# The Dark Side of Tax Breaks for Foreigners 

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Following Mirrlees (1982), the literature has focused on tax competition between Nation States, considering that, in a given country, all tax residents face the same income tax schedule (see, e.g., Blumkin, Sadka, Shem-Tov, 2014; Simula and Trannoy, 2010 and 2012; as well as Lehmann, Simula and Trannoy, 2015). We in contrast allow for the possibility of tax differentiation based on citizenship, in each country, and investigate the impact on well-being from a national as well as from an international perspective. Can it be optimal to rely on such a tax differentiation? Or should it be avoided? To investigate this issue, we use a world consisting of two (non-necessarily symmetric) countries. Individuals differ with respect to three dimensions of heterogeneity: native country, skill and cost of migration. The distribution of types is common knowledge. However, the skill and migration cost of a given individual are private information. In each country, a benevolent policymaker aims at redistributing incomes from the rich to the poor people. Following Mirrlees (1971), the government is unable to observe the skill nor the cost of migration of a given individual. In addition, the latter can only levy taxes on residents. However, contrary to Lehmann, Simula and Trannoy (2014) and the previous literature, it is not constrained to treat natives and foreigners in the same way. Some form of ?tagging? (Akerlof, 1978), based on citizenship, is thus allowed. This radically changes the problem each government faces. Indeed, given these circumstances, each policymaker designs two income tax schedules, one for the native residents and one for the resident expatriates. We first characterize the tax schedules of each country in a Nash equilibrium, assuming that skills (but not migration costs) can be observed. This intermediary situation, referred to as the ?Tiebout best? in Lehmann, Simula and Trannoy (2014), will give us insights into the second best, in which skills as well as migration costs are private information. We then turn to the characterization of the second best. We contrast the equilibrium schedules and profiles of well-being with those obtained when each country is not allowed to implement tax breaks for foreigners. Numerical illustrations, calibrated using US and Danish data, are then provided, to quantify the reduction in social welfare resulting from the introduction of tax breaks for foreigners. JEL Codes: D82, H21, H87, F22.

[^0]
## I. INTRODUCTION

The globalization process has not only made the mobility of capital easier. The transmission of ideas, meanings and values across national borders associated with the decrease in transportation costs has also reduced the barriers to international labor mobility. In this context, individuals are more likely to vote with their feet in response to high income taxes. This is in particular the case for highly skilled workers. The latter represent about $35 \%$ of the OECD immigration stock, for only $11.3 \%$ of the world labour force. In this context, a highly skilled is six times more likely to emigrate than a low-skilled agent Docquier and Marfouk (2005). There are still few empirical studies estimating the migration responses to taxation; however, they all suggest that highly skilled are very sensitive to tax differences when choosing where to locate (cf. Liebig et al. (2007), Kleven et al. (2013) and Kleven et al. (2014)). Consequently, the possibility of tax-driven migrations appears as an important policy issue and must be taken into account as a salient constraint when thinking about the design of taxes and benefits affecting households.

In this context, we observe two trends regarding top income tax rates payable to central governments: a tendency to fall down until the financial crisis of 2007-2008 and the introduction, and development in the last few years, of specific tax cuts for highly skilled or rich foreigners. These tax breaks for foreigners exist in all Scandinavian countries, in Germany, in the Netherlands, in Belgium, in France, in the UK, in Switzerland, but also in Australia or in Canada, etc. In addition, they have recently been extended in France ("impatriates" can benefit from it during 8 years, instead of 3 years) or in the UK (to avoid the departure of expatriates after the "Brexit").

Following Mirrlees (1982), the literature has focused on tax competition between Nation States, considering that, in a given country, all tax residents face the same income tax schedule (see, e.g., Blumkin et al. (2012); Simula and Trannoy (2010); as well as Lehmann et al. (2014)). We in contrast allow for the possibility of tax differentiation based on citizenship, in each country, and investigate the impact on well-being from a national as well as from an international perspective. Can it be optimal to rely on such tax differentiation? Or should it be avoided? Our ambition is to determine whether and to which extent this form of tax differentiation is detrimental in terms of social welfare when it is implemented in a variety of countries. If the results of our analysis confirm our preliminary investigations and intuition, our research would cast light on the non-sustainability of tax breaks for highly skilled foreigners and, thus, call for a change of direction regarding ongoing public policy practices.

To investigate this issue, we use a world consisting of two (non-necessarily symmetric) countries. Individuals differ with respect to three dimensions of heterogeneity: native country, skill and cost of migration. The distribution of types is common knowledge. However, the skill and migration cost of a given individual are private information. In each country, a benevolent policymaker aims at redistributing incomes from the rich to the poor people. Following Mirrlees (1971), the government is unable to observe the skill nor the cost of migration of a given individual. In addition, the latter can only levy taxes on residents. However, contrary to Lehmann, Simula and Trannoy (2014) and the previous literature, it is not constrained to treat natives and foreigners in the same
way. Some form of "tagging" (Akerlof, 1978), based on citizenship, is thus allowed. This radically changes the problem each government faces. Indeed, given these circumstances, each policymaker designs two income tax schedules, one for the native residents and one for the resident expatriates. Given that our focus is on tax-driven migrations, we consider that the production technology is the same in each country and exhibits constant returns to scale. Hence, each individual's wage is equal to his/her productivity and does not vary in case of migration. We also consider that each agent is constrained to consume and work in a same country. In addition, we consider that, in each country, the policymaker does not account for the well-being of the expatriates in the social welfare function. In other words, expatriates are mainly regarded (by the State) as net taxpayers, i.e., as agents from which taxes can be collected. This assumption can be motivated by the actual deterioration of government balance in a large number of developed countries. Our analysis proceeds in steps. We first characterize the tax schedules of each country in a Nash equilibrium, assuming that skills (but not migration costs) can be observed. This intermediary situation, referred to as the "Tiebout best" in Lehmann et al. (2014), will give us insights into the second best, in which skills as well as migration costs are private information. We then turn to the characterization of the second best. We contrast the equilibrium schedules and profiles of well-being with those obtained when each country is not allowed to implement tax breaks for foreigners.

## II. A MODEL ALLOWING FOR TAX BREAKS ON FOREIGNERS

We consider an economy consisting of two countries, indexed by $i=\{A, B\}$. The same constant return to scales technology is available in both countries. Each worker is characterized by three characteristics: her native country $i \in\{A, B\}$, her productivity (or skill) $w \in\left[w_{0}, w_{1}\right]$, and the migration cost $m \in \mathbb{R}^{+}$she supports if she decides to live abroad. Note that $w_{1}$ may be either finite or infinite, $w_{0}$ is nonnegative, and any agent's productivity $w$ is unaltered in case of migration. In addition, the empirical evidence that some people are immobile is captured by the possibility of infinitely large migration costs. This in particular implies that there will always be a mass of natives of skills $w$ in each country. The migration cost corresponds to a loss in utility, due to various material and psychic costs of moving: application fees, transportation of persons and household's goods, forgone earnings, costs of speaking a different language and adapting to another culture, costs of leaving one's family and friends, and so on. We do not make any restriction on the correlation between skills and migration costs. We simply consider that there is a distribution of migration costs for each possible skill level.

In country $i=\{A, B\}$, we denote by $h_{i}(w)$ the continuous skill density, by $H_{i}(w) \equiv \int_{w_{0}}^{w} h_{i}(w) d x$ the corresponding cumulative distribution function (CDF) and by $N_{i}$ the size of the population. For each skill $w, g_{i}(m \mid w)$ denotes the conditional density of the migration cost and $G_{i}(m \mid w) \equiv$ $\int_{0}^{m} g_{i}(x \mid w) d x$ the conditional CDF. The initial joint density of $(m, w)$ is thus $g_{i}(m \mid w) h_{i}(w)$ while $G_{i}(m \mid w) h_{i}(w)$ is the initial mass of individuals of skill $w$ with migration costs lower than $m$.

Following Mirrlees (1971), the government does not observe individual types ( $w, m$ ). We refer
to this information setting as the "second best"; we will also examine alternative information settings, but with the only aim of illuminating second-best allocations. In addition, each policymaker can only levy taxes on residents. However, contrary to Lehmann, Simula and Trannoy (2014), the latter is not constrained to treat natives and foreigners in the same way. Some form of tagging, based on citizenship, is thus allowed. Given these circumstances, each policymaker designs two income tax schedules, for the native residents on the one hand and for the resident expatriates on the other hand.

## II.1. Individual Choices

Every worker derives utility from consumption $c$, and disutility from effort and migration, if any. Effort captures the quantity as well as the intensity of labor supply. The choice of effort corresponds to an intensive margin and the migration choice to an extensive margin. Let $v(y ; w)$ be the disutility of a worker of skill $w$ to obtain pre-tax earnings $y \geq 0$ with $v_{y}^{\prime}>0>v_{w}^{\prime}$ and $v_{y y}^{\prime \prime}>0>v_{y w}^{\prime \prime}$. Let $\mathbb{1}$ be equal to 1 if she decides to migrate, and to zero otherwise. Individual preferences are described by the quasi-linear utility function:

$$
\begin{equation*}
c-v(y ; w)-\mathbb{1} \cdot m \tag{1}
\end{equation*}
$$

Note that the Spence-Mirrlees single-crossing condition holds because $v_{y w}^{\prime \prime}<0$. The quasi-linearity in consumption implies that there is no income effect on taxable income and appears as a reasonable approximation.

## Intensive Margin

We focus on income tax competition while allowing each country to implement tax differentiation between native and foreign residents. Every native living in country $i$ is liable to an income tax $T_{i}^{N}(\cdot)$; every foreign resident faces the income $\operatorname{tax} T_{i}^{M}(\cdot)$, where the superscript $j=\{N, M\}$ stands for "natives" and "mercenaries" respectively. Hence, an agent of skill $w$, who has chosen to work in country $i$, solves:

$$
\begin{equation*}
U_{i}^{j}(w) \equiv \max _{y} y-T_{i}^{j}(y)-v(y ; w) \tag{2}
\end{equation*}
$$

We refer to $U_{i}^{j}(w)$ as the corresponding gross utility. It is the net utility level for a native and the utility level absent migration cost for a mercenary. We call $Y_{i}^{j}(w)$ the solution to program (2) and $C_{i}^{j}(w)=Y_{i}^{j}(w)-T_{i}^{j}\left(Y_{i}^{j}(w)\right)$ the agent's consumption level. ${ }^{1}$ The first-order condition can be written as:

$$
\begin{equation*}
1-\operatorname{MTR}_{i}^{j}\left(Y_{i}^{j}(w)\right)=v_{y}^{\prime}\left(Y_{i}^{j}(w) ; w\right) \tag{3}
\end{equation*}
$$

[^1]where $\operatorname{MTR}_{i}^{j}\left(Y_{i}^{j}(w)\right) \equiv d T_{i}^{j}\left(Y_{i}^{j}(w)\right) / d Y_{i}^{j}$ denotes the marginal tax rate. Differentiating (3), we obtain the elasticity of gross earnings with respect to the retention rate $1-M T R_{i}^{j}$,
\[

$$
\begin{equation*}
\varepsilon_{i}^{j}(w) \equiv \frac{1-M T R_{i}^{j}\left(Y_{i}^{j}(w)\right)}{Y_{i}^{j}(w)} \frac{\partial Y_{i}^{j}(w)}{\partial\left(1-M T R_{i}^{j}\left(Y_{i}^{j}(w)\right)\right.}=\frac{v_{y}^{\prime}\left(Y_{i}^{j}(w) ; w\right)}{Y_{i}^{j}(w) v_{y y}^{\prime \prime}\left(Y_{i}^{j}(w) ; w\right)} \tag{4}
\end{equation*}
$$

\]

and the elasticity of gross earnings with respect to productivity $w$ :

$$
\begin{equation*}
\alpha_{i}^{j}(w) \equiv \frac{w}{Y_{i}^{j}(w)} \frac{\partial Y_{i}^{j}(w)}{\partial w}=-\frac{w v_{y w}^{\prime \prime}\left(Y_{i}^{j}(w) ; w\right)}{Y_{i}(w) v_{y y}^{\prime \prime}\left(Y_{i}^{j}(w) ; w\right)} \tag{5}
\end{equation*}
$$

## The Migration Margin

A native of country $i$, of type $(w, m)$, gets utility $U_{i}^{N}(w)$ in country $i$ and utility $U_{-i}^{M}(w)-m$ in country $-i$. She therefore emigrates if and only if $U_{-i}^{M}(w)-m>U_{i}^{N}(w)$, i.e., when $m<U_{-i}^{M}(w)-$ $U_{i}^{N}(w)$. We define:

$$
\begin{equation*}
\Delta_{i} \equiv U_{i}^{N}(w)-U_{-i}^{M}(w) \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta_{-i} \equiv U_{-i}^{N}(w)-U_{i}^{M}(w) \tag{7}
\end{equation*}
$$

The "number" of natives with skill $w$ staying in country $i$ is given by:

$$
\varphi_{i}^{N}\left(\Delta_{i} ; w\right) \equiv \begin{cases}h_{i}(w) N_{i} & \text { if } \Delta_{i} \geq 0  \tag{8}\\ \left(1-G_{i}\left(-\Delta_{i} \mid w\right)\right) h_{i}(w) N_{i} & \text { if } \Delta_{i}<0\end{cases}
$$

which is increasing in $\Delta_{i}$. Because migration costs have support $\mathbb{R}_{+}$, we know that $\varphi_{i}^{N}\left(\Delta_{i} ; w\right)>0$ at every $w$ : there is always a positive measure of natives of skill $w$. The "number" of foreigners in country $i$ ("tax mercenaries") is obtained as:

$$
\varphi_{i}^{M}\left(\Delta_{-i} ; w\right) \equiv \begin{cases}0 & \text { if } \Delta_{-i} \geq 0  \tag{9}\\ G_{-i}\left(-\Delta_{-i} \mid w\right) h_{-i}(w) N_{-i} & \text { if } \Delta_{-i}<0\end{cases}
$$

This non-negative quantity decreases in $\Delta_{-i}$ for $\Delta_{-i}<0$. Combining natives and foreigners, there are altogether $\varphi_{i}^{N}\left(\Delta_{i} ; w\right)+\varphi_{i}^{M}\left(\Delta_{-i} ; w\right)$ agents of skill $w$ staying in country $i$, and a total population of $\int_{w_{0}}^{w_{1}}\left[\varphi_{i}^{N}\left(\Delta_{i} ; w\right)+\varphi_{i}^{M}\left(\Delta_{-i} ; w\right)\right] d w$ agents in this country.

It is useful to summarize responses along the migration margin in terms of semi-elasticities.

When $\Delta_{i}<0$ at a given $w$, we define the semi-elasticity of emigration as:

$$
\begin{equation*}
\eta_{i}^{N}\left(\Delta_{i} ; w\right) \equiv \frac{\partial \varphi_{i}^{N}\left(\Delta_{i} ; w\right)}{\partial \Delta_{i}} \frac{1}{\varphi_{i}^{N}\left(\Delta_{i} ; w\right)}=\frac{g_{i}\left(-\Delta_{i} \mid w\right)}{1-G_{i}\left(-\Delta_{i} \mid w\right)} . \tag{10}
\end{equation*}
$$

We otherwise set $\eta_{i}^{N}\left(\Delta_{i} ; w\right)=0$. Because of quasi-linearity in consumption, this semi-elasticity corresponds to the percentage change in the density of native taxpayers with skill $w$ when their consumption $C_{i}^{N}(w)$ is increased at the margin. When $\Delta_{-i}<0$ at a given $w$, we introduce the semi-elasticity of expatriation from country $-i$ to country $i$ to summarize extensive responses by tax mercenaries; we denote it by:

$$
\begin{equation*}
\eta_{i}^{M}\left(\Delta_{-i} ; w\right) \equiv \frac{\partial \varphi_{i}^{M}\left(\Delta_{-i} ; w\right)}{\partial \Delta_{-i}} \frac{1}{\varphi_{i}^{M}\left(\Delta_{-i} ; w\right)}=-\frac{g_{-i}\left(-\Delta_{-i} \mid w\right)}{G_{-i}\left(-\Delta_{-i} \mid w\right)} . \tag{11}
\end{equation*}
$$

We otherwise set $\eta_{i}^{M}\left(\Delta_{-i} ; w\right)=0$. At skill $w$, the semi-elasticity of expatriation corresponds to the percentage change in the density of foreign taxpayers in country $i$ when their consumption $C_{i}^{M}(w)$ is increased at the margin.

## II.2. Governments

In country $i=\{A, B\}$, a benevolent policymaker designs the tax system to maximize the welfare of the worst-off natives ("maximin"). The maximin tax policy is the most redistributive one given the set of natives, as it corresponds to an infinite aversion to income inequality. The focus on the worst-off is primarily introduced to simplify the technical analysis. Alternatively, we could have considered a weighted sum of the indirect utilities of the natives, irrespective of their country of residence, or of the resident natives only. In the second case, the set of agents whose welfare is to count would be endogenous. This would raise further difficulties.

What is crucial to our setting is the idea that foreigners are regarded by each government as a potential source of tax receipts, which can be extracted through the implementation of a specific tax scheme. Whatever the tax schedule abroad, the assumption that the cost of migration has support $\mathbb{R}^{+}$ensures that a positive measure of natives always stay in country $i$ at every skill level $w$, thus preventing an "empty jurisdiction" issue skill by skill. This moreover corresponds to the empirical observation that there are no holes in the distribution of skills in a given country. The same reasoning does not apply to the set of tax mercenaries. In all generality, this set may either be empty, an interval, or the union of several intervals.

As mentioned earlier, in each country, the tax system consists of two non-linear income tax schedules, $T_{i}^{N}$ and $T_{i}^{M}$, offered to the native and foreign residents respectively. The budget con-
straint faced by country $i$ 's government is therefore:

$$
\begin{array}{r}
\int_{w_{0}}^{w_{1}} T_{i}^{N}\left(Y_{i}^{N}(w)\right) \varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right) d w \\
+\int_{w_{0}}^{w_{1}} T_{i}^{M}\left(Y_{i}^{M}(w)\right) \varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right) d w \geq E \tag{12}
\end{array}
$$

where $E \geq 0$ is an exogenous amount of public expenditures to finance. For simplicity, we concentrate on purely redistributive tax policies, in which case $E=0$.

## III. BEST RESPONSES

## III.1. Definition

We start with the characterization of each policymaker's best response. Because a taxpayer interacts with only one policymaker at the same time, it is easy to show that the standard taxation principle holds. Hence, in each country, it is equivalent to choose the non-linear income taxes $T_{i}^{N}$ and $T_{i}^{M}$, taking individual choices into account, or to directly select an allocation satisfying the incentive-compatible constraints

$$
\begin{equation*}
C_{i}^{j}(w)-v\left(Y_{i}^{j}(w) ; w\right) \geq C_{i}^{j}(x)-v\left(Y_{i}^{j}(x) ; w\right) \tag{13}
\end{equation*}
$$

for every $(w, x) \in\left[w_{0}, w_{1}\right]^{2}$ and $j=\{M, N\}$. Note that, because citizenship is observable, mimicking is impossible between native and foreign residents with a same skill level. Together with the single-crossing condition, the former implies that an allocation is incentive compatible if and only if:

$$
\begin{array}{r}
U_{i}^{N^{\prime}}(w)=-v_{w}^{\prime}\left(Y_{i}^{N}(w) ; w\right), \\
U_{i}^{M^{\prime}}(w)=-v_{w}^{\prime}\left(Y_{i}^{M}(w) ; w\right), \\
Y_{i}^{N}(w) \text { non-decreasing, } \\
Y_{i}^{M}(w) \text { non-decreasing. } \tag{17}
\end{array}
$$

Conditions (14) and (15) are the first-order conditions for incentive compatibility. Because of (14), the indirect utility is increasing in $w$ within the set of native residents. In addition, natives only choose to emigrate abroad if they get larger utility. Consequently, maximizing the utility of the worst-off natives amounts to setting $U_{i}^{N}\left(w_{0}\right)$ to its maximum value. Because of (15), the indirect utility is also increasing in $w$ within the set of "tax mercenaries". The monotonicity constraints (16) and (17) correspond to the second-order conditions for incentive compatibility. We below adopt the so-called "first-order approach" and neglect second-order conditions when deriving qualitative results. These conditions can be checked ex post when performing numerical simulations.

Because of the taxation principle, as explained above, choosing a tax policy $\left(T_{i}^{N}, T_{i}^{M}\right)$ is equiva-
lent to determining an incentive-compatible allocation $w \rightarrow\left(Y_{i}^{N}(w), U_{i}^{N}(w), Y_{i}^{M}(w), U_{i}^{M}(w)\right)$. The best-response allocation of government $i$ is therefore solution to:

$$
\begin{align*}
& \max _{\left\{U_{i}^{j}(w), Y_{i}^{j}(w)\right\}_{j=\{M, N\}}} U_{i}^{N}\left(w_{0}\right) \text { s.t. } U_{i}^{j \prime}(w)=-v_{w}^{\prime}\left(Y_{i}^{j}(w) ; w\right), j=\{N, M\} \text { and }  \tag{18}\\
& \int_{w_{0}}^{w_{1}}\left(Y_{i}^{N}(w)-v\left(Y_{i}^{N}(w) ; w\right)-U_{i}^{N}(w)\right) \varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right) d w \\
& +\int_{w_{0}}^{w_{1}}\left(Y_{i}^{M}(w)-v\left(Y_{i}^{M}(w) ; w\right)-U_{i}^{M}(w)\right) \varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right) d w \geq 0 .
\end{align*}
$$

## Optimality Conditions

We use the dual problem to characterize best responses, namely:

$$
\begin{align*}
& \max \int_{w_{0}}^{w_{1}}\left(Y_{i}^{N}(w)-v\left(Y_{i}^{N}(w) ; w\right)-U_{i}^{N}(w)\right) \varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right) d w  \tag{19}\\
& \quad+\int_{w_{0}}^{w_{1}}\left(Y_{i}^{M}(w)-v\left(Y_{i}^{M}(w) ; w\right)-U_{i}^{M}(w)\right) \varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right) d w \\
& \quad \text { s.t. } U_{i}^{j \prime}(w)=-v_{w}^{\prime}\left(Y_{i}^{j}(w) ; w\right), j=\{N, M\} \\
& \quad \text { and } U_{i}^{N}\left(w_{0}\right) \geq \underline{\mathrm{U}}_{i}^{N} .
\end{align*}
$$

In a given country, the objective function consists therefore of two separate parts: taxes collected from the National residents, on the one hand, and taxes collected from the Mercenaries, on the other hand. This situation is analogous to that encountered when a duopoly designing nonlinear price schedules offered to two distinct set of customers. In a best response, the price schedule offered to a set is independent to that offered to the other set of customers. This is through the best response of the other firm that there is a link between the two schedules a given firm designs.

When solving for optimization problem (19), we treat $Y_{i}^{N}(w)$ and $Y_{i}^{M}(w)$ as control variables, while $U_{i}^{N}(w)$ and $U_{i}^{M}(w)$ are state variables. Denoting the co-state variables associated with the incentive compatibility conditions (14) and (15) by $q_{i}^{N}(w)$ and $q_{i}^{M}(w)$ respectively, the Hamiltonian corresponding to Problem (19) may be written as:

$$
\begin{align*}
\mathcal{H}= & \left(Y_{i}^{N}(w)-v\left(Y_{i}^{N}(w) ; w\right)-U_{i}^{N}(w)\right) \varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right) \\
& +\left(Y_{i}^{M}(w)-v\left(Y_{i}^{M}(w) ; w\right)-U_{i}^{M}(w)\right) \varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right) \\
& -q_{i}^{N}(w) v_{w}^{\prime}\left(Y_{i}^{N}(w) ; w\right)-q_{i}^{M}(w) v_{w}^{\prime}\left(Y_{i}^{M}(w) ; w\right) \tag{20}
\end{align*}
$$

The following Proposition summarizes the associated optimality conditions.

Proposition 1. The optimality conditions are:

$$
\begin{gather*}
0=\left(1-v_{Y}^{\prime}\left(Y_{i}^{N}(w) ; w\right)\right) \varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right)-q_{i}^{N}(w) v_{y w}^{\prime \prime}\left(Y_{i}^{N}(w) ; w\right)  \tag{21}\\
0=\left(1-v_{Y}^{\prime}\left(Y_{i}^{M}(w) ; w\right)\right) \varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right)-q_{i}^{M}(w) v_{y w}^{\prime \prime}\left(Y_{i}^{M}(w) ; w\right)  \tag{22}\\
q_{i}^{\prime N}(w)=\varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right)-T_{i}^{N}\left(Y_{i}^{N}(w)\right) \frac{\partial \varphi_{i}^{N}\left(U_{i}^{N}(w)-U_{-i}^{M}(w) ; w\right)}{\partial U_{i}^{N}}  \tag{23}\\
q_{i}^{\prime M}(w)=\varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right)-T_{i}^{M}\left(Y_{i}^{M}(w)\right) \frac{\partial \varphi_{i}^{M}\left(U_{i}^{M}(w)-U_{-i}^{N}(w) ; w\right)}{\partial U_{i}^{M}}  \tag{24}\\
q_{i}^{N}\left(w_{0}\right) \leq 0  \tag{25}\\
q_{i}^{N}\left(w_{1}\right)=0 \text { when } w_{1}<\infty \text { and } q^{N}\left(w_{1}\right) \rightarrow 0 \text { when } w_{1} \rightarrow \infty  \tag{26}\\
q_{i}^{M}\left(w_{0}\right)=0  \tag{27}\\
q_{i}^{M}\left(w_{1}\right)=0 \text { when } w_{1}<\infty \text { and } q_{i}^{M}\left(w_{1}\right) \rightarrow 0 \text { when } w_{1} \rightarrow \infty \tag{28}
\end{gather*}
$$

## III.2. Best Response Taxation

In a best-response, $U_{-i}^{N}$ and $U_{-i}^{M}$ are exogenously given. Consequently, National taxation in country $i$ is determined by equations (21), (23), (25) and (26), while Mercenary taxation depends on equations (22), (24), (27) and (28). These two sets of equations are independent; it is therefore possible to decompose the best-response problem into two independent sub-problems: the first one determines best-response National Taxation and the second one best-response Mercenary Taxation. We now investigate these independent sub-problems in turn. ${ }^{2}$

## National Taxation

We now turn to the characterization of the tax schedule country $i$ offers to its national residents in a best response. Rearranging (21), we see that:

$$
\begin{equation*}
1-v_{Y}^{\prime}\left(Y_{i}^{N}(w) ; w\right)=\frac{q_{i}^{N}(w)}{\varphi_{i}^{N}\left(\Delta_{i} ; w\right)} v_{y w}^{\prime \prime}\left(Y_{i}^{N}(w) ; w\right) \tag{29}
\end{equation*}
$$

This expression is always well defined because $\varphi_{i}^{N}\left(\Delta_{i} ; w\right)>0$ at every $w$ as explained above. Combining the Euler condition (23) and the transversality condition (26), we obtain:

$$
\begin{align*}
q_{i}^{N}(w) & =q_{i}^{N}\left(w_{1}\right)-\int_{w}^{w_{1}} q_{i}^{\prime N}(x) d x \\
& =\int_{w}^{w_{1}}\left[T_{i}^{N}\left(Y_{i}^{N}(x)\right) \frac{\partial \varphi_{i}^{N}\left(\Delta_{i} ; x\right)}{\partial U_{i}^{N}}-\varphi_{i}^{N}\left(\Delta_{i} ; x\right)\right] d x . \tag{30}
\end{align*}
$$

[^2]We define:

$$
\begin{align*}
& X_{i}^{N}(w) \equiv \int_{w}^{w_{1}}\left[\varphi_{i}^{N}\left(\Delta_{i} ; x\right)-T_{i}^{N}\left(Y_{i}^{N}(x)\right) \frac{\partial \varphi_{i}^{N}\left(\Delta_{i} ; x\right)}{\partial U_{i}^{N}}\right] d x \\
& \quad=\int_{w}^{w_{1}}\left[1-\frac{T_{i}^{N}\left(Y_{i}^{N}(x)\right)}{\varphi_{i}^{N}\left(\Delta_{-i} ; x\right)} \frac{\partial \varphi_{i}^{N}\left(\Delta_{i} ; x\right)}{\partial U_{i}^{N}}\right] \varphi_{i}^{N}\left(\Delta_{i} ; x\right) d x \\
& =\int_{w}^{w_{1}}\left[1-T_{i}^{N}\left(Y_{i}^{N}(x)\right) \eta_{i}^{N}\left(\Delta_{i} ; x\right)\right] \varphi_{i}^{N}\left(\Delta_{i} ; x\right) d x \tag{31}
\end{align*}
$$

Note that, if $\Delta_{i}(x) \geq 0$ for any skill $x \geq w$, then $X_{i}^{N}(w)=N_{i}\left(1-H_{i}(w)\right)$. We can now substitute (31) into (29) and rearrange.

PROPOSITION 2. In a best response, marginal tax rates faced in country $i=\{A, B\}$ by National residents are given by:

$$
\begin{equation*}
\frac{\operatorname{MRT}_{i}^{N}\left(Y_{i}^{N}(w)\right)}{1-M T R_{i}^{N}\left(Y_{i}^{N}(w)\right)}=\frac{\alpha_{i}^{N}(w)}{\varepsilon_{i}^{N}(w)} \frac{X_{i}^{N}(w)}{w \varphi_{i}^{N}\left(\Delta_{i}(w) ; w\right)} \tag{32}
\end{equation*}
$$

## Mercenary Taxation

We now turn to the tax schedule country $i$ offers to the people it wants to attract as Mercenaries in a best response. Rearranging (22), we see that:

$$
\begin{equation*}
\left[1-v_{Y}^{\prime}\left(Y_{i}^{M}(w) ; w\right)\right] \varphi_{i}^{M}\left(-\Delta_{-i} ; w\right)=q_{i}^{M}(w) v_{y w}^{\prime \prime}\left(Y_{i}^{M}(w) ; w\right) \tag{33}
\end{equation*}
$$

Combining the Euler condition (24) and the transversality condition (28), we obtain:

$$
\begin{align*}
q_{i}^{M}(w) & =q_{i}^{M}\left(w_{1}\right)-\int_{w}^{w_{1}} q_{i}^{\prime M}(x) d x \\
& =\int_{w}^{w_{1}}\left[T_{i}^{M}\left(Y_{i}^{M}(x)\right) \frac{\partial \varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)}{\partial U_{i}^{M}}-\varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)\right] d x \tag{34}
\end{align*}
$$

We define:

$$
\begin{gather*}
X_{i}^{M}(w) \equiv \int_{w}^{w_{1}}\left[\varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)-T_{i}^{M}\left(Y_{i}^{M}(x)\right) \frac{\partial \varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)}{\partial U_{i}^{M}}\right] d x \\
=\int_{w}^{w_{1}}\left[1-\frac{T_{i}^{M}\left(Y_{i}^{M}(x)\right)}{\varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)} \frac{\partial \varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)}{\partial U_{i}^{M}}\right] \varphi_{i}^{M}\left(-\Delta_{-i} ; x\right) d x \\
=\int_{w}^{w_{1}}\left[1+\frac{T_{i}^{M}\left(Y_{i}^{M}(x)\right)}{\varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)} \frac{\partial \varphi_{i}^{M}\left(-\Delta_{-i} ; x\right)}{\partial \Delta_{-i}}\right] \varphi_{i}^{M}\left(-\Delta_{-i} ; x\right) d x \\
=\int_{w}^{w_{1}}\left[1+T_{i}^{M}\left(Y_{i}^{M}(x)\right) \eta_{i}^{M}\left(\Delta_{-i} ; x\right)\right] \varphi_{i}^{M}\left(\Delta_{-i} ; x\right) d x \tag{35}
\end{gather*}
$$

We can now substitute (35) into (33) and rearrange to obtain the following result.

Proposition 3. In a best response, marginal tax rates faced in country $i=\{A, B\}$ by foreign Mercenaries are given by:

$$
\begin{equation*}
\frac{\operatorname{MTR}_{i}^{M}\left(Y_{i}^{M}(w)\right)}{1-M T R_{i}^{M}\left(Y_{i}^{M}(w)\right)}=\frac{\alpha_{i}^{M}(w)}{\varepsilon_{i}^{M}(w)} \frac{X_{i}^{M}(w)}{w \varphi_{i}^{M}\left(\Delta_{-i}(w) ; w\right)} \text { if } \varphi_{i}^{M}\left(\Delta_{-i}(w) ; w\right)>0, \tag{36}
\end{equation*}
$$

and $\operatorname{MTR}_{i}^{M}\left(Y_{i}^{M}(w)\right)=+\infty$ otherwise.
Given (36), the marginal tax rates $M T R_{i}^{M}\left(Y_{i}^{M}(w)\right)$ faced by mercenaries in country $i$ are only well defined at $w$ for which some agents actually move from country $-i$ to country $i$ (i.e., $\varphi_{i}^{M}\left(\Delta_{-i}(w) ; w\right)>$ 0 ). At skill levels $w$ for which $\varphi_{i}^{M}\left(\Delta_{-i}(w) ; w\right)=0$ instead, it is not desirable for country $i$ to attract foreigners; without any loss in generality, we therefore set $M T R_{i}^{M}\left(Y_{i}^{M}(w)\right)=+\infty$ to capture this fact.

## IV. THE TIEBOUT BEST

To gain further insights into the second-best optimal tax policy, we investigate the situation in which each policymaker observes $w$ but is unable to recover the value of the migration cost $m$ faced by a given individual. We refer to this information setting as the "Tiebout best" in honor of Tiebout's seminal contribution to the analysis of migration in the field of public finance.

## IV.1. General Features of Any Nash Equilibrium

We first establish that, in any Nash equilibrium, Native and Foreign residents face different tax schedules.

Proposition 4. In the Tiebout best, at any Nash equilibrium, each country has an incentive to offer differentiated tax schedules to National and Foreign residents.

Proof The proof is by contradiction. Let us assume that there is a Nash equilibrium in which neither country uses tax differentiation. Hence, $\tilde{T}_{A}^{N}(w)=\tilde{T}_{A}^{M}(w)$ and $\tilde{T}_{B}^{N}(w)=\tilde{T}_{B}^{M}(w)$ at any $w$. Given the social objective in country $i$, there are at least some $w$ at which $\tilde{T}_{i}^{N}(w)<0$ and some other $w$ at which $\tilde{T}_{i}^{N}(w)>0, i=\{A, B\}$. There are therefore four cases to consider.
(i) $\tilde{T}_{i}^{N}(w)<0$ and $\tilde{T}_{-i}^{N}(w)<0$ at a given $w$ (possibly with $\tilde{T}_{i}^{N}(w)=\tilde{T}_{-i}^{N}(w)$ ). Setting $\tilde{T}_{k}^{M}(w)=0$ is clearly welfare improving in each country, $k=\{i,-i\}$. Hence, $\tilde{T}_{k}^{N}(w)=\tilde{T}_{k}^{M}(w)$ is not a best response.
(ii) $\tilde{T}_{i}^{N}(w)<0$ and $0<\tilde{T}_{-i}^{N}(w)$ at a given $w$. Then, the same argument as in Case 1 applies to country $i$.
(iii) $0<\tilde{T}_{i}^{N}(w)<\tilde{T}_{-i}^{N}(w)$ at a given $w$. Then, there are no foreign taxpayer of skill $w$ in country $-i$. Now let us consider that country $-i$ diminishes $\tilde{T}_{-i}^{M}(w)$ and set $\tilde{T}_{-i}^{M}(w)=\tilde{T}_{i}^{N}(w)-\epsilon$, with $0<\epsilon<\tilde{T}_{i}^{N}(w)$. This attracts a positive measure of taxpayers, moving from country $i$
to country $-i$, and increases collected taxes. This extra amount of taxes can be redistributed to the worst-off national in a lump-sum manner, which increases $U_{-i}^{N}\left(w_{0}\right)$. Hence, $\tilde{T}_{-i}^{N}(w)=$ $\tilde{T}_{-i}^{M}(w)$ is not a best response.
(iv) $0<\tilde{T}_{i}^{N}(w)=\tilde{T}_{-i}^{N}(w)$ at a given $w$. In that case, no taxpayer decides to move abroad. We can apply the same argument as in Case 1 to either country: given $\tilde{T}_{-i}^{N}(w)$, country $i$ has an incentive to deviate and offer $\tilde{T}_{i}^{M}(w)=\tilde{T}_{-i}^{N}(w)-\epsilon$, with $0<\epsilon<\tilde{T}_{-i}^{N}(w)$. Consequently, $\tilde{T}_{k}^{N}(w)=\tilde{T}_{k}^{M}(w)$ is not a best response, for $k=\{i,-i\}$.

## QED

We now provide additional features that must be verified by any Nash equilibrium.
Lemma 1. In the Tiebout best, and at any Nash equilibrium, the lowest skilled Natives of country $i$, with $w=w_{0}$, remain in country $i$.

Proof The lowest skilled Natives of country $i$ receive a net transfer in country $i$, i.e., $\tilde{T}_{i}^{N}\left(w_{0}\right)<0$. Mercenary taxation in country $-i$ proceeds from the maximization of tax receipts on the set of Mercenaries. Consequently, $\tilde{T}_{-i}^{M}\left(w_{0}\right)>0$. This implies: $U_{i}^{N}\left(w_{0}\right)>U_{-i}^{M}\left(w_{0}\right)$. QED

Lemma 2. If $\tilde{T}_{-i}^{N}$ is non-decreasing in $w$, or equivalently if $\eta_{i}^{N}$ is non-increasing in $w$, then $\varphi_{i}^{M}$ is non-decreasing in $w$.
Proof $\varphi_{i}^{M}$ is an implicit function of $\tilde{T}_{-i}^{N}$, with $\varphi_{i}^{M}=\varphi_{i}^{M}\left(\Delta_{-i}\left(\tilde{T}_{-i}^{N}\right)\right)$. By the chain rule, $\partial \varphi_{i}^{M} / \partial \tilde{T}_{-i}^{N}=$ $\left(\partial \varphi_{i}^{M} / \partial \Delta_{-i}\right) \times\left(\partial \Delta_{-i} / \partial \tilde{T}_{-i}^{N}\right)$. We already know that $\partial \varphi_{i}^{M} / \partial \Delta_{-i} \leq 0$. In addition, by definition of $\Delta_{i}$, we have:

$$
\begin{equation*}
\Delta_{-i}=Y_{-i}^{N}(w)-\tilde{T}_{-i}^{N}\left(Y_{-i}^{N}(w)\right)-v\left(Y_{-i}^{N}(w) ; w\right)-Y_{i}^{M}(w)+\tilde{T}_{i}^{M}\left(Y_{i}^{M}(w)\right)+v\left(Y_{i}^{M}(w) ; w\right) \tag{37}
\end{equation*}
$$

Therefore, $\partial \Delta_{-i} / \partial \tilde{T}_{-i}^{N}=-1$. Hence, $\partial \varphi_{i}^{M} / \partial \tilde{T}_{-i}^{N} \geq 0$. QED

Combining both Lemmas and Proposition 6, we obtain the following result:
Proposition 5. Let $\tilde{T}_{-i}^{N}$ be non-decreasing in $w$, or equivalently $\eta_{i}^{N}$ be non-increasing in $w$. Then, in country $i(i=\{A, B\})$, there exists a cutoff level $\hat{w}_{i}$, with $w_{0} \leq \hat{w}_{i}<+\infty$, such that:
(i) $\tilde{T}_{i}^{M}(w)=+\infty$ for $w<\hat{w}$;
(ii) $\tilde{T}_{i}^{M}(w)=\frac{G_{-i}\left(-\Delta_{-i}(w) \mid w\right)}{g_{-i}\left(-\Delta_{-i}(w) \mid w\right)}$ for $w>\hat{w}_{i}$.

Lemma 3. In the Tiebout best, any Nash equilibrium involves $\hat{w}=w_{0}$.
Proof Assume the is a Nash equilibrium for which $\varphi_{i}^{M}(w)=0$ for any $w>w_{0}$ and $i=\{A, B\}$. For every $w>w_{0}, \tilde{T}_{i}^{N}(w)>0$. Because $m$ has support $\mathbb{R}^{+}$, it is always possible for country $i$ to offer $\tilde{T}_{i}^{M}(w)=\tilde{T}_{-i}^{N}(w)-\epsilon$, with $\epsilon>0$, and attract additional net taxpayers in country $i$. This increases collected taxes. A contradiction. Therefore, $\varphi_{i}^{M}(w)=0$ for any $w>w_{0}$, implying $\hat{w}=w_{0}$. QED

## IV.2. Best-Response Taxation

In the Tiebout best, $\tilde{T}_{i}^{\prime j}\left(Y_{i}^{j}(w)\right)=0$ at each $w$, which implies $X^{j}(w)=0, j=\{M, N\}$. To show that the tax schedule directly depends on $w$, we below write $\tilde{T}_{i}^{\prime \prime}(w)$. Using (31) and (35), the following result is obtained.

Proposition 6. In the Tiebout best, and in a best response, tax liabilities in country $i=\{A, B\}$ are given by:

$$
\begin{equation*}
\tilde{T}_{i}^{N}(w)=\frac{1}{\eta_{i}^{N}\left(\Delta_{i} ; w\right)}=\frac{1-G_{i}\left(-\Delta_{i} \mid w\right)}{g_{i}\left(-\Delta_{i} \mid w\right)} \text { for every } w>w_{0} \tag{38}
\end{equation*}
$$

with an upward jump-discontinuity at $w=w_{0}$, and

$$
\tilde{T}_{i}^{M}(w)=-\frac{1}{\eta_{i}^{M}\left(\Delta_{-i}(w) ; w\right)}= \begin{cases}\frac{G_{-i}\left(-\Delta_{-i}(w) \mid w\right)}{g_{-i}\left(-\Delta_{-i}(w) \mid w\right)} & \text { if } \Delta_{-i}>0,  \tag{39}\\ +\infty & \text { otherwise. }{ }^{3}\end{cases}
$$

The tax liabilities paid in country $i$ by the National residents (with $w>w_{0}$ ) and the foreign Mercenaries are respectively equal to the inverse of the semi-elasticity of emigration and to the inverse of the semi-elasticity of expatriation. The National residents with $w=w_{0}$ then receive a lump-sum transfer which equally splits collected taxes among them. There is therefore a discontinuity in the Tiebout-best National tax schedule at $w_{0}$. By contrast, the Tiebout-best Mercenary schedule is continuous at every $w$ for which $\tilde{T}_{i}^{M}(w)$ is finite. When policymakers are not allowed to differentiate taxes based on citizenship, as in Lehmann et al. (2014), any agent of skill $w$ staying country $i$ pays a tax liability equal to the inverse of the average semi-elasticity of migration computed for National citizens as well as Foreigners.

## IV.3. Symmetric Countries

We now focus on the situation in which both countries are symmetric. If there is a Nash equilibrium, then it must be such that $\tilde{T}_{i}^{N}(w)=\tilde{T}_{-i}^{N}(w), \tilde{T}_{i}^{M}(w)=\tilde{T}_{-i}^{M}(w), \eta_{i}^{N}=\eta_{-i}^{N}$ and $\eta_{i}^{M}=\eta_{-i}^{M}$ at any $w$. We therefore in this subsection drop the $i$-subscripts. We moreover assume that $\eta^{N}$ is non-increasing in $w$, so that the characterization of Proposition 5 applies. To save on notations, we drop the $i$ and $-i$ subscripts in the rest of this subsection. Without any loss of generality, we let the size of the population in each country $N$ be normalized to 1 .

For any $w<\hat{w}$, there are no Mercenary, and thus $\Delta>0$. At any $w>\hat{w}$ instead, there are Mercenaries, and thus $\Delta<0$; moreover, it must be the case that $\tilde{T}^{M}(w)<\tilde{T}^{N}(w)$. Because preferences are quasilinear, linear in net income, we have: $\Delta(w)=U^{N}(w)-U^{M}(w)=T^{M}(w)-T^{N}(w)$. Choices made by the Natives of country $i$ can thus be summarized as follows:
(i) For $w<\hat{w}, \Delta(w)=T^{M}(w)-T^{N}(w)>0$; hence, all agents stay in their home country;
(ii) For $w>\hat{w}, \Delta(w)=T^{M}(w)-T^{N}(w)<0$ so that:
(ii.a) Agents with $m<T^{N}(w)-T^{M}(w)$ decide to become tax Mercenaries abroad;
(ii.b) Agents with $m>T^{N}(w)-T^{M}(w)$ stay in their home country.

## Illustrative Example

To gain further insights, we now specify the distribution of migration cost. For simplicity, we assume that it is described by an exponential distribution. More precisely, we consider $g(m \mid w)=$ $\lambda(w) \exp (-m \lambda(w))$ and $G(m \mid w)=1-\exp (-m \lambda(w))$, where $\lambda(w)>0$ is a $\mathcal{C}^{1}$ scale function. By definition of the semi-elasticity of emigration, we have: $\eta^{N}(w)=\lambda(w)$. In order satisfy the condition under which Proposition 5 applies, we need to verify $\lambda^{\prime}(w) \leq 0$. For illustrative purposes, we now consider $\lambda(w)=1 / w$.

- Using Proposition 6, we obtain:
(i) for any $w, \tilde{T}^{N}(w)=w$;
(ii) for $w>\hat{w}, \tilde{T}^{M}(w)=w \exp (-\Delta / w)[1-\exp (\Delta / w)]$.
- Therefore, for $w>\hat{w}, \Delta(w)=\tilde{T}^{M}(w)-\tilde{T}^{N}(w)$. Solving the latter equation for $\Delta$, we get $\Delta(w)=-0,442854 w$, from which $\tilde{T}^{M}(w)=0.557146 w$ and $\eta^{M}(w)=-1,79486 / w$.
- Given that $\Delta(w)<0$ for any $w, \hat{w}=w_{0}$. Substituting the different expressions into $\varphi^{M}$, the "number" of Mercenaries of skill $w$ is given by $\varphi^{M}(\Delta(w) ; w)=0,357799 h(w)$.

The $w_{0}$ natives always stay in their home country, where they receive a positive lump-sum transfer $\left|\tilde{T}^{N}\left(w_{0}\right)\right|$. In each country, $35,77 \%$ of the initial population -with the lowest migration costs- move abroad to benefit from the Mercenary tax schedule $\tilde{T}^{M}(w)$; the rest of the population stays at home and pays $\tilde{T}^{N}(w)$ in taxes.

Let us now consider that $w$ is uniformly distributed over $[0,10]$. In a given country, collected taxes from National residents amount to 1.789 and collected taxes from Mercenaries to 0.9967 . Therefore, 2.7857 are redistributed to the $w_{0}$ Native residents.

It is interesting to contrast these results with those obtained in a world in which each country's policymaker is unable to differentiate taxes between Native and Foreign residents. This situation was examined in Lehmann et al. (2014). Contrary to the present setting, no agent moves in the Nash equilibrium; each agents with $w>w_{0}$ pays taxes equal to $w$. The social budget thus amounts to 5 . Given the maximin objective, the introduction of tax breaks for foreigners therefore results in social welfare being reduced by about $45 \%$ in each country.

## V. THE SECOND BEST

To save on notations, we from now on denote the skill densities of taxpayers and the semi-elasticities in the Nash equilibrium by $f_{i, j}^{*} \equiv \varphi_{i}^{j}$ and $\eta_{i, j}^{*}$ respectively, $j=\{N, M\}$.

## V.1. National Taxation

The Tiebout-best tax schedule provides insights into the second-best solution, where both skills and migration costs are private information. Using (38), Equation (31) can be rewritten as:

$$
\begin{equation*}
X_{i}^{N}(w)=\int_{w}^{w_{1}}\left[\tilde{T}_{i}^{N}(w)-T_{i}^{N}\left(Y_{i}^{N}(x)\right)\right] \eta_{i, N}^{*}(x) f_{i, N}^{*}(x) d x \tag{40}
\end{equation*}
$$

Hence, for National residents, $X_{i}^{N}(w)$ is a weighted sum of the difference between the Tiebout-best tax liabilities and second-best tax liabilities for all skill levels $x$ above $w$. The weights are given by the product of the semi-elasticity of emigration (from country $i$ to country $-i$ ) and the skill density of country $i$ 's Native population, i.e. by the mass of pivotal Native individuals of skill $w$, who are indifferent between migrating or not.

In the Tiebout best, the mechanical and migration effects of a change in tax liabilities cancel out. Therefore, the Tiebout-best tax schedule defines a target for the policymaker in the second best, where distortions along the intensive margin have also to be minimized. The second-best solution thus proceeds from the reconciliation of three underlying forces: $i$ ) maximizing the welfare of the worst-off Native residents; $i$ i) being as close as possible to the Tiebout-best tax liability to limit the distortions stemming from the migration responses; iii) being as flat at possible to mitigate the distortions coming from the intensive margin. In the second-best, these three goals cannot be pursued independently because of the incentive constraints (14).

The following proposition is established in the Appendix, but we below provide graphs that cast light on the main intuitions.

Proposition 7. In a Nash equilibrium:
i) if $\eta_{i, N}^{* \prime}(\cdot)=0$, then $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right)>0$ and $T_{i}^{N}\left(Y_{i}^{N}(w)\right)<1 / \eta_{i, N}^{*}$ for all $w \in\left(w_{0}, w_{1}\right)$;
ii) if $\eta_{i, N}^{* \prime}(\cdot)<0$, then $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right)>0$ for all $w \in\left(w_{0}, w_{1}\right)$;
iii) if $\eta_{i, N}^{*,}(\cdot)>0$, then either
(a) $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right) \geq 0$ for all $w \in\left(w_{0}, w_{1}\right)$;
(b) or there exists a threshold $\breve{w} \in\left[w_{0}, w_{1}\right)$ such that $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right) \geq 0$ for all $w \in\left(w_{0}, \breve{w}\right)$ and $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right)<0$ for all $w \in\left(\breve{w}, w_{1}\right)$.
iv) if $\eta_{i, N}^{* \prime}(\cdot)>0$ and $\lim _{w \rightarrow \infty} \eta_{i, N}^{*}(w)=+\infty$, then there exists a threshold $\breve{w} \in\left(w_{0}, w_{1}\right)$ such that $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right) \geq 0$ for all $w \in\left(w_{0}, \breve{w}\right)$ and $\operatorname{MTR}_{i}^{N}\left(Y_{i}^{N}(w)\right)<0$ for all $w \in\left(\breve{w}, w_{1}\right)$.

Proof Because of the separability of the social planner's problem into two independ sub-problems, this proof is a direct adaptation of Proposition 3 in Lehmann et al. (2014). QED

This proposition casts light on the part played by the slope of the semi-elasticity of emigration. It considers the three natural benchmarks that come to mind when thinking about it. First, the
costs of migration may be independent of $w$ as in Blumkin et al. (2012) and Morelli et al. (2012), implying a constant semi-elasticity in a symmetric equilibrium. This makes sense, in particular, if most relocation costs are material (moving costs, flight tickets, etc.). ${ }^{4}$ Second, one might want to consider a constant elasticity of emigration, as in Brewer et al. (2010) and Piketty and Saez (2012). In this case, the semi-elasticity must be decreasing: if everyone receives one extra unit of consumption in country $i$, then the relative increase in the number of taxpayers becomes smaller for more skilled individuals. Third, the costs of migration may be decreasing in $w$. This seems to be supported by the empirical evidence that highly skilled are more likely to emigrate than low skilled (Docquier and Marfouk, 2006). This suggests that the semi-elasticity of emigration may be increasing in skills. A special case is investigated in Simula and Trannoy $(2010,2011)$, with a semi-elasticity equal to zero up to a threshold and infinite above.

## V.2. Mercenary Taxation

We now turn to the characterization of the second-best Mercenary tax schedule. We first establish that the poorest Nationals from country - $i$ never have an incentive to move abroad and settle down in country $i$. In other words, there is no Nash equilibrium with Mercenaries of skill $w_{0}$.

Lemma 4. In any Nash equilibrium, the lowest skilled Natives of country $i$, with $w=w_{0}$, remain in country $i$.

Proof The lowest skilled Natives of country $i$ receive a net transfer in country $i$, i.e., $T_{i}^{N}\left(Y_{i}^{N}\left(w_{0}\right)\right)<$ 0 . Mercenary taxation in country $-i$ proceeds from the maximization of tax receipts on the set of Mercenaries. Consequently, $T_{-i}^{M}\left(Y_{-i}^{M}\left(w_{0}\right)\right)>0$. Consequently, $U_{i}^{N}\left(w_{0}\right)>U_{-i}^{M}\left(w_{0}\right)$. QED

We now show that, if the National tax schedule offered in country $-i$ increases in skills, the proportion of Mercenaries in country $i$ increases in $w$.

Lemma 5. If $T_{-i}^{N}$ is non-decreasing in $w, \varphi_{i}^{M}$ is non-decreasing in $w$.
Proof $\varphi_{i}^{M}$ is an implicit function of $T_{-i}^{N}$, with $\varphi_{i}^{M}=\varphi_{i}^{M}\left(\Delta_{-i}\left(T_{-i}^{N}\right)\right)$. By the chain rule, $\partial \varphi_{i}^{M} / \partial T_{-i}^{N}=$ $\left(\partial \varphi_{i}^{M} / \partial \Delta_{-i}\right) \times\left(\partial \Delta_{-i} / \partial T_{-i}^{N}\right)$. We already know that $\partial \varphi_{i}^{M} / \partial \Delta_{-i} \leq 0$. In addition, by definition of $\Delta_{i}$, we have:

$$
\begin{equation*}
\Delta_{-i}=Y_{-i}^{N}(w)-T_{-i}^{N}\left(Y_{-i}^{N}(w)\right)-v\left(Y_{-i}^{N}(w) ; w\right)-Y_{i}^{M}(w)+T_{i}^{M}\left(Y_{i}^{M}(w)\right)+v\left(Y_{i}^{M}(w) ; w\right) . \tag{41}
\end{equation*}
$$

Therefore, $\partial \Delta_{-i} / \partial T_{-i}^{N}=-1$. Hence, $\partial \varphi_{i}^{M} / \partial T_{-i}^{N} \geq 0$. QED

[^3]This Lemma implies that, if $\varphi_{i}^{M}>0$ at a given $w^{\prime}$, then $\varphi_{i}^{M}>0$ at any $w>w^{\prime}$. As a result, in any Nash equilibrium, the population of Mercenaries in a given country is convex. Combining both Lemmas and Proposition 6, we obtain the following Proposition:

Proposition 8. If $T_{-i}^{N}$ is non-decreasing in $w$, there is a $\hat{w}$ such that:
(i) for $w<\hat{w}, f_{i, M}^{*}(w)=0, \eta_{i, M}^{*}=0$ and $T_{i}^{M}\left(Y_{i}^{M}(w)\right)=+\infty$;
(ii) for $w>\hat{w}, f_{i, M}^{*}(w)>0, f_{i, M}^{\prime *}(w)>0, \eta_{i, M}^{*}>0$ and

$$
\begin{equation*}
\frac{M T R_{i}^{M}\left(Y_{i}^{M}(w)\right)}{1-M T R_{i}^{M}\left(Y_{i}^{M}(w)\right)}=\frac{\alpha_{i}^{M}(w)}{\varepsilon_{i}^{M}(w)} \frac{X_{i}^{M}(w)}{w f_{i, M}^{*}(w)}<\infty \tag{42}
\end{equation*}
$$

with

$$
\begin{equation*}
X_{i}^{M}(w)=\int_{w}^{w_{1}}\left[T_{i}^{M}\left(Y_{i}^{N}(x)\right)-\hat{T}_{i}^{M}(x)\right] \eta_{i, M}^{*}(w) f_{i, M}^{*}(w) d x \tag{43}
\end{equation*}
$$

By Proposition 7, we know that $T_{-i}^{N}$ is non-decreasing in $w$ if $\eta_{i, N}^{* \prime}(\cdot)=0$ or $\eta_{i, N}^{* \prime}(\cdot)<0$. Consequently, Proposition 8 holds in both cases.

## VI. Numerical Illustration

This section numerically implements the equilibrium optimal tax formula.
For simplicity, we consider that the world consists of two symmetric countries. The distribution of the skill levels is based on the CPS data (2007) extended by a Pareto tail, so that the top $1 \%$ of the population gets $18 \%$ of total income, as in the US. The disutility of effort is given by $v(y ; w)=$ $(y / w)^{1+1 / \epsilon}$. This specification implies a constant elasticity of gross earnings with respect to the retention rate $\epsilon$, as in Diamond (1998) and Saez (2001). We choose $\epsilon=0.25$, which is a reasonable value based on the survey by Saez et al. (2012).

Even though the potential impact of income taxation on migration choices has been extensively discussed in the theoretical literature, there are still few empirical studies estimating the migration responses to taxation. A first set of studies consider the determinants of migration across US states (see Barro and Sala-i Martin (1992); Barro and Sala-I-Martin (1991), Ganong and Shoag (2013) and Suarez Serrato and Zidar (2013)). They find that per capita income has a positive effect on net migration rates into a state. This conclusion is entirely compatible with an explanation based on tax differences between US states, but may also be due to other differences (e.g., in productivities, housing rents, amenities or public goods). Strong structural assumptions are therefore required to disentangle the pure tax component. A second set of studies focuses exclusively on migration responses to taxation. Liebig et al. (2007) use differences across Swiss cantons and compute migration elasticities for different subpopulations, in particular for different groups in terms of education. Young and Varner (2011) use a millionaire tax specific to New Jersey. Because the salience of this millionaire tax is limited, their estimates of the causal effect of taxation on migration are
not statistically significant, except for extremely specific subpopulations. Still, their results suggest that the elasticity of migration is increasing in the upper part of the income distribution. Only two studies are devoted to the estimation of migration elasticities between countries. Kleven et al. (2013) examine tax-induced mobility of football players in Europe and find substantial mobility elasticities. More specifically, the mobility of domestic players with respect to domestic tax rate is rather small around 0.15 , but the mobility of foreign players is much larger, around 1 . Kleven et al. (2014) confirm that these large estimates apply to the broader market of highly skilled foreign workers and not only to football players. They find an elasticity above 1 in Denmark. In a given country, the number of foreigners at the top is however relatively small. Hence, these findings would translate into a global elasticity at the top of about 0.25 (see Piketty and Saez (2012)).

Our model pertains to international migrations and based on our survey of the empirical literature, we believe that the best we can do is to use an average elasticity of 0.25 for the national residents of the top $1 \%$ of the income distribution; and of 1 for the mercenaries.

We are presently working on the simulations, the results of which will be obtained within the next few weeks.

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[^1]:    ${ }^{1}$ If (2) admits more than one solution, we make the tie-breaking assumption that individuals choose the one preferred by the government.

[^2]:    ${ }^{2}$ This separability into two independent subproblems is not due to the maximin social objective; but to the asymmetric tax treatment of Native and Foreign residents in any given country.

[^3]:    ${ }^{4}$ Morelli et al. (2012) compare a unified nonlinear optimal taxation with the equilibrium taxation that would be chosen by two competing tax authorities if the same economy were divided into two States. In their conclusion, they discuss the possible implications of modifying this independence assumption and consider that allowing for a negative correlation might be more reasonable.

