

Strategic tax competition and public good provision*

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Abstract

This paper examines the issue of income tax competition between two jurisdictions in the presence of a public good under various informational assumptions. If the information on individuals' income is public, then introducing public good in an income tax competition model increases significantly the possibility of income redistribution by competing government. If the information on individual income is private, then the possibility for redistributions is more limited. TO BE COMPLETED

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1 Introduction

Income tax competition driven by mobile taxpayers is often seen as a serious impediment to governments' ability to redistribute income and/or to finance public good provision. It

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is indeed clear that the mobility of tax payers imposes on the tax authority's optimizing problem an additional "participation constraint". Tax payers must be willing to pay the tax in exchange for the public service they receive. If they are perfectly mobile, they will do so only if the package of tax and public good that is offered to them is more attractive than the one they could find elsewhere. In the last fifteen years or so, a significant body of research has explored the consequence of observing several government engaging themselves in tax competition. Each government, taking the tax policy of its competitor as given, chooses its own tax policy. One can then look for Nash equilibria of this game, that are combinations of tax policies – one for every government – that do not give anyone (government and tax payers) incentive to deviate. A common feature of these Nash equilibria, when they do exist, is the phenomenon known as the "race to the bottom". Every jurisdiction reduces its tax so as to attract highly skilled or productive tax payers, and the resulting provision of public good and tax tend to be inefficiently low (see, e.g., Cremer and Pestieau (2004) for a survey of the literature on this). While this "race to the bottom" phenomenon has been extensively analyzed in the context of capital taxation, it is only recently that it has been examined for individual optimal income taxation. THERE IS A LITERATURE ON COMPETITION AND LINEAR TAXATION, EVEN WITH PUBLIC GOODS. Two representative pieces of the literature on this issue are Birbrauer, Brett, and Weymark (2013) and Lehmann, Simula, and Trannoy (2014).

A specific difficulty raised by the (non-cooperative) game theoretic modeling of individual income tax competition is the resulting endogeneity of the population of residents of a given jurisdiction. When choosing a tax policy, a government chooses also the population that it represents, and on the behalf of which it takes decision. Hence, income tax competition becomes also a competition for attracting "good" individuals, and chasing away the "bad" ones. This makes the analysis of the underlying game complex, and questions even the consistency of the game theoretic approach to the issue. Indeed, games so defined may not have any equilibria, and may therefore not lead to definite predictions.

Yet, when do, they also suggests that income tax competition with free individual mobility lead to somewhat aggressive competition. In a model with two possible jurisdictions, and a Mirrlees (1971) setting with finitely many types of individuals and two governments who choose simultaneously their tax schedule by maximizing the average utility of their residents, Birbrauer, Brett, and Weymark (2013) show that in no equilibria of the game can one have a type paying a positive tax in one jurisdiction and having a larger utility than the average

utility of the other jurisdiction. In particular, this implies that no taxation at all of the high type can be observed with income tax competition. While this result is obtained in a setting *à la* Mirrlees (1971) with imperfect information, it is not difficult to show – as done herein – that it does not at all ride on the Mirrleesian framework. It would also hold in a situation where the taxing authority could observe perfectly individual income. Two average utilitarian competing governments who are taxing perfectly mobile individuals to finance a (possibly null) exogenous requirement of public good (who could be zero) will never tax individuals whose utility is higher than the average utility of the other jurisdiction.

This rather sharp prediction of the analysis of Birbrauer, Brett, and Weymark (2013) is clearly at odds with observations, that do show a significant income taxation of high earners. A possible way to adapt the Mirrleesian tax competition to empirical reality, taken by Lehmann, Simula, and Trannoy (2014) (see also Simula and Trannoy (2010) and Simula and Trannoy (2012)), is to introduce mobility cost in the model. In this paper, we consider instead introducing public good provision. It is indeed clear that taxation is only one face of a coin. As is well-known since at least Tiebout (1956), when choosing their place of residence, and even when they are perfectly mobile, individuals balance the tax they are asked to pay with the public good that they receive in exchange. In this paper, we therefore study competition for people by two governments *à la* Birbrauer, Brett, and Weymark (2013) or Lehmann, Simula, and Trannoy (2014) when these governments choose both public good provision and taxes. We show that introducing the public good in the analysis modifies significantly the nature of tax competition. For one thing, with public goods, high type individuals can be taxed, and actually will be taxed quite commonly in the Nash equilibria of the game. TO BE COMPLETED.

2 The model

There are $n \geq 2$ individuals taken from a set N who consume a public good (z) and a private good (x). These individuals are split into k types (with $k \leq n$) according to their income. We denote by N_h and n_h (with $n_h > 0$) the set and the number respectively of individuals of type h in the population (for $h = 1, \dots, k$). All individuals of type h have an exogenous income $w_h \in \mathbb{R}_+$, and the types are strictly ordered by their income in such a way that $w_h < w_{h+1}$ for $h = 1, \dots, k - 1$. We denote by $\bar{w} = \sum_{h=1}^k n_h w_h / n$ the average (or per capita) income in the population.

All individuals evaluate the alternative combinations of private and public goods by the same continuously differentiable, strictly increasing and concave utility function $U : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ that we assume to be cardinally meaningful. For further use, we denote by V ($V : \mathbb{R}_{++}^3 \rightarrow \mathbb{R}$) the individual's indirect utility function defined as usual by:

$$V(p_z, p_x, R) = \max_{z, x} U(z, x) \text{ subject to } p_z z + p_x x \leq R. \quad (1)$$

We denote the solution of this program by $z^M(p_z, p_x, R)$ and $x^M(p_z, p_x, R)$, the (Marshallian) demands for public good and private consumption (respectively). In the same vein, we denote by $E(p_z, p_x, u)$ the expenditure function defined by the dual program:

$$E(p_z, p_x, u) = \min_{z, x} p_z z + p_x x \text{ subject to } U(z, x) \geq u, \quad (2)$$

the solution of which, denoted by $z^H(p_z, p_x, u)$ and $x^H(p_z, p_x, u)$, are the standard Hicksian demands for public good and private consumption (respectively). The assumptions imposed on U are sufficient for the Marshallian and Hicksian demands, indirect utility and expenditure functions to be differentiable with respect to prices, income and/or utility. We make the additional assumption that the two goods are normal so that their Marshallian demands are both increasing in income.

To avoid triviality (in particular, the possibility that some individuals may prefer living with zero public good rather than paying even a tiny bit of tax), we also assume that $V(1/n, 1, \bar{w}) > U(0, w_h)$ for every h .

An individual with income w_h who has preferences for public and private good represented by the utility function U has also preference for public good and taxes $t \in -\infty, w_h]$ that are represented by the function $U(\bar{z}, w_i - \bar{t})$. These preferences differ across types (but not across individuals of the same type). The domain of definition of these preferences is also specific to the individual type. Under the assumed normality of the public and the private goods, these preferences satisfy the familiar single-crossing property that the slope of the indifference curves passing through any combination of public good and taxes that are feasible for two types is increasing with respect to the type, as illustrated on Figure ??.

There are two countries (or jurisdictions) in which these individuals can live, labeled A and B . The public authorities of these jurisdictions compete for attracting residents by offering them packages of public good and individual taxes that satisfy their budget constraint.

Specifically, if jurisdiction j is inhabited by the type-specific populations N_h^j (for $j = A, B$, and $h = 1, \dots, k$), the public authorities of j can offer them any package of public good z^j and type-specific individual taxes t_h^j (for $i = 1, \dots, \#N_h^j$, $h = 1, \dots, k$) that satisfy

$$z^j \leq \sum_{h=1}^k n_h^j t_h^j, \quad (3)$$

where $n_h^j = \#N_h^j$ denotes the number of individuals of type h in jurisdiction j , and

$$t_h^j \leq w_h$$

for all $h = 1, \dots, k$.

In line with the recent literature (see, e.g., Birbrauer, Brett, and Weymark (2013) or Lehmann, Simula, and Trannoy (2014)), we model the competition between the two public authorities as a non-cooperative game in normal form. The players of this game are the (public authorities of the) two jurisdictions A and B . The strategies of player j ($j = A, B$) are the levels of public good $z^j \in \mathbb{R}_+$ and the type-specific individual taxes t_h^j (for $h = 1, \dots, k$). The ability of the public authority to individualize taxation within a jurisdiction will, however, depend upon the information that it has about individuals. The payoff of every jurisdiction is the average utility of the population that it attracts by its public good and tax offer. Specifically, if jurisdiction j attracts the populations $N^j = (N_1^j, \dots, N_k^j)$ by offering taxes $t^j = (t_1^j, \dots, t_k^j)$ and public good z^j , it obtains the payoff $p(N^j; t^j, z^j)$ defined by:

$$p(N^j; t^j, z^j) = \frac{\sum_{h=1}^k n_h^j U(z^j, w_h - t_h^j)}{n^j} \text{ if } \exists h \text{ such that } N_h^j \neq \emptyset \text{ and } \sum_{h=1}^k n_h^j t_h^j \geq z^j \quad (4a)$$

$$= -\infty \text{ otherwise} \quad (4b)$$

where $n_h^j = \#N_h^j$ for $h = 1, \dots, k$ and $n^j = n_1^j + \dots + n_k^j$. In order to avoid the possibility of having trivial equilibria in which an empty jurisdiction (who would therefore suffer infinitely negative payoff) could nonetheless choose to offer positive public good provision that it is unable to finance, we explicitly rule out this later possibility. Any jurisdiction that is empty at some equilibrium (if any) must offer 0 public good provision (and 0 taxes).

We assume that an individual lives in the jurisdiction that offers him/her the best public

good and tax package (and assign themselves randomly to one of the two jurisdictions if they are indifferent). Specifically, given any pair of tax and public good packages (t^A, z^A) and (t^B, z^B) for jurisdictions A and B , we say that a partition $\{N_h^A, N_h^B\}$ (for $h = 1, \dots, k$) of the population between the two jurisdictions is individually rational for that pair of tax and public good packages if, for every h , one has

$$U(z^A, w_h - t_h^A) \geq U(z^B, w_h - t_h^B) \quad (5)$$

for every $i \in N_h^A$, and

$$U(z^B, w_h - t_h^B) \geq U(z^A, w_h - t_h^A) \quad (6)$$

for every $i \in N_h^B$. In words, a partition of the population between the two jurisdictions is individually rational given the tax and public good packages if no individual has a strict incentive to move from his/her jurisdiction of residence given his/her type. Notice that there may be many partitions of the population in the two jurisdictions that are individually rational for a given pair of policies. Inspection of inequalities (5) and (6) show that if an individually rational partition of the population assigns a strictly positive number of individuals of a given type to the two jurisdictions, then these individuals must be indifferent between the two jurisdictions. Put differently, if an individual strictly prefers his/her jurisdiction of residence to the other in an individually rational partition, then all individuals of his/her type must be living in his/her jurisdiction.

We are now equipped to define what we mean by an Income Tax Competition with Public Good (ITCPG) equilibrium.

Definition 1 *A pair of public good and tax packages $(z^{A*}, t_h^{A*}; z^{B*}, t_h^{B*})$ and a partition $\{N_h^{A*}, N_h^{B*}\}$ (for $h = 1, \dots, k$) of the population are said to be an Income Tax Competition with Public Good equilibrium iff*

- (1) *The partition $\{N_h^{A*}, N_h^{B*}\}$ for $h = 1, \dots, k$ is individually rational for the pair of public good and tax packages $(z^{A*}, t_h^{A*}; z^{B*}, t_h^{B*})$, and*
- (2) *For any jurisdiction $j \in \{A, B\}$ and any tax package (z^j, t_h^j) , there exists a partition $\{N_h^A, N_h^B\}$ (for $h = 1, \dots, k$) of the population that is individually rational for $(z^j, t_h^j; z^{j'*}, t_h^{j'*})$ (with $j' \in \{A, B\}$ and $j' \neq j$) such that $p(N_1^j, \dots, N_k^j; t_1^j, \dots, t_k^j, z^j) \leq p(N_1^{j'*}, \dots, N_k^{j'*}; t_1^{j'*}, \dots, t_k^{j'*}, z^{j'*})$.*

In words, an ITCPG equilibrium is a pair of tax schedule and public good provision – one

such schedule and public good provision for every jurisdiction – along with a partition of the population between the two jurisdictions that satisfy two conditions. First, each individual must prefer weakly the public good and tax package offered in his/her jurisdiction of residence to that offered in the other. Second, each jurisdiction achieves a (weakly) larger average welfare with its tax policy and public good provision than with any alternative policy, even when anticipating the impact that the alternative policy will have on its own population.

Before turning to the analysis of an ITCPG in the case of perfect information, we find useful to compare our framework with the situation that would prevail *without* public good. To be specific, assume momentarily that the utility function U does not at all depend upon the public good, so that it can be written as $U(z, x) = \Phi(x)$ for some increasing and concave function $\Phi : \mathbb{R}_+ \rightarrow \mathbb{R}$. In that case, the payoff function of jurisdiction j 's authority would write:

$$\begin{aligned} \widehat{p}(N^j; t^j) &= \frac{\sum_{h=1}^k n_h^j \Phi(w_h - t_h^j)}{n^j} \text{ if } \exists h \text{ such that } N_h^j \neq \emptyset \text{ and } \sum_{h=1}^k n_h^j t_h^j \geq g & (7a) \\ &= -\infty \text{ otherwise,} \end{aligned}$$

where g is an exogenous public expenditure requirement, assumed to be the same in the two jurisdictions. We can then apply the definition of individual rationality to this specification and define as follows the notion of an Income Tax Competition (ITC) equilibrium.

Definition 2 *A pair of tax schedules (t_h^{A*}, t_h^{B*}) and a partition $\{N_h^{A*}, N_h^{B*}\}$ (for $h = 1, \dots, k$) of the population are said to be an Income Tax Competition equilibrium iff*

- (1) *The partition $\{N_h^{A*}, N_h^{B*}\}$ is individually rational for the pair of tax schedules (t_h^{A*}, t_h^{B*}) (for $h = 1, \dots, k$), and*
- (2) *For any jurisdiction $j \in \{A, B\}$ and tax schedule (t_h^j) , there exists a partition $\{N_h^A, N_h^B\}$ (for $h = 1, \dots, k$) of the population that is individually rational for $(t_h^j; t_h^{j'*})$ (with $j' \in \{A, B\}$ and $j' \neq j$) such that $\widehat{p}(N_1^j, \dots, N_k^j; t_1^j, \dots, t_k^j) \leq \widehat{p}(N_1^{j'*}, \dots, N_k^{j'*}; t_1^{j'*}, \dots, t_k^{j'*})$.*

This case is somewhat similar to that considered in Birbrauer, Brett, and Weymark (2013), with the importance difference that income is assumed here to be perfectly observable an exogenous so that we abstract from the leisure-consumption choice by individuals. Yet, it happens that the main result obtained by Birbrauer, Brett, and Weymark (2013) – namely that we never observe a type paying a positive tax in one jurisdiction while having a larger

utility than the average utility of the other jurisdiction – holds here as well. Hence, the significant limitation to redistribution brought about by income tax competition between jurisdictions with perfectly mobile individuals does not depend at all upon the Mirleesian structure assumed in Birbrauer, Brett, and Weymark (2013). The formal statement of this result is as follows.

Proposition 1 *Assume that (t_h^{A*}, t_h^{B*}) and the partition of the population $\{N_h^{A*}, N_h^{B*}\}$ (for $h = 1, \dots, k$) is an ITC equilibrium. Then if $\Phi(w_h - t_h^j) > \widehat{p}(N^{j'}; t^{j'})$ for some $j, j' \in \{A, B\}$ such that $j \neq j'$ and some h such that $N_h^j \neq \emptyset$, it must be that $t_h^{j*} = 0$.*

Proof. *By contradiction, let the tax schedules (t_h^{A*}, t_h^{B*}) and the partition of the population $\{N_h^{A*}, N_h^{B*}\}$ (for $h = 1, \dots, k$) be an ITC equilibrium for which there are two distinct $j, j' \in \{A, B\}$ and a type h such that $N_h^j \neq \emptyset, t_h^{j*} > 0$ and $\Phi(w_h - t_h^j) > \widehat{p}(N^{j'}; t^{j'})$. But, then jurisdiction j' would attract individuals of type h located at j by offering them to pay zero taxes. This would not hurt anyone, and would not affect the budget constraint of jurisdiction j' . And attracting individuals of type h would increase average utility by assumption, a contradiction. ■*

Hence, without public good, tax competition rules out the possibility, for example, of taxing positively the richest individual (or for that matter any individual that has more utility than the average utility of the other). In the next section, we show that this limit to redistribution brought about by tax competition disappears if a public good is introduced. As a matter of fact, the only possible equilibrium that can arise in this case will entail complete redistribution of income.

3 The case of public information

We first consider as a benchmark the case where the information on the individual income is publicly available. In this case, the two jurisdictions' authorities can base individual taxation on individual's type. Before we handle this case, we find useful to characterize, for any jurisdiction $j \in A, B$, and any population $N_h^j \subset N_h$ (for $h = 1, \dots, k$) of type-indexed individuals living in j (and satisfying $\bigcup_{h=1}^k N_h^j \neq \emptyset$ to avoid triviality), the optimal first-best public good and tax policy that would be chosen by the jurisdiction's authority in the absence of any tax competition. This case is obviously ideal but it constitutes a natural

benchmark, with respect to which the introduction of tax competition can be compared. Without tax competition, the jurisdiction authority would solve the following program:

$$\bar{U}^*(N_1^j, \dots, N_k^j) = \max_{z, t_1, \dots, t_k \in \mathbb{R}_+ \times]-\infty, w_1] \times \dots \times]-\infty, w_k]} \frac{\sum_{h=1}^k n_h^j U(z, w_h - t_h)}{\sum_{h=1}^k n_h^j} \text{ s.t. } \sum_{h=1}^k n_h^j t_h \geq z. \quad (8)$$

This program is well-defined if $\bigcup_{h=1}^k N_h^j \neq \emptyset$. Denote also by $n(N_1^j, \dots, N_k^j) = n_1^j + \dots + n_k^j$ and $\bar{w}(N_1^j, \dots, N_k^j) = (\sum_{h=1}^k n_h^j w_h) / n(N_1^j, \dots, N_k^j)$, the population size and per capita income (respectively) in jurisdiction j when its population is $\{N_1^j, \dots, N_k^j\}$. It is immediate from the first-order conditions of this maximization that all individuals consume the same amount of the private good (and obviously of the public good), that is, $w_h - t_h^* = w_i - t_i^* = c^*$ for all $h, i \in \{N_1^j, \dots, N_k^j\}$. Given this, the (binding) budget constraint can be rewritten as

$$\sum_{h=1}^k n_h^j t_h = \sum_{h=1}^k n_h^j (w_h - c^*) = z \text{ or } \frac{z}{n(N_1^j, \dots, N_k^j)} + c^* = \bar{w}(N_1^j, \dots, N_k^j).$$

This last expression is precisely the (binding) budget constraint of an individual maximizing his/her utility while having income $\bar{w}(N_1^j, \dots, N_k^j)$ and facing prices $p_z = 1/n(N_1^j, \dots, N_k^j)$ and $p_x = 1$. Since all individuals have the same utility function, this implies that

$$\bar{U}^*(N_1^j, \dots, N_k^j) = V \left(\frac{1}{n(N_1^j, \dots, N_k^j)}, 1, \bar{w}(N_1^j, \dots, N_k^j) \right).$$

We now establish the main result of this section. Namely, we show that any ITCPG equilibrium under public information will involve one jurisdiction being empty, and the other jurisdiction gathering the whole population, and offering it the average utilitarian optimal allocation of private and public goods. We state this formally as follows.

Proposition 2 *Suppose that individual income is perfectly observable and that the local public good and tax policies $(z^{A^*}, t_h^{A^*}; z^{B^*}, t_h^{B^*})$ and the partition of the population $\{N_h^{A^*}, N_h^{B^*}\}$ (for $h = 1, \dots, k$) form an ITCPG equilibrium. Then one has either $N_h^{A^*} = \emptyset$, $N_h^{B^*} = N_h$ and*

$$(z^{B^*}, t_1^{B^*}, \dots, t_k^{B^*}) \in \arg \max_{z, t_1, \dots, t_k \in \mathbb{R}_+ \times]-\infty, w_1] \times \dots \times]-\infty, w_k]} \frac{\sum_{h=1}^k n_h U(z, w_h - t_h)}{n} \text{ s.t. } \sum_{h=1}^k n_h t_h \geq z,$$

or $N_h^{B^*} = \emptyset$ and $N_h^{A^*} = N_h$, and

$$(z^{A^*}, t_1^{A^*}, \dots, t_k^{A^*}) \in \arg \max_{z, t_1, \dots, t_k \in \mathbb{R}_+ \times [-\infty, w_1] \times \dots \times [-\infty, w_k]} \frac{\sum_{h=1}^k n_h U(z, w_h - t_h)}{n} \text{ s.t. } \sum_{h=1}^k n_h t_h \geq z.$$

Proof. By contradiction, assume that $(z^{A^*}, t_h^{A^*}; z^{B^*}, t_h^{B^*})$ and the partition $\{N_h^{A^*}, N_h^{B^*}\}$ (for $h = 1, \dots, k$) is an ITCPB equilibrium for which there exists $h \in \{1, \dots, k\}$ such that $n_h^{A^*} \geq 1$ and $g \in \{1, \dots, k\}$ such that $n_g^{B^*} \geq 1$. For $j = A, B$, define \bar{U}^{j^*} by

$$\bar{U}^{j^*} = \frac{\sum_{h=1}^k n_h^{j^*} U(z^{j^*}, w_h - t_h^{j^*})}{\sum_{h=1}^k n_h^{j^*}}. \quad (9)$$

These numbers are well-defined because the denominator of the right hand side of (9) is strictly positive by assumption. Without loss of generality (up to a relabeling of A and B), assume that $\bar{U}^{A^*} \geq \bar{U}^{B^*}$. Consider then the possibility for jurisdiction B to offering the local public good $z^{A^*} + z^{B^*}$ and tax policies \hat{t}_h for $h = 1, \dots, k$ defined by

$$\hat{t}_h = \begin{cases} t_h^{A^*} & \text{if } n_h^{A^*} = n_h \\ \max(t_h^{A^*}, t_h^{B^*}) & \text{if } 1 \leq n_h^{A^*} < n_h \\ t_h^{B^*} & \text{if } n_h^{A^*} = 0 \end{cases}$$

Observe that this local public good and tax policy satisfy jurisdiction B budget constraint because

$$\begin{aligned} \sum_{h=1}^k n_h \hat{t}_h &= \sum_{h: n_h^{A^*} = n_h} n_h^{A^*} t_h^{A^*} + \sum_{h: 1 \leq n_h^{A^*} < n_h} (n_h^{A^*} + n_h^{B^*}) \max(t_h^{A^*}, t_h^{B^*}) + \sum_{h: n_h^{A^*} = 0} n_h^{B^*} t_h^{B^*} \\ &\geq \sum_{h: n_h^{A^*} = n_h} n_h^{A^*} t_h^{A^*} + \sum_{h: 1 \leq n_h^{A^*} < n_h} (n_h^{A^*} t_h^{A^*} + n_h^{B^*} t_h^{B^*}) + \sum_{h: n_h^{A^*} = 0} n_h^{B^*} t_h^{B^*} \\ &= \sum_{h=1}^k n_h^{A^*} t_h^{A^*} + \sum_{h=1}^k n_h^{B^*} t_h^{B^*} \\ &\geq z^{A^*} + z^{B^*} \end{aligned}$$

if $(z^{A^*}, t_h^{A^*}; z^{B^*}, t_h^{B^*})$ and the partition $\{N_h^{A^*}, N_h^{B^*}\}$ (for $h = 1, \dots, k$) constitute an ITCPB equilibrium. Observe now that if z^{A^*} and z^{B^*} are both strictly positive, then everybody in either of the two jurisdictions strictly prefers the new tax policy offered by jurisdiction B to the one that was prevailing at his/her place of residence. Consider indeed a type h for

which $n_h^{j^*} \geq 1$ for any $j = A, B$. If $n_h^{j^*} = n_h$ for some $j \in \{A, B\}$, then an individual of type h pays the same tax $t_h^{j^*}$ as before. However, such a person now gets strictly more public good. Suppose now that $1 \leq n_h^{j^*} < n_h$ and, therefore, that individuals of type h are present in the two jurisdictions. In this case, for the partition of the population $\{N_h^{A^*}, N_h^{B^*}\}$ (for $h = 1, \dots, k$) to be individually rational, one must have

$$U(z^{A^*}, w_h - t_h^{A^*}) = U(z^{B^*}, w_h - t_h^{B^*}). \quad (10)$$

Let $j^* \in \arg \max(t_h^{A^*}, t_h^{B^*})$. Then $U(z^{j^*}, w_h - t_h^{j^*}) = U(z^{A^*}, w_h - t_h^{A^*}) = U(z^{B^*}, w_h - t_h^{B^*}) < U(z^{A^*} + z^{B^*}, w_h - t_h^{j^*})$ if both z^{A^*} and z^{B^*} are strictly positive. Hence, for any $j = A, B$, and any h such that $n_h^{j^*} \geq 1$, one has

$$U(z^{j^*}, w_h - t_h^{j^*}) < U(z^{A^*} + z^{B^*}, w_h - \hat{t}_h^j),$$

so that all individuals will move from A to B . Moreover,

$$\begin{aligned} \frac{\sum_{h=1}^k n_h^* U(z^{A^*} + z^{B^*}, w_h - \hat{t}_h)}{n} &= \frac{\sum_{h=1}^k n_h^{A^*} \sum_{h=1}^k n_h^{A^*} U(z^{A^*} + z^{B^*}, w_h - \hat{t}_h)}{n \sum_{h=1}^k n_h^{A^*}} \\ &\quad + \frac{\sum_{h=1}^k n_h^{B^*} \sum_{h=1}^k n_h^{B^*} U(z^{A^*} + z^{B^*}, w_h - \hat{t}_h)}{n \sum_{h=1}^k n_h^{B^*}} \\ &> \frac{\sum_{h=1}^k n_h^{A^*} \bar{U}^{A^*}}{n} + \frac{\sum_{h=1}^k n_h^{B^*} \bar{U}^{B^*}}{n} \\ &\geq \bar{U}^{B^*} \end{aligned}$$

Hence the move from the policy $(z^{B^*}, t_h^{B^*})$ to the policy $(z^{A^*} + z^{B^*}, \hat{t}_h)$ (for $h = 1, \dots, k$) strictly increases jurisdiction B 's payoff, which is a contradiction of the definition of ITCPB equilibrium. We leave to the reader the task of proving that one cannot have $z^{A^*} = 0$ or $z^{B^*} = 0$ if $(z^{A^*}, t_h^{A^*}; z^{B^*}, t_h^{B^*})$ and the partition $\{N_h^{A^*}, N_h^{B^*}\}$ (for $h = 1, \dots, k$) is an ITCPB equilibrium for which there exists $h \in \{1, \dots, k\}$ such that $n_h^{A^*} \geq 1$ and $g \in \{1, \dots, k\}$ such that $n_g^{B^*} \geq 1$. Hence, if $(z^{A^*}, t_h^{A^*}; z^{B^*}, t_h^{B^*})$ and the partition $\{N_h^{A^*}, N_h^{B^*}\}$ (for $h = 1, \dots, k$) is an ITCPB equilibrium, then one of the two jurisdictions must be empty. By the budget constraint, the empty jurisdiction can only offer zero unit of public good and non-negative tax to everybody. By assumption $V(1/n, 1, \bar{w}) > U(0, w_h)$ for every h . Hence, the grand jurisdiction's average utilitarian maximizing choice is to pick $z^* = z^M(1/n, 1, \bar{w})$ and the taxes $w_h - x^M(1/n, 1, \bar{w})$. Doing any other choice would reduce its payoff, given the other

jurisdiction choice. ■

Proposition 2 states strong restrictions on the equilibrium outcome that can result from tax competition with a public good. If such an outcome exists at all, it is such that the whole population is living in one jurisdiction, and the other is empty (and offering, as indicated above, zero public good provision). In fact, the equilibrium outcome is the Pareto optimal allocation where the public good is made available to the greatest number of individuals.

Observe also that, contrary to what is the case without public good, competition between jurisdictions is compatible with full income redistribution. Indeed, if an equilibrium exists, consumption of the private good is equalized across all individuals. This implies, among other things, that the marginal rate of taxation of the richest individuals is 100%.

These two results are in sharp contrast with the results obtained in a similar model without public good. In the case of public information, the presence of the public good reverses the conclusion about the impact of fiscal competition. With a public good, fiscal competition is not detrimental. Full redistribution of income remains possible and the mobility of agents makes it possible to exploit the full benefits of the public good by gathering all individuals in the same jurisdictions.

Proposition 2, however, does not guarantee the existence of an ITCPG equilibrium. In the next proposition, we give necessary and sufficient conditions for a ITCPG to exist. Recall that

$$\bar{U}^*(N_1^j, \dots, N_k^j) = V \left(\frac{1}{n(N_1^j, \dots, N_k^j)}, 1, \bar{w}(N_1^j, \dots, N_k^j) \right).$$

Proposition 3 *A ITCPG equilibrium exists if and only if there is no partition of individuals $\{N_1, \dots, N_k\}$ such that*

$$V \left(\frac{1}{n(N_1^j, \dots, N_k^j)}, 1, \bar{w}(N_1^j, \dots, N_k^j) \right) > V \left(\frac{1}{n}, 1, \bar{w} \right).$$

Proof. *We first prove necessity. Suppose a ITCPG equilibrium exists. From Proposition 2, we know that the whole population is in, say, jurisdiction A. If this is an equilibrium, this implies that jurisdiction B cannot deviate to a tax and public good package such that it attracts any subset of individuals that would be strictly better off than if they remained in jurisdiction A. This implies that the condition in the proposition must be satisfied, otherwise*

there would exist a successful deviation by jurisdiction B.

Now with sufficiency. Suppose the condition in the proposition is satisfied. Then, having say jurisdiction A offering $(z^{A*}, t_1^{A*}, \dots, t_k^{A*})$ and jurisdiction B offering $(z^{B*} = 0, t_1^{B*} = 0, \dots, t_k^{B*} = 0)$ is an equilibrium. By assumption, all individuals prefer the allocation of jurisdiction A to that of jurisdiction B. By definition, jurisdiction A maximizes its average welfare given that it attracts the whole population. And if the condition of the proposition is satisfied, jurisdiction B cannot deviate to successfully attract a subset of types. Hence, the proposed strategies form a ITCPG equilibrium. ■

4 The case of private information

We now consider the more interesting case when the jurisdiction authorities cannot observe the income of the individuals. Given the simple structure of our model, this implies that, in any given jurisdiction, all individuals, regardless of their income, pay the same tax and consume the same amount of public good. Suppose that jurisdiction j attracts a partition $\{N_1^j, \dots, N_k^j\}$. The optimal allocation for this partition solves the following maximization program.

$$\hat{U}^*(N_1^j, \dots, N_k^j) = \max_{z, t \in \mathbb{R}_+ \times [-\infty, w_a]} \frac{\sum_{h=1}^k n_h^j U(z, w_h - t)}{\sum_{h=1}^k n_h^j} \text{ s.t. } \sum_{h=1}^k n_h^j t \geq z \quad (11)$$

where a is the index of the smallest income individual in the partition $\{N_1^j, \dots, N_k^j\}$. The first-order conditions for an interior solution to this problem yields

$$\sum_{h=1}^k n_h^j \left(\frac{\sum_{h=1}^k n_h^j U_z(z, w_h - t)}{\sum_{h=1}^k n_h^j U_x(z, w_h - t)} \right) = 1.$$

This is a modified Samuelson condition that states that the sum over all individuals in the partition of a “pseudo” marginal rate of substitution is equal to the marginal cost of the public good (equal to 1). This pseudo marginal rate of substitution corresponds to the ratio of the average marginal utility of the public good and the average marginal utility of the private good. At the solution, define by $\hat{U}_h^*(N_1^j, \dots, N_k^j)$, the utility obtained by a type h individual. Note that, unlike the case with public information, this utility varies with w_h since all individuals pay the same tax (but have different incomes).

Before providing equilibrium characterizations, we first state two lemmas.

Lemma 1 *In any equilibrium, if $n_h^j > 0$, then $n_h^{j'} = 0$ for $j' \neq j$.*

Proof.

■

This lemma says that, in any equilibrium, if an individual belongs to one jurisdiction, all individuals with the same income belong to the same jurisdiction. We now show that if two individuals with different incomes belong to one jurisdiction, then all individuals with income between their two incomes also belong to the same jurisdiction.

Lemma 2 *In any equilibrium, if $n_h^j > 0$ and $n_{h+c}^j > 0$ for some integer $c > 1$, then $n_{h+b}^j > 0$ for all integers $0 < b < c$.*

Proof.

■

When combining the two lemmas, we have that any equilibrium partition includes an interval of types and that all individuals of the included types belong to the same jurisdiction. Note that these results were trivially satisfied in the case of public information where only the whole population can constitute an equilibrium partition.

We can now provide a characterization of ITCPG equilibria. It turns out that there can be two classes of equilibrium. In the first class, all individuals belong to one jurisdiction and the other jurisdiction is empty. In the second class, there are individuals in the two jurisdictions. With two jurisdictions, the two lemmas imply that there is segregation with poorer individuals in one jurisdiction and richer ones in the other. The two lemmas preclude having some very rich and very poor individuals in one jurisdiction and some middle class individuals in the other jurisdiction. Note that this would also be impossible for three or more jurisdictions, but, in this case, there could exist a middle class jurisdiction.

Proposition 4 *Suppose that individual income is unobservable and that the local public good and tax policies $(z^{A*}, t^{A*}; z^{B*}, t^{B*})$ and the partition of the population $\{N_h^{A*}, N_h^{B*}\}$ (for $h = 1, \dots, k$) form an ITCPG equilibrium. Then there exist parameter values for incomes and*

populations such that one has either $N_h^{A^*} = \emptyset$, $N_h^{B^*} = N_h$ and

$$(z^{B^*}, t^{B^*}) \in \arg \max_{z, t \in \mathbb{R}_+ \times [-\infty, w_1]} \frac{\sum_{h=1}^k n_h U(z, w_h - t)}{n} \text{ s.t. } \sum_{h=1}^k n_h t \geq z,$$

or $N_h^{B^*} = \emptyset$ and $N_h^{A^*} = N_h$, and

$$(z^{A^*}, t^{A^*}) \in \arg \max_{z, t \in \mathbb{R}_+ \times [-\infty, w_1]} \frac{\sum_{h=1}^k n_h U(z, w_h - t)}{n} \text{ s.t. } \sum_{h=1}^k n_h t \geq z.$$

Proof.

■

For such an equilibrium to exist, it has to be the case that the empty jurisdiction, say B , cannot succeed in attracting a subset of types. A possible deviation could be to attract a single type. Among the deviations that attract only one type, only those that attract either type 1 or type k need to be considered. Given that the proposed equilibrium allocation has all types paying the same tax and consuming the same amount of public goods, if a type $1 < h < k$ is attracted by a deviating offer by B , then so is some type smaller than h or larger than h . A necessary condition for the equilibrium of Proposition 6 to exist is that

$$\hat{U}_h^*(N_h) \leq \hat{U}_h^*(N_1, \dots, N_k) \text{ for } h \in \{1, k\}, \quad (12)$$

where we abuse notation when using N_h to denote the partition including exclusively all individuals of type h . Note that $\{N_1, \dots, N_k\}$ represents the grand coalition. If this condition is satisfied, then jurisdiction B cannot successfully attract only one type of individuals. It may, nonetheless, seek to attract a subset of types. Doing so, and the same argument as above, if B attracts a subset of types $[a, b]$, it necessarily attracts either all types larger than b or all those smaller than a . This implies that we only need to consider deviations attracting types $[1, b]$ for some $1 < b < k$ and deviations attracting types $[a, k]$ for some $1 < a < k$. Denote by $\{N_a, \dots, N_b\}$ as the partition including only all individuals of types in the interval $[a, b]$. A necessary condition for the equilibrium of Proposition 6 to exist is that, for all $h \in [a, b]$ and for $a = 1$ and all $1 < b < k$, or for all $1 < a < k$ and $b = k$,

$$\hat{U}_h^*(N_a, \dots, N_b) \leq \hat{U}_h^*(N_1, \dots, N_k). \quad (13)$$

We can now give necessary and sufficient conditions for the equilibrium of Proposition 6 to exist.

Proposition 5 *A ITCPG equilibrium with one jurisdiction attracting the whole population exists if and only if conditions 12 and 13 are satisfied.*

Proof. *We first prove necessity. Suppose a ITCPG equilibrium exists where the whole population is in, say, jurisdiction A. If this is an equilibrium, this implies that jurisdiction B cannot deviate to a tax and public good package such that it attracts any subset of individuals that would be strictly better off than if they remained in jurisdiction A. This implies that the conditions in the proposition must be satisfied, otherwise there would exist a successful deviation by jurisdiction B.*

Now with sufficiency. Suppose the conditions in the proposition are satisfied. Then, having say jurisdiction A offering (z^{A}, t^{A*}) and jurisdiction B offering $(z^{B*} = 0, t^{B*} = 0)$ is an equilibrium. By assumption, all individuals prefer the allocation of jurisdiction A to that of jurisdiction B. By definition, jurisdiction A maximizes its average welfare given that it attracts the whole population. And if the conditions of the proposition are satisfied, jurisdiction B cannot deviate to successfully attract a subset of types. Hence, the proposed strategies form a ITCPG equilibrium. ■*

It turns out that, with private information, there may exist segregated equilibria.

Proposition 6 *Suppose that individual income is unobservable and that the local public good and tax policies $(z^{A*}, t^{A*}; z^{B*}, t^{B*})$ and the partition of the population $\{N_h^{A*}, N_h^{B*}\}$ (for $h = 1, \dots, k$) form an ITCPG equilibrium. Then there are parameter values such that there exist a type $1 < \hat{h} < k$ such that $n_h^{j*} > 0$ and $n_h^{j'*} = 0$ for all $1 \leq h \leq \hat{h}$, and $n_h^{j*} = 0$ and $n_h^{j'*} > 0$ for all $\hat{h} < h \leq k$, for $j, j' = A, B$ with $j \neq j'$. The equilibrium public good and tax packages are given by*

$$(z^{j*}, t^{j*}) \in \arg \max_{z, t \in \mathbb{R}_+ \times [-\infty, w_1]} \frac{\sum_{h=1}^{\hat{h}} n_h U(z, w_h - t)}{\sum_{h=1}^{\hat{h}} n_h} \text{ s.t. } \sum_{h=1}^{\hat{h}} n_h t \geq z,$$

$$(z^{j'*}, t^{j'*}) \in \arg \max_{z, t \in \mathbb{R}_+ \times [-\infty, w_{\hat{h}}]} \frac{\sum_{h=\hat{h}+1}^k n_h U(z, w_h - t)}{\sum_{h=\hat{h}+1}^k n_h} \text{ s.t. } \sum_{h=\hat{h}+1}^k n_h t \geq z.$$

Proof.

■

In a segregated equilibrium, the poorer individuals reside in one jurisdiction while the richer ones reside in the other. Each jurisdiction offers the optimal public good and tax package given the partition of residents it attracts. The proposition shows that such segregated equilibria are possible.

Suppose there is a segregated equilibrium where jurisdiction j attracts the partition $\{N_1^{j*}, \dots, N_{\hat{h}}^{j*}\}$, with $1 < \hat{h} < k$, that is, it attracts the poorer individuals. Jurisdiction $j' \neq j$ attracts the richer types $[\hat{h} + 1, k]$. For such equilibrium to exist, it has to be the case that no jurisdiction, given the equilibrium package of the other jurisdiction, can attract an individually rational partition that yields a higher average social welfare than its equilibrium one. There are different deviations that can occur.

We first consider deviations by jurisdiction j . If jurisdiction j deviates, it obviously withdraws its equilibrium package, so that individuals partition themselves according to jurisdiction j 's deviation and jurisdiction j 's equilibrium package. It turns out that only two types of deviations have to be considered. Jurisdiction j can deviate to attract types $[1, a]$ for some $a > \hat{h}$, or deviate to attract types $[b, k]$ for some $b > 1$.

For some type $a > \hat{h}$, define the set $\mathcal{J}_1(a)$ such that, for all $(z, t) \in \mathcal{J}_1(a)$, we have

$$\sum_{h=1}^a n_h t - z \geq 0,$$

$$\hat{U}_h^*(N_{\hat{h}+1}, \dots, N_k) < U(z, w_h - t) \text{ for all } \hat{h} < h \leq a, \text{ and}$$

$$\hat{U}_h^*(N_{\hat{h}+1}, \dots, N_k) \geq U(z, w_h - t) \text{ for all } h > a.$$

The set $\mathcal{J}_1(a)$ is the set of financially feasible allocations (z, t) such that types in $[\hat{h} + 1, a]$ prefer these allocations to j 's equilibrium allocation and all higher types prefer j 's equilibrium allocation. It is clear that this implies that all types $[1, \hat{h}]$ also prefer the allocations in $\mathcal{J}_1(a)$ than j 's equilibrium allocation. Note that we can restrict our attention to types $a > \hat{h}$ since it would never be profitable for jurisdiction j to attract a poorer subset of types than the one it attracts in equilibrium. If it was, jurisdiction j could offer the same deviating package to its equilibrium subset of types $[1, \hat{h}]$. This would provide a higher average welfare than the deviating one since it adds higher utility types to the average. But this is impossible since the equilibrium allocation is optimal for the subset of types $[1, \hat{h}]$.

For some $b > 1$, define the set $\mathcal{J}^k(b)$ such that, for all $(z, t) \in \mathcal{J}^k(b)$, we have

$$\begin{aligned} \sum_{h=b}^k n_h t - z &\geq 0, \\ \hat{U}_h^*(N_1, \dots, N_{\hat{h}}) &\geq U(z, w_h - t) \text{ for all } 1 \leq h < \min\{b, \hat{h}\}, \\ \hat{U}_h^*(N_{\hat{h}+1}, \dots, N_k) &\geq U(z, w_h - t) \text{ for all } \hat{h} \leq h < b \text{ or} \\ \hat{U}_h^*(N_1, \dots, N_{\hat{h}}) &\geq U(z, w_h - t) \text{ for all } b \leq h < \hat{h}, \text{ and} \\ \hat{U}_h^*(N_{\hat{h}+1}, \dots, N_k) &< U(z, w_h - t) \text{ for all } \max\{b, \hat{h}\} \leq h \leq k. \end{aligned}$$

The set $\mathcal{J}^k(b)$ is the set of financially feasible allocations (z, t) such that types in $[b, k]$ prefer these allocations to j' 's equilibrium allocation and all lower types prefer j' 's equilibrium allocation.

So for the segregated equilibrium to be robust to a deviation by jurisdiction j , it is necessary that, for all $a > \hat{h}$, and all $(z, t) \in \mathcal{J}_1(a)$,

$$\frac{\sum_{h=1}^{\hat{h}} n_h \hat{U}_h^*(N_1, \dots, N_{\hat{h}})}{\sum_{h=1}^{\hat{h}} n_h} \geq \frac{\sum_{h=1}^a n_h U(z, t)}{\sum_{h=1}^a n_h}.$$

It is also necessary that, for all $b > 1$, and all $(z, t) \in \mathcal{J}^k(b)$,

$$\frac{\sum_{h=1}^{\hat{h}} n_h \hat{U}_h^*(N_1, \dots, N_{\hat{h}})}{\sum_{h=1}^{\hat{h}} n_h} \geq \frac{\sum_{h=b}^k n_h U(z, t)}{\sum_{h=b}^k n_h}.$$

If one inequality is not satisfied, then jurisdiction j can deviate by offering some package $(z, t) \in \mathcal{J}_1(a) \cup \mathcal{J}^k(b)$ thus increasing its average welfare.

We now consider deviations by jurisdiction j' . TO BE COMPLETED...

5 Conclusion

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