So close yet so unequal: Reconsidering spatial inequality in U.S. cities*

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March 2017

Abstract

Two new Gini-type spatial inequality indices are introduced to measure the average degree of income inequality within individual neighborhoods and the inequality of average incomes among individual neighborhoods. Connections with geostatistics and the asymptotic distributions of these indices are investigated. A rich database from the U.S. census is used to study spatial inequality in the 50 largest American metro areas during the last 35 years. Four different types of cities are identified along the lines of inequality between and within individual neighborhoods. This latter is shown to be associated with lifelong economic opportunities and life expectancy of urban residents.

Keywords: Neighborhood inequality, Gini, individual neighborhood, variogram, geostatistics, census, ACS, causal neighborhood effects, life expectancy, divided city, mixed city.

JEL codes: D31, D63, C21, R23, J62, I14.

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*We are grateful to Rolf Aaberge, Nathaniel Baum-Snow, Alberto Bisin, Martina Menon, Patrick Moyes, John Östh, Alain Trannoy, Claudio Zoli and seminar participants at the Catholic University of Milan, LISER and the 2017 Canazei Winter School for valuable comments on a preliminary draft.

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1 Introduction

The growing debate on the spatial dimension of income inequality in the U.S. has made it clear that not all cities are equal (Chetty, Hendren, Kline and Saez 2014). Income inequality in some cities has skyrocketed in the last decades, while remaining relatively low in others. For instance, the Gini index of disposable equivalent household income in New York City in 2014 is over 0.5, while it is below 0.4 in other major cities such as Washington, DC. The differences of income inequality across major U.S. cities can be explained by the distribution of skills and human capital across the cities (Glaeser, Resseger and Tobio 2009, Moretti 2013), as well as by the composition of local amenities (Albouy 2016). Important consequences arise for local policies, for targeting program participation based on location and for designing the federal redistribution of resources (Sampson 2008, Reardon and Bischoff 2011).

What this picture fails to show is that not all places in the same city are made equally unequal. Contributions at the frontier of economics, sociology and urban geography have recognized that inequality at the local level, i.e. measured among close neighbors, is generally not representative of citywide inequality in U.S. cities. This is a consequence of underlying behavior of households who sort across the urban space on the basis of their income (de Bartolome and Ross 2003, Brueckner, Thisse and Zenou 1999), thus producing a spatial pattern of income inequality. This paper contributes to the measurement of spatial income inequality and to assess its long-term implications on economic mobility and health perspectives of residents in American cities.

The features of spatial inequality at the urban level are accounted for in the literature by focusing on differences in incomes within and between neighborhoods, identified by the administrative partition of the urban space (Shorrocks and Wan 2005, Dawkins 2007, Wheeler and La Jeunesse 2008, Kim and Jargowsky 2009). Evaluations based on this approach, however, put the administrative neighborhood and not the individual, who is responsible for sorting decisions, at the center. The geography of incomes can be better taken into account by adopting the notion of the individual neighborhood, corresponding to the set of neighbors living within a certain distance of the individual, thereby placed at the center of his own neighborhood.[1]

In this paper, the features of inequality at the urban level are assessed by studying how incomes are distributed within and across individual neighborhoods. Both within and between dimensions of inequality arise from differences in income among individuals. More precisely, inequality *within* an individual neighborhood arises when the income of an urban resident is different from the income of her close neighbors. Inequality *between* individual neighborhoods arises, instead, from differences in average neighborhood incomes among all urban residents. The two dimensions account for separate consequences of income sorting across the urban space and, differently from existing approaches, they do not stem from a decomposition of citywide inequality. Rather, citywide inequality can be seen as an asymptotic case of spatial inequality where the role of individual neighborhood is weakened to the extreme. These points are made clear with an intuitive example, based

[1] Galster (2001) and Clark, Anderson, Östh and Malmberg (2015) develop this notion in geographic analysis. On the one hand, individual neighborhoods capture the relevant space where factors such as the housing market, amenities, preferences and social interactions (Schelling 1969) combine to shape the sorting of high income and low income people across the city. On the other hand, the notion of individual neighborhood is used to identify the extent of external effects exerted by neighbors on the individual's behavior and income (Durlauf 2004, Sampson 2008, Ludwig, Duncan, Gennetian, Katz, Kessler, Kling and Sanbonmatsu 2013, Chetty, Hendren and Katz 2016).

on the spatial distributions of incomes in three hypothetical cities shown in Figure 1.

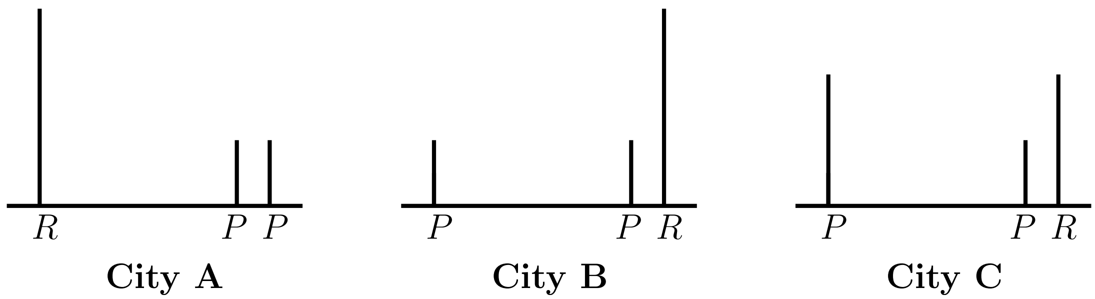
Consider first the two stylized cities *City A* and *City B*. There are three people living in each city, two poor (P) and one rich person (R). The two cities display the same overall income inequality, but differ in the way people are located in the urban space: in City A the poor persons are close neighbors and the rich person is isolated, while in City B a poor person lives near the rich one, and the other poor person is isolated. To evaluate spatial inequality, we first identify individual neighborhoods by drawing circles of given diameter around each individual, and then we study the income distribution within and among individual neighborhoods. The size (or equivalently, the degree of inclusiveness) of the individual neighborhood can vary. When the size is not too large,[2] the average degree of inequality (captured by the extent of income differences) within the individual neighborhoods is smaller in City A than in City B. This occurs because the rich person in City A lives isolated and the other two persons are equally poor.

Conversely, when the size of the individual neighborhood is not too large, the inequality between average incomes observed in each individual neighborhood of City B is smaller than the inequality observed in City A. This occurs because some income inequality between the rich and the poor person in City B is averaged out when computing the mean incomes at the individual neighborhood level.

In this framework, a *movement of people* across locations of a city might give rise to

[2]In Figure 1, individual neighborhoods are delimited by intervals centered on each individual. When each interval includes just one individual, there is no inequality within the neighborhood and inequality between neighborhoods coincides with citywide inequality. When each individual neighborhood is large enough to comprise the remaining two individuals, spatial inequality within the neighborhood coincides with citywide inequality, while inequality between neighborhoods is zero (since average incomes coincide across individual neighborhoods). All in-between cases, individual neighborhoods are not "too large", that is at least one individual neighborhood comprises two individuals.

Figure 1: Spatial distribution of incomes (vertical spikes) among the poor P, and the rich R in three linear cities.



changes in spatial inequality that are not trivial, although citywide income distribution would not be affected by this displacement. For instance, if individuals R and P living at the extremes of City A exchange their location, the resulting spatial distribution of incomes would be that of City B, implying an raise (a drop) in within (between) inequality.[3] Spatial inequality can also be affected by *income movements*. Consider now *City C* in Figure 1. This hypothetical city represents the spatial distribution of R and P people after a rich-to-poor transfer of income has eliminated income differences between the two individuals located at the outskirts of the city. Arguably, this transfer reduces citywide inequality, irrespective of whether the initial distribution is that of City A or of City B. It also turns out that spatial inequality between individual neighborhoods is reduced by the transfer. The implications for spatial inequality within individual neighborhoods, however, are ambiguous. Spatial inequality would have been reduced by the transfer if the starting configuration were as in City B, whereas it would have been increased if the starting configuration were as in City A. This highlights that even rich-to-poor income

[3]This movement reflects the implications of gentrification occurred in the last decades across all major American cities, where rich people previously isolated in wealthy suburbs moved towards a more densely populated area of the city, pricing out the poor people who are then marginalized.

transfer might give rise to divergent patterns of spatial inequality when the location of the population is taken into account.

In more realistic settings, it is not straightforward to identify the patterns of spatial inequality when people, incomes, or both change at once. The aim of this paper is to model and to measure spatial inequality using individual neighborhoods as primitive information, and to assess its patterns and implications. The first contribution is on the measurement side. In Section 2, we introduce two new spatial inequality measures, the *Gini Individual Neighborhood Inequality* (GINI) indices, that explicitly account for the urban geography of incomes. The first GINI-type index measures the average level of income inequality *within* individual neighborhoods. The second GINI-type index measures instead the inequality in average incomes *between* individual neighborhoods. The statistical foundations of the GINI indices are established building on connections with geostatistics literature. A methodological appendix develops innovative asymptotic results based on stationarity assumptions common in this literature. The advantages of the GINI indices are discussed, and differences with alternative measurement frameworks, such as those involving within-between decomposition techniques and income segregation indices, are highlighted.

The second contribution of this paper demonstrates the empirical relevance of the methodology utilizing GINI indices. In Section 3 the pattern of spatial inequality across the 50 most populated American cities is assessed by making use of a rich income database constructed from U.S. census data spanning almost four decades. We compute the values of GINI indices at any meaningful distance threshold (from zero to the size of the city). Very strong resemblances in patterns of spatial inequality among the 50 cities are documented, with high levels of within neighborhood inequality steadily increasing over time. Patterns of spatial inequality between individual neighborhoods are consistent across cities and always display a peak in the 90s and a subsequent decline over the following 25 years. These findings match with evidence presented in the literature (Hardman and Ioannides 2004, Wheeler and La Jeunesse 2008), despite being obtained from a substantially different methodological perspective. Instead of being separate elements of the Gini index decomposition, the GINI indices capture different aspects of spatial inequality that turn out to be orthogonal in cross-metros comparisons. This allows to build a taxonomy of cities along the dimensions of spatial inequality (see Sections 2.4 and 3.3 for a discussion).

Changes in spatial inequality are difficult to evaluate on purely normative grounds. For instance, when negative externalities arise from deprivation and envy (Luttmer 2005), within neighborhood inequality probably is the relevant dimension to look at to capture these externalities. A policy aiming to mitigate the incidence of these externalities should focus on reducing inequalities within the neighborhood, for instance by implementing local redistribution or by increasing the distance between rich and poor people. On this premise, the spatial distribution of incomes in City A should be preferred to that of City B, despite the same citywide inequality. However, the proximity among people of different social status might raise ambitions and generate opportunities for the poor and also benefit the rich person (Ellen, Mertens Horn and O' Regan 2013). In this case, the spatial distribution in Cities B should be preferred. Evaluations become even more complex when looking at the implications of neighborhood inequality on lifelong individual outcomes. In Section 4 we highlight that separate channels should be driving the long-term implications of inequality within the neighborhood, which is shown to be negatively associated with improvements in children's income prospects (Chetty and Hendren 2016) while being positively associated with adult health outcomes (Chetty, Stepner, Abraham, Lin, Scuderi, Turner, Bergeron and Cutler 2016) of people growing up and living in major American cities. These empirical correlations turn out to be robust with respect to the most relevant confounding factors. Section 5 concludes summarizing the results.

2 Spatial inequality measurement

2.1 The GINI indices

Given a population of $n \geq 3$ individuals, indexed by $i = 1, ..., n$, let $y_i \in \mathbb{R}_+$ be the income of individual i and $\mathbf{y} = (y_1, y_2, ..., y_n)$ the income vector with average $\mu > 0$. A popular measure of income inequality is the Gini coefficient, defined as

$$G(\mathbf{y}) = \frac{1}{2n(n-1)\mu} \sum_i \sum_j |y_j - y_i|.$$

In what follows, information on the income distribution is assumed to come with information about the location of each income recipient in the urban space.[4] For any individual, neighbors are identified as the group of people located at most as far as d distance units from this individual. The Euclidian distance is used to determine the extent of the neigh-

[4]For the sake of simplicity, we refer to the income-location distribution of individuals on the city map as an income distribution.

borhood.[5] The set of neighbors located within a distance d from individual i is designated as d_i, such that $j \in d_i$ if the distance between individuals i and j is less than or equal to d. The cardinality of d_i is denoted n_{id}, that is the number of people living within a range d from i (including i). The average income of individual i's neighborhood of length d, capturing the neighborhood's affluence, is $\mu_{id} = \frac{\sum_{j \in d_i} y_j}{n_{id}}$.

The first spatial inequality index that we consider is the Gini Individual Neighborhood Inequality within index, indicated by $GINI_W$. It measures the average degree of relative income inequality within individual neighborhoods. The $GINI_W$ index is inspired by Pyatt (1976), who provides a probabilistic interpretation of the Gini inequality index. According to Pyatt, the Gini index can be seen as the expected gain accruing to a randomly chosen individual from the income distribution if her income is replaced with the income of another individual randomly drawn from the same distribution. The $GINI_W$ index assumes that income comparisons are carried over exclusively within individual neighborhoods of a given size. For each individual i, the average distance between i's income and the income of her neighbors is computed and then this quantity is scaled by the neighborhood average income so that it ranges over the unit interval. This gives:

$$\Delta_i(\mathbf{y}, d) = \frac{1}{\mu_{id}} \sum_{j \in d_i} \frac{|y_i - y_j|}{n_{id}}.$$

Given the relevant notion of individual neighborhood parametrized by d, there are $1/n_{id}$ chances of drawing a neighbor from $i's$ neighborhood with whom i can compare her income. This probability changes across individuals, reflecting the population density of

[5] For a discussion of the use of multidimensional notions of distance, see Conley and Topa (2002).

individual neighborhoods. The $GINI_W$ index averages the normalized mean income gaps Δ_i across the whole population:

$$GINI_W(\mathbf{y}, d) = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{n} \Delta_i(\mathbf{y}, d).$$

The $GINI_W$ index hence captures the overall degree of inequality that would be observed if income comparisons were limited only to neighbors located at a distance smaller than d. The index is bounded, with $GINI_W(\mathbf{y}, d) \in [0, 1]$ for any $\mathbf{y}$ and d. Moreover, $GINI_W(\mathbf{y}, d) = 0$ if and only if all incomes within individual neighborhoods of size d are equal. Notice that this cannot exclude inequalities among people located at a distance larger than d. Additionally, $GINI_W(\mathbf{y}, d)$ can take values that are either larger or smaller than $G(\mathbf{y})$.[6] When d reaches the size of the city, spatial inequality coincides with citywide inequality, that is $GINI_W(\mathbf{y}, \infty) = G(\mathbf{y})$.

The $GINI_W$ index captures a relative concept of inequality, since income distances within each neighborhood are divided by the neighborhood average income. This implies that even if inequality in relatively small neighborhoods approaches citywide inequality the distribution of incomes within the individual neighborhood does not necessarily resemble that of the city as a whole. In fact, average incomes might substantially differ across individual neighborhoods. Inequality between individual neighborhoods average incomes can be valued by the Gini index for the vector $(\mu_{1d}, \ldots, \mu_{nd})$. The elements of

[6] Consider, for instance, the following distribution of incomes among four individuals: $(\$0, \$0, \$1000, \$2000)$. The Gini inequality index of this income distribution is 0.77. Suppose these individuals are distributed in space such that each of the two poor individuals lives close to a non-poor person, while the two pairs are far apart one from the other. Then, spatial inequality within the neighborhoods is maximal (i.e., $GINI_W(., d) = 1$ for d small) and larger than citywide inequality.

this vector depend upon individuals' locations and proximity. For instance, if a high-income person lives near to many low-income people, her income contributes to rising the mean income not only in the high-income person neighborhood, but also in the individual neighborhoods of all her low-income neighbors. However, if the high-income person is located at an isolated point on the urban map, her income does not generate any positive effect on other people's average neighborhood income, provided that the notion of individual neighborhood is sufficiently exclusive. As a consequence, the average value of the vector $(\mu_{1d},..,\mu_{nd})$, designated μ_d, generally differs from μ. The between dimension of spatial inequality is captured by the Gini Individual Neighborhood Inequality between index, $GINI_B$, defined as:

$$GINI_B(\mathbf{y},d) = \frac{1}{2n(n-1)\mu_d} \sum_i \sum_j |\mu_{id} - \mu_{jd}|.$$

As expected, $GINI_B(\mathbf{y}, d) \in [0, 1]$ for any $\mathbf{y}$ and d. The index is equal to $G(\mathbf{y})$ at a zero-distance and whenever all incomes within each individual neighborhood of length d are equal. $GINI_B$ converges to zero when d approaches the size of the city.

A simple and insightful picture of within and between spatial inequality patterns can be drawn by computing GINI indices for different values of d and plotting their values on a graph against d (on the horizontal axis). The curve interpolating these points is called the spatial inequality curve, generated by either the $GINI_W$ or the $GINI_B$ index. More precisely, the curve derived from $GINI_B$ takes the value of the overall Gini index when each individual is considered as isolated (that is, when $d = 0$) and approaches 0 when each individual neighborhood spans the whole city. The curve originated by $GINI_W$ can exhibit a less predictable shape. First, it can locally decrease or increase in d according to the spatial distribution of incomes. Second, when each individual neighborhood is large enough to include the whole population of the city, then $GINI_W(\mathbf{y}, d)$ approaches $G(\mathbf{y})$. Third, the graph of $GINI_W(\mathbf{y}, d)$ can be flat, meaning that incomes are randomized across locations and the spatial component of inequality is irrelevant. Fourth, the curve could increase with d, indicating that individuals with similar incomes tend to sort themselves in the city. The shape of the spatial inequality curves also suggests the degree to which citywide income inequality can be correctly inferred from randomly sampling individuals from the city.[7]

For a given size of the individual neighborhood, spatial inequality comparisons can be carried over by looking at the level of the GINI within or between index at the corresponding distance threshold. Each of these evaluations generates a complete ranking of the income distributions, although these rankings may contradict each others. Comparisons of spatial inequality curves allow evaluations that are robust *vis-à-vis* the size of the neighborhood. We propose to use these curves to carry over robust spatial inequality assessments.

The GINI indices are measures of inequality that account for the spatial association of unequal incomes.In the following section we establish the GINI indices and the way in which spatial association is treated in *geostatistics* literature.

[7]When the role played by space is negligible, i.e. the spatial inequality curves are rather flat, any random sample of individuals taken from a given point in the space is representative of overall inequality. When space is relevant and people locations are stratified according to income, then a sample of neighbors randomly drawn could underestimate the level of citywide inequality.

2.2 Spatial inequality and geostatistics

A spatial income distribution can be represented through its data generating process $\{Y_s : s \in \mathcal{S}\}$. This process is a collection of random variables Y_s located over the random field $\mathcal{S}$, which serves as a model of the relevant urban space. The process is distributed as $F_{\mathcal{S}}$, the joint distribution function combining information on the marginal income distributions in each location and the degree of spatial dependence of incomes on $\mathcal{S}$. Through geolocalization, it is possible to compute the distance "$||.||$" between locations $s, v \in \mathcal{S}$. Let $||s - v|| \leq d$ indicate that the distance between the two locations is smaller than d, or equivalently $v \in d_s$. The cardinality of the set of locations d_s is n_{d_s}, while n is the total number of locations. The observed income distribution $\mathbf{y}$ is a particular realization of the process, where only one income observation i occurs in a given location s.

Consider first the $GINI_W$ index of the spatial process $F_{\mathcal{S}}$. It can be written in terms of first order moments of the random variables Y_s as follows:[8]

$$GINI_W(F_{\mathcal{S}}, d) = \sum_{s} \sum_{v \in d_s} \frac{1}{2n\, n_{d_s}} \frac{\mathbb{E}[|Y_s - Y_v|]}{\mathbb{E}[Y_v]}.$$

The degree of spatial dependence represented by $F_{\mathcal{S}}$ enters in the $GINI_W$ formula through the expectation terms conditional on $\mathcal{S}$. Consider first the case displaying no spatial dependence in incomes, that is, the random variables Y_s and Y_v are i.i.d. for any $s, v \in \mathcal{S}$. One direct implication is that $GINI_W(F_{\mathcal{S}}, d) = \frac{\mathbb{E}[|Y_s - Y_v|]}{\mathbb{E}[Y_v]}$, which coincides with the

[8] Biondi and Qeadan (2008) use a related estimator to assess dependency across time in paleorecords observed in a given location.

definition of the standard Gini inequality coefficient (see for instance Muliere and Scarsini 1989).

If, instead, spatial dependence is at stake, then the expectation $\mathbb{E}[|Y_s - Y_v|]$ varies across locations and cannot be identified and estimated from the observation of just one data point in each location. It is standard in geostatistics to rely on assumptions about the stationarity of $F_{\mathcal{S}}$ (Cressie and Hawkins 1980, Cressie 1991). The first assumption is that the random variables Y_s have stationary expectations over the random field, i.e., $\mathbb{E}[Y_v] = \mu$ for any v. The second assumption is that the spatial dependence in incomes between two locations s and v only depends on the distance between the two locations, $||s - v||$, and not on their position in the random field. Here, we consider radial distance measures for simplicity, so that $||s - v|| = d$. This gives $\mathbb{E}[(Y_s - Y_v)^2] = 2\gamma(||s - v||) = 2\gamma(d)$, where the function 2γ is the *variogram* of the distribution $F_{\mathcal{S}}$ (Matheron 1963).

The function $2\gamma(d)$ is informative of the correlation between two random variables that are exactly d distance units away one from one other. The slope of the graph of the variogram function displays the extent to which spatial association affects the joint variability of the elements of the process. Generally, $2\gamma(d) \to 0$ as d approaches 0, indicating that random variables that are very close in space tend to be strongly spatially correlated and variability in incomes at the very local scale is small. Conversely, $2\gamma(d) \to 2\sigma^2$ when d is sufficiently large, indicating spatial independence between two random variables Y_s and Y_v far apart on the random field.

Together, the two assumptions listed above depict a form of *intrinsic stationarity* of the data generating process (Cressie and Hawkins 1980, Cressie 1991, Chilès and Delfiner

2012). If, additionally, Y_s is assumed to be Gaussian with mean μ and variance σ^2, $\forall s \in \mathcal{S}$, it is possible to show that the $GINI_W$ index is a function of the variogram, as follows:

$$GINI_W(F_{\mathcal{S}}, d) = \sum_{s} \sum_{v \in d_s} \frac{1}{\sqrt{\pi}} \frac{1}{n\, n_{d_s}} \frac{\sqrt{\gamma(||s-v||)}}{\mu}.$$

With some additional algebra, it is also possible to show that the $GINI_B$ index is a function of the variogram under stationarity and the Gaussian assumptions. Both indices can hence be described as averages, taken over the space of distances between locations, of distance-sensitive coefficients of variation. All results are formally derived in the appendix.

The possibility of expressing the GINI indices as transformations of the variogram leads to two considerations. The first is that the GINI indices measure spatial inequality as a direct expression of the spatial dependence in the data generating process, represented under stationarity assumptions by the variogram, without imposing external normative hypotheses about the interactions between incomes, income inequality and space. The second consideration is that the empirical counterpart of the variogram sets the basis for estimating asymptotic standard errors of the GINI indices. These results are used to test hypothesis on the extent and dynamics of spatial inequality.

2.3 Testing hypotheses about spatial inequality

The empirical estimators of the GINI spatial inequality curves (presented in the appendix) can be used to test hypotheses about the shape and dynamics of spatial inequality. (i) By contrasting the level of spatial inequality measured by the GINI curves at a given distance

d with the overall level of inequality captured by the Gini index, it is possible to assess if, and to what extent, average income inequality experienced within a neighborhood of size d is different from the level of inequality in the city. (ii) Moreover, by contrasting the level of the GINI curves at d and at $d' > d$, it is possible to state if, by how much, and at which speed, local inequality converges with citywide inequality. (iii) Lastly, by comparing the levels of the GINI curves at distance d registered in different periods within the same city, it is possible to reach conclusions about the dynamics of spatial inequality.[9]

In the appendix, distribution free, non-parametric estimators for the GINI indices are estimated in a general setting where sample information about the process $F_{\mathcal{S}}$ is available.[10] Under the intrinsic stationarity and the Gaussian assumptions, asymptotically valid standard errors for the GINI indices estimators are also derived. We find that the GINI estimators sampling distribution is asymptotically normal,[11] with standard errors that can be fully characterized by the variogram function.[12] Before moving to the empirical section, the novelties and advantages of the methods proposed above are compared with the existing literature on spatial inequality measurement.

[9]One is compelled to conclude in favor of spatial inequality only if there is strong evidence against the null hypothesis that the level of the GINI curve at d is the same as the Gini inequality index, and that the level of spatial inequality captured by the GINI curves does not change with d. When comparing two GINI (either between or within) curves, a strong increase or reduction in spatial inequality cannot be rejected if there is strong evidence against the null hypothesis that the two curves coincide *at every* d.

[10]The $GINI_B$ index estimator can be computed as a plug-in estimator as in Binder and Kovacevic (1995) and Bhattacharya (2007), provided individual neighborhood averages are properly estimated. On the contrary, the $GINI_W$ estimator involves comparisons of individual income realizations.

[11]Standard errors for GINI indices are derived using results for ratio-measures estimators (see Hoeffding 1948, Goodman and Hartley 1958, Tin 1965, Xu 2007, Davidson 2009) under intrinsic stationarity and normality (Cressie and Hawkins 1980, Cressie 1985). The latter assumption does not immediately translate into normality of the GINI estimators, which are highly non-linear functions of the underlying stochastic process. Rather, on this assumption, we can show that the GINI estimators are linear in the variogram, implying asymptotic normality.

[12]A Stata routine implementing the $GINI_W$ and the $GINI_B$ indices and spatial inequality curves curves, along with their standard error estimators, is made available by the authors.

2.4 Discussion

The inequality literature largely agrees that relative inequality indices should satisfy at least four normatively relevant properties (Atkinson 1970): (i) invariance with respect to population replication; (ii) invariance to the measurement scale; (iii) anonymity, that is, invariance to any permutation of the incomes across the income recipients; (iv) the Pigou-Dalton principle, implying that every rich-to-poor income transfer should not increase inequality. While properties (i) and (ii) have desirable implications for the measurement of spatial inequality and are satisfied by the GINI indices,[13] anonymity strongly conflicts with the idea that location matters in spatial inequality evaluations.[14] Consider, for instance, the income distribution in Figure 1 in the Introduction. The spatial configuration of incomes in City B can be obtained from that in City A by permuting the incomes of the individuals living at the margins of the city. Anonymity, which judges City A and City B as equal from a citywide inequality perspective, does not extend to spatial inequality, which instead rises in the within dimension and decreases in the between dimension after the permutation.

For the reasons mentioned above, anonymity should be weakened in spatial inequality comparisons. One method, predominant in the literature, is to constraint spatial inequality assessments to comparisons involving inequality between neighborhoods, originating from a well-defined partition of the city into administrative areas, such as urban blocks,

[13]Direct implications of these properties are that populations of different size and different average incomes can be made comparable. Replication invariance, in particular, guarantees that replacing single individuals by equally-sized groups in given locations does not affect spatial inequality. Both properties are satisfied by the GINI indices by standardizing income gaps by individual neighborhood-specific population counts and average incomes.

[14]Anonymity would not be a concern if incomes *and* locations were both permuted across individuals. Rather, we refer to anonymity as permutations of incomes alone.

census tracts, etc. Some contributions (Shorrocks and Wan 2005, Dawkins 2007, Wheeler and La Jeunesse 2008) have proposed the use of measures of inequality and earning volatility. Some authors have focused on a particular aspect of spatial inequality, called income segregation (by analogy with racial segregation), which is insensitive to the overall income distribution in the city and rather captures how rich and poor people sort unevenly across neighborhoods. In this spirit, Kim and Jargowsky (2009) suggested breaking down overall inequality in the components associated to within and between neighborhoods variability in incomes, and to assess spatial segregation as the share of citywide inequality due to the between component. Reardon and Bischoff (2011) focus instead on the degree of disproportionality of rich and poor individuals across neighborhoods.

The above approaches retain anonymity at two levels: first, among individuals living in the same neighborhood; second, in terms of average incomes across neighborhoods. Consequently, measures of spatial inequality consistent with this approach put the emphasis on the neighborhood as the unit of analysis, and are hence subject to the Modifiable Areal Units Problem (MAUP, see Openshaw 1983, Wong 2009), which arises from "scaling" and "zonation" issues. To overcome the scaling issue, some authors have proposed assessing inequality between neighborhoods at different scales of aggregation of the initial partition (Hardman and Ioannides 2004, Shorrocks and Wan 2005, Wheeler and La Jeunesse 2008). With a less refined partition of the urban space, the size of the neighborhood increases and extends anonymity to a larger number of people within the neighborhood. To avoid the zonation issues, Dawkins (2007) has proposed measures that account for the dependence of income segregation on the spatial arrangement of administrative neighborhoods.

The approach based on the GINI indices differs from this literature in using individual neighborhoods as primitives. In fact, individual neighborhoods do not originate from a partition of the urban space, but rather can display some degree of overlapping: the fact that individual k is in the neighborhood of individual i *and* of individual j does not imply that i and j are also neighbors. This logic discards anonymity within individual neighborhoods regardless of their size (permuting the incomes of any two neighbors might have substantial implications for other individual neighborhoods) and proves robust in relation to the issue of zonation. Furthermore, considering individual neighborhoods of different size, the degree of inclusiveness of individual neighborhoods can increase without strengthening anonymity within the neighborhoods.

Anonymity (also called symmetry) is a necessary condition for Schur-convexity, a mathematical property satisfied by all inequality indices consistent with the Pigou-Dalton transfer principle (see Marshall and Olkin 1979, p.54). By weakening anonymity, both rich-to-poor redistribution and relocation policies switching the position of poor and rich people across the city (without affecting citywide inequality) may give rise to unpredictable implications for spatial inequality, who are likely amplified by the behavioral responses of people who can sort across space along the income dimension (Durlauf 2004).

Contrary to standard practice in the literature, which breaks down citywide inequality into a within and between component, the GINI indices capture two distinct aspects of spatial inequality: the average inequality within individual neighborhoods and the degree of inequality in average incomes between individual neighborhoods. These two aspects are not necessarily intertwined, for any selected neighborhood size. Consequently, our

		$GINI_W$	
		Low	*High*
$GINI_B$	*High*	Polarized city	Unstable city
	Low	Even city	Mixed city

Table 1: Taxonomy of cities by spatial inequality.

methodology offers one additional degree of freedom compared to the traditional between-within decomposition techniques, where a high degree of within inequality mechanically involves low between inequality and vice-versa, for given citywide inequality. Building on these arguments, cities can be classified according to between and within dimensions of spatial inequality. Table 6 highlights four types of cities.

Low levels of the $GINI_W$ and $GINI_B$ indices mean that inequality within individual neighborhoods is low and that neighborhoods resemble each other in terms of income composition. This setting mirrors the homogeneous social structure of an "even city" characterized by relatively low citywide income inequality and strong income mixing (for a broader discussion of the *Just City*, see Fainstein 2010). In some situations, low $GINI_B$ index values can be paired with high levels of the $GINI_W$ index. This case identifies cities with mixed neighborhoods comprising people with different incomes (hence citywide inequality) who are evenly spread across the urban space. The "mixed city" model is a recurrent typology widely discussed in the urban planning literature (Sarkissian 1976) that can be conceptualized both as the outcome of gentrification processes (Lees 2008), and a stimulus for socio-economic opportunities for the residents (Musterd and Andersson 2005, Manley, van Ham and Doherty 2012).

High levels of the $GINI_B$ index occur in presence of citywide inequality and spatial sorting patterns that separate poor from rich people across the urban space. The image of a "divided city" provided in the recent Habitat (2016) report (chapter 4) and anticipated in van Kempen (2007) evokes the implicit social tensions in the urban fabric arising because of strong differences in incomes across neighborhoods. We further distinguish two cases within the "divided city" typology. The first type of cities, where high values of the $GINI_B$ index are paired with low levels of the $GINI_W$ index, is that of a "polarized city" with rich and poor people separated both in income and spatial dimensions.[15] The second type, the "unstable city", displays high levels of both GINI indices. In this case, high income heterogeneity within the neighborhood suggests that dimensions other than income (such as ethnicity) play a significant role (Boal 2010, Scholar 2006, Deaton and Lubotsky 2003) and might amplify the implications of income inequality in the sorting process.

In the following section the extent of these traits of spatial inequality and their effects on individual outcomes are investigated. The case of Chicago, IL, serves to illustrate the spatial dimension of inequality in a large U.S. metropolitan area. We also provide stylized facts about patterns of spatial inequality across the 50 largest U.S. cities, which are grouped into four distinct categories depending on the spatial inequality they display.

[15]Duclos, Esteban and Ray (2004) describe polarization through the concept of alienation between groups, here captured by the size of the individual neighborhood in relation to a relevant attribute, such as income. Alienation is stronger when groups are more homogeneous and cohesive (i.e., the lowest is inequality within the individual neighborhood) and more diverse (i.e., there is a high degree of inequality between neighborhoods).

3 Spatial inequality in U.S. cities: 1980-2014

3.1 Data

We use information on incomes distributions within U.S. cities over four decades, drawing on the census files of the U.S. Census Bureau for 1980, 1990 and 2000. Information about population counts, income levels and family composition at a very fine spatial grid is taken from the decennial census Summary Tape File 3A.[16] Due to anonymization issues, the STF 3A data are given in the form of statistical tables representative at the block group level, the finest available statistical partition of the American territory. After 2000, the STF 3A files have been replaced with survey-based estimates of the income tables from the American Community Survey (ACS), which runs annually since 2001 on representative samples of the U.S. resident population. We focus on the 2010-2014 5-years Estimates ACS module. Sampling rates in ACS vary independently at the census block level according to 2010 census population counts, covering on average 2% of the U.S. population over the 2010/14 period. As far as we know, ACS 2010/14 wave has not yet been used for empirical analysis of urban inequality.

The units of analysis are households with one or more income recipients. The focus is on the gross household income distribution. There are two available sources of information that can be used to model the income distribution at the block group level. The first set of tables displays aggregate income at the block group level. The second set of tables shows

[16]The Census STF 3A provides cross-sectional data for all U.S. States and their subareas in hierarchical sequence down to the block group level (the finest urban space partition available in the census). The geography of the block group partition changes over the decades to keep track with demographic changes within the Counties of each State.

instead counts of households per income interval at the block group level.[17] There are 17 income intervals in the census 1980, 25 in the census 1990 and 16 in the census 2000 and in the ACS. In all cases, the highest income bracket is not top-coded. We use a methodology based on Pareto distribution fitting as in Nielsen and Alderson (1997), to convert tables of household counts across income intervals into a vector of representative incomes for each income interval, along with the associated vector of households frequencies corresponding to these incomes.[18] Estimates of incomes and household frequencies vary across block groups, implying strong heterogeneity within the city in block-group specific household gross income distributions.

The STF 3A files and the ACS also provide tables of household counts by size (scoring from 1 to 7 or more household members) for each block group. To draw conclusions about the distribution of income across block groups that differ in households demographics, we construct equivalence scales that are representative at the block group level (the square root of average household composition in the block group level, obtained from households counts information). We can hence convert the representative incomes at the block group

[17]The ACS estimates of population counts should be interpreted as average measures across the 2010-2014 time frame. The survey runs over a five years period to guarantee the representativeness of income and demographic estimates at the block group level.

[18]The procedure consists in fitting a Pareto distribution to the grouped data (population shares and income thresholds) and then estimating references incomes within each interval. For income intervals below the median, the estimated reference income is the midpoint of the interval. For other intervals, estimates are obtained under the constraint that estimated average income at the block-group level should coincide with the observed average income in the data. Estimated medians for top income intervals are used as reference incomes, and empirical population counts as weights. Fitting methods consist in GMM (preferred) and quantile estimation as in Quandt (1966). Alternative estimation methods draw instead from the log-normality assumption, as in Wheeler and La Jeunesse (2008). Incomes estimates based on the preferred method display an MSA-year level average correlation of 95.2% with quantile fitting income estimates (MSA-years population weighted correlations range between $min = 76\%$ and $max = 98.9\%$, with 95% of the correlations larger than 89.3%), and 90.4% average correlation with log-normal fitting income estimates at the block group level (MSA-years population weighted correlations range between $min = 45.6\%$ and $max = 97.1\%$, with 95% of the correlations larger than 85.1%).

City	Year	# Blocks	Hh/block	Eq. scale	Equivalent household income				
					Mean	*20%*	*80%*	*Gini*	*90%/10%*
Chicago (IL)	1980	3756	1122	1.630	13794	5798	20602	0.434	11.351
	1990	4444	1217	2.029	21859	9132	32316	0.461	11.903
	2000	4691	1173	1.625	41193	16076	61667	0.473	11.533
	2010/14	4763	1060	1.575	55710	20022	89856	0.486	13.452

Table 2: The household equivalent gross income distribution in Chicago, IL

level into the corresponding equivalized incomes by scaling the estimated reference income values by the block group-specific equivalence scale.

Income reference levels, population frequencies associated to these levels and equivalence scales are estimated separately for each block group of a city in each census and ACS years. All block groups are georeferenced, and measures of distance between the block groups centroids can therefore be constructed. All income observations within the same block group are assumed to occur on its centroid. To identify the relevant urban space, defining the extension of a city, we resort to the Census definition of a Metropolitan Statistical Area (MSA) based on the 1980 Census definition.[19] For each city-year pair we therefore obtain an income database consisting of strings of incomes and frequency weights at each geocoded location on the map. Thus, weighted variants of the GINI index estimators can be used to evaluate facts about spatial inequality at various distance scales.

3.2 Spatial inequality in Chicago, IL

The extent of the Chicago metro area, based on 1980 definition of Chicago primary MSA, comprises Cook County, Du Page County and McHenry County surface.[20] Table 2 provides summary information of the household population and the respective income distribution in Chicago.[21] Average equivalent household income increased fourfold over 1980-2014 in nominal terms, corresponding to a 73% increase in real terms. Table 2 shows that the top 10% to bottom 10% income ratio sharply increased from 11.53 in 2000 to almost 13.5 in the 2010/2014 period, indicating increased dispersion at the tails of the distribution. The relative gap between the low income (bottom 20%) and high income households (top 20%) has increased at a constant yet lower pace. The citywide Gini index increased from 0.43 to 0.48 over the same period.

Values of the $GINI_W$ and the $GINI_B$ indices are computed for 1980, 1990, 2000 and 2010/2014 waves to assess the evolution of equivalent household income across individual neighborhoods. At distance zero up to approximately 0.2 miles, the $GINI_W$ index captures the average inequality in estimated income levels within block groups. Data confirm substantial income inequality within block groups in 2000,[22] with the Gini index fluctuat-

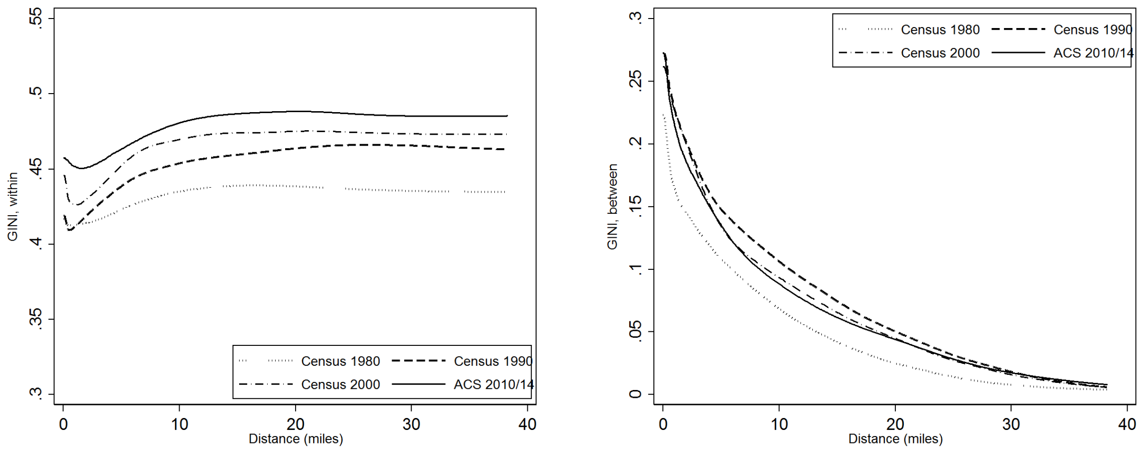
[19] The U.S. counties defining the MSAa in 1980 can be found at this link: *http://www.census.gov/population/metro/files/lists/historical/80mfips.txt*. The 1980 Census definition of MSA guarantees comparability of estimates across urban areas that are expanding or shrinking over the 35 years considered in this study.

[20] For some of the block groups of 1980 Census it is not possible to establish geocoded references. Hence, these units cannot be included in the index computation and might have an impact on the estimation of the GINI patterns. Reardon and Bischoff (2011) and other contributions have demonstrated, however, that the impact of this kind of missing information is negligible on overall trends of inequality within the 100 largest U.S. Commuting Zones.

[21] Throughout the four decades considered in this study, the block group partition of Chicago has become finer, with the number of block groups increasing 1000 units. This change keeps track of the demographic boom in Chicago, implying a roughly stable demographic composition in each block group (around 1100 households on average).

[22] For this year block group level estimates of inequality are collected in the census tables.

Figure 2: Spatial GINI indices of income inequality for Chicago (IL), 1980, 1990, 2000 and 2010/14



(a) $GINI_W$

(b) $GINI_B$

Note: Authors analysis of U.S. Census and ACS data.

ing between 0.2 to above 0.6, and standing at 0.4 when averaging across Chicago's block groups. This explains the relatively high intercept of the $GINI_W$ curves shown in Figure 2.(a). Inequality slightly decreases as the neighborhood size reaches two miles and then quickly rises to reach its city-wide level when the size of the neighborhood is larger than 20 miles. Comparing the $GINI_W$ curves of the different decades, within neighborhood inequality appears to increase over time, for any size of the neighborhood.

The between individual neighborhood inequality curves from 1980 to 2010/14 are plotted in Figure 2.(b). For individual neighborhoods of narrow size, the $GINI_B$ index values are generally smaller than 0.3. As expected, between neighborhood inequality decreases with the size of the neighborhood, but in a very smooth manner. For neighborhoods smaller than two miles, the $GINI_B$ index is generally larger than 0.25. It decreases to 0.1 only for neighborhoods of at least 16 miles range. Overall, this pattern is robust across Census years. Contrasting the $GINI_B$ curves over the last three decades, it can be noted that between inequality is on the rise up to 1990, decreases in 2000 and stabilizes thereafter.

Are these patterns statistically significant? Estimates for small size individual neighborhoods might in fact be biased by the approximations used to estimate block-group level income distributions. To assess significance, we first compute empirical estimators of the variograms based on geolocalized income data, and then derive standard errors and confidence intervals of the GINI within and between indices at pre-selected distance thresholds. Confidence bounds are drawn for each spatial inequality curve, and dominance relations across spatial inequality curves are tested making use of t-statistics at selected distance ranges.[23] Overall, we find evidence of the following patterns of spatial inequality in Chicago: i) for neighborhoods of small size (below 2 miles), the $GINI_W$ index ranges from 0.41 in 1980 to 0.45 in 2010/2014, and increases slightly with the size of the neighborhood; ii) the $GINI_B$ index decreases smoothly with neighborhood size and reaches 0.1 only for relatively large (more than 10 miles) neighborhoods, hence indicating persistence of inequality across the urban space; iii) the $GINI_W$ index is constantly on the rise over the period considered at any distance threshold, although there is little statistical evidence supporting these changes; iv) the $GINI_B$ index is on the rise during 1980-1990, it slightly (yet significatively) declined in 2000 and has remaind stable thereafter. The changes we

[23]To do so, we compute all pairwise differences in spatial inequality curves across all the decades under analysis. These differences, measured at pre-determined distance abscissae (along with the associated confidence bounds), are then plotted on a graph. If the horizontal line passing from the origin of the graph (indicating the null hypothesis of no differences in spatial inequality at every distance threshold) falls within these bounds, we conclude that the gap in the spatial inequality curves under scrutiny are not significant at standard confidence levels. For a detailed description of results, see online appendix B.

describe are robust over the entire domain of the neighborhood size parameter.

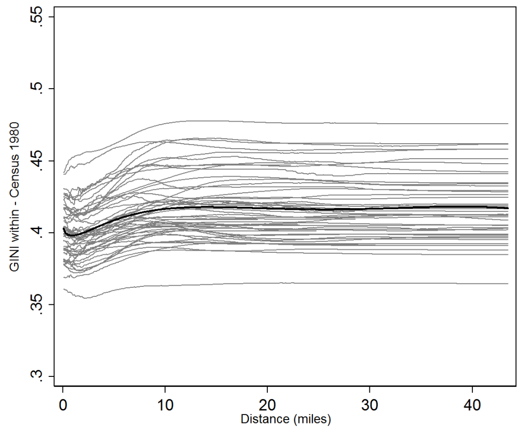
The spatial inequality patterns described above could be explained, on the one hand, by the changes in the citywide income distribution observed over the part 35 years. As shown in Table 2, the citywide Gini index of gross equivalent household income in Chicago was on the rise over the period and relative income gaps between the top and bottom income quintile grew considerably, while the top-to-bottom income decile ratio remained stable over 1980-2000 and increased afterward. If the spatial arrangement of households were completely random, the income distribution observed within any individual neighborhood would reflect the citywide income distribution, and the distributional changes in the citywide distribution would spread evenly over the urban space. However, this scenario is inconsistent with observed patterns of the spatial inequality between neighborhoods curve, which should rather be flat. One alternative explanation relies on the fact that households are stratified in space according to their incomes, with clusters of rich, medium class and poor households. This spatial configuration would give rise to substantial inequality within individual neighborhood of average size, if clusters are evenly distributed across the urban space.

The patterns described above reciprocally reinforce their implications for spatial inequality. In fact, changes in citywide inequality between 1980 and 2000 were driven by divergent growth of income along the income distribution, with rich and poor people moving far apart on the income distribution. This distributional change might produce effects that are consistent with patterns of between individual neighborhood inequality, which was on the rise until the Nineties, if, on average, high income households get richer in those neighborhoods where high income households are over-represented and where middle class households' income grew at a slower pace.

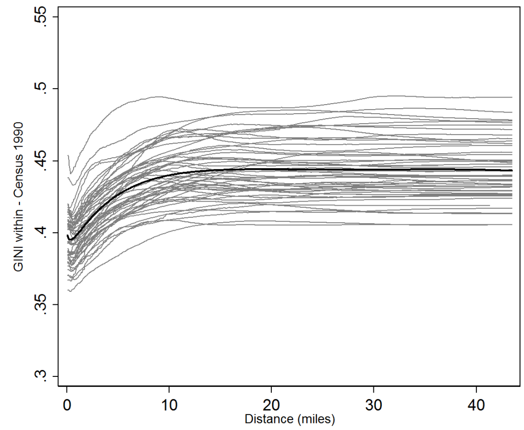
Since 2000, income inequality between individual neighborhoods has fallen despite growing citywide inequality, suggesting a role for changes in the spatial distribution of rich and poor households in Chicago. Spatial inequality between neighborhoods decreases when rich households move closer to the middle class households, pricing out poor households from neighborhoods historically occupied by the poor, who are then obliged to move farther away. This change could generate increasing inequality within individual neighborhoods, leveraging on the increasing disparity in incomes between rich households and the rest, and simultaneously could reduce inequality between individual neighborhoods. This configuration provide robust empirical evidence of the consequences of local and citywide income distribution of recent waves of gentrification in major US cities documented in Ehrenhalt (2012).[24] This phenomenon -the movement of wealthy, skilled people from suburbia to inner city- is referred to as the Great Inversion and is accompanied by the reconcentration of income poverty in suburbs, far away from central business districts and from the wealthy and the middle-class households (Kneebone 2016). The two demographic phenomena seem to have dominated the dynamics of urban evolution in major U.S. cities (including Chicago) since 2000. The $GINI_B$ index consistently show that spatial inequality has decreased (irrespectively of the underlying individual neighborhood size) despite the increasing divide of top and bottom deciles of Chicago income distribution after 2000.

[24] Oversimplifying, this type of change in the spatial distribution of households and incomes can be intuitively associated with the gentrification process exemplified in Figure 1, where a rich and isolated person in City A moves towards the densely populated area of the city, forcing the poor to relocate elsewhere (as in City B).

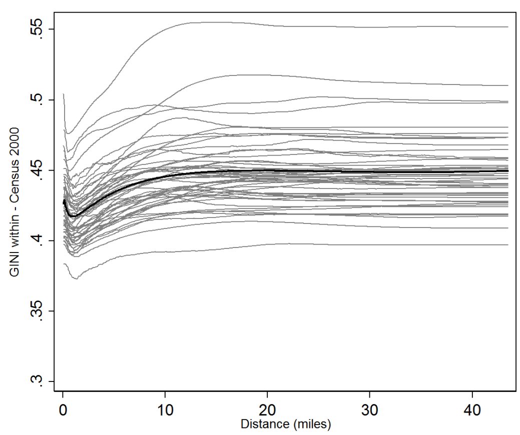
Figure 3: Inequality within individual neighborhood for 50 largest U.S. metro areas



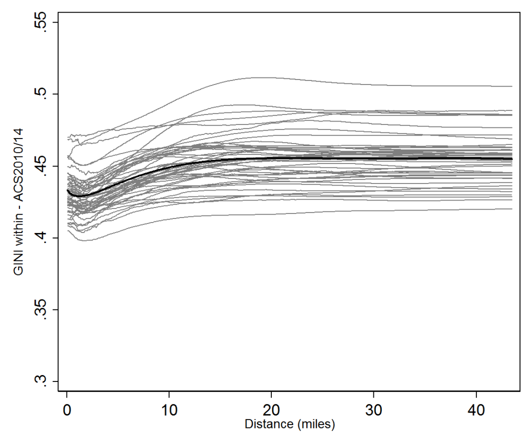
(a) Census 1980



(b) Census 1990



(c) Census 2000

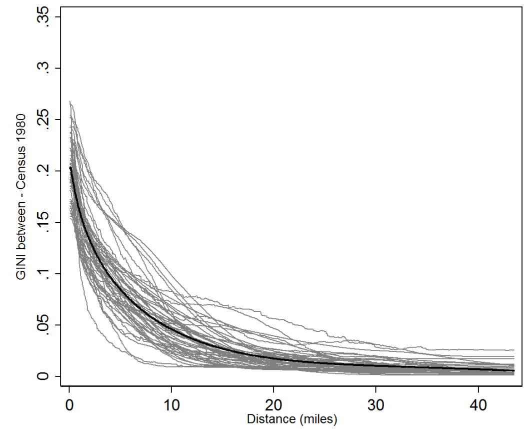


(d) ACS 2010/2014

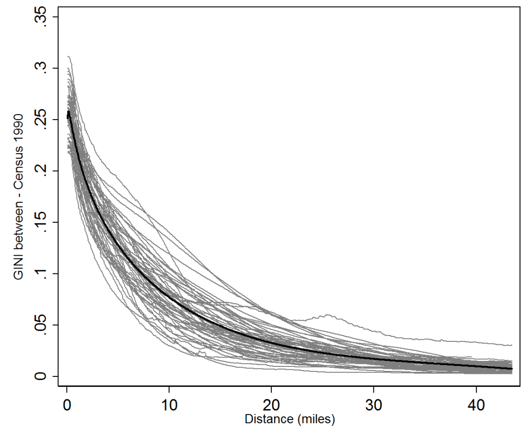
Note: Authors analysis of U.S. Census and ACS data.

In what follows, we provide new robust evidence that the patterns described above by investigating the extent and the evolution of spatial inequality in the 50 largest US metropolitan areas.

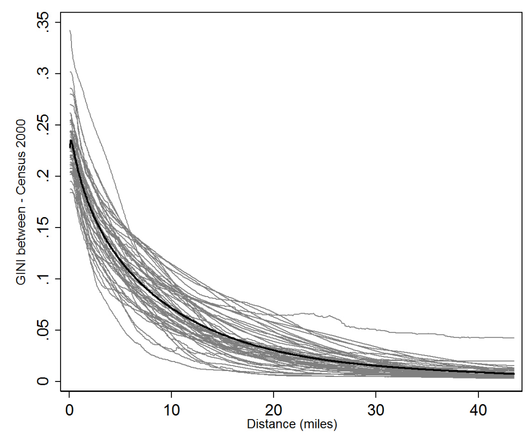
Figure 4: Inequality between individual neighborhoods for 50 largest U.S. metro areas



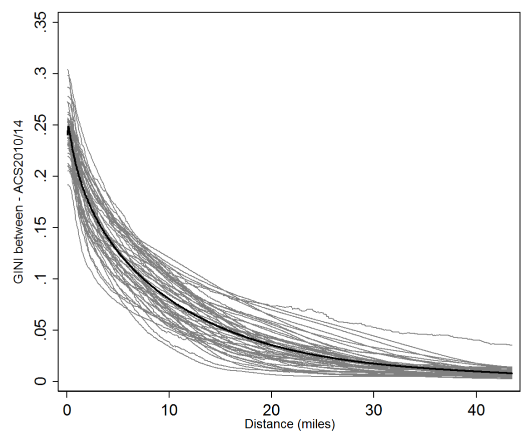
(a) Census 1980



(b) Census 1990



(c) Census 2000



(d) ACS 2010/2014

Note: Authors analysis of U.S. census and ACS data.

3.3 Stylized facts about spatial inequality in U.S. cities

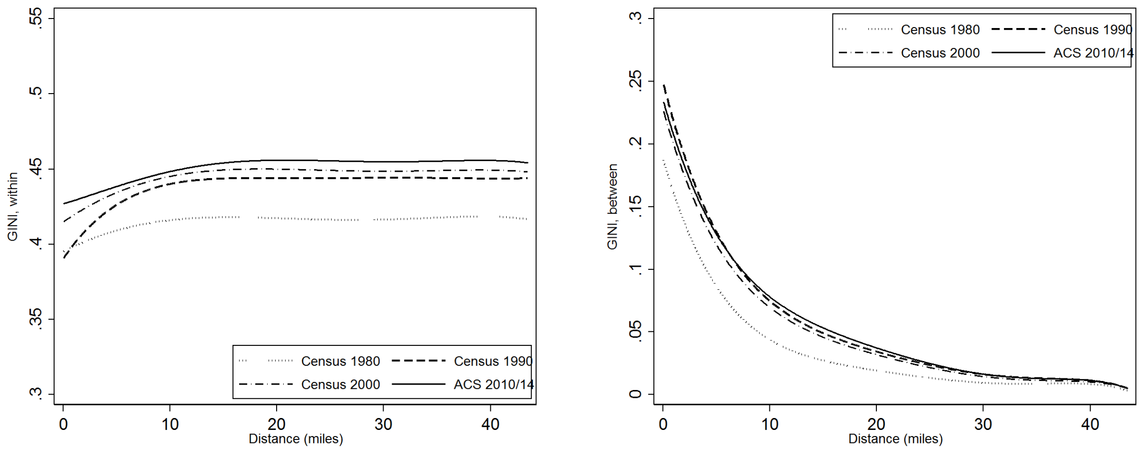
Figures 3 and 4 show spatial inequality curves for the years 1980, 1990, 2000 and 2010/2014. There are 50 curves in each plot, one for each city.[25] The patterns of the curves shown

[25] Data on demographic size of the 50 largest U.S. MSA are from the Census Bureau and can be downloaded from: *http://factfinder.census.gov/faces/tableservices/jsf/pages/productview.xhtml?src=bkmk*. The list of cities, ordered by their size, can be found in table 6 in the online appendix C. We stick to the 1980 Census definition of metropolitan statistical areas for each of these cities to define the relevant urban space. In this way, within-city patterns of spatial inequality can be meaningfully compared across decades.

in the figures indicate three basic facts. First, spatial inequality within and between individual neighborhoods was larger in 2010/2014 than in 1980, at every distance abscissa. Second, the patterns of spatial inequality displayed by the between and within curves of the 50 largest U.S. cities are similar to those recorded for Chicago. The $GINI_W$ index estimates are high even for small distances and rapidly converge to the citywide level of inequality. The $GINI_B$ index estimates fluctuate around 0.3 and smoothly converge to zero for substantially large (more than 15 miles) individual neighborhoods. The bold dark curves in the figure represent a fifth degree polynomial fit of the relation between the values of GINI within and between indices and the neighborhood size. The shape of this curve is remarkably consistent with spatial inequality curves identified for each city.

The third and final fact is that there is substantial heterogeneity in the levels of spatial inequality across the 50 cities. This heterogeneity does not reflect heterogeneity in citywide inequality observed across the 50 cities when the neighborhood size is smaller than 10 miles. For individual neighborhoods of larger size, heterogeneity in individual neighborhood inequality turns out to have only an "intercept" dimension, meaning that the degree of heterogeneity around the common trend is uniform across the distance spectrum over which GINI indices are calculated, while the distance gradient on spatial inequality is similar across cities. The intercept dimension of heterogeneity may be explained by differences in fundamentals across cities, such as the distribution of skills across local labor markets (Baum-Snow and Pavan 2013, Moretti 2013), rather than by city-specific characteristics that might have relevant implications for the sorting patterns of low and high income households. Differences in gradients may represent city specific spatial patterns

Figure 5: Spatial inequality in major U.S. metro areas (average), 1980, 1990, 2000 and 2010/14



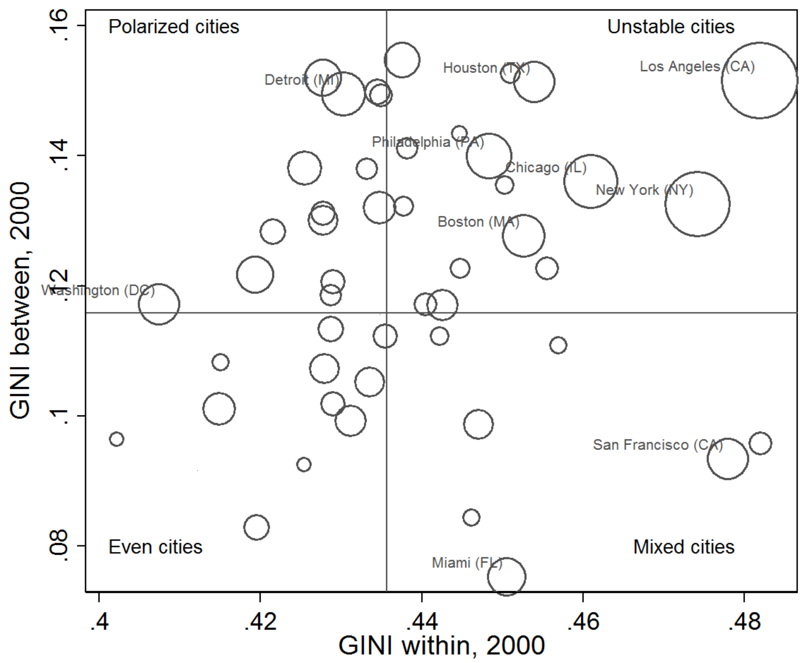
(a) $GINI_W$ (b) $GINI_B$

Note: Authors analysis of U.S. census and ACS data. Year-specific polynomial fittings of $GINI_W$ and $GINI_B$ across 50 largest U.S. metro areas.

in the distribution of rich and poor households. We associate shrinking heterogeneity of city-specific spatial inequality patterns around the common trend with convergence in fundamentals across the cities.

The dynamic of spatial inequality identified for Chicago reflects a general trend of spatial inequality across major U.S. metro areas. Figure 5 sets out polynomial fits of spatial inequality curves for the 50 largest cities generated by the GINI-within and between indices for 1980, 1990, 2000 and 2010/2014. Average trends confirm the stylized facts about spatial inequality: spatial inequality within individual neighborhoods of the average American metro area is high even in small-scale neighborhoods and has been on the rise over the last 35 years. The spatial inequality curve generated by the GINI-between index of the average American metro area converges to zero smoothly. Inequality between individual neighborhoods increased in 1990 and stagnated afterwards. The results support

Figure 6: Taxonomy of major U.S. metro areas, census 2000



Note: Authors analysis of 2000 U.S. census data. Spatial inequality at the city level is obtained by averaging the GINI indices values over the distance spectrum with uniform weighting across distance levels. The maximum distance is set to 20 miles. Metro areas are grouped according to the GINI indices levels. High/low GINI values are computed with respect to the a polynomial fitting of $GINI_W$ and $GINI_B$ values across 50 largest U.S. metro areas.

the previous findings of Wheeler and La Jeunesse (2008), while employing a completely different methodology. Wheeler and La Jeunesse (2008) considered two different exogenous spatial partitions of US metropolitan areas and reported high and persistent levels of spatial inequality within block groups. They also pointed out that the major changes over 1980-2000 were driven by the between component of inequality. This is reflected in the pattern of the GINI-between index.

We find slight evidence of correlation between spatial GINI indices across the 50 cities, suggesting that the two indices probably capture different features of cross sectional spatial inequality. The 50 metro areas can hence be grouped accordingly to the taxonomy induced by the level of within and between spatial inequality in year 2000, which serve as benchmark. Figure 6 displays the arrangement of cities across the four categories. The 10 largest American cities can be categorized in three groups. Detroit, for instance, is a polarized city, with relatively low inequality within the individual neighborhood and high inequality between neighborhoods. Los Angeles, New York and Chicago are classified as unstable cities by our taxonomy based on average trends of spatial inequality. Among the largest cities, San Francisco and Miami fall into the mixed cities category. None of the 10 largest U.S. cities fits in the even city typology.

Spatial inequality, either within or between individual neighborhoods, displays some positive association with citywide inequality, although evidence is less conclusive in 2000 and in more recent ACS waves. Furthermore, spatial inequality is not associated with citywide affluence (captured by the average household income in the city).[26] We conclude that the GINI indices capture separate aspects of inequality that, on the one hand, are rather stable across larger metropolitan areas in the U.S., but, on the other hand, cannot be anticipated from the sole knowledge of citywide income distribution features.

4 Income inequality in American neighborhoods and its long-term consequences

The unequal spatial distribution of incomes across the urban space may affect the long-term prospects of urban residents in different ways. While the extent of spatial inequality

[26] See the Online Appendix for an in-depth discussion of these correlations.

between individual neighborhoods is likely associated with sorting motivations and might have implication on segregation and quality of life of the residents, spatial inequality within individual neighborhood seems instead related to those mechanisms explaining how the place where one has grown up or lives has implications for one's lifelong achievements. Here, we are specifically interested in assessing the potential role of the neighborhood on future outcomes. For this reason, we focus on the within neighborhood aspect of spatial inequality.

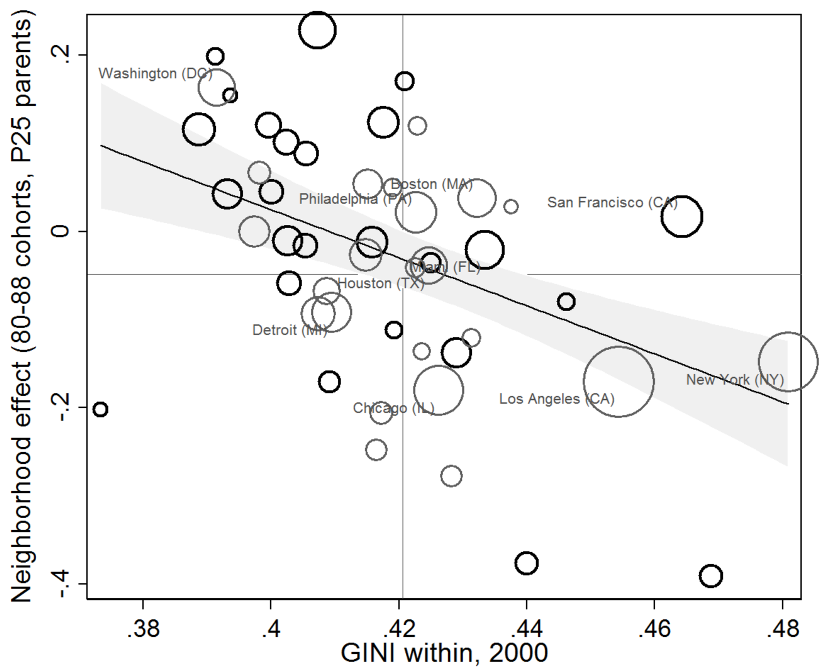
Recent literature has highlighted that inequality at the very local scale seems to play a key role in two important outcomes: prospects for upward mobility of the children raised in poor families and life expectancy of poor adults. Chetty et al. (2014) have documented substantial heterogeneity in income mobility prospects across American commuting zones. Chetty and Hendren (2016) exploit quasi-experimental approximations to identify and estimate the causal effect of the neighborhood people were exposed to in young age on their income mobility prospects in adulthood. Their identification strategy aims at disentangling the causal effect of neighborhood of residence during childhood from implications related to sorting of people with different income prospects across commuting zones. To do so, they focus on people whose parents moved across commuting zones during their childhood, the timing of the move being an exogenous treatment to the children.[27] Chetty and Hendren (2016) estimates are at the metro level reflect the average consequences of the neighborhood composition in any given metro area. They find that the upward mobil-

[27]Chetty and Hendren (2016) use administrative data to measure mobility prospects by the fraction of the difference in earnings of children living in the commuting zone of destination (relative to the earning of children who did not move from the commuting zone of departure) that a child would attain by moving in early age during childhood.

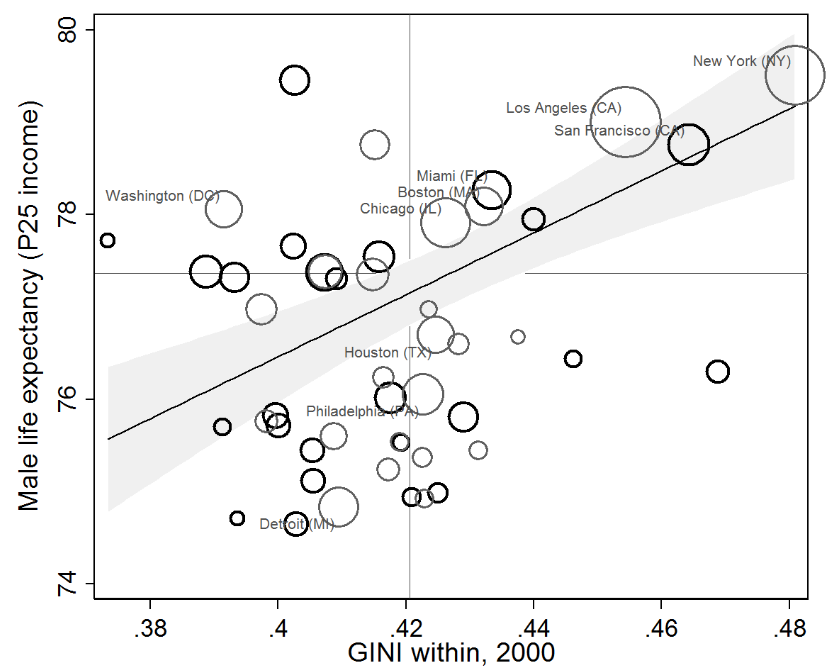
ity estimate for the most disadvantaged children, a measure of the opportunities faced by these worse-off children, only weakly correlates with citywide income inequality experienced during childhood. There are, nevertheless, many potential mechanisms pointing to the fact that the socioeconomic composition of the neighborhood, rather than characteristics of the city as a whole, affects future mobility prospects and might. Some mechanisms have to do with social interactions among neighbors, others with environmental and institutional factors (see for instance Leventhal and Brooks-Gunn (2000) and Ch. 12 in Shonkoff and Phillips (2000)). The extent to which these mechanisms produce effects is likely related to the consequences of the social composition of the neighborhood, which we believe can be picked up by income inequality observed on the very small geographic scale rather than at the citywide level. This suggests a pathway of correlation of spatial inequality within individual neighborhoods among parents and the geography of causal estimates of upward mobility prospects of their children. In line with Chetty and Hendren (2016) identification strategy, we propose using the $GINI_W$ index to measure the average degree of income inequality that children of moving families face in the city of destination.

Figure 7 shows the association between income inequality within individual neighborhoods of small size (less than 2 miles) and the long-term implications of the neighborhood of residence across major U.S. MSAs. Panel (a) displays empirical correlations between causal neighborhood effects estimated in Chetty and Hendren (2016) and the values of $GINI_W$ index for the sample of cities included in this study. The $GINI_W$ index for gross household equivalent income in 2000 is used to measure spatial inequality in the city of

Figure 7: Spatial inequality within the neighborhood and lifelong individual outcomes across U.S. cities.



(a) Neighborhood effects



(b) Life expectancy of the poor

Note: Authors processing of 2000 U.S. Census for 50 largest U.S. Metropolitan Statistical Areas in 2014. Spatial inequality computed at distance range of two miles. Causal neighborhood effects measure the gain or loss in income at age 26 from spending one more year of childhood in a given commuting zone for people whose parents were in the bottom quartile of the national income distribution. Data are as in Chetty and Hendren (2016), variable *causal_p25_czkr26*. Life expectancy is the point estimate of life expectancy in years calculated at age 40 for men at the bottom quartile of household income distribution. Data are as in Chetty, Stepner, Abraham, Lin, Scuderi, Turner, Bergeron and Cutler (2016), variable *le_raceadj_q1_M*. All data are freely accessible on the web from the authors webpages (extracted from *http://www.equality-of-opportunity.org/data/* on October 21, 2016). Causal neighborhood effects and life expectancy estimates are on the commuting zone scale (generally larger than MSA concepts of cities used here). Commuting Zone estimates for Norfolk, VA, are not available. The horizontal gray lines correspond to weighted averages of causal neighborhood effects and life expectancy estimates. The vertical gray lines correspond to average levels of $GINI_W$ in 2000, while black (resp., gray) circles refer to cities with $GINI_B$ below (resp., above) the sample average for 2000 (see reading note Figure 5). The shaded area indicated the 95% confidence bounds of regression predictions.

destination at the moment the parents move.[28] We find significant evidence that causal neighborhood effects on upward mobility of treated children are negatively associated with $GINI_W$ computed for neighborhoods of relatively small size.[29] The negative relation suggests the existence of a Great Gatsby curve (Corak 2013) at the individual neighborhood level, with cities where low-income parents experience on average less unequal income composition in their close neighborhood being also the cities where their children have larger upward mobility prospects.

It has been suggested in the literature that the implications of the neighborhood of residence extend to individual health outcomes, such as life expectancy. Chetty, Stepner, Abraham, Lin, Scuderi, Turner, Bergeron and Cutler (2016) use administrative data on incomes and mortality rates that are representative for the U.S. population for the period 2001-2014, to recover patterns of life expectancy of high and low income people across U.S. commuting zones. They found sharp differences in life expectancy between low and high income individuals, irrespective of gender. While life expectancy does not significantly vary across commuting zones for high income individuals, geography is a strong predictor of longevity for the poor. The authors found positive associations between life expectancy estimates and differences in healthy lifestyle, education and affluence across U.S. commuting zones. Based on this evidence, it can be conjectured that low income

[28] Causal neighborhood effects in Chetty and Hendren (2016) are estimated by the percentage gain (or loss) in income at age 26 attributed to spending one more additional year during childhood in a given commuting zone. These estimates refer to children born 1980-88 whose parents moved to another commuting zone in 1996-2012, i.e., when the children was nine or older. Spatial income inequality in 2000 is used to represent the average composition of a neighborhood at the moment of the move.

[29] This evidence suggests that neighborhood effects on children of poor families are also negatively associated with the degree of inequality between parental individual neighborhoods. This correlation might capture the implications of negative externalities of neighbors' income on child performance. Poor parents that move to cities with a high values of the $GINI_B$ index are more likely to be located in poor areas of the city, with negative external effects due to the economic status of the local community.

people benefit from the presence of more educated and affluent neighbors, who might serve as role models for a healthy lifestyle and consumption (Manley et al. 2012). Panels (b) of Figure 7 display correlation between the longevity at age 40 for low income males (from Chetty, Stepner, Abraham, Lin, Scuderi, Turner, Bergeron and Cutler 2016) and the $GINI_W$ index values in the selected sample of cities. The values of $GINI_W$ in year 2000 are used to measure inequality in the neighborhood experienced by the population for which more reliable longevity estimates are available. We find evidence of a positive association of spatial inequality within the neighborhood and longevity of poor, long-term residents.

The correlations visualized in Figure 7 are robust and their sign and significance remain after controlling for relevant features of the citywide income distribution. There is a concern in the U.S. that differences in income inequalities registered within the cities might mask implications of racial composition and racial segregation in the city. Deaton and Lubotsky (2003), for instance, highlight that the positive association between citywide income inequality and mortality found in the literature is indeed confounded by the effect of racial segregation. Table 3 shows partial effects of spatial inequality on upward mobility prospects and life expectancy estimates after controlling for the ethnic composition of the city and the degree of segregation (measured by the dissimilarity index) of the white population compared to blacks, latinos and asians. Controlling for ethnic size and composition in the cities does not affect the sign and the significance of the spatial inequality effects on upward mobility prospects. The sign and significance of the spatial inequality coefficient on life expectancy regression also survives after controlling for

Indep. var.:	Neighborhood effects				Life expectancy of the poor			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
GINI within	-2.727**	-2.709**	-2.139**	-1.964**	33.567**	33.912**	19.129**	10.989
	(0.60)	(0.60)	(0.67)	(0.82)	(6.63)	(6.35)	(6.61)	(7.08)
% Black		-0.002	-0.005**	-0.004		-0.047**	-0.000	0.006
		(0.00)	(0.00)	(0.00)		(0.02)	(0.02)	(0.02)
% Hispanic			-0.004**	-0.005**			0.015	-0.002
			(0.00)	(0.00)			(0.01)	(0.01)
% Asian			0.004	0.006*			0.132**	0.135**
			(0.00)	(0.00)			(0.03)	(0.03)
Ethnic segregation: dissimilarity of Whites wrt:								
- Blacks				0.000				-0.025
				(0.00)				(0.02)
- Hispanics				0.002				0.074**
				(0.00)				(0.02)
- Asians				-0.005				0.001
				(0.00)				(0.03)
Constant	1.116**	1.143**	0.971**	0.963**	63.035**	63.550**	68.049**	69.431**
	(0.26)	(0.26)	(0.26)	(0.27)	(2.84)	(2.72)	(2.58)	(2.39)
R-squared	0.310	0.334	0.468	0.494	0.358	0.424	0.607	0.709
N	48	48	48	48	48	48	48	48

Table 3: Spatial inequality within the neighborhood and lifelong individual outcomes across U.S. cities.

Note: Authors analysis of U.S. Census data. Dependent variables are defined as in Figure 7. Data on ethnic composition within MSA and dissimilarity index values for Whites with respect to Blacks, Hispanics and Asians are taken from the Diversity and Disparities website hosted by Brown University, Residential Segregation page (see *https://s4.ad.brown.edu/projects/diversity/Data/Download1.htm*). Significance levels: $* = 10\%$ and $** = 5\%$.

citywide racial composition. There is a loss of statical power when controlling also for racial segregation, although the magnitude of the coefficient of spatial inequality remains sizable.

Results in Figure 7 suggest that inequality within the individual neighborhood might be a relevant policy target if the objective is to improve the income prospects of young people or the life expectancy of poor residents. However, within neighborhood inequality is associated with opposite effects on people of different age. For children of poor parents exposed to neighborhood inequality in young age, less inequality within the neighborhood is associated with positive and large upward mobility gains. When the degree of income inequality within the individual neighborhood is larger than the country average, causal upward mobility estimates at the metro level tend to be small or even negative. This association seems to reflect the prevalence of social interaction mechanisms, such as social contagion or collective socialization among peers. Contagion has positive implications for the future economic prospects of children exposed to an advantageous context, the effect being stronger if the local social structure is more cohesive. The average income inequality within the neighborhood, measured in the place of destination at the moment when treated children move from one metro area to another, might capture aspects of neighborhood cohesiveness that are relevant for children born in poor families. In places characterized by low $GINI_W$ values, children of poor parents who decide to move across commuting zones end up in neighborhoods that are on average more cohesive, hence expect stronger positive neighborhood effects. The effect attenuates as the expected degree of inequality within the neighborhood rises.

The implications of spatial inequality for life expectancy prospects of long-term residents are reversed. Being exposed to a mixed income neighborhood seems to increase, on average, the life expectancy of low-income residents. The correlation might be a direct consequence of mechanisms that are directly related to the income mix in the neighborhood (which might arise from a wider range of opportunities and role models present in the neighborhood) or can be fostered by other institutional mechanisms (such as stigmatization of bad behaviors or a more balanced presence within the city of services that promote healthy behaviors and consumption) that seem to be more effective in heterogeneous communities.

5 Concluding remarks

Spatial inequality at the urban level is studied from the perspective of the individual. Information about the income distribution in the neighborhood surrounding each individual is exploited to derive new spatial inequality measures connected to the Gini index. Investigating the patterns of spatial inequality in the 50 largest U.S. cities from 1980 to 2014, new evidence is established: i) inequality within individual neighborhoods is high also for individual neighborhoods of small size; ii) inequality between individual neighborhoods is also high and decreases smoothly with the size of the individual neighborhood; iii) inequality between individual neighborhoods has risen over the last four decades reflecting the trends of the "Great Inversion" (Ehrenhalt 2012); iv) spatial inequality is poorly associated with citywide average income and inequality; vi) American cities can be classified into four distinct groups, on the basis of the values of the within and between GINI indices; v) spatial inequality within individual neighborhoods matters for upward mobility prospects of young people raised in poor families and for life expectancy of low-income residents in America's cities.

Results i)-iii) highlight the increasing importance of income sorting. Despite increasing citywide inequality registered throughout the US largest metro area in recent decades, income sorting seems not to have mitigated inequality at the local scale. Decomposing the changes in inequality by skill and labor attachment type might shed light on the implications for the neighborhood income distribution of households sorting along those dimensions that are more relevant for explaining differences in inequality across cities (Baum-Snow and Pavan 2013). Result iv) suggests that the income mix among neighbors is stronger in cities of larger size, which are all clustered in the "Divided City" typology. Result v) suggests that the desirability of more spatial inequality within the neighborhood is conditional on the timing when this inequality is experienced. Despite more income mix in the neighborhood might be beneficial to low-income, long-term adult residents, it is also significantly associated with smaller and even negative upward mobility gains experienced by children raised in disadvantaged families. Along these lines, policies that aim at improving the chances of success of poor American children should prove more effective by exploiting income targeting to move poor households with young children into cohesive and wealthy neighborhoods, rather than promoting income-mixed local communities. This message, based on cross-metro areas evidence, aligns with findings from randomization studies such as MTO and provides a basis for their generalization.

References

Albouy, D. (2016). What are cities worth? land rents, local productivity, and the total value of amenities, *The Review of Economics and Statistics* **98**(3): 477–487.

Atkinson, T. B. (1970). On the measurement of inequality, *Journal of Economic Theory* **2**: 244–263.

Baum-Snow, N. and Pavan, R. (2013). Inequality and city size, *The Review of Economics and Statistics* **95**(5): 1535–1548.

Bhattacharya, D. (2007). Inference on inequality from household survey data, *Journal of Econometrics* **137**(2): 674 – 707.

Binder, D. and Kovacevic, M. (1995). Estimating some measures of income inequality from survey data: An application of the estimating equations approach., *Survey Methodology* **21**: 137–45.

Biondi, F. and Qeadan, F. (2008). Inequality in paleorecords, *Ecology* **89**(4): 1056–1067.

Bishop, J. A., Chakraborti, S. and Thistle, P. D. (1989). Asymptotically distribution-free statistical inference for generalized Lorenz curves, *The Review of Economics and Statistics* **71**(4): pp. 725–727.

Boal, F. W. (2010). From undivided cities to undivided cities:assimilation to ethnic cleansing, *Housing Studies* **14**(5): 585–600.

Brueckner, J. K., Thisse, J.-F. and Zenou, Y. (1999). Why is central paris rich and downtown detroit poor?: An amenity-based theory, *European Economic Review* **43**(1): 91 – 107.

Chetty, R. and Hendren, N. (2016). The impacts of neighborhoods on intergenerational mobility i: Childhood exposure effects. mimeo.

Chetty, R., Hendren, N. and Katz, L. F. (2016). The effects of exposure to better neighborhoods on children: New evidence from the Moving to Opportunity experiment, *American Economic Review* **106**(4): 855–902.

Chetty, R., Hendren, N., Kline, P. and Saez, E. (2014). Where is the land of opportunity? The geography of intergenerational mobility in the United States, *The Quarterly Journal of Economics* **129**(4): 1553–1623.

Chetty, R., Stepner, M., Abraham, S., Lin, S., Scuderi, B., Turner, N., Bergeron, A. and Cutler, D. (2016). The association between income and life expectancy in the United States, 2001-2014., *The Journal of the American Medical Association* **315**(14): 1750–1766.

Chilès, J.-P. and Delfiner, P. (2012). *Geostatistics: Modeling Spatial Uncertainty*, John Wiley & Sons, New York.

Clark, W. A. V., Anderson, E., Östh, J. and Malmberg, B. (2015). A multiscalar analysis of neighborhood composition in Los Angeles, 2000-2010: A location-based approach to segregation and diversity, *Annals of the Association of American Geographers* **105**(6): 1260–1284.

Conley, T. G. and Topa, G. (2002). Socio-economic distance and spatial patterns in unemployment, *Journal of Applied Econometrics* **17**(4): 303–327.

Corak, M. (2013). Income inequality, equality of opportunity, and intergenerational mobility, *Journal of Economic Perspectives* **27**(3): 79–102.

Cressie, N. (1985). Fitting variogram models by weighted least squares, *Journal of the International Association for Mathematical Geology* **17**(5): 563–586.

Cressie, N. A. C. (1991). *Statistics for Spatal Data*, John Wiley & Sons, New York.

Cressie, N. and Hawkins, D. M. (1980). Robust estimation of the variogram: I, *Journal of the International Association for Mathematical Geology* **12**(2): 115–125.

Dardanoni, V. and Forcina, A. (1999). Inference for Lorenz curve orderings, *Econometrics Journal* **2**: 49–75.

Davidson, R. (2009). Reliable inference for the Gini index, *Journal of Econometrics* **150**(1): 30 – 40.

Dawkins, C. J. (2007). Space and the measurement of income segregation, *Journal of Regional Science* **47**: 255–272.

de Bartolome, C. A. and Ross, S. L. (2003). Equilibria with local governments and commuting: income sorting vs income mixing, *Journal of Urban Economics* **54**(1): 1 – 20.

Deaton, A. and Lubotsky, D. (2003). Mortality, inequality and race in american cities and states, *Social Science & Medicine* **56**: 1139–1153.

Duclos, J.-Y., Esteban, J. and Ray, D. (2004). Polarization: Concepts, measurement, estimation, *Econometrica* **72**(6): 1737–1772.

Durlauf, S. N. (2004). *Neighborhood effects*, Vol. 4 of *Handbook of Regional and Urban Economics*, Elsevier, chapter 50, pp. 2173–2242.

Ehrenhalt, A. (2012). *The Great Inversion and the Future of the American City*, New York: Alfred A. Knopf.

Ellen, I. G., Mertens Horn, K. and O' Regan, K. M. (2013). Why do higher-income households choose low-income neighbourhoods? Pioneering or thrift?, *Urban Studies* **50**(12): 2478–2495.

Fainstein, S. S. (2010). *The Just City*, Cornell University Press.

Galster, G. (2001). On the nature of neighbourhood, *Urban Studies* **38**(12): 2111–2124.

Glaeser, E., Resseger, M. and Tobio, K. (2009). Inequality in cities, *Journal of Regional Science* **49**(4): 617–646.

Goodman, L. A. and Hartley, H. O. (1958). The precision of unbiased ratio-type estimators, *Journal of the American Statistical Association* **53**(282): 491–508.

Habitat, U. (2016). World cities report 2016, *Technical report*.

Hardman, A. and Ioannides, Y. (2004). Neighbors' incom distribution: Economic segregation and mixing in US urban neighborhoods, *Journal of Housing Economics* **13**(4): 368–382.

Hoeffding, W. (1948). A class of statistics with asymptotically normal distribution, *The Annals of Mathematical Statistics* **19**(3): 293–325.

Kim, J. and Jargowsky, P. A. (2009). *The Gini-coefficient and segregation on a continuous variable*, Vol. Occupational and Residential Segregation of *Research on Economic Inequality*, Emerald Group Publishing Limited, pp. 57 – 70.

Kneebone, E. (2016). *The changing geography of disadvantage*, Shared Prosperity in America's Communities, University of Pennsylvania Press, Philadelphia, chapter 3, pp. 41–56.

Lees, L. (2008). Gentrification and social mixing: Towards an inclusive urban renaissence, *Urban Studies* **45**(12): 2449–2470.

Leone, F. C., Nelson, L. S. and Nottingham, R. B. (1961). The folded normal distribution, *Technometrics* **3**(4): 543–550.

Leventhal, T. and Brooks-Gunn, J. (2000). The neighborhoods they live in: The effects of neighborhood residenceon child and adolescent outcomes, *Psychological Bulletin* **126**(2): 309–337.

Ludwig, J., Duncan, G. J., Gennetian, L. A., Katz, L. F., Kessler, R. C., Kling, J. R. and Sanbonmatsu, L. (2013). Long-term neighborhood effects on low-income families: Evidence from Moving to Opportunity, *American Economic Review* **103**(3): 226–31.

Luttmer, E. F. (2005). Neighbors as negatives: Relative earnings and well-being, *The Quarterly Journal of Economics* **120**(3): 963–1002.

Manley, D., van Ham, M. and Doherty, J. (2012). *Social mixing as a nure for negative neighbourhood effects: Evidence based policy or urban myth?*, Vol. Mixed Communities. Gentrification by Stealth?, The Policy Press, Bristol UK, chapter 11.

Marshall, A. W. and Olkin, I. (1979). *Inequalities: Theory of Majorization and Its Applications*, Springer.

Matheron, G. (1963). Principles of geostatistics, *Economic Geology* **58**(8): 1246–1266.

Moretti, E. (2013). Real wage inequality, *American Economic Journal: Applied Economics* **5**(1): 65–103.

Muliere, P. and Scarsini, M. (1989). A note on stochastic dominance and inequality measures, *Journal of Economic Theory* **49**(2): 314 – 323.

Musterd, S. and Andersson, R. (2005). Housing mix, social mix and social opportunities, *Urban Affairs Review* **40**(6): 1–30.

Nielsen, F. and Alderson, A. S. (1997). The Kuznets curve and the great u-turn: Income inequality in U.S. counties, 1970 to 1990, *American Sociological Review* **62**(1): 12–33.

Openshaw, S. (1983). *The modifiable areal unit problem*, Norwick: Geo Books.

Pyatt, G. (1976). On the interpretation and disaggregation of Gini coefficients, *The Economic Journal* **86**(342): 243–255.

Quandt, R. (1966). Old and new methods of estimation and the Pareto distribution, *Metrika* **10**: 55–82.

Reardon, S. F. and Bischoff, K. (2011). Income inequality and income segregation, *American Journal of Sociology* **116**(4): 1092–1153.

Sampson, R. J. (2008). Moving to inequality: Neighborhood eeffects and experiments meet social structure, *American Journal of Sociology* **114**(1): 189–231.

Sarkissian, W. (1976). The idea of social mix in town planning: An historical review, *Urban Studies* **13**: 231–246.

Schelling, T. C. (1969). Models of segregation, *The American Economic Review* **59**(2): pp. 488–493.

Scholar, R. E. (2006). *Divided Cities*, Oxford University Press, Oxford, UK.

Shonkoff, J. P. and Phillips, D. A. (2000). *From Neurons to Neighborhoods: The Science of Early Childhood Development*, National Research Council and Institute of Medicine, National Academic Press, Washington, D.C.

Shorrocks, A. and Wan, G. (2005). Spatial decomposition of inequality, *Journal of Economic Geography* **5**(1): 59–81.

Tin, M. (1965). Comparison of some ratio estimators, *Journal of the American Statistical Association* **60**(309): 294–307.

van Kempen, R. (2007). Divided cities in the 21st century: Challenging the importance of globalisation, *journal of Housing and the Built Environment* **22**: 13–31.

Wheeler, C. H. and La Jeunesse, E. A. (2008). Trends in neighborhood income inequality in the U.S.: 1980–2000, *Journal of Regional Science* **48**(5): 879–891.

Wong, D. (2009). The modifiable areal unit problem (MAUP), *The SAGE handbook of spatial analysis* pp. 105–124.

Xu, K. (2007). U-statistics and their asymptotic results for some inequality and poverty measures, *Econometric Reviews* **26**(5): 567–577.

Appendix for online publication

A Standard errors and confidence bounds for spatial inequality measures

A.1 Setting

Let $\mathcal{S}$ denote a random field. The spatial process $\{Y_s : s = 1, \dots, n\}$ with $s \in \mathcal{S}$ is defined on the random field and is jointly distributed as $F_{\mathcal{S}}$. Suppose data come equally spaced on a grid, so that for any two points $s, v \in \mathcal{S}$ such that $||v - s|| = h$ we write $v = s + h$. The process distributed as $F_{\mathcal{S}}$ is said to display *intrinsic (second-order) stationarity* if $E[Y_s] = \mu$, $Var[Y_s] = \sigma^2$ and $Cov[Y_s, Y_v] = c(h)$ when the covariance function is isotropic and $v = s + h$. Under these circumstances, let $Var[Y_{s+h} - Y_s] = E[(Y_{s+h} - Y_s)^2] = 2\sigma^2 - 2c(h) = 2\gamma(h)$ denote the variogram of the process at distance lag h.

Noticing that $E[Y_{s+h} \cdot Y_s] = \sigma^2 - \gamma(h) + \mu^2$, the covariance between differences in random variables can be written as $Cov[(Y_{s+h_1} - Y_s), (Y_{v+h_2} - Y_v)] = \gamma(s - v + h_1) + \gamma(s - v - h_2) - \gamma(s - v) - \gamma(s - v + h_1 - h_2)$ as in Cressie and Hawkins (1980). Let first assume that the spatial data occur on a transect. Denote $s - v = h$ where h indicates that the random variables are located within a distance cutoff of h units. It follows that $Cov[(Y_{s+h_1} - Y_s), (Y_{v+h_2} - Y_v)] = \gamma(|h + \min\{h_1, h_2\}|) + \gamma(|h - \max\{h_1, h_2\}|) - \gamma(|h|) - \gamma(|h - |h_1 - h_2||)$, which yields the formula above when $h_1 > h_2$. We adopt the convention that $\gamma(-h) = \gamma(h)$ in what follows.

We now introduce one additional distributional assumption: Y_s is gaussian with mean μ and variance σ^2. The random variable $(Y_{s+h} - Y_s)$ is also gaussian with variance $2\gamma(h)$, which implies $|Y_{s+h} - Y_s|$ is *folded-normal* distributed (Leone, Nelson and Nottingham 1961) and its first and second moment depend exclusively on the variogram, having expectation $E[|Y_{s+h} - Y_s|] = \sqrt{2/\pi Var[Y_{s+h} - Y_s]} = 2\sqrt{\gamma(h)/\pi}$ and variance $Var[|Y_{s+h} - Y_s|] = (1 - 2/\pi)2\gamma(h)$.

A.2 GINI indices and the variogram

Under the assumptions above, the GINI indices of spatial inequality in the population can be written as explicit functions of the variogram. We maintain the assumption that the spatial random process is defined on a transect, and occurs at equally spaced lags. For given d, we can thus partition the distance spectrum $[0, d]$ into B_d intervals of fixed size d/B_d. Each interval is denoted by the index b with $b = 1, \ldots, B_d$. We also denote with d_{bi} the set of locations at interval b (and thus distant $b \cdot d/B_d$) within the range d from location s_i. The cardinality of this set is $n_{d_{bi}} \leq n_{d_i} \leq n$. Under all the prevous assumptions, the $GINI_W$ index rewrites

$$\begin{aligned}
GINI_W(F_{\mathcal{S}}, d) &= \sum_i \sum_{j \in d_i} \frac{1}{2n\, n_{d_i}} \frac{E[|Y_{s_j} - Y_{s_i}|]}{\mu} \\
&= \sum_i \sum_{j \in d_i} \frac{1}{2n\, n_{d_i}} \frac{\sqrt{4\gamma(||s_j - s_i||)/\pi}}{\mu} \\
&= \sum_i \frac{1}{n} \sum_{b=1}^{B_d} \frac{n_{d_{bi}}}{n_{d_i}} \sum_{j \in d_{bi}} \frac{1}{2\, n_{d_{bi}}} \frac{\sqrt{4\gamma(s_i + b - s_i)/\pi}}{\mu} \\
&= \frac{1}{2} \sum_{b=1}^{B_d} \left(\sum_i \frac{n_{d_{bi}}}{n\, n_{d_i}} \right) \frac{\sqrt{4\gamma(b)/\pi}}{\mu}. \qquad (1)
\end{aligned}$$

The $GINI_W$ index is an average of a concave transformation of the (semi)variogram function, weighted by the average density of observed incomes at given distance cutoff b on the transect. This average is then normalized by the average income, to produce a scale-invariant measure of inequality. The index can be also conceptualized as an average of coefficients of variation, where the standard deviation is replaced by a measure of dispersion that accounts for the spatial dependence of the underlying process.

Similarly, also the spatial $GINI_B$ index can be written as a function of the variogram. This can be shown under the assumption that the process Y_s is gaussian, as above, which implies that $\mu_{s_i d} = \frac{1}{n_{di}} \left(Y_{s_i} + \sum_{j \in d_i} Y_{s_j} \right)$ is also gaussian under the intrinsic stationarity assumption, with expectation $E[\mu_{s_i d}] = \mu$ for any i. From this, it follows that the difference in random variables $|\mu_{s_i d} - \mu_{s_\ell d}|$ occurring in two locations s_i and s_ℓ is a folded-normal distributed random variable with expectation $E[|\mu_{s_i d} - \mu_{s_\ell d}|] = \sqrt{2/\pi\, Var[\mu_{s_i d} - \mu_{s_\ell d}]}$. The variance term can be decomposed as follows:

$$Var[\mu_{s_i d} - \mu_{s_\ell d}] \;=\; Var[\mu_{s_i d}] + Var[\mu_{s_\ell d}] - 2Cov[\mu_{s_i d};\ \mu_{s_\ell d}]. \tag{2}$$

Developing the variance and covariance terms we obtain:

$$\begin{aligned}
Var[\mu_{s_i d}] &= Var\left[\frac{1}{n_{di}}\left(Y_{s_i} + \sum_{j\in d_i} Y_{s_j}\right)\right] = \frac{1}{n_{di}^2}\sum_{j\in d_i\cup\{i\}}\ \sum_{k\in d_i\cup\{i\}} E[Y_{s_j}Y_{s_k}] - \mu^2 \\
&= \frac{1}{n_{di}^2}\sum_{j\in d_i\cup\{i\}}\ \sum_{k\in d_i\cup\{i\}} c(||s_j - s_k||) \qquad (3) \\
&= \sum_{b=1}^{B_d}\sum_{j\in d_{bi}}\frac{1}{n_{di}}\sum_{b'=1}^{B_d}\sum_{k\in d_{b'i}}\frac{1}{n_{di}} c(|s_i + b - (s_i + b')|) \qquad (4) \\
&= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d}\frac{n_{d_{bi}} n_{d_{b'i}}}{n_{di}^2}\gamma(b - b'), \qquad (5)
\end{aligned}$$

where (3) follows from the definition of the covariogram, (4) is a consequence of the assumption that the process can be represented on a transect and, for simplicity, it is assumed that the set of location at $b = 1$ is $d_{1i} \cup \{i\}$ with cardinality $n_{d_{bi}}$, while (5) follows from the definition of the variogram. Similarly, the covariance term in (2) can be manipulated to obtain the following:

$$\begin{aligned}
Cov[\mu_{s_i d};\ \mu_{s_\ell d}] &= \sum_{j\in d_i}\sum_{k\in d_\ell}\frac{1}{n_{d_i} n_{d_\ell}} E[Y_j Y_k] - \mu^2 \\
&= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d}\frac{n_{d_{bi}} n_{d_{b'\ell}}}{n_{d_i} n_{d_\ell}}\gamma(s_i - s_\ell + |b - b'|), \qquad (6)
\end{aligned}$$

where the assumption that the process can be represented on a transect allows to write the variogram as a function of $s_i - s_\ell$. Plugging (5) and (6) into (2), and denoting $i - \ell = m$ to recall that the gap between locations s_i and s_ℓ is m, with m positive integer such that $m \leq B$ with B being the maximal distance between any two locations on the transect, we obtain

$$
\begin{aligned}
Var[\mu_{s_i d} - \mu_{s_\ell d}] &= \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} 2\frac{n_{d_{bi}} n_{d_{b'\ell}}}{n_{d_i} n_{d_\ell}} \gamma(s_i - s_\ell + |b-b'|) - \\
&\quad - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \left(\frac{n_{d_{bi}} n_{d_{b'i}}}{n_{d_i}^2} + \frac{n_{d_{b\ell}} n_{d_{b'\ell}}}{n_{d_\ell}^2} \right) \gamma(b-b') \\
&= \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} 2\frac{n_{d_{bi}} n_{d_{b'\ i+m}}}{n_{d_i} n_{d_{i+m}}} \gamma(m + |b-b'|) - \\
&\quad - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \left(\frac{n_{d_{bi}} n_{d_{b'i}}}{n_{d_i}^2} + \frac{n_{d_{b\ i+m}} n_{d_{b'\ i+m}}}{n_{d_{i+m}}^2} \right) \gamma(b-b') \\
&= V(\gamma, i, m). \qquad (7)
\end{aligned}
$$

Variogram models suggested in the literatures (for a review, see Cressie 1991) guarantee that $V(\gamma, i, m) > 0$. Using (7), we derive an alternative formulation of the $GINI_B$ index:

$$
\begin{aligned}
GINI_B(F_{\mathcal{S}}, d) &= \frac{1}{2}\sum_i \sum_{\ell \neq i} \frac{1}{n(n-1)} \frac{E[|\mu_{s_i d} - \mu_{s_\ell d}|]}{\mu} \\
&= \frac{1}{2}\sum_i \frac{1}{n} \sum_{m=1}^{B} \sum_{\ell \in n_{mi}} \frac{1}{(n-1)} \frac{E[|\mu_{s_i d} - \mu_{s_\ell d}|]}{\mu} \\
&= \frac{1}{2}\sum_{m=1}^{B} \frac{\left(\sum_i \frac{1}{n}\frac{n_{mi}}{(n-1)}\sqrt{2V(\gamma,i,m)/\pi}\right)}{\mu}. \qquad (8)
\end{aligned}
$$

Under stationarity assumptions we can show that the $GINI_B$ index of spatial inequality can be written as an average of coefficients of variations, each discounted by a weight controlling for the spatial density of income observations.

Formulations of the GINI within and between indices in (1) and (8) clarify the role of spatial dependence (as modeled by the variogram function) on the measurement of spatial inequality. Standard errors and confidence intervals of the GINI indices can be calculated exploiting the variogram properties.

A.3 Standard errors for the $GINI_W$ index

Confidence interval bounds for the $GINI_W$ index are derived under three key assumptions: 1) the underling spatial process is supposed to be stationary; 2) the spatial process occurs on a transect at equally spaced points; 3) the Gaussian law. This allows to build confidence intervals for the empirical $GINI_W$ index estimator of the form $G\hat{I}NI_W(\mathbf{y}, d) \pm z_\alpha SE_{W\,d}$, where z_α is the standard normal critical value for confidence level $1-\alpha$ (for instance, 95%) and $SE_{W\,d}$ is the standard error of the $GINI_W$ estimator. For a given empirical income distribution, the confidence interval changes as a function of the distance parameter selected. Hence, the confidence interval estimator can be used to derive confidence bounds for the spatial inequality curve originated from the $GINI_W$ index. Null hypothesis of dominance or equality for the spatial inequality curves can be tested by using confidence bounds, which define the rejection region (alike to statistical tests for strong forms of stochastic dominance relations, as in Bishop, Chakraborti and Thistle 1989, Dardanoni and Forcina 1999).

Asymptotic standard errors (SE in brief) are derived for the weighted $GINI_W$ index. We assume that the random field $\mathcal{S}$ is limited to n locations. We denote these locations for simplicity by i such that $i = 1, \ldots, n$. The spatial process is then a collection of n random variables $\{Y_i : i = 1, \ldots, n\}$ that are spatially correlated. The joint distribution of the process is F. Each location is associated with a weight $w_i \geq 0$ with $w = \sum_i w_i$, which might reflect the underling population density at a given location. These weights are assumed to be non-stochastic. We also assume intrinsic stationarity as before. The first implication is that, asymptotically, the random variable $\mu_{id} = \sum_{j\in d_i} \frac{w_j}{\sum_{j\in d_i} w_j} Y_j$ is equivalent in expectation to $\tilde{\mu} = \sum_i \frac{w_i}{\sum_i w_i} Y_i$, i.e., $E[\tilde{\mu}] = \mu$. The second implication is that the spatial correlation exhibited by F is stationary in d and can be represented through the variogram of F, denoted $2\gamma(d)$.

An asymptotically equivalent version of the weighted $GINI_W$ index of the process distributed as F where individual neighborhood have size d is

$$GINI_W(F, d) = \frac{1}{2\mu} \sum_{i=1}^{n} \sum_{j\in d_i} \frac{w_i w_j}{2\,w\,\sum_{j\in d_i} w_j} |Y_i - Y_j| = \frac{1}{2\mu} \Delta_{W\,d}. \tag{9}$$

The $GINI_W$ index can thus be expressed as a ratio of two random variables. Asymptotic SE for ratios of random variables have been developed in Goodman and Hartley (1958) and Tin (1965). These SE can be equivalently retrieved from the U-statistics estimators pioneered in Hoeffding (1948) and adopted to derive asymptotic SE for the Gini coefficient of inequality under simple and complex random sampling by Xu (2007) and Davidson (2009). We exploit these results to write the asymptotic variance of the $GINI_W$ index in (9) as:

$$\begin{aligned} Var\left[GINI_W(F,d)\right] \quad = \quad & \frac{1}{4n\mu^2}Var[\Delta_{W\,d}] + \frac{(GINI_W(F,d))^2}{n\mu^2}Var[\tilde{\mu}] - \\ & \frac{GINI_W(F,d)}{n\mu^2}Cov[\Delta_{W\,d},\tilde{\mu}] + O(n^{-2}), \end{aligned} \tag{10}$$

where the asymptotic SE is $SE_{W\,d} = \sqrt{Var\left[GINI_W(F,d)\right]}$ at any d.

To link the variance and covariance terms in (10) and the variogram, the additional assumptions reported above are introduced. The first assumption is that the process distributed as F occurs on a transect, as explained before. We use scalars m, b, b' and so on to identify equally spaced points on the transect. Second, we assume that Y_i is Gaussian with expectation μ and variance σ^2, $\forall i$. These assumptions are taken from Cressie and Hawkins (1980). Under these assumptions, the variance of $\tilde{\mu}$ writes

$$\begin{aligned} Var[\tilde{\mu}] \quad &= \quad \sum_i \frac{w_i}{w}\sum_j \frac{w_j}{w}E[Y_iY_j] - \mu^2 \\ &= \quad \sum_i \frac{w_i}{w}\sum_{m=1}^{B}\frac{\sum_{j\in d_{mi}} w_j}{w}\sum_{j\in d_{mi}}\frac{w_j}{\sum_{j\in d_{mi}} w_j}c(||s_i - s_j||) && (11) \\ &= \quad \sum_{m=1}^{B}\left(\sum_i \frac{w_i}{w}\frac{\sum_{j\in d_{mi}} w_j}{w}c(|m|)\right) && (12) \\ &= \quad \sigma^2 - \sum_{m=1}^{B}\omega(m)\gamma(m), && (13) \end{aligned}$$

where (13) is obtained from (12) by renaming the weight scores so that $\sum_{m=1}^{B}\omega(m) = 1$, and by using the definition of the variogram.

The second variance component of (10) can be written as follows:

$$
\begin{aligned}
Var[\Delta_{W\,d}] &= \sum_{i=1}^{n}\sum_{j\in d_i}\frac{w_i w_j}{w\,\sum_{j\in d_i} w_j}\sum_{\ell=1}^{n}\sum_{k\in d_\ell}\frac{w_\ell w_k}{w\,\sum_{k\in d_\ell} w_k}E[|Y_i-Y_j||Y_\ell-Y_k|] \\
&\quad -\left(\sum_i \frac{w_i}{w}\sum_{j\in d_i}\frac{w_j}{\sum_{j\in d_i} w_j}E[|Y_j-Y_i|]\right)^2.
\end{aligned}
$$

The first component of $Var[\Delta_{W\,d}]$ cannot be further simplified, as the absolute value operator enters the expectation term in multiplicative way. Under the Gaussian assumption, the expectation can be nevertheless simulated, since the random vector (Y_j, Y_i, Y_k, Y_ℓ) is normally distributed with expectations (μ, μ, μ, μ) and its variance-covariance matrix $Cov[(Y_j, Y_i, Y_k, Y_\ell)]$ is:

$$
Cov[(Y_j, Y_i, Y_k, Y_\ell)] = \begin{pmatrix} \sigma^2 & c(||s_j-s_i||) & c(||s_j-s_k||) & c(||s_j-s_\ell||) \\ & \sigma^2 & c(||s_i-s_k||) & c(||s_i-s_\ell||) \\ & & \sigma^2 & c(||s_k-s_\ell||) \\ & & & \sigma^2 \end{pmatrix}.
$$

Data occur on a transect at equally spaced points, where $s_j = s_i + b$ and $s_k = s_\ell + b'$ for the positive integers $b \leq B_d$ and $b' \leq B_d$. We take the convention that $b' > b$ and we further assume that there is a positive gap m, with $m \leq B$ between points s_i and s_ℓ. Using this notation, we can express the variance-covariance matrix as a function of the variogram

$$
Cov[(Y_j, Y_i, Y_k, Y_\ell)] = \begin{pmatrix} \sigma^2 & \sigma^2-\gamma(b) & \sigma^2-\gamma(m-|b'-b|) & \sigma^2-\gamma(m+\min\{b',b\}) \\ & \sigma^2 & \sigma^2-\gamma(m-\max\{b',b\}) & \sigma^2-\gamma(m) \\ & & \sigma^2 & \sigma^2-\gamma(b') \\ & & & \sigma^2 \end{pmatrix}.
$$

The expectation $E[|Y_i - Y_j||Y_\ell - Y_k|]$ can be simulated from a large number S (say, $S = 10{,}000$) of independent draws $(y_{1s}, y_{2s}, y_{3s}, y_{4s})$, with $s = 1, \ldots, S$, from the random vector (Y_j, Y_i, Y_k, Y_ℓ). The simulated expectation is a function of the variogram parameters

m, b, b' and d and of σ^2. It is denoted $\theta_W(m, b, b', d, \sigma^2)$ and estimated as follows:

$$\theta_W(m, b, b', d, \sigma^2) = \frac{1}{S}\sum_{s=1}^{S} |y_{2s} - y_{1s}| \cdot |y_{4s} - y_{3s}|.$$

With some algebra, and using the fact that $E[|Y_\ell - Y_i|] = 2\sqrt{\gamma(m)/\pi}$ for locations ℓ and i at distance $m \leq B$ one from each other, it is then possible to write the term $Var[\Delta_{W\,d}]$ as follows:

$$\begin{aligned} Var[\Delta_{W\,d}] &= \sum_{m=1}^{B}\sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega(m, b, b', d)\theta_W(m, b, b', d, \sigma^2) \\ &\quad -4\left(\sum_{m}^{B_d} \omega(m, d)\sqrt{\gamma(m)/\pi}\right)^2. \end{aligned} \tag{14}$$

In the formula, $\omega(m, b, b', d) = \sum_i \frac{w_i}{w} \sum_{j\in d_{bi}} \frac{w_j}{\sum_{j\in d_i} w_j} \sum_{\ell\in d_{mi}} \frac{w_\ell}{w} \sum_{k\in d_{b'\ell}} \frac{w_k}{\sum_{k\in d_\ell} w_k}$ while $\omega(m, d) = \sum_i \frac{w_i}{w} \sum_{j\in d_{mi}} \frac{w_j}{\sum_{j\in d_i} w_j}$ are calculated as before.

The third component of (10) is the covariance term. It can be written as a function of the variogram. To show this, we take as given that the process is defined on the transect and i and j are separated by b units of spacing while i and ℓ are separated by m unit of spacing, as we rely on the following equivalence:

$$\begin{aligned} E[|Y_j - Y_i|Y_\ell] &= E[|Y_jY_\ell - Y_iY_\ell|] = E[Y_jY_\ell] - E[Y_iY_\ell] - 2E[\min\{Y_jY_\ell - Y_iY_\ell, 0\}] \\ &= c(||s_j - s_\ell||) + \mu^2 - c(||s_i - s_\ell||) - \mu^2 - 2E[\min\{Y_jY_\ell - Y_iY_\ell, 0\}] \\ &= \gamma(m) - \gamma(m - b) - 2E[\min\{Y_jY_\ell - Y_iY_\ell, 0\}]. \end{aligned} \tag{15}$$

The expectation $E[\min\{Y_jY_\ell - Y_iY_\ell, 0\}]$ is non-liner in the underlying random variables. Under the Gaussian hypothesis it can be nevertheless simulated from a large number S (say, $S = 10,000$) of independent draws (y_{1s}, y_{2s}, y_{3s}), with $s = 1, \ldots, S$, from the random vector (Y_j, Y_i, Y_ℓ) which is normally distributed with expectations (μ, μ, μ) and variance-covariance matrix $Cov[(Y_j, Y_i, Y_\ell)]$. As the process occurs on the transect, the variance-covariance matrix writes

$$Cov[(Y_j, Y_i, Y_\ell)] = \begin{pmatrix} \sigma^2 & \sigma^2 - \gamma(b) & \sigma^2 - \gamma(m) \\ & \sigma^2 & \sigma^2 - \gamma(m-b) \\ & & \sigma^2 \end{pmatrix}$$

for given m, b and d. The resulting simulated expectation is denoted $\phi_W(m, b, d, \sigma^2)$ and computed as follows:

$$\phi_W(m, b, d, \sigma^2) = \frac{1}{S}\sum_{s=1}^{S} \min\{y_{2s}y_{3s} - y_{1s}y_{3s}, 0\}.$$

Based on this result, the covariance term in (10) is:

$$\begin{aligned} Cov[\Delta_{W\,d}, \tilde{\mu}] &= \sum_i \frac{w_i}{w} \sum_{j\in d_i} \frac{w_j}{\sum_{j\in d_i} w_j} \sum_\ell \frac{w_\ell}{w} E[|Y_j - Y_i|Y_\ell] \\ &\quad -\mu \sum_i \frac{w_i}{w} \sum_{j\in d_i} \frac{w_j}{\sum_{j\in d_i} w_j} E[|Y_j - Y_i|] \\ &= \sum_{m=1}^{B}\sum_{b=1}^{B_d} \omega(m, b, d)\left[\gamma(m) - \gamma(m-b) - 2\phi_W(m, b, d, \sigma^2)\right] \\ &\quad -2\mu \sum_{m=1}^{B_d} \omega(m, d)\sqrt{\gamma(m)/\pi}. \end{aligned} \tag{16}$$

The weights in (16) coincide respectively with $\omega(m, b, d) = \sum_i \frac{w_i}{w} \sum_{\ell\in d_{mi}} \frac{w_\ell}{w} \sum_{j\in d_{bi}} \frac{w_j}{\sum_{j\in d_i} w_j}$ and $\omega(m, d) = \sum_i \frac{w_i}{w} \sum_{j\in d_{mi}} \frac{w_j}{\sum_{j\in d_i} w_j}$.

A consistent estimator for the SE, denoted $\hat{SE}_{W\,d}$, is obtained by plugging into (10) the empirical counterparts of the variogram and the lag-dependent weights, using the formulas in (13), (14) and (16). These estimators are discussed in section A.5.

A.4 Standard errors for the $GINI_B$ index

Confidence interval bounds $\hat{GINI}_B(\mathbf{y}, d) \pm z_\alpha SE_{B\,d}$ for the $GINI_B$ index are obtained under the same assumptions outlined in the previous section. We assume that the spatial process $\{Y_s : s \in \mathcal{S}\}$ is limited to n locations. We index these locations for simplicity by

i such that $i = 1, \ldots, n$. The spatial process is then a collection of n random variables $\{Y_i : i = 1, \ldots, n\}$ which are spatially correlated. The joint distribution of the process is F. Each location is associated with a weight $w_i \geq 0$ with $w = \sum_i w_i$. These weights are assumed to be non-stochastic.

Under stationary assumptions, the neighborhood averages $\mu_{id} = \sum_{j \in d_i} \frac{w_j}{\sum_{j \in d_i} w_j} Y_j$ and $\mu_d = \sum_i \frac{w_i}{w} \mu_{id}$ are equivalent in distribution to $\tilde{\mu}$, and hence $\tilde{\mu}$ can be used to assess $Var[\mu_d]$, as $Var[\mu_d] = Var[\tilde{\mu}]$. These equivalences apply as well to μ_d.

An asymptotically equivalent version of the weighted GINI index for inequality between individual neighborhoods of the process distributed as F where individual neighborhood have size d is:

$$GINI_W(F, d) = \frac{1}{2\mu} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{w_i w_j}{w^2} |\mu_{id} - \mu_{jd}| = \frac{1}{2\mu} \Delta_{B\,d}. \tag{17}$$

We use results on variance estimators for ratios to derive the SE of (17) as follows:

$$\begin{aligned} Var\left[GINI_B(F, d)\right] &= \frac{1}{4n\mu^2} Var[\Delta_{B\,d}] + \frac{(GINI_B(F, d))^2}{n\mu^2} Var[\tilde{\mu}] - \\ &\quad - \frac{GINI_B(F, d)}{n\mu^2} Cov[\Delta_{B\,d}, \mu_d] + O(n^{-2}), \end{aligned} \tag{18}$$

where the asymptotic SE is $SE_{B\,d} = \sqrt{Var\left[GINI_B(F, d)\right]}$ at any d. The variance and covariance terms in (18) are shown to be related to the variogram. We show that this holds for each of the three elements adding up to (18), under two assumptions. Assume first that the process distributed as F occurs on a transect, as explained before. We use scalars m, b, b' and so on to identify equally spaced points on the transect. Second, assume that Y_i is Gaussian with expectation μ and variance σ^2, $\forall i$.

The variance term $Var[\tilde{\mu}]$ in (18) is given as in (12).

The second variance component in (18) can be written as follows:

$$
\begin{aligned}
Var[\Delta_{B\,d}] &= \sum_i \sum_j \frac{w_i w_j}{w^2} \sum_\ell \sum_k \frac{w_\ell w_k}{w^2} E[|\mu_{id} - \mu_{jd}||\mu_{\ell d} - \mu_{kd}|] \\
&\quad - \left(\sum_i \frac{w_i}{w} \sum_j \frac{w_j}{w} E[|\mu_{jd} - \mu_{id}|] \right)^2 . \qquad (19)
\end{aligned}
$$

The first component of $Var[\Delta_{B\,d}]$ cannot be further simplified as the absolute value operator enters the expectation term in multiplicative way. Under the Gaussian assumption, the expectation can be nevertheless simulated. This can be done since the random vector $(\mu_{jd}, \mu_{id}, \mu_{kd}, \mu_{\ell d})$ is normally distributed with expectations (μ, μ, μ, μ) and variance-covariance matrix $\mathbf{C}$ of size 4×4. The cells in the matrix $\mathbf{C}$ are indexed accordingly to vector $(\mu_{jd}, \mu_{id}, \mu_{kd}, \mu_{\ell d})$, so that element C_{12}, for instance, is used to indicate the covariance between the random variables μ_{jd} and μ_{id}. The sample occurs on a transect. We use scalars b and b' to denote a well defined distance gap between any location indexed by $\{j, i, k, \ell\}$ and any other location that is b or b' units away from it, within a distance range d. We use scalars m to indicate the gap between i and ℓ, so that $\ell = i + m$; we use m' to indicate the gap between i and j, so that $j = i + m'$ and we use m'' to indicate the gap between k and ℓ, so that $\ell = k + m''$. Based on this notation, we can construct a weighted analog of (6) to explicitly write the elements of $\mathbf{C}$ as transformations of the variogram. This gives:

$$
\begin{aligned}
C_{11} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_1(b,d)\omega_1(b',d)\gamma(b-b'),\\
C_{22} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_2(b,d)\omega_2(b',d)\gamma(b-b'),\\
C_{33} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_3(b,d)\omega_3(b',d)\gamma(b-b'),\\
C_{44} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_4(b,d)\omega_4(b',d)\gamma(b-b'),\\
C_{12} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_1(b,d)\omega_2(b',d)\gamma(m'+|b-b'|),\\
C_{13} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_1(b,d)\omega_3(b',d)\gamma(m+|b-b'|),\\
C_{14} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_1(b,d)\omega_4(b',d)\gamma(m+|b-b'|),\\
C_{23} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_2(b,d)\omega_3(b',d)\gamma(m+|b-b'|),\\
C_{24} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_2(b,d)\omega_4(b',d)\gamma(m+|b-b'|),\\
C_{34} &= \sigma^2 - \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} \omega_3(b,d)\omega_4(b',d)\gamma(m''+|b-b'|),
\end{aligned}
$$

where we denote, for instance, $\omega_1(b,d) = \sum_j \frac{w_j}{w} \sum_{j' \in d_{b\,j}} \frac{w_{j'}}{\sum_{j' \in d_j} w_{j'}}$ and similarly for the other elements.

The expectation $E[|\mu_{jd} - \mu_{id}||\mu_{kd} - \mu_{\ell d}|]$ can be simulated from a large number S (say, $S = 10,000$) of independent draws $(\bar{y}_{1s}, \bar{y}_{2s}, \bar{y}_{3s}, \bar{y}_{4s})$, with $s = 1, \ldots, S$, of the random vector $(\mu_{jd}, \mu_{id}, \mu_{kd}, \mu_{\ell d})$. The simulated expectation will be a function of the variogram parameters m, m', m'' and d and of σ^2. It is denoted $\theta_B(m, m', m'', d, \sigma^2)$ and estimated as follows:

$$\theta_B(m, m', m'', d, \sigma^2) = \frac{1}{S}\sum_{s=1}^{S} |\bar{y}_{2s} - \bar{y}_{1s}| \cdot |\bar{y}_{4s} - \bar{y}_{3s}|.$$

The summations in $Var[\Delta_{B\,d}]$ run over four indices i, j, k, ℓ. These can be equivalently represented through summations at given distance lags m, m', m''. For instance, we write $\sum_i \frac{w_i}{w} \sum_j \frac{w_j}{w} = \sum_{m'=1}^{B} \sum_i \frac{w_i}{w} \sum_{j \in d_{m'i}} \frac{w_j}{w}$ to indicate that i and j are separated by a lag of m' units on the transect. Repeating this for each of the three pairs of indices i, j and ℓ, k and i, ℓ we end up with three summations over m', m'' and m respectively, where the aggregate weight is denoted

$$\omega(m, m', m'', d) = \sum_i \frac{w_i}{w} \sum_{j \in d_{m'i}} \frac{w_j}{w} \cdot \sum_{\ell} \frac{w_\ell}{w} \sum_{k \in d_{m''\ell}} \frac{w_k}{w} \cdot \sum_i \frac{w_i}{w} \sum_{\ell \in d_{mi}} \frac{w_\ell}{w}.$$

The first term of $Var[\Delta_{B\,d}]$, $\sum_i \sum_j \frac{w_i w_j}{w^2} \sum_\ell \sum_k \frac{w_\ell w_k}{w^2} E[|\mu_{id} - \mu_{jd}||\mu_{\ell d} - \mu_{kd}|]$, can be written as follows:

$$\sum_{m=1}^{B} \sum_{m'=1}^{B} \sum_{m''=1}^{B} \omega(m, m', m'', d)\theta_B(m, m', m'', d, \sigma^2). \tag{20}$$

The second term of $Var[\Delta_{B\,d}]$, we make use of the gaussian assumption and the variogram properties to express the square of the expectation as a weighted analog of (8), that is

$$\begin{aligned} Var[\Delta_{B\,d}] &= E\left[\sum_i \sum_j \frac{w_i w_j}{w^2} |\mu_{id} - \mu_{jd}|\right]^2 \\ &= \left(\sum_i \sum_j \frac{w_i w_j}{w^2} E[|\mu_{id} - \mu_{jd}|]\right)^2 \\ &= \left(\sum_i \sum_j \frac{w_i w_j}{w^2} \sqrt{Var[|\mu_{id} - \mu_{jd}|]}\sqrt{\frac{2}{\pi}}\right)^2 \\ &= \frac{2}{\pi}\left(\sum_i \frac{w_i}{w} \sum_{m'=1}^{B} \frac{\sum_{j \in d_{mi}} w_j}{w} \sum_{j \in d_{mi}} \frac{w_j}{\sum_{j \in d_{mi}} w_j} \sqrt{Var[|\mu_{id} - \mu_{jd}|]}\right)^2 \\ &= \frac{2}{\pi}\left(\sum_{m'=1}^{B} \sum_i \sum_{j \in d_{mi}} \omega_{ij}(m, d)\sqrt{Var[|\mu_{id} - \mu_{jd}|]}\right)^2 \end{aligned} \tag{21}$$

where

$$Var[|\mu_{id} - \mu_{jd}|] = \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} 2\omega_{ij}(b, b', d)\gamma(m - |b - b'|) - (\omega_i(b, b', d) + \omega_j(b, b', d))\gamma(b - b').$$

Both weighting schemes in (20) and in (21) cannot be easily estimated in reasonable computation time: they involve multiple loops across the observed locations, so that the length of estimation increases exponentially with the density of the spatial structure. In section A.5 we discuss estimators of the weights $\omega_{ij}(m, m', m'', d)$, $\omega_{ij}(m, d)$, $\omega_{ij}(b, b', d)$, $\omega_i(b, b', d)$ and $\omega_j(b, b', d)$ that are feasible, and provide the empirical estimator of the variance $Var[\Delta_{B\,d}]$.

The third component of (18) is the covariance term $Cov[\Delta_{B\,d}, \mu_d]$. The indices i, j, ℓ identify three locations and the average income in a neighborhood of size d in each of the three location is represented by the vector $(\mu_{id}, \mu_{jd}, \mu_{\ell d})$. Under normality and stationarity assumptions, we can write the covariance term as follows

$$\begin{aligned} Cov[\Delta_{B\,d}, \mu_d] &= Cov[\sum_i \sum_j \frac{w_i w_j}{w^2} |\mu_{id} - \mu_{jd}|, \sum_\ell \frac{w_\ell}{w} \mu_{\ell d}] \\ &= \sum_\ell \frac{w_\ell}{w} Cov[\sum_i \sum_j \frac{w_i w_j}{w^2} |\mu_{id} - \mu_{jd}|, \mu_{\ell d}] \\ &= \sum_\ell \frac{w_\ell}{w} \sum_i \sum_j \frac{w_i w_j}{w^2} E[|\mu_{id} - \mu_{jd}| \mu_{\ell d}] - \\ &\quad - \sum_\ell \frac{w_\ell}{w} E[\mu_{\ell d}] \sum_i \sum_j \frac{w_i w_j}{w^2} E[|\mu_{id} - \mu_{jd}|] \\ &= \sum_\ell \frac{w_\ell}{w} \sum_i \sum_j \frac{w_i w_j}{w^2} E[|\mu_{id} - \mu_{jd}| \mu_{\ell d}] - \\ &\quad - \sqrt{\frac{2}{\pi}} \mu \sum_i \sum_j \frac{w_i w_j}{w^2} \sqrt{Var[\mu_{id} - \mu_{jd}]}. \end{aligned} \tag{22}$$

The first term of (22) is the expectation of a non-linear function of convex combinations of normally distributed random variables. Under the Gaussian hypothesis, the expectation $E[|\mu_{id} - \mu_{jd}| \mu_{\ell d}]$ can be nevertheless simulated from a large number S (say, $S = 10,000$) of independent draws $(\bar{y}_{1s}, \bar{y}_{2s}, \bar{y}_{3s})$ with $s = 1, \ldots, S$ from the random vector $(\mu_{id}, \mu_{jd}, \mu_{\ell d})$ which is normally distributed with expectations (μ, μ, μ) and variance-covariance matrix

$\mathbf{C}$ of size 3×3. Let use scalars b and b' to denote a well defined distance gap between any observation indexed by $\{i, j, \ell\}$ and any other observation that is b or b' units away from it, within a distance boundary d. We use scalars m' to indicate the gap between i and j, so that $j = i + m'$; we use m'' to indicate the gap between i and ℓ, so that $\ell = i + m''$. Based on this notation, we obtain a convenient formulation for the covariances of mean neighborhood incomes that are weighted analog of (6), thus giving:

$$
\begin{aligned}
C_{11} &= \sigma^2 - \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} \omega_1(b,d)\omega_1(b',d)\gamma(b-b'), \\
C_{22} &= \sigma^2 - \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} \omega_2(b,d)\omega_2(b',d)\gamma(b-b'), \\
C_{33} &= \sigma^2 - \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} \omega_3(b,d)\omega_3(b',d)\gamma(b-b'), \\
C_{12} &= \sigma^2 - \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} \omega_1(m'+b,d)\omega_2(b',d)\gamma(m'+|b-b'|), \\
C_{13} &= \sigma^2 - \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} \omega_1(b,d)\omega_3(m''+b',d)\gamma(m''+|b-b'|), \\
C_{23} &= \sigma^2 - \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} \omega_2(m'+b,d)\omega_3(m''+b',d)\gamma(|m'-m''|+|b-b'|),
\end{aligned}
$$

where we denote, for instance, $\omega_1(b,d) = \sum_i \frac{w_i}{w} \sum_{i' \in d_{b\,i}} \frac{w_{i'}}{\sum_{i' \in d_i} w_{i'}}$ and similarly for the other elements. See previous notation for further details. The expectation $E[|\mu_{jd} - \mu_{id}|\mu_{\ell d}]$ is simulated from a number S of independent draws $(\bar{y}_{1s}, \bar{y}_{2s}, \bar{y}_{3s})$ with $s = 1, \ldots, S$ of the random vector $(\mu_{jd}, \mu_{id}, \mu_{\ell d})$. The simulated expectation will be a function of the variogram parameters m', m'' and d and of σ^2. It is denoted $\phi_B(m, m', m'', d, \sigma^2)$ and estimated as follows:

$$
\phi_B(m', m'', d, \sigma^2) = \frac{1}{S} \sum_{s=1}^{S} |\bar{y}_{2s} - \bar{y}_{1s}|\bar{y}_{3s}.
$$

This element is constant over m' and m''. Hence, we use $\phi_B(m', m'', d, \sigma^2)$ as a simulated analog for $E[|\mu_{id}-\mu_{jd}|\mu_{\ell d}]$, so that the covariance term $\sum_\ell \frac{w_\ell}{w}\sum_i\sum_j \frac{w_i w_j}{w^2}E[|\mu_{id}-\mu_{jd}|\mu_{\ell d}]$ writes $\sum_\ell \frac{w_\ell}{w}\sum_i\sum_j \frac{w_i w_j}{w^2}\phi_B(m',m'',d,\sigma^2)$, or equivalently

$$\sum_i \frac{w_i}{w}\sum_{m'=1}^{B}\frac{\sum_{j\in d_{m'i}} w_j}{w}\sum_{m''=1}^{B}\frac{\sum_{\ell\in d_{m''i}} w_\ell}{w}\phi_B(m',m'',d,\sigma^2),$$

which is denoted $\sum_{m'=1}^{B}\sum_{m''=1}^{B}\omega(m',m'',d)\phi_B(m',m'',d,\sigma^2)$.

The second term of (22) is calculated as in (21). Overall, we are now allowed to write the covariance term as follows:

$$\begin{aligned} Cov[\Delta_{B\,d},\mu_d] \;=\; & \sum_{m'=1}^{B}\sum_{m''=1}^{B}\omega(m',m'',d)\phi_B(m',m'',d,\sigma^2) \\ & -\sqrt{\frac{2}{\pi}}\mu\sum_{m'=1}^{B}\sum_i\sum_{j\in d_{mi}}\omega_{ij}(m,d)\sqrt{Var[|\mu_{id}-\mu_{jd}|]}, \end{aligned} \tag{23}$$

where

$$Var[|\mu_{id}-\mu_{jd}|] = \sum_{b=1}^{B_d}\sum_{b'=1}^{B_d} 2\omega_{ij}(b,b',d)\gamma(m-|b-b'|) - (\omega_i(b,b',d)+\omega_j(b,b',d))\gamma(b-b')$$

The weights have been already defined in (21). Plugging (13), (20), (21) and (23) into (18) we derive an estimator for the $GINI_B$ index SE. The last section discuss feasible estimators.

A.5 Implementation

Consider a sample of size n of income realizations y_i with $i = 1,\ldots,n$. The income vector $\mathbf{y} = (y_1,\ldots,y_n)$ is a draw from the spatial random process $\{Y_s : s \in \mathcal{S}\}$, while for each location $s \in \mathcal{S}$ we assume to observe, at most, one income realization. Information about location of an observation i in the geographic space $\mathcal{S}$ under analysis is denoted by $s_i \in \mathcal{S}$, so that a location s identifies a precise point on a map. Information about latitude and longitude coordinates of s_i are given. In this way, distance measures between locations can be easily constructed. In applications involving geographic representations, the latitude and longitude coordinates of any pair of incomes y_i, y_j can be combined to obtain the geodesic distance among the locations of i and of j. Furthermore, observed incomes are associated with weights $w_i \geq 0$ and are indexed according to the sample units, with $w = \sum_i w_i$. It is often the case that the sample weights give the inverse probability of selection of an observation from the population.

The mean income within an individual neighborhood of range d, μ_{id}, is estimated by $\widehat{\mu}_{id} = \sum_{j=1}^{n} \hat{w}_j y_j$ where

$$\hat{w}_j := \frac{w_j \cdot \mathbf{1}(||s_i - s_j|| \leq d)}{\sum_j w_j \cdot \mathbf{1}(||s_i - s_j|| \leq d)}$$

so that $\sum_j \hat{w}_j = 1$, and $\mathbf{1}(.)$ is the indicator function. The estimator of the average neighborhood mean income is instead $\hat{\mu}_d = \sum_{i=1}^{n} \frac{w_i}{w} \widehat{\mu}_{id}$. The estimator of the $GINI_B$ index of spatial inequality, denoted $G\hat{I}NI_B(\mathbf{y}, d)$, is the Gini inequality index of the vector of estimated average incomes $(\hat{\mu}_{1d}, \ldots, \hat{\mu}_{nd})$, indexed by the size d of the individual neighborhood. It can be computed by mean of the plug-in estimators as in Binder and Kovacevic (1995) and Bhattacharya (2007). The estimator of the $GINI_W$ index of spatial inequality, denoted $G\hat{I}NI_W(\mathbf{y}, d)$, is the sample weighted average of the mean absolute deviation of the income realization in location s from the income realization in location s', with $||s - s'|| \leq d$. Formally

$$G\hat{I}NI_W(\mathbf{y}, d) = \sum_{i=1}^{n} \frac{w_i}{w} \frac{1}{2\hat{\mu}_{id}} \sum_{j=1}^{n} \hat{w}_j \, |y_i - y_j|,$$

where $\hat{w}_j$ is defined as above.

The estimation of the GINI indices is conditional on d, which is a parameter under control of the researcher. The distance d is conventionally reported in miles and is meant to capture a continuous measure of the extent of an individual neighborhood. In practice, however, one cannot produce estimates of spatial inequality for a continuum of neighborhoods, and so in applications the neighborhood size is parametrized by the product of the number and size of lags between observations. The GINI indices are estimated for a finite number of lags and for a given size of the lags. The maximum number of lags indicates the point at which distance between observations is large enough that the spatial GINI indices converge to their respective asymptotic values. For a given neighborhood of size d, we can then partition the distance interval $[0, d]$, defining the size of a neighborhood, into K intervals $d_0, d_1, \ldots, d_K$ of equal size, with $d_0 = 0$. We always use d_k to denote the distance between any pair of observations i and j located at distance $d_{k-1} < ||s_i - s_j|| \leq d_k$ one from the other. The pairs $(d_k, \hat{GINI}_B(\mathbf{y}, d_k))$ and $(d_k, \hat{GINI}_W(\mathbf{y}, d_k))$ for any $k = 1, \ldots, K$ can be hence plotted on a graph. The curves resulting by linearly interpolating these points are the empirical equivalent of the GINI spatial inequality curves.

A plug-in estimator for the asymptotic standard error of the GINI indices can be derived under the assumptions listed in the previous sections. The SE estimator crucially depend on four components: (i) the consistent estimator for the average $\tilde{\mu}$, denoted $\hat{\mu}$, which coincides with the sample average; (ii) the consistent estimator for variance σ^2, denoted $\hat{\sigma}^2$, which is given by the sample variance; (iii) the consistent estimator for the variogram; (iv) the estimator of the weighting schemes.

Empirical estimators $\hat{\mu}$ and $\hat{\sigma}^2$ are standard. The robust non-parametric estimator of the variogram proposed by Cressie and Hawkins (1980) can be used to assess the pattern of spatial dependency from spatial data on income realizations. The empirical variogram is defined for given spatial lags, meaning that it produces a measure of spatial dependence among observations that are located at a given distance lag one from the others. Under the assumption that data occur on the transect at equally spaced points, we use $b = 1, \ldots, B$ to partition the empirical spectrum of distances between observed locations into equally spaced lags, and we estimate the variogram on each of these lags. This means that $2\gamma(b)$ refers to the correlation between incomes placed at distance lags of exactly b distance units. It is understood that the size of the sample is large compared to B, in the sense that the sampling rate per unit area remains constant when the partition into lags becomes finer. This assumption allows to estimate a non-parametric version of the variogram at every distance lag. Following Cressie (1985), we use weighted least squares to fit a theoretical variogram model to the empirical variogram estimates. The theoretical model consists in a continuous parametric function mapping distance into the corresponding variogram level. In our empirical analysis of spatial inequality in the 50 largest U.S. metro areas, we choose the spherical variogram model for γ (see Cressie 1985). We also assume that $\gamma(0) \to 0$ and that $\gamma(a) = \sigma^2$, where a is the so-called range level: beyond distance a, the random variables Y_{s+h} and Y_s with $h > a$ are spatially uncorrelated. Under the assumption that data occur on a transect, we set the max number of lags B so that $B = 2a$. Parameters of the variogram are estimated by fitting via weighted least squares a parametric variogram model to the non-parametric variogram sample estimates at pre-determined distance cutoffs. The estimated parameters are then used to draw predictions for the estimator $2\hat{\gamma}$ of the variogram at each distance cutoff. The predictions are then plugged into the GINI indices SE estimators. Cressie (1985) has shown that this methodology leads to consistent estimates of the true variogram function under the stationarity assumptions mentioned above.

Finally, SE estimation requires to produce reliable estimators of the weights ω. These can be non-parametrically identified from the formulas provided above. In some cases, however, computation of the exact weights requires looping several times across observations. The overall computation time thus increases exponentially in the number of observations and the procedure becomes quickly unfeasible. We propose alternative, feasible estimator for these weights, denoted $\hat{\omega}$, that are expressed as linear averages. The computational time is, nevertheless, quadratic in the number of observations as it requires at least one loop across all observations.

We consider here only the weights that appear in the estimators $\hat{SE}_{W\,d}$ in (10) and $\hat{SE}_{B\,d}$ in (18) that cannot be directly inferred (i.e., are computationally unfeasible) from observed weights. For a given observation i, define $w(b,i) = \sum_{j \in d_{bi}} w_j$ for any gap $b = 1, \ldots, B_d, \ldots, B$ the weight associated with income realizations that are exactly located b lags away from i. Then, denote $w(d,i) = \sum_{j \in d_i} w_j = \sum_{b=1}^{B_d} w(b,i)$. We construct the following estimators for the weights appeasing in the $GINI_W$ SE estimator:

$$\text{For (14)} \quad : \quad \hat{\omega}(m,b,b',d) = \sum_i \frac{w_i}{w} \frac{w(b,i)}{w(d,i)} \frac{w(m,i)}{w} \frac{w(m+b',i)}{w(m+d,i)},$$

$$\text{For (16)} \quad : \quad \hat{\omega}(m,b',d) = \sum_i \frac{w_i}{w} \frac{w(m,i)}{w} \frac{w(b',i)}{w(d,i)},$$

To compute these weights, one has to loop over all observations twice, and assign to each observation i the total weight $w(b,i)$ of those observations $j \neq i$ that are located exactly at distance b from i. Then, $\hat{\omega}(m,b,b',d)$ and $\hat{\omega}(m,b',d)$ are obtained by averaging these weights across i's. The key feature of these estimators is that second-order loops across observations occurring at distance b' from an observation at distance m from i are estimated by averaging across all observations i the relative weight of observations at distance $m+b'$ from i.

For the computation of the $GINI_B$ index, one needs to construct the relative weights by taking as a reference the maximum distance achievable, and not the reference abscissa d for which the index is calculated. We hence assume that beyond the threshold $\overline{d}$, indicating half of the the maximum distance achievable in the sample, spatial correlation is negligible and weights can thus be set to zero. We implicitly maintain that $d \leq \overline{d}$. We then propose the following estimators:

$$
\begin{aligned}
\text{For (20)} &: \quad \hat{\omega}(m,m',m'',d) = \sum_i \frac{w_i}{w} \frac{w(m',i)}{w(\overline{d},i)} \frac{w(m,i)}{w(\overline{d},i)} \frac{w(m+m'',i)}{w(m+\overline{d},i)} \\
\text{For (21)} &: \quad \hat{\omega}_{ij}(b,b',d) = \frac{w(b,i)}{w(d,i)} \frac{w(m+b',i)}{w(m+d,i)} \\
\text{For (21)} &: \quad \hat{\omega}_{i}(b,b',d) = \frac{w(b,i)}{w(d,i)} \frac{w(b',i)}{w(d,i)} \\
\text{For (21)} &: \quad \hat{\omega}_{j}(b,b',d) = \frac{w(m+b,i)}{w(m+d,i)} \frac{w(m+b',i)}{w(m+d,i)} \\
\text{For (21)} &: \quad \sum_i \sum_{j \in d_{mi}} \hat{\omega}_{ij}(m,d) = \sum_i \frac{w_i}{w} \frac{w(m,i)}{w}
\end{aligned}
$$

By plugging these estimators into (19) we obtain the implementable estimator of the variance component $Var[\Delta_{B\,d}]$, defined as follows:

$$
\begin{aligned}
\widehat{Var}[\Delta_{B\,d}] \;=\; & \sum_{m=1}^{B} \sum_{m'}^{B} \sum_{m''=1}^{B} \hat{\omega}(m,m',m'',d)\theta_B(m,m',m'',d,\hat{\sigma}^2) - \\
& -\frac{2}{\pi} \left(\sum_{m'=1}^{B} \sum_i \frac{w_i}{w} \frac{w(m,i)}{w} \sqrt{\widehat{Var}[|\mu_{id} - \mu_{jd}|]} \right)^2
\end{aligned} \tag{24}
$$

where

$$\widehat{Var}[|\mu_{id} - \mu_{jd}|] = \sum_{b=1}^{B_d} \sum_{b'=1}^{B_d} 2\hat{\omega}_{ij}(b, b', d)\hat{\gamma}(m - |b - b'|) - (\hat{\omega}_i(b, b', d) + \hat{\omega}_j(b, b', d))\hat{\gamma}(b - b').$$

An equivalent procedure, based on analogous weighting scheme, has to be replicated to determine the empirical estimator for (23).

B Additional results

B.1 Inference results for spatial inequality curves, Chicago (IL)

Figure 8 shows that the gap in $GINI_W$ indices is small over time and never significant, not even at 10% confidence level. Essentially, there is no statistical support to conclude that the GINI curves for within spatial inequality have changed across time, a result which holds irrespectively of the extent of individual neighborhood. We draw a different conclusion for what concerns changes associated to the spatial inequality curves generated by $GINI_B$. Pairwise differences across these curves, along with their confidence intervals, are reported in Figure 9. The differences in inequality curves compared to the spatial inequality curve of the year 1980 (panels (a), (b) and (c) of the figure) are generally positive and significant at 5% confidence level. This indicates that spatial inequality between individual neighborhoods has increased compared to the initial period, roughly homogenously with respect to the individual neighborhood spatial extension. After that period, data display very little statistical support to changes in inequality across the 1990' and 2000'. Spatial between inequality has slightly increased after 1990 (panels (d) and (e)), while it has remained stable after 2000 (panel (f)). In the latter case, the confidence bounds of the difference in spatial inequality curves of years 2010/2014 and 2000 fluctuates around the horizontal axis.

Estimates of the GINI standard errors allow to study the pattern of the spatial inequality curves. More specifically, differences in $GINI_W(d)$ or $GINI_B(d)$ indices are first computed at various abscissae d. Then, the standard errors of these differences are derived, and finally it is checked if these differences are significantly different than zero. If they are, we study how spatial inequality evolves with the size of the neighborhood. In particular, the sign of these differences predicts the direction of the change in spatial inequality. We refer to five distance thresholds referring to neighborhoods that are very small (100 meters, 300 meters), relatively large (1km, 5km), and very inclusive neighborhoods (10km, 25km), which include most of the urban space under analysis. The resulting differences are reported in Table 4. The dip in the spatial inequality curve associated with

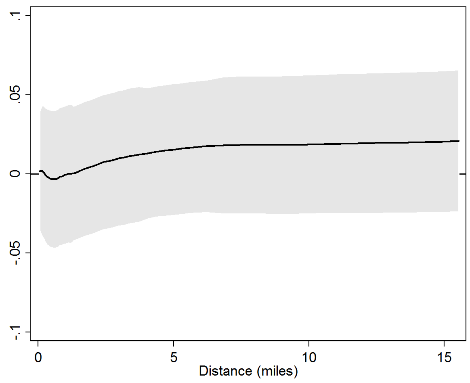
Index	Year	Differences across distances						
		300m vs 100m	1km vs 100m	5km vs 100m	25km vs 100m	10km vs 2km	25km vs 2km	25km vs 10km
$GINI_W$	1980	-0.004	-0.012	-0.006	0.015	0.013	0.025	0.012
		(0.019)	(0.020)	(0.020)	(0.021)	(0.021)	(0.023)	(0.023)
	1990	-0.006	-0.019	0.003	0.037*	0.036	0.051**	0.015
		(0.022)	(0.022)	(0.021)	(0.021)	(0.022)	(0.023)	(0.021)
	2000	-0.004	-0.016	-0.002	0.035*	0.034	0.050**	0.016
		(0.017)	(0.017)	(0.020)	(0.021)	(0.021)	(0.022)	(0.024)
	2010	-0.000	-0.004	0.001	0.033	0.019	0.036	0.017
		(0.017)	(0.018)	(0.019)	(0.021)	(0.021)	(0.023)	(0.024)
$GINI_B$	1980	-0.020**	-0.087**	-0.151**	-0.239**	-0.061**	-0.120**	-0.059**
		(0.002)	(0.002)	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)
	1990	-0.012**	-0.084**	-0.171**	-0.280**	-0.097**	-0.160**	-0.064**
		(0.004)	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.004)
	2000	-0.009**	-0.060**	-0.130**	-0.237**	-0.095**	-0.152**	-0.057**
		(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)
	2010	-0.019**	-0.083**	-0.160**	-0.261**	-0.084**	-0.141**	-0.058**
		(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)

Table 4: Patterns of GINI indices across distance levels

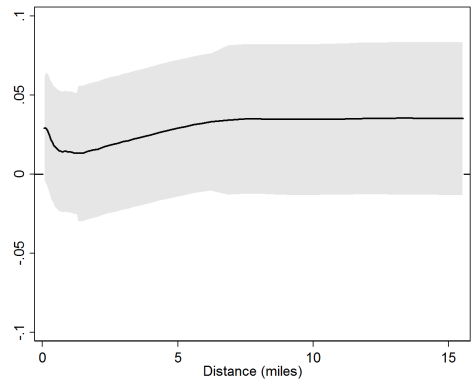
Note: Authors analysis of U.S. Census data. Each column report differences in GINI indices at various distance thresholds. SE of the distance estimate are reported in brackets. Significance levels: $* = 10\%$ and $** = 5\%$.

the $GINI_W$ is not statistically significant, since most of the changes in spatial inequality in very large neighborhoods is substantially equivalent to the spatial inequality observed for very small neighborhoods (between 100 to 300 meters of size). For 1990 and 2000, we find a statistically significant increase in inequality when average size neighborhoods (1km of radius) are compared with very large concepts of neighborhoods. Overall, the $GINI_W$ index pattern is substantially flat when the distance increases beyond 5km. The pattern registered for the $GINI_B$ index is much more clear-cut: generally, the spatial inequality curve constructed from the index is decreasing in distance (differences in $GINI_B$ are always negative), and the patterns of changes are also significant at 5%, indicating strong reliability on the pattern of heterogeneity in average income distribution across neighborhoods.

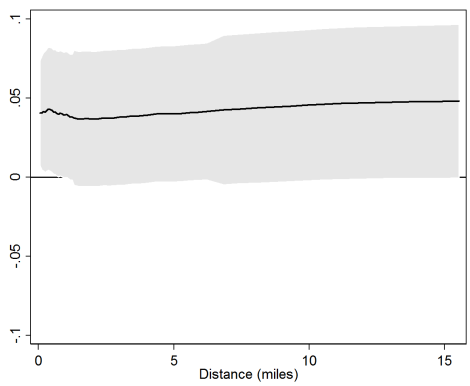
Figure 8: Differences in $GINI_W$ estimates over four decades, Chicago (IL)



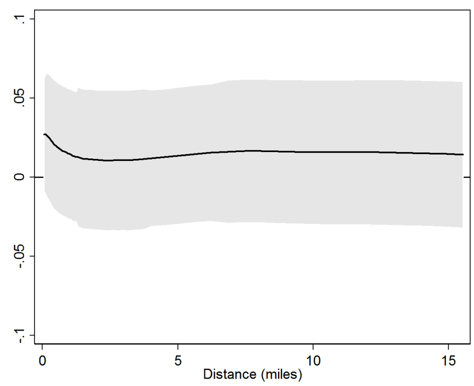
(a) $GINI_W$ 1990 - $GINI_W$ 1980



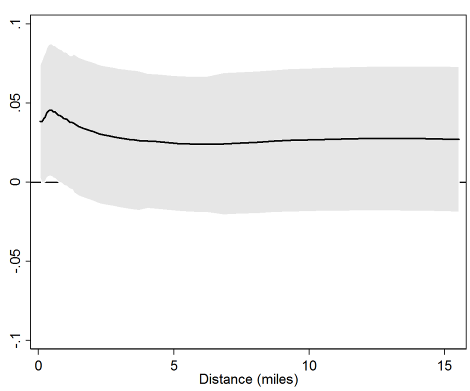
(b) $GINI_W$ 2000 - $GINI_W$ 1980



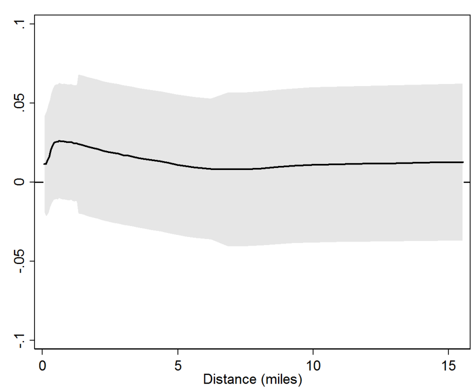
(c) $GINI_W$ 2014 - $GINI_W$ 1980



(d) $GINI_W$ 2000 - $GINI_W$ 1990



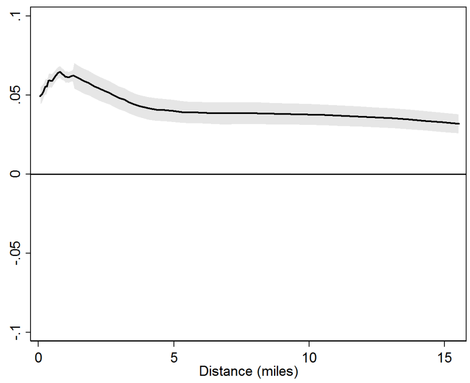
(e) $GINI_W$ 2014 - $GINI_W$ 1990



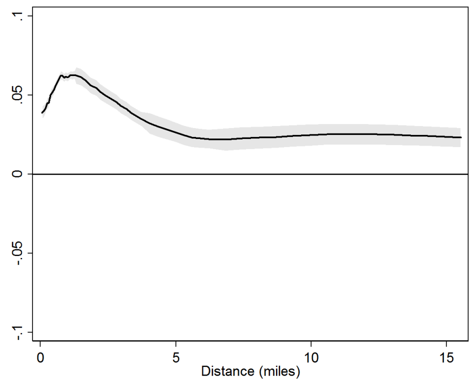
(f) $GINI_W$ 2014 - $GINI_W$ 2000

Note: Authors analysis of U.S. decennial Census data and 2010/14 CS data. The income concept is equivalent gross annual household income. Confidence bounds at 95% are based on standard error estimators discussed in the appendix A.

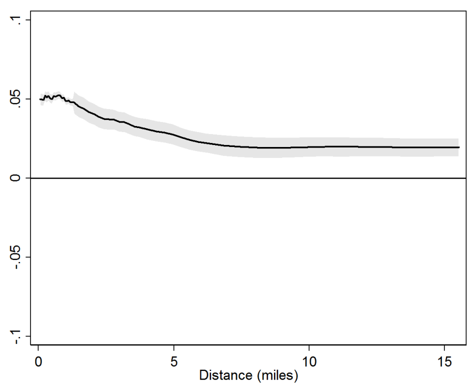
Figure 9: Differences in $GINI_B$ estimates over four decades, Chicago (IL)



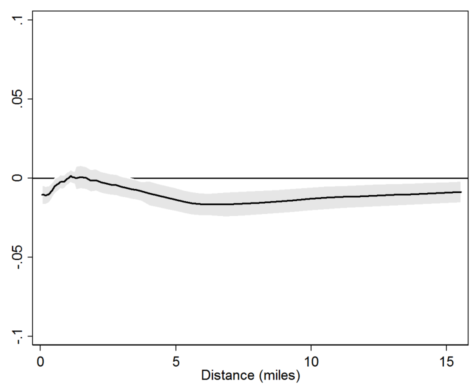
(a) $GINI_B$ 1990 - $GINI_B$ 1980



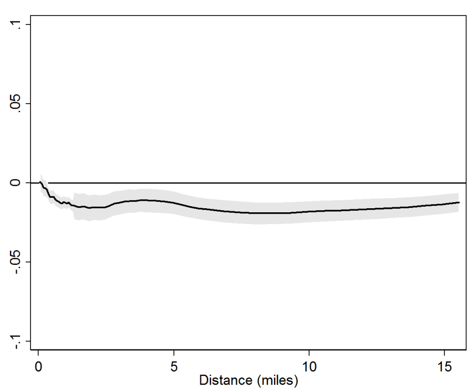
(b) $GINI_B$ 2000 - $GINI_B$ 1980



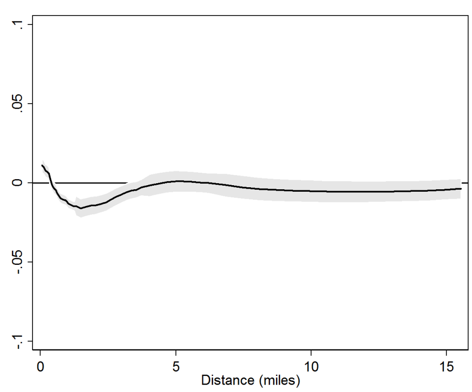
(c) $GINI_B$ 2014 - $GINI_B$ 1980



(d) $GINI_B$ 2000 - $GINI_B$ 1990



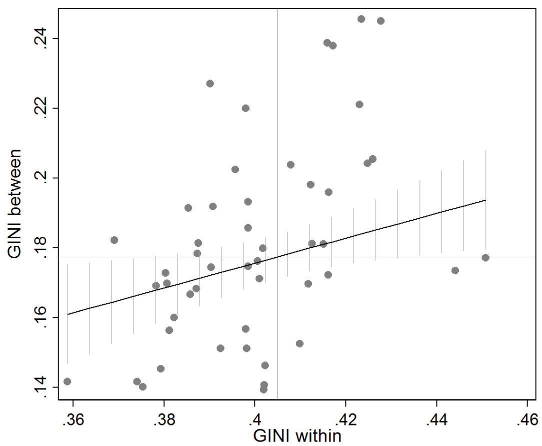
(e) $GINI_B$ 2014 - $GINI_B$ 1990



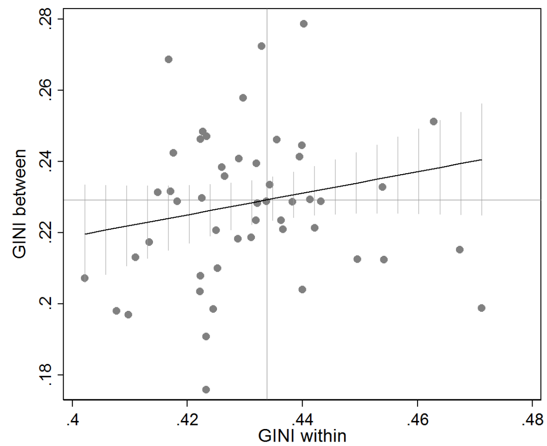
(f) $GINI_B$ 2014 - $GINI_B$ 2000

Note: Authors analysis of U.S. decennial Census data and 2010/14 CS data. The income concept is equivalent gross annual household income. Confidence bounds at 95% are based on standard error estimators discussed in the appendix A.

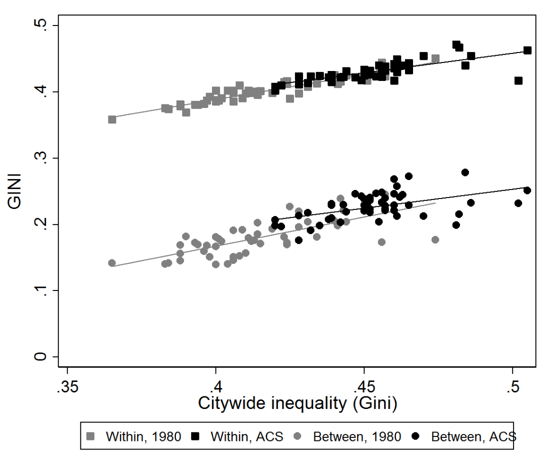
Figure 10: Spatial inequality, income inequality and average incomes across U.S. cities.



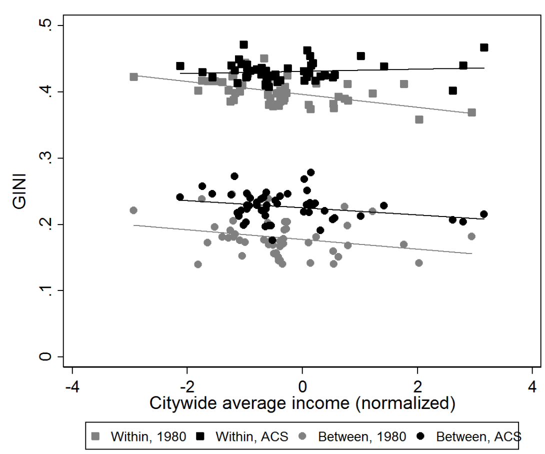
(a) Within and between GINI, 1980



(b) Within and between GINI, 2010/14



(c) Spatial and citywide inequality



(d) Spatial inequality and citywide income

Note: Authors analysis of U.S. Census and ACS data for 50 largest U.S. cities in 2014. Spatial inequality computed at a distance range of two kilometers. Citywide income inequality and average incomes are based on block-group level household equivalent gross income estimates. Average income is normalized to have zero average and unit standard deviation over the weighted selected sample of 50 cities. Gray lines correspond to sample weighted averages of within and between GINI indices. Vertical spikes identify the 95% confidence bounds of regression predictions.

B.2 Spatial inequality in the largest U.S. metro areas

Figure 3 and Figure 4 report patterns of spatial inequality measured by GINI within and between indices for the 50 largest U.S. metro area (as of 2014). At any given distance abscissa, the graphs display substantial heterogeneity in measured spatial inequality across the metro areas. We correlate variability observed at a given distance threshold of two kilometers with characteristics of the city. We find that the GINI within and between indices capture dimensions of inequality that are not necessarily interconnected. Although both indices should converge to precise values when the neighborhood size is very small or very large, the in-between patterns capture different aspects of the joint distribution of incomes and locations. In panel (a) an (b) of Figure 10 we display the joint pattern of the two indices computed for the spatial distributions of incomes in the 50 largest U.S. cities. In this way, we capture substantial heterogeneity both in the geography and the inequality of urban income distributions. We compute both indices for individual neighborhoods of size 1km using 1980 Census data and 2010/14 ACS data. As the figure shows, the two dimensions of spatial inequality seem slightly positively correlated in 1980, although there is little statistical support for this claim. The 2010/14 ACS data do not reveal significance correlations of within and between GINI indices.

Figure 10.(c) displays the empirical relation between citywide inequality (measured by the Gini index) and spatial inequality. The degree of association is visualized by the slopes of the regression lines. We examine both within and between spatial inequality for the Census year 1980 and for ACS 2010/14 data, for an individual neighborhood of size two kilometers. As expected, the citywide Gini index and the GINI indices are positively correlated. Heterogeneity in $GINI_B$ around the regression lines is, however, substantially larger than heterogeneity in $GINI_W$, thus indicating less reliability in these latter correlations. In both cases, the degree of association between spatial and citywide inequality is slightly decreasing over time. Figure 10.(d) shows the association among GINI indices and city affluence (measured by the normalized average equivalent income in each city). Results are less clear-cut and we do not detect a remarkable association between city affluence and GINI spatial inequality, both in the within and the between

form. This is somehow expected, as the GINI indices capture relative notions of inequality (thus improving comparability across cities that differ in affluence).

C Statistics for selected U.S. cities

City	Year	# Blocks	Hh/block	Eq. scale	Equivalent household income				
					Mean	*20%*	*80%*	*Gini*	*90%/10%*
New York (NY)	1980	6319	1318	1.572	12289	4601	19034	0.474	11.247
	1990	6774	1664	2.058	22763	7799	35924	0.507	13.013
	2000	6618	1537	1.604	41061	12196	66542	0.549	25.913
	2010/14	7182	1140	1.566	56558	19749	92656	0.502	17.323
Los Angeles (CA)	1980	5059	1052	1.615	14697	6167	22248	0.441	10.735
	1990	5905	1585	2.012	26434	10509	41048	0.475	12.391
	2000	6103	1158	1.690	38844	13720	59767	0.509	19.256
	2010/14	6385	1107	1.649	55224	19056	90324	0.505	13.628
Chicago (IL)	1980	3756	1122	1.630	13794	5798	20602	0.434	11.351
	1990	4444	1217	2.029	21859	9132	32316	0.461	11.903
	2000	4691	1173	1.625	41193	16076	61667	0.473	11.533
	2010/14	4763	1060	1.575	55710	20022	89856	0.486	13.452
Houston (TX)	1980	1238	1253	1.624	15419	6900	22718	0.428	10.233
	1990	2531	1291	1.994	22827	10203	33287	0.462	11.771
	2000	2318	1418	1.667	39231	16619	57539	0.472	10.736
	2010/14	2781	2148	1.644	55841	22156	88033	0.484	12.394
Philadelphia (PA)	1980	3978	855	1.650	12651	5589	18557	0.410	10.245
	1990	3300	1384	2.001	21816	9601	31606	0.442	11.788
	2000	4212	982	1.602	38995	15788	57841	0.454	10.972
	2010/14	3819	1124	1.566	56205	21567	89602	0.465	13.174
Phoenix (AZ)	1980	697	1155	1.609	12854	5920	18741	0.401	8.972
	1990	1857	961	1.970	21233	9831	30732	0.439	9.803
	2000	1984	1222	1.622	37860	17098	54998	0.437	8.541
	2010/14	2494	1110	1.590	48194	20218	73509	0.456	10.906
San Antonio (TX)	1980	597	891	1.686	10501	4364	15399	0.451	10.206
	1990	1101	890	1.983	17350	7569	25243	0.455	9.903
	2000	1065	1189	1.651	31592	13726	45517	0.454	16.081
	2010/14	1220	1307	1.623	44773	19048	68074	0.454	11.225
San Diego (CA)	1980	908	1471	1.577	12759	5628	18338	0.412	8.893
	1990	1628	1473	1.961	24194	11007	35191	0.434	11.239
	2000	1678	1172	1.637	39537	16698	57219	0.451	9.644
	2010/14	1789	1546	1.615	55564	21947	88783	0.452	11.978

Table 6: Income and population distribution across block groups, U.S. 50 largest cities

Continued

City	Year	# Blocks	Hh/block	Eq. scale	Equivalent household income				
					Mean	*20%*	*80%*	*Gini*	*90%/10%*
Dallas (TX)	1980	1141	931	1.620	14614	6759	21494	0.425	9.522
	1990	2310	965	1.993	24074	11287	35141	0.454	11.691
	2000	2189	1251	1.633	43913	19306	65158	0.464	10.093
	2010/14	2696	1251	1.625	54729	23689	84291	0.460	11.163
San Jose (CA)	1980	571	1417	1.633	16762	8441	24258	0.365	7.215
	1990	1016	1400	1.954	32120	15598	47103	0.405	8.339
	2000	965	1169	1.689	59428	24663	91637	0.433	9.465
	2010/14	1071	1427	1.664	82154	30785	137435	0.455	14.295
Austin (TX)	1980	296	1084	1.517	11407	4867	17064	0.440	9.902
	1990	718	1345	2.019	18968	8497	27339	0.461	10.522
	2000	644	1416	1.569	38993	17418	55766	0.442	9.455
	2010/14	899	1662	1.576	55093	23478	85981	0.443	11.403
Jacksonville (FL)	1980	434	1000	1.622	10868	4602	15546	0.428	9.415
	1990	628	1509	1.973	19217	8365	27219	0.435	9.512
	2000	505	2358	1.590	34398	14528	49341	0.434	8.629
	2010/14	688	1757	1.550	46517	18370	71941	0.450	10.883
San Francisco (CA)	1980	1083	1166	1.514	16322	6927	24339	0.424	9.864
	1990	1226	1477	2.040	28783	11624	44191	0.467	13.379
	2000	1105	1316	1.549	60967	20961	97430	0.494	13.179
	2010/14	1210	1328	1.525	85755	28440	145763	0.482	16.858
Indianapolis (IN)	1980	730	1073	1.617	12550	5958	18183	0.388	9.032
	1990	1029	1395	1.985	20996	9806	29406	0.425	9.515
	2000	944	1395	1.573	37021	16392	52896	0.423	8.317
	2010/14	1030	1639	1.568	47262	19870	71036	0.450	10.624
Columbus (OH)	1980	758	1105	1.593	12427	5984	17840	0.394	8.874
	1990	1281	1128	1.988	19865	9262	28819	0.427	9.649
	2000	1140	986	1.553	35926	16152	51815	0.431	8.848
	2010/14	1269	1293	1.560	48270	21115	72778	0.439	11.633
Fort Worth (TX)	1980	640	650	1.615	12873	5870	18794	0.409	9.169
	1990	1203	956	1.972	21517	10428	30620	0.424	9.835
	2000	1101	1147	1.638	37074	17140	52607	0.429	8.719
	2010/14	1326	1294	1.625	50540	21830	75565	0.449	10.553
Charlotte (NC)	1980	346	1169	1.614	11411	5203	16277	0.400	8.864
	1990	930	1032	1.959	20366	8961	29519	0.424	9.445
	2000	856	1195	1.583	39683	16640	59188	0.451	9.145
	2010/14	1172	1299	1.579	47697	19231	74717	0.452	11.757
Detroit (MI)	1980	2184	764	1.638	12853	5587	19246	0.415	10.783
	1990	4531	974	1.990	22673	10194	33441	0.445	12.181
	2000	3954	963	1.603	40742	17362	59654	0.439	9.817
	2010/14	3798	986	1.560	46492	18592	71604	0.456	11.856
El Paso (TX)	1980	218	897	1.759	8525	3572	12373	0.443	9.182
	1990	425	1042	1.969	15009	6372	21601	0.456	8.963
	2000	418	960	1.750	23862	9095	33972	0.476	16.668
	2010/14	511	1142	1.694	33277	13049	51000	0.462	11.060

Continued

City	Year	# Blocks	Hh/block	Eq. scale	Equivalent household income				
					Mean	*20%*	*80%*	*Gini*	*90%/10%*
Seattle (WA)	1980	1405	885	1.540	14437	6481	21204	0.398	8.514
	1990	2255	1004	1.984	22563	10601	31905	0.416	10.514
	2000	2473	855	1.568	42386	18650	60276	0.427	8.448
	2010/14	2475	1087	1.555	59626	24442	92751	0.438	10.314
Denver (CO)	1980	1054	899	1.575	14283	6866	20352	0.396	8.081
	1990	1694	983	2.005	22072	10791	31410	0.432	11.069
	2000	1711	1038	1.578	43300	20142	62101	0.425	8.100
	2010/14	1908	1230	1.561	58203	24081	90216	0.450	10.751
Washington (DC)	1980	1580	1608	1.619	18273	9281	26315	0.390	8.361
	1990	2540	2193	1.968	32091	16818	45700	0.404	7.758
	2000	2642	1409	1.603	53263	24898	78715	0.425	8.968
	2010/14	3335	1360	1.600	80366	35929	124973	0.420	10.665
Memphis (TN)	1980	478	1021	1.639	11370	4852	16693	0.457	10.804
	1990	920	903	1.997	17888	8072	26052	0.471	10.945
	2000	783	1153	1.605	33086	13753	47853	0.471	18.640
	2010/14	764	1380	1.573	42700	17702	65757	0.465	11.492
Boston (MA)	1980	3662	809	1.622	12696	5417	18790	0.406	10.048
	1990	4497	1032	1.997	24633	10314	37112	0.436	12.226
	2000	3963	961	1.584	43840	16776	66109	0.458	11.004
	2010/14	4082	1058	1.566	64422	23196	105048	0.470	13.712
Nashville (TN)	1980	375	1043	1.605	12416	5382	18373	0.442	10.358
	1990	755	1260	1.979	19811	8712	28653	0.442	9.710
	2000	723	1374	1.555	36360	15118	52565	0.448	9.000
	2010/14	911	1535	1.568	49714	20024	76735	0.452	10.444
Baltimore (MD)	1980	1517	900	1.641	12751	5932	18442	0.400	10.075
	1990	1965	1269	1.972	23987	11302	34591	0.426	11.780
	2000	1780	1204	1.588	38615	16954	55517	0.431	9.565
	2010/14	1932	1182	1.567	59954	25171	93398	0.439	11.158
Oklahoma City (OK)	1980	709	720	1.573	12933	5777	18878	0.419	9.075
	1990	1034	854	1.993	17551	7499	26072	0.445	9.616
	2000	880	941	1.557	30578	12488	44422	0.447	15.739
	2010/14	1015	1021	1.562	45377	18504	68795	0.457	10.504
Portland (OR)	1980	696	1077	1.526	12819	5411	18704	0.404	9.155
	1990	1145	1131	1.991	19987	8840	28511	0.424	9.403
	2000	1141	1111	1.586	37618	16409	53854	0.417	8.385
	2010/14	1374	1211	1.567	49201	19927	74485	0.428	10.490
Las Vegas (NV)	1980	150	2018	1.554	12756	5568	17713	0.406	8.542
	1990	318	2570	1.976	20006	8888	27960	0.431	9.310
	2000	796	1396	1.620	36442	16095	51823	0.430	8.202
	2010/14	1284	1215	1.592	44657	18771	66044	0.442	9.525
Louisville (KY)	1980	582	873	1.592	11451	5036	17218	0.414	9.188
	1990	957	938	1.990	18323	7864	27067	0.445	9.771
	2000	742	1021	1.542	32264	13213	46595	0.444	15.196
	2010/14	840	1087	1.536	45220	17798	69576	0.451	10.739

Continued

City	Year	# Blocks	Hh/block	Eq. scale	Equivalent household income				
					Mean	*20%*	*80%*	*Gini*	*90%/10%*
Milwaukee (WI)	1980	1125	788	1.606	13629	6277	19823	0.384	8.008
	1990	1540	935	1.994	20192	9430	29189	0.420	9.621
	2000	1389	883	1.575	36437	15855	52408	0.426	8.692
	2010/14	1465	927	1.540	48088	19198	72556	0.452	10.903
Albuquerque (NM)	1980	278	957	1.629	11593	5209	16795	0.413	9.366
	1990	430	884	1.992	18125	8120	26181	0.444	9.886
	2000	404	941	1.558	33181	13980	47243	0.440	9.523
	2010/14	434	1176	1.533	43410	17042	66070	0.461	11.785
Tucson (AZ)	1980	306	810	1.578	10384	4601	15056	0.400	8.130
	1990	561	1029	2.000	16834	7279	24236	0.461	9.772
	2000	601	1045	1.551	30864	12504	44934	0.460	15.544
	2010/14	614	1423	1.534	42082	16637	64100	0.463	11.018
Fresno (CA)	1980	571	1417	1.633	16762	8441	24258	0.365	7.215
	1990	532	1044	1.989	18020	7467	26327	0.463	9.649
	2000	546	933	1.730	27064	10878	38272	0.471	16.750
	2010/14	587	1094	1.714	37117	15473	56226	0.461	11.747
Sacramento (CA)	1980	423	1148	1.529	11659	4941	17097	0.408	9.032
	1990	1031	1557	1.968	21357	9535	30607	0.421	10.800
	2000	1094	1199	1.616	36344	15452	52005	0.434	9.269
	2010/14	1369	1143	1.606	49000	20048	75343	0.435	11.883
Kansas City (MO-KS)	1980	1006	991	1.587	13577	6444	19645	0.393	9.056
	1990	1465	1043	1.991	20820	9844	29980	0.426	9.736
	2000	1352	1005	1.575	38395	17532	54896	0.426	8.529
	2010/14	1468	1111	1.562	50056	21337	76139	0.439	10.496
Atlanta (GA)	1980	840	1150	1.591	11821	4837	17433	0.457	10.792
	1990	1962	1650	1.959	24596	11684	35257	0.431	11.546
	2000	1639	1826	1.628	43435	19191	63050	0.438	9.395
	2010/14	2379	1631	1.598	51857	20271	80941	0.460	12.044
Norfolk (VA)	1980	541	1142	1.666	11265	5156	16109	0.411	9.453
	1990	903	1531	1.951	19181	9208	27018	0.405	9.323
	2000	892	1189	1.619	32543	15069	45638	0.412	7.757
	2010/14	1089	1135	1.572	48576	21406	72037	0.420	9.538
Omaha (NE-IA)	1980	399	814	1.616	12576	5952	17858	0.388	8.192
	1990	626	728	1.991	19465	9546	27285	0.424	9.462
	2000	650	626	1.584	35338	16484	49614	0.417	7.904
	2010/14	745	801	1.570	47979	21411	70100	0.428	9.776
Colorado Springs (CO)	1980	159	961	1.583	11320	5290	16547	0.406	8.194
	1990	308	1077	1.970	19034	9441	26299	0.408	9.125
	2000	303	1174	1.612	35946	18023	49660	0.391	7.238
	2010/14	362	1506	1.590	47967	21394	72013	0.422	9.581
Raleigh (NC)	1980	237	1331	1.563	12403	5620	18069	0.414	9.799
	1990	499	1623	1.981	21517	9825	30516	0.421	11.087
	2000	430	1545	1.553	40050	16738	57936	0.445	9.987
	2010/14	707	1679	1.567	54607	22647	84366	0.444	10.753

Continued

City	Year	# Blocks	Hh/block	Eq. scale	Equivalent household income				
					Mean	*20%*	*80%*	*Gini*	*90%/10%*
Miami (FL)	1980	1307	2022	1.559	12962	5246	18895	0.444	9.980
	1990	1549	3062	2.008	19659	7405	28802	0.477	10.503
	2000	638	1987	1.556	35599	14112	51177	0.451	9.572
	2010/14	936	1474	1.557	47343	18170	73153	0.457	11.349
Oakland (CA)	1980	1376	1007	1.589	14714	6930	21331	0.397	9.819
	1990	1636	1673	1.972	27737	13353	40200	0.428	11.701
	2000	1488	1277	1.631	47663	20554	71300	0.443	11.010
	2010/14	1676	1289	1.622	68482	27490	110290	0.457	13.566
Minneapolis (MN)	1980	1704	829	1.593	14300	6794	20511	0.383	7.374
	1990	2239	1096	1.986	23220	11170	33176	0.411	10.532
	2000	2105	1136	1.593	43427	20413	61659	0.408	7.339
	2010/14	2244	1231	1.570	57533	24116	88819	0.432	9.900
Tulsa (OK)	1980	340	823	1.546	12889	5475	19014	0.431	9.341
	1990	730	779	1.990	18258	7716	26596	0.455	9.883
	2000	541	980	1.566	33077	13504	48629	0.446	8.419
	2010/14	599	1154	1.566	44777	17354	68006	0.457	10.355
Cleveland (OH)	1980	1654	867	1.631	12466	5551	18359	0.402	9.899
	1990	2691	1052	2.005	19509	8388	28706	0.446	10.056
	2000	2272	1029	1.563	35221	14392	50973	0.443	9.109
	2010/14	2238	1085	1.519	44764	17146	68783	0.460	11.080
Wichita (KS)	1980	289	704	1.576	12717	5768	18455	0.388	8.499
	1990	451	896	1.989	19303	8801	27625	0.428	9.526
	2000	371	954	1.590	33430	15421	47101	0.414	7.812
	2010/14	411	1133	1.575	43162	18600	64259	0.431	9.672
New Orleans (LA)	1980	938	960	1.623	11743	4629	17279	0.456	11.116
	1990	1215	1113	2.015	15751	5944	23640	0.484	26.274
	2000	974	1009	1.597	29996	10495	43919	0.490	18.694
	2010/14	1053	924	1.532	44250	15342	69804	0.481	13.121
Bakersfield (CA)	1980	169	810	1.635	11081	4431	15901	0.423	9.342
	1990	374	1170	1.965	18526	8018	26588	0.433	9.347
	2000	353	1171	1.723	27908	11092	39953	0.459	16.969
	2010/14	450	1319	1.723	38846	16404	59346	0.447	11.251
Tampa (FL)	1980	903	1300	1.515	10663	4430	15388	0.424	8.280
	1990	1547	1620	1.980	17140	7176	24448	0.440	9.216
	2000	1448	1307	1.530	32815	13303	46343	0.448	8.451
	2010/14	2002	1131	1.506	43788	17047	66315	0.460	10.445