Mitigating Upcoding: Incentive Schemes for Risk Adjusted Payment Contracts in Health Care Markets

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Abstract

Health care providers are almost always universally reimbursed by third party purchasers. As a result, health care purchasers are faced with risk selection challenges. In response, risk adjustment methods are introduced in the reimbursement for providers’ services. However, health care providers under this arrangement have incentives to manipulate the risk element in an attempt to obtain larger payments from the purchasers i.e. the realisation of risk adjuster becomes sensitive to the providers’ upcoding behaviour. In such a scenario, we analyse two types of incentive schemes: one where the treatment intensity, along with the payment schedule, is contractible, and the other in which it is not. We show that both schemes have strong incentive effects and induce honest behaviour by providing positive rents (rewards) to the honest provider compared to the full information case although the channels via which such incentive effects work are quite different. With the non-contractible treatment intensity, the payment schedule is distorted in favour of the provider who treats patients with worse medical conditions. With contractible treatment intensity, however, the intensity level is reduced compared to the first best one although such a contract resembles the one under full-information with a modified ex-post utility function. Our analysis has strong policy implications for health care markets pervaded by upcoding issues.

Keywords: Upcoding, Asymmetric Information, Health Contracts, Risk Adjuster, Treatment Intensity

JEL Classification: I11, D82

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1 Introduction.

In this paper we analyse the problem of designing optimal incentive schemes when the health care providers exhibit tendencies to extract higher reimbursements from the health care purchasers through manipulation of patients’ diagnostic information.

Health care providers in developed nations are almost always universally reimbursed by third party purchasers. In the UK National Health Services (NHS) primary care, the approaches adopted include the introduction of capitation with monitoring of provider’s performance. The primary care trusts and general practices are funded through the arrangement of capitation based on the resource allocation formulas that take into account demographic factors such as sex and age (Carr-Hill and Sheldon, 1992). In the US health care system, fixed budgets and monitoring of performance are widely adopted to improve the efficiency of health care provision. In the US system, diagnosis-based risk adjustment methods have been developed to take into account the difference in case-mix and morbidity of primary care population. The primary reason for introducing such a method is to ensure efficiency and fairness. By making a certain amount of payments to providers, conditional on patients’ characteristics, such a system aims to ensure an efficient allocation of resources in healthcare. Furthermore, by taking into account the morbidity of each individual patient, the system aims to ensure fairness. Unfortunately, such risk-adjustment payment methods also have several serious unintended consequences as discussed below.

A crucial feature of such a payment method is the diagnostic coding of each patient. Payments to providers are then based on the diagnosis information aggregated into a risk score. The data elements used in the risk adjustment systems such as age, sex, and diagnoses, are routinely collected from the administrative records. They therefore suffer from challenges such as incomplete or inaccurate coding of diagnostic data. Coding errors can occur in the recording procedures along the ‘paper trail’: such error sources can include errors occurring in electronic and written records, coder training and experience, facility quality control efforts, and unintentional and intentional coder errors.

However, the ‘error’ that is of very serious concern is not just the one that occurs along the above lines, but rather the one where health care providers deliberately engage in manipulative behaviour in order to increase their risk-adjusted capitation payment. Upcoding occurs whenever the coding of patients’ diagnosis related groups (DRGs) is shifted to another DRG that makes a higher reimbursement to the providers. Examples of such upcoding behaviour include recording additional diagnoses or reclassifying patients’ diagnoses (Weiner et al,
withholding medically necessary diagnosis for patients with complex illnesses to save on costs of treatments (patients ‘dumping’ as pointed out by Ma, 1994) or treating patients with less complex health problems (‘cream skimming’ as pointed out by Barros, 2003). It is indeed very difficult to pinpoint the provider’s discretion to engage in upcoding behaviour since there are so many sources through which coding errors can occur. Further, it is perhaps even more difficult to question providers’ experience, qualification or authority, even if upcoding is suspected. This unobservability and non-verifiability of coding behaviour then create informational advantages to the providers which they can use to manipulate patients’ information in order to secure higher payments from the purchasers. Such informational problem was indeed first noted by Arrow who wrote "Because medical knowledge is so complicated, the information possessed by the physician as to the consequences and possibilities of treatment is necessarily very much greater than that of the patient, or at least as it is believed by both parties. Further, both parties are aware of this informational inequality, and their relation is colored by this knowledge" (Arrow, 1963, page 951). The fact that the tendency for upcoding arising out of physician’s information advantages generates market power was also noted by McQuire (2000).

Such fraudulent behaviour however is hugely wasteful. In the USA, where roughly 17% of GDP is spent on healthcare, a study by Donald Berwick (a former head of the Centres for Medicare and Medicaid Services (CMS)) and Andrew Hackbarth (RAND Corporation), estimated that fraud (and the extra rules and inspections required to fight it) added as much as $98 billion, or roughly 10%, to annual Medicare and Medicaid spending—and up to $272 billion across the entire health system (The Economist, 31 May 2014). According to the report of the Office of Inspector General (OIG) of the US Department of Health and Human Services (HHS), in Fiscal year 2016, the government recovered $2.5 billion as a result of health care fraud judgments, settlements and additional administrative impositions in health care fraud cases and proceedings. Since its inception in 1997, the Health Care Fraud and Abuse Control (HCFAC) Program has returned more than $29.4 billion to the Medicare Trust Funds.

Recently, some empirical research has documented the extent of upcoding in risk-adjustment methods in different countries. According to the estimates provided by Geruso and Layton (2015), in the US market, private medicare plans generate 6% to 16% higher diagnosis-based risk scores than they would generate if payments were not affected by diagnostic codes. Jürges and Köberlin(2015), using data between 2003 and 2010, provide evidence of DRG upcoding in German neonatal care. They conclude that hospitals received an excess reimbursement in the neighbourhood of €114 million during the study period. Barros and Braun (2016)
shows evidence of upcoding in Portugal NHS that led to higher budgets being allocated to the hospitals. O’Malley et al (2005) has examined the potential sources of errors occurring in the international classification of disease (ICD) coding process. They discussed that the errors along the ‘patient trajectory’ relate to the communication among patients and providers: the quality and quantity of information exchanged between patients and admitting clerks or treating clinicians, the clinician’s knowledge and experience with illness and the clinician’s attention to detail are all critical determinants of coding accuracy.

Motivated by above facts, in this paper, we analyse the problem of designing optimal (second-best) contracts between health care purchasers and providers, where the the purchaser’s payoff is based on the realization of risk adjustment variable that depends not only upon the nature of (true) illness but also on the degree of providers’ upcoding behavior. It may seem that the obvious solution to the upcoding problem is to invest in resources for auditing coding procedures and use that information in the contract: if coding behavior could be perfectly observable, the purchaser could indeed require the provider to record the diagnostic codes honestly and the first best could be achieved. However, observing such behaviour is highly costly and the outcomes of such actions are not guaranteed. Indeed, every year the US Department of Justice (DOJ) and the Office of Inspector General (OIG) of the US Department of HHS spend huge amount of resources on health care fraud investigation. They often use the Federal false claims acts as a means to prevent fraudulent claims in the health care industry (see e.g. Lorence et al 2002 and Salcido 2003). Despite the significant level of spending in different programmes (e.g. in the US Medicare integrity programme which was worth $714 million in 2005, 29% of this value was spent on auditing cost reports), the total funding allocated to such fraud detection is only a very small proportion of the entire Medicaid and Medicare spending. This simply points to the fact that not only that a high percentage of fraudulent activities goes undetected, that happens because the full observation of coding behaviour is so tremendously costly that it is almost impossible to implement.

In this paper, we take up a different approach to alleviating the problem of upcoding. Instead of focussing on health care purchaser’s audit and punishment strategies for detecting

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1ACG Morbidity Index could be used as an example of the risk adjuster. In the next section, we will briefly introduce how the index is created, based on the diagnostic information from ACG system.

2For example, In FY 2016, investigations conducted by HHS’ Office of Inspector General (HHS-OIG) resulted in 765 criminal actions against individuals or entities that engaged in crimes related to Medicare and Medicaid (OIG report 2016)

3According to Lorence et al. (2001, page 423) improper payments due to misreporting can cost the government as much as $12 billion implying that the government has clear incentives to prevent such behaviour through appropriate punishment. In fact, in some cases penalties can be as harsh as imposing prison sentences.
and punishing upcoding behaviour, an approach which has recently been undertaken to address the problem of upcoding (see Kuhn and Siciliani, 2013)\(^4\), we investigate whether the purchaser can design a payment scheme that can automatically induce honest behaviour (in equilibrium). In any case, the actual act of audit is not directly carried out by the purchaser. Rather, such acts are carried out by external public inspectors (such as the ones hired by the DoJ and OIG) who conduct such (perhaps random) audits and impose (exogenous) punishment (if fraudulent behaviour is detected).\(^5\) So the dishonest provider still faces the threat of an exogenously imposed punishment if caught by the external auditor, the value of which is known by all parties. By taking into consideration such facts, the purchaser then attempts to design suitable contract terms with regard to providers’ reimbursement schedule and/or patients’ treatment intensity over which they have direct control.

We find several interesting and strong results. First of all, we are able to show that any tendency for upcoding can be eliminated by providing a positive rent or ”reward” for exhibiting honest behaviour regardless of the type of contracts offered (see below for details on the type of contract). What is more interesting is that we show that the reward that could be awarded to the honest provider is directly proportional to the value of potential punishment that a dishonest provider faces if he was caught by the external auditor. This trade-off between potential punishment and the reward is indeed very crucial in our model as purchasers use this ‘carrot and stick’ approach to induce honest behaviour. We show that in equilibrium such reward scheme eliminates any incentives for upcoding as not only that the honest provider receives larger expected utility than the dishonest provider does in equilibrium, his level of his ex-post of utility is also higher than that under the first best.

Such trade-offs between punishment and reward schemes are not uncommon in the health care industry. The potential penalties for upcoding behavior could stem from the threat of government prosecution or result from the concerns about the damage to providers’ reputation.\(^6\) The potential rewards of coding health care information used on reimbursement and the risk of discovery have been widely investigated by authors like Silverman and Skinner\(^7\).

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\(^4\)To the best of our knowledge, Kuhn and Siciliani (2013) is the only paper that analyses upcoding behaviour from a theoretical perspective.

\(^5\)In this sense our model shares the similar spirit as that in the tax evasion literature (Allingham and Sandmo (1971), Srinivasan (1973), Yitzhaki (1974)) which assumes that individuals face a given probability of detection and given tax and penalty function. However, our focus is primarily on designing (second best) incentive contracts.

\(^6\)Interested readers are invited to check out the web page of the Office of Inspector General (https://oig.hhs.gov/) for various interesting information about health care fraud detection, reports, and alerts. It also contains intriguing information about OIG’s ”most wanted fugitives” amongst various others!
We consider two types of payment schemes adopted by the purchaser. One is where the purchaser reimburses the provider based only on the observed realization of the risk adjuster (non-contractible treatment intensity) - in this case the risk of punishment is the only instrument for controlling moral hazard. The other is where the purchaser’s payoff depends not only upon the risk adjuster but also upon the treatment intensity (contractible treatment intensity). Since the latter approach requires that the provider must deliver the treatment package relevant for patients’ diagnoses, the choice of treatment intensity then provides, besides the interactions between potential reward and punishment, another channel for reducing the motivation for upcoding.

We show that both schemes have strong incentive effects as, under both schemes, the honest provider receives the same reward (and obtain higher expected utility in comparison to the full information case whilst the dishonest provider receives zero), although the channels through which these incentives mechanisms work are very different. We show that the distortion in transfer payment and the ex-post utility effect associated with the contract where treatment intensity is non-contractible, lead resources to be allocated to patients with worse medical conditions. In constrast, the contract where remuneration depends also upon the treatment intensity can lead to a reduction in treatment intensity for all patients and hence can lower the benefit of consuming health care services. The optimal contract under this scheme however resembles the full information one (with a modified ex-post utility function) and hence may be socially preferrable. The ultimate choice of the contract therefore boils down to the normative issue of identifying societal preferences as well as determining what is best for the society.

Finally, a word about similarities of our work with some other fields is warranted. There are a few other areas with which our model has some relations. The first one is the costly state verification models that involve either deterministic auditing (e.g. Townsend (1979), Gale and Hellwig (1985), or random auditing (e.g. Mookherji and Png (1989)) while focusing on finite penalties. The second one is the issue of information manipulation that has drawn widespread attention in the financial and accounting literature. Empirical studies in this area show evidence that performance-based compensation provides managers with incentives to manipulate information in order to increase their payment at a cost to shareholders (e.g. Burns and Kedia (2005), Bergstresser and Philippon (2005), Johnson et al. (2005), Sadka
Our assumption of exogenous punishment imposed by the public inspector, and manipulation of information about patients' illness can be contrasted with these.

The rest of the paper is organised as follows. After giving a brief description about risk adjustment system in section 2, we propose our model in section 3. Section 4 provides the full information benchmark. Sections 5 considers the asymmetric information contract with contractible treatment intensity; while section 6 considers the one without. Section 7 concludes. All proofs are in the appendix.

2 A Brief Introduction to the ACG System and the ACG Morbidity Index

The most intensive research on risk adjustment is concentrated on diagnosis-based methods. A leading risk adjustment system, called Adjusted Clinical Group (ACG), has been developed by Jonathan Weiner and his colleagues at Johns Hopkins University (Buntin and Newhouse, 1998). The ACG system has been applied to adjust capitation payment rates and for physician profiling in the United States (see, for example, Adams et al 2002 and Knutson 1998) and Canada (see, for example, Reid et al 2002 and Verhulst et al 2001). In Canada, this case-mix system has been validated as a predictor of subsequent health care expenditures by Reid and his colleagues in Manitoba and British Columbia (Reid et al, 1999 and 2002). Hutchison et al (2006) applied this system to assess the usability of neighbourhood level variations in illness burden.

Johns Hopkins ACG case-mix system is applied to characterize population illness burden at the small area level. This system categorizes diagnostic information from administrative health records (e.g. ICD-9/ICD-9-CM) into 32 clinically meaningful groups (ADGs) based on expected clinical outcomes and resource uses. These 32 ADGs are then further collapsed into 12 ‘collapsed’ ADGs (CADGs). According to the combination of CADGs and the individual’s age/sex structure, the individual is assigned one of Adjuster Clinical Groups (ACGs) that are mutually exclusive terminal groups.

ACG Morbidity Index was created by Reid and co-worker (Reid et al, 1999 and 2002) to convert the ACG assignment at an individual level to a population-based measure of health need, which therefore can be used as the measure of a risk adjuster for the purpose of reimbursement. This approach first assigns ACGs to the users in each cluster. The expected
costs were then obtained by assigning ACG costs (illness weights) that were derived from actual resources used by the users in the ACG category. Hutchison et al (2006) applied and assessed this measure in Ontario scenario. They concluded that the index generated by the ACG case-mix adjustment system can be used to assess the relative need for primary care services and for health services planning at the neighborhood and local community level.

3 The Model

We consider a health care provider who provides health care services to patients within a certain specified area during a particular time horizon, say one year. The area could either be a small geographic area, a health plan or a primary care trust. Given particular medical conditions, the provider records diagnostic codes and chooses the appropriate treatment strategies. The purchaser converts all of the diagnostic codes for all patients within the time period into a summary measure of medical conditions. For the purpose of explanation, we consider ACG Morbidity Index (AMI) as the measure for such medical condition.9

Although AMI itself is a stochastic variable, its realization is measurable as explained above. The purchaser reimburses the provider for his services based on the value of the realization, denoted by $x$, where the realization of $x$ can take any value within the compact interval $[x, \overline{x}]$ where $f(x)$ and $F(x)$ denote respectively the (unconditional) density and distribution functions.10 As AMI indicates the illness burden of patients, higher values of AMI imply worse health status and vice versa. That is, patients with $\overline{x}$ have the worst health status, while those with $x$ have the best. Let $t$ denote the payoff received by the provider, then the payoff function is $t = t(x)$.

**The provider’s utility function.** The provider’s preference is represented by the utility function that depends upon the transfer payment received and the intensity of treatment provided i.e. the utility function is represented by $U(t, y)$ where the variable $y$ measures the treatment intensity and $t$ is the payment received. The treatment intensity corresponds to the diagnostic codes, based on which AMI is created. The intensity variable $y$ can be thought of as an index measurement characterizing all treatment episodes delivered by the provider within the particular time period. Therefore, the intensity index $y$ is also the function of $x$,
say $y = y(x)$. We assume that the provider’s utility increases in the payoff $t(x)$ and decreases in the intensity index $y(x)$. Specifically, we assume that

$$U_t(t, y) > 0, U_{tt}(t, y) < 0, U_y(t, y) < 0 \text{ and } U_{yy}(t, y) < 0.$$  

We also assume that the provider’s utility at the boundary value, i.e. at $x = \bar{x}$, is not less than zero as shown below by assumption $A1$.

**A1** $\bar{U} = U(\bar{t}, \bar{y}) \geq 0$, where $\bar{t} = t(\bar{x})$ and $\bar{y} = y(\bar{x})$.

The above assumption ensures that health care services could be provided to the patients with the worst medical conditions.

**The purchaser’s benefit function.** The purchaser is concerned about the patients’ health gain resulting from the consumptions of the health care services. Patients with AMI $x$ receiving treatment intensity $y$ will have a gain in health status, say $h(x, y)$. We assume that the purchasers’ benefit from health care service is directly related to the patients’ health gain i.e.

$$b(x, y) \equiv b(h(x, y))$$

Further, we assume that the benefit increases and is strictly concave in $y$ and decreases in $x$, i.e.

$$b_y(x, y) > 0, b_{yy}(x, y) < 0, \text{ and } b_x(x, y) < 0$$

The assumption $b_x(x, y) < 0$ implies, for a given a treatment intensity, that worse the patient’s medical conditions are, the lower is the benefit obtained.\(^{11}\) The purchaser maximizes the benefit received from "consuming" the health care services minus the monetary transfers to the provider, namely,

$$V = b(x, y) - t(x)$$

To simplify the moral hazard problem, we assume that the provider’s upcoding behaviour is represented by the variable $a$ taking binary values 0 and 1: i.e. $a = \{0, 1\}$, where $a = 1$ implies that the provider exerts upcoding, whilst $a = 0$ indicates ‘no upcoding’ i.e. honest behaviour.\(^{12}\) The variable $x$ is then distributed with conditional distribution function $F(x|a)$,

\(^{11}\)See proposition 1 and the discussion after, for the sign of the cross derivative $b_{yx}$.

\(^{12}\)Hence, in this model, we do not deal with the extent of upcoding as a choice variable. Kuhn and Siciliani (2013) models the extent of manipulation as a decision variable. However the focus of their model is different to ours as they as they focus on the purchaser’s auditing strategy which directly affects the purchaser’s payoff.
and the density function \( f(x|a) \), that depend not only upon the nature of medical conditions but also upon the variable \( a \).\(^{13}\)

We assume that there is a public inspector, such as the one employed by the United States Department of Justice (DoJ) and/or the Office of Inspector General (OIG) of the Department of Health and Human Services, who can potentially detect any upcoding behaviour. Thus, auditing does not cost the purchaser anything directly in this model.\(^{14}\) Further, the performance of the public inspector is assumed to be imperfect: this is motivated by the fact that perfect auditing that entails the first best solution by eliminating any dishonest behaviour requires auditing at such a high intensity and probability (perhaps with probability one) that it is extremely costly to implement. Instead, DoJ or OIG can only inspect provider’s claims randomly and hence upcoding behavior will be caught only with a probability. We assume that the audit probabilities are known by both the purchaser and the provider.\(^{15}\) Thus, in our model, punishment can be treated as an exogenous variable with a known (expected) value as it is imposed by an external organization rather than the purchaser. We denote this value of punishment (or the utility loss associated with this punishment) by \( \psi \).\(^{16}\) The value of this potential punishment is assumed to be known by both the purchaser and the provider.

The constraints. Any second best mechanism that attempts to induce zero upcoding behaviour must satisfy the following participation constraints

\[
\int_{-x}^{x} U(t,y) f(x|0)dx \geq 0
\]

\[
\int_{-x}^{x} U(t,y) f(x|1)dx \geq 0
\]

\(^{13}\)Thus, similar to the standard models on moral hazard (e.g. Holmstrom, 1979 and Mirrlees, 1999) we assume that, given the degree of upcoding behavior, the distributions of risk adjuster is common knowledge.

\(^{14}\)The issue of collusion between the auditor and the provider can be safely ignored in this model: since the public inspector is hired by the Government directly (and there is no contractual relationship between the auditor and the purchaser), we can safely assume that only honest auditors are hired by the Federal bodies otherwise the government’s reputation will be at stake.

\(^{15}\)Similarly, the issue of non-commitment to auditing strategies (i.e. non-commitment to audit probabilities) does not arise in this model either.

\(^{16}\)Since an outside auditor monitors randomly the upcoding behavior and such behaviour is caught with a certain (known) probability, note that \( \psi \) is in fact the expected value of punishment To illustrate this, suppose that upcoding is caught with a probability \( \pi \) and given that upcoding is a binary decision in our model the subsequent punishment (if detected) is a constant, say \( k \) (and zero otherwise). Hence the expected value of punishment is \( \psi = \pi k \). Such a punishment then entails a fixed amount of utility loss of \( \pi = f(\psi) \) which can also be normalised to \( \psi \) for simplicity.
where (1) refers to the participation constraint of the honest provider and (2) refers to the participation constraint of the dishonest provider. Such a mechanism must also satisfy the incentive compatibility constraint (ICC) where the utility from exerting honest behaviour should (weakly) exceed the utility from exerting upcoding behaviour plus any potential utility loss associated with such a behaviour as measured by the value of punishment. Hence the ICC takes the following form:

$$\int_{x} U(t, y) f(x|0)dx \geq \int_{x} U(t, y) f(x|1)dx + \psi$$

Note that the slightly non-standard formulation of the incentive compatibility constraint in this model. In contrast to the standard incentive theory that focuses on the trade-off between inducing higher output and higher effort (measured in terms of ‘costs’ or agent’s disutility from performing higher effort) e.g. as in LaFont and Martimort (2002), here the upcoding behaviour is the ‘effort’ the purchaser hopes to avoid.\(^{17}\) Thus, this incentive compatibility constraint implies as if the purchaser will reward the honest provider in order to induce zero upcoding behaviour.\(^{18}\) While, we acknowledge that setting up this monitoring system will incur fixed costs, without any loss of generality, we ignore this fixed setting up cost as it does not alter our results.

To investigate the incentive effect of the mechanism, we consider two types of contractual environments.

1. **Contractible treatment intensity.** In this scenario, the payment scheme is based on the realization of \(x\) and the treatment intensity is contractible, which means the intensity index \(y\) must be contingent upon the realization of \(x\).\(^{19}\)

2. **Non-contractible treatment intensity.** In this second scenario, the payment scheme is based on the realization of \(x\) but the treatment intensity is not contractible, which for instance is determined based on medical professionals’ experiences during the previous periods.

\(^{17}\)Writing the constraint alternatively as $$\int_{x} U(t, y) f(x|0)dx - \psi \geq \int_{x} U(t, y) f(x|1)dx$$

we see that this implies as if the provider’s utility is quasi-linear in the value of punishment (‘disutility of punishment’): The cost of ‘effort’ in exerting honest behaviour is strictly positive whereas it is zero in case of dishonest behaviour. Such quasi-linear formulation of utility function is quite standard in the contract theory.

\(^{18}\)Indeed, we show that in equilibrium, the honest provider receives an expected utility exactly equaling \(\psi (\psi > 0)\) whilst the dishonest one receives zero expected utility (see sections 5 and 6).

\(^{19}\)In principle, the contract requires implicitly a monitoring system that ensures that resources used are consistent with the recorded diagnoses. In practice, the monitoring is conducted by comparing medical records with the use of resource records.
Timing. The timing of the game is as follows. To start with, the purchaser designs a contract to induce honest behavior. If the provider chooses to accept the contract, he will decide whether or not to conduct upcoding and will treat patients with the chosen treatment intensities. Finally, the contracts are implemented. For the first contract, payment is based on both the realization of $x$ and the treatment intensity $y$, and for the second contract, payment is made based only on the realization of $x$.

4 The Full Information Benchmark

In this case, since the provider’s upcoding behavior (i.e. recording of patient’s information) is perfectly observable, the purchaser will require the provider to behave honestly. The distribution of $x$ is hence affected only by the nature of the patients’ medical conditions which are under scrutiny. We denote this unconditional density function by $f(x)$. The purchaser’s problem is to maximize the expected benefit subject to the participation constraint that ensures that the provider is willing to provide the necessary care services. Namely, the maximization problem (P) is:

$$\max_{t(x), y(x)} \int_{x}^{x^*} \left[ b(x, y) - t(x) \right] f(x) \, dx$$

$$\text{s.t.} \int_{x}^{x^*} U(t, y) f(x) \, dx \geq 0$$

Since in the purchaser’s objective function the transfer $t(x)$ reduces her benefits and the provider’s expected utility increases in $t(x)$, the purchaser will design the payment scheme such that participation constraint binds: this is also verified in the proof of proposition 1 in the appendix. The properties of the optimal solution are summarized in the following proposition.

**Proposition 1** Given that the provider’s upcoding behavior is perfectly observable and the distribution of the random variable $x$ is common knowledge with the density function $f(x)$, the purchaser is able to design an optimal contract such that:

1. The first best trajectories of transfer $t(x)$ and treatment intensity index $y(x)$ are deter-
mined by the following conditions:

\[ b_y(x, y) = -\frac{U_y(t, y)}{U_t(t, y)} \]
\[ \int_x^\pi U(t, y) f(x) \, dx = 0 \]

2. Given \( \bar{U} \geq 0 \) (assumption A1), the maximum value of the provider’s utility function is non-decreasing in \( x \). That is, for any \( x \in [\underline{x}, \bar{x}] \),

\[ \frac{dU(t, y)}{dx} \geq 0 \quad \text{for } \bar{U} \geq 0 \]

3. The sign of the slopes of the trajectory \( t(x) \) and \( y(x) \) depends on that of \( b_{yx} \). That is,

\[ \frac{dt(x)}{dx}, \quad \frac{dy(x)}{dx} \begin{cases} > 0 \text{ if } b_{yx}(x, y) > 0 \\ < 0 \text{ if } b_{yx}(x, y) < 0 \end{cases} \]

**Proof.** See the Appendix. \( \blacksquare \)

Part 1 of the proposition states that the first best trajectories must satisfy the necessary condition that the purchaser’s marginal benefit of the treatment must be equal to the marginal rate of substitution between intensity \( y(x) \) and transfer \( t(x) \) for the provider. Part 2 of the proposition says that when the boundary value \( \bar{U} \) is larger than zero, the provider’s utility monotonically increases in the realization of \( x \): this indicates that the provider has the motivation in treating patients with worse medical conditions. Had the boundary value been set to zero, the ex-post utility would be extracted to zero for all values of \( x \). Recall that \( b_{yx} \) indicates the change of marginal benefit resulting from treatment in medical conditions. Therefore, \( b_{yx} > 0 \) means treating patients with worse medical conditions will produce more marginal benefit to the purchaser (or more marginal health status gain for population).

Finally, part 3 of the proposition reveals that, while both the \( t(x) \) and \( y(x) \) change with \( x \) in the same direction, their slopes depend on the sign of \( b_{yx} \). If the marginal benefit from treatment is larger for those with worse medical conditions the contract will allow more transfer to the provider, along with more intensive treatment, who provides services to patients with worse conditions. Similarly, if the marginal benefit from treatment decreases in \( x \) the payoff also decreases in \( x \).
Obviously, the purchaser in practice prefers the former to the latter. We thereafter restrict our discussion on the case where treating worse patients results in a larger marginal benefit. Formally, we assume the purchaser’s preference is characterized by \( b_{yx} > 0 \).

5 Asymmetry of Information—Contractible Treatment Intensity

We now consider the situation where upcoding (or patients’ data recording) behavior is not observable to the purchaser. If the first best contract were to be offered, the provider would have an incentive to exert upcoding behavior that results in the right skew in AMI distribution. To solve this moral hazard problem, the purchaser must now solve the following problem:

\[
\begin{align*}
\max_{t(x), y(x)} & \int \left[ b(x, y) - t \right] f(x|0) dx \\
\text{s.t.} & \int U(t, y) f(x|1) dx \geq 0 \\
& \int U(t, y) f(x|0) dx \geq \int U(t, y) f(x|1) dx + \psi
\end{align*}
\]

where the first constraint is the participation constraint, given upcoding behavior, which says that the provider’s expected utility cannot be less than zero, while the second constraint is the incentive compatibility constraint that ensures that the provider’s expected utility with no upcoding behavior must not be less than that with upcoding. Note that there is the other participation constraint \( \int U(t, y) f(x|0) dx \geq 0 \) for the honest provider. We will show that this is satisfied in equilibrium. Solution to the above problem is summarized in proposition 2.

**Proposition 2** In the presence of moral hazard, the optimal contract offered to the provider is equivalent to the first best outcome for a provider with the ex-post utility \( U(t, y) - \psi \). That is, given the boundary condition \( U \geq \psi \), all features stated in proposition 1 still hold except

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\(^{20}\)Henceforth, we restrict our discussion on the case where \( b_{yx} > 0 \) as this is, in practice, purchaser’s preference. Similar approach could be applied to the assumption of \( b_{yx} < 0 \).
that the optimal trajectories \( t(x) \) and \( y(x) \) are now determined by:

\[
b_y (x, y) = -\frac{U_y (t, y)}{U_t (t, y)} \\
\int \limits_{x}^{\pi} U (t, y) f (x|0) dx = \psi
\]

**Proof.** See the Appendix. ■

Thus, there are two interpretations to the solution characterized by this proposition. In the first interpretation, the provider is characterized by the *ex-post* utility function \( U(t, y) \) and the information structure is asymmetric, while the second one is the full information contract offered to the provider with *ex-post* utility function \( U(t, y) \) minus \( \psi \). In other words, the first best contract designed for the provider with *ex-post* utility \( U(t, y) - \psi \) can eliminate the moral hazard problem associated with the provider with *ex-post* utility \( U(t, y) \). To see that, rewrite \( \int \limits_{x}^{\pi} U (t, y) f (x|0) dx = \psi \) as:

\[
\int \limits_{x}^{\pi} [U (t, y) - \psi] f (x|0) dx = 0
\]

Then define the following utility function:

\[
W (t, y) = U (t, y) - \psi
\]

Since \( W_t (t, y) \equiv U_t (t, y) \) and \( W_y (t, y) \equiv U_y (t, y) \), the solution (i.e. the equations in part 2 of proposition 2) are further rewritten as:

\[
b_y (x, y) = -\frac{W_y (t, y)}{W_t (t, y)} \\
\int \limits_{x}^{\pi} W (t, y) f (x|0) dx = 0
\]

As indicated in proposition 1, these equations entail the first best mechanism for the provider with ex-post utility \( W (t, y) \).

**Insert Figure 1 here**

Figure 1 indicates how the trajectories \( t(x) \) and \( y(x) \) are distorted in the presence of moral hazard, compared to the full information case. From the definition of \( W(t, y) \), we learn that,
in $t - y$ space, the indifference curve of $U(t, y)$ is on North-West side of $W(t, y)$. $U(t, y)$ stands for the indifference curve with moral hazard and $W(t, y)$ for that with full information. For a given realization of $x$, any intensity index, say $y_0(x)$, corresponds to transfers $t_A(x)$ on the curve $W(t, y)$ and $t_B(x)$ on the curve $U(t, y)$. As $t_B(x) > t_A(x)$, this indicates that the asymmetry of information distorts transfer towards the provider if he exerts honest behaviour. That is to say, in the presence of moral hazard, the purchaser will make more transfers to the provider in an attempt to induce the honest behaviour. Similarly, any transfer, say $t_0(x)$, corresponds to two intensity indices $y_A(x)$, $y_B(x)$ with $y_A(x) > y_B(x)$. Synthetically, the second best outcomes in the presence of moral hazard are characterized by raising transfers to the provider and reducing the treatment intensity, compared to the full information scenario.

We can understand the distortion by investigating the constraints. Binding participation and incentive compatibility constraints for the dishonest provider (see the proof of proposition 2 in the appendix) implies, we have:

$$\int_\mathcal{X} U(t, y) f(x|0) \, dx = \psi$$

Thus the honest provider receives a rent or reward equaling $\psi > 0$ compared to the scenario under full information where her rent was zero. This additional utility gain result from either more transfer or less treatment intensity or both. Despite that, the honest provider is still indifferent between upcoding and non-upcoding although in equilibrium the honest provider’s expected utility is strictly larger than that of the dishonest provider.

6 Asymmetry of Information—Non-Contractible Treatment Intensity

When the treatment is not contractible, transfer arrangement is the only instrument for the purchaser to cope with moral hazard. The determination of treatment intensity then depends upon the provider’s professional experiences. Professional reputation may then be the main consideration in choosing treatment intensity. In this situation, the purchaser solves the

---

21 The slopes of the indifference curves are positive: $dU(t, y) = U_t(t, y) dt + U_y(t, y) dy = 0 \Rightarrow \frac{dy}{dx} = -\frac{U_t(t, y)}{U_y(t, y)} > 0$. 

---

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following problem:

\[
\max_{t(x)} \int_{\mathbb{R}} [b(x, Y) - t] f(x) dx \\
\text{s.t. } \int_{\mathbb{R}} U(t, Y) f(x|1) dx \geq 0 \\
\int_{\mathbb{R}} U(t, Y) f(x|0) dx \geq \int_{\mathbb{R}} U(t, Y) f(x|1) dx + \psi
\]

where the treatment intensity \( Y \) is now assumed to be of constant value as it is non-contractible.

As in the previous section, the first constraint is a participation constraint for the dishonest provider and the second one is incentive compatibility constraint. The participation constraint for the honest provider is omitted: we can easily verify that this is satisfied in equilibrium. To solve this problem we first introduce the following Lemma.

**Lemma** Given that the density functions \( f(x|0) \) and \( f(x|1) \) satisfy the Monotone Likelihood Ratio Property (MLRP), i.e. \( \frac{d}{dx} \left( \frac{f(x|1)}{f(x|0)} \right) > 0 \), there exists a value of \( x \), say \( x_0 \in [\underline{x}, \bar{x}] \), such that:

\[
\begin{align*}
\frac{f(x|1)}{f(x|0)} & < 1 \text{ if } x \in [\underline{x}, x_0) \\
& = 1 \text{ if } x = x_0 \\
& > 1 \text{ if } x \in (x_0, \bar{x}]
\end{align*}
\]

**Proof.** See the Appendix. ■

According to Proposition 2 of Milgrom (1981), the assumption that \( f(x|0) \) and \( f(x|1) \) satisfy MLRP indicates that the distribution conditional upon behaviour 1 is skewed to the right. Lemma 1 implies \( f(x|1) \) crosses \( f(x|0) \) only once. The solution to the above maximization problem is summarized in proposition 3.

**Proposition 3** In the presence of moral hazard the optimal contract with non-contractible treatment intensity entails that:

1. The second best trajectory \( t(x) \) is determined from the equations:

\[
\frac{1}{U_t(t, Y)} = \lambda \frac{f(x|1)}{f(x|0)} + \mu \frac{f(x|0) - f(x|1)}{f(x|0)} \\
\int_{\mathbb{R}} U(t, Y) f(x|0) dx = \psi
\]
\[
\int_{\underline{x}}^{\bar{x}} U(t,Y) f(\cdot|1) \, dx = 0
\]

2. Given that MLRP holds, there exists a value of \( x \), say \( x_0 \in (\underline{x}, \bar{x}) \), such that:

\[
t^{SB}(x) \begin{cases} 
< t^*(x) & \text{if } x < x_0 \\
= t^*(x) & \text{if } x = x_0 \\
> t^*(x) & \text{if } x > x_0 
\end{cases}
\]

where \( t^*(x) \) and \( t^{SB}(x) \) are the first and second best trajectories respectively.

**Proof.** See the Appendix. ■

Thus proposition 3 indicates that when the treatment intensity is non-contractible, the second best trajectory \( t^{SB}(x) \) is *distorted in favour* of the provider who delivers services to patients with worse medical conditions. This result is illustrated in Figure 5-2. Point A corresponds the case that \( x = x_0 \), where the first best transfer is equal to the second best transfer, i.e. \( t^*(x_0) = t^{SB}(x_0) \). If \( x < x_0 \), \( \frac{1}{U_t} \) is on the left side of A, say point B, where the value of \( \frac{1}{U_t} \) is less than that at A. Correspondingly, \( t^{SB}(x)|_{x<x_0} < t^*(x) \). Similarly, at point C, the value of \( \frac{1}{U_t} \) is larger than that at A and \( t^{SB}(x)|_{x>x_0} > t^*(x) \).

Figure 5-3 presents the optimal trajectory for the transfer \( t(x) \) under full and asymmetric information. The provider receives less transfer if \( x < x_0 \) and more if \( x > x_0 \), compared to the full information case. This contract then distorts transfer in favour of those who provide services to patients with worse medical conditions.

This payment arrangement results in the incentive effect that induces honest behaviour. Under the asymmetry of information, the provider who behaves honestly is rewarded by receiving an expected utility that is equal to \( \psi \), whilst under the full information his expected utility is brought down to zero. Formally, the expected utility under asymmetric information is given by \( \int_{\underline{x}}^{\bar{x}} U(t,Y) f(\cdot|0) \, dx = \psi \) and, that under full information is given by \( \int_{\underline{x}}^{\bar{x}} U(t,Y) f(\cdot|0) \, dx = 0^{22} \).

---

\(^{22}\)In the benchmark section, the density function is \( f(x) \). For the purpose of comparison here we rewrite it as \( f(x|0) \). This does not change the results because both represent the distribution without ‘upcoding’ behaviour.
For $U_i(t, Y) > 0$, in comparison with the first best case, this contract characterizes that the ex-post utility curve is steeper. That is to say, the ex-post utility increases for any $x \in (x_0, \bar{x}]$ and decreases for any $x \in [x, x_0]$. Whilst the expected utility for the honest provider has been raised to be equal to $\psi$, the ex-post utility he received will be less (or more) than that in the first best if the realization of $x$ is small (or large) enough. This mechanism offers the provider the incentive to treat patients with worse medical conditions.

7 Concluding Comments

Risk selection is an important concern in health policy. The health authority, health insurers, and private employers who purchase care services for their employees are all faced with risk selection challenges. In response, risk adjustment methods have been introduced in the reimbursement for services. Whilst the risk adjustment approach helps to ensure that the morbidity of individual patients is taken into account, the health care providers under this payment arrangement have the tendency to manipulate patients’ diagnostic codes in order to secure greater payoffs from the purchasers. Such upcoding behavior is possible because of the uncertainties involved with assessing inappropriate diagnoses.

Faced with such problems, the purchaser uses certain pieces of information to alleviate the problem of moral hazard. ACG Morbidity Index, which is developed based on diagnostic information, is often applied as a proxy for measuring morbidity burden and hence, as the risk adjuster. Whilst the index itself is a stochastic variable due to the influence of factors such as the uncertainty of medical conditions, the coding inaccuracy and so forth, its realization however can be measured by collecting health care administrative data (e.g. ICD-9 codes). Another useful piece of information is the distribution of the index. Based on past experiences, this distribution pattern is assumed to be common knowledge.

In this scenario, we have demonstrated that the problem of upcoding can be alleviated if an appropriate contract is offered to the provider. We have considered two types of contracts: one in which the treatment intensity is contractible and the other in which it is not. In the first type of contract, the purchaser designs a payment scheme and imposes appropriate treatment intensity as the regulatory instruments whilst the second type of contract is based only on the payment scheme. We show that both types of schemes have strong incentive effects whereby both schemes offer the provider with incentives to record diagnostic codes honestly. Indeed,
under both types of incentive schemes, the honest provider receives positive rent (or reward) compared to the full information scenario where he receives nothing. Furthermore, under both incentive mechanisms, the rewards that the honest providers receive are the same which equal the value of expected punishment, $\psi$ ($\psi > 0$) where the value of $\psi$ depends upon how the society (i.e. the purchasers and the providers) measures the value of potential punishment if any diagnostic codes manipulation behaviour is detected. In this sense, both mechanisms have the same strengths of incentives as they both imply the same increase in expected utilities.

From the standpoint of incentive effects, the two types of contracts achieve their goals of inducing honest behavior via different channels. The first contract provides incentives to the provider to act honestly either by reducing the treatment intensity relative to the first best for a certain payment level, or by increasing payments relative to the first best for a certain treatment intensity level, or by some combination of both. Such a tendency to lower the treatment intensity along with perhaps increased payments in an attempt to induce zero upcoding, however, can have a negative impact on the society as it potentially lowers the benefit from enjoying health care services. On the other hand, the second contract has the feature that provider receives higher (lower) payments (relative to the first best scenario) when patients with worse (better) medical conditions are treated. Thus, there is a bias in transfer payments (relative to the first best) that lead to resources being allocated to the treating patients with worse medical conditions. Depending upon societal perspective and preference, this can be indeed be a ‘good’ outcome.

As far as the problem of the implementation of contract is concerned, we believe that the first mechanism which is based on both the payment scheme and the treatment intensity, is perhaps superior. Given that the second mechanism is related to the distribution conditional upon upcoding behavior, under this mechanism, the payment scheme depends not only upon the distribution without upcoding but also upon the conditional distribution, which is assumed to be known at the beginning of the game. The provider in practice therefore still has the discretion to shift the conditional distribution through varying the degree of upcoding behavior, even after he has accepted the contract. Such a tendency then will disrupt the equilibrium. On the other hand, under the first mechanism, since the optimal solution to the second best contract with moral hazard is equivalent to the first-best solution with a modified utility function (i.e. $U(t, y) - \psi$), any distributional concerns arising out of upcoding behaviour is fully eliminated. Given that the distribution without upcoding reflects the nature of true medical conditions, it can then be determined on the basis of population medical history. However, this contract requires that the treatment intensity must correspond to the realiza-
tion of morbidity index (i.e. $x$). Hence it implicitly assumes that the treatment intensity is observable. In practice however, the treatment intensity usually is not observed with accuracy and hence monitoring the intensity level may still be necessary to ensure that the treatment synchronizes with the diagnostic information, which will incur extra monitoring costs.

**References**


Appendix

Proof of Proposition 1.

The purchaser solves the problem:

$$\max_{t(x), y(x)} \int_{x}^{L} [b(x, y) - t] f(x) dx$$

s.t. $$\int_{x}^{L} U(t, y) f(x) dx \geq 0$$
The combined functional $\mathcal{L}(x)$ is introduced as:

$$\mathcal{L}(x) = \int_{\underline{x}}^{\bar{x}} [b(x, y) - t] f(x) \, dx + \lambda \int_{\underline{x}}^{\bar{x}} U(t, y) f(x) \, dx$$

The optimal solution satisfies the functional derivative equations $\frac{\delta \mathcal{L}(x)}{\delta t(x)} = 0$ and $\frac{\delta \mathcal{L}(x)}{\delta y(x)} = 0$. Namely,

$$- f(x) + \lambda U_t(t, y) f(x) = 0 \quad (3)$$

$$b_y(x, y) f(x) + \lambda U_y(t, y) f(x) = 0 \quad (4)$$

Or

$$\lambda = \frac{1}{U_t(t, y)} > 0$$

indicating that the participation constraint binds i.e.

$$\int_{\underline{x}}^{\bar{x}} U(t, y) f(x) \, dx = 0 \quad (5)$$

From equation (4) we have

$$b_y(x, y) = - \frac{U_y(t, y)}{U_t(t, y)} \quad (6)$$

Next we investigate the slopes of trajectory $t(x)$ and $y(x)$. Integrating equation (5) by part, obtain:

$$U(t, y) F(x) \bigg|_{\underline{x}}^{\bar{x}} - \int_{\underline{x}}^{\bar{x}} F(x) \left[ U_t(t, y) \frac{dt}{dx} + U_y(t, y) \frac{dy}{dx} \right] \, dx = 0$$

Note that $F(\underline{x}) \equiv 0$ and $F(\bar{x}) \equiv 1$. Hence we have:

$$\int_{\underline{x}}^{\bar{x}} F(x) \left[ U_t(t, y) \frac{dt}{dx} + U_y(t, y) \frac{dy}{dx} \right] \, dx = \bar{U}$$

where $\bar{U} \equiv U(t(\bar{x}), y(\bar{x}))$ is the ex-post utility as $x = \bar{x}$. And $\bar{x} \geq 0$ by assumption. Because of $F(x) > 0$ for any $x \in (\underline{x}, \bar{x}]$, we have:

$$U_t(t, y) \frac{dt}{dx} + U_y(t, y) \frac{dy}{dx} \geq 0 \text{ according as } \bar{U} \geq 0 \quad (7)$$
Now, \( \{ U_t (t, y) \frac{dt}{dx} + U_y (t, y) \frac{dy}{dx} \} = \frac{dU(t,y)}{dx} \). Hence
\[
\frac{dU(t,y)}{dx} \geq 0
\]

Therefore (7) implies:
\[
\frac{dt}{dx} \geq b_y \frac{dy}{dx} \tag{8}
\]

taking derivative of equation (6) with respect to \( x \), we get:
\[
\left\{ \begin{array}{l}
U_{yy} (t, y) \frac{dy}{dx} + U_{yt} (t, y) \frac{dy}{dx} + b_y \left[ U_{tt} (t, y) \frac{dt}{dx} + U_{ty} (t, y) \frac{dy}{dx} \right] \\
+ U_t (t, y) \left[ b_{yy} (t, y) \frac{dy}{dx} + b_{yx} (x, y) \right]
\end{array} \right\} = 0
\]

or
\[
\left\{ \begin{array}{l}
\frac{dy}{dx} [U_{yy} (t, y) + b_y U_{ty} (t, y) + U_t (t, y) b_{yy} (t, y)] \\
+ \frac{dt}{dx} [U_{yt} (t, y) + b_y U_{tt} (t, y)]
\end{array} \right\} = -U_t (t, y) b_{yx} (x, y)
\]

Substituting (8) into the equation above, we have:
\[
\frac{dt}{dx} \left[ U_{yy} (t, y) + b_{yy} (t, y) \right] \geq -U_t (t, y) b_{yx} (x, y) \tag{9}
\]

or
\[
\left\{ \begin{array}{l}
\frac{dy}{dx} \left[ U_{yy} (t, y) + b_y U_{ty} (t, y) + U_t (t, y) b_{yy} (t, y) \right] \\
+ b_y U_{yt} (t, y) + b^2_y U_{tt} (t, y)
\end{array} \right\} \leq -U_t (t, y) b_{yx} (x, y) \tag{10}
\]

Note that by assumption each term in the square bracket of equations (9) and (10) are negative. Therefore, from (8) and (10), we have:
\[
\frac{dy}{dx} > 0 \text{ and hence } \frac{dt}{dx} > 0 \text{ if } b_{yx} (x, y) > 0
\]

And from (8) and (9), we have:
\[
\frac{dt}{dx} < 0 \text{ and hence } \frac{dy}{dx} < 0 \text{ if } b_{yx} (x, y) < 0
\]

Q.E.D.

Proof of Proposition 2.
The Lagrange for this problem is:

\[
\mathcal{L}(x) = \left\{ \int_0^\infty \left[ b(x, y) - t \right] f(x|0) \, dx + \lambda \int_0^\infty U(t, y) f(x|1) \, dx \right. \\
+ \mu \int_0^\infty U(t, y) \left[ f(x|0) - f(x|1) \right] \, dx - \mu \psi \right\}
\]

The first order conditions are:

\[
\frac{\partial \mathcal{L}(x)}{\partial t} = 0 \Rightarrow -f(x|0) + \lambda U_t(t, y) f(x|1) + \mu U_t(t, y) \left[ f(x|0) - f(x|1) \right] = 0
\]

and

\[
\frac{\partial \mathcal{L}(x)}{\partial y} = 0 \Rightarrow \left\{ \begin{array}{c} b_y(x, y) f(x|0) + \lambda U_y(t, y) f(x|1) \\
+ \mu U_y(t, y) \left[ f(x|0) - f(x|1) \right] \end{array} \right\} = 0
\]

Rewriting the above first order conditions, we have:

\[
\frac{f(x|1)}{U_t(t, y)} = \lambda \frac{f(x|0)}{f(x|0)} + \mu \frac{f(x|0) - f(x|1)}{f(x|0)}
\]

and

\[
\frac{b_y(x, y)}{U_y(t, y)} = -\lambda \frac{f(x|1)}{f(x|0)} - \mu \frac{f(x|0) - f(x|1)}{f(x|0)}
\]

Next we show that \( \lambda, \mu > 0 \). Integrating equation (11) obtain:

\[
\int_0^\infty \frac{f(x|0)}{U_t(t, y)} \, dx = \lambda \int_0^\infty f(x|1) \, dx + \mu \int_0^\infty \left[ f(x|0) - f(x|1) \right] \, dx
\]

Noting that \( \int_0^\infty f(x|0) \, dx = \int_0^\infty f(x|1) \, dx = 1 \) and \( \int_0^\infty \frac{f(x|0)}{U_t(t, y)} \, dx \equiv E \left( \frac{1}{U_t(t, y)} \right) \), we have:

\[
\lambda = E \left( \frac{1}{U_t(t, y)} \right) > 0
\]

Substituting equation (13) into (11), we have:

\[
\frac{1}{U_t(t, y)} = E \left( \frac{1}{U_t(t, y)} \right) \frac{f(x|1)}{f(x|0)} + \mu \frac{f(x|0) - f(x|1)}{f(x|0)}
\]

Multiplying both sides of the equation by \( U(t, y) \) and \( f(x|0) \), and then integrating it, we
have:
\[
\int_{\mathcal{X}} U(t, y) f(x|0) \, dx = \left\{ \int_{\mathcal{X}} E \left( \frac{1}{U_t(t, y)} \right) U(t, y) f(x|1) \, dx \right\} + \mu \int_{\mathcal{X}} U(t, y) [f(x|0) - f(x|1)] \, dx
\]

(15)

From Kuhn-Tucker condition:
\[
\mu \left\{ \int_{\mathcal{X}} U(t, y) [f(x|0) - f(x|1)] \, dx - \psi \right\} = 0
\]
or
\[
\mu \psi = \mu \int_{\mathcal{X}} U(t, y) [f(x|0) - f(x|1)] \, dx
\]

(16)

Equations (15) and (16) therefore yield:
\[
\mu \psi = \int_{\mathcal{X}} U(t, y) f(x|0) \, dx - E \left( \frac{1}{U_t(t, y)} \right) \int_{\mathcal{X}} U(t, y) f(x|1) \, dx
\]

Using the definition covariation\textsuperscript{23}, we have:
\[
\mu \psi = \int_{\mathcal{X}} \frac{U(t, y)}{U_t(t, y)} f(x|0) \, dx - E \left( \frac{1}{U_t(t, y)} \right) \int_{\mathcal{X}} U(t, y) f(x|1) \, dx
\]
\[
> \int_{\mathcal{X}} \frac{U(t, y)}{U_t(t, y)} f(x|0) \, dx - E \left( \frac{1}{U_t(t, y)} \right) \int_{\mathcal{X}} U(t, y) f(x|0) \, dx
\]
\[
= E \left( \frac{U(t, y)}{U_t(t, y)} \right) - E \left( \frac{1}{U_t(t, y)} \right) E(U(t, y))
\]
\[
= Cov \left( U(t, y), \frac{1}{U_t(t, y)} \right)
\]

Here we note that, from incentive compatibility constraint,
\[
\int_{\mathcal{X}} U(t, y) f(x|1) \, dx < \int_{\mathcal{X}} U(t, y) f(x|0) \, dx.
\]

\textsuperscript{23}That is, the covariation of random variables $\xi, \eta$ is defined as
\[
cov(\xi, \eta) = E[(\xi - E\xi)(\eta - E\eta)] = E(\xi\eta) - E\xi E\eta
\]
Taking into account the fact that $U(t, y)$ and $\frac{1}{U_t(t, y)}$ change in the same direction, we have:

$$\mu \psi = Cov \left( U(t, y), \frac{1}{U_t(t, y)} \right) > 0 \Rightarrow \mu > 0$$

Therefore, both the participation constraint and incentive compatibility constraint bind. The parameter $\lambda$ and $\mu$, and trajectories $t(x)$ and $y(x)$ can then be determined by using these constraints (holding with equality) and the first order conditions given by equations (11) and (12). Equations (11) and (12) together yield:

$$b_y(x, y) = -\frac{U_y(t, y)}{U_t(t, y)}$$

(17)

and the both constraints holding with equality imply:

$$\int_{x}^{x} U(t, y) f(x|0) dx = \psi$$

(18)

Hence the trajectories $t(x)$ and $y(x)$ are determined by using the equation above. Next we show this solution is equivalent to the first best. Rewrite equation (18) as:

$$\int_{x}^{x} [U(t, y) - \psi] f(x|0) dx = 0$$

and define the utility function:

$$W(t, y) \equiv U(t, y) - \psi$$

Note that $W_t(t, y) \equiv U_t(t, y)$ and $W_y(t, y) \equiv U_y(t, y)$. Therefore the optimal trajectories are determined by the equations below:

$$\int_{x}^{x} W(t, y) f(x|0) dx = 0$$

$$b_y(x, y) = -\frac{W_y(t, y)}{W_t(t, y)}$$

Noting that $f(x|0) \equiv f(x)$ (as both represent the density function without upcoding behaviour), it immediately follows that the first best mechanism can be used in the case. Therefore, given the boundary condition $\bar{W} \equiv \bar{U} - \psi \geq 0$, all of results in proposition 1 can be achieved here.  

Q.E.D.

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Proof of Lemma 1.

Given MLRP held, i.e. \( \frac{d}{dx} \left( \frac{f(x|1)}{f(x|0)} \right) > 0 \), there are three possible relations between \( f(x|1) \) and \( f(x|0) \), which are listed below:

1. \( f(x|1) > f(x|0) \) for any \( x \in [x, \bar{x}] \)
2. \( f(x|1) < f(x|0) \) for any \( x \in [x, \bar{x}] \)
3. There exists \( x_0 \in (x, \bar{x}) \), such that:

\[
\frac{f(x|1)}{f(x|0)} = \begin{cases} 
< 1 & \text{if } x \in [x, x_0) \\
1 & \text{if } x = x_0 \\
> 1 & \text{if } x \in (x_0, \bar{x}] 
\end{cases}
\]

Obviously, 1 and 2 can not be true. To see that, in case 1 for example, integrating both sides yields:

\[
\int_{x}^{\bar{x}} f(x|1) \, dx > \int_{x}^{\bar{x}} f(x|0) \, dx
\]

But this cannot be true since by the definition of density functions, both sides are the probabilities that must be equal to 1. Similarly for case 2. Therefore only case 3 can be true.

Q.E.D.

Proof of Proposition 3.

The purchaser solves the problem:

\[
\max_{t(x)} \int_{x}^{\bar{x}} [b(x, Y) - t] f(x|0) \, dx \\
\text{s.t. } \int_{x}^{\bar{x}} U(t, Y) f(x|1) \, dx \geq 0 \\
\int_{x}^{\bar{x}} U(t, Y) f(x|0) \, dx \geq \int_{x}^{\bar{x}} U(t, Y) f(x|1) \, dx + \psi
\]

The first order condition is:

\[
-f(x|0) + \lambda U_t(t, Y) f(x|1) + \mu U_t(t, Y) [f(x|0) - f(x|1)] = 0
\]

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\[
\frac{1}{U_t(t,Y)} = \lambda \frac{f(x|1)}{f(x|0)} + \mu \frac{f(x|0) - f(x|1)}{f(x|0)}
\] (19)

Using the same approach as that in the proof of proposition 2, we see \(\lambda, \mu > 0\). Therefore parameters \(\lambda\) and \(\mu\), and the trajectory \(t(x)\) can be determined by using equation (19) and the constraints (now holding with equality). Next, we examine the relation between the first best trajectory \(t^*(x)\) and the second best one \(t^{SB}(x)\). Rewriting equation (19), we have:

\[
\frac{1}{U_t(t,Y)} = \lambda + (\mu - \lambda) \left[ 1 - \frac{f(x|1)}{f(x|0)} \right]
\] (20)

or

\[
\frac{1}{U_t(t,Y)} - E \left( \frac{1}{U_t(t,Y)} \right) = (\mu - \lambda) \left[ 1 - \frac{f(x|1)}{f(x|0)} \right]
\] (21)

We will show \(\mu - \lambda < 0\) if \(x \neq x_0\), where \(x_0\) is the value discussed in proof of Lemma 1. Using the same approach as in the proof of the proposition 1, we can show \(\frac{dt}{dx} > 0\). Considering this and the assumption \(U_{tt}(t,Y) < 0\), we immediately learn that \(\frac{1}{U_t(t,Y)} - E \left( \frac{1}{U_t(t,Y)} \right)\) monotonously increases in \(x\). We also know from Lemma 1 that \(f(x|1) = f(x|0)\) if \(x = x_0\), which implies \(\frac{1}{U_t(t,Y)} - E \left( \frac{1}{U_t(t,Y)} \right) = 0\) at the point \(x = x_0\). Therefore, we have:

\[
\frac{1}{U_t(t,Y)} - E \left( \frac{1}{U_t(t,Y)} \right) \begin{cases} < 0 & \text{if } x < x_0 \\ = 0 & \text{if } x = x_0 \\ > 0 & \text{if } x > x_0 
\end{cases}
\]

The above combined with Lemma 1 and equation (21) implies \(\mu - \lambda < 0\) if \(x \neq x_0\).

Obviously, the first order condition in the full information can be written as:

\[
\frac{1}{U_t(t,Y)} = \lambda
\] (22)

Comparing equation (20) with equation (22), we conclude:

\[
t^{SB}(x) = \begin{cases} < t^*(x) & \text{if } x < x_0 \\ = t^*(x) & \text{if } x = x_0 \\ > t^*(x) & \text{if } x > x_0
\end{cases}
\]

Q.E.D.
Figure 1
Figure 2
Figure 3