The importance of considering optimal government policy when social norms matter for the private provision of public goods

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Abstract

Social pressure can help overcome the free rider problem associated to public good provision. In the social norms literature concerned with the private provision of public goods there seems to be an implicit belief that it is always best to induce all agents to adhere to the 'good' social norm of contribution. We challenge this view and study optimal government policy in a reference model (**?**) of public good provision and social approval in a dynamic setting. We show that even if complete adherence to the social norm is a potential equilibrium it is by no means necessarily optimal to push society towards it. We show that optimal government policy may consist of inducing any conceivable level of the social norm, from pushing everyone out of the norm to interior solutions or inducing everyone to adhere to the social norm. In addition, we demonstrate that the way one models government policy has crucial implications for the optimal equilibria. Thus, we argue that extreme care must be taken when formulating policies and subsequent results will fully depend on this formulation.

Keywords: government policy; social norms; public goods; optimal policy.

JEL:

1 Introduction

In the social norms literature that is concerned with the private provision of public goods there seems to be an implicit belief that it is always best to have all agents adhere to the 'good' social norm where everyone participates in the contribution of the public good. Thus, this literature instigates an overall strong support for a government policy that induces society to move towards the equilibrium where all contribute to the public good. This is especially true for the case where society would otherwise be stuck in a bad social norm of no one contributing. The question that we ask in this article is whether and when it is actually optimal from a society's point of view that a government nudges society to take up a certain social norm.

We start from a simple, yet widely accepted model of social norms (**?**), allow for optimal government policy, and then study how far this augments the tacit idea that a government should try to enforce an otherwise socially beneficial norm. We view the contribution of this article as emphasizing the need to move away from the simple crowding in and crowding out story and instead study endogenous government policies. Additionally, we send a note of caution by giving some first insights into how different setups of the government policy lead to strikingly different results on the optimal level of the social norm.

Up to now, the literature on social norms and public goods has mostly concentrated on exogenous government policy. As a result of this focus on exogenous policy, researchers have emphasized whether or not a public policy would crowd in or crowd out private provisions. This terminology only makes sense if we study the impact of exogenous policy. Government policy is, however, never exogenous. It is directed at achieving a specific target. And this target should not be whether or not it crowds in or crowds out private contributions, but instead it should be undertaken to induce a socially optimal level of a social norm. Once one accepts that government policy should be fully endogenized, then, as we show below, many of the commonly accepted results from the exogenous policy literature must be viewed in a new light.

The present work is at the crossroads of several strands of the literature, namely the analysis of voluntary contributions to public goods, social approval and the dynamic diffusion of pro-social norms. Several authors have proposed explanations for the observed fact that, even in large societies, people do contribute to public goods such as charities. Even though pure altruism can explain that people contribute to public goods, economic theory predicts that incentives to contribute vanish with increasing numbers of potential contributors (see ??). ? and ? argue that there is a private benefit from contributing to public good, related to one's own contribution, that helps explain common patterns observed and soften the possible crowding out from public provision. ? proposes that individuals maximizes their utility subject to a moral constraint.

? suggests the existence of a social effect and considers that people care about their relative contribution to a public good. ? introduce the idea that people compare their contribution to an ideal one. In a static framework, they emphasize the possible adverse effect on contribution of a public policy that would operate via a deterioration of the ideal effort. ? model the signaling effect of contributing to a public good, altruism being socially rewarded and signaled through high contributions, where they emphasize that a subsidy to contributions reduces the altruism signaling incentive. ? study the optimal government regulation in such circumstances. All these models are static and do not study the diffusion of the pro-social behavior.

Under rather general conditions the social effect can give rise to multiplicity of equilibria, a feature that translates into historical lock-in in dynamic frameworks. ? emphasizes that social approval can induce a multiplicity of equilibria because the more people contribute the higher the social reward of contributing (and vice versa). When considering public policy she stresses that a subsidy can help unlock society from a zero-contributor situation and push it toward a full contribution equilibrium. ? provide a model of green consumption with social approval that shares similar features. In dynamic settings, the existence of multiple equilibria associated with social approval as also been studied by ?, who considers that the amplitude of the social approval can be soften by the introduction of a subsidy. These authors again consider the ability of public policy to shift society to the full participation equilibria without providing a welfare discussion.

We instead argue that it can be a significant mistake to induce full participation equilibria. In fact, we show that in the case of a linear deadweight loss from raising public funding it is never optimal from society's perspective to induce the full participation equilibrium. However, we also show that this result is only upheld if the deadweight loss is approximately linear in taxes raised. If this is not the case, then the optimal equilibrium can be the full participation one, but it does not need to be. In fact, depending the parameter combination there may be a multitude of equilibria, some of which may even be optimal at the same time. In other words, we observe the existence of Skiba points which implies that initial conditions may be equally important as parameter conditions. Additionally, we find that there may also be social traps, meaning that despite the government's ability to shape the social norm in any way it wants, it may be optimal to approach the one or other equilibrium depending on the initial conditions.

The article is set up as follows. In the next section 2 we start by introducing **?**'s original model. We then extend her model in section 3 by introducing endogenous government policy which comes at a linear deadweight loss. This modeling assumption allows us to get explicit solutions which help us to discuss the implication of optimal policy. In section 4 we look into further aspects such as a comparison to the Pigouvian tax or non-linear deadweight loss. Section 5 concludes.

2 The background model

In this section we present the main ideas behind **?**'s model of a social norm that influences the incentives for the private provision of a public good, discuss the results on the exogenous government policy, and then study the implication of endogenizing this policy. We fully follow the notation in **?** for simplicity.

In **?** there exists a continuum of agents (on $[0, 1]$) who decide to either contribute ($g_i = 1$) or not contribute ($g_i = 0$) to a public good. x is the share of contributors, and $w(x)$ is the benefit of the public good. Agents have income I that they may use for consumption (at price 1) or for the public good at price $p > 0$, and they are also affected by a social norm $q_i(x)$. The utility function is assumed to be linear and of the form $U_i = I + w(x) - pg_i + q_i(x)$.

The social norm arises from the interaction of agents. A non-contributor feels disapproval, whereas a contributor feels approval if he is observed by a contributor. A person feels neither approval nor disapproval from non-contributors. The magnitude of the approval or disapproval feeling depends on the frequency of the behavior in society; it is proportional to the benefits for society of the social norm (by a factor $\lambda \equiv w(1) - p > 0$); and it depends on how many agents someone meets from one's own type, the share of which is $k \in (0, 1/2)$. This yields[1] a social approval of $q_i(x) = \lambda(1 - x)(k + (1 - k)x)$ for contributors. while non-contributors obtain a disapproval of $q_i(x) = -\lambda x^2(1 - k)$.

Individuals then play a coordination game in which they choose whether to be a contributor by maximizing their utility, the difference of utility being

$$\Delta U(x) = U^1(x) - U^2(x) = \lambda(k + (1 - 2k)x) - p. \tag{1}$$

Let us define $\bar{x}$ the share of contributors that cancels this difference:

$$\bar{x} \equiv \frac{p - \lambda k}{\lambda(1 - 2k)}.$$

Because of social approval the difference in utility is increasing in the share of contributors. If the share of contributors is larger than $\bar{x}$, then everyone prefers to be a contributor, while no one prefers to contribute otherwise. As stated in Proposition 1 in **?**, if $\bar{x} \in (0, 1)$ there are three Nash equilibriums of the static game: $x = 0$, $x = 1$ and $x = \bar{x}$. This is the case if the following assumption is fulfilled:

Assumption 1 *We assume $p > \lambda k$ and $p < \lambda(1 - k)$.*

? took this model a step further and, based on **?**, **?** and **?**, allowed the social norm to evolve dynamically and endogenously. Clearly, if agents were fully aware of their

[1] For the derivation we refer the reader to **?**.

preferences and could adapt to the social norm instantly, then the social norm would also instantly spread through society. This, however, is not realistic and in general agents are not well-informed about their preferences and only learn over time what is best for them. This idea of dynamic evolution can be captured through the so-called replicator dynamics and is given by the following equation

$$\dot{x}(t) = x(t)(1 - x(t))\Delta U(x(t)). \tag{2}$$

Based on this evolutionary dynamics **?** shows that the three potential equilibria as identified above are still possible, but only two are stable ($x(t) = 0$ and $x(t) = 1$) while $x(t) = \bar{x}$ is unstable. For $x(t) < \bar{x}$ society converges over time to the non-contributor equilibrium, while for $x(t) > \bar{x}$ society converges to the equilibrium where everyone contributes. In other words, if there are too few contributors in society then non-contributors have not enough incentives to become contributors. Similarly, contributors are not sufficiently approved as they meet too few other contributors, so that it may simply not be worthwhile for them to contribute any longer. As a result, society converges to a norm where nobody contributes. In contrast, if there are sufficient contributors in society then social approval and disapproval motivates non-contributors to become contributors.

? then investigates if the government, by introducing price subsidies s that are paid for via income taxes xs, can instigate a change in society that may induce society to converge to the equilibrium with everyone contributing. We thus rewrite the utility of contributors as

$$U^1(x, s) = I - xs + w(x) - p + s + \lambda(k + (1 - k)x)(1 - x), \tag{3}$$

where $p - s$ is the original price p with the subsidy s, and xs is the income tax, while the utility of non-contributors becomes

$$U^2(x, s) = I - xs + w(x) - \lambda(1 - k)x^2. \tag{4}$$

This augments the utility difference, which now becomes

$$\Delta U(x, s) = s + \lambda(k + (1 - 2k)x) - p. \tag{5}$$

Thus, if the subsidy is large enough ($s > p - \lambda k$) then this utility difference will be positive and society can converge to the high equilibrium. Furthermore, once the subsidy has been put in place for long enough such that the share of the contributors exceeds a certain threshold ($\bar{x}$), then even without this subsidy society would continue to converge to the equilibrium where everyone contributes to the public good. Thus, a subsidy will crowd-in voluntary contributions.

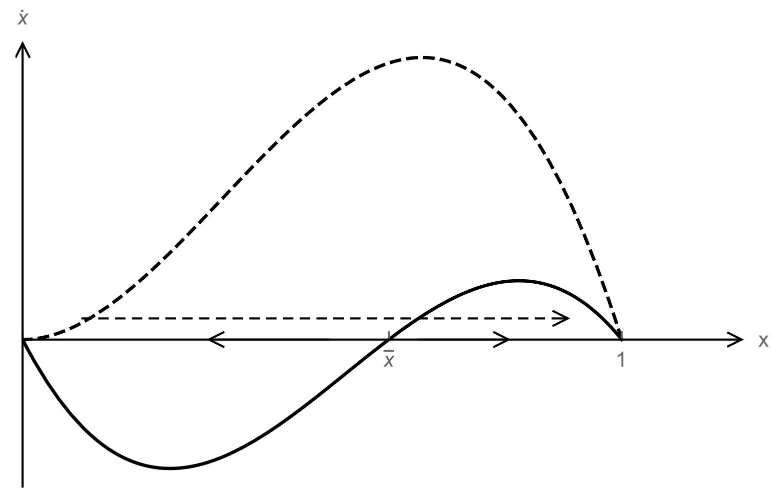

Figure 1: Influence of a fixed subsidy $s > p - \lambda k$ on the dynamic of the share of contributors. Source: **?**

While it is good to know that government policy can induce changes in equilibria, it is also important to know whether and when this is actually optimal. We now turn to our contribution.

3 The implication of endogenous policy

In general when economists are faced with any kind of externalities they resort to the use of Pigouvian taxes. These taxes are intended to correct market inefficiencies by internalizing externalities. Practically speaking it is, however, not always easy to identify all the externalities that one would wish to address. So we start by assuming that a policy maker has only limited information on the social norm, and we call this policy maker *norm ignorant.* We then argue that even policy makers may understand that there is a social norm involved and that they want to also address the externality that results from the norm. We then take this model all the way by assuming that a policy maker also faces constraints when introducing a policy and study how these constraint impact optimal decisions.

It is clear that our norm ignorant policy maker would only see one externality, namely that some agents do not contribute to the public good. He would thus subsidize the price of the public good as contributing to it yields a public benefit. He would do so by setting the subsidy just high enough so that those who previously would not want to benefit would want to. It is easy to see that the level of subsidy required to make those individuals contribute is $s^i \geq p$. Intuitively this means that the policy maker would reduce the effective price of the public good to zero (or marginally negative). As the price p of the public good in this model is constant, this implies that the subsidy is constant. As more and more individuals will start to adhere to the social norm, the per capita tax ($x_t s^i$) to finance this subsidy will have to increase until finally everyone adheres to the social norm. A policy maker who is fully norm ignorant would, even in this case, continue to subsidize the public good simply because he is not aware that there is a social norm in the background that actually does not require the subsidy any longer for full contributions. Hence, a norm ignorant policy maker would set a constant subsidy such that the public good is provided for free even though over time all agents would adhere to the norm.

It is clear that even policy makers may understand that social norms play a vital role when it comes to the private provision of public goods. Thus we shall assume that they can fully evaluate the role of the social norm and understand how it interacts with the incentives for the private provision. However, assume that our policy maker is unaware of the actual underlying dynamics of the norm. Let us call this case the *norm informed* policy maker. Our norm informed policy maker will know that the social pressure affects utility and thus he will set the subsidy with this in mind at the level $s^n \geq p - \lambda(k + (1-2k)x_t)$. It is clear that $s^n < s^i$ as now the social norm now helps in achieving his goal. Furthermore, as $p > \lambda k$ but $p < \lambda(1-k)$ then we know that $s^n = 0$ for $x_t \geq \bar{x}$. Hence, our norm informed policy maker would always set $s^n \leq s^i$, where the subsidy s^n declines over time and reaches zero when the norm itself is sufficient to induce individuals to accept the norm over time.

We now take this a step further and fully endogenize the government policy. There is clearly a variety of ways in which one could set up an optimal government policy program. We are here going to take the simplest possible setting as this already yields the insights that we are after. We assume that agents continue to act myopically, continue to believe that they have no impact on either the social norm nor the public good, but we assume that there exists a government who can introduce a policy. This policy maker is forward looking, has perfect foresight, and maximizes the infinite stream of the agents' utilities by appropriately setting incentives. These incentives come in the form of price subsidies $s(t)$ that can be positive or negative. Allowing for also negative subsidies gives the policy maker complete freedom over the direction in which he wants to push the consumption of the public good, and thus indirectly also the social norm. We assume the existence of exogenous bounds on $s(t)$ in the form of $\underline{s} \leq s(t) \leq \bar{s}$, with $\underline{s} < 0$. We shall, for simplicity, assume that $\underline{s} \leq p - \lambda(1-k)$ and $\bar{s} \geq p - \lambda k$.

Total instantaneous felicity is given by $V = xU^1(x,s) + (1-x)U^2(x,s)$ which only depends on x and after substitution this yields

$$V(x) = I + w(x) - px + \lambda x(1-x)k. \tag{6}$$

In each period the policy maker balances the budget but whenever he raises taxes to pay for the price subsidy then he incurs a deadweight loss. There is a shadow cost of public funds $\gamma > 0$, i.e. raising \$1 of public money costs society \$$(1+\gamma)$ because of distortionary taxation. Conversely, if the government taxes the public good ($s < 0$) then this reduces the taxpayers' burden by $sx(\gamma+1)$. This, in a sense, gives this model a bit of a partial equilibrium character, but it significantly simplifies the analysis.[2]

[2] In a later section we consider a non-linear deadweight loss which provides a more general equilibrium character.

Hence we can now write the policy maker's objective function, which is given by

$$W = \int_{t=0}^{+\infty} e^{-\phi t} \Big[V(x(t)) - \gamma x(t) s(t) \Big] dt, \tag{7}$$

where $\phi > 0$ is the discount rate. Based on this setup there is only one state equation

$$\dot{x} = x(1-x)\Delta U(x,s). \tag{8}$$

The policy maker then maximizes equation (23) with respect to $s(t)$, subject to the constraint (24) and with bounds on subsidies in the form of $\underline{s} \leq s(t) \leq \bar{s}$.

Using equation (5) and subsituting s into the expression (23) of the objective function yields

$$W = \int_{t=0}^{\infty} e^{-\phi t} \left\{ V(x) - \gamma x(t) \Big[p - \lambda(k + (1-2k))x(t) \Big] - \gamma \frac{\dot{x}}{1-x} \right\} dt \tag{9}$$

Due to the linearity of the control $s(t)$ we know that the solution to this is a *Most Rapid Approach Path*. Maximizing the objective function we obtain the Euler equation, and denoting the optimal solution by x^*, we then get

$$V'(x^*) - \gamma \Big[p - \lambda k - 2\lambda(1-2k)x^* \Big] - \frac{\gamma \phi}{1-x^*} = 0. \tag{10}$$

There are three terms. The first is the effect on per period welfare V of an increase in the number of contributors. This term can be further decomposed as $V' = w' - p + \lambda(1 - 2x)k$ the sum of the marginal benefits from the public good, the cost of contributing and the marginal social approval effect. Note that this last social effect can be either positive or negative depending on how many people are already contributing to the public good.

The second term is the effect on the per period cost of the total subsidy xs. The subsidy necessary to sustain a given share of contributors is decreasing in the share of contributors, an effect which is due to the social approval.

The third term is related to the impatience of the social planner, it is the discounted cost of the subsidy. The more impatient the social planner the lower the benefits from increasing the number of contributors. This time effect crucially hinges on the existence of the deadweight loss that the policy maker bears each instance. At the steady state the corresponding subsidy is positive or negative. If it is positive, the cost of further increasing the subsidy should be equalized with the discounted value of the benefits from increasing the share of contributors. If it is negative, the public good is taxed and the benefits from this tax should be compared with the losses from decreasing the pool of contributors. This marginal cost grows to infinity as x becomes close to 1, for then the norm spreads slowly in the society (there are few non-contributors to be converted) and it becomes very costly to further spread the norm.

Assumption 2 *We assume that $w'(0) > \gamma\phi + (1+\gamma)(p - \lambda k)$ and $w'''(x) = 0$.*

This insures that there is a unique interior solution to x^*.

Proposition 1 *The optimal solution to the maximization problem (9) is a Most Rapid Approach Path.*

Given Assumptions 1 and 2, the optimal policy consists in :

- *If $x(t) < x^*$ then $s(t) = \bar{s}$, and $\dot{x} > 0$;*
- *If $x(t) > x^*$ then $s(t) = \underline{s}$, and $\dot{x} < 0$;*
- *And once $x(t) = x^*$ the steady state solution is*

$$s^* = p - \lambda(k + (1-2k)x^*) \tag{11}$$

in which x^* solves (10).

Proof 1 Due to the linearity of the control $s(t)$ it is clear that the solution to the optimal control problem is a Most Rapid Approach Path. The Euler equation then is given by (10). The properties of this Euler equation are as follows. Define $SL(x) \equiv w'(x) - (1+\gamma)(p - \lambda k) - 2\lambda x(k - \gamma(1-2k))$, and $SR(x) \equiv \frac{\gamma\phi}{1-x}$. Then we obtain $SR(0) = \gamma\phi > 0$, $SR(1) = \infty$, $SR'(x) = \frac{\gamma\phi}{(1-x)^2} > 0$ and $SR''(x) = 2\frac{\gamma\phi}{(1-x)^3} > 0$. Furthermore,

$SL(0) = w'(0) - (1+\gamma)(p - \lambda k)$, $SL(1) = w'(1) - (1+\gamma)(p - \lambda k) - 2\lambda(k - \gamma(1-2k))$, $SL'(x) = w''(x) - 2\lambda(k - \gamma(1-2k))$ and $SL''(x) = w'''(x)$. Thus it is clear that Assumption 2 insures a unique interior equilibrium. Then it is straight forward to see that for $x(t) < x^*$ we have $s(t) = \bar{s}$, while for $x(t) > x^*$ we obtain $s(t) = \underline{s}$. For $x(t) = x^*$ we have the steady state solution $s^* = p - \lambda(k + (1-2k)x^*)$, where x^* solves (10). ■

We can directly see that $s^* < 0$ if and only if $x^* > \bar{x}$. The policy maker applies a tax if $x(t) > x^*$, or a subsidy if $x(t) < x^*$. This is an important difference to what we found previously. In fact, we find that if $x^* < \bar{x}$ then it may even be optimal to tax the public good cum social norm. This stands in stark contrast to the previous cases where a tax (a negative subsidy) was never optimal.

It is never optimal to push society toward $x = 1$ because of the cost of public fund and the diffusion dynamics. If x is close to 1 then there are few non-contributors left, and it is too expensive to subsidize contribution to convert them into contributors. Conversely, to tax contributors has a small effect on the dynamic of the norm and is then justified by the cost of public funds.

3.1 Relation with Pigouvian taxation

In order to clarify the relationship with standard Pigouvian taxation it is worth rewriting the equation (10) satisfied by the optimal share x^* injecting the value of the optimal subsidy:

$$w'(x^*) - \lambda x^* - (1+\gamma)s^* - \gamma\lambda(1-2k)x^* - \frac{\gamma\phi}{1-x^*}. \tag{12}$$

or

$$s^* = \frac{w'(x^*) - \lambda x^*}{1+\gamma} - \frac{\gamma}{1+\gamma}\left[\lambda(1-2k)x^* + \frac{\phi}{1-x^*}\right]. \tag{13}$$

This formula can then be related to standard Pigouvian taxation in which the optimal subsidy should be set equal to the marginal benefits at the optimum share. It helps understand the role of three ingredients, in addition to the public good: the cost of public fund, the social norm and the replicator dynamic. The first is relatively standard the two others are not. Let us look at equation () and switching on each of these components in turn to understand the role played by each and how they interact.

First, let us consider that λ and γ are equal to zero, in such a case the equation boils down to $w'(x^*) = s^*$, the optimal subsidy is the Pigouvian subsidy equal to the marginal external benefit. Indeed, if $x^* < 1$ this optimal policy is exactly equal to the marginal cost of contributing p, otherwise it is larger.

Second, if $\lambda > 0$ and $\gamma = 0$ the equation (12) becomes: $w'(x^*) - \lambda x^* = s^*$ the optimal subsidy encompasses two pigouvian terms: the public good external benefit and the social external cost, none of them being internalize when actions lead to $\Delta U(x, s) = 0$. To better grasp the origin of the term $-\lambda x$ it is illuminating to write V as the pondered sum $xU^1(x) + (1-x)U^2(x)$ so that

$$\begin{aligned} V' &= \Delta U(x) + xU_x^1(x) + (1-x)U_x^2(x) \\ &= w'(x) + \Delta U(x,s) - s \\ &\quad + \lambda \Big\{ x\Big[(1-k)(1-x) - (k + (1-k)x)\Big] - (1-x)2(1-k)x \Big\} \end{aligned} \tag{14}$$

The braced term is the social effect of an increase of the share of contributors. It is the sum of the effect on contributors and the effect on non-contributors. For each category it is the sum of the increased probability of meeting a contributor and the influence on approval. For non-contributors, the diffusion of the contributory behavior has a cost: they meet more contributors and feel more disapproved. For contributors the effect is ambiguous: they benefit from meeting more contributors but their feeling of approval is reduced for each encounter. This social effect is negative: $V' = w'(x) + \Delta U(x,s) - s - \lambda x$.

Note that even though there is a negative social externality and an apparently lower optimal subsidy, the optimal share of contributors can still be larger with the social norm than without it.

Third, if $\lambda > 0$ and $\gamma > 0$ and $\phi = 0$, so that the replicator dynamic plays no role on the optimal subsidy, the equation (12) becomes $w'(x^*) - \lambda x^* - (1+\gamma)s^* = \gamma\lambda(1-2k)x^*$ which is often written, if $s^* \neq p$:

$$\frac{1}{p - s^*}\left[\frac{w'(x^*) - \lambda x^*}{1+\gamma} - s^*\right] = -\frac{\gamma}{1+\gamma}\frac{1}{\epsilon}$$

in which

$$\epsilon = \frac{p - s^*}{\lambda(1-2k)x^*}$$

corresponds to the elasticity of the steady state share of contributors with respect to the cost of contributing. This way to write the relationship between s and the associated steady state x helps relate to a Ramsey like formula of the optimal subsidy. The term $(w'(x) - \lambda x)/(1+\gamma)$ is the marginal benefit from the public good in public monetary units, its difference with the optimal subsidy being the implicit tax on contributory behavior. The ratio between this implicit tax and the price of the public good is proportional to the inverse of the elasticity. The peculiarity being that this elasticity is positive because of the social norm, that is, the interior share of contributors is decreasing with respect to the subsidy.

3.2 Social norm and public policy interaction

In presence of a public good economic agents under-provide the public good and there is an economic rationale for government intervention to improve social welfare by setting a subsidy or quota. With a social norm, agents are incentivize to contribute because of social approval and disapproval, it may help overcome the free-rider problem. However, the social norm introduces other externalities, positive and negative, that blur the picture of the desirability of the contributory behaviour.

The regulatory intervention and the social norm seem to be a priori substitute ways to overcome free-riding and increase the provision of the public good. What is the optimal level of contribution depends on the policy tool available and the costs associated to any of these tools.

Let us first define the first best share of contributors as the share x^{FB} that maximizes $V(x)$ defined by equation (6). If $\lambda k > w'(1) - p > 0$ then $x^{FB} < 1$ with the social norm whereas $x^{FB} = 1$ without the social norm (negative externality of the social norm). The following assumption ensures that $x^{FB} = 1$ even with the social norm.

Assumption 3 $w'(1) - p > \lambda k$

Hence we assume that we are in a world where full adherence to the norm seems socially desirable.

If the social norm parameter λ is sufficiently low, it is then desirable to have full contribution, and the social norm can potentially moves society toward this situation if $x(0) > \bar{x}$.[3] In such a case, it is true that the full contribution equilibrium made possible by the social norm is desirable.

In the dynamic setting, with the replicator dynamic assumption, full contribution is never reached but society converges toward it, if $x(0) > \bar{x}$, thanks to the social norm and welfare progressively increases.

If one considers regulatory intervention, in a second-best setting the tools and the costs associated to the regulation influences the level of the optimal share of contributors. This optimal share becomes contingent on the regulatory tools used. If Assumption 3 is true then

- $x^{FB} = 1$ and the social norm helps reach this situation (if $x(0) > \bar{x}$ only)
- without costly public fund the regulator sets a high subsidy to move society toward $x = 1$ as fast as possible
- with costly public fund the regulator never pushes society toward the $x = 1$ equilibrium, and the presence of the social norm does not necessarily advocate for a high level of contribution

[3]Note that with $\lambda = w(1) - p$, as assumed by **?**, and $k < 1/2$, the above assumption is satisfied if w is linear with respect to x.

The social norm has two effects on the optimal share of contributors, when policy is implemented: a direct effect on the marginal welare $V'(x)$, and an indirect via the cost of public funds. The first effect is $\lambda(1-2x)k$ which is maximized at $x = 1/2$. The second effect operate via the subsidy and cost of public funds, this effect is positive. If the optimal share of contributors os large the social norm can be associated with a lower share of contributors because of the negative social effects. However, since public funds are costly the social norm reduces the cost to reach a given contributor shares.

Proposition 2 *An increase of λ has a positive effect on the steady state optimal level of the public good x^* if and only if*

$$x^*[k - \gamma(1-2k)] < \frac{1}{2}(\gamma+1)k$$

Proof 2 *rewrite the equation (10) as $SL(x^*, \lambda) = SR(x^*)$. The influence of λ on x^* is $SL_\lambda/[SR_x - SL_x]$ the denominator is negative at x^* and the numerator is*

$$SL_\lambda = (1-2x)k + \gamma(k + 2(1-2k)x) = (\gamma+1)k - 2x[k - \gamma(1-2k)]$$

It is not fully clear whether social approval effect should be integrated within a welfare function. Still social approval influence the diffusion of the behavior and the subsidy required to push society.

Proposition 3 *A social planner who does not take the social norm into account maximizes*

$$W = \int_0^{+\infty} e^{-\phi t}\Big[(w(x) - px) - \gamma x s\Big]dt.$$

Thus he uses a most rapid approach path toward a steady state optimal level x^{**} that is larger than x^* if and only if $x^* > 1/2$, and increasing with respect to λ.

Proof 3 *The Euler equation satisfied by the steady state level x^{**} is*

$$w'(x^{**}) - p - \gamma\Big[p - \lambda k - 2\lambda(1-2k)x^{**}\Big] = \frac{\gamma\phi}{1-x}$$

there is one solution to this equation, it is x^{**} (thanks to Assumption 2) for lower values the left hand side is larger than the right hand side and it is so for x^* iff $x^* < 1/2$

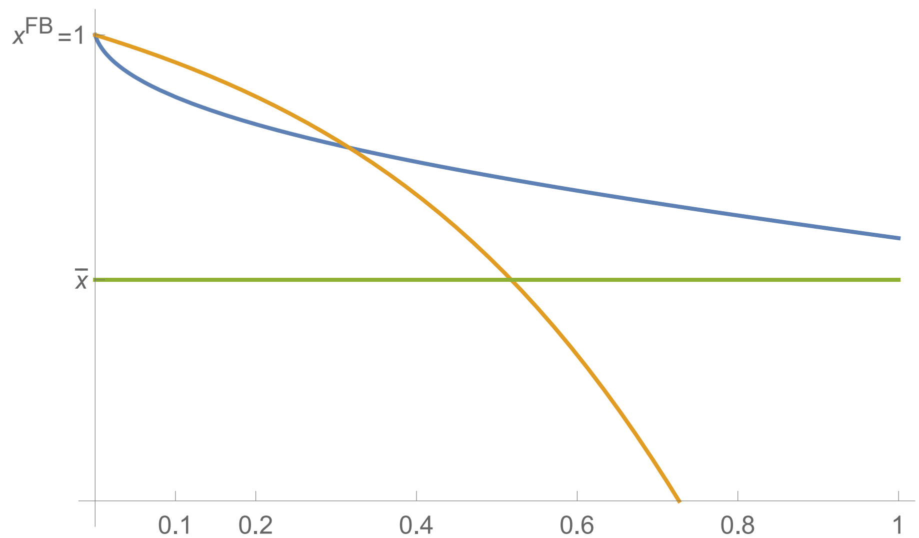

Figure 2: Optimal share of contributors as a function of the costs of public funds with the social norm (blue line) and without it (orange line)

NB: the figure is obtained for sufficiently large k.

Finally, to conclude this comparative static exercice let us look at the influence of the cost of public fund. Costly public funds are key to our result that it is never optimal to reach full participation, but they are not always antagonistic with public good provision.

If it is not optimal to have full participation at the steady state without costly public funds, then an increase in the cost of public funds can justify an increase in the share of contributors. Because of the social effect, it can be worthwhile to increase the share of contributors to reduce the steady subsidy (or increase the tax). This case is illustrated on Figure 3.

Corollary 1 *An increase in the cost of public funds increases the steady state share of contributors if and only if*

$$2x^* > \bar{x} + \frac{\phi}{1-x^*}\frac{1}{\lambda(1-2k)} \tag{15}$$

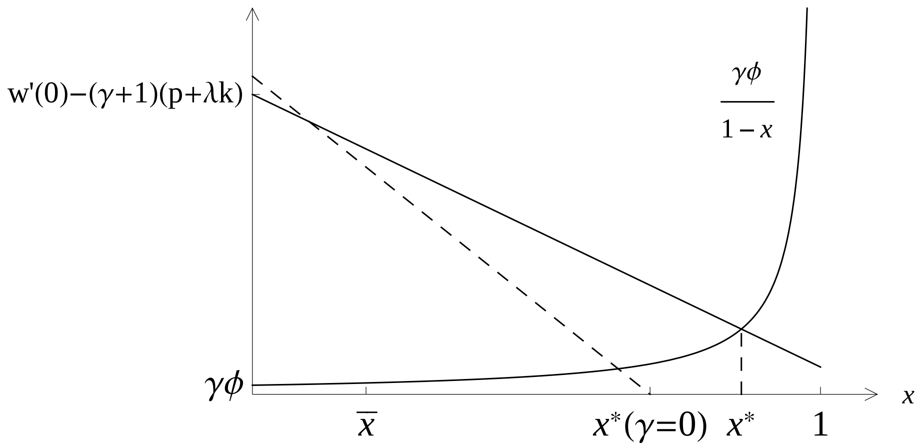

Figure 3: Influence of the cost of public fund. The dashed line corresponds to the marginal flow welfare with $\gamma = 0$

4 Further analysis of the government policy

Several kind of regulators associated to objective functions.

1. Norm ignorant regulator would maximizes

$$\int e^{-\phi t}[V(x) - \gamma x s]dt$$

with $\dot{x} = x(1-x)(s-p)$

it gives a MRAP toward the solution of

$$V'(x) = \frac{\gamma\phi}{1-x}$$

1.0. without costly public funds it gives $V'(x) \geq 0$ with equality if $x < 1$

1.1 with costly public funds it gives a Ramsey like formula adapted to the replicator dynamic

in any cases it is decentralized with $s = p$ at the steady state, and any moves from the steady state is corrected through bang-bang changes of the subsidy

2. The regulator know that there is a social norm that can help enforce contribution but does not set value to social approval/disapproval. Leads to Proposition

$$\int e^{-\phi t}[V(x) - \gamma xs]dt$$

with $\dot{x} = x(1-x)[s - p + \lambda()]$

4.1 Public provision of the public good

4.2 The bounds

In the basic analysis above we assumed the bounds were large enough to insure that the policy maker can shift the norm in any way that he wants. This obviously does not need to be the case.

If the range of subsidy available is smaller, the two extremes being closer to zero, it might be impossible for the social planner to move the share of contributors away from extremes social situations. If $\bar{s} < p - \lambda k$ the social planner is unable to prevent the share of contributors to move toward zero if it is small. similarly if $\underline{s} > p - \lambda(1-k)$ the social planner setting the larger possible tax on contributors would still not prevent the share of contributors to increase if x is close to 1.

4.3 The deadweight loss

In the preceding sections we assumed that the cost of public funds is a linear function of the total subsidy. We assumed a linear function for simplicity and in order to take the analytical results as far as possible. However, it is also clear that it is unlikely that the deadweight loss should be linear in subsidies. This linearity also led to the result that it is never optimal to push society toward the $x = 1$ equilibrium. In order to motivate this linearity we relied on a kind of partial equilibrium explanation.

In contrast, in a more general equilibrium setting, one can consider that whether or not the subsidy is positive or negative, in both cases there is a deadweight loss that arises due to distortions, collection costs, or any standard argument related to deadweight losses from public interventions.

Thus, our objective here is threefold. First, since we want to motivate readers to further investigate the role of optimal policy in the social norm literature, we want to show that assuming linear or non-linear costs of public interventions can lead to substantial differences in the results. For example, we shall show that moving to a non-linear modelling of the deadweight loss can make the $x = 1$ social norm level, in contrast to the linear case, an optimal policy. Second, our motivation is to move away from this somewhat partial equilibrium argument that founded our linear deadweight loss model, towards a general equilibrium model. While this may be an argument of semantics mostly, it may be a more appealing setting to the macroeconomic readership. Third, this non-linear case allows us to derive additional results regarding the choice to move between equilibria and the importance of initial conditions.

Let us assume that there is a cost to collect subsidies that is a quadratic function of the subsidy per individual so that the total deadweight loss is now given by $x\gamma s^2/2$. The objective of the social planner is then to maximize

$$W(t) = \int_{t=0}^{\infty} \Big(I + w(x(t)) - px(t) + \lambda x(t)(1 - x(t))k - \gamma x(t)s(t)^2/2 \Big) e^{-\phi t}. \tag{16}$$

The policy maker then maximizes equation (23) subject to $s(t)$ and the constraint (24). We write the Hamiltonian as

$$\mathcal{H} = I + w(x) - px + \lambda x(1 - x)k - \gamma x s^2/2 + \mu x(1 - x)\Big(s - p + \lambda(k + (1 - 2k)x) \Big).$$

First order conditions yield

$$s = \frac{\mu(1 - x)}{\gamma}, \tag{17}$$

$$\dot{\mu} = \mu\Big(\phi - (1 - 2x)\big(s - p + \lambda k\big) - (2 - 3x)\lambda(1 - 2k)x \Big) - w'(x) + p + \gamma s^2/2 - \lambda(1 - 2x)k \tag{18}$$

It needs to be emphasized that equation (18) does not apply at $x(t) = 0$. In this case the policy maker is essentially free to choose any (finite) level of $s(t)$ as the dead-weight loss will be zero.

Differentiating equation (18) wrt time and solving for $\dot{\mu}$ yields

$$\dot{\mu} = \frac{\gamma}{1-x}\dot{s} + \frac{\gamma s}{(1-x)^2}\dot{x}.$$

We now obtain a system of differential equations in $\{x, s\}$, which is given by

$$\dot{x} = x(1-x)\Big(s - p + \lambda(k + (1-2k)x)\Big), \tag{19}$$

$$\begin{aligned} \dot{s} = & -xs\big(s - p + \lambda(k + (1-2k)x)\big) \\ & + s\big(\phi - (1-2x)\big(s - p + \lambda k\big) - (2-3x)\lambda(1-2k)x\big) \\ & + \frac{1-x}{\gamma}\Big(\gamma s^2/2 - \lambda(1-2x)k - w'(x) + p\Big). \end{aligned} \tag{20}$$

There are three immediate candidates for steady states:

candidate 1: $x = 0$

While we know that $x(t) = 0$ is one of the potential steady state solutions for $\dot{x}(t) = 0$, we also know from the first-order conditions that $s(t)$ at $x(t) = 0$ is a variable that the policy maker can choose freely as it does not entail a social cost. Thus, we need to figure out whether the dynamic system would make us approach this steady state. After all, in the convergence to this steady state we will still have that the necessary conditions must be fulfilled. However, at the steady state we know that this need not be the case. We can, nevertheless, hypothesize that if it is optimal to approach this steady state, then it would not make sense to impose a tax or subsidy when $x(t)$ reaches zero as we then obtained our optimal level of the social norm. Thus, it is clear that whenever the steady state $x = 0$ is optimal, then $s = 0$ at the steady state while $s(t)$ follows the necessary conditions during the convergence to this steady state. Conclusively, whether or not convergence to this steady state is optimal will depend on the shape of the phase curves and thus the associated dynamics.

candidate 2: $x = 1$

This is the steady state where the policy maker would push for the highest level of the social norm in society. Substituting the $x(t) = 1$ solution into the dynamic system yields the logical optimal solution $s(t) = 0$. As we know that from the threshold level $\bar{x}$ onwards the social norm is self-enforcing, then it is clear that positive subsidies after $x(t)$ crossed this threshold are only necessary in order to push $x(t)$ faster towards its steady state. Intuitively, the reason for a positive level of $s(t)$ for $x(t) > \bar{x}$ is then only that the deadweight loss is small compared to the higher social norm (which would anyway have occurred). In this case it should also be clear that the losses from not undertaken a policy if $x(t) > \bar{x}$ should not be too high, as both with and without a subsidy the optimal steady state $x(t) = 1$ is achieved. In contrast, if $x(t) > \bar{x}$ but if it were that e.g. the $x = 0$ steady state would be optimal, then not undertaking a policy would yield substantial losses to social welfare.

The Jacobian around the $\{x_2, s_2\} = \{1, 0\}$ steady state is given by

$$\mathcal{J}\Big|_{(x_2,s_2)} = \begin{bmatrix} p-(1-k)\lambda & 0 \\ \frac{w'(1)-p-k\lambda}{\gamma} & \phi \end{bmatrix}.$$

As this is a lower triangular matrix we have that the eigenvalues are given by $EV_1 = p-(1-k)\lambda$ and $EV_2 = \phi$. This steady state is saddle path stable if $p < (1-k)\lambda$, which applies given Assumption 1. Conclusively, it is now possible that this steady state is optimal, which stands in stark contrast to the linear deadweight loss case. Hence we find that not only is it important to acknowledge that there is a wider need to study the implication of optimal policy in the social norms literature, but we also observe that the way we model this public intervention can yield to vastly different results given the optimal strategy that a policy maker may want to pursue.

candidate 3: $x = \frac{p-s-\lambda k}{(1-2k)\lambda}$, which can be re-written as $s = p - x(1-2k)\lambda - \lambda k$. Substitute this into equation (22) evaluated at steady state gives

$$(1-x) = \frac{\gamma(p-(x+k(1-2x))\lambda)((1-2k)(1-x)x\lambda - \phi)}{p-k(1-2x)\lambda + \frac{1}{2}\gamma(p-(k+x-2kx)\lambda)^2 - w'(x)}. \tag{21}$$

This interior steady state equation is rather complicated and allows for a multitude of combinations of interior steady states with a wide variety of dynamics. The interior steady state can be unique and stable or unstable, or there can exist multiple steady states which are stable or unstable with or without complex dynamics, or it is also possible that no interior steady state exists at all. Finally, there is the possibility of Skiba points, such that there exists an initial condition $x(0)$ for which it is optimal to converge to either of the various equilibria.

5 Conclusion

The literature on the private provision of public goods has mostly settled on social norms as a reason for why private agents would provide public goods. In this literature it has also been emphasized that there is room for public policy to induce the 'good' social norm of everyone contributing. As a result, the literature has to a large extend focused on whether or not public policy crowds-in or out private provisions.

In this article we have argued that it is a mistake to focus on whether or not public policy crowds-in or out private provisions as an argument for or against public policy. Instead, we argued that public policy needs to be assessed on the grounds of whether or not it is actually optimal from a society's perspective. This the reshapes the debate from whether or not public policy crowds-in or out private provisions to whether or not, and if yes then when, it should crowd-in or out private provisions.

In order to study this we extended the rather general model introduced in **?** and introduced endogenous public policy. We showed that in very simple settings where the public policy is subject to a linear deadweight loss then it is not optimal at all to induce everyone to adhere to the social norm. Furthermore, we have shown that a Most Rapid Approach Path is the optimal solution and thus convergence and speed of convergence depends on the bounds of the public policy. In other words, results depend on inhowfar the policy maker can subsidize or tax private contributors. Optimality of the corner or interior solutions in the social norm then naturally depend on a variety of parameters.

Extending this simple model of a social norm to the case of non-linear deadweight losses turns out to have surprisingly complicated dynamics once a policy maker wants to take optimal policy into account. Here more or less anything is possible, from a no interior equilibrium to a unique interior equilibrium, Skiba points and traps in social norms. Furthermore, while the equilibrium where everyone fully adheres to the social norm is always a potential equilbrium and it is, in fact, always locally stable, it does not need to be the optimal equilibrium.

In practical terms this result suggests that it is a mistake to not investigate the social optimality of government interventions in social norms. This has significant implications for example for the fashionable nudging (?), for the analysis of social norms and public policy, and for inhowfar the government should intervene when it comes to the private provision of public goods. Furthermore the simple analytical results on multiple steady states and various dynamics show already that the practical difficulties of judging the optimal policies could be very large. This points certainly set a stringent research agenda.

the objective is to maximize

$$W = \int_{t=0}^{+\infty} e^{-\phi t} \big[V(x(t)) - \gamma x(t) s(t) \big] dt, \tag{22}$$

with

$$\dot{x} = x(1-x)\Delta U(x,s). \tag{23}$$

The policy maker then maximizes equation (23) with respect to $s(t)$, subject to the constraint (24) and with bounds on subsidies in the form of $\underline{s} \leq s(t) \leq \bar{s}$.

the Hamiltonien is

$$\mathcal{H}(x,s,\mu) = V(x) - \gamma x s + \mu x(1-x)\Delta U(x,s).$$

then

the derivative with respect to s is

$$\mathcal{H}_s = -\gamma x + \mu x(1-x)$$

so that

$$s = \begin{cases} \bar{s} & \text{if } \mu(1-x) > \gamma \\ \underline{s} & \text{if } \mu(1-x) < \gamma \\ \textit{indeterminate} & \text{if } \mu(1-x) = \gamma \end{cases} \tag{24}$$

$$\Delta U(x,s) = s + \lambda(k + (1-2k)x) - p.$$

that is $s = \Delta U - x\Delta U_x + (p - \lambda k)$ and the evolution of μ is given by

$$\dot{\mu} = \phi\mu - \mathcal{H}_x = \mu\phi - \mu\big[(1-2x)\Delta U + x(1-x)\Delta U_x\big] - \big(V'(x) - \gamma s\big) \tag{25}$$

$$= \mu\phi - \mu\big[(1-2x)\Delta U + x(1-x)\Delta U_x\big] - V'(x) + \gamma\big[\Delta U - x\Delta U_x + (p-\lambda k)\big] \tag{26}$$

$$= \mu\phi + [\gamma - \mu(1-2x)]\Delta U - [\gamma + \mu(1-x)]x\Delta U_x - V'(x) + \gamma(p - \lambda k) \tag{27}$$

so that if $s \in (0,1)$ then $\mu(1-x) = \gamma$ or $\mu = \gamma/(1-x)$ and $\dot{\mu} = \gamma\dot{x}/(1-x)^2 = x\Delta U/(1-x)$ injecting this in the above equation gives

$$\frac{x}{(1-x)}\Delta U = \frac{\gamma\phi}{1-x} - \frac{\gamma}{1-x}\Bigg[x\Delta U - 2(1-x)\Delta U_x\Bigg] - V'(x) + \gamma(p - \lambda k) \tag{28}$$

that is

$$V'(x) - \gamma\big[p - \lambda k - 2\lambda(1-2k)x\big] = \frac{\gamma\phi}{1-x} \tag{29}$$