Environmental Taxation, Frictional Unemployment and Migration in a Two-Region Model

Diane Aubert*

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Abstract

This paper investigates how a rise in the pollution tax rate may affect unemployment, migration and welfare in a Harris–Todaro (HT) model. We build a two-regional/two-sectorial model with imperfect labor markets, pollution externalities and non-homothetic preferences on polluting consumption. This analysis shows that frictional unemployment and non-homothetic preferences bring about inter-region wage differential. Thus, an economy almost always exhibits distortions in the absence of the government intervention. Green tax may exacerbate these distortions by generating spillovers, if the labor market is initially more frictional in the region where the subsistence level of the polluting good is the lowest one. Wages subsidies and transfers among regions are explored as the solution to remove distortions.

JEL classification: - H23 - J64 - Q52 - Q56 - R13.

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*Université Paris 1 & Paris School of Economics

1 Introduction

Facing the growing concern about global warming, the international community is beginning to feel a sense of urgency about the need to reduce the emissions of greenhouse gases (GHG). But the way of implementing environmental policies and the fear of their negative consequences give rise to many debates and criticismes, as most recently reflected by the United States withdrawal from the environmental agreement at the COP 21. Among the different policy instruments to achieve environmental quality objectives, the economists traditionally promote the efficiency of the market-based instruments such as green taxes or cap-and trade market. Yet, the introduction of such a carbon price is still lively discussed. Recently, even among economists community, a debate emerged about the relevance of a global carbon price. Some argue that this would be a perfect tool to promote a universal participation and that it would avoided "free-rider" behavior. Many opponents answer that the tax burden of such a price is likely to differ between countries or even areas, underlining the local component of the regressivity of green taxes.

Emerging countries, such as China, India, Mexico, and Chile, have attempted to reduce domestic poverty through industrialization inducing urban-rural migration but also environmental degradation. It is thus of fundamental and practical importance to understand whether poverty reduction through industrialization is consistent with environmental preservation in a emerging economy.

On the other side, inside developed countries, opponents to a carbon tax underline its regressivity, as the energy part in the total expenditure of the poor households is larger compared to the one of rich households (Metcalf [1999]). Moreover, poorer people seem to have less substitution possibilities between clean and dirty goods, because they live far from city centers and thus do not have access to public transportation or city gas for instance. Because of these differences in "access cost", the project of a global carbon tax is often considered unfair. In addition, workers in polluting industries are sometimes considered poorer than the others (Fullerton [2013]). An increase in environmental regulation could generate more unemployment among those industries and then raise pre-existing inequalities.

To offset the potential negative impacts of green taxes, economists suggest using other fiscal tools beside. Implementing transfers and subsidies between areas or industries, based on income and localization criteria can be part of the solution. However, this policy requires information on the local distributive pattern of green taxes.

This paper aims at providing a better understanding on the distributive characteristics of a global green tax, implemented in two different areas. More specifically, we would like to study the impact of such an instrument on unemployment, welfare, and the reallocation of workers between areas/sectors. Does green tax induce more inefficiency? What are the fiscal instrument needed beside in order to correct the negative externalities of green taxes? Can migration between areas represent a solution of adaptation? Does the heterogeneity of access to clean goods matter for the the efficiency of the environmental green ?

Several empirical papers already addressed this issue, underlying the local dimension in the regressivity of green taxes. Sterner [2012] shows that the elasticity of substitution between clean and dirty good seems higher in a rural area compared to urban area in a developed country. More recently, Carraro and Zatti [2014], using a micro-simulation model, show that geographic and social-economic features of households greatly influence redistributive patterns of duties on fuel sources and vehicle taxes. Rural households and large families tend to be more affected within each income quintile. Moreover, richer households are normally those capable of shifting towards more fuel-efficient vehicles. Ciaschini [2012], William [2014], and Hassett [2007] confirm these results by using CGE model.

Yet, to the best of our knowledge, the distribution of green taxes burden has not been extensively analyzed from the perspective of regional inequalities in a theoretical framework. Some environmental economics theoretical works investigate the difficulties of setting an optimal green tax in an economic federation with different regions. These papers refer to fiscal externalities of local governments who compete for workers or capital and generate spillovers (Oates [2002]). This last point was also studied between national and local governments when the latter transfers its tax burden on the former (Aronsson & al [2004], Williams [2011]). But this emerging literature dealing with environmental federalism does not focus on the disparities in wealth and access to clean goods.

Papers introducing environmental concern in Harris-Todaro models may represent a contribution. Harris and Todaro [1970] generalize a general equilibrium model of two-sectors introducing migration and difference in wealth between regions. This paper and related studies have provided a series of models that constitute the received theory of rural-urban migration. Workers are assumed to compare expected incomes in cities with agricultural wages and to migrate if the former exceeds the latter. Migration is the equilibrating force which equates the two expected incomes. Equilibrium is attained when they are equalized and there is no migration. Although there is an abundant literature about Harris-Todaro model, few studies consider the environmental problem faced by the developed and developing countries. Wang [1990], building on this standard model, demonstrates that a raise in a green tax increases the agricultural wage and lowers urban unemployment by producing backward migration to the agricultural sector. Recently Daitoh [2003], in a model in which urban manufacturing production exhibits a negative externality on consumers' utility function, derives the sufficient condition for a rise in the pollution tax rate on urban manufacturing to improve national income.

We intend to complement this short stream of literature to get a larger picture of the regional distributive and efficiency consequences of an environmental tax reform. Contrary to the main papers in Harris-Todaro model, we focuse on developed countries in which we assume disparities between regions. To do so, we mainly focus on the paper of Daitoh [2003], to which we add two fondamental assumptions. First, households have a subsistence level of polluting goods that we allow to differ among regions (Jacobs and van der

Ploeg [2016l]). Second, there is frictional unemployment in both sectors. In fact, our paper incorporates some features borrowed from papers that merge search generated unemployment literature (introduced by Pissarides [1998]) within a two regions migration framework (see for example Sato [2004], Kuralbayeva [2013], Satchi and Temple [2009]). In contrast to the previous studies in the Harris-Todaro framework, they show that migration toward city induces frictional urban unemployment that causes an inter-sector wage disparity. Because of the frictional externalities, the allocation of agents between regions / sectors can be sub-optimal. Thus, the original model of Harris-Todaro has been adapted in order to match the developed economies concern.

One of our key contributions is to combine frictional unemployment with non-homothetic preferences for the polluting good in a "Harris-Todaro" economy. We show that a difference in subsistence level of the polluting good among regions may exacerbate the sector-wage disparities due to frictions and this may generate spillovers. Moreover, these specifications allow us to work in an ideal framework in order to study the trade-off between efficiency (employment), inter-regional equity (due to perfect mobility) and environmental welfare of an environmental tax reform. Our paper then joints the literature with the traditional Double Dividend literature (see Bovenberg and de Mooij [1994], Goulder [1995], Bovenberg and van der Ploeg [1998]).

In a general equilibrium framework, we build a two-sector model, within labor is assumed perfectly mobile (Harris-Todaro [1970]) and there is a pollution externality. As in Bovenberg and De Mooij [1994], pollution is due to two different sources: the use, in the production processes, of a polluting input; and the consumption of polluting commodities by household. We represent this pollution commodity as a necessity and we allow its subsistence level to differ between regions. Both sectors present structural unemployment caused by hiring costs, and we use a static search and matching model to formulate frictions on labor markets with individual worker-firm bargaining. The model is fully solved analytically as we have specified, in the simpliest way, preferences and technologies. The main results are the following: frictional unemployment and non-homothetic preferences bring about an inter-region wage differential. Thus, an economy almost always exhibits distortions in the absence of government intervention. A green tax may exacerbate these distortions by generating spillovers, if the labor market is initially more frictional in the region where the subsistence level of the polluting good is lower.

The remainder of the paper is set out as follows. In the next section, we present the basic features of our model. Section 3 solves the general equilibrium, and analyzes the effect of an increase in a green tax on wage disparities, unemployment and migration.

2 The model

The model assumes a closed economy made up of two regions (indexed by $i = 1, 2$, in the following). Each of them is specialized in the production of one good, denoted X_i for region i that are assumed to be imperfect substitutes[1]. We treat the good X_2 as a numeraire and let p_1 denote the relative price of the good X_1.

There is a continuum of workers of exogenous size $\bar{L}$ in this country. L_1 workers are living in region 1 and involved in the production of sector X_1, whereas:

$$L_2 = \bar{L} - L_1 \tag{1}$$

reside in the region 2 and work in the corresponding sector X_2. We assume structural unemployment in both areas caused by hiring costs, and we use a search and matching model to formulate frictions on labor markets with individual worker-firm bargaining. In order to make the analysis as simple as possible, we adopt a static framework[2] . We refer to l_i to specify employed workers in the sector X_i. Besides labor considered as perfectly mobile between sectors, a second input (E_i) enters the production process of X_i. This input causes environmental damage when used in production. Accordingly, it is called the 'polluting' input (Daitoh [2004]) and we can consider it as energy. Households setting in region i also consume "C_i^E " as commodities that also harms the environment. We assume, for simplicity, that the market for E does not exist and that supra-national government imposes a specific tax t_E on the use of E by firms and consumers (Copeland and Taylor [2004], Rapanos [2007], Daitoh & al [2011]). This government uses these tax revenues to provide lump-sum transfers T to each household.

2.1 Households behavior

We use a simple static search and matching framework of labor market to model unemployment among workers. There are heterogeneities (or mismatches) in the labor market that make it costly for a worker or a firm to find a partner with whom they can produce sufficiently high returns (Pissarides [1998]). Labor market heterogeneities are summarized in the matching function that gives the rate at which good matches are formed in the labor market. Given a mass L_i of workers searching a job in the area i, and the number of vacant jobs v_i in this area, in its simplest form, the matching function is defined as: $M_i = m_i(v_i, 1)$, with positive first partial derivatives, negative second derivatives and constant returns to scale. The matching

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[1] Throughout the paper, we will refer to different regions. But this is just convenient terminology: the model is general, and could just as easily represent areas like urban/rural areas, or even just two sectors if consumers preferences are identiques (See the section 3). Yet it can be inconvenient for nations inside a Federal System as European Union: we do not assume different searching cost for migrants in our model (see for instance Combes et al. [2016] for a theoretical framework of search and match with migrants discrimination).

[2] As Diamond [1982] showed, we can describe the essence of job search and recruiting externalities using a static model. For examples of static search and matching models, see Sato [2004], Keuschnigg and Ribi [2008].

function implies that a firm looking for a worker finds one with a probability less than one, equals to $\frac{M_i}{v_i}$, even if there are enough jobs to satisfy all workers. Denoting $\theta_i = \frac{v_i}{L_i}$, the tightness ratio of the labor market, we can rewrite this probability as: $q_i(\theta_i) = \frac{M_i}{v_i} = m_i(1, 1/\theta_i)$. It represents the Poisson matching probability of a vacant job *ie* the rate at which vacant jobs are filled. Symmetrically, the rate at which an unemployed worker finds a job is given by $\theta_i q_i(\theta_i) = M_i$. Then, for workers in the i area, $\theta_i q_i(\theta_i) L_i = l_i$ are employed in sector X_i and $[L_i - l_i]$ are unemployed. Thus, if u_i denotes the unemployment rate in region i, the standard Beveridge curve is defined as:

$$u_i = \left[\frac{L_i - \theta_i q_i(\theta_i) L_i}{L_i}\right] = 1 - \theta_i q_i(\theta_i) \tag{2}$$

In the remain of this paper, we will assume for simplicity that [3] :

$q_i(\theta_i) = \frac{M_i}{v_i} = \mu_i \theta_i^{-\xi_i}$, where $0 < \xi_i = -\frac{\partial q(\theta_i)}{\partial \theta_i} * \frac{\theta_i}{q_i(\theta_i)} < 1$ represents the elasticity of the matching function and $\mu_i > 0$ the efficiency of the process.

Consumption preferences

Each individual worker supplies one unit of labor inelastically and consumes three goods. Let $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E denote the consumption of regional goods and of the polluting good respectively. They are assumed imperfect substitutes in the following consumption utility function Z_i :

$$Z_i = z_i\left(C_i^E, v(C_i^{X_1}, C_i^{X_2})\right)$$

As the environmental degradation acts as an externality, we assume households ignore the adverse effect of their demand for polluting goods on the quality of the environment. Consequently, households i choose $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E in order to maximize utility Z subject to their budget constraint: $p_1 C_i^{X_1} + C_i^{X_2} + t_E C_i^E = I_i$ (with I_i denoting the income of households i).

We assume functional separability between the polluting good and the regional goods in the joint utility function of consumption. Hence, the consumption function is denoted as $Z_i = z_i\left(C_i^E, v(C_i^{X_1}, C_i^{X_2})\right)$ where the first argument of $z_i(\cdot;\cdot)$ is the polluting good C_i^E and the second argument is the conjoint regional consumption goods $v(C_i^{X_1}, C_i^{X_2})$. This specification is similar to the one used by Copeland and Taylor [2004] and it allows us to solve the model analytically. Functional separation implicitly assumes that the price of the polluting good does not impact the ratio of prices between both regional goods.

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[3] Pissarides [1998]0, [1986] and Blanchard and Diamond [1989] have shown that a reasonable approximation to the matching function is a Cobb-Douglas function.

Agents are all assumed risk neutral, meaning that z_i and v_i are assumed to be linear in income. Moreover we assume v homothetic (the aggregated demand of regional goods is independant of income distribution), but, in contrast to the standard literature, we do not allow Z to be linearly homogeneous in C_i^E (but v is homothetic). In fact, usual quasi linear and homothetic preferences imply that the elasticity of substitution between polluting goods and regional goods is constant and thus independent on individual revenues. It results in constant expenditures shares of polluting goods. Hence, in most of the models, the revenue of agents does not affect the allocation between goods, and the green tax on the dirty good is superfluous as a distributional device (Jacobs and van der Ploeg [2016]). However, poor households, but also rural households seem to devote a larger fraction of their consumption to the dirty goods than rich households (Ruiz and Trannoy [2008], Metcalf [1999]). They are therefore the ones most likely to be hurt by an increase in carbon tax. To make the trade-off between redistribution and the efficiency of green tax reforms more realistic and thus more relevant from a policy point of view, we assume Gorman Polar form (1961) preferences: its corresponding indirect utility function can be written as:

$$Z_i^* = \frac{[I_i - \varphi_i(t_E)]}{P_{Z(t_E)}}$$

where I_i is net disposable income of agent i, $\frac{1}{P_{(t_E)}}$ denotes the marginal sub-utility of income $\partial Z_i / \partial I_i$, $P_{Z(t_E)}$ is the marginal price of consumption. $P_{Z(t_E)}$ is the same for all individuals. $\varphi_i(t_E)$ represents the minimum expenditure of the good C_i^E to obtain an utility equals to zero. We allow it to differ between regions. The consumption of the polluting good is determined as:

$$C_i^{E*} = \frac{P'_{(t_E)}}{P_{Z(t_E)}} [I_i - \varphi_i(t_E)] + \varphi'_i(t_E)$$

The Gorman Polar form (1961) preferences makes it possible to model a share of consumption that is not responsive to price changes ($\varphi'_i(t_E)$) and another share that can adapt instantaneously to price variations ($C_i^E - \varphi'_i(t_E)$). This specification allows us to represent the polluting goods as necessities (their income elasticity is less than unity). Moreover, these elasticities now depend on areas, which capture regional disparities (Deaton and Muellbauer [1980], Chung [1994], Jacobs and van der Ploeg [2016]). The consumer first purchases a subsistence level of the polluting good and then allocates the leftover income ($I_i - \varphi_i(t_E)$), in fixed proportions to each good according to their respective preference parameter (as a classical Cobb douglas utility function). The assumption of households' risk neutrality implies that their indirect consumption utility is defined as the purchasing power of their leftover income.

Then in this model, households differ with respect to their income (that depends on their sector market activity and on the government transfers), and their consumption commodities tastes depending on their area.

Income and welfare

Workers supply one unit of labor at wage w_i, if employed in sector i. Both unemployed and employed workers receive a same amount of transfers T from national government. The reservation wage, for which a household is indifferent between being employed or unemployed is then driven to zero[4]. Because we consider a static framework of matching, the *ex ante* probability of being unemployed u_i in the areas i, is equal to the *ex post* unemployment rate (Sato [2004]). The expected indirect utility of workers can be represented by:

$$V_i = u_i * [Z_i^* (T)] + (1 - u_i) * Z_i^*(w_i + T) - \psi [E_{tot}]$$

where $-\psi [E_{tot}]$ denotes the disutility due to the environmental degradation, $E_{tot} = E_1 + E_2 + C_1^E + C_2^E$, the aggregated energy demand being the source of global pollution.

Denoting from now on the relevant variable of the areas i, with the subscript e or u, depending on whether workers are employed or unemployed, this function can be rewritten as:

$$V_i = u_i * [V_i^u] + (1 - u_i) * V_i^e - \psi [E_{tot}] \tag{3}$$

Migration

As in Harris and Todaro (1970) and many others studies, we assume that workers are perfectly mobile between sectors and areas, and that migration occurs so as to equate the expected indirect utility betwen areas. Then we obtain:

$$V_1 = V_2$$

Using (2) and (3), this condition is reduced to:

$$\theta_1 q_1(\theta_1) * [w_1] + V_1^u = \theta_2 q_2(\theta_2) * [w_2] + V_2^u \tag{4}$$

We refer to this condition as the no-migration condition.

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[4] We could have introduced unemployment benefit and utility of leisure for unemployed worker that would have defined their reservation wage. But because, in our economy, global prices are equal between regions, there is no reason for different reservation wages between areas. Then, unemployment-benefit modelling becomes superfluous.

2.2 Firm's behavior

Technology

Following Sato [2004], the production in sector X_i consists of f_i many small firms, each of which can employ only one worker. Firms need to post a vacancy in order to hire workers.

Let c_i denotes the exogenous cost of this vacancy, which is represented in terms of the numeraire good. It can be interpreted as the fixed cost of labor recruitment which is represented in term of the good X_i. Analogously to Sato [2004], Helsey and Strange [1990] or Montfort and Ottaviano[2000], before paying the cost of posting a vacancy, a firm is not sure to be matched with a worker: remember that due to frictions, a vacant job is matched to an unemployed worker with a probability $q_i(\theta_i) < 1$ [5].

If the job of the firm is occupied, firms demand a polluting factor of production E_i at a price t_E and pay their unique worker a wage w_i. Consequently, the amount of output per firm in the sector i is then: $x_i = F_i\left(e_i, 1\right)$ where F_i is concave and displays decreasing return to scale with respect to e_i, the demand of polluting good per firm. As in Bovenberg and Van der Ploeg [1998], the aggregate production function amounts to $X_i = l_i x_i = F_i^*\left(l_i e_i, l_i\right)$ where F_i^* is concave and features constant returns to scale.

Expected profit of each firm is then:

$$\underset{e_i}{max}\, \pi_i = [q_i(\theta_i)(p_i x_i - w_i - t_E e_i)]\text{-}c_i \text{ s.t. } x_i = F_i\left(e_i, 1\right)$$

The firm's polluting good demand (e_i^*) condition is:

$$p_i \frac{\partial F_i\left(e_i^*, 1\right)}{\partial e_i} = t_E \tag{5}$$

Denoting ε_i the elasticity of the production function x_i with respect to e_i, we can rewrite this condition as:

$$\varepsilon_i \frac{p_i x_i}{e_i^*} = t_E \tag{6}$$

Assuming free entry of firms, in the steady state, the expected profit from an occupied job equals the expected costs of filling a vacancy, that gives:

$$p_{Li} = p_i x_i - t_E e_i^* = (1-\alpha_i)\, p_i x_i = w_i + \frac{c_i}{q_i(\theta_i)} \tag{7}$$

Where p_{L_i} denotes the productivity of labor in sector X_i. Equation (7) represents the traditional job

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[5] Then $f_i = \frac{l_i}{q_i(\theta_i)} = \theta_i L_i$.

creation condition: the marginal cost of investing in a job vacancy must correspond to the expected job rent. In contrast to a competitive labor market where firms hire until marginal productivity is equal to the wage, the total cost of worker exceeds the wage by a recruitment cost.

Wage determination

Once a suitably worker is found, a job rent appears that corresponds to the sum of the expected search and hiring costs for the firm and the worker. Wage needs to share this economic (local-monopoly) rent, in addition to compensating each side for its assets from forming the job. We assume a decentralized Nash bargain, which imposes a particular splitting of the matching surplus between the two parties involved according to the relative bargaining power between them. For a worker, the matching surplus is the difference between its expected utility when employed and that when unemployed: $Z_i^* \left(w_i + T\right) - Z_i^*(T) = \frac{w_i}{P_Q}$. For a firm, the matching surplus is the difference between the profit when it fills a vacancy and when it remains with vacancy: $(p_{L_i} - w_i - c_i) - (-c_i) = p_{L_i} - w_i$.
w_i is determined as: $w_i = argmax \left\{ \left(\frac{w_i}{P_Q}\right)^{\beta_i} (p_{L_i} - w_i)^{1-\beta_i} \right\}$ with β_i the worker's bargaining power of the sector is.

Appendix A.I shows that the first-order condition for the maximization of the Nash product implies the following expression of the wage:

$$w_i = \frac{\beta_i}{1-\beta_i} * \left[\frac{c_i}{q_i(\theta_i)}\right] = \beta p_{Li} \tag{8}$$

If hiring costs are zero ($c_i = 0$), in equilibrium $w_i = 0$. Thus, positive hiring costs increase the gap between the utility of employment and that of unemployment. Similarly, a drop in job vacancies (or θ_i) decreases the expected value of the firm's hiring costs ($\frac{c_i}{q_i(\theta_i)}$). This reduces the rents from the job match and decreases as well the wage. If the bargaining power of the low-skilled worker equals one (*i-e* $\beta = 1$), then the low-skilled wage equals the productivity of low-skilled labor (similary to competitive labor market), and labor demand doesn't depend at all of hiring costs.

2.3 The government budget constraint and the Equilibrium in the good market

To abstract from revenue-recycling approach, in line with Harberger (1962), and Daitoh [2004], we assume that the government transfers the tax revenue to consumers in a lump-sum fashion. In this context, the government revenue from pollution tax will not include any kind of redistribution that can affects firms or household location. The government budget constraint is thus decribed as:

$$T\bar{L} = t_E E_{tot} \tag{9}$$

To close the model, we need to determine the ratio of prices that is determined by the demand consumptions. Because X_2 is assumed to be the numeraire, we only need to determine the price of X_1. Remember that we assumed functional separability between the pollution good and the regional goods in the joint utility function of consumption: thus, the price of the polluting good does not impact the ratio of prices between both regional goods. Moreover $v(C_i^{X_1}, C_i^{X_2})$ is homothetic and thus has constant returns to scale. This implies that each individual chooses $\frac{C_i^{X_2}}{C_i^{X_1}} = \rho\left(\frac{p_1}{p_2}\right)$ where ρ is an increasing function. We can thus express $\frac{C_{tot}^{X_2}}{C_{tot}^{X_1}} = \rho(p_1)$ with $C_{tot}^{X_1}, C_{tot}^{X_2}$ respectively the aggregate private demand for good 1 and 2.

The equilibrium on good market requires that the total demand equals to the total supply, it means that: $X_1 = x_1 l_1 = C_{tot}^{X_1} + \frac{l_1 c_1}{p_1 q_1(\theta_1)}$ and $X_2 = l_2 x_2 = C_{tot}^{X_2} + \frac{l_2 c_2}{q_2(\theta_2)}$
The second term in each demand represents the aggregated search costs.
Dividing the two last equations and replacing $\frac{C_{tot}^{X_2}}{C_{tot}^{X_1}} = \rho(p_1)$, we obtain:

$$\left(x_1 l_1 - \frac{l_1 c_1}{p_1 q_1(\theta_1)}\right) = \frac{1}{\rho(p_1)}\left(l_2 x_2 - \frac{l_2 c_2}{q_2(\theta_2)}\right)$$

Noting that $\frac{l_i c_i}{q_i(\theta_i)} = c_i f_i$, the equilibrium on the good market is given by the following condition condition:

$$\frac{\rho(p_1)}{p_1}(p_1 l_1 x_1 - c_1 f_1) = l_2 x_2 - c_2 f_2 \tag{10}$$

3 General Equilibrium

The equilibrium of the model is defined as a tuple $(L_i^*,\, l_i^*,\, e_i*,\, \theta_i^*,\, w_i^*, p_i^*)$ for $i = 1, 2$, of 6*2 variables that satisfies the following conditions: the job creation conditions (7), the wage markup equations (8), the Beveridge curves (2), the firms' energy demands (6) , the no-migration condition (4), the total labour endowment equation (1), the price equation (10) and the price normalization $p_1 = 1$, that we will call (C1).

For a clear understanding of the mechanisms behind our model, we will first compute the general equilibrium assuming that (i) the preferences of households do not differ between areas, (ii) preferences are Cobb-Douglas types (iii) the output of firms take the form of $F_i(e_i, 1) = e_i^{\alpha_i}$. These assumptions are not realistic, but have the adavantages to allow us to solve the model in level. Moreover, assuming that preferences do not differ between households allow us to refer to a well known situation: a simple general equilibrium model where workers can choose freely in which sector to work. In the subsection 2, these assumptions are released and the model will be fully solved by log-linearisation. The subsection 3 compares the results of subsection 1 and 2.

3.1 General Equilibrium in a particular case: same preferences for households

We assume a specific form to our utility function given by:

$$Z_i = z\left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E})^{\gamma}\left(\left(C_i^{X_1}\right)^{\sigma}\left(C_i^{X_2}\right)^{1-\sigma}\right)^{1-\gamma}$$

The function v is a type of Cobb-Douglas and z is a Stone-Geary utility function (a particular case of Gorman-Polar form). Here $\varphi_i(t_E) = t_E\bar{E}$ where $\bar{E}$ is assumed to be a minimum of polluting good consumption . Appendix A.2 gives the solution of the consumer problem.

Before starting to describe the results of the general equilibrium, it is convenient to explain in wich specific case we work. We consider a particular case of our model in which prices, taxes and the minimum consumption of energy are identical between areas. Then the indirect utility of unemployment workers does not differ between areas. In fact, the only difference for workers comes from wages and unemployment rates. This situation is similar to a model with one area in which workers can choose freely in wich sector to work. If moreover labor markets are perfect, we already know from the literature [6], that free mobility between sectors will requiert the equality of wages. But because, in our framework, we introduce unemployment, this condition is now $\theta_1 q_1(\theta_1) * [w_1] = \theta_2 q_2(\theta_2) * [w_2]$, that is nothing else than the "migration" condition for $V_1^u = V_2^u$. This condition requiers that the expected wage of workers, taking account the probability of unemployment, should be the same in both sectors.

Comparatives statics

We start to consider than p_1 and L_i are fixed. We want to analyze the impact of an uncompensated raise of green taxes on both labour markets. From (C1), (7) and (8), (6) yields the equilibrium demand of the polluting input : $e_i^* = \left[\frac{p_i\alpha_i}{t_E}\right]^{\frac{1}{1-\alpha_i}}$. Obviously, an increase of green taxes, decreases the energy or the polluting input. Remembering that $p_{L_i} = (1-\alpha_i)\, p_i x_i$, and substituting into (8) gives :

$$p_{L_i}^* = (1-\alpha_i)\, p_i^{\frac{1}{1-\alpha_i}}\left[\frac{\alpha_i}{t_E}\right]^{\frac{\alpha_i}{1-\alpha_i}} \tag{11}$$

$$w_i^* = \beta_i p_{L_i}^* = \beta_i\,(1-\alpha_i)\, p_i^{\frac{1}{1-\alpha_i}}\left[\frac{\alpha_i}{t_E}\right]^{\frac{\alpha_i}{1-\alpha_i}} \tag{12}$$

$$q(\theta_i^*) = \frac{c_i}{(1-\beta_i)p_{L_i}} = \frac{c_i}{(1-\beta_i)\,(1-\alpha_i)} p_i^{-\frac{1}{1-\alpha_i}}\left[\frac{t_E}{\alpha_i}\right]^{\frac{\alpha_i}{1-\alpha_i}} \tag{13}$$

Intuitively, increasing pollution tax, because it increases the energy factor price, lowers the productivity of labour (that is complementary to energy) (see (11)). Thus wages decrease according to (12). Yet, it

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[6] (see Copeland and Taylor [2004])

is not enough to overcome the raise of energy prices. The profit of a functional firm (with a job filled) decreases. Because, in the long run, the expected profit is always equivalent to the expected costs of opening a vacancy that is fixed, the zero profit condition leads to an increase of $q_i(\theta_i^*)$: the probability of finding a worker for a firm in region i is increasing with the pollution tax (13). In other words, an increase of the pollution tax decreases the probability (θ_i^*) for an unemployer worker to find a job .

Thus, in a partial equilibrium, an increase of green tax lowers wage and energy input but increases unemployment. If production functions where identical, obviously prices will not change, and workers will not be incitated to move between sectors. But we assume assymetrie : α_i the energy intensity, c_i the hiring cost of firms, ξ_i the elasticity of the matching function differ between sectors. What are the impacts on p_1 and L_1 in this context?

General Equilibrium

Equations (11), (12), and (13) give us immediatly $p_{L_2}^*$, w_2^* $q(\theta_2^*)$, because $p_2 = 1$. From the migration condition (8), we finally find θ_1^* in function of θ_2^*: $\theta_1^* = \frac{c_2\beta_2(1-\beta_1)}{c_1\beta_1(1-\beta_2)}\theta_2^*$. And replacing in (11), (12), and (13), we finally have:

$$p_1^* = \left(\frac{t_E}{\alpha_1}\right)^{\alpha_1 - \frac{\alpha_2(1-\alpha_1)\xi_1}{(1-\alpha_2)\xi_2}} \left[\left(\frac{c_1}{\chi_1}\right)\left(\frac{\chi_2}{c_2}\right)^{\frac{\xi_1}{\xi_2}}\left(\frac{c_2\beta_2(1-\beta_1)}{c_1\beta_1(1-\beta_2)}\right)^{\xi_1}\right]^{(1-\alpha_1)} \tag{14}$$

where $\chi_i = \left(\mu_i(1-\beta_i)\,(1-\alpha_i)\,\alpha_i^{\frac{\alpha_i}{(1-\alpha_i)}}\right)$

Thus, $\frac{dp_1^*}{dt_E} > 0$ if and only if: $\frac{\alpha_1}{1-\alpha_1}\frac{1}{\xi_1} > \frac{1}{\xi_2}\frac{\alpha_2}{1-\alpha_2}$. This results is really intuitive: both production sectors use pollution, the relative price of goods thus depends explicitly of their relative intensity. This is what we can call the *pollution-intensity effect*. The higher the intensity of sector 1 is, the more the green tax impacts the sector 1 relatively to the sector 2. Expected wages have to be equalized between sectors due to the no-migration condition. Thus, firms of region 1 must increase their prices in order to insure the zero profit condition. Finally, the productivity of labor will not change except if $\xi_1 \neq \xi_1$. The same reasoning can be apply for the elasticity of the macting function, that characterizes the frictions.

We can rewrite the migration condition, noting that $\theta_1 q(\theta_1) = \frac{l_1}{L_1}$ and substituting (10) into (11), we finally obtain:

$$\frac{L-L_1}{L_1} = \frac{L_2}{L_1} = \frac{(1-\alpha_1)\,[1-(1-\alpha_1)(1-\beta_2)]}{(1-\alpha_2)\,[1-(1-\alpha_2)\,(1-\beta_1)]} * \frac{\sigma}{1-\sigma} \tag{15}$$

The previous equation shows us that the ratio of the number of households in each region does not depend on t_E . In this case, the green tax has no incidence at all on the reallocation of workers between sectors. This result might seem a bit surprising regarding to the previous models of Harris Todaro that deal with environmental issues. In reality, because we have $V_2^u = V_1^u$, we can link our results to the theorem of Pissarides [1989]: With Cobb-Douglas utility function, there is no possibility for a reallocation between sectors. Here, the ratio of the demand on regional good ($\frac{\rho(p_1)}{p_1}$) is exogenous. In consequences, the adjustment of prices is enough to insure the zero profit condition, the no-migration condition and the equilibrium on the good market. This theorem holds with the Stone-Geary utility function because our introduction of a Stone-Geary utility does not have any incidence on global price (weak separability). Moreover, labor frictions have to satisfy the "migration condition". A change of labor market characteristics impacts the number of workers per sector but not trough green taxes. The following propositions summarize the above arguments.

Proposition 1: *If (i) preferences of households do not differ between areas, (ii) preferences of regional goods are of type Cobb-Douglas types, and (iii) the elasticity of energy input to taxes is fixed, then an increase of green tax:*

1. *decreases wages and increases unemployment in both sectors.*
2. *decreases the relative price of goods (p_1^*) if and only if the ratio of the elasticity of the production function with respect to energy over the elasticity of the matching fonction is higher in the sector X_1 than in the sector X_2 ($\frac{1}{\xi_1}\frac{\alpha_1}{1-\alpha_1} > \frac{1}{\xi_2}\frac{\alpha_2}{1-\alpha_2}$).*
3. *does not influence the reallocation of workers between sectors.*

As discussed previously, the result (3) depends on assumption (i), (ii) and (iii). In the next sub-section, we identify and disentangle the impacts of the release of these assumptions on the reallocation of workers between sectors. We show that the release of one of these assumptions is enough to lead to a variation of L_i.

3.2 General case: log-linearization of our model

We consider the general consumption utility function :

$$Z_i = z\left(C_i^E, v(C_i^{X_1}, C_i^{X_2})\right).$$

where $v(C_i^{X_1}, C_i^{X_2})$ is homothetic with constant return to scale, and z is from Gorman Polar form [1961]. Thus the indirect utility function and the consumption of the polluting good can be written as:

$$C_i^{E*} = \frac{P'_{(t_E)}}{P_{z(t_E)}} \left[I_i - \varphi_i(t_E)\right] + \varphi_i'(t_E) \qquad\qquad Z_i^* = \frac{[I_i - \varphi_i(t_E)]}{P_{z(t_E)}}$$

where $\varphi_i(t_E)$ can be seen as the minimum of polluting good consumption, necessary to have a positive utility function. In contrast to the previous section, we allow for $\varphi_i(t_E)$ distinct between regions. In consequence, the migration condition depends on the difference of this level and we obtain :

$$V_1 = V_2 \Leftrightarrow \theta_1 q_1(\theta_1) * [w_1] = \theta_2 q_2(\theta_2) * [w_2] + \triangle\varphi(t_E) \qquad with\, \triangle\varphi(t_E) = \varphi_1(t_E) - \varphi_2(t_E)$$

This equation can give rise to two differents interpretations. The first one is to consider two areas that differs by their need of polluting good. We can assume for example that the public transport of region 2 provides full coverage and that is not the case of the other one. Thus, the need of polluting good in region 1 is higher ($\triangle\varphi_i(t_E) > 0$), in order to compensated the lack of public transportation. Switching from region 2 to region 1 implies to be shure that expected wages in area 1 overcomes the additional cost of the polluting good. An other explanation, that is maybe more intuitive, is to consider that $\triangle\varphi_i(t_E)$ is the cost of migration or the cost of mobility. It depends on the price on polluting good (transportation cost). Imagine that $\triangle\varphi_i(t_E)$>0. In this configuration, the expected wage is higher in region 1, than in region 2 ($\theta_1 q_1(\theta_1) * [w_1] > \theta_2 q_2(\theta_2) * [w_2]$). Workers who live in area 2 have thus an incentive to commute and work in area 1 for a higher expected wage. If they commute, they have to pay a transport cost equals to $\triangle\varphi_i(t_E)$>0. Until $\theta_1 q_1(\theta_1) * [w_1] > \theta_2 q_2(\theta_2) * [w_2] + \triangle\varphi_i(t_E)$,workers living in 2 have an interest to work in area 1. Thus the equilibrium is find where the expected wage in 1 is equal to the expected wage 2 plus the additional cost of commuting. In both of these interpretations, the migration has a cost that depends on the green tax.

Because we assume non explicit utility function, the model is solved through log-linearization. Appendix A.III gives the details of the computations. The tilde $\tilde{()}$ denotes percentage changes relative to initial values, i.e.$\tilde{l} = \frac{dl}{l}$. Exceptions to this definition are separately indicated. Detoning $\omega_2 = \frac{t_E e_2}{p_{L_2}}$; $\omega_1 = \frac{p_{L_1}}{x_1 p_1}$, we find:

Table 1: The Log-linearization solutions of the model

Energy input of region 2	$\widetilde{e_2} = -\varepsilon_2 \widetilde{t_E}$
Tightness ratio of region 2	$\tilde{\theta}_2 = -\left[\frac{\omega_2}{\xi_2}\right] \widetilde{t_E}$ with
Wage of region 2	$\widetilde{w_2} = -\omega_2 \widetilde{t_E}$
Tightness ratio of region 1	$\widetilde{\theta_1} = -\left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} \frac{\omega_2}{\xi_2} - \frac{t_E \triangle \varphi_i'(t_E)}{\theta_1 q_1(\theta_1) w_1}\right] \widetilde{t_E}$
Wage of region 1	$\widetilde{w_1} = -\xi_1 \left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} \frac{\omega_2}{\xi_2} - \frac{t_E \triangle \varphi_i'(t_E)}{\theta_1 q_1(\theta_1) w_1}\right] \widetilde{t_E}$
Energy input of region 1	$\widetilde{e_1} = -\varepsilon_1 \omega_1 \left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} \frac{\omega_2 \xi_1}{\xi_2} - \frac{\xi_1 t_E \triangle \varphi_i'(t_E)}{\theta_1 q_1(\theta_1) w_1} + 1\right] \widetilde{t_E}$ with
Ratio of prices	$\widetilde{p_1} = \left((1 - \omega_1) - \omega_1 \xi_1 \left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} \frac{\omega_2}{\xi_2} - \frac{t_E \triangle \varphi_i'(t_E)}{\theta_1 q_1(\theta_1) w_1}\right]\right) \widetilde{t_E}$

We need the variation of total workers between regions to complete the general equilibrium solutions. We want to compare our results with the results of the first subsection. Remember the denotation of the main assumptions in proposition 1*:*(i) preferences of households do not differ between areas; (ii) preferences of regional goods are of type Cobb-Douglas types; (iii) the elasticity of energy input to taxes is fixed (α_i). We try to disentangle the effect of a drop of each assumption on the reallocation of workers. Loglinearizing the equation (10) gives us:

$$\widetilde{L_1} - \widetilde{L_2} = \left(\underbrace{(\widetilde{\theta_2} - \widetilde{\theta_1})}_{=0\, under\ (i)} + \underbrace{(1 - \xi_{p_1})\, \widetilde{p_1}}_{=0\, under\ (ii)} + \underbrace{(\widetilde{\varepsilon_2} - \widetilde{\varepsilon_1})}_{=0\, under\ (iii)} \right) \tag{16}$$

where $\xi_{p_1} = \frac{\partial \rho(p_1)}{\partial p_1} * \frac{p_1}{\rho(p_1)}$ is the relative-price elasticity of relative demand for the regional goods (see proof in the appendix A.III). Thus, the release of one of the three assumptions is enough to lead to a reallocation of workers between areas/sectors.

The impact of assumption (iii) is very intuitive. Assumption (iii) means that elasticity of energy input is fixed. Thus ($\widetilde{\varepsilon_2} = \widetilde{\varepsilon_1} = 0$). Releasing this assumption, and assuming that $\widetilde{\varepsilon_1} > \widetilde{\varepsilon_2}$, leads to a higher impact of energy tax in sector 1 than in sector 2. Yet, the variation of elasticities do not affect the wages nor labor productivities. Thus, the reallocation of workers will be from region 1 to region 2 with no ambiguity $(\widetilde{L_2} > \widetilde{L_1} \Leftrightarrow d(\frac{L_2}{L_1}) > 0$).

With (ii), $\frac{\rho(p_1)}{p_1}$ is fixed and then $\xi_{p_1} = 1$ (by definition of the Cobb-Douglas function). Droping this assumption, an increase of the relative price p_1 will have two distinctive impacts on the reallocation of workers. First, a raise in p_1 increases the relative productivity of labor and thus the relative wage of region 1 to region 2. This will incite workers from region 2 to move to region 1 until search and match frictions equalized expected wages. Finally, region 1 will present a higher wage but a higher unemployment rate. In the other side, an increase of p_1 decreases the relative demand for the good X_1 and thus decreases the demand of firms in area 1 at the elasticity ξ_{p_1}. Because each firm hires one worker, this second effect tends to lower the demand of workers in region 1. If the first effect dominates the second one, that is if $(1 > \xi_{p_1})$, then an increase of the relative price pushes workers to move from region 2 to region 1. Note that under (i) and (iii), $\widetilde{p_1} = \left((1-\omega_1) - \omega_1 \left[\frac{\xi_1 \omega_2}{\xi_2}\right]\right) \widetilde{t_E}$. It gives $\widetilde{p_1} > 0$ if an only if $\frac{1}{\xi_1}\frac{t_E e_1}{p_{L_1}} > \frac{1}{\xi_2}\frac{t_E e_2}{p_{L_2}}$. It means that when the ratio of input prices in sector 1 is more intensive pollution, an increase of green tax raises the relative price (Proposition 1). Then, assuming than sector 1 is more "intensive in polluting good", an increase in green taxes raises p_1 and raises L_1 if $(1 > \xi_{p_1})$. In this situation, workers reallocate from the less-intensive polluting industry to the more- polluting intensive industrie. This resul is similar to the one in Daitoh [2004]. Green tax may increase the total labor in the more intensive energy sector, if the relative-price elasticity of relative demand for the good intensive in pollution (ξ_{p_1}) is small.

The drop of the assumption (i) is new in this context. If $\triangle\varphi(t_E) = 0$, the migration condition is given by $\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} = 1$. From the equations of the tightness ratio of the labor market (Table 1), we find immediately that $\widetilde{\theta_2} = \widetilde{\theta_1}$. The no-migration condition imposes similar variation of the labor tightness. It is no more the case as soon as we introduce a migration cost that depends on the green tax. We obtain a relative variation of the ratio of θ_i as follow:

$$\widetilde{\theta_2} - \widetilde{\theta_1} = -\frac{1}{\theta_1 q_1(\theta_1) w_1}\left[(\triangle\varphi(t_E))\frac{\omega_2}{\xi_2} + t_E \triangle\varphi'(t_E)\right]\widetilde{t_E} < 0 \qquad if \triangle\varphi(t_E) > 0. \tag{17}$$

In contrast to assumption (i) and (ii), the non-homothetic utility function q impacts directly the formation of wages and unemployment. If $\triangle\varphi(t_E)$ is initialy positive, the initial expected wage of region 1 is higher than the one in region 2. Noting that$\widetilde{\theta_2} - \widetilde{\theta_1} = \widetilde{\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}}$, equation (17) shows that an increase of green taxes raises the pre-existing expected wage gap between regions. Expected wages in region 1 have to overcome the additional surplus of migration cost, inducing by the raise of green taxes. Thus, it becomes more relatively costly for firms to hire a worker in region 1 than in region 2, and finally firms and thus workers are reallocated from sector 1 to sector 2 $(\widetilde{L_1} - \widetilde{L_2} < 0)$. The difference in the subsistence level of the polluting good consumption implies necessarily a wage disparity that amplifies frictions in the region where it is the highest one. Here, if high wages gap is initially due to high labor market frictions, raising green taxes may lead to inefficient reallocation of workers and contribute to generate negative spillovers. The difference in subsistence level of production should be interpreted as an additional labor distortion. In other words, the no-migration condition, driven by wage disparities, causes an additional misallocation of labor between the two regions. Finally we have:

$$\widetilde{L_1} - \widetilde{L_2} = -\left(\left[\frac{(\triangle\varphi(t_E))}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2} + (1-\xi_{p_1})\xi_1\left(\omega_1 + t_E\triangle\varphi'(t_E)\right)\right] + ((1-\xi_{p_1})(1-\omega_1)) + (\widetilde{\varepsilon_2} - \widetilde{\varepsilon_1})\right)\widetilde{t_E} \tag{18}$$

Thus, if $\triangle\varphi(t_E) > 0$ and $(1-\xi_{p_1}) > 0$, the presence of a subsitence level of pollution always tends to reallocate workers from region 1 to region 2, independently of initial frictions in region 2. The two following propositions summarize the above arguments

Proposition 2: *In a more general framework, assuming that sector 1 is the relatively polluting intensive sector, a raise of green tax has an ambigous impact on the reallocation of workers between regions. Workers will tend to move from sector 1 to 2 the higher:*

- *the relative elasticity of the polluting good to the tax between region 1 and region 2 is.* $(\widetilde{\varepsilon_1} > \widetilde{\varepsilon_2})$
- *the relative-price elasticity of relative demand for the regional goods* (ξ_{p_1}) *is.*
- *the relative subsistence level of the polluting bewteen region 1 and 2* $(\triangle\varphi(t_E))$ *is.*

Because we assume that the revenue of green taxes is recycle in the lum-sum fashion way[7], and because in this framework green taxes increase unemployment in both sectors, there is no possibility for obtaining a decrease of the general level of unemployment with an environmental policy. Still, we are able to predict if the reallocation of workers among regions, induced by green taxes, generates negative or positive spillovers on employment.

Proposition 3: *We assume that sector 1 is the relatively polluting intensive sector and presents also the lowest expected wage due to a high level of unemployment in this industry (thus*$\triangle\varphi(t_E)<0$*) . Then an increase of green taxes will always lead to an inefficency reallocation of workers that contributes to higher the global level of unemployment if and only if :*

$$\left(\left[\frac{(-\triangle\varphi(t_E))}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2} + (1-\xi_{p_1})\xi_1(-\triangle t_E\varphi'(t_E))\right] > (1-\xi_{p_1})\left(-(\xi_1+1)\omega_1 - 1\right) + (\widetilde{\varepsilon_1} - \widetilde{\varepsilon_2})\right)$$

If $\triangle\varphi(t_E) < 0$, there is a migration cost for workers in sector 1 to move to sector 2. Raising green taxes

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[7] This assumption will be remove in the next section

exacerbate this cost. Basically, workers will be stocked in region 1 and less able to move. This tends in favor of a reallocation of workers from sector 2 to 1. The impact on the relative price p_1 is ambiguous. In one side, because sector 1 is more intensive in pollution, p_1 is suceptible to raise. On the other side, the expected wages of region 2 raises due to the increase of the migration cost and tends to lower p_1. If the first effect overcome the second, assuming $\xi_{p_1} < 1$ and $(\widetilde{\varepsilon_1} - \widetilde{\varepsilon_2})$ small, then green taxes will lead to a reallocation of workers from sector 2 to 1. Yet, the unemployment tax rate whithin this region is already the highest one. The reallocation is then inefficient in term of employment. Moreover, it could also lead to an inefficiency in term of environment if

We have shown that if we relax the standard assumptions, green taxes may impact the reallocation of workers between areas. Moreover, assuming a subsistence level of polluting consumption, green tax may induce a misleading reallocation of workers and may generate a negative spillovers. How to remove this inefficiency? What are the instruments needed? The next section tries to give some elements of answers.

4 First-best and second best allocation of agents

We have shown in the preceding section, that the general equilibirum of our economie suffers from search and environmental externalities. The first subsection focus on the optimal control problem and the implementation of the first-best allocation. The second sub-section deals with wages taxes and subsidies that the Government can set in order to be as close as possible to the first best. We derive numerical results in order to compare the first best and the second-best policy for different scenarii.

4.1 First best allocation

We assume the social planner able to fix how many workers are in both of sectors, and how many firms operate. However, the planner is assumed to take the wage and price equations as given and to be unable to directly alter payments to individuals.

We define the optimal allocation of agents as a tuple (L_i^*, f_i^*,w_i^*, p_i^*,l_i^*) that maximizes with respect to l_i and f_i:

$$\underset{L_1,f_1,f_2}{max}\, Wtot = \sum \left[(L_i - l_i) * V_i^u + l_i * V_i^e\right] - \bar{L}\psi\left[E_{tot}\right]$$

Under the wage equations $w_i^* = \left[\frac{\beta}{1-\beta}\frac{c_1}{q_1(\theta_1^*)}\right]$, and the price equation (10). Substituting these equations into the W, $\Pi = f_2\pi_2 + f_1\pi_1$ and $\frac{l_i}{q(\theta_i)} = f_i$ into (11) gives us: $(l_1w_1 + l_2w_2 - \varphi_i(t_E)\bar{L}) + \pi_{tot} + T\bar{L} = (p_1l_1x_1 - c_1f_1) + (l_2x_2 - c_2f_2) = (l_2x_2 - c_2f_2)$ (1)

$$\underset{L_1,f_1,f_2}{max}\, Wtot = \frac{1}{P_{Z(t_E)}}\left[(p_1 l_1 x_1 - c_1 f_1) + (l_2 x_2 - c_2 f_2)\right](1-\psi \bar{L} P'_{(t_E)}) - \psi \bar{L}\left[l_1 \varphi_1'(t_E) + l_2 \varphi_2'(t_E) + l_2 x_2 \frac{\epsilon_2}{t_E} + l_1 x_1 \frac{\epsilon_1}{t_E}\right]$$

$$s.t \qquad \frac{\rho(p_1)}{p_1}(p_1 l_1 x_1 - c_1 f_1) = l_2 x_2 - c_2 f_2$$

The Lagrangian associated to this maximizing problem is the following one:

$Lagragien = \frac{1}{P_{Z(t_E)}}\left[(p_1 l_1 x_1 - c_1 f_1) + (l_2 x_2 - c_2 f_2)\right](1-\psi \bar{L} P'_{(t_E)}) - \psi \bar{L}\left[l_1 \varphi_1'(t_E) + l_2 \varphi_2'(t_E) + l_2 x_2 \frac{\epsilon_2}{t_E} + l_1 x_1 \frac{\epsilon_1}{t_E}\right] - \lambda(\frac{\rho(p_1)}{p_1}(p_1 l_1 x_1 - c_1 f_1) - l_2 x_2 - c_2 f_2)$The first-order conditions for maximizing W with respect to L_1 and f_1,f_2 are in tthe Appendix A.III. Solving these equations together gives the optimal number of workers and firms (l_i^* and f_i^*):

Corollary *: If $\psi = 0$, the equilibrium does not attain the optimal allocation of agents, except if $\xi_2 = (1-\epsilon_2)(1-\beta_2)$, $\xi_1 = (1-\epsilon_2)(1-\beta_1)$ and $\alpha_1 = c_2 \frac{\alpha_2}{c_1}$. That means that the initial equilibrium almost exibits distorsions even in a case with no environmental externalities.*

Interpretation: Remember that there are two central market failures in the matching model : congestion externalities and appropriability problems. The congestion externalities are as follows. Workers fail to internalize the fact that should they look for a job, they generate extra jobs at a rate lower than their own probability to find a job. This externality leads to too much worker search, i.e. too much unemployment.The appropriability problems come from the process of wage bargaining, when workers and firms are engaged in a process to share the surplus of accepting a job. Workers only appropriate a fraction of the private value of the jobs they find. Hence the value of looking for a job (*i.e.* the opportunity cost of working) is underestimated. This is the appropriability problem which leads to too little worker search, *i.e.* too little unemployment. Under the Hosios Condition, the low-skilled employment equilibrium is optimal: the appropriability and congestion problems exactly balance each other. With our constant returns to scale assumption on the matching function, Here the Hosios condition is satisfied if the workers' share in the surplus of a match (β) times $(1-\epsilon)$ is equal to the elasticity of the matching function (ξ).
In this case, we show that the equilibrium attain the optimal allocation under the Hosios conditions. What is interesting here, it is that there exists a case where even if the sector the most intensive in energy presents the highest employment rate, a green tax can lead to an allocation that is closer to the optimal allocation if in its sector $(1-\epsilon)(1-\beta) < \xi$. It means that the initial number of firms in this sector is initially to high.

4.2 The Second best policies: an empirical illustration

The second sub-section deals with wages taxes and subsidies that the Government can set in order to be as close as possible to the first best. Due to the complexity of the model, we are not able to derive analytical

results that gives good intuition. We propose then numerical results in order to compare the second-best policy for different scenarii.
We assume z a Stone-Geary utility function with $\bar{E_1} = \bar{E_2}$, and v a standard CES. Thus Z can be written as:

$$Z_i = z\left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E_i})^\gamma \left(\left(\left(\delta C_i^{X_1}\right)^\sigma \left((1-\delta)C_i^{X_2}\right)^\sigma\right)^{\frac{1}{\sigma}}\right)^{1-\gamma}$$

These last assumptions are enough to insure migration due to variations of price and the minimum of consumption. Yet for simplicity we assume $F_i(e_i, 1) = e_i^{\alpha_i}$. The government is allow to introduce s_i, a wage subvention in sectors in order to allow reallocation.
We start in a configuartion where, the region 1 is the more intensive in pollution $\alpha_1 > \alpha_2$, the expected wage of the region 1 is lower than in the region 2 due to high level of unemployment ($\beta_1 > \beta_2$) and the variation of $\bar{E_1} < \bar{E_2}$. We wants to identify the optimal wage suventions and transfers that results from the adaptation of a given environmental tax.
The table 2 presents the value of parameters of this model.

Table 2: Parameters of the model

Energy input intensity parameters	$\alpha_1 = 0.6$; $\alpha_2 = 0.4$
Labor market parameters	$\mu_1 = \mu_2 = 0.37$; $\beta_1 = 0.6 > \beta_2 = 0.5$; $\xi_1 = \xi_2 = 0.5$; $c_i = 0.1w_i$
Consumption preferences parameter	$\gamma = 0.1$;$\sigma_1 = 0.5$;$\sigma_1 = 0.6$; $\delta = 0.5$
Minimum of polluting good	$\bar{E_1} - \bar{E_2} = -1$
The green tax rate	$t_E = 0.1$

We calibrate $\mu_1 = \mu_2 = 0.37$ in order to have a unemployment rate at 7% in area 2 and 14.2% in area 1. Thus for the baseline scenario (see the first row of the next table), it gives a total unemployment rate at 10%. Moreover in the baseline scenario, we fixed the environmental dammage weight ($\psi(E_{tot})$) in order to have an optimal environmental tax rate at 0.01. The Table 3 presents the value of optimal wages subventions/taxes, and green taxes. Value of the total employment, and the total welfare are also reported.

Table 3: optimal tax structure[8]

	baseline	Higher dammages	Optimal Allocation
t_1	-0.54	-0.62	-
t_2	0.22	0.2	-
t_E	0.1	0.15	0.1
L_1	0.64	0.68	Forthcoming
E_{tot}	0.34	0.25	Forthcoming
$\frac{\bar{L}-(l_1+l_2)}{\bar{L}}$	0.1	0.12	Forthcoming
T	3.54	4.23	Forthcoming

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As we can see, with the initial configuration, an implementation of green taxes implies to subsidise wages in the region 1 and to tax wage in region2, in order to internalize the spillovers. This will contribute inside our model to moderate the initial wage disparities due green taxes, and to increase employment. An increase of dammages (or pollution externalities), increases the environmental tax, the wages subsidies but still lowers the total employment highlighting the importance of the trade-off between environment and employment. Other instruments are thus needed to

5 Conclusion

Based on the Harris-Todaro framework, our model contains several features that contribute to better understand the distribution of green taxes burden from the perspective of regional inequalities. In contrast to the previous studies in the Harris-Todaro framework, pollution is due to the use of a dirty input in the production processes of the two goods, that can also be consumed by households. Commodities tastes differ among areas and we assume non-homothetic preferences for the polluting good consumption. It allows us to represent the dirty good as a necessity. Finally, we introduced frictional unemployment in both sectors. Thus, we allow regions/areas to differ with respect to three components: (i) the subsistence level of the dirty consumption of their residents, (ii) the pollution intensity of their production sector, and (iii) the level of frictions on their labor market.

We find that green taxes tend to decrease wages and increase unemployment in both sectors. Under non-homothecy and/or non Cobb-Douglas utility function assumptions, a change of the relative price of goods is not enough to insure the no-migration condition; a reallocation of workers between regions/sectors

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[8] For the moment, the calibration of the model is not totally robust: estimations of the optimal tax structure are too sensitive to the labor market characteristics (in particular to μ). Moreover, the first simulations of the optimal reallocation gave some results that we are not able to interpret. We choose to do not present the results yet. Estimations and calibration will be improved in a next version.

appears. Moroever, frictional unemployment and non-homothetic preferences bring about inter-region wages differential. Typically, non homothetic preferences introduce a cost of migration that depends on green taxes. Thus, an economy almost always exhibits distortions in the absence of the government intervention. Green tax may exacerbate these distortions by generating spillovers, if the labor market is initially more frictional in the region where the subsistence level of the polluting good is the lowest one, and if the elasticity of the relative price is small. In consequence, the "natural" reallocation of workers in the long-run is inefficient and contributes to increase unemployment. The government needs to use other instruments in order to internalize these negative spillovers. Wages subsidies are explored as the solution to remove distortions. Simulations are done in order to compare the first best case, 'the optimal allocation of agents', with the second best situation. We find (with priliminary simulations), that these instruments are not enough to overcome the total negative impact of green taxes on unemployment.

This paper is still in progress and we intend to complement it with further investigations. If an employment dividend is not possible in this framework, an increase of the global welfare may be feasible with the introduction of different lump-sum transfers between areas. It would be interesting to study more deeply this last point in a theoretical way.

Appendix

A.I : Wage bargaigning

Wage of worker_i is determined as: $w_L = argmax\left\{\left(Z_i^*\left(I_i^E\right) - Z(I_i^U)\right)^{\beta}\left(p_{Li} - w_L\right)^{1-\beta}\right\}$

where $Z^*\left(I_i^E\right) = \left(\frac{w_i+T_i-t_E\bar{E}_i}{P_Q}\right)$ and $\left[Z^*(I_i^U)\right] = \left(\frac{T_i-t_E\bar{E}_i}{P_Q}\right)$

This is equivalent to $w_L = argmax\left\{\beta\left(lnZ*\left(I_i^E\right) - \left[Z^*(I_i^U)\right]\right) + (1-\beta)ln\left(p_{Li} - w_L\right)\right\}$. First order condition gives: $\beta\left[\frac{1}{P_Q\left[Z_i^{E*}-Z_i^{U*}\right]}\right] - (1-\beta)\left[\frac{1}{p_{L_i}-w_L}\right] = 0$. And with equation (7) we obtain:

$P_Q\left[Z_i^{E*} - Z_i^{U*}\right] = w_i = \frac{\beta}{1-\beta} * \left[p_{L_i} - w_{Li}\right] = \frac{\beta}{1-\beta} * \left[\frac{c}{q(\theta)}\right]$ (A.1)

A.II : The general equilibrium: The Cobb-Douglas example.

$$Z_i = z\left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E})^{\gamma}\left(\left(C_i^{X_1}\right)^{\sigma}\left(C_i^{X_2}\right)^{1-\sigma}\right)^{1-\gamma}$$

From the first-order conditions of the maximization of Z, we obtain the uncompensated demand for good $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E and the indirect utility of consumption.

$$C_i^{E*} = \frac{\gamma}{t_E}\left[I_i - t_E\bar{E}_i\right] + \bar{E}_i$$

$$C_i^{X_1*} = (1-\gamma)\frac{1}{\sigma p_1}\left[I_i - t_E\bar{E}_i\right] = \frac{(1-\sigma)\sigma}{\sigma p_1}C_i^{X_2*}$$

$$Z_i^* = \frac{\left[I_i - t_E\bar{E}_i\right]}{P_Z}$$

where P_Z represents the price index defined as : $\left(\frac{t_E}{\gamma}\right)^{\gamma}\left[\frac{(\sigma p_1)^{\sigma}(1-\sigma)^{1-\sigma}}{(1-\gamma)}\right]^{1-\gamma}$.
Thus:

$$\frac{C_i^{X_1*}}{C_i^{X_2*}} = \frac{\sigma p_1}{(1-\sigma)}$$

A.III : The general equilibrium: The log-linearization.

The price equation (10) is given by:

$\left(x_1 l_1 - \frac{l_1 c_1}{p_1 q_1(\theta_1)}\right) = \frac{1}{\rho(p_1)}\left(l_2 x_2 - \frac{l_2 c_2}{q_2(\theta_2)}\right)$

Replacing $\frac{l_i c_i}{q_i(\theta_i)}$ with the wage equation (8) and using the definition of the labour productivity gives us:

$(l_1 w_1) = \frac{p_1}{\rho(p_1)}\left(\frac{(1-\alpha_1)}{(1-\alpha_2)}\frac{[1-(1-\alpha_1)(1-\beta_2)]}{[1-(1-\alpha_2)(1-\beta_1)]}\right)(l_2 w_2)$

$\frac{L_1}{L_2} = \frac{(\theta_2 q_2(\theta_2) w_2)}{(\theta_1 q_1(\theta_1) w_1)}\frac{p_1}{\rho(p_1)}\left(\frac{(1-\alpha_1)}{(1-\alpha_2)}\frac{[1-(1-\alpha_1)(1-\beta_2)]}{[1-(1-\alpha_2)(1-\beta_1)]}\right)$

Noting that $\widetilde{\theta_2} - \widetilde{\theta_1} = \frac{\widetilde{\theta_2 q_2(\theta_2) w_2}}{\theta_1 q_1(\theta_1) w_1}$, the log-linearization of the last equation gives

$$\widetilde{L_1} - \widetilde{L_2} = \left(\underbrace{(\widetilde{\theta_2} - \widetilde{\theta_1})}_{=0\ under\ (i)} + \underbrace{(1-\xi_{p_1})\,\widetilde{p_1}}_{=0\ under\ (ii)} + \underbrace{(\widetilde{\varepsilon_2} - \widetilde{\varepsilon_1})}_{=0\ under\ (iii)}\right) \tag{19}$$

where $\xi_{p_1} = \frac{\partial\rho(p_1)}{\partial p_1} * \frac{p_1}{\rho(p_1)}$ is the relative-price elasticity of relative demand for the regional goods (see proof in the appendix A.III). Thus,

A.IV: The optimal reallocation

$\underset{L_1,f_1,f_2}{max}\, Wtot = \frac{1}{P_{Z(t_E)}}\left[(p_1 l_1 x_1 - c_1 f_1) + (l_2 x_2 - c_2 f_2)\right](1-\psi\bar{L}P'_{(t_E)}) - \psi\bar{L}\left[l_1\varphi_1'(t_E) + l_2\varphi_2'(t_E) + l_2 x_2\frac{\epsilon_2}{t_E} + l_1 x_1\frac{\epsilon_1}{t_E}\right]$

$s.t \qquad \frac{\rho(p_1)}{p_1}(p_1 l_1 x_1 - c_1 f_1) = l_2 x_2 - c_2 f_2;$

$\frac{\partial W}{\partial L_1} = A\,(l_i x_1)^{1-\sigma}(l_2 x_2 - c_1 f_1 - c_2 f_2)^{\sigma}\left((1-\sigma)\frac{\xi_1}{L_1} + \frac{\sigma l_2 x_2/L_2}{l_2 x_2 - c_1 f_1 - c_2 f_2}\right) - \bar{L}\psi'\left[E_{tot}\right]\left(\frac{\epsilon_1 - \epsilon_2}{t_E}\right)\frac{l_2}{L_2}x_2\frac{\sigma}{1-\sigma} = 0$

$\frac{\partial W}{\partial f_1} = A\,(l_i x_1)^{1-\sigma}(l_2 x_2 - c_1 f_1 - c_2 f_2)^{\sigma}\left((1-\sigma)\frac{(1-\xi_1)}{f_1} + \frac{\sigma c_1}{l_2 x_2 - c_1 f_1 - c_2 f_2}\right) - \bar{L}\psi'\left[E_{tot}\right]\left(\frac{\epsilon_1}{t_E}\right)\frac{\sigma}{1-\sigma} = 0$

$\frac{\partial W}{\partial f_2} = A\,(l_i x_1)^{1-\sigma}(l_2 x_2 - c_1 f_1 - c_2 f_2)^{\sigma}\left[(1-\xi_2)\,x_2 l_2 + c f_2\right] - \bar{L}\psi'\left[E_{tot}\right]\left(\frac{\epsilon_2}{t_E}\right)\frac{\sigma}{1-\sigma} = 0$

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