SAFETY IN NUMBERS? SELF-PROTECTION AS A LOCAL PUBLIC GOOD*

By

Clive D Fraser**

Leicester University

February 2017

ABSTRACT. In many contexts with endogenous risks - e.g., the household, a neighbourhood's traffic calming measures, quality control on production runs - risk reduction is a local public good. The decision-maker's incentive to reduce risk then naturally depends on the protected population's size. Modelling risk as a sequence of i.i.d. Bernoulli trials with endogenous "success" probability, we give sufficient conditions for safety to increase with the number protected. We utilise an elementary recursive decomposition of a covariance involving a monotonic function of a binomially distributed variable and first degree stochastic dominance (FSD). Because "protection" problems are generally non-concave, we give a detailed treatment of the second-order condition, again via FSD.

Keywords: risk, local public good, binomial variate, covariance, stochastic dominance

JEL Classifications: *C60; D10; D80; H41*

* This work is still preliminary. Thanks to Chris Skeels (ANU) for a helpful conversation on related material and also to Dilip Bhattacharyya and workshop participants at Leicester. The usual caveats apply.

**Economics Division, University of Leicester Business School, Leicester LE1 7RH, UK. E-mail: cdf2@le.ac.uk

SAFETY IN NUMBERS? SELF-PROTECTION AS A LOCAL PUBLIC GOOD

1. INTRODUCTION

Because of their simple intuitive binary structure, Bernoullian variables and binomial distributions are often used in representing risk in economics and related subjects. The following are some examples: (i) in manufacturing and distributing a commodity of uncertain quality, each of the ex ante identical units in a batch might be satisfactory with a common probability that depends on specifics of the production process, such as the age of the machinery and the investment in maintenance and quality control. The distribution of the number of unsatisfactory items in the batch is then binomial with parameters given by the batch size and the endogenous probability of an item being unsatisfactory; (ii) in a locality, the risk of an arbitrary pedestrian having a road accident in a period, dependent on the municipality's investment in road safety features, might be constant across all pedestrians. A binomial distribution, now for the number of pedestrian accident victims in the period, will result; (iii) the risk of accidental death or injury to any child in a household might be a scalar which depends on the parents' investment in protective activities. Yet again, for a household with one or more children, the distribution of family size or uninjured members at the end of a period is binomial.

In such examples, the parameter representing the accident or failure risk (equivalently, the level of "safety") can be taken to be endogenous. The economic problem is then to analyse the factors which determine this risk from the societal, firm or household's standpoint and the consequences of this level of risk. Moreover, any investment which affects the level of risk is a local public good in that it affects all the relevant risks equally. Thus, e.g., if investment in a machine reduces the risk of an item from a batch being defective, it reduces the risk equally for all items in the batch. The firm, household or society's incentive to make such risk-reducing investments can be analysed using expected utility theory, where utility is household utility, company profits or social welfare and can be taken to be monotonically decreasing in the number of accidents, losses or failures, depending on the context. Because the decision-maker can spend to reduce the probability of loss, the environment is one of self-protection (SP) as analysed by Erhlich and Becker (1972).

One particularly important issue that arises, and which is our main focus, is how the incentive for risk reduction depends on the size of the population protected. I.e., how does the extent of publicness of risk reduction, defined here as the number of people who benefit from it, affect the incentive to undertake it? In analysing this, it transpires that how utility covaries with the number of losses or accidents in the relevant population and how this covariance changes as the population size increases play pivotal roles. We provide a simple recursive formula for this covariance which yields a considerable simplification of subsequent analysis incorporating it.

It is worth noting that there is an extensive urban economics literature which studies how population size and growth affect, among other things, per capita public expenditure on public safety. This literature is largely empirically orientated. Regarding safety, it concentrates mainly on interjurisdictional comparisons of protection against crime and fire. It stresses the importance of a community's socio-economic characteristics and the nature of the congestion function for shared goods, hence their degree of publicness, for determining expenditure. See, e.g., Clark and Cosgrove (1990), Ladd (1992), McGreer and McMillan (1993) - who also consider highway maintenance - and Schwab and Zampelli (1987).

An early notable exception to this empirical focus is the neglected paper of Kolm (1976). Kolm presents a general analysis in which the physical risk someone faces depends on two types of safety expenditures: first, one with an effect which is purely private to the individual; second, one which also reduces the risks for others and thus has a degree of publicness. He considers the socially optimal rate of substitution between these two types and the dependence of the optimal relationship between impurely public and purely public safety expenditure on whether the former is a complement or substitute for safety expenditures with a purely private effect. However, Kolm did not consider directly how optimal purely public safety expenditure changes as the population protected by it increases, our main concern. This neglect of the influence of the size of the protected population on optimal safety decisions is pervasive in both the theoretical literature and, as Viscusi (1995) observes, policy-making[1].

More recently, several papers have examined public good aspects of self-protection (e.g., Fraser, 1996 WP, Kunreuther and Heal, 2003; Heal and Kunreuther, 2005; Muermann and Kunreuther, 2008; Lohse, Robledo and Schmidt, 2012). Barring Fraser, these authors consider scenarios where safety decisions are made by several agents with interdependent risks. The focus is on the free-riding behaviour that might then arises. By contrast, as in this paper, Fraser (1996) considers scenarios wherein a single decision-maker, or decision-makers acting collectively, make a safety decision that influences the risk faced by several people. Thus, I do not consider free-riding incentives. However, the framework that I employ can be extended straightforwardly to consider such incentives, with the agents analysed in this paper each being an agent in a Nash equilibrium model.

Section 2 of this paper introduces my model in the context of endogenous physical safety within the household. I show how the covariance between utility and the number of accidents is important for how the incentive to engage in risk reduction varies with the size of the population protected. Section 3 provides the recursive relationship linking the covariances between utility and the number of accidents for protected populations of different sizes. We give sufficient conditions for this covariance to decrease with the size of the protected population via this recursion relationship. In an Appendix, we consider an alternative approach based on first degree stochastic dominance (FSD) to show that our approach using this recursion relationship yields stronger sufficient conditions than the latter. Because "self-protection" problems are not generally concave, we also give a detailed treatment of the second-order condition for optimal safety investment, again via FSD. Finally, we bring together the preceding analysis to obtain sufficient conditions for the investment in safety to increase with the size of the population protected. Section 4 concludes. The Appendix contains two proofs.

2. HOW THE COVARIANCE CAN ARISE

Without loss of generality, we will specialise the analysis initially to endogenous safety within the household. Some of the details which follow are not in themselves important. Rather, they are minutiae illustrative of a context in which our analysis applies. As we note below, it applies in production and other contexts as well. Thus, consider a two-parent household with n children in which each child at risk faces an independent and identical probability $p(e)$ of suffering death or injury (taken to be death for simplicity) during a given time period. This probability is endogenous and depends on the parents' expenditure of protective "effort" and/or money e, satisfying

(2.1) $$p'(e) \le 0,\ p''(e) \ge 0,\ e \in E \text{ (non-increasing returns)}$$

Here E is the compact interval of the non-negative real line within which e can lie. As e affects identically the risk confronting all children considered at risk in the household (*which need not be all of the* n *)*, it is a local public good.

The parents, who command a household income M, fixed for simplicity, are assumed to derive utility from both the household's material standard of living and its size. We assume that their utility subsumes their children's for, despite the many models questioning parents' altruism towards each other or their children[2], it seems reasonable to assume that parents at least agree on the desirability of their children's safety. Because of the children's accident risk, the number of children in the household at the end of the period and hence the household size are binomially distributed random variables, denoted $\tilde{n}$ and $\tilde{h} = \tilde{n} + 2$, respectively. For simplicity, the material standard of living is proxied by the household's per capita income at the end of the period, denoted $\tilde{x} \equiv M / \tilde{h}$, and is again random ex ante. Moreover, there is an obvious trade-off between the household's size and its material standard of living.

As bereavement is distressing, it gives parents an extra incentive to protect their children. Thus, we assume that their joint utility function is state-dependent and index the states of the world by $r \equiv$ the number of children lost to accidents. Then U^r, r = $0,1,2,\ldots,n,$ denotes their utility function conditional on the number of children lost. We will follow much of the insurance, moral hazard and incentives literature in assuming that this utility function is quasi-linear in the level of "effort" parents expend in protection[3]. Then utility in state r will be given by

(2.2) $$U^r(x^r, h^r, e) \equiv u^r[M/(2+n-r), 2+n-r] - e$$

It seems plausible to assume that, whether or not they are state-dependent, the functions U^r and u^r will be monotonically decreasing in r. I.e., other things equal,

(A.1) $$U^r > U^{r+1},\ r = 1,2,\ldots,j-1.$$

One formulation that I have used elsewhere (Fraser 1996a, 1996b, 2001) is

(A.1') $$u^r\left[M/(2+n-r),2+n-r\right]\equiv u[M/(2+n-r),2+n-r,r],du/dr<0$$ [4].

However, for the rest of this paper, the specifics of the utility function do not matter beyond that (A.1) and another assumption to be introduced below, (A.2), hold.

Suppose $j \le n$ of the household's children are at risk. For a given level of e, $\binom{j}{r} p(e)^r (1-p(e))^{j-r}$ is the probability that exactly r of these children will be killed, $r = 0,1,2,\ldots, j$, $\binom{j}{r}$ being the binomial coefficient giving the number of ways r items can be chosen from j. The parents can be thought to choose the level of e to solve the following conditional expected utility (CEU)[5] maximisation problem:

(2.3) $$\underset{e}{Max.} \left\{ CEU^j(e) \equiv \sum_0^j \binom{j}{r} p(e)^r (1-p(e))^{j-r} u^r - e \right\}$$

This problem is in the class of self-protection problems (cf. Becker and Ehrlich (1972)). Such problems are not usually concave in e [6]. Thus, for a calculus solution, we must assume or show that the second-order condition (SOC) holds at any e satisfying the first-order condition (FOC). One of our innovations below is a detailed analysis of the SOC which yields sufficient conditions for the problem's concavity. Granted this, an interior optimal choice for e, denoted e^j, satisfies:

(2.4) (FOC:) $CEU_e^j(e^j) =$

$$-p'(e^j)\left\{\sum_{r=0}^{j}\binom{j}{r}\left[(j-r)p(e^j)^r(1-p(e^j))^{j-r-1}-rp(e^j)^{r-1}(1-p(e^j))^{j-r}\right]u^r\right\}-1=0$$

Note that this is the Samuelson condition for self-protection in the household.

So, suppressing functional arguments and rearranging, we have:

$$\text{(2.5)} \qquad -p'\left\{\sum_{r=0}^{j}\binom{j}{r}\left[p^r(1-p)^{j-r}(jp-r)\right]u^r\right\}=p(1-p)$$

In (2.4), 1 is the constant marginal cost of reducing the child mortality risk and $-p'\left\{\sum_{r=0}^{j}\binom{j}{r}\left[(j-r)p^r(1-p)^{j-r-1}-rp^{r-1}(1-p)^{j-r}\right]u^r\right\}$ is the parents' perceived marginal benefit of this expenditure. In (2.5), given (A.1), as the joint distribution of r and u^r is identical to r's, $-\left\{\sum_{r=0}^{j}\binom{j}{r}\left[p^r(1-p)^{j-r}(jp-r)\right]u^r\right\}$ = $Cov(r,u^r)$, where $Cov(v,w)$ is the covariance between the random variables v and w. Thus, from (2.5), the parents' marginal benefit from protective expenditure is proportional to $Cov(r,u^r)$. How $Cov(r,u^r)$ behaves *at a given* e as the number of children at risk increases, the subject of section 3, determines whether an increase in the degree of publicness of protective expenditure increases the incentive to make that expenditure.

3. A RECURSIVE DECOMPOSITION OF $Cov(r,u^r)$ AND ITS IMPLICATIONS

It is convenient to use the following notation in the sequel. Let $Cov(r, u^{f(r)})_a^b \equiv$ the covariance between the random variables r and $u^{f(r)}$, where $f(r)$ and $u^{f(r)}$ are some functions of r and r ranges from a to b. It is also convenient to assume:

(A.2) u^r is the same at a given r, *irrespective of the number of children protected by a given risk-reducing expenditure*.

Assumption (A.2) is quite strong.[7] It ensures that the u^r appearing in $Cov(r,u^r)_0^{j+1} = \sum_0^{j+1}\binom{j+1}{r}p^r(1-p)^{j+1-r}\left[r-(j+1)p\right]u^r$ and in $Cov(r,u^r)_0^j = \sum_0^j\binom{j}{r}p^r(1-p)^{j-r}\left[r-jp\right]u^r$ are identical. In the household context, the assumption is plausible. It merely requires that parents initially with n children assign the same utility to losing r of them if j of them are at risk, $r \le j \le n$, as when $j+1$ are, $r \le j+1 \le n$. It is also plausible in other contexts. E.g., for an expected vote maximising local authority undertaking public safety measures, other things equal, the endogenous popularity function, say, would need to be uniquely determined by the number of accidents in the locality and not by the number of people actually at risk. In a production context, it would require, e.g., the profits of an expected profit maximising firm to be uniquely related to the number of defective items sold[8].

Even with (A.2), how large $Cov(r,u^r)_0^{j+1}$ is relative to $Cov(r,u^r)_0^j$ is unclear. On one hand, $Cov(r,u^r)_0^{j+1}$ has the extra term in $u^{j+1}\left[j+1-(j+1)p\right]$ compared to $Cov(r,u^r)_0^j$. On the other, without further restrictions on j and p, we cannot say whether the term in an arbitrary $u^r, r = 0,1,..,j$, increases going from $Cov(r,u^r)_0^j$ to $Cov(r,u^r)_0^{j+1}$. Thus we need a more systematic investigation of how the covariance behaves as the size of the population increases. Our core result here is presented next.

3.1. *The Main Theorem*

THEOREM 3.1. *Suppose $r \le k$ is the number of "successes" in a series of k Bernoulli trials, each with success probability p, and u^r is a state-dependent function of r satisfying $u^r > u^{r+1}$, $r, r+1 \le k$. Then, for $k = j+1$ trials,*

(3.1) $$Cov(r,u^r)_0^{j+1} = (1-p)\left(\frac{j+1}{j}\right)Cov(r,u^r)_0^j + p\left(\frac{j+1}{j}\right)Cov(r,u^{r+1})_0^j$$

PROOF. Consider an arbitrary $j, j = 1,2,\ldots \le n-1$. By definition, given (A.1),

(3.2) $$Cov(r,u^r)_0^{j+1} = \sum_0^{j+1}\binom{j+1}{r}p^r(1-p)^{j+1-r}\left[r-(j+1)p\right]u^r$$

$$=\left(\frac{j+1}{j}\right)(1-p)\sum_0^j\binom{j}{r}p^r(1-p)^{j-r}(r-jp)u^r +$$

$$+\sum_0^{j+1}\binom{j+1}{r}p^r(1-p)^{j+1-r}\left[r-(j+1)p\right]u^r - \left(\frac{j+1}{j}\right)(1-p)\sum_0^j\binom{j}{r}p^r(1-p)^{j-r}(r-jp)u^r .$$

Consider the last two summands in the second equality. Collecting like terms, the coefficient of u^0 is $-\left[\binom{j+1}{0}(1-p)^{j+1}(j+1)p - \left(\frac{j+1}{j}\right)(1-p)\binom{j}{0}(1-p)^j jp\right] =$

$-(1-p)^{j+1}p\left[\binom{j+1}{0}(j+1) - \left(\frac{j+1}{j}\right)\binom{j}{0}j\right] = 0$, because $\binom{j+1}{0} = \binom{j}{0}$. Again, collecting like terms from the last two summands in (3.2), the coefficient of an arbitrary $u^r, r = 1,2,\ldots, j,$ is

$$p^r(1-p)^{j+1-r}\left\{\binom{j+1}{r}\left[r-(j+1)p\right]-\left(\frac{j+1}{j}\right)\binom{j}{r}\left[r-jp\right]\right\} =$$

$$\frac{p^r(1-p)^{j+1-r}}{j}\left\{\binom{j+1}{r}j\left[r-(j+1)p\right]-\binom{j}{r}(j+1)\left[r-jp\right]\right\} = \left(\text{using } \binom{j}{r} = \left(\frac{j+1-r}{j+1}\right)\binom{j+1}{r}\right) =$$

$$\frac{p^r(1-p)^{j+1-r}}{j}\binom{j+1}{r}\left\{j\left[r-(j+1)p\right]-\left(\frac{j+1-r}{j+1}\right)(j+1)\left[r-jp\right]\right\} =$$

$$\frac{p^r(1-p)^{j+1-r}}{j}\binom{j+1}{r}r\left[(r-1)-jp\right] = \left(\text{using } \frac{r}{j}\binom{j+1}{r} = \frac{(j+1)!r}{r!(j+1-r)!j} = \frac{(j+1)j!}{(r-1)!(j-(r-1))!j} =\right.$$

$$\left.\left(\frac{j+1}{j}\right)\binom{j}{r-1}\right) = p\left(\frac{j+1}{j}\right)\binom{j}{r-1}p^{r-1}(1-p)^{j-(r-1)}\left[(r-1)-jp\right].$$

The coefficient of the term in u^{j+1} is $\binom{j+1}{j+1}p^{j+1}(1-p)(j+1) = p\left(\frac{j+1}{j}\right)p^j(1-p)j$. Using these derived coefficients in the last two summands of (3.2) implies

$$\sum_0^{j+1}\binom{j+1}{r}p^r(1-p)^{j+1-r}\left[r-(j+1)p\right]u^r - \left(\frac{j+1}{j}\right)(1-p)\sum_0^j\binom{j}{r}p^r(1-p)^{j-r}(r-jp)u^r$$

$$= p\left(\frac{j+1}{j}\right)\sum_1^{j+1}\binom{j}{r-1}p^{r-1}(1-p)^{j-(r-1)}\left[r-1-jp\right]u^r = p\left(\frac{j+1}{j}\right)Cov(r,u^{r+1})_0^j.$$ As

$$\left(\frac{j+1}{j}\right)(1-p)\sum_0^j\binom{j}{r}p^r(1-p)^{j-r}(r-jp)u^r = (1-p)\left(\frac{j+1}{j}\right)Cov(r,u^r)_0^j,$$ we are done. Q.E.D.

Four remarks are in order before we apply Theorem 3.1 to investment in safety.

REMARK 1. With r binomial and u^r non-monotonic, $\sum_0^j\binom{j}{r}p^r(1-p)^{j-r}\left[r-jp\right]u^r$ is the first moment about zero of the function $(r-jp)u^r$, denoted $\mathrm{E}_r\left[(r-jp)u^r\right]_0^j$ (E_r indicating expectation over r), not a covariance. Substituting $\mathrm{E}_r\left[(r-jp)u^r\right]_0^j$ in place of $Cov(r,u^r)_0^j$, and so on in (3.1), the basic recursion formula in (3.1) will continue to apply.

REMARK 2. The recursion formula (3.1) can be applied successively to obtain

(3.3) $$Cov(r,u^r)_0^{j+1} = (1-p)^l \binom{j+1}{j-(l-1)} Cov(r,u^r)_0^{j-(l-1)}$$

$$+p\sum_{k=0}^{l-1}\binom{j+1}{j-l}(1-p)^k Cov(r,u^{r+1})_0^{j-k}\ ,\ l=1,\ldots,j.$$

This formula might be of independent interest although we will not use it in this paper.

REMARK 3. Interest in recurrence relationships for the moments of binomial distributions in the literature has centred on those between successive moments, such as the mean and variance[9].

REMARK 4. If u^r is linear, not nonlinear, in r, we can normalise so that $u^r=-r$ without loss of generality. Then, in notation analogous to that introduced above, $Cov(r,u^r)_0^{j+1}=Cov(r,-r)_0^{j+1}=-Var(r)_0^{j+1}=-(1-p)p(j+1)$. Thus we can see that formula (3.1) is a proper generalisation of the variance for a binomial variate because, now,

$$Var(r)_0^{j+1}=(1-p)p(j+1) = \binom{j+1}{j}\left[(1-p)(1-p)pj+p(1-p)pj\right]$$

$$=\binom{j+1}{j}\left[(1-p)Var(r)_0^j+pVar(r+1)_0^j\right].$$

3.2 *Some Corollaries*

The main implications of Theorem 3.1 for parents' incentive to invest in their children's safety as the number protected increases are derived from the following two corollaries.

COROLLARY 3.1. $\frac{1}{j+1} \geq p$ *is sufficient for* $Cov(r,u^r)_0^{j+1} < Cov(r,u^r)_0^j (< 0)$.

PROOF. Given $Cov(r,u^r)_0^j < 0$, $\left(\frac{j+1}{j}\right)(1-p)Cov(r,u^r)_0^j \leq Cov(r,u^r)_0^j$ if

$\left(\frac{j+1}{j}\right)(1-p) \geq 1$ or, on rearrangement, if $\frac{1}{j+1} \geq p$. But then, from (3.1), $Cov(r,u^r)_0^{j+1} =$

$\left(\frac{j+1}{j}\right)\left[(1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j\right] <$ $Cov(r,u^r)_0^j$. Q.E.D.

In the Appendix, we will prove a slightly weaker analogue of Corollary 3.1, Corollary 3.1A, by a stochastic dominance technique also utilised below in Section 3.3. Here, the inequality $\frac{1}{j+1} > p$ is the condition for a binomial density based on j i.i.d. Bernoulli trials to have its mode at $r = 0$. For the major physical risks to children usually encountered in the household, $\frac{1}{j+1} > p$ is likely to be a reasonable assumption because, typically, both j and p will be small. Hence, "no accident" is the highest probability event. But in other contexts where we might apply our model, such as in analysing quality control in production where, in effect, there are numerous trials, this need not be the case. Also, as p is endogenous, it might not always be thought appropriate to put such an *a priori* restriction on it. Thus we need a more general result, such as in Corollary 3.2.

COROLLARY 3.2. *If* u^r *is monotonically decreasing and concave in* r*, then, at the same* p*,* $Cov(r,u^r)_0^{j+1} < Cov(r,u^r)_0^j (< 0)$.

PROOF. Let $\Delta u^k \equiv u^{k+1} - u^k$. Then

(3.4) $$Cov(r,u^{r+1})_0^j - Cov(r,u^r)_0^j = \sum_0^j \binom{j}{r} p^r (1-p)^{j-r} (r-jp)\left[u^{r+1} - u^r\right] =$$

$$\sum_0^j \binom{j}{r} p^r (1-p)^{j-r} (r-jp)\Delta u^r .$$

By our earlier argument, if Δu^r is monotonic in r, $\sum_0^j \binom{j}{r} p^r (1-p)^{j-r} (r-jp)\Delta u^r =$

$Cov(r,\Delta u^r)$. If Δu^r is also increasing in r, then $Cov(r,\Delta u^r) \geq 0$ and, from (3.4), $Cov(r,u^{r+1})_0^j$

$\geq$

$Cov(r,u^r)_0^j$. Likewise, if Δu^r is decreasing in r, then $Cov(r,u^r)_0^j \geq Cov(r,u^{r+1})_0^j$. Now, if

Δu^r is increasing in r, then $\Delta u^{r+1} \geq \Delta u^r \Leftrightarrow u^{r+2} - u^{r+1} \geq u^{r+1} - u^r \Leftrightarrow$

$(1/2)\left[u^{r+2} + u^r\right] \geq u^{r+1}$. The last two inequalities define convexity of u^r in r. Likewise, if u^r is concave in r, Δu^r is decreasing in r and, hence, $Cov(r,u^r)_0^j \geq Cov(r,u^{r+1})_0^j$. Thus, if u^r is concave in r, $Cov(r,u^r)_0^j \geq (1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j$, $\forall\ p \in (0,1)$, hence

$Cov(r,u^r)_0^j \geq (1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j >$

$\left(\frac{j+1}{j}\right)\left[(1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j\right] = Cov(r,u^r)_0^{j+1}$ by Theorem 3.1. Q.E.D.

REMARK 5. If Δu^r is increasing in r, hence u^r is convex in r, thus $Cov(r,u^r)_0^j \leq$ $Cov(r,u^{r+1})_0^j$, then $Cov(r,u^r)_0^j \leq (1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j$. But, for $j > 0$, as

$(1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j > \left(\frac{j+1}{j}\right)\left[(1-p)Cov(r,u^r)_0^j + pCov(r,u^{r+1})_0^j\right] =$

$Cov(r,u^r)_0^{j+1}$, it is now still possible that $Cov(r,u^r)_0^j > Cov(r,u^r)_0^{j+1}$. Thus, that u^r is concave and decreasing in r is sufficient but unnecessary for $Cov(r,u^r)_0^j > Cov(r,u^r)_0^{j+1}$.

It seems intuitively reasonable in the household context that u^r will be concave and decreasing in r - equivalently that $u^{r+1} - u^{r+2} \geq u^r - u^{r+1}$. This states merely that the parents' fall in utility from losing the additional child is no less the more children are lost.

3.3. *Existence and Uniqueness of a Protective Equilibrium by a First-Order Approach*

We noted above that the protection problem is generally non-concave. It is thus important to investigate the restrictions on the utility and risk functions which ensure that the SOC is always satisfied and, hence, the FOC identifies a unique protective equilibrium.

The second-order condition for problem (2.3) requires

(3.5) (SOC:)
$$CEU_{ee}^j(e^j) = \frac{p''}{(1-p)p}\sum_0^j \binom{j}{r} p^r(1-p)^{j-r}(r-jp)u^r +$$
$$\frac{p'p'}{(1-p)p}\sum_0^j \binom{j}{r}\left[p^r(1-p)^{j-r}(-j) + rp^{r-1}(1-p)^{j-r}(r-jp) - (j-r)p^r(1-p)^{j-r-1}(r-jp)\right]u^r$$
$$-\frac{p'p'(1-2p)}{(1-p)^2p^2}\sum_0^j \binom{j}{r} p^r(1-p)^{j-r}(r-jp)u^r < 0$$

Thus, multiplying through by $(1-p)^2p^2$ and simplifying the middle term, we require

$$(1-p)^2p^2\,CEU_{ee}^j(e^j) = (1-p)pp''\sum_0^j \binom{j}{r} p^r(1-p)^{j-r}(r-jp)u^r +$$

$$p'p'\sum_0^j \binom{j}{r} p^r (1-p)^{j-r}\left[(r-jp)^2-(1-p)pj\right]u^r -$$

$$p'p'(1-2p)\sum_0^j \binom{j}{r} p^r (1-p)^{j-r}(r-jp)u^r < 0$$

Equivalently, combining the first and last terms before the inequality sign and using the well-known result that, for the binomial distribution, $(1-p)pj = \mathrm{E}_r(r-jp)^2$, we need

(3.6) $$(1-p)^2 p^2\, CEU_{ee}^j(e^j) = \left[p(1-p)p'' - p'p'(1-2p)\right]Cov(r,u^r)_0^j +$$

$$p'p'\sum_0^j \binom{j}{r} p^r (1-p)^{j-r}\left[(r-jp)^2 - \mathrm{E}_r(r-jp)^2\right]u^r < 0$$

Although $Cov(r,u^r)_0^j < 0$, it is not obvious what the signs of the two terms of $(1-p)^2 p^2$ $CEU_{ee}^j(e^j)$ are. The ensuing Theorem 3.2 shows that, if $\frac{1}{j+1} > p$ and $p(e) = \lambda\exp(-\gamma e)$, for scalars $\lambda, \gamma > 0$, they are both negative, thus the SOC will be satisfied.

THEOREM 3.2. *If* $\frac{1}{j+1} > p$ *and* $p(e) = \lambda\exp(-\gamma e)$, *for some scalars* $\lambda, \gamma > 0$, *then* $(1-p)^2 p^2\, CEU_{ee}^j(e^j) < 0$.

PROOF. If $p(e) = \lambda\exp(-\gamma e)$, the first term after the equality sign in (3.6) is

(3.7) $$\left[ppp'' + pp''(1-2p) - p'p'(1-2p)\right]Cov(r,u^r)_0^j = ppp''Cov(r,u^r) \le 0,$$

given (2.1) and $Cov(r,u^r)$ <0. The second term can be rewritten as

$$\mathrm{E}_r(r-jp)^2\left[\sum_0^j \binom{j}{r} p^r(1-p)^{j-r}\left\{\frac{(r-jp)^2}{\mathrm{E}_r(r-jp)^2}-1\right\}u^r\right].$$

But $\sum_0^j \binom{j}{r} p^r(1-p)^{j-r}\ \frac{(r-jp)^2}{\mathrm{E}_r(r-jp)^2}\ = \frac{\mathrm{E}_r(r-jp)^2}{\mathrm{E}_r(r-jp)^2}=1$. Thus,

$\binom{j}{r} p^r(1-p)^{j-r}(r-jp)^2\,/\,\mathrm{E}_r(r-jp)^2\ \equiv \theta(r)$ can be taken as a new discrete probability density for r with a corresponding cumulative distribution $\Theta(r)$, compared with the original binomial density $\binom{j}{r} p^r(1-p)^{j-r}\equiv \beta(r)$ and cumulative $\mathrm{B}(r)$. Now, if $\frac{1}{j+1}>p$, then $\theta(0)/\beta(0)=(jp)^2\,/\left[jp(1-p)\right]<1$. Moreover, as $(r-jp)^2\,//\mathrm{E}_r(r-jp)^2$ is decreasing (increasing) in r as $r<(>)\ jp$, and $1>(j+1)p\ =jp+p$ by assumption, $r>jp$ and thus $(r-jp)^2$ $/\mathrm{E}_r(r-jp)^2$ is increasing in r for $r=1,2,\ldots,j$. Thus the graph of $\theta(r)$ must single-cross that of $\beta(r)$ from below. Hence

(3.8) $\Theta(r)\le \mathrm{B}(r), r=0,1,\ldots,j$, with strict equality only at $r=j$.

But (3.8) is the definition for a discrete distribution $\Theta(r)$ to strictly first degree stochasticallly dominate (FSD) another, $\mathrm{B}(r)$. Now, we know that if $\Theta(r)$ strictly FSD $\mathrm{B}(r)$ then $\left[\sum_0^j \binom{j}{r} p^r(1-p)^{j-r}\left\{\frac{(r-jp)^2}{\mathrm{E}_r(r-jp)^2}-1\right\}u^r\right]<0$ for any strictly decreasing function u^r. Combining this with (3.8) yields $(1-p)^2p^2\,CEU_{ee}^j(e^j)<0$. Q.E.D.

REMARK 6. Thus, if $p(e) = \lambda \exp(-\gamma e)$, provided the risk function is such that, irrespective of the level of e, the resulting binomial distribution has its mode at the origin, then the second-order condition for the parents' optimisation will be satisfied everywhere.

REMARK 7. If $\frac{1}{j+1} > p$, the second-order condition definitely holds if $p(e)$ satisfies the elasticity condition $p''/p' \leq p'/p$. The family of risk functions $p(e) = \lambda \exp(-\gamma e)$, $\lambda, \gamma > 0$, is the only one yielding non-increasing returns and $p''/p' = p'/p$.

REMARK 8. Treatment of the second-order condition in the literature has been somewhat perfunctory (e.g.., cf. Arnott and Stiglitz (1988)), perhaps for two reasons. First, in the simple binary models usually considered, if the separable specification which we have employed here is used alongside strict convexity of $p(e)$, then the second-order condition holds at any local extremum (Arnott and Stiglitz (1988, 390)). Second, the focus of earlier authors was primarily on moral hazard. Hence, as in Arnott and Stiglitz or Helpman and Laffont (1975), they were more concerned with the consequences of the problem being non-concave than with establishing conditions for its concavity.

3.4 *Safety in Numbers*

We can now draw together our earlier results to obtain sufficient conditions for an increase in the size of the protected population to result in increased protective expenditure when safety is a local public good. These are summarised in the following theorem:

THEOREM 3.3. (i) If $CEU^{j+1}(e)$ is concave in a neighbourhood containing both e^j and e^{j+1} and u^r is concave and monotonically decreasing in r, then $e^{j+1} > e^j$. (ii) If $p''/p' \leq p'/p$ and risk levels are such that $r = 0$ is the most likely event in the respective cases with j and $j+1$ protected, then $e^{j+1} > e^j$.

PROOF. (i) From (2.5) we know that $p'(e^j)Cov(r,u^r)_0^j \ / \ p(e^j)\left(1-p(e^j)\right) = 1$ at an interior e^j while, from Corollary 3.2, $(0>)Cov(r,u^r)_0^j > Cov(r,u^r)_0^{j+1}$ if u^r is concave and monotonically decreasing in r. Thus, under these conditions,

$$(3.9) \qquad CEU^{j+1}(e^j) = \frac{p'(e^j)Cov(r,u^r)_0^{j+1}}{p(e^j)\left(1-p(e^j)\right)} - 1 \ > \ \frac{p'(e^j)Cov(r,u^r)_0^j}{p(e^j)\left(1-p(e^j)\right)} - 1 = 0.$$

Hence, if $CEU^{j+1}(e)$ is concave in a neighbourhood containing both e^j and e^{j+1}, then $e^{j+1} >$ e^j. (ii) If risk levels are such that $r = 0$ is the most likely event, then $\frac{1}{j+2} > p$, hence $\frac{1}{j+1} > p$ and, from Corollary 3.1, $Cov(r,u^r)_0^{j+1} < Cov(r,u^r)_0^j$. Thus (3.9) again holds. Moreover, from Theorem 3.2 and Remark 7, if the risk function satisfies $p''/p' \le p'/p$ also, $CEU_{ee}^{j+1}(e) < 0$ holds. Then, by concavity and (3.9), $e^{j+1} > e^j$. Q.E.D.

REMARK 9. By Corollary 3.1.A in the Appendix, we can prove $0 > Cov(r,u^r)_0^j$ $> Cov(r,u^r)_0^{j+1}$, hence (3.9) and its implications, via FSD if $\frac{1}{j+2} > p$. Our covariance decomposition method provides a more general approach because: (a) via Corollary 3.1, we can show $(0>)Cov(r,u^r)_0^j > Cov(r,u^r)_0^{j+1}$ provided only that the weaker inequality $\frac{1}{j+1} \ge p$ holds; (b) even when this inequality does not hold, we can show $(0>)Cov(r,u^r)_0^j > Cov(r,u^r)_0^{j+1}$ via Corollary 3.2 if u^r is concave and monotonically decreasing in r.

4. CONCLUSION.

We have studied an archetypal problem in which members of a population each confront an identical risk and the risks are modelled as a series of i.i.d. Bernoulli trials with endogenous "success" probability. This probability is determined by a decision-maker's protective expenditure, hence this expenditure is a local public good. We have posed and answered the question: how does the size of the population protected by a risk reduction affect the incentive to undertake it? Although such a protection problem is generally non-concave, we show that it is concave under plausible assumptions for some important environments. Moreover, the concavity issue turns out to be intertwined with that of how the protected population's size influences the decision-maker's spending on risk reduction. Although, surprisingly, an increase in this size does not necessarily result in greater protective expenditure being optimal, it does so unambiguously when we can establish that the protection problem is concave. While we have not focused explicitly on moral hazard here, our analysis suggests that an increase in the public good aspect of protection might mitigate the moral hazard problem. This conjecture will need to be elucidated in the context of an explicit model of the insurance market.

The scenario which we have examined has been fairly basic and has relied on a crucial simplifying assumption. This is that the utility associated with a given number of accidents is the same regardless of the size of the protected population. We have deliberately structured the problem in this manner for two reasons. Not only is it realistic, at least in the household context, but also it ensures that any incentive to make greater protective expenditure as the population increases does not depend on an increased willingness to pay because a larger population has more resources than a smaller one. However, as the latter force will be of considerable relevance in some contexts, the implications of relaxing this simplifying assumption, as well as the assumption that the size of the protected population is exogenous, will be pursued elsewhere. Endogeneity of the population size might be important in the household context if parents seek to determine simultaneously the number of their children and the level of protection afforded them.

We specialised much of our discussion to the case of parents seeking to protect some of their children. We saw that the conditions for the protection problem to be concave, hence for the protective expenditure to increase as the number of children protected increases, would then be expected to be satisfied. Thus, in this context, alongside the many reasons which might be advanced for children to prefer to have many siblings, this paper has placed another: there is greater safety in numbers.

APPENDIX.

AN ALTERNATIVE PROOF OF THEOREM 3.1. Using $\binom{j+1}{r} = \binom{j}{r} + \binom{j}{r-1}$ (Pascal's Triangle), $Cov(r,u^r)_0^{j+1} = \sum_0^{j+1}\binom{j+1}{r}p^r(1-p)^{j+1-r}[r-(j+1)p]u^r =$

$$= \sum_0^{j+1}\binom{j}{r}p^r(1-p)^{j+1-r}[r-(j+1)p]u^r + \sum_0^{j+1}\binom{j}{r-1}p^r(1-p)^{j+1-r}[r-(j+1)p]u^r =$$

$$\sum_0^{j}\binom{j}{r}p^r(1-p)^{j+1-r}[r-(j+1)p]u^r + \sum_1^{j+1}\binom{j}{r-1}p^r(1-p)^{j+1-r}[r-(j+1)p]u^r =$$

$$(1-p)\sum_0^{j}\binom{j}{r}p^r(1-p)^{j-r}[r-(j+1)p]u^r + p\sum_1^{j+1}\binom{j}{r-1}p^{r-1}(1-p)^{j-(r-1)}[r-(j+1)p]u^r$$

(using $\binom{j}{r} = 0$ if $r < 0$ or if $r > j$ **).** Now, using $r-(j+1)p = \left(\frac{j+1}{j}\right)[r-jp] - \frac{r}{j}$ and $r-(j+1)p = \left(\frac{j+1}{j}\right)[r-1-jp] + \frac{(j+1-r)}{j}$ in the last equation yields $Cov(r,u^r)_0^{j+1} =$

$$(1-p)\left(\frac{j+1}{j}\right)\sum_0^{j}\binom{j}{r}p^r(1-p)^{j-r}[r-jp]u^r + p\left(\frac{j+1}{j}\right)\sum_1^{j+1}\binom{j}{r-1}p^{r-1}(1-p)^{j-(r-1)}[r-1-jp]u^r$$

$$-(1-p)\sum_0^{j}\binom{j}{r}\left(\frac{r}{j}\right)p^r(1-p)^{j-r}u^r + p\sum_1^{j+1}\binom{j}{r-1}\left(\frac{J+1-r}{j}\right)p^{r-1}(1-p)^{j-(r-1)}u^r$$. As both

$$\left(\frac{j+1-r}{j}\right)\binom{j}{r-1} = \left(\frac{j-(r-1)}{j}\right)\frac{j!}{(j-(r-1))!(r-1)!} = \binom{j-1}{r-1}$$ and

$$\binom{j}{r}\left(\frac{r}{j}\right) = \left(\frac{r}{j}\right)\frac{j!}{r!(j-1-(r-1))!} = \binom{j-1}{r-1},$$

$$-(1-p)\sum_0^j \binom{j}{r}\left(\frac{r}{j}\right)p^r(1-p)^{j-r}u^r + p\sum_1^{j+1}\binom{j}{r-1}\left(\frac{j+1-r}{j}\right)p^{r-1}(1-p)^{j-(r-1)}u^r =$$

$$-\sum_0^j \binom{j-1}{r-1}p^r(1-p)^{j+1-r}u^r + \sum_1^{j+1}\binom{j-1}{r-1}p^r(1-p)^{j+1-r}u^r = 0$$, again using

$\binom{j}{r} = 0$ if $r < 0$ or if $r > j$ to eliminate the terms in u^0 and u^{j+1}. Thus $Cov(r,u^r)_0^{j+1} =$

$$(1-p)\left(\frac{j+1}{j}\right)\sum_0^j \binom{j}{r}p^r(1-p)^{j-r}[r-jp]u^r + p\left(\frac{j+1}{j}\right)\sum_1^{j+1}\binom{j}{r-1}p^{r-1}(1-p)^{j-(r-1)}[r-1-jp]u^r =$$

$$(1-p)\left(\frac{j+1}{j}\right)Cov(r,u^r)_0^j + p\left(\frac{j+1}{j}\right)Cov(r,u^{r+1})_0^j.$$ Q.E.D.

COROLLARY 3.1A. *If* $1/(j+2) > p$, *then* $Cov(r,u^r)_0^j > Cov(r,u^r)_0^{j+1}$.

PROOF.

(a.1) $$Cov(r,u^r)_0^{j+1} - Cov(r,u^r)_0^j =$$

$$\sum_0^{j+1} p^r(1-p)^{j+1-r}\left[\binom{j+1}{r}(r-(j+1)p) - \binom{j}{r}\frac{(r-jp)}{(1-p)}\right]u^r + \frac{p^{j+1}}{(1-p)}(j+1-jp)\binom{j}{j+1}u^r$$

$$= \sum_0^{j+1} p^r(1-p)^{j+1-r}\left[\binom{j+1}{r}(r-(j+1)p) - \binom{j}{r}\frac{(r-jp)}{(1-p)}\right]u^r$$

(using $\binom{j}{j+1} = 0$). Now, $\binom{j+1}{r}(r-(j+1)p) - \binom{j}{r}\frac{(r-jp)}{(1-p)} =$

$\binom{j+1}{r}\left[(r-(j+1)p) - \left(\frac{r-jp}{1-p}\right)\left(\frac{j+1-r}{j+1}\right)\right]$. Let $\left[(r-(j+1)p) - \left(\frac{r-jp}{1-p}\right)\left(\frac{j+1-r}{j+1}\right)\right]$

$= \dfrac{(r-(j+1)p)(1-p)(j+1)-(r-jp)(j+1-r)}{(1-p)(j+1)} \equiv \dfrac{N_1}{D_1}$. Expanding, $N_1 =$

$\big(r-(j+1)p\big)(j+1)-p(j+1)\big(r-(j+1)p\big)-\big(r-(j+1)p\big)(j+1)+r\big(r-(j+1)p\big)-p(j+1-r) =$

$\big(r-(j+1)p\big)^2 - p(j+1-r) = \big(r-(j+1)p\big)^2 -(1-p)p(j+1)+(1-p)pr - pp(j+1-r) =$

$\Big[\big(r-(j+1)p\big)^2 -(1-p)p(j+1)\Big] + p\big(r-(j+1)p\big)$. Resubstituting for N_1 / D_1 into (a.1),

(a.2) $\quad Cov(r,u^r)_0^{j+1} - Cov(r,u^r)_0^{j}$

$$= \frac{1}{(1-p)(j+1)}\sum_0^{j+1} p^r(1-p)^{j+1-r}\binom{j+1}{r}\Big[\big(r-(j+1)p\big)^2-(1-p)p(j+1)\Big]u^r$$

$$+ \frac{p}{(1-p)(j+1)}\sum_0^{j+1} p^r(1-p)^{j+1-r}\binom{j+1}{r}\big(r-(j+1)p\big)u^r$$

$$= p\sum_0^{j+1} p^r(1-p)^{j+1-r}\binom{j+1}{r}\left[\frac{(r-(j+1)p)^2}{(1-p)p(j+1)}-1\right]u^r + \frac{p}{(1-p)(j+1)}Cov(r,u^r)_0^{j+1}.$$

Now, applying the same argument used in the proof of Theorem 3.2 in the text, as

$(1-p)p(j+1) = \mathrm{E}_r\Big[\big(r-(j+1)p\big)^2\Big]$, $p^r(1-p)^{j+1-r}\binom{j+1}{r}\big(r-(j+1)p\big)^2 / (1-p)p(j+1) \equiv \delta(r)$ can be treated as a new probability density for r with cumulative $\Delta(r)$. Then, if $1/(j+2) > p$, $\Delta(r)$ single-crosses the binomial distribution function once from below. Hence, from our earlier argument, $\Delta(r)\,\mathrm{FSD}\ \mathrm{B}(r)$ (where $r = 0,1,2,\ldots, j+1$ now). Hence,

$p\sum_0^{j+1} p^r(1-p)^{j+1-r}\binom{j+1}{r}\left[\dfrac{(r-(j+1)p)^2}{(1-p)p(j+1)}-1\right]u^r < 0$ for any monotonicly decreasing in r function u^r. Inserting this together with $Cov(r,u^r)_0^{j+1} < 0$ in (a.2) yields $Cov(r,u^r)_0^{j+1} - Cov(r,u^r)_0^{j} < 0$.

Q.E.D.

Footnotes.

1. Viscusi (1995, 50) notes : “Regulatory agencies are generally concerned with the risk...In contrast, the number of people exposed to the risk plays a much less prominent role in regulatory decisions. The standard regulatory policy trigger is typically linked to a probability of an adverse outcome as opposed to an expected body count...In the course of the detailed policy analysis prepared for each Superfund site, EPA never assesses the size of the population exposed to the risk.”

2. The seminal contributions of Manser and Brown (1980) and McElroy and Horney (1981) on parents’ non-altruistic preferences have now spawned numerous offsprings. These are reviewed in Lundberg and Pollak (1994, 1995).

3. This assumption lets us abstract from income effects in determining e. Our results would be unchanged if we replaced $-e$ by $-\psi(e)$ for some convex function $\psi(.)$ as in, e.g., Arnott and Stiglitz (1988), Mas-Colell, Whinston and Green (1995) and references therein.

4. A justification for (A.1’) would be that, if parents have chosen the optimal family size as a decision prior to and separate from the protection decision, bereavement must decrease their utility even if it increases the household’s material living standard.

5. Luce and Krantz (1971) provide perhaps the first rigorous justification for CEU.

6. Perhaps the most detailed treatment of the implications of the non-concavity of protection problems is provided by Arnott and Stiglitz (1988).

7. Suppose that the utility function takes the form $U^r\left[x\left(h(n,r)\right), h(n,r), r\ \right]$, where $x\left(h(n,r)\right)$ is the “equivalent” level of household consumption and $h(n,r)$ is the household size, dependent on the initial number of children and the number lost. Then, for a differential change in n, satisfaction of (A.2) can be shown to require $[U_x^r/U_h^r] = -(x')^{-1}$.

8. One instance in which the latter would be true is when the firm is a price -taker selling a fixed quantity of items, the reliability of each of which, $1-p(e)$, can be affected by investment e in some shared finishing machine. If P is the per unit price of the commodity and either there is no loss of goodwill from selling defective items or defective items are detected before sale , the firm’s expected

profit from a batch of size j will be $\pi^j(e) = P\left[\sum_0^j \binom{j}{r} p(e)^r (1-p(e))^r (j-r)\right] - e =$

$P\left[j - \sum_0^j \binom{j}{r} p(e)^r (1-p(e))^r r\right] - e$. To maximise expected profits, the firm would seek to minimise

$-P\sum_0^j \binom{j}{r} p(e)^r (1-p(e))^r r - e$. If defective items could only be detected in use (i.e., the product is an "experience good") and the firm suffered a loss of goodwill from the sale of defective items which was non-linearly and monotonically related to r, the number of defectives sold, and denoted $g(r)$, then it would seek to minimise $-\sum_0^j \binom{j}{r} p(e)^r (1-p(e))^r \left(rP + g(r)\right) - e$. The last two formulations would yield problems identical to those considered in the text.

9. A. Aygangar (1934), R. Frisch (1925), J. Riordan (1937) and V. Romanovsky (1923) are just a few of the authors who have derived or refined recurrence relations for binomial and related distributions. Patil et al (1984) provide a useful bibliography of work in this area. More recent references include Renyi (2005).

REFERENCES.

ARNOTT, R. J. AND J.E. STIGLITZ (1988): "The Basic Analytics of Moral Hazard," *Scandinavian Journal of Economics*, 90, 383-413.

AYGANGAR, A. A. K. (1934): "Note on the Recurrence Formulae for the Moments of the Point Binomial," *Biometrika*, 26, 262-265.

CLARK. D. E., AND J. C. COSGROVE (1990): "Hedonic Prices, Identification, and the Demand for Public Safety," *Journal of Regional Science*, 30, 105-121.

EHRLICH, I., AND G.S. BECKER (1992): "Market Insurance, Self-Insurance and Self-Protection," *Journal of Political Economy*, 80, 623-648.

FRASER, C. D. (1996a): “On Tort as an Implicit Insurance System with State-Dependent Preferences: The Case of Child Mortality Risks,” *International Review of Law and Economics*, 16, 449-460.

_______ (1996b) Safety in Numbers, Public Sector Research Centre Working Paper, University of Leicester.

______(2001), Income Risk, the Tax-Benefit System and the Demand for Children, *Economica*, 68, 105-125.

FRISCH, R. (1925): “Recurrence Formulae for the Moments of the Point Binomial,” *Biometrika,* 17, 165-171.

HEAL, G., AND H. KUNREUTHER (2005). “You Can Only Die Once,” in: H. W. Richardson, P. Gordon, and J. E. Moore II, eds., *The Economic Impacts of Terrorist Attacks* (Cheltenham: Edward Elgar).

HELPMAN, E., AND J.J. LAFFONT (1975): “On Moral Hazard in General EquilibriumTheory,” *Journal of Economic Theory*, 10, 8-23.

JOHNSTON, N. L., S. KOTZ AND A. W. KEMP (1992): *Univariate Discrete Distributions*, Second Edition, Chichester: John Wiley-Interscience.

KOLM, S-C. (1976): “Public Safety,” *American Economic Review*, 66, 382-387.

KUNREUTHER, H., AND G. HEAL (2003): “Interdependent Security,” *Journal of Risk and Uncertainty,* 26, 231-249.

LADD, H. F. (1992): “Population Growth, Density and the Cost of Providing Public Services,” *Urban Studies*, 29, 273-295.

LOHSE, T., ROBLEDO, J.R., AND U. SCHMIDT (2012): “Self-Insurance and Self-Protection as Public Goods,” *Journal of Risk and Uncertainty*, 79, 57-76.

LUNDBERG, S., AND R. A. POLLAK (1994): “Noncooperative Bargaining Models of Marriage,” *American Economic Review (Papers and Proceeding),* 84, 132-137.

______(1995), "Bargaining and Distribution in Marriage," mimeo., University of Washington.

LUCE, D., AND D. H. KRANTZ (1971): "Conditional Expected Utility," *Econometrica*, 31, 253-271.

MANSER, M., AND M. BROWN (1980): "Marriage and Household Decision Making: A Bargaining Analysis," *International Economic Review*, 21, 31-44.

MCELROY, M. B., AND M. J. HORNEY (1981): "Nash Bargained Household Decision," *International Economic Review*, 22, 333-349.

MCGREER, E., AND M. J. MCMILLAN (1993): "Public Output Demand from Alternative Congestion Functions," *Journal of Urban Economics*, 33, 95-114.

MAS-COLELL, A., M. D. WHINSTON AND J. R. GREEN (1995): *Microeconomic Theory*, Oxford: Oxford University Press.

MUERMANN, A., AND H. KUNREUTHER (2008): "Self-Protection and Insurance with Interdependencies," *Journal of Risk and Uncertainty*, 36, 103-123.

PATIL, G. P., M. T. BOSWELL, S. W. JOSHI AND M. V. RATNAPARKHI (1984): *Dictionary and Classified Bibliography of Statistical Distributions in Scientific Work, Volume 1: Discrete Distributions*, Fairland, Maryland: International Co-operative Publishing House.

RENYI, A. (2005): "A Recursive Formula for Moments of the Binomial Distribution," *The College Mathematics Journal*, 36, 68-72.

RIORDAN, J. (1937): "Moment Recurrence Relations for Binomial, Poisson and Hypergeometric Frequency Distributions," *Annals of Mathematical Statistics*, 8, 103-111.

ROMANOVSKY, V. (1923): "Note on the Moments of a Binomial $(p+q)^n$ about its Mean," *Biometrika*, 15, 410-412.

SCHWAB, R. M., AND E. M. ZAMPELLI (1987) "Disentangling the Demand Function from the Production Function for Local Public Services. The Case of Public Safety," *Journal of Public Economics*, 33, 245-260.

VISCUSI, W. K. (1995): “Carcinogen Regulation: Risk Characteristics and the Synthetic Risk Bias,” *American Economic Review (Papers & Proceedings)*, 85, 50-54.