Endogenous Timing and Income Inequality in the Voluntary Provision of Public Goods*

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Abstract

The main purpose of this paper is to investigate how heterogenous incomes as well as heterogenous preferences among potential donors affect the timing of contribution decisions when the timing decisions of contributions is endogenously determined by contributors themselves. More specifically, we consider how redistributions of income which alter the extent of income inequality affects the well-being of individuals as well as social welfare when potential donors are allowed for the endogenous choices of contribution timing as well as how much to contribute in the voluntary provision of public goods using a simple setting with two donors, Cobb-Duglas preferences and complete informations about the returns or utility from public and private consumptions..This paper demonstrates the following results. First, when the inequality of income among individuals becomes huge, the timing of providing public goods does not matter. That is, when it is extremely unequal, the timing between simultaneous and sequential moves is indifferent from the viewpoint of potential contributors who have different preferences toward public goods. Second, when the income gap is narrowed, the simultaneous move is more likely to arise as an equilibrium outcome, because all potential contributors prefer acting as a leader. In addition, the higher valued contributors prefer acting as a follower. Third, in the presence of multiple public goods the higher valued contributors for a particular public good tend to be a first contributor to that public good, although the impacts of income inequality affect the timing of contribution in the same way as in the model of a single public good.

Key words: Nash equilibrium, Stackelberg equilibrium, Public good, Endogenous timing, Voluntary provision, Income distribution

Classification Numbers: D31; H41; H42

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1 Introduction

The main purpose of this paper is to investigate how heterogenous incomes as well as heterogenous preferences among potential donors affect the timing of contribution decisions when the timing decisions of contributions is endogenously determined by contributors themselves. More specifically, we consider how redistributions of income which alter the extent of income inequality affects the well-being of individuals as well as social welfare when potential donors to pubic goods are allowed for the endogenous choices of contribution timing as well as how much to contribute in the voluntary provision of public goods using a simple setting with two donors, Cobb-Duglas preferences and complete informations about the returns or utility from public and private consumptions. This question is also very important from the social welfare prospective because sequential moves in contribution games has a detrimental effect on total supply of public goods compared with simultaneous move games, which was found by Varian (1994). Varian (1994) shows not only that in a sequential contribution game where each individual contributes after observing the contributions made by the earlier individual *in an exogenous order of moves*, it is possible that the higher valuation individuals move first, contributes zero and the lower valuation individual then contributes her individually optimal level so that the total contribution falls short of the total contribution if the individuals made their contribution decisions simultaneously. His result indicates that social welfare would be harmed by the sequential moves choices of contributors, although the first mover would be better off.

The analysis regarding the impacts of *heterogenous income* among donors have formed a core research agenda in the literature on the voluntary provision of public goods. In particular, the impact of redistributive income policies on voluntary contributions has been a central issue in government policies regarding privately provided public goods, since Warr (1983) found the counter-intuitieve neutrality theorem in which the Nash equilibrium provision level of a pure public good remains invariant to redistributions of income among an unchanged set of contributors. His finding leads to the strong policy implication in which equalizing redistributions of income among contributors would never increase total voluntary contributions. Olson (1965), on the other hand, claims the so-called *exploitation hypothesis* in that wealthier persons contribute more than poor persons; consequently, wealthier persons tend to bear a disproportional burden sharing. As group size becomes larger without bound, the class of contributors includes only the richest people. If the distribution of income is sufficiently unequal, very poor individuals contribute nothing. These two issues related to heterogenous incomes among potential contributors have been intensively investigated in the succeeding literatures using either simultaneous-move or a sequential-move contribution games whose timing of contributions are exogenously fixed.

This study adds to a series of papers which analyze an endogenous timing in the private provision model of public goods. There are various theoretical and experimental papers which have studied this issue in various settings. Bliss and Nalebuff (1984) is the first to investigate the endogenous timing of providing voluntarily public goods, and show that the individual who benefits the most is the one who must wait for others to supply the public goods, while the individual with the high cost will never supply the public good. Romano and Yildirim (2001) introduce warm-glow and snob effects in the utility function of donors in the sequential contribution game proposed by Varian (1994); hence, each donor not only cares about the total contribution or the total provision of the public good, but also the individual contribution levels of other donors. They show that there are three subgame perfect equilibria using the two-stage game, action-commitment game of Hamilton and Slutsky (1990) (more precisely, in the first stage, players simultaneously announce which role (leading or following) it prefers and are committed to their choices of contributions; after observing the profile of announcements, players makes their contribution decisions according to the resulting move ordering): simultaneous-move equilibrium where both donors contribute earlier and both Stackelberg equilibria where the leader contributes earlier and the follower contributes later. Kempf and Graziosi (2008) consider a two-jurisidiction model in which the policy makers of the two different countries or regions noncooperatively choose their preferred sequence of moves before providing local public goods, and find condition under which a first- or second-mover advantage emerges for each country using a quasi-linear utility function, *highlighting the role of spillovers and the complementary of substitutability of public goods.* Kempf and Graziosi (2008) as well as Romano and Yildirim (2001) use a timing game, or equivalently, the two-stage action commitment game proposed by Hamilton and Slutsky (1990). Nosenzo and Sefton (2011) introduce inequality aversion preferences following Fehr and Scmidt's (1999) specification of inequality aversion in the two-stage action commitment game of Hamilton and Slutsky (1990) in conjunction with a quasi-linear utility function and reveals the possibility that both players delay their contributions; that is, the presence of inequality aversion preferences considerably expands the set of equilibrium timing outcomes, and confirm this theoretical prediction in laboratory experiments. In the context of imperfect information, on the other hand, Vesterlund (2003) and Potters, Sefton and Vesterlund (2005) show theoretically that when some donors do not know the true quality of the public good, either the uninformed and uniformed donors contribute simultaneous or the informed contributes prior to the uninformed do, and their experiments also show that the donor predominantly chooses to contribute sequentially.

Nevertheless, in spite of the importance of their remarkable contributions, to best our knowledge there still remain being unexplored the fundametal question such as the impacts of heterogenous incomes on the endogenous timing of contributions. Our study is intended to fill this gap. This neglect arises partly because the above-mentioned authors have adopted quasi-linear utility preferences assumed by Varian (1984). Hence, it is natural to ask how *heterogenous incomes* among donors or redistributions of income (or preferences) affect the timing decisions of potential donors' contributions in the canonical voluntary contribution model of Bergstrom, Blume and Varian (1986) (afterwords, BBV) using more general utility functions which entail income effects. We use the two-stage action commitment game of Hamilton and Slutsky (1990) following the other researchers and investigate who is a leader (or follower) in providing public goods when potential contributors are heterogenous with respect to incomes and preferences. Furthermore, we ask how redistribution policies affect the well-beings of individuals as well as equilibrium outcome when individuals can voluntarily choose how much to contribute to public goods as well as the timing of providing public goods. We also consider the endogenous timing of providing public goods in the setting of multiple-public goods in which each contributor choose his or her order of moves for different public goods in order to highlight the role 9f heterogenous preferences in determining the timing of contributing to different public goods. In order to get sharper results, we focus on a two-staeg contribution game with a single or two public goods and with Cobb-Duglas preferences: at the first-stage individuals endogenously choose the timing of contribution to a single public goods or two different public goods: at the second-stage they choose their contributions to a single or two public goods according to the predetermined order of moved.

This paper demonstrates the following results. First, when the inequality of income among individuals becomes huge, the timing of providing public goods does not matter. That is, when it is extremely unequal, the timing between simultaneous and sequential moves is indifferent from the viewpoint of potential contributors who have different preferences toward public goods. Second, when the income gap is narrowed, the simultaneous move is more likely to arise as an equilibrium outcome, because all potential contributors prefer acting as a leader. In addition, the higher valued contributors prefer acting as a follower. Third, in the presence of multiple public goods the higher valued contributors for a particular public good tend to be a first contributor to that public good, although the impacts of income inequality affect the timing of contribution in the same way as in the model of a single public good.

The present paper is organized as follows. Section 2 presents our analytical model and characterizes its one-shot Nash solution in a single public good model. Section 3 investigates the equilibrium outcomes in a single public good model. In Section 4, we investigate a contribution game in the presence of multiple public goods. Section 5 concludes the paper with a brief discussion on extending our model.

2 The Model

There are two individuals indexed by $i = 1, 2$ (we could extend this model to the on with an arbitrary number of heterogenous individuals later). Each individual divides his/her own income between private consumption c_i and contributions toward the public good G, which is denoted by g_i. The preferences of individuals i are given by $u^i(x_i, G)$ for $i = 1, 2$. Following the literature, when individuals are assumed to make contribution $g_i \geq 0$, simultaneously and noncooperatively, we use a simultaneous-move Nash equilibrium as a solution concept.

Individual i's budget constraint is expressed by

$$x_i + g_i = m_i, \tag{1}$$

where m_i is the exogenously given income of individual i, and the relative prices (unit costs of production) of the public good G relative to the (numeraire) private good is fixed and normalized to one. We further assume that two individuals have the following non-identical Cobb-Douglas preferences but different income levels, respectively:[1]

$$\max U_1 = \ln x_1 + \beta \ln G, \quad \text{s.t.} \quad (1).$$

$$\max U_2 = \ln x_2 + \delta \ln G, \quad \text{s.t.} \quad (1).$$

where $G \equiv g_1 + g_2$, $m_1 = \rho$, and $m_2 = 1 - \rho$. The parameters $\beta \in (0,1)$ and $\delta \in (0,1)$ represent the weight towards public goods. For analytically simplicity, we further assume

Assumption 1 *(i)* $1 < \delta(1+\beta)$, *(ii)* $\beta > \delta$.

Assumption 1 implies that the preference of public good of individual 1 is higher than individual 2. Table 4 summarizes the equilibrium of each strategy. Solving this problem, the following first-order conditions are derived:

$$\frac{x_1 \beta}{G} \leq 1 \qquad \text{with equality if} \;\; g_1 > 0, \tag{2}$$

$$\frac{x_2 \delta}{G} \leq 1 \qquad \text{with equality if} \;\; g_2 > 0. \tag{3}$$

2.1 Nash equilibrium

We first consider a case where all individuals make contributions to the single public good G *simultaneously*. Elementary manipulations reveal that, depending on the distribution of

[1] When the preference is CES, the equilibrium level is same with log-linear preference. See Appendix A for details.

income, the Nash equilibrium would fall into any one of 3 regimes according to the varying pattern of equilibrium allocations. Table 1 summarizes these regimes:

Regime	Income distribution	x_1	x_2	g_1	g_2	G
I	$0 \leq \rho \leq \frac{\delta}{\delta(1+\beta)+\beta}$	ρ	$\frac{1-\rho}{1+\delta}$	0	$\frac{\delta(1-\rho)}{1+\delta}$	$\frac{\delta(1-\rho)}{1+\delta}$
II	$\frac{\delta}{\delta(1+\beta)+\beta} \leq \rho \leq \frac{\delta(1+\beta)}{\delta(1+\beta)+\beta}$	$\frac{\delta}{\beta(1+\delta)+\delta}$	$\frac{\beta}{\beta(1+\delta)+\delta}$	$\rho - \frac{\delta}{\beta(1+\delta)+\delta}$	$\frac{\delta(1+\beta)}{\beta(1+\delta)+\delta} - \rho$	$\frac{\beta\delta}{\beta(1+\delta)+\delta}$
III	$\frac{\delta(1+\beta)}{\delta(1+\beta)+\beta} \leq \rho \leq 1$	$\frac{\rho}{1+\beta}$	$1-\rho$	$\frac{\beta\rho}{1+\beta}$	0	$\frac{\beta\rho}{1+\beta}$

Table 1. The profile of equilibrium allocation in simultaneous moves

Note, however, that following an equilibrium refinement suggested by Hamiltonian and Sltutsky (1990), we focus only on equilibria that do not involve the use of a weakly dominated strategy in the reduced game. This refinement gives a sharp prediction in our game.

2.2 Stackelberg Equilibrium

In this subsection, we consider a Stackelberg equilibrium.

(i) Individual 1 acts as a leader, while Individual 2 acts as a follower

First, we assume that each individual plays the Stackelberg game of public good provision and that individual 1 acts as a Stackelberg leader. Given this order of the moves, individual 1 takes the reaction function of individual 2 into account when maximizing his/her own utility function:

$$U_1 = \ln(\rho - g_1) + \beta \ln(g_1 + \max\{R^2(g_1; 1-\rho), 0\}), \tag{4}$$

where $R^2(g_1; 1-\rho)$ represents the reaction function of individual 2 and is given by

$$R^2(g_1; 1-\rho) = \begin{cases} \frac{\delta(1-\rho)-g_1}{1+\delta}, & \text{if } \delta(1-\rho) \geq g_1, \\ 0, & \text{if } \delta(1-\rho) \leq g_1. \end{cases} \tag{5}$$

When $\delta(1-\rho) \geq g_1$, we have

$$g_1 = \begin{cases} \rho - \frac{1}{1+\beta}, & \text{if } \rho \geq \frac{1}{1+\beta}, \\ 0, & \text{if } \rho \leq \frac{1}{1+\beta}. \end{cases} \tag{6}$$

When $\delta(1-\rho) \leq g_1$, we have

$$g_1 = \frac{\beta}{1+\beta}\rho. \tag{7}$$

Substituting back (6) and (7) into (5) yields

$$g_1 = \begin{cases} \frac{\beta}{1+\beta}\rho, & \text{if } \frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)} \leq \rho \leq 1, \\ \rho - \frac{1}{1+\beta}, & \text{if} \frac{1}{1+\beta} \leq \rho \leq \frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)}, \\ 0, & \text{if } 0 \leq \rho \leq \frac{1}{1+\beta}, \end{cases} \tag{8}$$

and

$$g_2 = \begin{cases} 0, & \text{if } \frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)} \leq \rho \leq 1, \\ \frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)} - \rho, & \text{if} \frac{1}{1+\beta} \leq \rho \leq \frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)}, \\ \frac{\delta(1-\rho)}{1+\delta}, & \text{if } 0 \leq \rho \leq \frac{1}{1+\beta}, \end{cases} \tag{9}$$

Combining (8) and (9) together with (1), we can summarize the result as Table 2:

Regime	Income distribution	x_1	x_2	g_1	g_2	G
I	$0 \leq \rho \leq \frac{1}{1+\beta}$	ρ	$\frac{1-\rho}{1+\delta}$	0	$\frac{\delta(1-\rho)}{1+\delta}$	$\frac{\delta(1-\rho)}{1+\delta}$
II	$\frac{1}{1+\beta} \leq \rho \leq \frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)}$	$\frac{1}{1+\beta}$	$\frac{\beta}{(1+\delta)(1+\beta)}$	$\rho - \frac{1}{1+\beta}$	$\frac{\delta(1+\beta)+1}{(1+\delta)(1+\beta)} - \rho$	$\frac{\beta\delta}{(1+\delta)(1+\beta)}$
III	$\frac{1+\delta(1+\beta)}{(1+\delta)(1+\beta)} \leq \rho \leq 1$	$\frac{\rho}{1+\beta}$	$1-\rho$	$\frac{\beta\rho}{1+\beta}$	0	$\frac{\beta\rho}{1+\beta}$

Table 2. The profile of equilibrium allocation when 1 acts as a leader, while 2 a follower

(ii) Individual 1 acts as a follower, while individual 2 acts as a leader

Next, let us assume that individual 2 acts as a leader, while individual 1 as a follower. Following the same procedure as in Subsection 2.1.1, we have

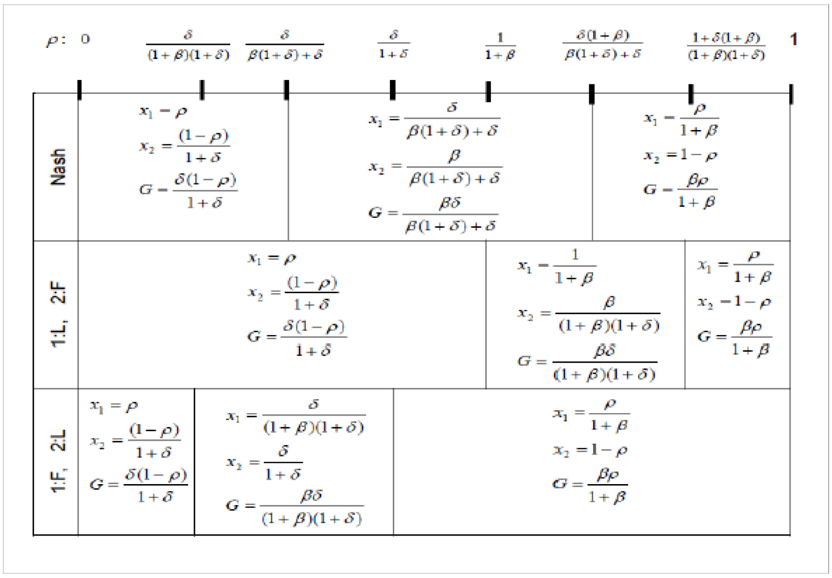
$$g_1 = \begin{cases} \frac{\beta}{1+\beta}\rho, & \text{if } \frac{\delta}{1+\beta} \leq \rho \leq 1, \\ \rho - \frac{\delta}{(1+\beta)(1+\delta)}, & \text{if} \frac{\delta}{(1+\beta)(1+\ \delta)} \leq \rho \leq \frac{\delta}{1+\delta}, \\ 0, & \text{if } 0 \leq \rho \leq \frac{\delta}{(1+\beta)(1+\beta)}, \end{cases}$$

$$g_2 = \begin{cases} 0, & \text{if } \frac{\delta}{1+\beta} \leq \rho \leq 1, \\ \frac{\delta}{1+\delta} - \rho & \text{if} \frac{\delta}{(1+\beta)(1+\delta)} \leq \rho \leq \frac{\delta}{1+\delta}, \\ \frac{\delta}{1+\delta}(1-\rho), & \text{if } 0 \leq \rho \leq \frac{\delta}{(1+\beta)(1+\beta)}. \end{cases}$$

Table 3 summarizes the equilibrium allocation:

Regime	Income distribution	x_1	x_2	g_1	g_2	G
I	$0 \leq \rho \leq \frac{\delta}{(1+\delta)(1+\beta)}$	ρ	$\frac{1-\rho}{1+\delta}$	0	$\frac{\delta(1-\rho)}{1+\delta}$	$\frac{\delta(1-\rho)}{1+\delta}$
II	$\frac{\delta}{(1+\delta)(1+\beta)} \leq \rho \leq \frac{\delta}{1+\delta}$	$\frac{\delta}{(1+\delta)(1+\beta)}$	$\frac{1}{1+\delta}$	$\rho - \frac{\delta}{(1+\delta)(1+\beta)}$	$\frac{\delta}{(1+\delta)} - \rho$	$\frac{\beta\delta}{(1+\delta)(1+\beta)}$
III	$\frac{\delta}{1+\delta} \leq \rho \leq 1$	$\frac{\rho}{1+\beta}$	$1-\rho$	$\frac{\beta\rho}{1+\beta}$	0	$\frac{\beta\rho}{1+\beta}$

Table 3. The profile of equilibrium allocation when 2 acts as a follower, while 1 a leader



Regime	ρ range	x_1	x_2	G
Nash	0 to $\frac{\delta}{\beta(1+\delta)+\delta}$	$x_1=\rho$	$x_2=\frac{(1-\rho)}{1+\delta}$	$G=\frac{\delta(1-\rho)}{1+\delta}$
Nash	$\frac{\delta}{\beta(1+\delta)+\delta}$ to $\frac{\delta(1+\beta)}{\beta(1+\delta)+\delta}$	$x_1=\frac{\delta}{\beta(1+\delta)+\delta}$	$x_2=\frac{\beta}{\beta(1+\delta)+\delta}$	$G=\frac{\beta\delta}{\beta(1+\delta)+\delta}$
Nash	$\frac{\delta(1+\beta)}{\beta(1+\delta)+\delta}$ to 1	$x_1=\frac{\rho}{1+\beta}$	$x_2=1-\rho$	$G=\frac{\beta\rho}{1+\beta}$
1:L, 2:F	0 to $\frac{1}{1+\beta}$	$x_1=\rho$	$x_2=\frac{(1-\rho)}{1+\delta}$	$G=\frac{\delta(1-\rho)}{1+\delta}$
1:L, 2:F	$\frac{1}{1+\beta}$ to $\frac{1+\delta(1+\beta)}{(1+\beta)(1+\delta)}$	$x_1=\frac{1}{1+\beta}$	$x_2=\frac{\beta}{(1+\beta)(1+\delta)}$	$G=\frac{\beta\delta}{(1+\beta)(1+\delta)}$
1:L, 2:F	$\frac{1+\delta(1+\beta)}{(1+\beta)(1+\delta)}$ to 1	$x_1=\frac{\rho}{1+\beta}$	$x_2=1-\rho$	$G=\frac{\beta\rho}{1+\beta}$
1:F, 2:L	0 to $\frac{\delta}{(1+\beta)(1+\delta)}$	$x_1=\rho$	$x_2=\frac{(1-\rho)}{1+\delta}$	$G=\frac{\delta(1-\rho)}{1+\delta}$
1:F, 2:L	$\frac{\delta}{(1+\beta)(1+\delta)}$ to $\frac{\delta}{1+\delta}$	$x_1=\frac{\delta}{(1+\beta)(1+\delta)}$	$x_2=\frac{\delta}{1+\delta}$	$G=\frac{\beta\delta}{(1+\beta)(1+\delta)}$
1:F, 2:L	$\frac{\delta}{1+\delta}$ to 1	$x_1=\frac{\rho}{1+\beta}$	$x_2=1-\rho$	$G=\frac{\beta\rho}{1+\beta}$

Table 4. Graph for equilibrium profile

3 Income distribution and endogenous timing

In Section 2, we have found how the equilibrium allocation of public goods and private consumption are related to the distribution of income. Because the optimal strategy depends on the level of income distribution and the strategy of other individual, we can identify the Nash equilibrium associated with a given distribution of income.

In the regime of $0 \leq \rho \leq \frac{\delta}{(1+\beta)(1+\delta)}$, the payoffs for each individual are summarized as a follows. The first number of each cell displays the payoff for individual 1 and the second number of each cell shows that of individual 2. The L (F) displays the strategy that an individual acts as a leader (a follower).

		Individual 2	
		L	F
Individual 1	L	V_1^*, V_2^*	V_1^*, V_2^*
	F	V_1^*, V_2^*	V_1^*, V_2^*

where $V_1 \equiv \ln\rho + \beta\ln\frac{\delta(1-\rho)}{1+\delta}$ and $V_2 \equiv \ln\frac{1-\rho}{1+\delta} + \delta\ln\frac{\delta(1-\rho)}{1+\delta}$. Since it is easily verified that the payoffs corresponding to all strategy profiles exhibit the same value, the strategy becomes indifferent from the viewpoint of each individual; consequently, all strategy profiles constitute a Nash equilibrium.

In the regime of $\frac{\delta}{(1+\beta)(1+\delta)} \leq \rho \leq \frac{\delta}{\beta(1+\delta)+\delta}$, the payoffs for each individual are summarized as follows:

Individual 1		Individual 2: L	Individual 2: F
	L	V_1^*, V_2^*	V_1^*, V_2^*
	F	W_1, W_2	V_1, V_2

where $W_1 \equiv \ln\frac{\delta}{(1+\beta)(1+\delta)} + \beta\ln\frac{\beta\delta}{(1+\beta)(1+\delta)}$ and $W_2 \equiv \ln\frac{1}{1+\delta} + \delta\ln\frac{\beta\delta}{(1+\beta)(1+\delta)}$.

Lemma 1 *$V_1 > W_1$ and $W_2 > V_2$.*

Proof. $V_1 - W_1 \equiv F(\rho) = \ln\rho - \ln\frac{\delta}{(1+\beta)(1+\delta)} + \beta(\ln\frac{\delta(1-\rho)}{1+\delta} - \ln\frac{\beta\delta}{(1+\beta)(1+\delta)})$. $\frac{\partial F(\rho)}{\partial\rho} = \frac{1}{\rho} - \frac{\beta}{1-\rho} > 0$ iff $\rho \leq \frac{\delta}{\beta(1+\delta)+\delta} < \frac{1}{1+\beta}$. $F(\frac{\delta}{(1+\beta)(1+\delta)}) = \beta(\ln\frac{\delta(1+\beta(1+\delta))}{(1+\delta)^2(1+\beta)} - \ln\frac{\beta\delta}{(1+\delta)(1+\beta)}) = \beta\ln\frac{1+\beta(1+\delta)}{(1+\delta)\beta} > 0$. Because $F(\rho)$ is increasing function in ρ, $F(\rho) > 0$ for all $\rho \in [\frac{\delta}{(1+\beta)(1+\delta)}, \frac{\delta}{\beta(1+\delta)+\delta}]$. $W_2 - V_2 \equiv F_2(\rho) = \ln\frac{1}{1+\delta} + \delta\ln\frac{\beta\delta}{(1+\beta)(1+\delta)} - (\ln\frac{1-\rho}{1+\delta} + \delta\ln\frac{\delta(1-\rho)}{1+\delta})$. $\frac{\partial F_2(\rho)}{\partial\rho} = \frac{1}{1-\rho} + \frac{\delta}{1-\rho} > 0$. $F_2(\frac{\delta}{(1+\beta)(1+\delta)}) = \ln\frac{(1+\beta)(1+\delta)}{1+\beta+\beta\delta} + \delta\ln\frac{\beta(1+\delta)}{1+\beta+\beta\delta}$. The first term is positive and the second term is negative, however, because $|\ln\frac{(1+\beta)(1+\delta)}{1+\beta+\beta\delta}| > |\ln\frac{\beta(1+\delta)}{1+\beta+\beta\delta}| > \delta|\ln\frac{\beta(1+\delta)}{1+\beta+\beta\delta}|$, we have $F_2(\frac{\delta}{(1+\beta)(1+\delta)}) > 0$. Because $F_2(\rho)$ is increasing function in ρ, $F_2(\rho) > 0$ for all $\rho \in [\frac{\delta}{(1+\beta)(1+\delta)}, \frac{\delta}{\beta(1+\delta)+\delta}]$. ■

In the regime of $\frac{\delta}{\beta(1+\delta)+\delta} \leq \rho \leq \frac{\delta}{1+\delta}$, the payoffs for each individual are summarized as follows.

Individual 1		Individual 2: L	Individual 2: F
	L	X_1^*, X_2^*	V_1, V_2
	F	W_1, W_2	X_1, X_2

where $X_1 \equiv \ln\frac{\delta}{\beta(1+\delta)+\delta} + \beta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta}$ and $X_2 \equiv \ln\frac{\beta}{\beta(1+\delta)+\delta} + \delta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta}$.

Lemma 2 *$X_1 > W_1$ and $X_2 > V_2$.*

Proof. $X_1 - W_1 = \ln\frac{\delta}{\beta(1+\delta)+\delta} + \beta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta} - (\ln\frac{\delta}{(1+\beta)(1+\delta)} + \beta\ln\frac{\beta\delta}{(1+\beta)(1+\delta)}) = (1+\beta)\ln\frac{(1+\beta)(1+\delta)}{\beta(1+\delta)+\delta} > 0$. $X_2 - V_2 = \ln\frac{\beta}{\beta(1+\delta)+\delta} + \delta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta} - (\ln\frac{1-\rho}{1+\delta} + \delta\ln\frac{\delta(1-\rho)}{1+\delta}) = (1+\delta)\ln\frac{\beta(1+\delta)}{(\beta(1+\delta)+\delta)(1-\rho)} > 0$ iff

$\rho > \frac{\delta}{(\beta(1+\delta)+\delta)}$. $W_2 - X_2 = \ln\frac{1}{1+\delta} + \delta\ln\frac{\beta\delta}{(1+\beta)(1+\delta)} - (\ln\frac{\beta}{\beta(1+\delta)+\delta} + \delta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta}) = \ln\frac{\beta(1+\delta)+\delta}{\beta(1+\delta)} + \delta\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)(1+\delta)}$. Since $|\ln\frac{\beta(1+\delta)+\delta}{\beta(1+\delta)}| > |\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)(1+\delta)}| > \delta|\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)(1+\delta)}|$, we have $W_2 > X_2$. ■

In the Regime $\frac{\delta}{1+\delta} \leq \rho \leq \frac{1}{1+\beta}$, the payoffs for each individual are summarized as follows.

Individual 1		Individual 2	
		L	F
	L	$X_1^*,\ X_2^*$	$V_1,\ V_2$
	F	$Z_1,\ Z_2$	$X_1,\ X_2$

where $Z_1 \equiv \ln\frac{\rho}{1+\beta} + \beta\ln\frac{\beta\rho}{1+\beta}$ and $Z_2 \equiv \ln(1-\rho) + \delta\ln\frac{\beta\rho}{1+\beta}$.

Lemma 3 *$X_1 > Z_1$ and $Z_2 > X_2$.*

Proof. $X_1 - Z_1 = \ln\frac{\delta}{\beta(1+\delta)+\delta} + \beta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta} - (\ln\frac{\rho}{1+\beta} + \beta\ln\frac{\beta\rho}{1+\beta}) = (1+\beta)\ln\frac{(1+\beta)\delta}{(\beta(1+\delta)+\delta)\rho} > 0$ iff $\rho < \frac{(1+\beta)\delta}{\beta(1+\delta)+\delta}$. $Z_2 - X_2 \equiv G(\rho) = \ln(1-\rho) + \delta\ln\frac{\beta\rho}{1+\beta} - (\ln\frac{\beta}{\beta(1+\delta)+\delta} + \delta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta}) = \ln\frac{(\beta(1+\delta)+\delta)(1-\rho)}{\beta} + \delta\ln\frac{(\beta(1+\delta)+\delta)\rho}{(1+\beta)\delta}$. $G'(\rho) = \frac{-1}{1-\rho} + \frac{\delta}{\rho} < 0$ iff $\rho > \frac{\delta}{1+\delta}$. Since $G(\frac{1}{1+\beta}) = \ln\frac{\beta(1+\delta)+\delta}{(1+\beta)} + \delta\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)^2\delta}$. Because $\ln\frac{\beta+\delta+\beta\delta}{1+\beta} > \ln\frac{\beta+\delta+\beta\delta}{(1+\beta)^2\delta}$, when $(1+\beta)\delta > 1$ (see Assumption 1) we have $|\ln\frac{\beta+\delta+\beta\delta}{1+\beta}| > \ln|\frac{\beta+\delta+\beta\delta}{(1+\beta)^2\delta}| > \delta|\ln\frac{\beta+\delta+\beta\delta}{(1+\beta)^2\delta}|$. It leads to $G(\rho) > 0$. ■

In the Regime $\frac{1}{1+\beta} \leq \rho \leq \frac{\delta(1+\beta)}{\beta(1+\delta)+\delta}$, the payoffs for each individual are summarized as follows:

Individual 1		Individual 2	
		L	F
	L	$X_1^*,\ X_2^*$	$Y_1,\ Y_2$
	F	$Z_1,\ Z_2$	$X_1,\ X_2$

where $Y_1 \equiv \ln\frac{1}{1+\beta} + \beta\ln\frac{\beta\delta}{(1+\delta)(1+\beta)}$ and $Y_2 \equiv \ln\frac{\beta}{(1+\delta)(1+\beta)} + \delta\ln\frac{\beta\delta}{(1+\delta)(1+\beta)}$.

Lemma 4 *$Y_1 > X_1$ and $X_2 > Y_2$.*

Proof. $Y_1 - X_1 = \ln\frac{1}{1+\beta} + \beta\ln\frac{\beta\delta}{(1+\delta)(1+\beta)} - (\ln\frac{\delta}{\beta(1+\delta)+\delta} + \beta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta}) = \ln\frac{\beta(1+\delta)+\delta}{(1+\beta)\delta} + \beta\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)(1+\delta)}$. Since $|\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)\delta}| > |\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)(1+\delta)}| > \beta|\ln\frac{\beta(1+\delta)+\delta}{(1+\beta)(1+\delta)}|$,we have $Y_1 > X_1$. $X_2 - Y_2 = \ln\frac{\beta}{\beta(1+\delta)+\delta} + \delta\ln\frac{\beta\delta}{\beta(1+\delta)+\delta} - (\ln\frac{\beta}{(1+\delta)(1+\beta)} + \delta\ln\frac{\beta\delta}{(1+\delta)(1+\beta)}) = (1+\delta)\ln\frac{(1+\beta)(1+\delta)}{\beta(1+\delta)+\delta} > 0$. ■

In the Regime $\frac{\delta(1+\beta)}{\beta(1+\delta)+\delta} \leq \rho \leq \frac{1+\delta(1+\beta)}{(1+\beta)(1+\delta)}$, the value function of each individual is summarized as follows:

Individual 1		Individual 2: L	Individual 2: F
	L	Z_1^*, Z_2^*	Y_1, Y_2
	F	Z_1^*, Z_2^*	Z_1^*, Z_2^*

Lemma 5 *$Y_1 > Z_1$ and $Z_2 > Y_2$.*

Proof. $Y_1 - Z_1 = \ln\frac{1}{1+\beta} + \beta\ln\frac{\beta\delta}{(1+\delta)(1+\beta)} - (\ln\frac{\rho}{1+\beta} + \beta\ln\frac{\beta\rho}{1+\beta}) = \ln\frac{1}{\rho} + \delta\ln\frac{\delta}{(1+\delta)\rho} < 0$. $Z_2 - Y_2 \equiv H(\rho) = \ln(1-\rho) + \delta\ln\frac{\beta\rho}{1+\beta} - (\ln\frac{\beta}{(1+\delta)(1+\beta)} + \delta\ln\frac{\beta\delta}{(1+\delta)(1+\beta)}) = \ln\frac{(1-\rho)(1+\delta)(1+\beta)}{\beta} + \delta\ln\frac{\rho(1+\delta)}{\delta}$, $\frac{\partial H(\rho)}{\partial\rho} = \frac{-1}{1-\rho} + \frac{\delta}{\rho} < 0$, iff $\rho > \frac{\delta}{1+\delta}$. $H(\frac{1+\delta(1+\beta)}{(1+\beta)(1+\delta)}) = \delta\ln\frac{1+\delta(1+\beta)}{(1+\beta)\delta} > 0$. Because $H(\rho)$ is decreasing function in respect to ρ and $H(\rho)$ is positive in the maximum level of ρ, $H(\rho) > 0$ for all $\rho \in [\frac{\delta(1+\beta)}{\beta(1+\delta)+\delta}, \frac{1+\delta(1+\beta)}{(1+\beta)(1+\delta)}$. ■

In the Regime $\frac{1+\delta(1+\beta)}{(1+\beta)(1+\delta)} \leq \rho \leq 1$, the payoffs for each individual are summarized as follows:

Individual 1		Individual 2: L	Individual 2: F
	L	Z_1^*, Z_2^*	Z_1^*, Z_2^*
	F	Z_1^*, Z_2^*	Z_1^*, Z_2^*

Proposition 1 *When the income gap is narrowed, the simultaneous move is more likely to arise as an equilibrium outcome, while as the degree of income inequality becomes greater, both the sequential and simultaneous moves are more likely to arise. Further, when it is extremely unequal, the timing between simultaneous and*

4 CES Utility function

In the Case of CES preferences:

$$U_1 = \frac{x_1^{1-\sigma}}{1-\sigma} + \beta \frac{G^{1-\sigma}}{1-\sigma},$$
$$U_2 = \frac{x_2^{1-\sigma}}{1-\sigma} + \delta \frac{G^{1-\sigma}}{1-\sigma}.$$

The first-order conditions are derived as

$$\beta^{-\sigma} \frac{x_1}{G} \leq 1,$$
$$\delta^{-\sigma} \frac{x_2}{G} \leq 1,$$

Because the above first-order conditions are the same as those under log utility in the previous section. Because the ratio between private consumption and public good consumption remains constant, the property of the equilibria are the same as those under log-linear preferences.

5 Multiple public goods

In this section, we consider the model with two individual and multiple public goods. For analytical convenience, we assume that there are only two public goods which are respectively denoted by G and H. Each individual divides his/her income between private consumption c_i and contributions toward two public good, G and H, which are respectively denoted by g_i and h_i. The preferences of individuals i are given by $u^i(x_i, G, H)$ for $i = 1, 2$. As in the previous sections, we successively consider two cases where individuals make contribution g_i and h_i, *simultaneously or sequentially.* Individual i's budget constraint is expressed by

$$x_i + g_i + h_i = m_i. \tag{10}$$

Furthermore, we assume that two individuals have the following non-identical Cobb-Douglas preferences but different income levels:[2]

$$\text{Max } U_1 = \ln x_1 + \beta_1 \ln G + \beta_2 \ln H, \quad \text{s.t. } (10),$$

$$\text{Max } U_2 = \ln x_2 + \delta_1 \ln G + \delta_2 \ln H, \quad \text{s.t. } (10),$$

where $G \equiv g_1 + g_2$, $H \equiv h_1 + h_2$, $m_1 = \rho$, and $m_1 = 1 - \rho$, while $\beta_i \in (0, 1)$ and $\delta_i \in (0, 1)$ are the preference parameters representing the strength towards the public goods G and H, respectively. By solving the above problems, respectively, we obtain the following first-order conditions:

$$\begin{aligned}
\frac{x_1\beta_1}{G} &\leq 1 \qquad \text{with equality if } \; g_1 > 0, \\
\frac{x_1\beta_2}{H} &\leq 1 \qquad \text{with equality if } \; h_1 > 0, \\
\frac{x_2\delta_1}{G} &\leq 1 \qquad \text{with equality if } \; g_2 > 0, \\
\frac{x_2\delta_2}{H} &\leq 1 \qquad \text{with equality if } \; h_2 > 0.
\end{aligned}$$

We list the following 16 cases, depending on the combinations between the strategies of two individuals:

	LL	FF	FL	LF
LL	*Case* 1	*Case* 2	*Case* 3	*Case* 4
FF	*Case* 5	*Case* 6	*Case* 7	*Case* 8
FL	*Case* 9	*Case* 10	*Case* 11	*Case* 12
LF	*Case* 13	*Case* 14	*Case* 15	*Case* 16

Table 5. List of combinations of strategies

LL (FF) means that the individual acts as a leader (follower) on providing both the public goods G and H, while FL means that the individual acts as a follower in providing the public

[2]When the preference is CES, the equilibrium level is same with log-linear preference. See Appendix A for details.

good G and also acts as a leader in providing public good H. LF means that individual acts as a leader in providing the public good G and also acts as a follower on that of public good L. Unfortunately, it is almost impossible to proceed the analysis with unspecified parameter values and income levels. We, therefore, employ a numerical analysis and focus on the three profiles of income distribution such as $\rho = 1/4$ and $\rho = 3/4$ (i.e. unequal income distribution), and $\rho = 1/2$ (i.e. equal income distribution).

When $\rho = 1/4$, the payoffs for each individual are summarized as follows.[3]

	LL	FF	FL	LF
LL	Γ_1, Γ_2	Λ_1, Λ_2	Λ_1, Λ_2	Γ_1, Γ_2
FF	Φ_1, Φ_2	Γ_1, Γ_2	Ω_1, Ω_2	Φ_1, Φ_2
FL	Ψ_1, Ψ_2	Γ_1, Γ_2	Ω_1, Ω_2	Φ_1, Φ_2
LF	Γ_1, Γ_2	Λ_1, Λ_2	Λ_1, Λ_2	Γ_1, Γ_2

Table 6. The payoffs for the two individuals when $\rho = 1/4$

where where $\Gamma_1 \equiv \ln \frac{\delta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)} + \beta_1 \ln \frac{\beta_1\delta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)} + \beta_2 \ln \frac{\beta_1\delta_2}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$, $\Gamma_2 \equiv \ln \frac{\beta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)} + \delta_1 \ln \frac{\beta_1\delta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)} + \delta_2 \ln \frac{\beta_1\delta_2}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$, $\Lambda_1 \equiv \ln \rho + \beta_1 \ln \frac{\delta_1(1-\rho)}{(1+\delta_1+\delta_2)} + \beta_2 \ln \frac{\delta_2(1-\rho)}{(1+\delta_1+\delta_2)}$, $\Lambda_2 \equiv \ln \frac{1-\rho}{(1+\delta_1+\delta_2)} + \delta_1 \ln \frac{\delta_1(1-\rho)}{(1+\delta_1+\delta_2)} + \delta_2 \ln \frac{\delta_2(1-\rho)}{(1+\delta_1+\delta_2)}$, $\Phi_1 \equiv \ln \frac{\rho}{1+\beta_1} + \beta_1 \ln \frac{\beta_1\rho}{1+\beta_1} + \beta_2 \ln \frac{\delta_2(1-\rho)}{1+\delta_2}$, $\Phi_2 \equiv \ln \frac{1-\rho}{1+\delta_2} + \delta_1 \ln \frac{\beta_1\delta_2}{\delta_1+\beta_1(1+\delta_1+\delta_2)} + \delta_2 \ln \frac{\delta_2(1-\rho)}{1+\delta_2}$, $\Omega_1 \equiv \ln \frac{\delta_1(1+\delta_1)}{D(\beta_1+\delta_1(1+\beta_1))} + \beta_1 \ln \frac{\beta_1\delta_1(1+\delta_1)}{D(\beta_1+\delta_1(1+\beta_1))} + \beta_2 \ln \frac{\delta_2}{(1+\delta_1+\delta_2)}$, $\Omega_2 \equiv \ln \frac{\beta_1(1+\delta_1)}{(1+\delta_1+\delta_2)(\beta_1+\delta_1(1+\beta_1))} + \delta_1 \ln \frac{\beta_1\delta_1(1+\delta_1)}{(1+\delta_1+\delta_2)(\beta_1+\delta_1(1+\beta_1))} + \delta_2 \ln \frac{\delta_2}{(1+\delta_1+\delta_2)}$, $\Psi_1 \equiv \ln \frac{\delta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)} + \beta_1 \ln \frac{\beta_1\delta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)} + \beta_2 \ln \frac{\beta_2\delta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)}$, $\Psi_2 \equiv \ln \frac{1+\beta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)} + \delta_1 \ln \frac{\beta_1\delta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)} + \delta_2 \ln \frac{\beta_2\delta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)}$.

Because it still does not allow for identifying an equilibrium solution, we further set the parameter values such as $\beta_1 = \frac{5}{3}$, $\beta_2 = \frac{8}{9}$, $\delta_1 = \frac{15}{32}$, and $\delta_2 = \frac{1}{2}$. Table 6 summarizes the resulting payoffs for each individual:

[3]See Appendix A for the deviation of the payoff.

	LL	FF	FL	LF
LL	$-6.09^{*}, -2.33^{*}$	$-5.73, -2.60$	$-5.73, -2.60$	$-6.09^{*}, -2.33^{*}$
FF	$-6.69, -2.54$	$-6.09, -2.33$	$-6.02, -2.29$	$-6.69, -2.54$
FL	$-7.16, -2.46$	$-6.09, -2.33$	$-6.02, -2.29$	$-6.69, -2.54$
LF	$-6.09^{*}, -2.33^{*}$	$-5.73, -2.60$	$-5.73, -2.60$	$-6.09^{*}, -2.33^{*}$

Table 7. The payoffs for the two individuals when $\rho = \frac{1}{4}, \beta_1 = \frac{5}{3}, \beta_2 = \frac{8}{9}, \delta_1 = \frac{15}{32}$, and $\delta_2 = \frac{1}{2}$

From Table 7, it is easy to identify Nash equilibria, each of which are given by the strategy profiles (LL, LL), (LL,LF), (LF, LL), and (LF, LF), respectively, where the first (second) sequences of the receptive strategy profiles represent the combinations of the strategies chosen by individual 1 (2).

When $\rho = 1/2$, the payoffs for each individual are summarized as follows.[4]

	LL	FF	FL	LF
LL	Φ_1, Φ_2	Φ_1, Φ_2	Φ_1, Φ_2	Φ_1, Φ_2
FF	Π_1, Π_2	Φ_1, Φ_2	Φ_1, Φ_2	Φ_1, Φ_2
FL	Π_1, Π_2	Φ_1, Φ_2	Φ_1, Φ_2	Φ_1, Φ_2
LF	Π_1, Π_2	Φ_1, Φ_2	Π_1, Π_2	Φ_1, Φ_2

Table 8. The payoffs for the two individuals when $\rho = 1/2$

where $\Pi_1 \equiv \ln \frac{\rho}{X} + \beta_1 \ln \frac{\beta_1 \rho}{X} + \beta_2 \ln \frac{\beta_2 \rho}{X}$, $\Pi_2 \equiv \ln(1-\rho) + \delta_1 \ln \frac{\beta_1 \rho}{X} + \delta_2 \ln \frac{\beta_2 \rho}{X}$. Table 7 summarizes the payoffs for the two individuals.

	LL	FF	FL	LF
LL	$-5.25^{*}, -2.54^{*}$	$-5.25^{*}, -2.54^{*}$	$-5.25, -2.54$	$-5.25^{*}, -2.54^{*}$
FF	$-6.23, -2.54$	$-5.25, -2.54$	$-5.25, -2.54$	$-5.25, -2.54$
FL	$-6.23, -2.54$	$-5.25, -2.54$	$-5.25, -2.54$	$-5.25, -2.54$
LF	$-6.23, -2.54$	$-5.25, -2.54$	$-6.23, -2.42$	$-5.25, -2.54$

Table 9. The payoffs for the two individuals when $\rho = \frac{1}{2}, \beta_1 = \frac{5}{3}, \beta_2 = \frac{8}{9}, \delta_1 = \frac{15}{32}$, and $\delta_2 = \frac{1}{2}$

By deriving the equilibrium, we have the following Nash equilibria: (LL, LL), (LL, FF), (LL, FL), and (LL, LF), where first (second-) sequence of letters means the strategy combination of individual 1 (2).

[4] See, Appendix B for the deviation of the value function.

When $\rho = 3/4$, the payoffs for each individual are summarized as follows.[5]

	LL	FF	FL	LF
LL	Π_1,Π_2	Φ_1, Φ_2	Π_1,Π_2	Φ_1, Φ_2
FF	Π_1,Π_2	Π_1,Π_2	Π_1,Π_2	Π_1,Π_2
FL	Π_1,Π_2	Φ_1, Φ_2	Π_1,Π_2	Φ_1, Φ_2
LF	Π_1,Π_2	Π_1,Π_2	Π_1,Π_2	Π_1,Π_2

Table 10. The payoffs for the two individuals when individuals $\rho = 3/4$

	LL	FF	FL	LF
LL	-4.79*, -2.71*	-4.74, -3.37	-4.79*, -2.71*	-4.74, -3.37
FF	-4.79*, -2.71*	-4.79, -2.71	-4.79*, -2.71*	-4.79, -2.71
FL	-4.79*, -2.71*	-4.74, -3.37	-4.79*, -2.71*	-4.74, -3.37
LF	-4.79*, -2.71*	-4.79, -2.71	-4.79*, -2.71*	-4.79, -2.71

Table 11. The payoffs for the two individuals when $\rho = 3/4, \beta_1 = \frac{5}{3}, \beta_2 = \frac{8}{9}, \delta_1 = \frac{15}{32}$, and $\delta_2 = \frac{1}{2}$

Inspection of Table 11 reveals that the Nash equilibria are given by (LL,LL), (LL,FL), (FF, LL), (FF, FL), (FL, LL), (FL, FL), (LF, LL), and (LF, FL), where left- (right-) hand side shows the strategy of individual 1 (2).

6 Concluding remarks

This paper reveals that the timing of providing public goods depends critically on the distribution of income in addition to the preferences toward public goods. Policy makers search for policy prescriptions to prevent free riding and increase voluntary giving to efficient levels. Nevertheless, as shown in Varian (1994), in the sequential moves the outcomes regarding allocations and welfare not only are much different from those in the simultaneous Nash equilibrium, but also produce detrimental impacts on total supply of public goods as well as on individual' well-being. From the viewpoint of policy makers, whether potential contributors

[5] See, Appendix C for the deviation of value function

take simultaneous or sequential moves matters in preventing from free-rider behavior and social perspectives. In other words, since potential contributors have two strategic choice variables; that is, the amount of individual contributions and the timing decisions for contributions. In response, policy marker regarding redistribution polices have to take care of their responses through those dual choices in order to attain the desired policy prescription. Given the fixed choice of timing decisions, if policy markers were to anticipate the neutrality property of redistributing policy, those contributors may change the timing of contribution and thus the anticipated neutrality property will not be realized, and thus the outcome of that policy would be different from the policy outcome would be anticipated. Hence, policy makers have to take care for those dual choices of contributors simultaneously when implementing redistribution polices. The second policy implication is that when the timing decision of contributors may have detrimental impacts in response to redistribution policies, policy makers or fund-raisers should regulate the order moves of contributors by annoucements and new donors of large leadership donors or examples which influence the choice of moves taken by contributors in the desired direction.

The results obtained in this paper critically rely on the restrictive structure of the present model (e.g., a two-individual model and Cobb Douglas preferences). First, experimental studies regarding the relationship between heterogenous incomes and the timing of contributions are urgently needed in order to confirm whether our theoretical predictions are valid or not. Secondly, future research should address the robustness of the results under more general functions. Owing to its complexity, we need to resort to a numerical analysis. Thirdly, an equally important extension is to consider a pubic good provision model with different supplying technology for public goods.

Appendix A

	LL	FF	FL	LF
LL	$E1$	$E2$	$E2$	$E1$
FF	$E3$	$E1$	$E4$	$E3$
FL	$E5$	$E1$	$E4$	$E3$
LF	$E1$	$E2$	$E2$	$E1$

Table 12. The profile of equilibrium allocation when $\rho = 1/4$

	LL	FF	FL	LF
LL	$E3$	$E3$	$E3$	$E3$
FF	$E6$	$E3$	$E6$	$E3$
FL	$E6$	$E3$	$E3$	$E3$
LF	$E6$	$E3$	$E6$	$E3$

Table 13. The payoffs for the two individuals when $\rho = 1/2$

	LL	FF	FL	LF
LL	$E6$	$E3$	$E6$	$E3$
FF	$E6$	$E6$	$E6$	$E6$
FL	$E6$	$E3$	$E6$	$E3$
LF	$E6$	$E6$	$E6$	$E6$

Table 14. The equilibrium profile when$\rho = 3/4$

	LL	FF	FL	LF
LL	$E6$	$E3$	$E6$	$E3$
FF	$E6$	$E6$	$E6$	$E3$
FL	$E6$	$E3$	$E6$	$E3$
LF	$E6$	$E6$	$E6$	$E6$

Table 15. The equilibrium profile when $\rho = 3/4$

where the profile of equilibrium allocation corresponding to each cell is given by the followings:

	x_1	x_2	g_1	g_2	h_1	h_2
$E1$	$\frac{\delta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$	$\frac{\beta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$	$\rho - \frac{\delta_1}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$	$\frac{\delta_1(1+\beta_1)}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$	0	$\frac{\beta_1\delta_2}{\delta_1+\beta_1(1+\delta_1+\delta_2)}$
$E2$	ρ	$\frac{1-\rho}{(1+\delta_1+\delta_2)}$	0	$\frac{\delta_1(1-\rho)}{(1+\delta_1+\delta_2)}$	0	$\frac{\delta_2(1-\rho)}{(1+\delta_1+\delta_2)}$
$E3$	$\frac{\rho}{1+\beta_1}$	$\frac{\rho}{1+\beta_1}$	$\frac{\beta\rho}{1+\beta}$	0	0	$\frac{\delta_2(1-\rho)}{1+\delta_2}$
$E4$	$\delta_1 A$	$\beta_1 A$	$\rho - \delta_1 A$	$\delta_1(1+\beta_1)A - \rho,$	0	$\frac{\delta_2}{D}$
$E5$	$\frac{\delta_1}{1+\beta_1+\delta_1 X}$	$\frac{1+\beta_1}{1+\beta_1+\delta_1 X}$	$\rho - (1+\beta_2)B$	$BX - \rho$	$\beta_2 B$	0
$E6$	$\frac{\rho}{(1+\beta_1+\beta_2)}$	$1-\rho$	$\frac{\beta_1\rho}{(1+\beta_1+\beta_2)}$	0	$\frac{\beta_2\rho}{(1+\beta_1+\beta_2)}$	0

where $A \equiv \dfrac{\delta_1(1+\delta_1)}{(1+\delta_1+\delta_2)(\beta_1+\delta_1(1+\beta_1))}$, $B \equiv \dfrac{\delta_1}{1+\beta_1+\delta_1(1+\beta_1+\beta_2)}$, $X \equiv$ and $D \equiv$

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