

Minimum Quality Standards and Compulsory Labeling when Environmental Quality is not Observable

Laura Birg*

Jan S. Voßwinkel**

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Abstract

This paper studies the effect of a minimum quality standard, a compulsory labeling scheme, and the combination of both instruments in a vertical differentiation model when not all quality dimensions can be observed by consumers. Both a minimum quality standard for the non-observable quality dimension and a labeling scheme that informs consumers about the non-observable quality dimension increase prices, and have no effect on the observable quality dimension and market shares. The combination of a minimum standard and a labeling scheme increases both the unobservable and the observable quality dimension, increases prices, and shifts market shares from the high quality firm to the low-quality firm. Social welfare is higher under the combination of both instruments than under no regulation, the minimum quality standard or labeling applied as only instrument.

JEL Classification: Q58, L13, L15

1 Introduction

This paper studies the effect of a minimum quality standard, a compulsory labeling scheme, and the combination of both instruments for goods with two quality dimensions: Conventional quality and environmental quality. We assume that conventional quality has experience good properties while environmental quality has credence good properties – it is neither observable nor verifiable for consumers.

*Department of Economics, University of Göttingen, Platz der Göttinger Sieben 3, 37073 Göttingen, Germany, laura.birg@wiwi.uni-goettingen.de.

**Department of Economics, NGU Nürtingen-Geislingen University, Neckarsteige 6-10, 72622 Nürtingen, Germany, jan.vosswinkel@hfwu.de.

Many consumer goods feature more than one quality dimension, but not all dimensions may be equally observable to consumers. A vacuum cleaner, for instance, is characterized by its cleaning performance, its noise while active, its design, and its energy efficiency. Consumers may base their choice of a vacuum cleaner on more than one quality dimension. For instance, they may have a positive valuation for both its design and its energy efficiency and they may be willing to forgo some “units” of energy efficiency if they are compensated by lower noise. But not all quality dimensions are directly observable for consumers. While the cleaning performance and the noise of a vacuum cleaner are experience goods, energy consumption (and especially the change in energy consumption over time) is not directly visible for consumers. They could invest in appliances to measure energy consumption, but for most consumers, this investment comes at prohibitively high cost.

Some quality dimensions are not only relevant to the user of a product, but also to third parties. The noise of a vacuum cleaner may be relevant to neighbors, increased energy usage may cause harmful externalities such as carbon dioxide. If (parts of) the benefits of investing in the observation of the hidden quality dimension accrue to third parties, this even lowers the interest in investing in the observation of hidden quality dimensions and aggravates the problem of non-observability of some quality dimensions.

But the interest of consumers in the non-observable quality dimension may also be grounded in their narrow self-interest (more energy efficient products help to save money over time) or in a preference for behaving environmental friendly (“green consumers”) or a combination thereof.

A positive willingness to pay for environmental quality is well established in the literature (see e.g. Chen, 2001; Brécard, 2011). A Flash Eurobarometer survey finds that a majority of EU citizens believe that environmentally friendly products are a “good value for money” and are willing to pay “somewhat more for products if they are confident that they are environmentally friendly” (European Commission, 2013; see also Chen, 2001, who reports for similar findings for Americans). We follow this finding and assume that consumers have positive willingness to pay not only for conventional quality, but also for environmental quality of products. If a product is characterized by multiple quality dimensions, it is natural to model consumer utility as the sum of all quality dimensions. We assume that consumers differ in their overall preference for product quality, but do not differ with respect to their preference for specific quality characteristics. We discuss the robustness of our results with respect to this assumption in a discussion section.

Usually the literature assumes that green consumers are able to observe the environ-

mental quality of products. But many EU citizens lack knowledge of the environmental impacts of the products they use (European Commission, 2013). Firms may have an incentive to signal the level of the non-observable quality dimension(s) of their products. But consumers may be reluctant to trust this information, because they may not distinguish informative from persuasive advertising and know that firms have an incentive to praise the quality of their products. This is why we model the environmental quality dimension of products as a credence good, for which consumers rely on information provided by compulsory standardized labeling. In this setting, a compulsory label changes product characteristics in the view of consumers. In addition, voluntary labeling may tempt consumers to consider this information to be not of high importance (Schmeiser, 2014). Compulsory labeling might help consumers to choose more energy efficient products by providing additional information and by communicating that the government considers this information to be important.

The externality and the credence good properties have given rise to a set of regulatory intervention of the European Union in the form of minimum quality standards and compulsory labeling schemes for energy efficiency of energy related products. Energy related products are products that either consume energy during use (such as a vacuum cleaner) or influence energy consumption (such as a window). The Ecodesign Directive (2009/125/EC) imposes minimum quality standards on energy-related products with regard to energy efficiency. The Energy Labeling Directive (2010/30/EU) obliges firms to label the energy efficiency of their products in a transparent manner. The label informs consumers about the energy efficiency characteristics of products that would be difficult to monitor for consumers otherwise.

An ever growing list of products falls under the Energy Labeling Directive and/or the Ecodesign Directive in the European Union. The “Working Plan 2016-2019 under the Ecodesign Directive” lists many groups of products for which an energy labeling scheme and/or a minimum energy efficiency standard already applies or is foreseen for the near future. Products in the Working Plan are e.g. fans, light bulbs, vacuum cleaners, dishwashers, and televisions. For these products, usually two quality dimensions are relevant, conventional quality and the environmental quality.

If a product is characterized by more than one quality dimension, or if environmental quality characteristics are relevant for several stages of a product life cycle, influencing the environmental aspects at one stage of the life cycle or for one quality dimension may result in unintended consequences. Chen (2001) analyzes the effect of environmental products standards when there is a trade-off between the traditional product attributes and environmental attributes. An (appropriate) environmental standard may benefit the

environment, but tightening the standard may also result in a lower level of total environmental quality. Bernard (2016) analyzes cross-relationships between various product quality dimensions over the whole life cycle of products. An instrument that is intended to increase the environmental quality at one stage of the life cycle may decrease the environmental quality at another stage of the life cycle and even decrease the overall environmental quality. In our paper the cost for both quality dimensions is independent, but by cost convexity substitutability from the perspective of consumers, regulation of one quality dimension may have an effect of another quality dimension as well. Especially we find that a minimum quality standard or labeling of the environmental quality dimension if applied as the only instrument does not affect conventional quality dimension, but the joint application of both instruments may increase conventional quality.

The literature has so far mainly focused on the analysis of the isolated effects of minimum quality standards or labeling schemes, but not on a comparison of both instruments and the effect of their combination. To our knowledge, the only paper comparing minimum quality standards and labeling is Baltzer (2012), who shows that a minimum quality standard may generate higher welfare than a labeling scheme if the labeling scheme induces market power. Other than Baltzer (2012), we assume that a labeling scheme makes the hidden quality dimension observable, but does not affect market power of firms. We also study the combined use of both a minimum quality standard and labeling.

The literature on labeling has focused on various aspects such as the impact of labeling on market structure, side-effects of costly certification, the trustworthiness of labels, mandatory vs. voluntary labeling etc. (see Bonroy & Constantatos, 2014). While our paper assumes compulsory labeling, because it focuses on the interaction of the Energy Labeling Directive and the Ecodesign Directive, Roe, Teisl & Deans (2014) present a detailed discussion of mandatory vs. voluntary labeling. Labels provide additional information for consumers about non-observable quality dimensions. While this should result in welfare increases at first glance, the results in the literature about labeling are mixed (Bonroy & Constantatos, 2014). Labeling may increase welfare if markets for labeled products are competitive. But labeling may reduce welfare if it enhances market power. This problem arises especially when the cost difference between quality levels is high (Zago & Pick, 2004). With a focus on consumers' attention, Lusk & Marette (2012) show that more information provided by labels is not always better for consumers if their attention is limited. Labels may distract consumers' attention from more important aspects of their decision problems by assigning an artificially high weight to labeled aspects. Lusk & Marette (2012) focus on the effect of one instrument and the choice between several products. Our analysis focuses on the effect of the simultaneous use of

two instruments, so we disregard the idea of distraction of consumers' attention.

We study the effect of both instruments, a minimum quality standard and compulsory labeling, and a combination thereof in a vertical differentiation model with two quality dimensions, an observable and a non-observable dimension. A minimum quality standard may alter the unobservable environmental quality dimension and internalize the externality, but consumers still cannot observe the environmental quality dimension. Labeling makes the environmental quality dimension visible for consumers, but that alone does not necessarily correct the externality, because consumers may not have a sufficiently high interest in correcting the externality. Using both instruments, like the European Union does for many products, may result in additional effects on conventional quality, and market shares compared to the use of one instrument only. We show that neither a minimum quality standard nor labeling alone have an effect on market shares. But the combined use of both instruments changes market shares. Social welfare is higher under the combined use of both instruments than under no regulation or one single instrument only.

The introduction of a minimum quality standard with respect to the unobservable dimension increases the quality level of the unobservable dimension, but does not cause vertical differentiation. Compulsory labeling of the environmental product characteristic has two effects: It informs consumers about this quality dimension and it signals that the government considers this quality dimension to be of relevance (as suggested by Schmeiser, 2014). This motivates firms to differentiate their products also with respect to this quality dimension. Compared to no regulation or a minimum quality standard, the profit of firms increases under labeling, because it allows firms to compete along an additional quality dimension and to skim off the willingness to pay of consumers.

The combination of a minimum quality standard and a compulsory labeling scheme supports product differentiation, but simultaneously reduces the vertical quality differentiation in the regulated quality dimension. This affects competition between firms, alters market shares, and may even increase firms' profits. Consumer surplus is higher under the combination of instruments than under no regulation or the use of one instrument only. Social welfare is higher under the combined use of both instruments compared to no regulation or the use of one single instrument only. This indicates that two instruments are better in addressing two problems – asymmetric information and a negative externality – than one instrument only.

The rest of the paper is organized as follows. In the next section, the vertical differentiation model is presented. Section 3 studies the case of no government intervention, the effects of a minimum quality standard, a compulsory labeling scheme, and the com-

bination of both. Section 4 analyzes welfare, section 5 discusses the robustness of our results with respect to our model specifications. Section 6 concludes.

2 The Model

We follow Ecchia & Lambertini (1997) in our basic model of a duopolistic market with vertical product differentiation. We extend this model by assuming that products are characterized by two quality dimensions, of which one is observable and the other is not observable. Consider two identical firms $i = H, L$ supplying a product that is characterized by two quality dimensions s and v . Quality dimension s represents conventional product quality such as the cleaning performance or the noise of a vacuum cleaner. Quality dimension v is environmental quality, which is assumed to be not observable and not verifiable for consumers.

In equilibrium, both firms choose to differentiate their quality levels to mitigate competition. One firm is a high quality firm and the other is a low quality firm. Assume without loss of generality, $s_H > s_L$ and $v_H > v_L$.

The production technology is characterized by variable cost, which is convex in quality and linear in quantity. For simplicity, we assume that the cost of quality improving is symmetric for quality dimensions s or v . Different cost functions for both quality dimensions would not result in qualitatively different results. Therefore, our results for the effect of a minimum quality standard and/or labeling do not depend on different cost functions for both quality dimensions. We discuss the effects of different cost functions in sections 5.

In section 5, we discuss alternative cost functions. The cost function is given as

$$C_i = \left(\frac{1}{2}s_i^2 + \frac{1}{2}v_i^2 \right) q_i. \quad (1)$$

We assume that the provision of quality entails no fixed cost for firms, but rather variable cost, similar to Motta (1993) and Crampes & Hollander (1995). Fixed cost of quality stem from quantity independent features like the design of the product or R&D investment. Variable cost of quality improvements may be related to higher quality materials or more complex production processes. For the list of products mentioned above, like electrical household appliances or insulation products, variable cost of quality improvements seem to be more relevant, as an enhanced quality level requires more complex production processes and higher quality materials.

Consumers differ in their preference for quality in both dimensions θ , which is uniformly distributed on the interval $[a, b]$ ¹. The heterogeneity in preference parameter θ may be interpreted as differences in income, in taste, or in frequency of usage. The way we model consumer heterogeneity implies that a consumer who is highly interested in the observable quality dimension is also highly interested in the unobservable quality dimension. Hence, θ captures an encompassing preference for quality.

Each consumer buys at most one unit of the most preferred good. We assume that s and v are substitutes for consumers if v is made observable by labeling. That is, consumers care about the energy efficiency of products and are willing to accept lower levels of s (e.g. higher noise of a vacuum cleaner) if they are compensated by higher levels of v . The assumption that both quality dimensions are perfect substitutes is a special case. However, our results are qualitatively similar for other utility functions, see our discussion of the effects of different utility functions in section 5.

The utility derived from no purchase is zero, while a consumer who buys one unit of the good at price p obtains a net utility of

$$U = \theta (s_i + \delta v_i) - p_i, \quad i = H, L, \quad (2)$$

where $\delta = 1$ if the consumer is informed about v by a label, and $\delta = 0$ otherwise.

The marginal consumer indifferent between purchasing the high quality good and the low quality good is given by $\theta^* = \frac{p_H - p_L}{(s_H + \delta v_H) - (s_L + \delta v_L)}$. Without compulsory labeling, consumers cannot observe the quality dimension v . Disregarding the quality dimension v for unlabeled goods simplifies the expression above to $\theta^* = \frac{p_H - p_L}{s_H - s_L}$. Demand for the good of firm H and the good of firm L , respectively, is given by

$$q_H = b - \frac{p_H - p_L}{(s_H + \delta v_H) - (s_L + \delta v_L)}, \quad q_L = \frac{p_H - p_L}{(s_H + \delta v_H) - (s_L + \delta v_L)} - a. \quad (3)$$

Firms' profits are given by

$$\pi_i = (p_i - (s_i^2 + v_i^2)) q_i. \quad (4)$$

After the government has decided whether to apply a minimum quality standard, a labeling scheme, or both instruments in combination, competition follows a two-stage game: In the first stage, firms choose quality levels. In the second stage, firms compete in prices.

¹ Assume $b = a + 1$ and $b \geq \frac{9}{4}$ to guarantee equilibrium existence.

3 Regulatory Scenarios

3.1 No Regulation

Consider first a system with no government intervention. Without regulation, the quality dimension v is not observable for consumers and is not relevant for their purchase decision. Table 1 presents prices, quantities, quality levels, and firm profits.

	H	L
price p	$\frac{8b(2b+1)+25}{32}$	$\frac{8b(2b-5)+49}{32}$
quantity q	$\frac{1}{2}$	$\frac{1}{2}$
quality level s	$\frac{4b+1}{4}$	$\frac{4b-5}{4}$
quality level v	0	0
profit π	$\frac{3}{8}$	$\frac{3}{8}$

Table 1: No Regulation. Prices, Quantities, Quality Levels, and Profits

Firms have no incentive to invest in v and set the quality level to $v_H = v_L = 0$. This is equivalent to the no regulation results of Ecchia & Lambertini (1997). In equilibrium, firms set quality levels s_H and s_L with $s_H > s_L$. Both quality levels increase in the maximum willingness to pay b .

Firms set equilibrium prices p_H and p_L . Both prices increase in the maximum willingness to pay b . Market shares and profits are identical for both firms because the model is symmetric.

3.2 Minimum Quality Standard

Now assume that the government introduces a minimum quality standard $V^\mu > 0$ with respect to quality dimension v . In the European Union, this scenario applies to computers, set-top boxes, and hot-water boilers.² As the minimum quality standard has no effect on the visibility (or relevance) for consumers v remains irrelevant for the purchase decision of consumers. Table 2 presents prices, quantities, quality levels, and firm profits. Firms have no incentive to invest in v more than necessary and set $v_H^\mu = v_L^\mu = V^\mu$. The minimum quality standard changes the quality level for of the non-observable quality dimension of high quality products and low quality products. But consumers cannot observe this change, and accordingly, quality competition between firms is not affected by the minimum quality standard. This is why equilibrium quality levels s_H , s_L are not

²https://ec.europa.eu/growth/single-market/european-standards/harmonised-standards/ecodesign_en.

	H	L
p	$\frac{8b(2b+1)+25+16V^2}{32}$	$\frac{8b(2b-5)+49+16V^2}{32}$
q	$\frac{1}{2}$	$\frac{1}{2}$
s	$\frac{(4b+1)}{4}$	$\frac{(4b-5)}{4}$
v	V^μ	V^μ
π	$\frac{3}{8}$	$\frac{3}{8}$

Table 2: Minimum Quality Standard on v . Prices, Quantities, Quality Levels, and Profits

affected by the minimum quality standard V and are the same as under no regulation: $s_H^\mu = s_H$, $s_L^\mu = s_L$. Since consumers cannot observe v , but only s , the quality of the product remains unchanged in their view. The degree of product differentiation in the market remains the same.

Firms set prices p_H^μ and p_L^μ . Both prices are higher than under no regulation ($p_H^\mu > p_H$, $p_L^\mu > p_L$) and increase in V^μ . Firms pass on the cost of investment in quality dimension v completely to consumers, i.e. $p_i^\mu = p_i + c_i(v_i)$.

Quantities are the same as under no regulation, the market remains symmetric ($q_H^\mu = q_L^\mu = q_H = q_L$).

Proposition 1 summarizes the effect of a minimum quality standard.

Proposition 1 *Suppose a minimum quality standard on quality dimension v is introduced. Then the minimum quality standard i) has no impact on quality dimension s , ii) increases both prices, and iii) has no impact on market shares.*

3.3 Labeling

Now consider that the government imposes a labeling scheme with respect to v that informs consumers about the level of this quality dimension. This scenario used to be the case for many household appliances and light bulbs prior to the introduction of the Ecodesign Directive. Firms remain free to choose v (no minimum quality standard applies), but now this quality dimension is visible and relevant for consumers. Table 3 presents prices, quantities, quality levels, and firm profits.

As v is now visible to consumers, they take v into account in making their purchase decision – and so do firms in their decision on quality levels. Both firms increase their quality levels for v ($v_H^\lambda > v_H$, $v_L^\lambda > v_H$). Since v and s are perfect substitutes for consumers and they cause identical convex productions cost, firms choose identical levels

	H	L
p	$\frac{8b(2b+1)+25}{16}$	$\frac{8b(2b-5)+49}{16}$
q	$\frac{1}{2}$	$\frac{1}{2}$
s	$\frac{(4b+1)}{4}$	$\frac{(4b-5)}{4}$
v	$\frac{(4b+1)}{4}$	$\frac{(4b-5)}{4}$
π	$\frac{3}{4}$	$\frac{3}{4}$

Table 3: Labeling. Prices, Quantities, Quality Levels, and Profits

for both dimensions $s_H^\lambda = v_H^\lambda$ and $s_L^\lambda = v_L^\lambda$. Under labeling, the quality level of s is the same as under no regulation and as under the minimum quality standard ($s_H^\lambda = s_H^\mu = s_H$, $s_L^\lambda = s_L^\mu = s_L$). This now visible quality dimension v does not change the preference for dimension s , which remains unaffected by v_i^λ .

Firms set prices p_H^λ and p_L^λ . Both prices are higher than under no regulation ($p_H^\lambda > p_H$, $p_L^\lambda > p_L$). From the perspective of consumers, the introduction of the labeling scheme adds a new quality dimension for which they have a positive willingness to pay. But the introduction of the labeling scheme does not alter the strategic interaction of both firms in principle. They now compete in an additional quality dimension, which is a perfect substitute to the conventional one. Quantities under compulsory labeling are the same as under no regulation ($q_H^\mu = q_H = q_L = q_L^\mu$). Profits of both firms increase, because they are now able to compete in a second quality dimension and to skim off their willingness to pay for quality.

Compared to the minimum quality standard, we can distinguish three cases: If $V^\mu < \frac{4b-5}{4}$, both firms choose a higher quality level for v under labeling than under the minimum quality standard ($v_H^\lambda > v_H^\mu$, $v_L^\lambda > v_L^\mu$). If $\frac{4b-5}{4} < V^\mu < \frac{4b+1}{4}$, the high quality firm chooses a higher quality level for v under labeling compared to the minimum quality standard ($v_H^\lambda > v_H^\mu$), the low quality firm chooses a lower quality level under labeling compared to the minimum quality standard ($v_L^\lambda < v_L^\mu$). If $V^\mu > \frac{4b+1}{4}$, both firms choose lower quality levels under labeling compared to the minimum quality standard ($v_H^\lambda < v_H^\mu$, $v_L^\lambda < v_L^\mu$). If $V^\mu > \frac{2b-1}{2}$ the average quality level of v is lower under labeling than under the minimum quality standard.

Compared to the minimum quality standard, the price of the high quality product is lower under labeling if the minimum quality level exceeds a threshold level of \widetilde{V}_H^μ ($p_H^\lambda < p_H^\mu$ if $V^\mu > \widetilde{V}_H^\mu = \sqrt{\frac{8b(2b+1)+25}{16}}$). The price of the low quality product is higher under labeling compared to the minimum quality standard if the minimum quality standard exceeds a threshold level of \widetilde{V}_L^μ ($p_L^\lambda > p_L^\mu$ if $V^\mu > \widetilde{V}_L^\mu = \sqrt{\frac{8b(2b-5)+49}{16}}$). This is, which

instrument raises prices by more, depends on the level of the minimum quality standard.

Market shares are the same under the minimum quality standard and the labeling scheme. The market remains symmetric.

Proposition 2 summarizes the effect of a labeling scheme.

Proposition 2 *Suppose a labeling scheme that informs consumers about quality dimension v is introduced. Then compared to no regulation the labeling scheme i) has no impact on quality dimension s , ii) increases both prices, and iii) has no impact on market shares.*

3.4 Combined Scheme – Minimum Standard and Labeling

Now assume that the government simultaneously imposes a minimum quality standard $V^{\mu\lambda} > 0$ and a labeling scheme with respect to v . In the EU, the combined scheme applies e.g. for air conditioners, dishwashers, lamps, televisions, and vacuum cleaners. Assume that the minimum quality standard is set according to $\underline{V^{\mu\lambda}} = \frac{4b-5}{4} \leq V^{\mu\lambda} \leq \overline{V^{\mu\lambda}} = \frac{2b+3\sqrt{2}-4}{2}$ to ensure that it is binding for the low quality firm compared to labeling only and to ensure that the low quality firm stays in the market.³ Table 4 presents prices, quantities, quality levels, and firm profits. The combination of both instruments has an

	H	L
p	$\frac{4\sqrt{2}\Psi(5V+4b-2)+16b(2b+7)+125+4V(17V+2b-37)}{108}$	$\frac{2\sqrt{2}\Psi(8V+b-5)+4V(19V+7b-44)+4b(b-1)+217}{108}$
q	$\frac{4(4b-4V+7-\sqrt{2}\Psi)}{36}$	$\frac{4(\sqrt{2}\Psi-4b+4V+2)}{36}$
s	$\frac{4V+2b-1+\sqrt{2}\Psi}{6}$	$\frac{4V+2b-10+\sqrt{2}\Psi}{6}$
v	$\frac{4V+2b-1+\sqrt{2}\Psi}{6}$	V
π	$\frac{(2(V(2V-4b-7)+b(2b+7)+23)-2\sqrt{2}\Psi(V-b+\frac{1}{2}))^2}{972(\frac{1}{2}\sqrt{2}\Psi+4-V+b)}$	$\frac{(2(2V(V-2b+1)+2b(b-1)-13)-2\sqrt{2}\Psi(V-b+5))^2}{972(\frac{1}{2}\sqrt{2}\Psi+4-V+b)}$

Table 4: Combined Scheme. Prices, Quantities, Quality Levels, and Profits, with $\Psi = \sqrt{2V(V-2b-8)+2b(b+8)+5}$

effect on vertical product differentiation. Both quality dimensions are now visible and relevant for consumers. In addition, firms have to comply with the minimum quality standard $V^{\mu\lambda}$ and are therefore restricted in choosing quality levels $v_i^{\mu\lambda}$. The L -firm sets v_L in accordance with the minimum quality standard $V^{\mu\lambda}$. Note that both quality levels are strategic substitutes between firms, so that an increase in one quality dimension by

³Note that if the minimum quality standard V exceeds a critical level of $V^{\max} = \frac{4b-1}{4} > \overline{V^{\mu\lambda}}$ the combined scheme collapses to the case of the minimum quality standard only, where both firms set the invisible quality dimension according to $V^{\mu\lambda}$ and choose their optimal quality levels for the visible quality dimension.

one firm results in an increase in the same quality dimension by the other firm. This is, the increase in v_L also raises v_H .

Both firms choose higher quality levels for both quality dimensions compared to no regulation ($s_H^{\mu\lambda} > s_H, s_L^{\mu\lambda} > s_L, v_H^{\mu\lambda} > v_H, v_L^{\mu\lambda} > v_L$). The minimum quality standard drives increases in the quality level of the hidden quality v . In addition, v is visible to consumers, so that quality differentiation in this dimension is attractive for both firms. Increasing marginal cost of quality improvement and the substitutability of both quality dimensions also induce an increase in the unregulated visible quality dimension s for both firms.

The equilibrium prices of both firms are higher than under no regulation ($p_H^{\mu\lambda} > p_H, p_L^{\mu\lambda} > p_L$).

Compared to no regulation, to the minimum quality standard, and to labeling, the high quality firm sells less under the combined scheme, the low quality firm sells more ($q_H^{\mu\lambda} < q_H = q_H^\mu = q_H^\lambda, q_L^{\mu\lambda} > q_L = q_L^\mu = q_L^\lambda$). The increase in quality of the low quality product and the price increases motivate some consumers to shift their demand from the high quality product to the low quality product.

Compared to the minimum quality standard, both firms choose higher quality levels for the visible quality dimension s under the combined scheme ($s_H^{\mu\lambda} > s_H^\mu, s_L^{\mu\lambda} > s_L^\mu$). Compared to the minimum quality standard, we distinguish two cases: If the minimum quality standard under the combined scheme is the same as under the minimum quality standard only ($V^{\mu\lambda} = V^\mu$), the high quality firm chooses a higher quality level on the quality dimension v compared to the minimum quality standard only ($v_H^{\mu\lambda} > v_H^\mu$). The low quality firm sets the same level of v under both the combined scheme and the minimum quality standard only ($v_L^{\mu\lambda} = v_L^\mu$). If the minimum quality standard under the combined scheme differs from the minimum quality standard used as the only instrument ($V^{\mu\lambda} \neq V^\mu$) both firms may set higher or lower levels of v ($v_H^{\mu\lambda} \geq v_H^\mu, v_L^{\mu\lambda} \geq v_L^\mu$). The regulator is unrestricted in imposing a minimum quality standard if this is the only instrument he applies. If combined with compulsory labeling, the regulator is restricted in the level of the minimum quality standard (see above) in order to guarantee that the minimum quality level is binding and that the low quality firm stays in the market. Therefore he may choose different minimum quality levels under both regimes.

Compared to the minimum quality standard only, prices of both firms are higher ($p_H^{\mu\lambda} > p_H^\mu, p_L^{\mu\lambda} > p_L^\mu$).

Compared to labeling only, both firms choose higher quality levels under the combined scheme for both quality dimensions ($v_H^{\mu\lambda} > v_H^\lambda, v_L^{\mu\lambda} > v_L^\lambda; s_H^{\mu\lambda} > s_H^\lambda, s_L^{\mu\lambda} > s_L^\lambda$).

The prices of the high quality product and the low quality product under the com-

bined scheme are higher than the prices under labeling only ($p_H^{\mu\lambda} > p_H^\lambda, p_L^{\mu\lambda} > p_L^\lambda$). This is a result of the higher quality level under the combined scheme than under labeling only.

Proposition 3 summarizes the effect of the combination of a minimum standard and a labeling scheme.

Proposition 3 *Suppose that the combination of a minimum standard and a labeling scheme is introduced. Compared to no regulation, a minimum quality standard (with an identical level of V), or labeling, this i) increases the quality levels of quality dimension s for both firms, ii) increases both prices, and iii) decreases the market share of the high quality firm and increases demand for the low quality firm.*

4 Welfare

Consider social cost ϕ that may be reduced by an enhanced quality level of products. This social cost may be caused by a negative externality such as harmful emissions of power generation stemming from a low level of v . The cost of the externality is given as $R = \phi - q_H v_H - q_L v_L$. We assume a constant marginal damage, which is quite common in the literature.⁴ The quality dimension v decreases the level of the externality. Energy efficiency is an example, where reduced energy consumption by products decrease external effects of energy production i.e. emissions of coal fired power plants or radioactive radiation of nuclear power plants.

Without any regulation with respect to v both firms set their quality level to $v_H = v_L = 0$, so $R = \phi$.

The regulator aims to maximize social welfare that is given as the sum of profits, consumer surplus, and the cost of the externality R ($W = \pi_H + \pi_L + CS - R$).

4.1 Minimum Quality Standard

If the government introduces a minimum quality standard on v , but no compulsory labeling, both firms set $v_H^\mu = v_L^\mu = V^\mu$. Consumer surplus, profits and the cost of the externality can be found in the Appendix. Compared to no regulation, the profit remains the same for both firms as the cost for investment in quality dimension v is passed on to consumers ($\pi_H^\mu = \pi_H, \pi_L^\mu = \pi_L$). Consumer surplus decreases under the minimum quality

⁴Assuming increasing marginal damage would place a greater weight on v_L and v_H and would result in a higher value of V under the minimum quality standard. This, in turn, would result in higher marginal cost and higher prices. Results would be qualitatively similar.

standard because of higher prices ($CS^\mu < CS$). The cost of the externality decreases under the minimum quality standard, because both firms increase the quality level of v ($R^\mu < R$). Social welfare is lower under the minimum quality standard ($W^\mu < W$).

Proposition 4 summarizes the effect of a minimum quality standard on social welfare.

Proposition 4 *Suppose a minimum quality standard V^μ is introduced. Then compared to no regulation i) profits are the same for both firms, ii) consumer surplus is lower, iii) the cost of the externality is lower, and iv) social welfare is lower.*

4.2 Labeling

Consumers are now informed about the quality dimension v . From their perspective, products are now characterized by an additional quality component for which they have an additional willingness to pay. Consumer surplus, profits and the cost of the externality can be found in the Appendix. As a result, profits for both firms double as they compete along a second quality dimension ($\pi_H^\lambda > \pi_H$, $\pi_L^\lambda > \pi_L$). Also consumer surplus doubles compared to no regulation ($CS^\lambda > CS$). The benefit from additional quality is higher than the negative effect from higher prices. Compared to no regulation, the cost of the externality decreases ($R^\lambda < R$). Social welfare increases compared to no regulation ($W^\lambda > W$).

Compared to the minimum quality standard, profits of both firms are higher under labeling ($\pi_H^\lambda > \pi_H^\mu$, $\pi_L^\lambda > \pi_L^\mu$). Under labeling, firms compete along two quality dimensions, whereas they compete only along one dimension under a minimum quality standard. Consumer surplus is higher under labeling than under the minimum quality standard only ($CS^\lambda > CS^\mu$). Under a minimum quality standard, consumers do not realize the increase in v . They only notice the price increase. Under labeling, they also realize the price increase, but in addition, total quality increases in their point of view. The cost of the externality is lower under labeling than under the minimum quality standard if the minimum quality standard is sufficiently low ($V^\mu < \frac{2b-1}{2}$, $R^\lambda < R^\mu$). Under this condition, the average quality under labeling exceeds the average quality under labeling and reduces social cost by more. Social welfare is higher under labeling than under the minimum quality standard ($W^\lambda > W^\mu$).

Proposition 5 summarizes the effect of a labeling scheme on social welfare.

Proposition 5 *Suppose a compulsory labeling scheme for quality dimension v is introduced. Then compared to no regulation i) profits of both firms are higher, ii) consumer surplus is higher, iii) the cost of the externality is lower, iv) social welfare is higher.*

Compared to the minimum quality standard, v) profits for both firms are higher, vi) consumer surplus is higher, vii) the cost of the externality is lower if the minimum quality standard V^μ is sufficiently low, viii) social welfare is higher under labeling.

4.3 Combined Scheme – Minimum Standard and Labeling

Consider now a combined scheme of a minimum quality standard and labeling (consumer surplus, profits and the cost of the externality can be found in the Appendix).

Compared to no regulation and to the minimum quality standard, the profit for the high quality firm is higher under the combined scheme if the minimum quality standard is sufficiently low ($\pi_H^{\mu\lambda} > \pi_H^\mu = \pi_H$, if $V^{\mu\lambda} < \widetilde{V}_\pi^\mu$, see Appendix). The profit of the low quality firm is higher compared to no regulation and to the minimum quality standard ($\pi_L^{\mu\lambda} > \pi_L^\mu = \pi_L$).

Compared to the labeling scheme, the profit of both the high quality firm is lower $\pi_H^{\mu\lambda} < \pi_H^\lambda$. The profit of the low quality firm is higher under the combined scheme than under labeling only if the minimum quality standard is sufficiently low ($\pi_L^{\mu\lambda} > \pi_L^\lambda$, if $V^{\mu\lambda} < \widetilde{V}_\pi^\lambda$, see Appendix).

On the one hand, the labeling-component of the combined scheme provides the firms with the possibility to extend competition to a second quality dimension. On the other hand, the minimum quality standard-component of the combined scheme distorts quality choices in both dimensions and increases cost.

The cost of the externality is lower under the combined scheme compared to the minimum quality standard only ($R^{\mu\lambda} < R^\mu$). It is lower than under labeling only if the minimum quality standard is sufficiently high ($R^{\mu\lambda} < R^\lambda$, if $V^{\mu\lambda} > \widetilde{V}_R$, see Appendix).

Consumer surplus is higher under the combined scheme than under no regulation, the minimum quality standard, and labeling only, ($CS^{\mu\lambda} > CS, CS^\mu, CS^\lambda$). The effect from higher quality levels exceeds the negative effect of higher prices.

Social welfare is higher under combined scheme than under no regulation, the minimum quality standard, and labeling only. The combination of both instruments is able to address two market failures: It addresses the negative externality via the minimum quality standard and the information asymmetry between firms and consumers.

Proposition 6 summarizes the effect of the combination of both instruments on welfare.

Proposition 6 *Suppose a combined scheme with labeling and a minimum quality standard for quality dimension v is introduced. Compared to no regulation i) the profit of the*

high quality firm is higher if the minimum quality standard is sufficiently low, the profit of the low quality firm is higher; ii) consumer surplus is higher, iii) decreases the cost of the externality, and iv) increases social welfare.

Compared to the minimum quality standard only, v) profit of the high quality firm is higher if the minimum quality standard is sufficiently low, the profit of the low quality firm is higher, vi) consumer surplus is higher, vii) the cost of the externality is lower, viii) total welfare is higher.

Compared to labeling only ix) the profit of the high quality firm is lower, the profit of the low quality firm is higher, if the minimum quality standard is sufficiently low, x) consumer surplus is higher, xi) the cost of the externality is lower if the minimum quality standard is sufficiently high, xii) total welfare higher.

5 Robustness

In this section, we discuss the robustness of our results with respect to the specifications of our model.

5.1 Utility Function

In our utility function, both quality dimensions are perfect substitutes. Consumers differ in their willingness to pay for both quality dimensions, but each consumer has the same willingness to pay for both quality dimensions. Although this is a plausible assumption for many cases, for instance if differences in θ are interpreted as differences in the general preference for quality, also other cases seem plausible.

Without labeling, consumers do not take the non-observable quality dimension v into account. Under no regulation as well as under the minimum quality standard utility depends only on the visible quality dimension s . Therefore, the effect of a minimum quality standard is independent of the specific functional form of utility.

Under labeling, we obtain similar results for different functional forms of the utility functions: $U = \theta(s + v)$, $U = \theta(sv)$, $U = \theta \min(s, v)$. The qualitative effect of labeling is independent of whether quality dimensions are perfect substitutes ($U = \theta(s + v)$), (imperfect) substitutes ($U = \theta(sv)$), or complements ($U = \theta \min(s, v)$). Firms differentiate quality levels for both quality dimensions $s_H = v_H > s_L = v_L$. This is, firms differentiate products along both dimensions to relax competition. Symmetric cost for both quality dimensions results in identical quality levels for both dimensions for each firm. This implies that (total) product differentiation is higher than for one quality dimension

only.

Also the effect of the combined scheme is similar under these utility functions. If quality dimensions are (perfect) substitutes or complements, firms set quality levels $s_H = v_H > v_L = V > s_L$ ($U = \theta(s + v)$, or $U = \theta(sv)$) or $s_H > v_H > v_L = V = s_L$ ($U = \theta \min(s, v)$). This is, firms sustain product differentiation also under the combined scheme, but the minimum quality standard under this combined scheme determines the lower quality bound and creates an asymmetry in quality levels if both dimensions are substitutes.

If consumers are heterogenous with respect to one quality dimension only ($U = \theta s + v$ or $U = \theta v + s$), firms differentiate quality levels only along the quality dimension for which consumers have different preferences under labeling and the combined scheme: $s_H > s_L$ and $v_H = v_L = V$ or $v_H > v_L = V$ and $s_H = s_L$.

5.2 Cost of Quality Increases

We assume a cost function that is linear in quantity and convex in quality $C_i = (\frac{1}{2}s_i^2 + \frac{1}{2}v_i^2) q_i$. We think that this assumption is plausible for many products covered by the Ecodesign Directive and the Energy Labeling Directive, because, e.g. an led light is more complex than a light bulb and needs more expensive components.

Alternatively, a cost function of the form $C_i = (\frac{1}{2}s_i^2 + \frac{1}{2}v_i^2 + \gamma s_i v_i) q_i$ could be assumed. $\gamma > 0$ ($\gamma < 0$) would imply diseconomies of scope (economies of scope). Diseconomies of scope (economies of scope) would result in lower (higher) quality levels of s_i if a minimum quality standard, labeling or the combination of both instruments increases the levels of v_i .

In addition, cost of quality increases could differ for both quality dimensions: $C_i = (\frac{1}{2}s_i^2 + \frac{\gamma}{2}v_i^2) q_i$. If $\gamma > 1$ ($\gamma < 1$), quality increases in v_i are more (less) expensive than quality increases in s_i . This would generate asymmetric quality levels for each firm, but product differentiation would be maintained. Results would be qualitatively similar to our analysis.

The analysis with the assumption of fixed cost of quality improvements requires a setting where the market is uncovered. In a covered market, fixed cost of quality improvement induce no strategic effect of quality choice by firms. In our setting of a covered market, we abstract from overall demand effects induced by a minimum quality standard or a labeling scheme. While the analysis of demand effects could be of some interest, demand effects for many products covered by the Ecodesign Directive and the Labeling Directive may be limited. Usually, consumers buy a new vacuum cleaner or a

new light if the old one is broken. This is why we assume a covered market and take only variable cost of quality improvement into account.

5.3 Damage Function

We assume a linear damage induced by the externality. In the literature, linear or convex damage functions are common (see for instance Petrakis & Xepapadeas, 1998). In our paper, the damage function is only relevant for the welfare analysis, because the minimum quality standard is set exogenously. A convex damage function would make a reduction of the externality by increasing the quality level v even more important for the regulator than under the linear damage function we use. Therefore, an increase in v , induced by a minimum quality standard, labeling or the combination of both instruments, would increase total welfare by more than under a linear damage function.

6 Conclusion

In this paper we have studied the effect of a minimum quality standard, a compulsory labeling scheme, and the combination of both instruments in a vertical differentiation model when environmental quality is not observable for consumers if it is not indicated by label. We have assumed variable cost of quality improvement. This may be an appropriate assumption for many products that are regulated by the Ecodesign Directive and the Labeling Directive of the EU.

Both a minimum quality standard on the non-observable quality dimension and a compulsory labeling scheme that informs consumers about the non-observable quality dimension increase prices, but have no effect on the observable quality dimension and market shares. The combination of a minimum standard and a labeling scheme, affects investment in quality dimension s , increases prices, shifts demand from the H -firm to the L -firm. This means that the combination of two instruments – which do not affect competition if applied separately – may have an effect on competition by increasing the market share of the low quality product.

The minimum quality standard is intended to address the negative externality, labeling addresses the information asymmetry between firms and consumers. The combination of both instruments results in an unintended side effect: It increases the quality of the unregulated quality dimension s for both firms. Thereby, it increases the market share of the low quality firm.

In our setting, an increase in s is not accompanied by negative side-effects such as a

negative externality. If s is assumed to cause negative externalities (i.e. bigger and more comfortable cars use more fuel), the combined scheme would increase this externality.

The combined scheme results in an increased market share for the low quality product. If high quality products result in co-benefits such as higher safety and security, the government could be interested in stimulating a higher market share of high quality products. The combined scheme would then counteract this policy goal by implying a higher market share of the low quality product.

Consumer surplus and social welfare are higher under the combination of both instruments than under no regulation or the use of one instrument only. This indicates that two instruments are better than one in addressing two problems: asymmetric information and a negative externality.

The results of this paper are based on a special assumption with respect to the production technology: Firms are able to set the quality levels of both dimensions s and v independently. In many cases alternative production technologies may be more realistic, where quality levels of both dimensions are interdependent, e.g. an increase in v may be associated with a decrease of s or an increase of the cost of s . In this case, an increase of V may lead to a decrease of the visible quality dimension s . The transition from the traditional light bulb to energy saving lamps may be an example: While the energy efficiency increased tremendously, the light quality decreased in the perspective of many consumers. Modern dishwashers use less water and less energy than old ones, but need more time to clean the dishes. An analysis of the effects of alternative production technologies seems to be a promising topic for future research.

We have limited our analysis to a duopolistic setting. Having more than two firms in the market could possibly lead to different results of a minimum quality standard, labeling, and a combination thereof (see Scarpa, 1988 for the analysis of a minimum quality standard in a setting with more than two firms). This is also a topic for further research.

References

- [1] Baltzer, K. (2012). Standards vs. Labels with Imperfect Competition and Asymmetric Information. *Economic Letters* 113, 61-63.
- [2] Bernard, S. (2016). Design choices and environmental policies. Cirano Working Paper No. 2016s-09.
- [3] Bonroy, O. & Constantatos, C. (2014). On the Economics of Labels: How their Introduction Affects the Functioning of Markets and the Welfare of All Participants. *American Journal of Agricultural Economics* 97, 239-259.
- [4] Brécard, D. (2011). Environmental tax in a green market. *Environmental and resource economics*, 49(3), 387-403.
- [5] Chen, C. (2001). Design for the environment: A quality-based model for green product development. *Management Science* 47 250-263.
- [6] Crampes, C. & Hollander, A. (1995). Duopoly and Quality Standards. *European Economic Review* 39, 71-82.
- [7] European Commission (2013). Attitudes of Europeans towards Building the Single Market for Green Products. Flash Eurobarometer 367.
- [8] Ecchia, G. & Lambertini, L. (1997). Minimum Quality Standards and Collusion. *The Journal of Industrial Economics* 65, 101-113.
- [9] Fulton, M. & Giannakas, K. (2004). Inserting GM products into the Food Chain: The Market and Welfare Effects of Different Labeling and Regulatory Regimes. *American Journal of Agricultural Economics* 86, 42-60.
- [10] Lusk, Jayson L. & Marette, Stéphane (2012). Can Labeling and Information Policies Harm Consumers?. *Journal of Agricultural & Food Industrial Organization* 10, 1-15.
- [11] Motta, M. (1993). Endogenous Quality Choice. Price versus Quantity Competition. *Journal of Industrial Economics* 41, 113-131.
- [12] Mussa, M. & Rosen, S. (1978). Monopoly and Product Quality. *Journal of Economic Theory* 18, 301-317.
- [13] Petrakis, E. & Xepapadeas, A. (1998). Does Government Precommitment Promote Environmental Innovation. mimeo.

- [14] Roe, B.E., Teisl, M.F. & Deans, C.R. (2014). The Economics of Voluntary Versus Mandatory Labels. *Annual Review of Resource Economics* 6, 1-21.
- [15] Ronnen, U. (1991). Minimum Quality Standards, Fixed Costs, and the Competition. *RAND Journal of Economics* 22, 490-504.
- [16] Scarpa, C. (1998). Minimum quality standards with more than two firm. *International Journal of Industrial Organization* 16, 665-676.
- [17] Schmeiser, S. (2014). Consumer Influence and the Regulation of Consumer Information. *International Journal of Industrial Organization* 37, 192-200.
- [18] Tirole, Jean. (1988). *The theory of industrial organization*. Cambridge, Massachusetts.
- [19] Zago, A.M. & Pick, D (2004). Labeling Policies in Food Markets: Private Incentives, Public Intervention, and Welfare Effects. *Journal of Agricultural and Resource Economics* 29, 150-165.

A Appendix

A.1 No regulation

$$\begin{aligned}
s_H &= \frac{(4b+1)}{4}, s_L = \frac{(4b-5)}{4} \\
\frac{\partial s_H}{\partial b} &> 0, \frac{\partial s_L}{\partial b} > 0 \\
p_H &= \frac{8b(2b+1)+25}{32}, p_L = \frac{8b(2b-5)+49}{32} \\
\frac{\partial p_H}{\partial b} &> 0, \frac{\partial p_L}{\partial b} > 0 \\
q_H &= q_L = \frac{1}{2} \\
\pi_H &= \pi_L = \frac{3}{8}
\end{aligned}$$

A.2 Minimum quality standard on v

$$\begin{aligned}
s_H^\mu &= \frac{(4b+1)}{4}, s_L^\mu = \frac{(4b-5)}{4} \\
p_H^\mu &= \frac{8b(2b+1)+25+16V^{\mu 2}}{32}, p_L^\mu = \frac{8b(2b-5)+49+16V^{\mu 2}}{32} \\
p_H^\mu &> p_H, p_L^\mu > p_L \\
\frac{\partial p_H^\mu}{\partial V^\mu} &> 0, \frac{\partial p_L^\mu}{\partial V^\mu} > 0 \\
q_H^\mu &= q_L^\mu = \frac{1}{2} \\
\pi_H^\mu &= \pi_L^\mu = \frac{3}{8}
\end{aligned}$$

A.3 Labeling

$$\begin{aligned}
s_H^\lambda &= v_H^\lambda = \frac{(4b+1)}{4}, s_L^\lambda = v_L^\lambda = \frac{(4b-5)}{4} \\
p_H^\lambda &= \frac{8b(2b+1)+25}{16}, p_L^\lambda = \frac{8b(2b-5)+49}{16} \\
\pi_H^\lambda &= \pi_L^\lambda = \frac{3}{4}
\end{aligned}$$

A.4 Combined Scheme

$$\begin{aligned}
s_H^{\mu\lambda} &= v_H^{\mu\lambda} = \frac{4V+2b-1+\sqrt{2}\sqrt{2V^{\mu\lambda}(V^{\mu\lambda}-2b-8)+2b(b+8)+5}}{6}, \\
s_L^{\mu\lambda} &= \frac{4V+2b-10+\sqrt{2}\sqrt{2V^{\mu\lambda}(V^{\mu\lambda}-2b-8)+2b(b+8)+5}}{6}, v_L^{\mu\lambda} = V^{\mu\lambda} \\
p_H^{\mu\lambda} &= \frac{4\sqrt{2}\sqrt{2V^{\mu\lambda}(V^{\mu\lambda}-2b-8)+2b(b+8)+5}(5V^{\mu\lambda}+4b-2)+16b(2b+7)+125+4V^{\mu\lambda}(17V^{\mu\lambda}+2b-37)}{108} \\
p_L^{\mu\lambda} &= \frac{2\sqrt{2}\sqrt{2V^{\mu\lambda}(V^{\mu\lambda}-2b-8)+2b(b+8)+5}(8V^{\mu\lambda}+b-5)+4V^{\mu\lambda}(19V^{\mu\lambda}+7b-44)+4b(b-1)+217}{108} \\
q_H^{\mu\lambda} &= \frac{4(4b-4V^{\mu\lambda}+7-\sqrt{2}\sqrt{2V^{\mu\lambda}(V^{\mu\lambda}-2b-8)+2b(b+8)+5})}{36} \\
q_L^{\mu\lambda} &= \frac{4(\sqrt{2}\sqrt{2V^{\mu\lambda}(V^{\mu\lambda}-2b-8)+2b(b+8)+5}-4b+4V^{\mu\lambda}+2)}{36} \\
\pi_H^{\mu\lambda} &= \frac{\left(2(V^{\mu\lambda}(2V^{\mu\lambda}-4b-7)+b(2b+7)+23)-2\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda 2}+2b^2+5}(V^{\mu\lambda}-b+\frac{1}{2})\right)^2}{972\left(\frac{1}{2}\sqrt{2}\sqrt{-2V^{\mu\lambda}(-V^{\mu\lambda}+2b+8)+2b(b+8)+5}+4-V+b\right)}
\end{aligned}$$

$$\pi_L^{\mu\lambda} = \frac{\left(2(2V^{\mu\lambda}(V^{\mu\lambda}-2b+1)+2b(b-1)-13)-2\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5}(V^{\mu\lambda}-b+5)\right)^2}{972\left(\frac{1}{2}\sqrt{2}\sqrt{-2V^{\mu\lambda}(-V^{\mu\lambda}+2b+8)+2b(b+8)+5+4-V^{\mu\lambda}+b}\right)}$$

A.5 Welfare

A.5.1 No Regulation

$$CS = \frac{16b(b-1)-23}{32}.$$

$$W = \pi_H + \pi_L + CS - R = \frac{16b(b-1)+1}{32} - \phi.$$

A.5.2 Minimum Quality Standard

$$R = \phi - V^\mu$$

$$CS = \frac{16b(b-1)-23-16V^{\mu^2}}{32}.$$

$$W = \frac{16b(b-1)+1-16V^\mu(V^\mu+2)}{32} - \phi.$$

A.5.3 Labeling

$$R = \phi + \frac{1}{2} - b$$

$$CS = \frac{16b(b-1)-23}{16}$$

$$W = \frac{(16b^2-7)}{16} - \phi$$

A.5.4 Combined Scheme

\widetilde{V}_π^μ is the solution to

$$f(V^{\mu\lambda}) = \frac{1}{2}\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5}\frac{27}{4}(4V^{\mu\lambda}-4b-8V^{\mu\lambda}b+4V^{\mu\lambda}2+4b^2-53)$$

$$-\frac{27}{32}(1176V^{\mu\lambda}-1176b+96V^{\mu\lambda}b^2-96V^{\mu\lambda}2b+192V^{\mu\lambda}b-96V^{\mu\lambda}2+32V^{\mu\lambda}3-96b^2-32b^3-599)$$

$\widetilde{V}_\pi^\lambda$ is the solution to

$$g(V^{\mu\lambda}) = \frac{1}{2}\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5}\frac{27(20V^{\mu\lambda}-20b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+23)}{2}$$

$$-\frac{27}{16}(408V^{\mu\lambda}-408b+48V^{\mu\lambda}b^2-48V^{\mu\lambda}2b-192V^{\mu\lambda}b+96V^{\mu\lambda}2+16V^{\mu\lambda}3+96b^2-16b^3+425)$$

$$R = \phi - \frac{2\left((4V^{\mu\lambda}(V^{\mu\lambda}-2b+19)+2b(2b-11)-17)-2\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5}(V^{\mu\lambda}-b-4)\right)}{108}.$$

\widetilde{V}_R is the solution to

$$h(V^{\mu\lambda}) = \frac{1}{2}(38V^{\mu\lambda}-38b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5)$$

$$-\frac{1}{2}\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5}(V^{\mu\lambda}-4-b)$$

$$CS = \frac{4\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda}2+2b^2+5}(V^{\mu\lambda}(V^{\mu\lambda}-2b-35)+b(b+35)-11)}{972}$$

$$-\frac{(4V^{\mu\lambda}(165V^{\mu\lambda}-330b-6V^{\mu\lambda}b+2V^{\mu\lambda}2+6b^2-120)-4b(78b+2b^2-363)+811)}{972}$$

$$W = \frac{54\sqrt{2}\sqrt{-16V^{\mu\lambda}+16b-4V^{\mu\lambda}b+2V^{\mu\lambda^2}+2b^2+5(2V^{\mu\lambda}(5V^{\mu\lambda}-10b-22)+2b(5b+22)+43)}}{26244} - \frac{27(4V^{\mu\lambda}(V^{\mu\lambda}(10V^{\mu\lambda}-30b+159)+6b(5b-53)-213)-4b(84b+10b^2-213)+149)}{26244} - \phi$$