of local market equilibria with shared demand and positive profits, but also characterizes completely its prices and reveals consumer personal preferences. The conditions are rather general and rely exclusively on the properties of the social profile through the social propensity index. Socially prone duopolies thus disrupt the Bertrand paradox and provides pure price solutions These solutions do not rely on heterogeneity to exist or to be assymetrical. Note that this work focus on local equilibria, so in order to obtain a global solution, one would need to discuss firms belief farther away from the local behavior of consumers. In these cases, firms need not coordinate on the same belief.

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