

OPTIMAL INCOME TAXATION WITHOUT FULL GOVERNMENT COMMITMENT

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(I) Introduction

Most of the literature on optimal income taxation presumes, often implicitly, that the government can commit to policies announced before individuals made economic decisions. Mirrlees (1971) and Stiglitz (1982) made that assumption in the context of a one period model. Brito, et al (1991), in extending that analysis to a multi-period model, continued to assume that the government could commit to announced policies, even across periods. In multi-period models, some work has been done analyzing situations in which the government cannot commit. A classic paper by Kydland and Prescott (1977) considered capital taxation when the government could not commit to imposing taxes only in the initial period. More recently, Berliant and Ledyard (2014) analyzed multi-period optimal income taxation without government commitment.

The multi-period setting seems to be a natural one to rule out the government's ability to commit since policies announced in one year can be changed in later periods. However, a central feature in income taxation models in the Mirrlees-Stiglitz tradition is the need for the government to acquire information about the type or ability of individuals. Extending this to multi-period setting is of most interest if ability is highly, if not perfectly, correlated across periods. If across period ability correlations are low, the government would need independent revelation in each period and would gain little from being able to commit in one period not to use the information it learned about an individual's ability in later periods.

Even in one period models, whether commitment is possible can be an important issue especially in countries in which policy makers are not restrained in their behavior by institutions such as an independent judiciary. Consider a farmer who must decide whether to truthfully inform an absolute monarch's tax collector the amount of a crop he has grown that year. The tax collector assures him that the monarch's share will be only 5%. The farmer may be as or even more worried that after hearing the amount the tax collector will say the monarch's share is really 25% than that the amount taken next year

will depend on this year's crop.

In this

paper, we consider optimal taxation in a Stiglitz two type framework when the government cannot commit to announced policies. One crucial factor in the results is the degree to which the government desires to carry out redistribution. If the redistributive motives of the government are too strong, its ability to redistribute may be lessened. This creates an extremist's dilemma where a voter who has a strong preference for redistribution may prefer to vote for a government whose preferences for redistribution are much less.

(II) The Model

We consider a standard two type model as in Stiglitz (1982). Denote the types as 1 and 2 where the type 2 individuals are assumed to have higher ability. Each individual consumes a bundle $X^i = (C^i, Y^i)$ where the first component is consumption and the second is income and has a utility function $U^i(X^i)$ which satisfies the following standard properties: (i) $U^i(C^i, Y^i)$ is quasiconcave and continuously differentiable. Since labor Y^i is a bad, $\partial U^i / \partial Y^i < 0$, while $\partial U^i / \partial C^i > 0$ so that indifference curves slope up (

$$MRS^i(C_i, Y_i) = -\frac{\partial U^i / \partial C_i}{\partial U^i / \partial Y_i} > 0).$$

- (ii) Let $(C^i(K), Y^i(K))$ be the bundle that solves $\text{Max } U^i(C, Y)$ s.t. $C - Y = K$. If $K' > K''$, then $Y^i(K') \leq Y^i(K'')$ and $C^i(K') \geq C^i(K'')$ so that neither consumption nor leisure is inferior.
- (iii) Individuals have maximum possible pre-tax incomes \bar{Y}_1 and \bar{Y}_2 . Furthermore,

$$\lim_{Y \rightarrow \bar{Y}_i} \frac{\partial U^i}{\partial Y} \rightarrow -\infty.$$

(iv) Each individual has a closed convex survival set S^i which places lower bounds on the bundle an individual can receive. $S^1 \subset S^2$ in consumption-income space since $S^1 = S^2$ is assumed in consumption-labor space. Any bundle on the lower boundary of S^i is inferior to any bundle on the interior. The line $pX = 0$ runs through the interior of S^i .

In addition, we make a standard single crossing assumption across types:

(v) At any bundle X , $MRS^1(X) > MRS^2(X)$, and hence $\bar{Y}_1 < \bar{Y}_2$.

Assumption (v) holds in the standard case in which individuals differ only in ability with type 2s more able than type 1s. Figure 1 displays indifference maps of the two types satisfying these assumptions.

There are N_1 type 1's and N_2 type 2's in the economy. Individuals know their own type but not that of any other individual and the government does not know the type of any particular individual. All individuals and the government know the numbers N_1 and N_2 . For $p = (1, -1)$, the resource constraint if every individual reveals their type truthfully is $N_1 pX^1 + N_2 pX^2 \leq 0$. This must be modified if some individuals of either type misreveal. The government has a social welfare function which is a weighted sum of the utilities of the individuals. Let α , lying between 0 and 1, specify the weight that it puts on an honest type 1 while β is the weight put on an individual of type 1 who the government believes has misrevealed. We assume that $\beta \leq \alpha$. For type 2's, $(1 - \alpha)$ is the weight placed on an individual believed to be truthful while $(1 - \gamma)$ is the weight on an individual of type 2 who is believed to have misrevealed with $\alpha \leq \gamma$. The standard model assumes that the government can commit to its tax policies. Thus, in the first period the government commits to a single tax schedule which is faced by every individual regardless of type. This schedule specifies the tax of the individual as a function of that individual's income independent of what other individuals do. Revelation of type could either arise by individuals choosing how much income to earn or, following the revelation principle, by simply

announcing their type to the government. Using the latter approach, the government chooses the X^i bundles subject to self-selection constraints and the budget constraint to maximize social welfare. This yields the following optimization problem:

$$\text{Max } \alpha N_1 U^1(X^1) + (1 - \alpha) N_2 U^2(X^2) \quad (1)$$

$$X^1, X^2$$

$$\text{s.t. } N_1 p X^1 + N_2 p X^2 \leq 0 \quad (2)$$

$$U^j(X^j) \geq U^j(X^i), i \neq j \quad (3)$$

This optimization has been studied extensively. Let $X^i(\alpha)$ denote the solution bundles as functions of the government's preference parameter. The following proposition states some useful results about its solution as given for example in Stiglitz(1982) and Brito, et al (1990).

Proposition: There exist α' , α^* , and α'' , with $\alpha' < \alpha^* < \alpha''$, such that:

$$(i) (a) pX^1(\alpha^*) = pX^2(\alpha^*) = 0, (b) pX^1(\alpha) < 0 \text{ and } pX^2(\alpha) > 0, \text{ for } \alpha < \alpha^* \text{ and}$$

$$(c) pX^1(\alpha) > 0 \text{ and } pX^2(\alpha) < 0, \text{ for } \alpha > \alpha^*$$

$$(ii) (a) \text{ For } \alpha' \leq \alpha \leq \alpha'', \text{ neither type's self-selection constraint binds and thus}$$

$$\text{both } X^1(\alpha) \text{ and } X^2(\alpha) \text{ are non-distorted bundles; (b) for } 0 \leq \alpha < \alpha', U^1(X^1(\alpha)) =$$

$$U^1(X^2(\alpha)) \text{ and } U^2(X^2(\alpha)) > U^2(X^1(\alpha)); \text{ and (c) for } \alpha'' < \alpha \leq 1, U^2(X^2(\alpha)) =$$

$$U^2(X^1(\alpha)) \text{ and } U^1(X^1(\alpha)) > U^1(X^2(\alpha)).$$

From part (i), there exists one value of α , called α^* , for which the government would do no redistribution of resources across the types. For α greater than that, redistribution is toward type 1's while for α less than that, it is toward type 2's. From part (ii), there is a region of α containing α^* in

which redistribution occurs but a full information non-distorted allocation is achieved. For α above this region, the type 2 bundle remains undistorted but the type 1 bundle is distorted. For α below this region, the reverse holds.

In this paper, we consider the allocations that result if the government cannot commit to policies. In this case, any initial announcements by the government are irrelevant cheap talk. Individuals are really the first movers who reveal their type and then the government chooses its policies after learning what individuals reveal. It now matters how individuals reveal. The results when individuals simply reveal their type, which we call *announcement revelation*, differ from when they reveal by choosing an income level which we call *action revelation*. For either type of revelation, we will consider subgame perfect Nash equilibria in which individuals, when making their revelation decisions in the first stage, look ahead to how these will affect the subsequent policy decisions by the government. These policy decisions are optimizing choices given what individuals have revealed and what values of α , β , and γ the government has. These values are not chosen strategically by the government but are preference parameters of the government throughout the game and are known to all individuals. They may have been chosen in an election prior to the start of the game.

Consider first the case of announcement revelation and assume that α is greater than α^* so that the government desires to redistribute from type 2's to type 1's. As recipients, the type 1's will always reveal truthfully but type 2's may have an incentive to misreveal their type since this could avoid a tax and gain them a transfer. A similar analysis would follow if α were small and redistribution was desired in the opposite direction. Assume that n_2 of the type 2 individuals have falsely revealed themselves to be of type 1. The government knows that n_2 of those who declare that they are of type 1 are really type 2's but it does not know which they are. It must treat all those who said they were type 1 the same. The government chooses bundles to solve the following optimization problem:

$$\begin{aligned} \text{Max } & \alpha N_1 U^1(X^1) + (1 - \gamma) n_2 U^2(X^1) + (1 - \alpha)(N_2 - n_2) \alpha U^2(X^2) \\ & X_1, X_2 \end{aligned} \tag{4}$$

$$\text{s.t. } (N_1 + n_2)pX^1 + (N_2 - n_2)pX^2 \leq 0 \quad (5)$$

Note that there are no self-selection constraints in this problem since individuals have already revealed their types and cannot alter what they revealed. Denote the solution bundles to this problem as $X^1(\alpha, \gamma, n_2)$ and $X^2(\alpha, \gamma, n_2)$. Now go back to the first stage and consider equilibrium values of n_2 .

$$U^2(X^2(\alpha, \gamma, n_2)) \geq U^2(X^1(\alpha, \gamma, n_2)) \quad \text{for } n_2 = 0 \quad (6)$$

$$U^2(X^2(\alpha, \gamma, n_2)) \leq U^2(X^1(\alpha, \gamma, n_2)) \quad \text{for } n_2 = N_2 \quad (7)$$

$$U^2(X^2(\alpha, \gamma, n_2)) = U^2(X^1(\alpha, \gamma, n_2)) \quad \text{for some } n_2 \text{ with } 0 < n_2 < N_2 \quad (8)$$

Condition (6) is required for an equilibrium in which all type 2's are truthful while condition (7) must hold when all type 2's misreveal. Condition (8) must be satisfied if there is partial misrevelation by type 2's. Note that a sufficient condition for (8) to hold is that $X^1(\alpha, \gamma, n_2)$ and $X^2(\alpha, \gamma, n_2)$ vary continuously in n_2 and that the inequalities in both (6) and (7) are violated. Given that, as in Stiglitz (1982), we assume that there is a continuum of each type of individual, this situation can be viewed as a mixed strategy equilibrium where each individual makes an independent random decision to misreveal with probability n_2/N_2 . Which of (6), (7), or (8) holds will depend on the value of α and γ . Let $n_2(\alpha, \gamma)$ denote how the equilibrium value of n_2 varies with α and γ . Action revelation is more complicated to model. The revelation variable (income) is continuous not binary as is type. In the final stage, the government may face an arbitrary distribution of incomes, $f(X_2)$. In a world where it can commit, the government could force individuals to choose among only two income levels by specifying extremely high tax rates on any other levels of income. Without commitment, the government would not carry out such threats and therefore a range of incomes might be observed. Given this distribution, the government must have beliefs about the type of an individual who chose to earn any particular income. In general, the government could believe that there was some probability of each level of income earned being type 1

where this probability would depend on the entire distribution. That is, $g(X_2/f(Z_2))$ would denote the probability that an individual who earned some income X_2 when the entire distribution of incomes was $f(Z_2)$ was actually of type 1. We assume that these beliefs are straightforward in that the bottom end of the distribution of mass N_1 are type 1's and the top end of mass N_2 are type 2's. Using these beliefs, the government would then choose a level of consumption for each income level to maximize expected social welfare given its preferences over types. It might put different weights on those who did not choose the initially specified income levels but only if it truly held beliefs that individuals who "misrevealed" should be discounted over those who revealed correctly.

(III) Equilibria under Announcement Revelation

For simplicity, we assume that α is greater than α^* so that the government desires to transfer resources from the more able type 2's to the less able type 1's. The results vary greatly depending on the value of α .

Theorem 1: For any α with $\alpha^* < \alpha < \alpha''$, the optimal policies under full commitment are an equilibrium under no commitment.

For these values of α , since the government desires to carry out only a small amount of redistribution, it can do this without distorting either type's bundle and without causing the higher ability type 2's to envy the bundle of the lower ability type 1's. Since self-selection constraints are not binding in this region, the government does not need the ability to commit in order to limit the amount of redistribution it will carry out. Type 2 individuals, knowing the value of α , will not fear that truthfully revealing their type will put them in jeopardy of being charged significantly higher taxes. This equilibrium exists regardless of the value of γ . All individuals are truthful in equilibrium and each individual is infinitesimal so the weight the government would put on someone who switched to

misrevealing does not affect an individual's decision of whether or not to misreveal when no one else is misrevealing.

When α exceeds α'' , the type 2 self-selection constraint strictly binds. The full commitment optimum would now violate condition (6) and hence would not be an equilibrium when commitment is not possible. In the commitment solution, the government limited how much redistribution it did in order to satisfy self-selection and induce type 2's to reveal truthfully. Once types have been revealed, the government is no longer bound by the self-selection constraint and would do additional redistribution. Knowing that this will happen, the type 2 individuals will be reluctant to reveal their type. However, for α near α'' , there could be partial revelation with imperfect redistribution.

Theorem 2: There may exist an α''' with $\alpha'' < \alpha''' < 1$, such that for α between α'' and α''' there is an equilibrium value of n_2 that satisfies condition (8). Whether there is such an α''' and, if there is, what its exact value is depend on the value of γ . The $N_2 - n_2$ type 2's who do reveal their true type are taxed ($pX^2(\alpha, \gamma, n_2) < 0$) and these resources are transferred to the mix of type 1's and the n_2 type 2's who have misrevealed.

For this range of α , there may be a partial pooling equilibrium in which some redistribution is carried out. However, it tends to be less than what would have been done under full commitment. There are fewer type 2's from whom resources can be extracted and these resources are not as effectively utilized since some of them are given to other type 2's and not the type 1's for whom they are intended. The $N_2 - n_2$ type 2's who reveal correctly will consume non-distorted bundles. If γ is not 1, then the bundle given to the mix of type 1's and misrevealing type 2's will lie between the optimal bundles of the two types on the $pX_1 = D$ budget line where D is the amount by which each such persons consumption exceeds income. Each type will be consuming a distorted bundle with the distortions in opposite

directions for the two types.
government cannot commit.

For α greater than α''' , no redistribution is possible when the

Theorem 3: For $\alpha > \alpha'''$, the equilibrium has $n_2 = N_2$ so that all individuals receive the same bundle $X^1(\alpha, \gamma, N_2)$ which satisfies $pX^1(\alpha, \gamma, N_2) = 0$. As γ increases to 1 (which would occur if α increases to 1), this bundle moves down the $pX = 0$ line converging to the most preferred bundle for type 1's on that line.

To see that a range of values exists as specified in the theorem, consider α near 1 which means that γ is also near 1. Then, no weight is given to type 2 individuals and the bundle that they are assigned if truthful will be on the lower boundary of S^2 . When n_2 is at or near N_2 , essentially no resources are available to be transferred to the mix of type 1's and the misrevealing type 2's. Hence, this mix will essentially be consuming on the $pX = 0$ line. The government will assign them the best bundle for type 1's on this line. From assumption (iv) this bundle will be in S^1 and hence also in S^2 . It will therefore be preferred to the bundle assigned to truthful type 2's. Thus, the condition in (7) is satisfied and there is an equilibrium in which all type 2's misreveal. In this equilibrium, everyone will consume a distorted bundle as for the mix of individuals in Theorem 2 when there is partial revelation. Only if α is 1, will the type 1's receive an undistorted bundle. When α is sufficiently below 1 so that truthful type 2's are not placed on the lower boundary of S^2 , the value of γ may have an effect on whether this type of equilibrium exists. Assume that α equals γ , that only an infinitesimal number of type 2's are truthful, and that condition (7) holds with equality. Then, for α greater than γ , the bundle assigned to the mix of type 1's and misrevealing type 2's will be closer to the bundle most preferred by type 1's and, hence, a small number of type 2's may prefer to be truthful preventing this from being an equilibrium. For α in this range, the government has a high desire to redistribute but has almost no ability to carry this out. Type 2 individuals, knowing of this desire and of the high taxes that would be imposed on them, will never be

willing to reveal their type. Note that, although in the equilibrium, no individual pays any taxes so that their consumption exactly equals their income, except for type 1's when α exactly equals 1, no one is at their optimal no-tax bundle. The bundle assigned is one that maximizes $\alpha N_1 U^1(X^1) + (1 - \gamma) N_2 U^2(X^1)$. This is not a *laissez-faire* economy with the government playing no role. The results in Theorems 1 to 3 lead to what could be called the *extremist's dilemma*. Assume that there is an earlier stage in which someone is elected to the position of social planner who will carry out the government policies using their own values of α and γ . Assume that all voters are perfectly informed of the preferences of candidates for social planner. Assume that all individuals and candidates have the same value of γ . Consider a voter whose personal value for social preferences is θ . Let $W(\alpha/\theta)$ be how the voter who has preferences θ evaluates the social outcome when the social planner has preferences α . Then:

$$W(\alpha/\theta) = \theta N_1 U^1(X^1(\alpha, n_2(\alpha))) + (1 - \gamma) n_2(\alpha) U^2(X^1(\alpha, n_2(\alpha))) + (1 - \theta)(N_2 - n_2(\alpha)) \alpha U^2(X^2(\alpha, n_2(\alpha))) \quad (9)$$

Theorem 4: For high values of θ , there is a range of values of α strictly below θ for which $W(\theta/\theta) < W(\alpha/\theta)$.

An individual who desires a large amount of redistribution would be better off if an individual who had more moderate preferences toward redistribution were elected. This can occur if θ is in the region consistent with Theorem 3 in which no redistribution is done. An individual with this preference would prefer the government to have a sufficiently lower value of α so that at least some redistribution would occur. This would be a form of commitment by the voter not to do extensive redistribution by electing a government which truly prefers less redistribution than does the voter. When being in this situation, voters would face a choice between purity and pragmatism in how they vote. However, it often seems difficult for individuals to make an optimizing choice between purity and pragmatism. This dilemma seems to be playing out in the current presidential nomination contests in both political parties in the United States. The divisions between Sanders and Clinton for Democrats and between Tea Party

adherents and more establishment candidates among Republicans both appear to relate to this dilemma. Finally, consider preferences over γ which can be interpreted as preferences over how harsh to treat individuals who lie about their type. Consider an individual who puts weight θ on type 1's and weight $1 - \mu$ on misrevealing type 2's. Let δ equal $\mu - \theta$ be the degree of harshness the voter has toward misrevealers. That is, if δ is 0, the individual is very soft, treating misrevealers the same as truth tellers, while if δ equals $1 - \theta$ the individual is very harsh, putting no weight on those who misreveal. Similarly, let a candidate's harshness be denoted by ε which equals $\gamma - \alpha$, where ε can run between 0 and $1 - \alpha$. If α is such that the no commitment equilibrium sustains the full commitment solution as given in Theorem 1, then harshness is irrelevant. If this is not true, then harshness matters both within the partial revelation and no redistribution regions given in Theorems 2 and 3 and also in affecting which of those cases occurs. To get some idea of how individuals would vote over candidates with these two dimensions, consider candidates that would lead the government to do no redistribution.

Theorem 5: Assume a set of candidates who all would lead to a no redistribution outcome satisfying condition (7) where all type 2's misreveal. Then a voter who has values θ and δ for the weight on type 1's and for harshness to misrevealing type 2's would be completely happy with any candidate for whom $\varepsilon = \delta + (\theta - \alpha)(1 - \delta)/\theta$.

This theorem follows from noting that in this region, the objective function for the individual $\theta N_1 U^1(X^1) + (1 - \mu)N_2 U^2(X^1)$ can be rewritten as $N_1 U^1(X^1) + ((1 - \mu)/\theta)N_2 U^2(X^1)$ and similarly the government's objective function can be rewritten as $N_1 U^1(X^1) + ((1 - \gamma)/\alpha)N_2 U^2(X^1)$. An ideal candidate for the individual would have $(1 - \mu)/\theta = (1 - \gamma)/\alpha$.

An ideal candidate could want more redistribution than the voter, $\theta - \alpha < 0$, but then that candidate would have to be less harsh, $\varepsilon < \delta$, or an ideal candidate could want less redistribution, $\theta - \alpha > 0$, but then would have to be harsher, $\varepsilon > \delta$. If wanting more redistribution and softness are "liberal"

positions and wanting less redistribution and harshness are “conservative” positions, then an individual would prefer candidates who were more liberal or more conservative on both dimensions to a candidate who was mixed, more liberal on one and more conservative on the other.

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