Optimal Environmental Taxation with Heterogeneous Households and Endogenous Productivities

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Work in progress, please do not quote

Abstract

How should redistributive governments set pollution taxes in a context of endogenous productivity through investment in education?

This paper investigates if the optimal green tax is still equal to the Pigouvian rate, in a context of endogenous productivity, or the distributive / efficient impact should be remove. In a Mirrlees partial equilibrium model based on Jacobs and de Mooij [2015], and Jacobs and Van der Ploeg [2010], we introduce endogenous productivities through investment in education. Each household differs by ability and chooses whether to acquire an education and become a high-skilled worker or remain unskilled. Given these assumptions, our economy is characterized by an ability cut-off such as those with education cost parameter strictly above will invest in education and become skilled. It determines endogenously the proportion of skilled and unskilled workers in the economy that is directly impacted by the government policy instruments. We derive the optimal carbon tax in conjunction with the optimal redistributive income tax and lumps sum transfers. We found that discriminating lump sum tax transfers between high and low skilled workers is necessary to allow the government to reach the first best pollution tax (the Pigouvian one). Tax subsidies on education have the same advantage. Yet, this result holds only under the homothetic properties of the human capital production function. Otherwise, neither lump-sum transfers, nor education subsidies allow reaching the first best pollution tax.

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Preliminary Motivations /Introduction

Environmental taxes, such as pollution emission taxes or energy consumption taxes, were introduced by Pigou [1920], in order to address the problem of emitters of greenhouse gases not facing the full social cost of their actions. Pigou taught us that to internalize a negative environmental externality; the optimal environmental tax must be equal to the marginal external damage arising from polluting activities. Even if economists traditionally recognize the efficiency of such a market-based instrument to achieve a better environmental quality, there are still debates on the desirability of a carbon price. The opponents to the carbon tax underline its macroeconomic gross cost measured as the welfare loss due to lower growth, consumption or employment. Indeed, as any indirect taxation, a green tax causes a loss of purchasing power for the consumers. This loss is somehow alleviated by the substitution effect that leads them to alter their consumption basket, but not completely offset. Beyond this fact, distributive elements matter also when we consider how costs of environmental policies are likely to be distributed among individuals with different incomes. Green taxes are usually said to be strongly regressive as the energy share in the total expenditure of the poor households is larger compared to the one of rich households (Metcalf [1999]). Thus, the project of a global carbon tax is often considered costly and unfair, and governments are reluctant to implement significant green tax shifts for fear of strong opposition from public opinion.

The "double dividend" literature emerged in this context. In order to legitimate the use of environmental taxes, economist tried to diminish the gross cost of environmental taxes, and even to make it negative. If the initial tax system is not optimal, governments can use the revenues of environmental taxes to decrease other distortionary taxes. Hence, an environmental tax may simultaneously improve the environmental quality and achieve a less distortionary tax system, i.e. it may lead to a double dividend, according to Goulder [1995]. Parry [1995] establishes the general condition which guarantees the existence of the second dividend: The revenue-recycling effect that reduces the existing tax distortions has to be greater than the tax interaction effect, which increases the welfare gross cost of the green tax through the mutual tax erosion. A number of studies have emphasized that the revenue-recycling effect renders a weak double dividend more likely, because reducing distortionary taxes is preferred to reducing non-distortionary lump-sum taxes (see e.g. Fullerton [2013] and Goulder et al. ([1997] and [1998]), Bovendberg and Van der Ploeg [1996]). Thus, the theory of second best has challenged the optimality of the Pigouvian tax. In the presence of distortionary taxes, the optimal Pigouvian tax should be set above or below the marginal environmental damage from pollution, depending on its capacity to reduce initial distortions.

Somewhat surprisingly, the analyses of the double dividend issue have until recently neglected the "distributional effect" associated with distortionary taxation. The recent debates on the regressive nature of green taxes prompted economists to differ from representative agent models and to allow for heterogeneous agents. It seems clear that, if a carbon tax reform is to be done, the way the tax proceeds are redistributed is of foremost importance for equity concerns. Papers on these issues are still few. Allowing for heterogeneous agents, Jacob and de Mooij [2015] have shown that one should not only look at the revenue-recycling effect, but also account for the so-called "distributional effect" that is associated with distortionary taxation. Whether a weak double dividend occurs depends on the balance of the revenue-recycling and distributional effects. But, on the optimal tax system, the two exactly offset each other and finally the optimal environmental tax should not differ from the Pigouvian rate. Jacob and Van der Ploeg [2016] generalize this result in a model with non-homothetic preferences to capture the potential regressivity of green taxes (the poor spend a large part of their income on dirty goods). They found surprisingly that the optimal carbon tax still amounts to the value of marginal climate damages (the Pigouvian tax) even though preferences are non-homothetic. In contrast to the large part of the literature on the double dividend issue, both of these papers advise an environmental tax equal to the Pigouvian one. Accordingly, countries with highly distortionary tax systems due to a strong preference for equality should not set lower taxes on, e.g. carbon emissions than countries with smaller tax distortions. Nevertheless, the government can always employ non-distortionary, non-individualized lump-sum taxes. Intuitively, non-individualized lump-sum taxes are always incentive-compatible, and should therefore be part of the instrument set of the government. The authors highlight the importance for the government to have access to sufficient policy instruments in order to internalize the trade-off between revenue-recycling and distributional effects that is still present. Moreover, the tax structure also matters. The progressivity of the income tax is one more instrument that can help to design an environmental tax reform with distributive objective." If there are no other sources of heterogeneity than in skills, the non-linear income tax is the informationally most efficient instrument to redistribute income, which renders indirect instruments for redistribution redundant" (Jacob and de Mooij [2015]). These results are in line with previous works that aim to assign a redistributive objective to an environmental tax reform (see Cremer and Ladoux [2003], Chiroleu-Assouline and Fodha [2014]).

Yet, as distorsive instrument, progressivity of income tax or the tax system (lump-sum tax) can lead also to adverse effects. As typically argued in public finance, tax progressivity may be detrimental to incentives on work-effort and lead to an inevitable trade-off between equality and efficiency. Its impact on growth and education level is also pointed out. Jacobs and Bovendberg [2010] have shown that the optimal marginal income taxes are lower with endogenous human capital formation than it would be with exogenous one. Intuitively, with endogenous learning, the efficiency costs of redistribution increase because positive marginal tax rates distort not only labor supply but also human capital accumulation. Caucutt, Imrohoroglu and Kumar [2006], builded a general equilibrium model of endogenous growth, in which there is heterogeneity in skill through education decision, income, and tax rates and evaluate the effect of progressivity of taxes on growth and welfare. They found that welfare is unambiguously higher in a flat rate system when comparisons are made across balanced growth equilibrium. Surprisingly a less progressive tax system, which is rarely perceived as an egalitarian measure, gives rise to an increase in growth, a decrease in inequalities, and a greater mobility for the poor in the long run, especially in light of contradicting claims in the literature regarding the connection between growth and inequality.

In regards with these papers, it seems that the potential adverse efficiency effect of the use of progressivity or lump sum transfers due to endogenous learning can be a serious barrier to the implementation of environmental policies. Typically if instruments besides green taxes (labor taxes and lump sum transfers) are not sufficient to internalize the trade-off between efficiency and equity, it could raise again the issue of green taxes levied at the Pigouvian tax rate. How does endogenous learning matter for the equity and the efficiency of the environmental green tax reform? The optimal green tax will be still equal to the Pigouvian rate, in a context of endogenous productivity, or the distributive/ efficient impact should be remove?

This paper aims at providing a way to answer these questions. To the best of our knowledge no paper tries to incorporate this feature in a model with environmental concern. We think this issue is relevant and may have several implications for a better understanding of the strong political opposition that environmental policies face now. Kempf and Rossignol [2010] address the important issue of the long-term impact of income distribution on the environment. They claim that income inequality is harmful for the environment insofar as a it generates a conflict of interest: relatively poor people are more interested in fostering physical growth at the expense of a clean environment, whereas relatively rich people are more concerned with the quality of the environment and are more willing to spend for depollution purposes, even if this means a less productive economy in the long run. Allowing for endogenous education decisions could provide another explanation and push us to consider some other instruments to encourage environmental policies, as for instance education subsidies.

In a Mirrlees partial equilibrium model based on Jacobs and de Mooij [2015], and Jacobs and Van der Ploeg [2010], we introduce endogenous productivities through investment in education. Each household differs by ability and chooses whether to acquire an education and become a high-skilled worker or remain unskilled. Given these assumptions, our economy is characterized by an ability cutoff such as those with education cost parameter strictly above will invest in education and become skilled. It determines endogenously the proportion of skilled and unskilled workers in the economy that is directly impacted by the government policy instruments. We derive the optimal carbon tax in conjunction with the optimal redistributive income tax and lumps sum transfers. We found that discriminating lump sum tax transfers between high and low skilled workers is necessary to allow the government to reach the first best pollution tax (the Pigouvian one). Tax subsidies on education have the same advantage. Yet, this result holds only under the homothetic properties of the human capital production function. Otherwise, neither lump-sum transfers, nor education subsidies allow reaching the first best pollution tax.

The first section describes the model. The second one derives the optimal carbon tax in conjunction with the optimal redistributive income tax and lumps sum transfers. The third section discuss about the implication of the human capital function.

1 Model

1.1 Household's Behaviour

In a partial equilibrium framework, we assume an economy populated by a continuum of measure one of households that differ by ability n, which is private information. The cumulative distribution function of ability is denoted by $G(\cdot)$ with the support being the interval [0;1]. The density function is denoted by $g(\cdot) = G'(\cdot)$. All households have one unit of labor time during their life, but are born without skills and thus with low productivity. Each household can choose whether to acquire an education and become a high-skilled worker (indexed by $i = H$), or instead remain unskilled ($i = L$, low-skilled worker). A household with ability n that has gone to college produces $(1 + pn)w = w_n^H$ units of consumption per unit of labor, whereas a household without a college degree has labor productivity $w^L = w$. Here $p > 0$ (the college premium for the most able type) is a fixed positive parameters and w stands for the wage rate per efficiency unit of skill[3]. The two labor types are assumed to be perfect substitutes in aggregate production. As a result, w is constant. We also assume a positive pecuniary cost of going to college wk (where $k > 0$ is a parameter), which is assumed to be tax deductible. Each individual derives utility from two types of commodities: the so-called polluting consumption commodity (b), that harms the environment when consumed, and the clean consumption commodity (c). We assume their prices (before taxes) are fixed and normalized to unity (the rates of transformation is assumed to be constant). In addition, individual derive disutility from supplying labor l_n^i. Assume further that the environmental externality enters linearly in the utility function, the utility function of an individual n, can be written as:

$U_n^i((c_n^i, b_n^i), 1 - l_n^i) + \varphi(E)$ for $i = L; H$

Where U_n^i is assumed strictly quasi-concave, weakly separable between labor and consumption and identical across individuals ($U_n^i = U$). So the marginal utility of income is decreasing. $\varphi(E)$ denotes the utility of a better environmental quality. We assume that the government imposes a tax t^i on the labor income of households of types and provides lump-sum transfers to individuals T^i[4]. Moreover the government fights pollution externalities by imposing a green tax q on the consumption of the dirty good. Households spend all their revenue on consumption of clean and dirty goods.

The individual budget constraint therefore reads as follows:

For an individual that did not go to college (low skilled):

$(1 + q)b^L + c^L = (1 - t^L)w^L + T^L$

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[3] We could also consider another situation where the labor supply of the high skilled is diminished by its time passed to study as in Jacobs and Bovendberg [2006]. Investment in human capital is denoted $e_n(for\ education)$ and the production function for human capital is homothetic and given by: $\square_n^H = w(1 + pn)\phi(e_n^H) = w(1 + n)(e_n^H)^\beta$. Where $\square_n^H$ denotes human capital that features decreasing returns with respect to investments. See the section 3 of the paper for a discussion.

[4] (We assume that individualized lump-sum transfers and taxes are not feasible, as the government can observe neither individual earnings ability nor labor effort, just the type of workers)

For an individual that has gone to college (high skilled):

$$(1+q)b_n^H + c_n^H = (1-t^H)(w_n^H l_n^H) - kw + T^H$$
$$= w((1-t^H)(1 + pn)\, l_n^H - k) + T^H$$

As the environmental degradation acts as an externality, we assume households ignore the adverse effect of their demand for polluting goods on the quality of the environment. Consequently, optimal choices of labor and consumption are governed by the following first-order conditions:

$$\frac{U_b^i}{U_c^i} = 1 + q;\ \ \frac{U_{1-l}^i}{U_c^i} = (1-t^i)w^i \quad \text{for } i = L; H$$

The marginal benefit of consuming one marginal unit of dirty good in monetary term, has to be equals to its marginal cost. Labor supply is expressed as a function of the after tax wage rate. More ability raises high skilled labor supply by increasing before tax-wage.

The indirect utility function is designated by $V_n^i(T,t,q,E) = U_n((c_n^{i*}, b_n^{i*}), 1 - l_n^{i*}) + \varphi(E)$ where stars denote the optimized values of each commodity and labor supply.

1.2 The cutoff determination

Given our assumptions, the previous solution for the household problem is characterized by an ability cutoff level denoted by n_C, that equalizes the indirect utility function of a low skilled and a high skilled.

$$V_{n_c}^H(T^H, t^H, q, E, n_c) \ = V^L(T^L, t^L, q, E)$$

The cutoff is defined such as those with education cost parameter strictly above n_C will invest in education and become skilled, while everyone else remains unskilled. It thus determines endogenously the proportion of skilled $(1 - G(n_C))$ and unskilled workers $(G(n_C))$ in the economy. Because we made the assumption of a strongly separable utility function between the environmental quality and consumption and leisure, the cut-off n_C only depends on the government policy instruments ($n_C\ (t^H; t^L;\ q;\ T^H; T^L)$. Differentiating the previous equation in n_C, and using the theorem of the implicit function gives us the variation of the cut-off due to the variation of each government instrument:

$$\frac{dn_C}{dx} = \left(\frac{\partial(V^L(T^L,t^L,q,E)) - \partial V_{n_C}^H(T^H,t^H,q,E,n_C)}{\partial x}\right)\left(\frac{\partial V_{n_C}^H(T,t,q,E,n_C)}{\partial n_C}\right)^{-1} \text{with } x\ =\ t^H; t^L;\ q;\ T^H; T^L$$

Using the Roy-lemma (see Appendix) we finally find: $\frac{dn_C}{dT^L} = (\lambda^L)\left(\frac{\partial V_{n_C}^H(T,t,q,E,n_C)}{\partial n_C}\right)^{-1} > 0$; $\frac{dn_C}{dT^H} = \left(-\lambda_{n_C}^H\right)\left(\frac{\partial V_{n_C}^H(T,t,q,E,n_C)}{\partial n_C}\right)^{-1} < 0$; $\frac{dn_C}{dt^L} = \left(-\frac{dn_C}{dT^L}\right) l^L w^L < 0$; $\frac{dn_C}{dt^H} =$

$\left(\left(-\frac{dn_C}{dT^H}\right)\left(l^H_{n_C} w^H_{n_C}\right)\right) > 0;\quad \frac{dn_C}{dq} = -\left(\left(\frac{dn_C}{dT^L}\right) b^L + \left(\frac{dn_C}{dT^H}\right) b^H_{n_C}\right) \lessgtr 0$ depending on the distributives properties of the pollution tax.

Where λ^i_n stands for the private marginal utility of income of type $i = L; H$ with ability n (the Lagrange multiplier associated with the budget constraint of household). The sign of the derivatives of n_C with respect to the government policy instruments is easy to interpret. A marginal increase of the low-skilled lump-sum transfer will benefit to low-skilled households without changing (all things equal) the welfare of high skilled workers. The opportunity cost of investing in education becomes lower: low skilled households have less incentive to go to school. Thus, the proportion of low skilled workers in the economy will increase meaning an increase of n_C . The same reasoning can be applied for $x = t^H; t^L;\ T^H$. Note that the sign of the derivatives of the cutoff (n_C) with respect to the pollution tax (q) is ambiguous and depends on the sign of $\lambda^H_{n_C}(1+q)b^H_{n_C} - \lambda^L(1+q)b^L$. If the utility from the polluting consumption good is higher for the indifferent household $(type\ of\ n_C)$ than for low skilled households, increasing the dirty tax will increase the cutoff ability level and then lower the number of high skilled households inside the economy. It thus depends on the distributive properties of the polluting good (Giffen goods for example). The next section will consider some the non homothetic utility function for this purpose (See proposition 5 further).

1.3 Environmental quality:

The environmental quality E is modeled as a pure public good.

$$E = E_0 - \alpha\left(\int_0^{\mathrm{n_C}} b^L\, \mathrm{dG(n)} + \int_{\mathrm{n_C}}^1 \mathrm{b^H_n}\, \mathrm{dG(n)}\right)$$

E_0 denotes the initial stock of environmental quality. Environmental quality is a linear function of aggregate consumption of dirty goods. The latter assumption ensures increasing marginal social damage from pollution. Alternatively, one can also interpret equation (5) as the production technology of environmental quality (Jacobs and de Mooij [2015]).

1.4 Government:

The government maximizes a Samuelson-Bergson social welfare function, which is a concave sum of individuals:

$$\int_0^{n_C} \psi(U^L)\, dG(n) + \int_{n_C}^1 \psi(U_n^H)\, dG(n)$$

We assume that the government uses its taxes revenue for financing some exogenous government spending $\bar{D}$ and to provide lumps sum transfers. Individualized lump-sum taxes are not feasible, as the government can observe neither individual earnings ability nor labor effort, just the type of worker (low skilled/high skilled). The government budget constraint can be written as follow:

$$\begin{aligned}\bar{D} + G(n_C)T^L + (1 - G(n_C))T^H \\ = \int_0^{n_C} (t^L w^L l^L)\, dG(n) + \int_{n_C}^1 t^H (w_n^H l_n^H)\, dG(n) \\ + q\left[\int_0^{n_C} b^L\, dG(n) + \int_{n_C}^1 b_n^H\, dG(n)\right]\end{aligned}$$

2 Optimal linear taxation

The Lagrangian for maximizing social welfare is given by:

$$\begin{aligned}\underset{\{t^H; t^L; q; T^H; T^L\}}{Max} \quad L = \int_0^{n_C} [\psi(V^L) + \eta(t^L w^L l^L) + (\eta q - \mu\alpha) b^L - \eta T^L]\, dG(n) \\ + \int_{n_C}^1 [\psi(V_n^H) + \eta t^H (w_n^H l_n^H) + (\eta q - \mu\alpha) b_n^H - \eta T^H]\, dG(n) \\ -\eta[\bar{D}] - \mu(E - E_0)\end{aligned}$$

Where η and μ stand respectively for the marginal social value of public resources (the Lagrange multiplier of the governmental budget constraint) and the marginal social cost (measured in social welfare units) of providing a better environmental quality E.

Equation (a.1) to (a.6) in the appendix (A.I) give the first-order conditions of this problem.

2.1 Social marginal value of income and Marginal cost of public funds

In line with Jacobs and de Mooij [2015] and following Diamond [1975], we define the net marginal social value of income of low skilled workers of type $n < n_C$, including the income effect on the tax base as:

$$\lambda^{*L} = \psi'(V^L)\lambda^L + \eta\left(t^L w^L \frac{\partial l^L}{\partial T^L}\right) + (\eta \mathrm{q} - \mu\alpha)\frac{\partial b^L}{\partial T^L}$$

We use the same decomposition of λ^{*L} than in de Mooij and Jacobs [2015]. First, if a low-skilled individual receives a marginal unit of income, his private welfare rises by λ^{*L}, and social welfare increases by $\psi'(V^L)\lambda^L$. Second, assuming leisure as a normal good, the low-skilled labor supplies increase if they receive a marginal unit of income ($\frac{\partial l^L}{\partial T^L} > 0$). In the case where low-skilled labor income is taxed, this increases the government tax revenues and then the social welfare by $\eta\left(t^L w^L \frac{\partial l^L}{\partial T^L}\right)$. Third, assuming the dirty good as a normal good, the low-skilled worker will consume more dirty goods with a marginal unit of income in addition. If dirty commodities are taxed, this will again increase the tax revenues of the government by $(\eta \mathrm{q})\frac{\partial b^L}{\partial T^L}$. Yet the impact on the social welfare will be ambiguous due to its negative impacts on environmental degradation $(-\mu\alpha)\frac{\partial b^L}{\partial T^L}$.

Similarly, the social marginal value of transferring a marginal unit of income to a high skilled workers ($n > n_c$) is given by:

$$\lambda_n^{*H} = \psi'(\mathrm{V_n^H})\lambda_\mathrm{n}^\mathrm{H} + \eta\left(t^H \mathrm{w_n^H} \frac{\partial \mathrm{l_n^H}}{\partial T^H}\right) + (\eta \mathrm{q} - \mu\alpha)\frac{\partial \mathrm{b_n^H}}{\partial T^H}$$

Yet, government lump-sum transfers have also an impact on the ratio of skilled and unskilled workers in the economy. Typically, for workers of ability n_C there is a trade-off between paying education cost and being paid more or remaining low skilled. For them, both high and low skilled lump-sum transfers matters for this decision: a high (low) skilled worker of ability n_C could be influence to disinvest (invest) in education and switch from high skilled status for a low skilled status if government increase (decrease) low skilled lump-sum transfers more than high-skilled one. In this case, the social marginal value of transferring a marginal unit of income to a worker of ability (n_c) is given by:

$$\lambda_{n_C}^* = -\Big[\Big(\psi\big(\mathrm{V_{n_C}^H}\big) - \psi(V^L)\Big) + \eta\big(t^H l_{n_C}^H w_{n_C}^H - t^L l^L wL\big) + \eta\left(\mathrm{q} - \frac{\mu}{\eta}\alpha\right)\big(b_{n_C}^H - b^L\big) +$$
$$\eta(T^H - T^L)\Big] * \left[\frac{\partial(\mathrm{n_C})}{\partial T^L} + \frac{\partial(\mathrm{n_C})}{\partial T^H}\right]\mathrm{y}$$
$$= -\eta[TR]\left[\frac{\partial(\mathrm{n_C})}{\partial T^L} + \frac{\partial(\mathrm{n_C})}{\partial T^H}\right] = -\big(\lambda^L - \lambda_{n_C}^H\big)\left[\eta[TR]\left(\frac{\partial V_{n_C}^H(T, t, q, E, n_c)}{\partial n_C}\right)^{-1}\right]$$

Where $\eta TR = \eta\left[\left(t^H l_{nc}^H w_{nc}^H - t^L l^L wL\right) - \left(T^H - T^L\right) + \left(\mathrm{q} - \frac{\mu}{\eta}\alpha\right)\left(b_{nc}^H - b^L\right)\right]$ denotes the marginal government tax revenue gains associated to an additional high skilled worker in the economy (i-e to one less low skilled worker). Thus the social welfare increase by $-\eta[TR]\left(\lambda^L - \lambda_{nc}^H\right)\left(\frac{\partial V_{nc}^H(T,t,q,E,n_C)}{\partial n_C}\right)^{-1}$. The sign is ambiguous and depends on the difference of the private marginal utility of income $\left[\lambda^L - \lambda_{nc}^H\right]$ between low and high skilled.

The average social marginal value of one private unit of private income is then:

$\overline{\lambda^*} = \int_0^{\mathrm{n_C}}(\lambda^{*L})\,\mathrm{dG(n)} + \int_{\mathrm{n_C}}^1(\lambda_n^{*H})\,\mathrm{dG(n)} + \left(\lambda_{nc}^*\right)\frac{dG(n_C)}{dn_C}$ i-e

$$\overline{\lambda^*} = \int_0^{\mathrm{n_C}}(\lambda^{*L})\,\mathrm{dG(n)} + \int_{\mathrm{n_C}}^1(\lambda_n^{*H})\,\mathrm{dG(n)} + \left(\lambda_{nc}^H - \lambda^L\right)\left[\eta[TR]g(\mathrm{n_C})\left(\frac{\partial V_{nc}^H}{\partial n_C}\right)^{-1}\right]$$

The marginal cost of public funds (MCF) is defined as the ratio between marginal social value of one unit of public income (η) and the average of the marginal social value of one unit of private income $\overline{\lambda^*}$.

$$MCF = \frac{(\eta)}{\overline{\lambda^*}}$$

2.2 The Feldstein distributional characteristics

<u>*Definition 1*</u>: The Feldstein (1972) distributional characteristic of labor income as the normalized covariance between the welfare weight the government attaches to income of a particular ability n and gross labor income:

$$\varepsilon_l = \frac{cov\left[\lambda_{\mathrm{n}}^{*\mathrm{i}}, \mathrm{w_n^i l_n^i}\right]}{\overline{\lambda^*} * \overline{\mathrm{wl}}}$$

$$= 1 - \frac{\int_0^{\mathrm{n_C}} \lambda^{*L} w^L l^L\,\mathrm{dG(n)} + \int_{\mathrm{n_C}}^1 \lambda_{\mathrm{n}}^{*\mathrm{H}} \mathrm{w_n^H l_n^H}\,\mathrm{dG(n)} + \left(\lambda_{nc}^H \mathrm{w}_{nc}^{\mathrm{H}} \mathrm{l}_{nc}^{\mathrm{H}} - \lambda^L w^L l^L\right)\left[\eta[TR]g(\mathrm{n_C})\left(\frac{\partial V_{nc}^H}{\partial n_C}\right)^{-1}\right]}{\eta\left[\int_0^{\mathrm{n_C}} w^L l^L\,\mathrm{dG(n)} + \int_{\mathrm{n_C}}^1(\mathrm{w_n^H l_n^H})\,\mathrm{dG(n)}\right]}$$

It corresponds to the normalized covariance of earnings of individual of type n, and the net social welfare weight of individual n. ε_l measures the marginal gain in social welfare (in monetary equivalents), expressed as a percentage of taxed labor income, of raising revenue via the tax on labor income. If TR was equal to zero, the covariance would have been positive because the covariance between labor earnings and welfare weights is negative: individuals with higher incomes feature lower welfare weights because of diminishing social marginal utility of income. This is caused by diminishing private marginal utility of income and the concavity of the social welfare function. But here, the distributional characteristic is here ambiguous because raising income tax impacts on the mass of low/high skilled households. The third term of the nominator can be interpret has the new skilling down term showing how a higher income tax rate *t* pushes up the cut-off level as we mention in section, narrows the high-income tax base and thus raises the costs of income taxation . A positive distributional characteristic ε_l implies that taxing labor income yields distributional benefits. A zero distributional characteristic is obtained either if the government is not interested in redistribution and attaches the same welfare weight $\lambda_{\mathrm{n}}^{*\mathrm{i}}$ to all n, or if taxable income $w_l^i l_n^i$ is the same for all n so that there is no inequality, or . The distributional characteristic reaches a maximum of one with the strongest possible distributional concerns, i.e., if the government has Rawlsian social preferences.

<u>*Definition 2*</u>: The Feldstein (1972) distributional characteristic of dirty goods consumption is defined as:

$$\varepsilon_b = \frac{cov[\lambda_{\mathrm{n}}^{*\mathrm{i}}, \mathrm{b}_{\mathrm{n}}^{\mathrm{i}}]}{\overline{\lambda^*} * \bar{\mathrm{b}}}$$

$$= 1 - \frac{\int_0^{\mathrm{n_C}} (\lambda^{*L}) b^L \,\mathrm{dG(n)} + \int_{\mathrm{n_C}}^1 (\lambda_{\mathrm{n}}^{*\mathrm{H}})(\mathrm{b}_{\mathrm{n}}^{\mathrm{H}}) \,\mathrm{dG(n)} + \left(\lambda_{n_C}^H \mathrm{b}_{n_C}^{\mathrm{H}} - \lambda^L b^L\right) \left[\eta [TR] g(\mathrm{n_C}) \left(\frac{\partial V_{n_C}^H}{\partial n_C}\right)^{-1}\right]}{\eta \left[\int_0^{\mathrm{n_C}} b^L \,\mathrm{dG(n)} + \int_{\mathrm{n_C}}^1 (\mathrm{b}_{\mathrm{n}}^{\mathrm{H}}) \,\mathrm{dG(n)}\right]}$$

It corresponds to the normalized covariance of earnings of individual of type n, and the net social welfare weight of individual n. The sign of ε_b is generally ambiguous. If individuals with a high ability (low ability) consume relatively more from the dirty good, the distributional characteristics is positive (negative). As above, the third term of the nominator can be interpret has the new skilling down term showing how a higher pollution tax rate *q* pushes up the cut-off level , and narrows the high skilled tax base and thus raises the welfare lost of pollution taxation.

Definition 3: The distributional characteristic of environmental quality is:

$$\varepsilon_E = 1 - \frac{\int_0^{\mathrm{nc}} \frac{U_E^L}{U_C^L} (\lambda^{*L})\, dG(n) + \int_{\mathrm{nc}}^1 \frac{U_{nE}^H}{U_{nC}^H} (\lambda_n^{*H})\, dG(n)}{\eta \left[\int_0^{\mathrm{nc}} \frac{U_E^L}{U_C^L} dG(n) + \int_{\mathrm{nc}}^1 \frac{U_{nE}^H}{U_{nC}^H} dG(n) \right]}$$

If mainly high-ability (low-ability) types benefit from a better environmental quality, then $\varepsilon_E > 0$ ($\varepsilon_E < 0$). $\varepsilon_E = 0$ if environmental quality is distributionally neutral. In that case, all individuals share the same willingness to pay for a better environment $\frac{U_{nE}^H}{U_{nC}^H} = \frac{U_E}{U_C^L}$. Notice, that because of our separability assumption of environmental damage in the utility function, the cut-off does not depend on the quality of environment and the numerator of the previous equation does not contain any skilling down term.

2.3 Optimal redistribution and corrective taxes:

Proposition 1: *The optimal pollution tax, the optimal Pigouvian tax, the optimal marginal income tax rate and the marginal cost of public funds are determined by*:

- $MCF = \frac{(\eta)}{\lambda^*} = 1$
- $\varepsilon_l = \frac{t}{1-t}\left(-\overline{\varepsilon_{l,t}}\right) + \frac{\left(q - \mu\frac{\alpha}{\eta}\right)}{1+q}\left(\overline{-\gamma \varepsilon_{l,q}}\right)$
- $\varepsilon_b = \frac{\left(q - \mu\frac{\alpha}{\eta}\right)}{1+q}\left(\frac{-\overline{\varepsilon_{b,q}}}{\overline{\gamma}}\right) + \frac{t}{1-t}\left(-\frac{\overline{\varepsilon_{b,t}}}{\overline{\gamma}}\right)$
- $(1 - \varepsilon_E)\left[\int_0^{\mathrm{nc}} \frac{U_E^L}{U_C^L} dG(n) + \int_{\mathrm{nc}}^1 \frac{U_{nE}^H}{U_{nC}^H} dG(n)\right] = \frac{\mu}{\eta}$

Where $\gamma^i(n) = \frac{(1+q)\, \mathrm{b}_\mathrm{n}^\mathrm{i}}{(1-t^i)\, \mathrm{w}_\mathrm{n}^\mathrm{i} \mathrm{l}_\mathrm{n}^\mathrm{i}}$ the share parameter, and $\bar{\gamma} = \frac{\int_0^{\mathrm{n_C}} (\gamma^L) w^L l^L \mathrm{dG(n)} + \int_{\mathrm{n_C}}^1 (\gamma_\mathrm{n}^\mathrm{H})(\mathrm{w_n^H l_n^H}) \mathrm{dG(n)}}{\overline{wl}}$.

Where $\varepsilon_{yn,t}^i = \frac{\partial y_n^i}{\partial x^i} \frac{x^i}{y_n^i}$ for $i = L; H$ and $y_n^i = b_n^i;\ l_n^i, and\ x^i = t^i, q,$ the compensated elasticity of dirty consumption (or labor supply) with respect to the labor or the environmental tax rate.

And $\overline{\varepsilon_{y,x}} = \left[\int_0^{\mathrm{n_C}} \varepsilon_{y,x}^L w^L l^L\, \mathrm{dG(n)} + \int_{\mathrm{n_C}}^1 (\varepsilon_{y,x}^H)(\mathrm{w_n^H l_n^H})\, \mathrm{dG(n)}\right] \left[[\overline{wl}]\right]^{-1}$, the income-weighted average of the compensated elasticity of dirty consumption with respect to the tax rate $x = q;\ t$.

The first equation ($MCF = \frac{(\eta)}{\overline{\lambda^*}} = 1$) tells us that the average social marginal benefit of a higher lump-sum transfer $\overline{\lambda}^*$, should equals to its costs η. Resources are equally valuable in public and private sector.

The left-hand side of the equation 2, shows the marginal benefit of redistribution using the income tax rate. The right-hand side shows the marginal cost consisting of a first term: the Ramsey term (the optimal income tax rate is inversely proportional to the wage elasticity of labour supply in accordance with the Ramsey considerations), and a Corlett-Hague term depending on the difference between the pollution tax and the Pigouvian pollution tax and on the cross elasticity of demand for dirty goods with respect to the income tax rate (the second term).

In the same line, the left-hand side of the equation 3, shows the marginal benefit of redistribution using the polluting tax rate. The right-hand side shows the marginal cost consisting of a Ramsey term (the first term), a Corlett-Hague term depending on the cross elasticity of the labor supply with respect to the pollution tax rate (the second term), and the new skilling down term showing how a higher polluting tax rate *q* pushes up the cut-off level (the third term).

The optimal tax structure of income and environmental taxes can be solved as functions of the elasticities and distributional terms. It gives the following corollary:

Corollary: *The optimal tax structure of the income tax and the pollution one are given by*:

➢ $\frac{t}{1-t} = \frac{\varepsilon_l + (\overline{\varepsilon_{b,t}})\left(\frac{\overline{\gamma\varepsilon_b}}{\overline{-\gamma\varepsilon_{b,q}}}\right)}{\overline{-\varepsilon_{l,t}} - (\overline{\varepsilon_{b,t}})\left(\frac{\overline{\varepsilon_{l,t}}}{\overline{-\gamma\varepsilon_{b,q}}}\right)}$

➢ $\frac{\left(q-\mu\frac{\alpha}{\eta}\right)}{1+q} = \frac{\overline{\gamma\varepsilon_b} + (\overline{\varepsilon_{l,q}})\left(\frac{\varepsilon_l}{\overline{-\gamma\varepsilon_{l,t}}}\right)}{\overline{-\gamma\varepsilon_{b,q}} - (\overline{\varepsilon_{l,q}})\left(\frac{\overline{\gamma\varepsilon_{b,t}}}{\overline{-\varepsilon_{l,t}}}\right)}$

Proof (solve equations system in the proposition 1 for $\frac{t}{1-t}$ and $\frac{\left(q-\mu\frac{\alpha}{\eta}\right)}{1+q}$)

2.4 First best and Second best:

Proposition 3: *In the absence of distributional concerns, the first-best policy rules for the environment can be obtained. The marginal income tax rate is set to zero ($t = 0$). The optimal environmental tax q satisfies the first-best Pigouvian tax rate $\left(q = \mu\frac{\alpha}{\eta}\right)$, and sustains a first-best level of environmental quality* $(1-\varepsilon_E)\left[\int_0^{n_C}\frac{U_E^L}{U_C^L}dG(n) + \int_{n_C}^1\frac{U_{nE}^H}{U_{nC}^H}dG(n)\right] = \frac{\mu}{\eta}$

This proposition demonstrates most clearly that positive marginal tax rates on labor income are introduced only to redistribute income.
Special cases can be derived where the optimal environmental tax in the second-best equals the first-best expression for the Pigouvian tax.

Proposition 4: *Conditions for first-best rules in the second-best: If the utility function is given by $U_n^i = U(v(c_n^i, b_n^i), 1-l_n^i) + \varphi(E)$ where $v(c_n^i, b_n^i)$ is of the Gorman Polar form and identical for all individuals, if government allows for education subsidies then the optimal income marginal rate is given by $\frac{\varepsilon_l}{(\overline{-\varepsilon_{l,t}})} = \frac{t}{1-t} = \frac{t^i}{1-t^i}$, the optimal pollution tax equals the first-best Pigouvian tax $\left(q = \mu\frac{\alpha}{\eta}\right)$, and environmental quality follows the first-best Samuelson rule $(1-\varepsilon_E)\left[\int_0^{n_C}\frac{U_E^L}{U_C^L}dG(n) + \int_{n_C}^1\frac{U_{nE}^H}{U_{nC}^H}dG(n)\right] = \frac{\mu}{\eta}$. These results are also true for an Utility function of type HARA.*

As in Jacobs and van der Ploeg [2010], the optimal environmental tax still amounts the Pigouvian tax rate i-e marginal climate damages even though preferences are non-homothetic (the poor spend a greater proportion of their income on dirty goods) and productivity is endogenous. Pushing the optimal carbon tax below the Pigouvian tax for income-distributional or for efficiency reasons is thus not optimal. The regressive nature of the dirt tax does not require an extra effort of the linear income tax to compensate for the adverse effects on inequality. Yet, its potential detrimental impact on education level needs to be internalized. The demonstration of the proposition is in appendix and shows that this results does not hold anymore if government cannot discriminate lump sum transfers between low and high skilled. Clearly, education costs generate distortions that have to be internalized by another instrument: here lumps-sum transfers or education costs subsidies as it is shown in the previous proposition. Differentiating the labor income tax rate in this case will not work, as the education cost is independent of labor income.

3 Extension and Discussion: Endogenous education investment:

As previously noticed in a footnote, we could also consider another situation where the labor supply of the high skilled is diminished by its time passed to study as in Jacobs and Bovendberg [2006]. Investment in human capital is denoted e_n and the production function for human capital is homothetic and given by: $h_n^H = w(1+pn)\phi(e_n^H) = w(1+n)(e_n^H)^\beta$. Where h_n^H denotes human capital that features decreasing returns with respect to investments. The Gross labor income: z_n^H would be the product of the number of efficiency units of human capital h_n^H and hours worked l_n^H , i-e $z_n^H = l_n^H h_n^H = l_n n(e_n^H)^\beta$. In the same time, education cost (that is subsidies at a rate s^H) depends also of ability.

Thus the budget constraint of an high skilled household would be:

$$(l_n^H n(e_n^H)^\beta.(1-t^H) + T^H - (1-s^H)kw(e_n^H) = (1+q)b_n^H + c_n^H$$

Taking the policy instruments of the government as given, individuals maximize utility by choosing $c_n^H, b_n^H, l_n^H, e_n^H$, subject to the household budget constraint.

First orders conditions for e_n^H and the production function of human capital l_n^H imply that gross labor income zn is proportional to education investment. With linear policy instruments, the proportionality factor does not depend on ability n, because the shares of e_n^H in human capital investment, are the same for all high skilled agents (homothetic function of human capital production).

Proposition 5: *If the function of human capital production is homothetic such as that gross labor income* zn *is proportional to education investment and the proportion is independent on ability, then proposition 4 and 5 still hold under this representation. Otherwise, neither lump-sum transfers, nor education subsidies allow to reach the first best pollution tax*.

3 Empirical illustration (to be done)

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Appendix

Household Behaviours

Roy's lemma:

$\frac{\partial V_n^H}{\partial q} = -\lambda_n^H b_n^H$; $\frac{\partial V^L}{\partial q} = -\lambda^L b^L$;
$\frac{\partial V_n^H}{\partial t} = -\lambda_n^H \left(l_n^H w_n^H\right)$; $\frac{\partial V^L}{\partial t} = -\lambda^L w^L l^L$;
$\frac{\partial V_n^H}{\partial T} = \lambda_n^H$; $\frac{\partial V^L}{\partial T} = \lambda^L$;
$\frac{\partial V^L}{\partial E} = -\frac{U_E^L}{U_C^L}\lambda^L$; $\frac{\partial V_H}{\partial E} = -\frac{U_E^H}{U_C^H}\lambda_n^H$

Slutzki equations:

$$\left(\frac{\partial C_i}{\partial p_j} = \left(\frac{\partial C_i^*}{\partial p_j}\right)_U + \frac{\frac{\partial V_i}{\partial p_i}}{\frac{\partial V_i}{\partial T}}\frac{\partial C_i}{\partial T}\right)$$

$$\left(\frac{\partial b_n^H}{\partial q} = \frac{\partial b_n^H *}{\partial q} - b_n^H \frac{\partial b_n^H}{\partial T}\right);\left(\frac{\partial b_n^H}{\partial t} = \frac{\partial b_n^H *}{\partial t} - w_n^H l_n^H \frac{\partial b_n^H}{\partial T}\right);\left(\frac{\partial l_n^H}{\partial q} = \frac{\partial l_n^H *}{\partial q} - b_n^H \frac{\partial l_n^H}{\partial T}\right);\left(\frac{\partial l_n^H}{\partial t} = \frac{\partial b_n^H *}{\partial t} - w_n^H l_n^H \frac{\partial l_n^H}{\partial T}\right)$$

$$\left(\frac{\partial b^L}{\partial q} = \frac{\partial b^L *}{\partial q} - b^L \frac{\partial b^L}{\partial T}\right);\left(\frac{\partial b^L}{\partial t} = \frac{\partial b^L *}{\partial t} - w^L l^L \frac{\partial b^L}{\partial T}\right);\left(\frac{\partial l^L}{\partial q} = \frac{\partial l^L *}{\partial q} - b^L \frac{\partial l^L}{\partial T}\right);\left(\frac{\partial l^L}{\partial t} = \frac{\partial l^L *}{\partial y} - w^L l^L \frac{\partial l^L}{\partial T}\right)$$

Compensated Elasticities:

A. I Firts-order-conditions of the government program

Tools for the F.O.C

Soit : $J = \int_o^{n_c(t,q)} f(t,n)dG(n)$;$\frac{dJ}{dt}$ =?
We can express $J(t)$ as fonction of two variables x and t (where $x = t$) as follow :

$$\left(J(t) = I(t,x) = \int_o^{n_c(t,q)} f(x,n)dG(n) = \int_o^{n_c(t,q)} f(x,n)g(n)dn\right)$$

$$\left(\frac{\partial It(x,t)}{\partial x}\right)_t = \int_o^{n_c(t,q)} \frac{\partial f(x,n)}{\partial x}dG(n) \text{ and } \left(\frac{\partial It(x,t)}{\partial t}\right)_x = f(x,n_c) * g(n_c) * \frac{\partial n_c(t,q)}{\partial t}$$

Finally we get:

$$dI = \left(\frac{\partial It(x,t)}{\partial x}\right)_t dx + \left(\frac{\partial It(x,t)}{\partial t}\right)_x dt = dx \int_o^{n_c(t,q)} \frac{\partial f(x,n)}{\partial x}dG(n) + \left(f(x,n_c) * g(n_c) * \frac{\partial n_c(t,q)}{\partial t}\right) dt$$

If x and t are the same variable:

$$dI\left(t,x\right) = dJ(t) = J'(t)dt = \left[\int_o^{n_c(t,q)} \frac{\partial f(t,n)}{\partial t}dG(n) + f(t,n_c) * g(n_c) * \frac{\partial n_c(t,q)}{\partial t}\right] dt$$

Then we get: $J'(t) = \left[\int_o^{n_c(t,q)} \frac{\partial f(t,n)}{\partial t}dG(n) + f(t,n_c) * g(n_c) * \frac{\partial n_c(t,q)}{\partial t}\right]$

Similary for $H'(t)=\int_{n_c(t,q)}^{1} f(t,n)dG(n)$ we get:

$H'=\int_{n_c(t,q)}^{1} \frac{\partial f(t,n)}{\partial t}dG(n) - f(t,n_c) * g(n_c) * \frac{\partial n_c(t,q)}{\partial t}$

Then we find:

$$\frac{\partial \mathcal{L}}{\partial T^L}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\Psi'(V^L)\lambda^L+\eta t w^L\frac{\partial l^L}{\partial T^L}+(q\eta-\alpha\mu)\frac{\partial b^L}{\partial T^L}\right]dG(n)+ \\ +\eta\left([TR]\frac{\partial G(n_c)}{\partial T^L}-G(n_c)\right) \qquad\qquad =0\end{array} \quad (A.1)$$

$$\frac{\partial \mathcal{L}}{\partial T^H}=0 \iff \begin{array}{c}\int_{n_c}^{1}\left[\Psi'(V_n^H)\lambda_n^H+\eta t w_n^H\frac{\partial l_n^H}{\partial T^H}+(q\eta-\alpha\mu)\frac{\partial b_n^H}{\partial T^H}\right]dG(n) \\ +\eta\left([TR]\frac{\partial G(n_c)}{\partial T^H}+(1-G(n_c))\right)=0\end{array} \quad (A.2)$$

$$\frac{\partial \mathcal{L}}{\partial t^L}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\left(-\Psi'(V^L)\lambda^L+\eta\right)w^L l^L+\eta t w^L\frac{\partial l^L}{\partial t^L}+(q\eta-\alpha\mu)\frac{\partial b^L}{\partial t^L}\right]dG(n)+ \\ +\eta\,[TR]\,\frac{\partial G(n_c)}{\partial t^L}=0\end{array} \quad (A.3)$$

$$\frac{\partial \mathcal{L}}{\partial t^H}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\left(\left(-\Psi'(V_n^H)\lambda_n^H+\eta\right)\left(w_n^H l_n^H\right)+\eta t w_n^H\frac{\partial l_n^H}{\partial t^H}+(q\eta-\alpha\mu)\frac{\partial b_n^H}{\partial t^H}\right)\right]dG(n) \\ +\eta\,[TR]\,\frac{\partial G(n_c)}{\partial t^H}=0\end{array} \quad \text{(A.4)}$$

$$\frac{\partial \mathcal{L}}{\partial q}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\left(-\Psi'(V^L)\lambda^L+\eta\right)b^L+\eta t w^L\frac{\partial l^L}{\partial q}+(q\eta-\alpha\mu)\frac{\partial b^L}{\partial q}\right]dG(n)+ \\ \int_0^{n_c}\left[\left(\left(-\Psi'(V_n^H)\lambda_n^H+\eta\right)\left(b_n^H\right)+\eta t w_n^H\frac{\partial l_n^H}{\partial q}+(q\eta-\alpha\mu)\frac{\partial b_n^H}{\partial q}\right)\right]dG(n) \quad +\eta\,[TR]\,\frac{\partial G(n_c)}{\partial q}=0\end{array} \quad \text{(A.5)}$$

$$\frac{\partial \mathcal{L}}{\partial E}=0 \iff \int_0^{n_c}\left[\left(-\Psi'(V^L)(U_E^L)/(U_C^L)\right)\right]dG(n)+ \quad \int_0^{n_c}\left[\left(-\Psi'(V_n^H)(U_E^L)/(U_C^L)\right)\right]dG(n)-\mu)=0 \quad \text{(A.6)}$$

A.II Proof of proposition 1

By replacing λ^{*L}, λ_n^{*H}, and$\lambda_{n_C}^*$ in the first-order conditions of the lump-sum transfers (equation a.1 and a. 2). We found $\eta=\overline{\lambda^*}$ => MCF=$\eta/\overline{\lambda^*}$ =1.

With The FOC for $t^i=t$,

$$\frac{\partial \mathcal{L}}{\partial t}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\left(-\Psi'(V^L)\lambda^L+\eta\right)w^L l^L+\eta t w^L\frac{\partial l^L}{\partial t}+(q\eta-\alpha\mu)\frac{\partial b^L}{\partial t}\right]dG(n)+ \\ \int_0^{n_c}\left[\left(\left(-\Psi'(V_n^H)\lambda_n^H+\eta\right)\left(w_n^H l_n^H\right)+\eta t w_n^H\frac{\partial l_n^H}{\partial t}+(q\eta-\alpha\mu)\frac{\partial b_n^H}{\partial t}\right)\right]dG(n) \qquad +\eta\,[TR]\,\frac{\partial G(n_c)}{\partial t}=0\end{array}$$

Using the Roy's lemma, the Sluzky equations for $\frac{l^L}{\partial t}$ and $\frac{\partial b^L}{\partial t}$ and substituting the marginal social value in the previous equation ($\lambda^{L*}$and λ_n^{H*})

$$\frac{\partial \mathcal{L}}{\partial t}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\left(-\left[\left(\Psi'(V^L)\lambda^L+\eta t w^L\frac{\partial l^L}{\partial T}+(q\eta-\alpha\mu)\frac{\partial b^L}{\partial T}\right)\right]+\eta\right)w^L l^L+\eta t w^L\left(\frac{\partial l^{L*}}{\partial t}\right)+(q\eta-\alpha\mu)\left(\frac{\partial b^{L*}}{\partial t}\right)\right]dG(n) \\ +\int_{n_c}^{1}\left[\left(-\left(\Psi'(V_n^H)\lambda_n^H+\eta t w_n^H\frac{\partial l_n^H}{\partial T}+(q\eta-\alpha\mu)\frac{\partial b_n^H}{\partial T}\right)+\eta\right)\left(w_n^H l_n^H\right)+\eta t w_n^H\left(\frac{\partial l_n^{H*}}{\partial t}\right)\right]dG(n) \\ +\int_{n_c}^{1}\left[(q\eta-\alpha\mu)\left(\frac{\partial b_n^{H*}}{\partial t}\right)\right]dG(n)+\eta\,[TR]\,\frac{\partial G(n_c)}{\partial t}=0\end{array}$$

$$\frac{\partial \mathcal{L}}{\partial t}=0 \iff \begin{array}{c}\int_0^{n_c}\left[\left(-\lambda^{L*}+\eta\right)w^L l^L+\eta\frac{t}{1-t}\left(\frac{\partial l^{L*}}{\partial t}\frac{1-t}{l^L}\right)w^L l^L+\frac{(q\eta-\alpha\mu)}{1+q}\left(\frac{\partial b^{L*}}{\partial t}\frac{1-t}{b^{L*}}\right)\left(\frac{1+q}{w^L l^L}\frac{b^L}{1-t}\right)\left(w^L l^L\right)\right]dG(n) \\ +\int_{n_c}^{1}\left[\left(-\lambda_n^{H*}+\eta\right)\left(w_n^H l_n^H\right)+\eta\frac{t}{1-t}\left(\frac{\partial l_n^{H*}}{\partial t}\frac{1-t}{l_n^H}\right)\left(\frac{l_n^H(1+pn)}{l_n^H(1+pn)-k}\right)\left(w_n^H l_n^H-kw\right)\right]dG(n) \\ +\int_{n_c}^{1}\left[(q\eta-\alpha\mu)\left(\frac{1+q}{(w_n^H l_n^H)}\frac{b_n^H}{1-t}\right)\left(w_n^H l_n^H\right)\left(\frac{\partial b_n^{H*}}{\partial t}\right)\right]dG(n)+\eta\,[TR]\,\frac{\partial G(n_c)}{\partial t}=0\end{array}$$

Using the definition of compensated elasticities and $\gamma_n^i = \left(\frac{1+q}{1-t}\right)\frac{b^i(n)}{w_n^i l_n^i}$, we get:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial t} = 0 \iff \quad & \int_0^{n_c}\left[\left(-\lambda^{L*}+\eta\right)w^L l^L + \eta\left[\frac{t}{1-t}\xi_{l,t}^L + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left(\xi_{b,t}^L\gamma^L\right)\right]w^L l^L\right]dG(n)\\
& + \int_{n_c}^1\left[\left(-\lambda_n^{H*}+\eta\right)\left(w_n^H l_n^H\right) + \eta\left[\frac{t}{1-t}\xi_{l,t}^H + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left(\xi_{b,t}^H * \gamma_n^H\right)\right]w_n^H l_n^H\right]dG(n)\\
& +\eta\left[TR\right]\frac{\partial G(n_c)}{\partial t} = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial t} = 0 \iff \quad & \int_0^{n_c}\left[\frac{t}{1-t}\xi_{l,t}^L + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left(\xi_{b,t}^L * \gamma^L\right)\right]w^L l^L dG(n)\\
& + \int_{n_c}^1\left[\frac{t}{1-t}\xi_{l,t}^H + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left(\xi_{b,t}^H * \gamma_n^H\right)\right]w_n^H l_n^H dG(n)\\
& = -\left(\int_0^{n_c} w^L l^L dG(n) + \int_{n_c}^1\left(w_n^H l_n^H\right)dG(n)\right) + \frac{\int_0^{n_c}\lambda_n^{L*}w^L l^L dG(n) + \int_{n_c}^1 \lambda_n^{H*}\left(w_n^H l_n^H\right)dG(n) - \eta[TR]\frac{\partial G(n_c)}{\partial t}}{\eta}\\
\frac{\partial \mathcal{L}}{\partial t} = 0 \iff \quad & \frac{t}{1-t}\left[-\int_0^{n_c}\xi_{l,t}^L w^L l^L dG(n) + \int_{n_c}^1 \Gamma_n^H \xi_{l,t}^H\left(w_n^H l_n^H\right)dG(n)\right] * \left[\int_0^{n_c} w^L l^L dG(n) + \int_{n_c}^1\left(w_n^H l_n^H\right)dG(n)\right]^{-1}\\
& + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left[-\int_0^{n_c}\gamma^L\xi_{b,t}^L w^L l^L dG(n) + \int_{n_c}^1\gamma_n^H\xi_{b,t}^H w_n^H l_n^H dG(n)\right] * \left[\int_0^{n_c} w^L l^L dG(n) + \int_{n_c}^1\left(w_n^H l_n^H\right)dG(n)\right]^{-1}\\
& = 1 - \frac{\int_0^{n_c}\lambda_n^{L*}w^L l^L dG(n) + \int_{n_c}^1\lambda_n^{H*}\left(w_n^H l_n^H\right)dG(n) - \eta g(n_c)[TR]\left(\lambda^L w^L l^L - \lambda_{n_c}^H\left(w_{n_c}^H l_{n_c}^H\right)\right)}{\eta\left[\int_0^{n_c} w^L l^L dG(n) + \int_{n_c}^1\left(w_n^H l_n^H\right)dG(n)\right]} = \xi_l
\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial t} = 0 \iff \xi_l = \frac{t}{1-t}\left(-\overline{\xi_{l,t}}\right) + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left(-\overline{\gamma\xi_{b,t}}\right)$$

Similary, we obtain ξ_q,ξ_E.

A.III proof of proposition 4

$q* = \frac{\alpha\mu}{\eta} \implies \frac{\partial \mathcal{L}}{\partial t} = 0$ **,** $\frac{\partial \mathcal{L}}{\partial q} = 0$**?**

Remember the First -order condition for the dirty good:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q} = 0 \iff \quad & \int_0^{n_c}\left[\left(\eta-\lambda^{L*}\right)b^L + \eta\left(\frac{t}{1-t}w^L l^L\left[\frac{1-t}{1+q}\right]\xi_{l,q}^L + \left(\frac{q-\alpha\frac{\mu}{\eta}}{1+q}\right)\xi_{b,q}^L b^L\right)\right]dG(n)\\
& + \int_{n_c}^1\left[\left(\eta-\lambda_n^{H*}\right)b_n^H + \eta\left(\frac{t}{1-t}w_n^H l_n^H\left[\frac{1-t}{1+q}\right]\xi_{l,q}^H + \frac{\left(q-\alpha\frac{\mu}{\eta}\right)}{1+q}\left(\xi_{b,q}^H\right)b_n^H\right)\right]dG(n)\\
& +\eta\left[TR\right]\frac{\partial G(n_c)}{\partial q} = 0
\end{aligned}$$

So if $q = \alpha\frac{\mu}{\eta}$, taking into account the variation of the cutoff we obtain:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q} = 0 \iff \quad & \int_0^{n_c}\left[\left(\eta-\lambda^{L*}\right)b^L + \eta\left(\frac{t}{1-t}w^L l^L\left[\frac{1-t}{1+q}\right]\xi_{l,q}^L\right)\right]dG(n)\\
& + \int_0^{n_c}\left[\left(\eta-\lambda_n^{H*}\right)b_n^H + \eta\left(\frac{t}{1-t}w_n^H l_n^H\left[\frac{1-t}{1+q}\right]\xi_{l,q}^H\right)\right]dG(n\\
& \eta\left[TR\right]g\left(n_c\right)\left(\lambda_n^H b_{n_c}^H - \lambda^L b^L\right)\left(\frac{\partial V^H}{\partial n_c}\right)^{-1} = 0
\end{aligned}$$

With a Gorman polar form, we get: $(1+q)b_n^i = \alpha(q) + \phi(q)R_n^i$

$$\frac{\partial\mathcal{L}}{\partial q}=0 \iff \int_0^{n_c}\left[\left(\eta-\lambda^{L*}\right)\left(\frac{\alpha(q)+\phi(q)\left(w^L l^L(1-t)+T^L\right)}{1+q}\right)+\eta\left(\frac{t}{1-t}w^L l^L\left[\frac{1-t}{1+q}\right]\xi^L_{l,q}\right)\right]dG(n)$$
$$+\int_{n_c}^1\left[\left(\eta-\lambda^{H*}_n\right)\left(\frac{\alpha(q)+\phi(q)\left(\left(w^H_n l^H_n\right)(1-t)+T^H-kw\right)}{1+q}\right)+\eta\left(\frac{t}{1-t}w^H_n l^H_n\left[\frac{1-t}{1+q}\right]\xi^H_{l,q}\right)\right]dG(n)$$
$$+\eta\left[TR\right]g\left(n_c\right)\left(\lambda^H_n\left(\frac{\alpha(q)+\phi(q)\left(\left(w^H_{n_c} l^H_{n_c}\right)(1-t)+T^H-kw\right)}{1+q}\right)-\lambda^L\left(\frac{\alpha(q)+\phi(q)\left(w^L l^L(1-t)+T\right)}{1+q}\right)\right)\left(\frac{\partial V^H}{\partial n_c}\right)^{-1}$$

$$\frac{\partial\mathcal{L}}{\partial q}=0 \iff \left[\frac{(1-t)\phi(q)}{1+q}\right]\int_0^{n_c}\left[\left(\eta-\lambda^{L*}\right)w^L l^L+\eta\left(\frac{t}{1-t}\frac{\xi^L_{l,q}}{\phi(q)}w^L l^L\right)\right]dG(n)$$
$$+\left[\frac{(1-t)\phi(q)}{1+q}\right]\int_{n_c}^1\left[\left(\eta-\lambda^{H*}_n\right)\left(w^H_n l^H_n\right)+\eta\left(\frac{t}{1-t}\frac{\xi^H_{l,q}}{\phi(q)}w^H_n l^H_n\right)\right]dG(n)$$
$$+\left[\frac{(1-t)\phi(q)}{1+q}\right]\eta\left[TR\right]g\left(n_c\right)\left(\lambda^H_n\left(w^H_{n_c} l^H_{n_c}\right)-\lambda^L\left(w^L l^L\right)\right)\left(\frac{\partial V^H}{\partial n_c}\right)^{-1}$$
$$+\left(\frac{\alpha(q)+\phi(q)\left(T^H-kw\right)}{1+q}\right)\left[\int_{n_c}^1\left[\left(\eta-\lambda^{H*}_n\right)\right]dG(n)+\eta\left[TR\right]g\left(n_c\right)\left(\lambda^H_{n_C}\right)\left(\frac{\partial V^H}{\partial n_c}\right)^{-1}\right]$$
$$+\left(\frac{\alpha(q)+\phi(q)\left(T^L\right)}{1+q}\right)\left[\int_0^{n_c}\left[\left(\eta-\lambda^{L*}\right)\right]dG(n)-\eta\left[TR\right]g\left(n_c\right)\left(\lambda^L\right)\left(\frac{\partial V^H}{\partial n_c}\right)^{-1}\right]=0$$

With the first -order-condition for the lump-sum, the two last term of the previous equation equals to zero. Moerover the compensated elasticities for a Gorman polar form is gives us: $\frac{\xi^i_{l,q}}{\phi(q)}=\xi^i_{l,t}$ thus we have:

$$\frac{\partial\mathcal{L}}{\partial q}=0 \iff \left[\frac{(1-t)\phi(q)}{1+q}\right]*\int_0^{n_c}\left[\left(\eta-\lambda^{L*}\right)w^L l^L+\eta\left(\frac{t}{1-t}\xi^H_{l,t}w^L l^L\right)\right]dG(n)$$
$$+\left[\frac{(1-t)\phi(q)}{1+q}\right]\int_0^{n_c}\left[\left(\eta-\lambda^{H*}_n\right)\left(w^H_n l^H_n-kw\right)+\eta\frac{t}{1-t}\xi^H_{l,t}w^H_n l^H_n\right]dG(n)$$
$$+\eta\left[TR\right]g\left(n_c\right)\left(\lambda^H_n\left(w^H_{n_c} l^H_{n_c}-kw\right)-\lambda^L\left(w^L l^L\right)\right)\left(\frac{\partial V^H}{\partial n_c}\right)^{-1}$$
$$=\left[\frac{(1-t)\phi(q)}{1+q}\right]\frac{\partial\mathcal{L}}{\partial t}\Big|_{\left(q*=\frac{\alpha\mu}{\eta}\right)}$$

At the optimum we have, $q*=\frac{\alpha\mu}{\eta}$. The compensated elasticities of labor supply (of dirty consumption) with respect to the labor and the environmental tax rate are defined as: $\xi^i_{y,t}=\frac{\partial y^i_n}{\partial t}\frac{1-t}{y^i_n}$ and $\xi^i_{y,q}=\frac{\partial y^i_n}{\partial q}\frac{1+q}{y^i_n}$ with $i=L,H$ and $y=b,l$