Self-reporting and market structure

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Abstract

This paper studies the effectiveness of self-reporting as an enforcement mechanism within the context of an oligopoly. Whereas, prior work only considered the effectiveness of self-reporting under perfect competition (e.g. Kaplow and Shavel, 1994). We show that accounting for market structure identifies an additional benefit to self-reporting that has not yet been identified in the literature. Specifically, if enforcement costs are fixed, then it is always optimal to audit firms under a self-reporting regime. However, in the no-reporting regime, if the level of competition is sufficiently high then the regulator will find it optimal to cease auditing completely. Additionally, we show that in the fixed cost case the regulator's optimal audit probability under self-reporting (and the compliance level) is lower than that under no-reporting only if the market if sufficiently concentrated. If the market is sufficiently competitive this result is reversed. If enforcement costs are proportional to market size, then the regulator's optimal audit probability under self-reporting (and the compliance level) is higher than that under no-reporting only if the market if sufficiently concentrated. Next, we show that the effectiveness of self-reporting also depends on the interaction between the level of competition and other market characteristics such as the strength of the product demand. Finally, we study a social planner who can choose both the optimal regime, auditing level, and the level of competition N. Here, we show that the level of competition chosen by a planner will be higher in a self reporting regime than in a no reporting regime. Thus, our paper shows that market structure affects many of the prior results identified in the self-reporting literature.

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1 Introduction

Kaplow and Shavell (1994) show that self-reporting is beneficial to a welfare maximizing regulator because the regulator can save on auditing costs by not having to audit those firms that confess to having committed a violation. Their paper studies the efficacy of self reporting for the case with a large number of (atomistic) agents or firms. However, it is not clear how these results concerning self-reporting extend to cases in which firms behave strategically. Specifically, in an oligopoly the behavior of one firm to self-report can affect its competitor's decision to report or pollute, thereby influencing the effectiveness of self-reporting mechanisms. The goal of this paper, therefore, is to study the effectiveness of self-reporting within an oligopoly.

We believe that it is important to study self reporting within oligopolies because self reporting is becoming widespread in many regulatory settings. For example, the U.S. Environmental Protection Agency (EPA) encourages firms to self-report oil-spills or other environmental "crimes." Similarly, in an era of reduced government expenditures, the U.S. Department of Agriculture has recently adopted self-reporting to regulate firms for compliance with food safety standards. Interestingly, many of these industries are not perfectly competitive. For example, the EPA utilizes self reporting for firms in the energy industry, which is clearly an oligopoly. Thus, since many regulatory agencies are introducing self-reporting schemes, we wish to examine how strategic market behavior, and competitive forces among firms, influences the effectiveness of self reporting.

To study this issue we develop a "Cournot-style" model in which oligopolistic firms generate a negative externality (pollution) during production. Firms can reduce this harm by investing in costly pollution abatement. However, since abatement is costly, in the absence of any regulation firms do not abate. To incentivize abatement firms are audited by a regulator who can choose either a self-reporting regime, or a non-reporting regime to fine firms for causing harm.

Within this framework our first result shows that accounting for market structure identifies an additional benefit to self-reporting (besides those already identified in the literature). Specifically, if enforcement costs are fixed and the market is sufficiently competitive, then under a no reporting regime the regulator finds it optimal (in a second best sense) to implement the laissez faire outcome. However, under self-reporting it is always optimal to regulate the firm. Thus, in other words self-reporting makes regulation optimal for cases where it would otherwise not be optimal to intervene in the market (under no reporting).

Additionally, we find that many prior results in the self-reporting literature depend on market competition. First, whether the socially optimal audit probability under self-reporting is higher or lower than the optimal probability in the no-reporting regime depends on the level of competition and whether enforcement costs are fixed or proportional to the industry's size. If costs are fixed, the optimal audit probability is higher under no reporting than under self-reporting only when the industry is sufficiently concentrated. Instead, if costs are proportional, then for highly concentrated industries the audit probability is lower under no reporting than under self-reporting. For highly competitive industries, this result is reversed. Relatedly, when competition is low, introducing self-reporting decreases the equilibrium level of pollution. While, when competition is intense, introducing self reporting increases the equilibrium level of pollution. Thus, we show that whether self-reporting involves a higher or lower audit probability and increases or decreases the harm (relative to no-reporting), depends on the level of industry competition.

We also derive some comparative statics with respect to the optimal audit probabilities. When product demand is strong, then the audit probability under the self reporting regime is higher than the probability under no self reporting even for relatively competitive industries. Whereas when demand is weak then the audit probability under self reporting is lower than the probability under no reporting even for relatively concentrated industries. Similarly, we also find that the relationship between the level of harm and the optimal audit probabilities depends on the level of competition. Thus, regulators must account for the level of competition when introducing self reporting auditing regimes.

It is worth relating these findings to the existing literature on self reporting. Kaplow and Shavell (1994) and Malik (1993) both find that the audit probability may be higher under self reporting versus no reporting, however, they do not determine analytically the exact conditions under which this will occur. By introducing market structure into this framework, we show that this outcome is determined by the level of competition and other market characteristics. Our findings are also related to Innes (1999, 2001) who finds two other benefits from self reporting (besides saving on audit costs). First, if firms can engage in clean up activities, then under self reporting firms always engage in clean up, whereas, under no reporting firms only clean up when they are caught. Since clean up is welfare improving, self reporting improves welfare for this additional reason. Second, if firms can invest in costly detection avoidance, then under self reporting there is less avoidance. Since avoidance is wasteful, self reporting enhances welfare.

Our results concerning competition are related to a growing body of research that studies the interaction between competition and law enforcement. Daughety and Reingenaum (2006) study the impact of various liability regimes on a firm's decision to reduce its pollution levels. Using an analytic framework similar to ours (oligopolistic firms generating a harm), they find that whether "strict liability" or "negligence rules" yields more compliance and less pollution, depends on the level of competition. Similarly, Dechenaux and Samuel (2015) study a regulator's decision to conduct surprise on announced inspections. They find that whether an announced inspection is preferred to a surprise inspection (in reducing pollution) depends on the level of competition. Thus, this paper contributes to the literature that studies the interaction between competition and regulatory enforcement.

The rest of this paper is organized as follows. Section 2 sets up the basic model as well as the market equilibrium. Section 3 studies the welfare maximization problem under self reporting and no-reporting. Section 4 conducts an analysis of the solutions to these problems, and section 5 concludes. All proofs are provided in the appendix.

2 The model

Consider a market with $N \geq 1$ oligopolistic firms that each produce q_i units of a product. The total market quantity $Q = \sum_{i=1}^{N} q_i$. The cost of producing each unit is c, and there are no fixed costs of producing q_i. Firms sell products to consumers with quasilinear utility function $U(\mathbf{q}, q_0; \mathbf{a}) = u(\mathbf{q}, \mathbf{a}) + q_0$ where good 0 is the numéraire, with $p_0 = 1$. We assume that U has the Bowley form

$$U(\mathbf{q}, q_0; \mathbf{a}) = \sum_{i=1}^{N} \beta q_i - \frac{\gamma}{2} \left(\sum_i q_i^2 + \sum_i \sum_{i \neq j} q_i q_j \right) + q_0$$

Maximizing this utility function with respect to a standard budget constraint yields the linear inverse demand,

$$P = \beta - \gamma Q$$

Where necessary we denote q_{-i} is the sum of all the firms' quantities other than firm i.

Besides the direct costs, producing q_i units imposes a total negative cost (externality) on society $q_i h$. This externality can be abated at the rate $a_i \in (0, 1]$, so that the harm $q_i h$ only occurs with probability $(1 - a_i)$. Abatement, however, costs $k(a_i)$ per unit where we assume that $\frac{k a_i^2}{2}$. Since abatement is costly, and the harm does not affect a firm's profits, a firm will not choose to abate unless there is some regulation. That is the laissez faire level of abatement $a_{LF} = 0$.

To incentivize abatement a welfare maximizing regulator audits the firm with probability ρ and fines it if it discovers that the firm has caused harm. Specifically, when harm occurs (with probability $1 - a_i$) in the *non-self reporting* (ϕ) regime, the firm is audited with probability ρ_ϕ and is fined $F \in [0, \overline{F}]$ per unit, were $\overline{F}$ is the maximal feasible fine. Thus, in the NSR regime a firm's profit is,

$$\pi_{i,\phi} = \left[\beta - \gamma Q - c - (1 - a_i)\rho_\phi F - \frac{k a_i^2}{2} \right] q_i.$$

In a self reporting regime (r) if harm occurs the firm self reports the occurrence of an accident with probability $\tau \in [0, 1]$, in which case it is fined f. In keeping with Kaplow and Shavell (1993) it is audited with probability ρ_r when it does not make a report (or reports no accident). Thus, a firm's profit in the SR regime is,

$$\pi_{i,r} = \left[\beta - \gamma Q - c - (1-\tau)(1-a_i)\rho_r F - \tau(1-a)f - \frac{ka_i^2}{2}\right] q_i.$$

Let $z = \{r, \phi\}$ index the two regimes, then the timing of both games is as follows

Stage 1. The regulator chooses ρ_z and f or F

Stage 2. The firm chooses a_z and q_z

Stage 3. Harm is realized or not

Stage 4. The firm self-reports in a r regime

Stage 5. The regulator audits

We first solve the model in the case of the SR regime. Working backwards, in stage 4 (taking quantities and abatement levels as given) the firm chooses τ to maximize profits, therefore the derivative of 2 with respect to τ is,

$$\rho_r(1-a_i)F - (1-a_i)f$$

Since $(1-a_i) \geq 0$, if $\rho_{SR}F \geq f$, the $\tau^* = 1$, otherwise, $\tau^* = 0$.

Clear, choosing $\rho_{SR}F < f$ is not optimal since ρ can be lowered (maintaining the equality) while also improving welfare. Also, $\rho_{SR}F > f$ cannot be a solution since in that case firms would never self report and the equilibrium would be identical to the NSR regime. Thus, $\rho_{SR}F = f$ is optimal so that firms always self report when harm occurs. Thus, 2 reduces to,

$$\pi_{i,r} = \left[\beta - \gamma Q - c - \tau(1-a)f - \frac{ka_i^2}{2}\right] q_i.$$

The derivative of the first order condition with respect to a_i yields the profit maximizing level of abatement in the SR regime

$$a^* = \frac{f}{k} = \frac{\rho F}{k}$$

To ensure an interior solution we make the following assumption.

Assumption 1 *The cost of abatement is sufficiently large $k > F$.*

Substituting the value for a^* into the profit function yields

$$\pi_{i,r} = \left[\beta - \gamma Q - c - f + \frac{f^2}{2k}\right] q_i.$$

Let m_i be the firm's full marginal cost, then using the fact that $f = \rho F$,

$$m_i = c + \rho F - \frac{(\rho F)^2}{2k},$$

Then (maximizing profits with respect to quantity) the firm's quantity in a symmetric equilibrium is,

$$q_r^* = \frac{\beta - m_i}{\gamma(N+1)},$$

and profits are,

$$\pi_r^* = \gamma(q^*)^2$$

Turning the ϕ regime. It is straightforward to observe that since $f = \rho_\phi F$, equations 2 and 2 are identical except that ρ_r is replaced by ρ_ϕ in 2. Hence the expressions for a^*, q_ϕ^*, and π_ϕ will be identical *given some* $\rho\{\rho_r, \rho_\phi\}$.

3 Welfare Analysis: constrained social planner

With this framework we now study the planner's welfare maximizing fines and audit probability. Here we assume that the regulator is a constrained social planner who takes the market size N as given. Further, the regulator acts as a"Stackelberg leader" that takes the firms' behavior as given. Thus, given the fines, the audit probability, and the regime, firms choose the symmetric Cournot oligopoly quantities and level of abatement derived in the previous section. The cost of enforcement for the regulator is given by $g(.)$ where,

$$g(\rho) = \rho_z g \sum\nolimits_j q_{j,z}^\delta a_{j,z}^{\mathbf{1}_{z=SR}},$$

where $\delta = [0,1]$. If $\delta = 0$ then costs are fixed in the sense that firm size does not affect enforcement costs, if $\rho = 1$ then costs are linear in firm size. Otherwise, they are concave in firm size.

In a self reporting regime, the regulator chooses ρ and F to maximize,

$$W_r = q_0 + \beta Q^* - \frac{\gamma}{2}(Q^*)^2 - Q[c + k(a) + (1-a)h] - \rho g N q^\gamma a.$$

while in a non-self reporting regime the regulator's welfare is,

$$W_\phi = q_0 + \beta Q^* - \frac{\gamma}{2}(Q^*)^2 - Q[c + k(a) + (1-a)h] - \rho g N q^\gamma,$$

where $\gamma = 1$ is the case where costs are proportionate to market size $Q = qN$ and $\gamma = 0$ the case where costs are fixed. In these two welfare functions, note that difference in costs in the two welfare functions is critical to the well-known result that self reporting is optimal.

Under self reporting the regulator only needs to audit the firm whenever no accident has occurred (with probability a). Under a no self reporting regime, the regulator must always audit.

Before proceeding to analyze the social optimal choices, we make the following assumptions to our model for any regime $z = \{r, \phi\}$.

Assumption 2 *The parameters in our model possess the following properties.*

a. Demand is sufficiently strong; that is,

$$\beta - c > k$$

so that full abatement is feasible (for the firm).

b. $h > \beta - c$

c. $hF - kg < kF$.

d. The fine $F < w$ *where* w *is the wealth of a given firm.*

Assumption 2 a is straightforward. For quantity (and hence profits to be positive) $q_z > 0$ when $a = 1$. Substituting $a = 1$ into the function for quantity yields $q_z = \frac{2(\beta - c - k)}{\gamma(N+1)} > 0$ or $\beta - c - k > 0$. 2 (b) is assumed so that society would prefer no industry to an unregulated industry; that is, $W_z(0) < q_0$. This implies

$$W_z(0) = q_0 - \frac{N[\beta - c]\{2h[1+N] - [\beta - c][2+N]\}}{2\gamma[1+N]^2}.$$

Hence, if

$$h > \max_N \frac{2+N}{1+N}[\beta - c] = \frac{3}{2}[\beta - c] > \beta - c$$

or $h > \beta - c$, society prefers regulation to the laissez faire outcome.

Assumption 3 (c)implies that for the regulator, full abatement is not socially optimal. To ensure this,

$$\left.\frac{\partial W_z}{\partial \rho_z}\right|_{\rho = k/F} = \left[\frac{N}{1+N}\right]\frac{[2[\beta - c] - k][Fh - k[F+g]]}{2\gamma k}$$

At $a = 1$ the above expression must be negative, or

$$h < \frac{k}{F}[F+g]$$

which implies that $hF - kg < kF$.

Given these assumptions the regulator chooses ρ and F to maximize

$$W_z$$

subject to the constraint that $\rho_z \leq k/F$ (for $a_z = 1$ when $\rho_z = k/F$ and it is never optimal to raise ρ_z once $a_z = 1$)

Under these results, our first step is to establish a result concerning the optimal fine in the ϕ regime.

Lemma 1 *Regardless of the cost structure, in the no reporting regime, the optimal fine is maximal.*

Given this result, for the rest of this paper we assume that the fine F is the maximal fine $\overline{F}$.

3.1 $\delta = 0$ Case

We first analyze the case where the enforcement costs are fixed with respect to firms' size. Later we offer a discussion of the case where $\delta = 1$. The optimal audit probability is characterized in the following proposition.

Proposition 1 *The optimal audit probabilities in the two regimes are,*

$$\rho_\phi = max\left\{0, \frac{F\{h[1+N]-k\}\{2[\beta-c]-k\} - 2\gamma gk[1+N]^2}{F^2N\{2[\beta-c]-k\}}\right\}$$

$$\rho_r = \frac{\{2[\beta-c]-k\}\{h[1+N]-k\}}{FN\{2[\beta-c]-k\}+4\gamma g[1+N]^2}.$$

Further, if enforcement costs are fixed, that is $\gamma = 0$, then there exists an $N' > 1$ such that for all $N > N'$, a regulator chooses $\rho_\phi = 0, a_\phi = 0$. However, $\rho_r > 0$ for all N.

The previous proposition shows that when enforcement costs are fixed, then if markets are sufficiently competitive, a regulator will not choose to regulate. Since $\rho_\phi = 0$, $a_\phi = 0$. That is, the laissez faire outcome is optimal in a second-best sense for a sufficiently large N. However, under self-reporting, $\rho_r > 0$ is optimal for any N. Thus, self-reporting "allows" the regulator to regulate even in circumstances where it would otherwise not be optimal to regulate. Consequently, under self-reporting the level of pollution will more closely approximate the first-best level of pollution.

We now present the following comparative static results with respect to ρ_ϕ and ρ_r.

Proposition 2 *The optimal audit probability*

- *ρ_z, for $z = \{\phi, r\}$ is*
  - *strictly decreasing in N and γ and increasing in h and $\beta - c$.*
  - *ambiguous with respect to k*
- *ρ_ϕ is ambiguous in $\overline{F}$, but ρ_r is increasing in $\overline{F}$.*

The comparative statics with respect to h and $\beta - c$ are intuitive. As the harm increases the regulator needs to increase the audit intensity. Similarly, when demand is strong (i.e. $\beta - c$ large) then quantity produces, and consequently the harm increases. Thus, audit intensity also rises. The effect of competition on the audit probability, however, is particularly interesting. As competition increases the gain in consumer surplus becomes larger. Since ρ reduces these gains, the marginal benefit of raising the audit probability falls. Consequently, the optimal audit probability declines with N. However, the intensity of the effect is different under the two regimes so that there exist cases where the audit probability falls more rapidly under the non reporting regime so that if the market is sufficiently concentrated the audit probability is higher in the non reporting regime, but when the market is sufficiently competitive the audit probability is lower in the non reporting regime. We characterize this result below.

Proposition 3 *If*

$$h > \frac{3}{4}k + \frac{4g\gamma k}{F(2(\beta - c) - k)},$$

then there exists a unique $N_0 \in (1, N')$ such that

$$\rho_\phi > \rho_r \Leftrightarrow a_\phi > a_r \Leftrightarrow> Q_\phi < Q_r \Leftrightarrow N < N_0.$$

Proposition 2 highlights the main result of our paper, which is that the qualitative difference of introducing a self-reporting policy is not trivial. If the market is sufficiently concentrated, then a regulator that implements self-reporting may introduce a higher level of crime even though it raises overall welfare because self reporting lowers enforcement costs. From the perspective of the public this higher level of crime may have more salience than the corresponding reduction in enforcement costs. If the market is sufficiently competitive, this result is reversed. Consequently, competition is "good" for self reporting in the sense that if markets are sufficiently competitive, the efficiency gains from self-reporting can be fully realized without raising crime. Indeed, if a regulator were constrained by the notion that any new policy implemented must lower crime, then self-reporting will be more likely to occur in very competitive markets.

Kaplow and Shavell (1993) show that in general the audit probability under self-reporting may be higher lower than the probability under no reporting. Since the fine is always maximal it implies that the level of crime may be higher or lower under self reporting than in the no reporting regime. This proposition "tightens" their result and shows that whether the audit probability in one regime is higher or lower than in the other depends on the level of competition (see figures 1 and 2). Further, because of this abatement a, is linear and increasing in ρ, whether the level of abatement will be higher or lower under the self-reporting regime relative to no reporting, also depends on the level of competition.

4 Welfare Analysis: Unconstrained social planner

We now assume that the social planner can choose both N as well as ρ in both the ϕ and the r regimes. Our main result is summarized in the following proposition.

Proposition 4 *Let $\hat{\rho}, \hat{N}$ denote the socially optimal level of auditing and market size. This socially optimal policy for an unconstrained social planner possesses the following characteristics.*

- $\hat{\rho}_\phi > \hat{\rho}_r$
- $\hat{N}_\phi < \hat{\rho}_r$

Proof. See Appendix ■ This rather strong result shows that a switch from a non reporting to a self reporting regime will also always increase market size while also reducing auditing. These leads to two rather surprising results. First, since compliance is increasing in ρ it implies that this regime switch will result in less compliance. But, second, it will result in a bigger market size while also raising compliance.

5 Conclusion

Many regulatory agencies are beginning to utilize self-reporting mechanisms to save on enforcement costs. In many cases the firms participating in these self-reporting programs belong to oligopolistic industries. Thus, their choice to self report and comply with regulations strategically depend on their competitors. Consequently, the value to the regulator of switching to self-reporting may depend on the strategic interactions between firms.

In this paper we show that self inspection regimes are, indeed, affected by strategic market forces. Specifically, whether the optimal audit probability under self-reporting is higher or lower than the audit probability under no reporting depends on the level of competition. This result is interesting in light of Kaplow and Shavell (1994) who find that the audit probability under the two regimes maybe higher or lower, but do not offer any analytic results regarding this issue. Here we show that competition determines whether the optimal probability will be higher or lower in the self-reporting versus the no reporting regime.

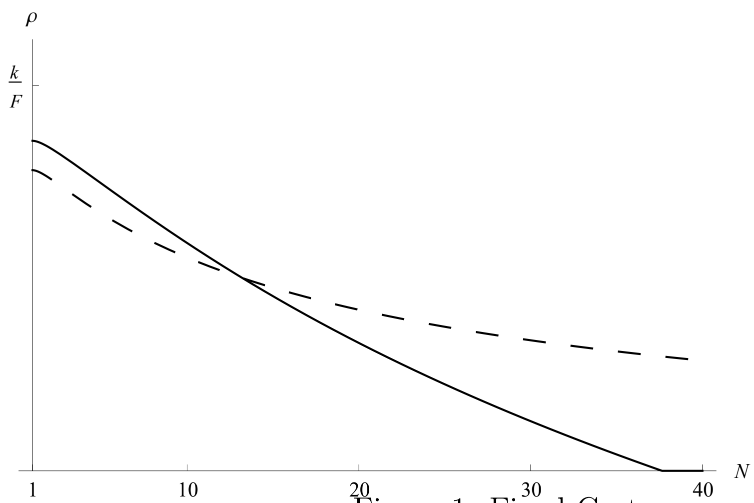

Figure 1: Fixed Costs, ρ_r: - - -, —— : ρ_ϕ

Second, we find that the level of compliance may be higher or lower in either regime, depending on the level of competition. This finding stands in sharp contrast to the prior literature on self-reporting (Kaplow and Shavell, 1993; Innes 1999) who find that the level of compliance under self-reporting is always higher than under no reporting. Finally, whether compliance or the optimal audit probability is higher or lower under self-reporting versus no reporting also depends on other market characteristics such as the strength of the consumer's demand. Taken together, these results suggest that regulators introducing self-reporting need to consider market forces in the industries into which these policies are being implemented.

Finally, we study the unconstrained social planner's problem who can choose both the optimal regime, auditing level, and the level of competition N. Here, we show that the level of competition N will be higher in a self reporting regime than in a no reporting regime. Thus, this result offers a new argument in support of self-reporting regimes. Namely, that they lead to a larger market size.

Appendix Appendix

Appendix.1 Figures

Appendix.2 ρ and F are substitutes.

Here we prove that ρ and F are substitutes w.r.t. Q (or q).

$$Q^* = \frac{\beta - c - \rho F + \frac{(\rho F)^2}{2k}}{\gamma(N+1)}.$$

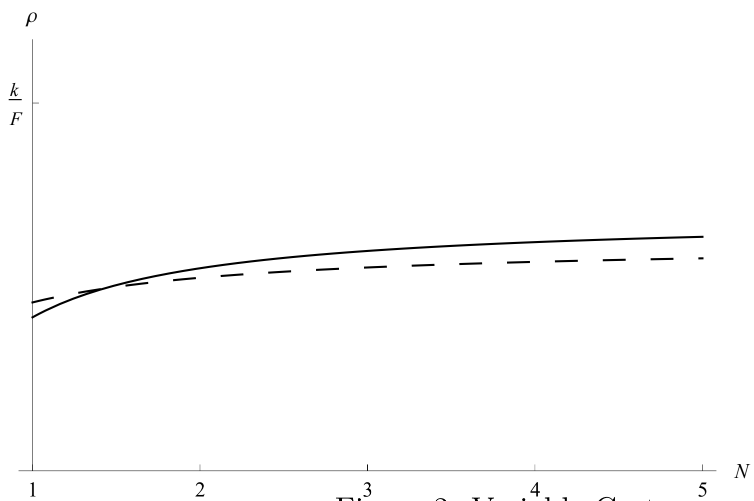

Figure 2: Variable Costs, ρ_r: - - -, —— : ρ_ϕ

Thus,

$$\frac{dQ^*}{d\rho} = N\left(\frac{1}{\gamma(N+1)}\left[-F + \frac{\rho F^2}{k}\right]\right)$$

Multiplying and dividing the above expression by ρ yields,

$$\frac{N}{\rho\gamma(N+1)}(-F + \frac{(\rho F)^2}{k})$$

$$\frac{dQ^*}{d\rho} = \frac{A}{\rho}$$

Multiplying both sides of the previous equation by $\frac{\rho}{Q}$ yields the elasticity,

$$\epsilon_{Q,\rho} = \frac{A}{Q}$$

Similarly,

$$\frac{dQ^*}{dF} = N\left(\frac{1}{\gamma(N+1)}\left[-F + \frac{\rho^2 F}{k}\right]\right)$$

Multiplying and dividing the above expression by F yields,

$$\frac{dQ^*}{dF} = \frac{A}{F}$$

Multiplying both sides of the previous equation by $\frac{F}{Q}$ yields the elasticity,

$$\epsilon_{Q,F} = \frac{A}{Q}$$

Therefore, the two parameters ρ and Q have identical effects on Q.

Next we turn our attention to $x = Q(1-a)h$, the total external harm generated in the industry. The following calculations show that the effect of ρ and F on x are identical.

$$\frac{dx}{d\rho} = \left(\frac{dQ}{d\rho}(1-a) - Q\frac{F}{k}\right) h$$

Multiplying both sides of this expression by $\frac{\rho}{x}$ yields,

$$\epsilon_{x,\rho} = \left(\frac{dQ}{d\rho}\rho(1-a) - \frac{QF\rho}{k}\right)\frac{h}{x}.$$

Using the expression for $dQ/d\rho$ from earlier, we have;

$$\left(\frac{A}{\rho}\rho(1-a) - \frac{Q\rho F}{k}\right)\frac{h}{x}$$

Now, turning to the elasticity for F.

$$\frac{dx}{dF} = \left(\frac{dQ}{dF}(1-a) - Q\frac{\rho}{k}\right) h$$

Multiplying both sides of this expression by $\frac{F}{x}$ yields,

$$\epsilon_{x,F} = \left(\frac{dQ}{dF}F(1-a) - \frac{QF\rho}{k}\right)\frac{h}{x}.$$

Using the expression for $dQ/d\rho$ from earlier, we have;

$$\left(\frac{A}{F}F(1-a) - \frac{Q\rho F}{k}\right)\frac{h}{x}$$

Hence, the two elasticities are equal.

Appendix.3 Proof of Lemma 1

Note that social welfare can be written as

$$W(.) = \frac{N+2}{2N}\gamma(Q^*)^2 + (1 - \frac{\rho F}{k})\rho F Q - x - g(\rho). \tag{A.1}$$

Let Φ denote the first two terms of the above expression. Then the first order conditions are,

$$\rho : \frac{dQ}{d\rho} - \frac{dx}{d\rho} \geq g'(\rho). \tag{A.2}$$

$$F : \frac{dQ}{dF} - \frac{dx}{dF} \geq 0. \tag{A.3}$$

We prove that F is maximal by contradiction. We know that the FOC for ρ must hold with a strict equality since we assume that we have an interior solution for $\rho \in (0, 1)$. Now suppose that F is also interior, then the FOC for F implies

$$\Phi'(\rho F) = \frac{dx}{dF}\frac{1}{\rho}$$

Substituting the previous expression for $\phi'(\rho F)$ into the first order condition for ρ yields,

$$\frac{dx}{dF}\frac{1}{\rho} - \frac{dx}{d\rho} = g'(\rho).$$

multiplying and dividing by ρ/x yields,

$$\frac{x}{x}\left(\frac{F}{\rho}\frac{dx}{dF} - \frac{dx}{d\rho}\right)$$

Or,

$$x\left(\frac{\epsilon_{x,F}}{\rho} - \frac{\epsilon_{x,\rho}}{\rho}\right) = 0.$$

This implies that the FOC for ρ cannot hold with equality since the RHS $g'(\rho) > 0$. Hence, F cannot be interior.

Appendix.4 Proof of Proposition 1

Appendix.4.1 Characterization of ρ_z

We first characterize the the optimal ρ_z for $z \in \{\phi, r\}$

Under the boundary restrictions given above there is a unique optimum for ρ_z satisfying

$$\frac{\partial W_z(\rho_z)}{\partial \rho_z} = \left[\beta - c - \frac{\gamma}{2}Q_z - \frac{k}{2}a_z^2 - [1 - a_z]\, h - \rho_z a_z^{\mathbf{1}_{z=r}} g\right]\frac{\partial Q_z}{\partial \rho_z} \tag{A.4}$$

$$+ \left[-\frac{\gamma}{2}\frac{\partial Q_z}{\partial \rho_z} + [h - k a_z]\frac{\partial a_z}{\partial \rho_z} - g\left[a_z^{\mathbf{1}_{z=r}} + \mathbf{1}_{z=r}\rho_z\frac{\partial a_z}{\partial \rho_z}\right]\right] Q_z = 0 \tag{A.5}$$

These are cubic equations in ρ_z. Note that the first order condition is satisfied for $\rho_z = k/F$ and $h = k\left[1 + 2^{\mathbf{1}_{z=r}} g/F\right]$. We expand $\partial W_z(\rho_z)/\partial \rho_z$ around this point in $\{h, \rho_z\}$ space using

$$
\begin{aligned}
f(x,y) &\approx f(a,b) + f_x(a,b)[x-a] + f_y(a,b)[y-b] \\
&+\frac{1}{2!}\left\{ f_{xx}(a,b)[x-a]^2 + 2f_{xy}(a,b)[x-a][y-b] + f_{yy}(a,b)[y-b]^2 \right\}
\end{aligned}
$$

to obtain

$$
\rho_z \approx \frac{F\{2[\beta-c]-k\}\{h[1+N-k]\} - gk[1+N]\{2[\beta-c]-3k\}\left\{-\frac{2k}{2[\beta-c]-3k}\right\}^{\mathbf{1}_{z=r}}}{F\left\{FN\{2[\beta-c]-k\} + 2gk[1+N]\left[\frac{2[\beta-c]}{k}\right]^{\mathbf{1}_{z=r}}\right\}}
$$

Appendix.4.2 Existence of N'

We provide the proof of the second part here. The first order condition (assuming an interior solution) with respect to ρ in the non reporting regime is:

$$
\rho : \frac{dQ}{d\rho}\frac{2\gamma(N+2)}{2N}Q - \frac{dx}{d\rho} = g'(\rho).
$$

Substituting the value of ρ this expression simplifies to,

$$
qN\left(\frac{F(h-k+\rho F}{k}\right) - \frac{F(\rho F-k)(1-a)h}{\gamma k} = gN
$$

To show the existence of N' it is sufficient to show that at $\rho = 0$ the LHS of the previous equation is strictly greater than the $LHS < RHS$. At $\rho = 0$, then LHS is,

$$
qN\left(\frac{F(h-k)}{k}\right) + \frac{Fh}{\gamma} = \frac{\beta-c}{N+1}NF(h-k) + Fhk
$$

while the RHS is gN. Further, both the LHS and the RHS are monotonically increasing in N. Clearly, as $N \to \infty$, the RHS approaches $+\infty$ while the LHS is finite and approaches $(\beta-c)F(h-k) + Fhk$. Thus, there exists an $N' \in (0, +\infty)$ such that for all $N > N'$ the $LHS < RHS$. That is, $\rho_\phi = 0$ for all $N \geq N'$.

The expression for N' may be derived explicitly. Set $z = \phi$ and $\rho_\phi = 0$. Then foc gives $\bar{F}\Phi(0) - gN' = 0$. So

$$
N' = \frac{F}{g}\Phi'(0).
$$

Appendix.4.3 $\rho_r > 0$

To show that $\rho_r > 0$ it is sufficient to show that the RHS of the first order condition is strictly less than the LHS of the first order condition at $\rho = 0$. The RHS of the first order condition is 0 at $\rho = 0$ is

$$\frac{\beta - c}{N+1} NF(h-k) + Fhk > 0 \text{for all} N.$$

However, the RHS =0 at $\rho = 0$. Hence, $\rho_r > 0 \forall N$.

Alternative, way of thinking about this: $\rho_r > 0$ as $N \to \infty$

From the foc under SR we have $\bar{F}\Phi'\left(\rho_r \bar{F}\right) - 2\rho_r \bar{F} g N/k = 0$. Hence

$$2Ng\rho_r = k\Phi'\left(\rho_r \bar{F}\right)$$

Suppose $\rho_r = 0$, then it must be that $0 = k\Phi'(0)$, equivalently $\Phi'(0) = 0$. But this is a contradiction as $\Phi'(0) > 0$ at an interior maximum. Hence $\rho_r \neq 0$. As ρ_r cannot be negative it follows that $\rho_r > 0$.

Appendix.5 Proof of Proposition 2

$$\begin{aligned}
\frac{\partial \rho_z}{\partial N} &= \frac{\Phi_{\rho N} - \frac{\Phi_\rho}{N}}{\frac{2gFN}{k}\mathbf{1}_{z=r} - \Phi_{\rho\rho}} \\
\frac{\partial \rho_z}{\partial g} &= -\frac{g\,[2a]^{\mathbf{1}_{z=r}}}{\frac{2gFN}{k}\mathbf{1}_{z=r} - \Phi_{\rho\rho}} \\
\frac{\partial \rho_z}{\partial k} &= \frac{\Phi_{\rho k} - \frac{\Phi_\rho}{k}\mathbf{1}_{z=r}}{\frac{2gFN}{k}\mathbf{1}_{z=r} - \Phi_{\rho\rho}} \\
\frac{\partial \rho_z}{\partial F} &= \frac{\Phi_{\rho F} + \frac{2\Phi_\rho}{F}\mathbf{1}_{z=r}}{\frac{2gFN}{k}\mathbf{1}_{z=r} - \Phi_{\rho\rho}} \\
\frac{\partial \rho_z}{\partial z} &= \frac{\Phi_{\rho,z}}{\frac{2gFN}{k}\mathbf{1}_{z=r} - \Phi_{\rho\rho}} \qquad z = [\beta - c]\,, \gamma, h
\end{aligned}$$

We now prove that $\Phi_{\rho N} - \frac{\Phi_\rho}{N} < 0$. We have

$$\Phi_{\rho N} - \frac{\Phi_\rho}{N} = \frac{\overline{F}\left\{\begin{array}{c} [2+3N]\left[\rho\overline{F}\right]^3 - 3\left\{h\,[1+N] + k\,[2N+1]\right\}\left[\rho\overline{F}\right]^2 \\ +2k\left\{3h\,[1+N] + N\,[\beta - c + k]\right\}\rho\overline{F} - 2k\left\{[\beta - c]\left\{h\,[1+N] - k\right\} + h\,[1+N]\,k\right\} \end{array}\right\}}{2\gamma k^2\,[1+N]^2}$$

Hence

$$\Phi_{\rho N} - \left.\frac{\Phi_\rho}{N}\right|_{\rho=0} = -2k\left\{[\beta-c]\left\{h[1+N]-k\right\}+h[1+N]k\right\} < 0;$$
$$\Phi_{\rho N} - \left.\frac{\Phi_\rho}{N}\right|_{\rho=k/F} = -\frac{\overline{F}[h-k][2[\beta-c]-k]}{2\gamma k[1+N]} < 0.$$

To understand the behaviour of $\Phi_{\rho N} - \frac{\Phi_\rho}{N}$ in the interval $(0, k/F)$ we find

$$\left.\frac{\partial\left[\Phi_{\rho N} - \frac{\Phi_\rho}{N}\right]}{\partial\rho}\right|_{\rho=0} = \frac{\overline{F}^2\left\{3h[1+N]+N[\beta-c+k]\right\}}{\gamma k[1+N]^2} > 0$$
$$\left.\frac{\partial\left[\Phi_{\rho N} - \frac{\Phi_\rho}{N}\right]}{\partial\rho}\right|_{\rho=k/F} = \frac{\overline{F}^2 N[2[\beta-c]-k]}{2\gamma k[1+N]^2} > 0$$

As the derivative $\partial\left[\Phi_{\rho N} - \frac{\Phi_\rho}{N}\right]/\partial\rho$ is a quadratic equation in ρ it can switch sign at most once, hence it can only be that $\Phi_{\rho N} - \frac{\Phi_\rho}{N}$ is monotonic on the interval $\rho \in (0, k/F)$. Accordingly, everywhere on this interval $\Phi_{\rho N} - \frac{\Phi_\rho}{N} < 0$, so $\partial\rho_z/\partial N < 0$ for $z = \phi, r$. Similar reasoning yields $\Phi_{\rho\rho} < 0$, $\Phi_{\rho\gamma} < 0$ and $\Phi_{\rho F} + \frac{2\Phi_\rho}{F} > 0$. Hence $\partial\rho_z/\partial\gamma > 0$ for $z = \phi, r$ and $\partial\rho_r/\partial\overline{F} > 0$. It can also be shown that

$$\Phi_{\rho,[\beta-c]} = \frac{\overline{F}N\left\{h-k+N\left[h-\rho_z\overline{F}\right]\right\}}{\gamma k[1+N]^2} \geq \frac{\overline{F}N[h-k]}{\gamma k[1+N]} > 0;$$
$$\Phi_{\rho h} = \frac{FN\left\{3\left[\rho\overline{F}\right]^2 - 6k\rho\overline{F} + 2k[\beta-c+k]\right\}}{2\gamma k^2[1+N]} \geq \frac{\overline{F}N\left\{2[\beta-c]-k\right\}}{2\gamma k^2[1+N]} > 0.$$

hence $\partial\rho_{[\beta-c]}/\partial\gamma > 0$ and $\partial\rho_h/\partial\gamma > 0$ for $z = \phi, r$. We cannot unambiguously determine the sign of $\Phi_{\rho k} - \frac{\Phi_\rho}{k}$ as although

$$\left.\frac{\partial\left[\Phi_{\rho k} - \frac{\Phi_\rho}{k}\right]}{\partial\rho}\right|_{\rho=0} = \frac{3N\left\{[\beta-c]\left\{h[1+N]-k\right\}+h[1+N]k\right\}}{\gamma k[1+N]^2} > 0$$

we have

$$\left.\frac{\partial\left[\Phi_{\rho k} - \frac{\Phi_\rho}{k}\right]}{\partial\rho}\right|_{\rho=k/\overline{F}} = -\frac{N\left\{2[\beta-c]-k\right\}\left\{k[3+4N]-3h[1+N]\right\}}{2\gamma k[1+N]^2}$$

which cannot be signed uniquely with the conditions in Assumption 2. Thus, $\partial\rho_h/\partial k$ is ambiguous in sign both with and without self reporting. A similar analysis applies to $\Phi_{\rho\overline{F}}$ which is again ambiguous in sign.

Appendix.6 Characterizing ρ_0

Using the definition $\rho_z(N_0) - k/[2F] = 0$ the implicit function theorem gives

$$\begin{aligned}
\frac{\partial N_0}{\partial k} &= -\left[\frac{\partial\rho_z(N_0)}{\partial N}\right]^{-1}\left[\frac{\partial\rho_z(N_0)}{\partial k} - \frac{1}{2F}\right] \\
\frac{\partial N_0}{\partial F} &= -\left[\frac{\partial\rho_z(N_0)}{\partial N}\right]^{-1}\left[\frac{\partial\rho_z(N_0)}{\partial F} + \frac{k}{[2F]^2}\right] \\
\frac{\partial N_0}{\partial z} &= -\left[\frac{\partial\rho_z(N_0)}{\partial N}\right]^{-1}\frac{\partial\rho_z(N_0)}{\partial z} \qquad z = \beta, \gamma, c, h.
\end{aligned}$$

Noting that $\frac{\partial\rho_z(N_0)/\partial N}{}$ we obtain

$$\frac{\partial N_0}{\partial k}? \qquad \frac{\partial N_0}{\partial F}? \qquad \frac{\partial N_0}{\partial[\beta - c]} > 0 \qquad \frac{\partial N_0}{\partial\gamma} < 0 \qquad \frac{\partial N_0}{\partial h} > 0??$$

old stuff below here

Subtracting the two focs from each other we get

$$\frac{gN\left\{4[\rho_z F]^3 - 9k[\rho_z F]^2 + 4kF[\beta - c + k] - 2k^2[\beta - c]\right\}}{2\gamma k^2[1 + N]} = 0$$

The crossing point $\rho_\phi = \rho_r = \rho_0$ must therefore satisfy

$$4[\rho_0 F]^3 - 9k[\rho_0 F]^2 + 4kF[\beta - c + k] - 2k^2[\beta - c] = 0 \tag{A.6}$$

Thus

$$\rho_0 = \frac{3^{1/3}[\Psi(\beta, c, k)]^2 + 9k\Psi(\beta, c, k) - 3^{2/3}k\{16[\beta - c] - 11k\}}{12F\Psi(\beta, c, k)}$$

where

$$\Psi(\beta, c, k) = \left\{\sqrt{3k^3}\sqrt{\{16[\beta - c] - 11k\}^3 + 27k\{8[\beta - c] - 3k\}^2} - [3k]^2\{8[\beta - c] - 3k\}\right\}^{\frac{1}{3}}$$

For $\Psi(\beta, c, k) \in \mathbb{R}$ must have

$$
\begin{aligned}
\{16[\beta-c]-11k\}^3+27k\{8[\beta-c]-3k\}^2 &> 0 \\
\sqrt{3k^3}\sqrt{\{16[\beta-c]-11k\}^3+27k\{8[\beta-c]-3k\}^2}-[3k]^2\{8[\beta-c]-3k\} &> 0
\end{aligned}
$$

these require, respectively,

$$
\begin{aligned}
\beta-c &> \frac{k}{2} \\
\beta-c &> \frac{11k}{16}
\end{aligned}
$$

second condition is stronger, so need

$$\beta-c>\frac{11k}{16}\left(>\frac{k}{2}\right)$$

This condition also necessary and sufficient for $\rho_0 F<k$.
Now define

$$
\begin{aligned}
\Phi(\beta,c,F,g,k,h,\rho_0) &= 2k\{[h-k][\beta-c]+hk\}-2k\rho_0\{3Fh+2g[\beta-c]\} \\
&\quad +2g\rho_0[3k-2\rho_0 F][\rho_0 F]+3[h+k][\rho_0 F]^2-2[\rho_0 F]^3
\end{aligned}
$$

Then

$$N_0=\frac{\Phi(\beta,c,F,g,k,h,\rho_0)}{[\rho_0 F-k]\left\{2k[\beta-c]-2k\rho_0 F+[\rho_0 F]^2\right\}-\Phi(\beta,c,F,g,k,h,\rho_0)}$$

For comparative statics a Taylor Series approximation is needed. Note that if $\beta-c=3k/8$ then

$$\rho_0=\frac{3k}{4F}$$

We expand the lhs of (A.6) around this point in $\{\beta-c,\rho_0\}$ space using

$$
\begin{aligned}
f(x,y) &\approx f(a,b)+f_x(a,b)[x-a]+f_y(a,b)[y-b] \\
&\quad +\frac{1}{2!}\left\{f_{xx}(a,b)[x-a]^2+2f_{xy}(a,b)[x-a][y-b]+f_{yy}(a,b)[y-b]^2\right\}
\end{aligned}
$$

to obtain

$$\rho_0 \approx \left\{ \frac{27k - 32\left[\beta - c\right]}{11k - 16\left[\beta - c\right]} \right\} \frac{k}{4F}.$$

Equating the approximations for ρ_ϕ and ρ_r we obtain

$$N_0 \approx \frac{2F\left[h-k\right]\left[2\left[\beta-c\right]-k\right] - 4gk\left[\beta-c-k\right]}{-F\left[2h-k\right]\left[2\left[\beta-c\right]-k\right] + 4gk\left[\beta-c-k\right]}.$$

$N_0 \geq 1$ if

$$\beta - c > \frac{k}{2}\frac{4Fh - 3Fk - 8gk}{4Fh - 3Fk - 4gk}$$

Then

$$\rho_\phi \gtreqless \rho_r \Leftrightarrow N \gtreqless N_0$$

Elasticity of Φ and N $\quad \varepsilon_{\Phi,N} < 1$

Φ can be written in the form $\Phi = Nqw\left(\rho_z \bar{F}, N\right)$, where

$$w\left(\rho_z \bar{F}, N\right) = \gamma q \left[\frac{N+2}{2}\right] - \left[1 - a_z\right]\left[h - \rho_z F\right]$$

so

$$\begin{aligned} \frac{\partial \Phi}{\partial N} &= qw\left(\rho_z \bar{F}, N\right) + N\left[q_N w\left(\rho_z \bar{F}, N\right) + qw_N\left(\rho_z \bar{F}, N\right)\right] \\ &= \frac{\Phi}{N} + N\left[q_N w\left(\rho_z \bar{F}, N\right) + qw_N\left(\rho_z \bar{F}, N\right)\right] \end{aligned}$$

As $w_N\left(\rho_z \bar{F}, N\right) < 0$ and $q_N < 0$ we have

$$N\left[q_N w\left(\rho_z \bar{F}, N\right) + qw_N\left(\rho_z \bar{F}, N\right)\right] < 0$$

So

$$\varepsilon_{\Phi,N} = 1 + \frac{N^2}{\Phi}\left[q_N w\left(\rho_z \bar{F}, N\right) + qw_N\left(\rho_z \bar{F}, N\right)\right] < 1$$

Appendix.7 Proof of Proposition 4

$W = \Phi - C$

$a = \frac{\rho \bar{F}}{k}$

$C = g\rho N\left\{1 - \left[1 - a\right]\tau\right\}$

$\Phi = Nq\left(\rho, N\right) w\left(\rho, N\right)$

$w(\rho, N) = \gamma q \left[\frac{N+2}{2}\right] - [1-a]\left[h - \rho\bar{F}\right]$
$q(\rho, N) = \frac{1}{\gamma[1+N]}\left\{\beta - c - \frac{a}{2}\left[2k - \rho\bar{F}\right]\right\}$
NR: $\tau = 0$
SR: $\tau = 1$
The focs for ρ, N can be written as

$$\begin{aligned}\Phi_\rho - C_\rho &= 0 \\ \Phi_N - C_N &= 0\end{aligned}$$

where

$$\begin{aligned}C_\rho &= gN\{1 + [2a-1]\tau\} \\ C_N &= g\rho\{1 - [1-a]\tau\} > 0\end{aligned}$$

1. expression for N'

Set $\rho = 0$ and $\tau = 0$. Then foc for ρ gives $\Phi_\rho(0, N') - gN' = 0$. So N' given implicitly by

$$N' = \frac{1}{g}\Phi_\rho(0, N').$$

2. $\rho > 0$ as $N \to \infty$ under SR
From the foc under SR ($\tau = 1$) we have $\Phi_\rho(\rho, N) - 2\rho\bar{F}gN/k = 0$. Hence

$$2N\bar{F}g\rho = k\Phi_\rho\left(\rho\bar{F}, N\right)$$

Suppose $\rho = 0$, then it must be that $0 = k\Phi_\rho(0, N)$, equivalently $\Phi_\rho(0, N) = 0$. But this is a contradiction as $\Phi_\rho(0, N) > 0$ at an interior maximum ($\Phi_\rho(0, N) > 0$ is a condition for an interior maximum). Hence $\rho \neq 0$. As ρ cannot be negative it follows that $\rho > 0$.

3. Φ_ρ is increasing and strictly concave in N at the optimal level of enforcement (i.e., the "indirect" benefit function is increasing and strictly concave in N)

We have (globally)

$$\begin{aligned}
\Phi_{\rho N} &= [qw_\rho + q_\rho w] + N\,[qw_{\rho N} + q_\rho w_N + q_N w_\rho + q_{\rho N} w] \\
&= \frac{\Phi_\rho}{N} + N\,[qw_{\rho N} + q_\rho w_N + q_N w_\rho + q_{\rho N} w] \\
\Phi_{\rho NN} &= 2\,[q_\rho + N q_{\rho N}]\, w_N + 2\,[q_N w_\rho + q_{\rho N} w + q w_{\rho N}] \\
&\quad + N\,[q_\rho w_{NN} + q_{NN} w_\rho + 2 q_N w_{\rho N} + q_{\rho NN} w + q w_{\rho NN}]
\end{aligned}$$

Then (globally)

$$\begin{aligned}
q_{\rho N} &= -\frac{1}{1+N} q_\rho \\
q_N &= -\frac{q}{1+N} \\
q_{NN} &= \frac{2q}{[1+N]^2} \\
q_{\rho NN} &= \frac{2q_\rho}{[1+N]^2} \\
w_{\rho N} &= -\frac{\gamma q_\rho}{2\,[1+N]} \\
w_{NN} &= \frac{\gamma q_{NN}}{2} \\
w_{\rho NN} &= \gamma q_{\rho N} + \frac{2+N}{2} \gamma q_{\rho NN} \\
w_N &= \frac{\gamma q_N}{2}
\end{aligned}$$

So (globally)

$$\begin{aligned}
\Phi_{\rho N} &= \frac{1}{1+N} \left[\frac{\Phi_\rho}{N} - \gamma N q q_\rho \right] \\
\Phi_{\rho NN} &= \frac{2}{[1+N]^2} \left\{ \gamma\,[N-1]\, q q_\rho - [w q_\rho + q w_\rho] \right\} \\
&= \frac{2}{[1+N]^2} \left\{ \gamma\,[N-1]\, q q_\rho - \frac{\Phi_\rho}{N} \right\}
\end{aligned}$$

Then, at the optimal level of enforcement,

$$\frac{\Phi_\rho}{N} = \frac{C_\rho}{N} > 0$$

Hence, the result follows.

4. $\varepsilon_{\Phi_\rho,N} < 1$ at the optimal level of enforcement

Φ_ρ is strictly concave in N at the optimal level of enforcement, from 3. Hence, for all $N_1 < N_2$, $\Phi_\rho(\rho, N_2) - [N_2 - N_1]\Phi_{\rho N}(\rho, N_2) > \Phi_\rho(\rho, N_1)$. Setting $N_1 = 0$ and noting that $\Phi_\rho(\rho, 0) = 0$ (as $\Phi_\rho(\rho, N) = C_\rho(\rho, N) = gN\{1 + [2a - 1]\tau\}$) we have that $\Phi_\rho(\rho, N_2) - N_2\Phi_{\rho N}(\rho, N_2) > 0$ for all N_2, which implies $1 - \varepsilon_{\Phi_\rho,N} > 0$.

5. $\frac{\partial\rho}{\partial N} < 0$ at the optimal level of enforcement; $\frac{\partial N}{\partial\rho} < 0$ at the optimal level of competition

The foc for ρ is $\Phi_\rho - C_\rho = 0$ so

$$\frac{\partial\rho}{\partial N} = \frac{-W_{N\rho}}{W_{\rho\rho}}$$

Then, using 4,

$$-W_{N\rho} = \frac{\Phi_\rho}{N} - \Phi_{\rho N} = \frac{\Phi_\rho}{N}\left[1 - \varepsilon_{\Phi_\rho,N}\right] > 0$$

so

$$\frac{\partial\rho}{\partial N} = \frac{\frac{\Phi_\rho}{N}\left[1 - \varepsilon_{\Phi_\rho,N}\right]}{W_{\rho\rho}} < 0$$

The foc for N is $\Phi_N - C_N = 0$ so

$$\frac{\partial N}{\partial\rho} = \frac{-W_{N\rho}}{W_{NN}} = \frac{W_{\rho\rho}}{W_{NN}}\frac{\partial\rho}{\partial N} < 0$$

6. $\frac{\partial N}{\partial\tau} > 0$ at the optimal level of competition

The focs for N can be written as $\Phi_N - g\rho\{1 - [1 - a]\tau\} = 0$. Then

$$\frac{\partial N}{\partial\tau} = -\frac{g\rho[1 - a]}{W_{NN}} = -\frac{g\rho[1 - a]}{W_{NN}} > 0.$$

7. ignore

8. $\frac{\partial\rho}{\partial\tau} < 0$ at a social optimum

We have (from 14) that $2a - 1 > 0$. So, (from foc ρ),

$$\frac{\partial\rho}{\partial\tau} = \frac{gN[2a - 1]}{W_{\rho\rho}} < 0.$$

9. $\rho > \rho_0$ and $N < N_0$ at a social maximum

From foc for ρ

$$2a - 1 > 0 \Leftrightarrow \rho > \rho_0 \Leftrightarrow N < N_0$$

10. $\hat{N}_\phi < \hat{N}_r < N_0$
We first prove $\hat{N}_r > \hat{N}_\phi$. The total effect on N of an increase in τ is given by (chain rule)

$$\frac{dN}{d\tau} = \frac{\partial N}{\partial \tau} + \frac{\partial \rho}{\partial \tau}\frac{\partial N}{\partial \rho} > 0$$

Hence $\hat{N}_r > \hat{N}_\phi$. Final part of inequality follows from 9.

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