Distributive justice in an AK model of growth

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Abstract

In an AK model of growth with fixed and uniform labour supply but heterogeneous asset endowments by agents, we explore the conflict that arises in relation to the functional (labour-capital) distribution of output. Negative externalities make the competitive equilibrium inefficient. We develop the efficient utility frontier; we then analyse the outcomes in terms of growth and distributive (in)equality delivered by different ways of determining the labour (equivalently: capital) share. A clear order arises between various normative and positive arrangements such that Rawlsian is more egalitarian than utilitarian is more egalitarian than contractarian (Nash solution) is more egalitarian than median voter. We also propose a novel fourth concept of distributive justice ('justice as minimal social friction') and show that it is more egalitarian than all except the Rawlsian solution.

Keywords: Growth, factor shares, social conflict, distributive justice, institutions.
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1. Introduction

This paper brings distributive justice considerations to bear upon a simple model of growth and distribution. The paper is motivated by the general observation that inequality is generally rising among advanced economies and that wealth inequality is more prominent than income inequality. At the intersection between the two, the 'functional' distribution of income, i.e. its division into labour or capital income, is often the source of much social conflict. This conflict is multi-faceted whereby various actors or groups attempt to capture a larger share of the output either directly (through wage negotiations or price rises given the nominal wages) or indirectly by manipulating the political system to achieve favourable transfers, regulations, and other redistributive policies. Despite the diversity of forms of the social conflict (Dahrendorf, 2007), its core aspect or pursuit is arguably a conflict over factor shares. The allocation of shares is among the key determinants of income distribution; additionally, as the resulting factor shares determine the marginal revenue productivity of capital, social conflict also directly impinges on growth. In other words, the conflict over the functional distribution interacts with macroeconomic outcomes; this is our departure point. This perspective has been explored in sociology and politics, particularly in analyses of inflation (Goldthorpe, 1978, 1984; Hirsch, 1978). In macroeconomics, issues of distributive justice and social conflict have received less attention; notable exceptions include Rowthorn (1977) and Benhabib and Rustichini (1996), while the broader issue of distribution is thoroughly examined in Bertola, Foellmi and Zweimüller (2006). Most closely to our analysis here, a couple of models of the functional distribution of income (Alesina and Rodrik, 1994; Bertola, 1993) have pointed out a basic conflict of interest between workers and owners of capital and analyse its implications for tax policy. We also build on Tsoukis and Tournemaine (2011), which provides a simple formal model of social conflict and analyses its implications in terms of growth, factor shares and broader income distribution.

The purpose of this paper is to analyse the macroeconomic outcomes (growth, income distribution, welfare) that arise under different resolutions of the conflict over the functional distribution of income. The setup is a simple AK model with agent heterogeneity over asset endowments; because of the lack of dynamics in this model, the initial heterogeneity of assets, which here are only physical capital, is perpetuated for ever. In every other respect all agents are identical: they all provide inelastically one unit of labour and have a common rate of time preference. In particular, there are two classes of otherwise identical capital-rich and capital-poor agents. Agents consume their identical wage, equal to the labour share (as agents are of unit mass), plus a constant fraction of their assets (physical capital); this is standard in an AK model. The labour share however decreases the marginal profitability of capital and thereby growth, consistently with the Euler equation of consumption growth. So, a basic trade-off is that a higher labour income increases the wage but decreases growth. Because capital-poor (-rich) agents rely relatively more (less) on labour income, they prefer a higher (lower) labour share. This is the main source of conflict in this model and different arrangements are invoked as to how it is resolved. Highlighting the conflict over the function distribution of income, bringing it to bear upon the determination of growth and exploring positive and normative ways of resolving it is the nexus of contributions of this paper.

Our starting assumption is that the labour share is determined by socio-political factors as well as economic forces. The potential socio-political arrangements that determine the labour share and resolve the related conflict include a simple median-voter electoral outcome (Downs, 1957) and a 'social-contractarian' outcome that is analytically based on the Nash solution. Here, two parties that represent 'capitalists' and 'workers' (capital-rich and -poor agents, respectively) fight in the political arena but the class sizes do not map neatly into electoral outcomes; rather, other intermediate mechanisms such as culture, media, organisation strength of 'corporatist' associations such as labour or employer unions, play a role. We abstract from details and only postulate a political bargaining strength that affects outcomes. Aside from these positive arrangements, we also analyse normative criteria of justice such as the utilitarian criterion applied by a benevolent dictator; and Rawls's (1971) maximin criterion of distributive justice that entails maximising the welfare of the weakest.[1] Furthermore, we propose a criterion of justice as 'minimum social friction', 'friction' defined as difference in welfare between the two groups normalised by aggregate welfare (gains in it over a certain benchmark). The benevolent dictator that might adopt this criterion will need to find the optimal labour share that reduces welfare differences but without compromising the aggregate outcome too much. We map these arrangements in terms of labour shares, growth and group welfares that they deliver. To pre-amble, the Rawlsian criterion is the most egalitarian (almost by definition), followed by our 'minimal social friction' criterion, followed by the contractarian outcome, followed by the utilitarian outcome and finally by the median voter outcome. Thus, a clear pecking order arises and this is the biggest contribution of our paper.

As stated, the conflict between the two groups is highlighted with reference to the 'utility possibility frontier' (UPF) that shows the trade-off in intertemporal welfare between the two groups as the labour share varies. A key assumption is that this UPF everywhere dominates the competitive outcome which would arise if the labour share was fixed through perfect competition at the

[1] We assume that all agents are fully aware of the implications of growth for their intertemporal welfare, therefore we apply the maximin criterion to intertemporal welfare rather than current income (*contra* Rawls, 1971).

(implicit) technical coefficients of capital and labour in the production function.[2] The reason is the existence of negative externalities in welfare in the form of status comparisons in consumption ('keeping up with the Joneses') that has been amply highlighted in macroeconomics (Abel, 1990; Gali, 1994; Tsoukis, 2007; Tournemaine and Tsoukis, 2008, 2009). We show how such sociological considerations interact with the various politico-economic outcomes we explore here. Finally by pointing out the macroeconomic implications of different institutional setups, whether positive or normative, the paper also makes a contribution to analyses of institutional design (Acemoglou, 2006; Aghion, Alesina and Trebbi, 2004; Persson and Tabellini, 2005; Tricchi and Vindingni, 2010).

The remainder of the paper is organised as follows. In Section 2, we develop the model. In Section 3, we highlight the nature of the conflict via the emerging UPF. In Sections 4 and 5, we analyse the positive and normative outcomes. We conclude in Section 6.

2. A model of growth and the functional distribution of income

The model is essentially an AK one. We postulate an economy in continuous time with a unit mass of infinitely lived agents. The agents are identical in all respects except their physical capital (which is the only form of asset) endowments; more on that below. All workers are endowed with one unit of labour which they supply inelastically. Between them, and in proportion to their capital endowments, the agents own the unit mass of identical firms in the economy. Because of these conventions, the distinction between firm-specific and aggregate variables is immaterial, so it will not be emphasised (except when accounting for the externality in production below). Output, y_t, can be consumed or saved to give new units of capital. Each firm's technology of production is given by

$$y_t = A\left(k_t\right)^{1-\lambda}\left(l_t B_t\right)^{\lambda} \quad , \tag{1}$$

where $0<\lambda<1$ is the elasticity of labour in production, so that there are constant returns to the privately-owned factors; $A>0$ is a productivity parameter due to such factors as human capital, skills, or general purpose technology; $l_t \equiv 1$ is labour; k_t is physical capital; and $B_t \equiv \bar{k}_t$ proxies

[2] In other words, we assume that the AK model arises because of 'learning-by-doing' positive externalities (or other similar externalities such as those arising from public capital) that guarantee constant returns to scale to an aggregate concept of capital coupled with fixed labour. In this way, constant growth is fully consistent with technical coefficients for labour and (private) capital in production of $0 < \lambda$, $1-\lambda < 1$, respectively.

the level of productivity by mean (with overbar) capital: as mentioned, endogenous productivity is promoted by a ‘learning-by-doing’ externality (Romer, 1986) or productive public services (Barro, 1990) or some other largely equivalent interpretation. From national income accounting, capital accumulation, assuming away depreciation and in obvious notation, is given by:

$$k_{t+1} - k_t = g_t k_t = y_t - c_t \; . \tag{2}$$

where g_t is the growth rate of capital (on which more below). Importantly, this equation describes the period resource constraint for the national economy and the representative firm, where the variables are identical (due to unit mass of firms). If we add a generic agent superscript k, the same equation gives the budget constraint of each individual k:

$$k_{t+1}^k - k_t^k = g_t k_t^k = y_t^k - c_t^k \, . \tag{2’}$$

As is well known, in the AK framework there is no short-term dynamics; variables jump instantaneously to their steady states and the growth rate in (2) is the common and constant growth rate of the balanced growth path, g_t=g. This is a great simplifying device in two respects: The growth rate of physical capital is uniform across agents; and due to the absence of dynamics, it is constant across time. Thus, the initial heterogeneity in asset endowments between agents (classes thereof) is preserved unaltered for ever. Moreover, the choice of labour share, γ_t, however determined, becomes one-off: the decision variable stays constant at the value decided at the beginning of time for all time. Therefore $\gamma_t = \gamma$ for all t.

All agents (generically subscripted k) maximise intertemporal utility:

$$\text{Max} \quad W_0^k \equiv \sum_{t=0}^{\infty} (1+\rho)^{-t} U_t^k \; , \tag{3}$$

where ρ>0 is the common rate of time preference, and U^K is the period utility:

$$U_t^k = \log\left(c_t^k - \alpha c_t\right), \tag{4}$$

where $0<\alpha<1$ indicates the strength of the status or 'keeping up with the Joneses' externality relative to mean consumption (unsuperscripted c). Maximisation takes place subject to the sequence of constraints (2').

In this setup, with a constant labour share $0<\gamma<1$ and since labour is normalised to unity, we have the following marginal revenue product of capital (MRPK):

$$\mathrm{MRPK} = (1-\gamma)\mathrm{A}. \tag{5}$$

Thus, the Euler equation ("Keynes-Ramsey rule of consumption growth") reads:

$$1+\mathrm{g} = \frac{1+(1-\gamma)\mathrm{A}}{1+\rho}, \tag{6}$$

where g denotes the common growth rate of consumption, capital and output. Log-linearising, we can approximate (6) in the standard way as:

$$g = (1-\gamma)A - \rho. \tag{6'}$$

Equation (6) (or equivalently 6') is the key growth equation in this simple world. Our key assumption is that it is determined according to the arrangements in place.

We now turn to describe capital endowments, the only source of agent heterogeneity as stated. To keep things simple but also meaningful, we assume that individuals belong to two classes, (k=i, j) of fixed size $0<\theta<1$ and $1-\theta$, respectively; for concreteness, we shall assume that the poor are more numerous, $0<\theta<0.5$. The capital holdings relative to the mean are $\chi^i > 1 > \chi^j$. The two of course obey $\theta\chi^i + (1-\theta)\chi^j = 1$ (noting the unit size of the population).

In this economy, it is well known that consumptions of the two classes are given by:

$$c_t^i = \chi^i \rho k_t + \gamma A k_t \tag{7a}$$

$$c_t^j = \chi^j \rho k_t + \gamma A k_t \tag{7b}$$

Consumption of each individual is given by their fraction of the labour share (effectively, the wage times the unit of labour) plus a constant fraction of their capital holdings determined by their rate of impatience. Aggregating over time as instructed by (3) and (4), normalising initial aggregate capital to one, $k_0=1$, we have:

$$W_0^i = (1+\rho)g/\rho^2 + \log\left[A\gamma(1-\alpha)+\rho(\chi^i-\alpha)\right]/\rho \tag{8a}$$

$$W_0^j = (1+\rho)g/\rho^2 + \log\left[A\gamma(1-\alpha)+\rho(\chi^j-\alpha)\right]/\rho \tag{8b}$$

In the next Section, we shall show how varying the labour share, γ, affects the welfare of the two groups when growth (6') is taken into account ($g=(1-\gamma)A-\rho$). The central idea of (8a, b) is that a greater labour share is more egalitarian (as labour income is equally shared) but it decreases growth (it decreases the marginal profitability of capital). But as we show next, even the 'capitalist' class would like a labour share above the minimum lowest (the competitive) because they have a labour income (in real world-terms, they want some of the revenues of the business distributed as wages, because they get those); symmetrically, the 'labour' class does not desire a maximum labour share (γ=1) as they also care about growth.

As in Tournemaine and Tsoukis (2011), we find it instructive to normalise welfare to what will be achieved in competitive equilibrium, when $\gamma=\lambda$. Firstly, observe that due to various imperfections, such as the status externality (explicitly modelled here), the 'learning-by-doing' production externality (implicit) and other such effects, the competitive outcome is not Pareto efficient: Welfare of both groups can be greater than what may be achieved at the competitive equilibrium. Thus, quasi-subtracting (8a, b) from the counterparts with $\gamma=\lambda$, we re-write welfare for the groups as:

$$W_0^i = (1+\rho)A(\lambda-\gamma)/\rho^2 + \log\left[A(\gamma-\lambda)(1-\alpha)+\rho(\chi^i-\alpha)\right]/\rho \tag{8a'}$$

$$W_0^j = (1+\rho)A(\lambda-\gamma)/\rho^2 + \log\left[A(\gamma-\lambda)(1-\alpha)+\rho(\chi^j-\alpha)\right]/\rho \tag{8b'}$$

3. Distributive conflict and the Utility Possibility Frontier

3.1: Partisan solutions

For future reference, it is useful to establish unilaterally the solutions that each group would select if it cared only about its own welfare; this arises from maximising (8a') and (8b'), respectively, with respect to γ. Group j, for instance would set:

$$\text{Max}_{\gamma} \qquad W_0^j = (1+\rho)g/\rho^2 + \log\left[A(\gamma-\lambda)(1-\alpha)+\rho(\chi^j-\alpha)\right]/\rho$$

implying

$$\rho\frac{\partial W_0^j}{\partial \gamma} = \frac{A(1-\alpha)}{(\gamma-\lambda)A(1-\alpha)+\rho(\chi^j-\alpha)} - \frac{(1+\rho)A}{\rho} = 0$$

This yields:

$$\gamma_j^* = \lambda + \frac{\rho(1-\alpha)/(1+\rho)-\rho(\chi^j-\alpha)}{A(1-\alpha)} = \lambda + \frac{-\rho^2/(1+\rho)+\theta\rho(\chi^i-\chi^j)/(1-\alpha)}{A} \tag{9a}$$

Symmetrically, for group i:

$$\gamma_i^* = \lambda + \frac{\rho(1-\alpha)/(1+\rho)-\rho(\chi^i-\alpha)}{A(1-\alpha)} = \lambda + \frac{-\rho^2/(1+\rho)+(1-\theta)\rho(\chi^i-\chi^j)/(1-\alpha)}{A} \tag{9b}$$

We note that $\gamma_j^* > \gamma_i^*$, the poorer (less capital-endowed) group prefers a higher labour share as it is more dependent on labour income. As alluded to above, neither group would desire extremes of the labour share; in fact, under mild assumptions, both partisan optimums are greater than λ, so that we have:

$$\gamma_j^* > \gamma_i^* > \lambda > 0 \tag{10}$$

We also note that $\frac{\partial W_0^i}{\partial \gamma} \lessgtr 0$ as $\gamma \gtrless \gamma_i^*$ and $\frac{\partial W_0^j}{\partial \gamma} \lessgtr 0$. Therefore, both derivatives will be positive for $\gamma < \gamma_i^*$ and negative for $\gamma > \gamma_j^*$, while the signs will be mixed for $\gamma_i^* < \gamma < \gamma_j^*$. It is straightforward to argue then that no criterion will yield an optimal labour share outside of the region $[\gamma_\iota^*, \gamma_j^*]$. As a result, we can restrict attention to this range of labour shares. In terms of Figure 1 below, this implies restricting attention to the region northeast of the dotted lines.

3.2: The Utility Possibility Frontier (UPF)

In the interesting region $\gamma_\iota^* < \gamma < \gamma_j^*$, as the labour share varies, there is a trade-off between between the welfares of the two groups – the UPF. This is the downward-sloping concave line of Figure 1. Its slope is:

$$\frac{dW_0^i}{dW_0^j} = \frac{-(1+\rho)/\rho^2 + \dfrac{A(1-\alpha)}{\left[A(\gamma-\lambda)(1-\alpha)+\rho(\chi^i-\alpha)\right]\rho}}{-(1+\rho)/\rho^2 + \dfrac{A(1-\alpha)}{\left[A(\gamma-\lambda)(1-\alpha)+\rho(\chi^j-\alpha)\right]\rho}} < 0 \tag{11}$$

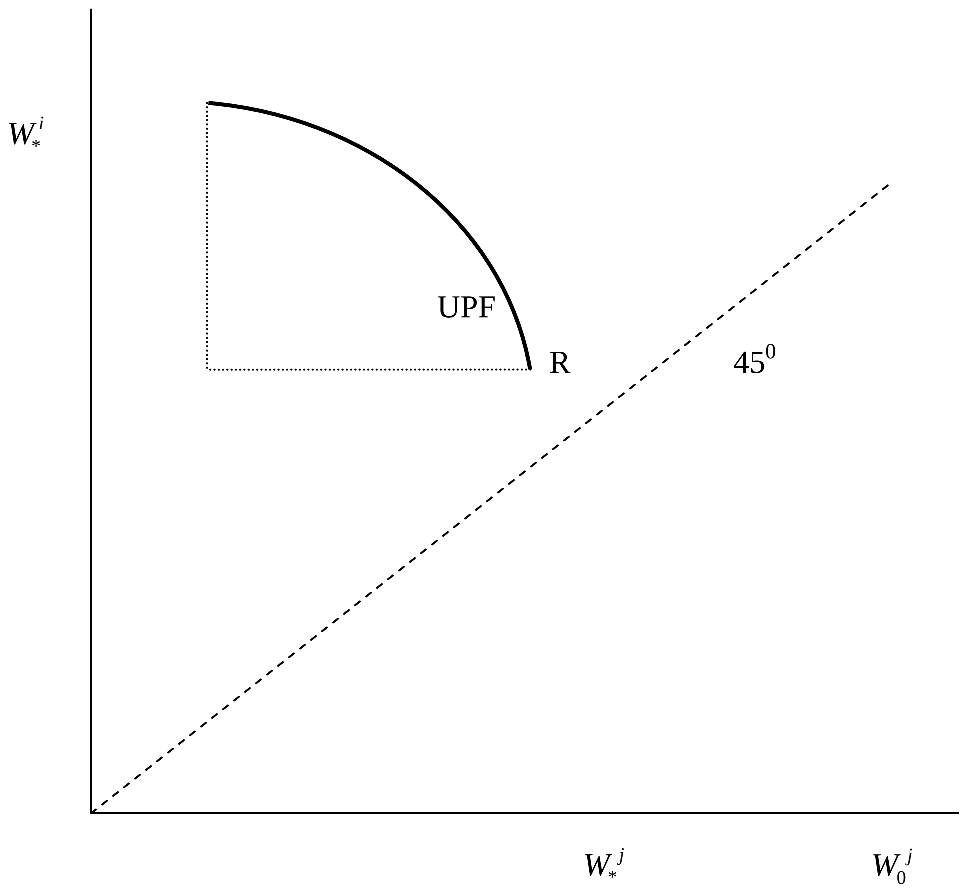
To see the negative sign, note that (11) may be re-expressed as:

$$\frac{dW_0^i}{dW_0^j} = \frac{(1+\rho)/\rho - \dfrac{1}{\left[(\gamma-\gamma_i^*)+\rho/(1+\rho)A\right]}}{(1+\rho)/\rho - \dfrac{1}{\left[(\gamma-\gamma_j^*)+\rho/(1+\rho)A\right]}} < 0 \tag{11'}$$

The negative sign follows from comparison with (9a, b).

Figure 1 depicts the UPF over the range in which conflict arises, i.e. $\gamma_\iota^* < \gamma < \gamma_j^*$. W_*^j is the maximised utility of group j with the partisan labour share ($\gamma = \gamma_j^*$) and likewise for i. The concave locus traces the effects on relative utilities as γ changes; from a low of γ_ι^* at the upper end to a high of γ_j^* at its lowest. The curve is concave as the numerator increases in absolute value as γ increases (as we are climbing down) while the denominator declines, so the curve becomes steeper at its lower end. The range of conflict is above the 45^0, a reflection of the fact that the welfare of group I (rich) is higher than that of group j (poor) – by definition. The graph reflects the fact that even with a labour share that suits their partisan preferences, the poor will not be as well off as the rich – but at point R, corresponding to $\gamma = \gamma_j^*$, they will have closed the welfare gap as much as possible.

Figure 1: The Utility Possibility Frontier



The main question is what point on the UPF is picked up. We present and compare two positive and two normative criteria below. These points are codified in Proposition 1:

Proposition 1: The UPF and conflict over factor shares

In the range $\gamma_i^* < \gamma < \gamma_j^*$, we have $\dfrac{dW_0^i(\gamma)}{dW_0^j(\gamma)} < 0$: There is conflict between the groups over the labour (equivalently: capital) share.

4. Two positive outcomes and two normative criteria

4.1: Two positive outcomes: The median voter and the 'contractarian' outcome

Strictly speaking, the median voter analysis (Downs, 1957) is trivial here. As there are two classes of identical individuals, the more numerous class would always win the political contest; since our maintained assumption is that this is the 'workers', the median (working-class) voter would always select $\gamma = \gamma_j^*$. But, *in the spirit* of the median-voter analysis, we might argue that the political competition would deliver a weighted-average of the two partisan choices, γ_i^* and γ_j^*:

$$\gamma^{MV} = \theta\gamma_i^* + (1-\theta)\gamma_j^* \tag{12}$$

The median voter outcome is denoted 'MV' in Figure 2.

The 'contractarian' outcome is in fact partly a positive and partly a normative criterion (see Rawls, 1971); we choose to emphasise the former interpretation here. Accordingly, this outcome is given by the 'social contract' as determined by a multi-faceted bargain between the two groups, when the bargain encompasses straightforward politics, but also cultural and ideological influence (e.g. through the media, education, etc.), ability to fund campaigns, 'corporatist' organisation (trade unions and employer organisations), laws and regulations, etc. Analytically, it is delivered by the Nash solution: maximisation of the product of welfares as improvements over the 'outside option', namely the competitive outcome. Accordingly, it is given by:

$$\text{Max}_\gamma \quad (W_0^i)^{\Theta}(W_0^j)^{1-\Theta} \tag{13}$$

where $0<\Theta\equiv\varphi\theta<1$: This assumes that the bargaining power of each group is equal to their population share times the strength of the factors mentioned above: degree of organisation and strength of the 'capitalist' political party(ies) and employers' union, and all features that favour 'capitalists' such as labour market institutions and laws, product market regulation and degree of competition/monopoly power of firms, etc. The capitalists' (i) relative organisational power is φ, so their overall power is Θ; by implication, the workers' power is $1-\Theta$ (rather than the more correct but also more cumbersome $(1-\varphi)(1-\theta)$).

The first-order condition implies:

$$\Theta \frac{\partial W_0^i}{\partial \gamma}\frac{1}{W_0^i} + (1-\Theta)\frac{\partial W_0^j}{\partial \gamma}\frac{1}{W_0^j} = 0 \tag{14}$$

The slope of the indifference curve will be:

$$\frac{dW_0^i}{dW_0^j} = -\frac{(1-\Theta)W_0^i}{\Theta W_0^j} \qquad (14')$$

This is a convex line. The contractarian outcome is denoted ‘C’ in Figure 2.

4.2: Two normative criteria: ‘Rawlsian’ justice and the utilitarian outcome

Rawls’s (1971) celebrated maximin criterion suggests that, behind the ‘veil of ignorance’, all individuals agree to adopt policies that maximise the welfare of the poorer class (j), as no individual knows what class they belong to. This is tantamount to extreme risk aversion in institutional design. Therefore, the criterion entails: Max $_\gamma$ W_0^j and hence, the Rawlsian outcome (indicated by R in Figures 1 and 2) coincides with the partisan choice of group j (9a):

$$\gamma^R = \gamma_j^* \qquad (15)$$

Under the utilitarian outcome, a weighted average of welfares is maximised, with the weights being the group sizes. This notion of the ‘common good’ may be rationalised as being pursued by a benevolent dictator; it may be also envisaged as institutional design behind a Rawlsian ‘veil of ignorance’ when in fact risk neutrality characterising every individual’s preferences. The criterion entails: Max$_\gamma$ $\theta W_0^i + (1-\theta) W_0^j$, which requires:

$$\theta \frac{\partial W_0^i}{\partial \gamma} + (1-\theta)\frac{\partial W_0^j}{\partial \gamma} = 0 \qquad (16)$$

Therefore the indifference curve is a straight line with slope:

$$\frac{dW_0^i}{dW_0^j} = -\frac{(1-\theta)}{\theta} \qquad (16')$$

The utilitarian outcome is indicated as ‘U’ in Figure 2.

4.3: Comparison

The first comparison will be between the median voter and utilitarian outcomes. Note that the former can be re-expressed, from (12), as:

$$\frac{\gamma^{MV}-\gamma_i^*}{\gamma_j^*-\gamma^{MV}}=\frac{1-\theta}{\theta} \tag{12'}$$

To compare this with the utilitarian outcome, note that differentiating (8a', 8b'), the utilitarian indifference curves (16') may be re-expressed as:

$$\frac{dW_0^i}{dW_0^j}=\frac{\partial W_0^i/\partial\gamma}{\partial W_0^j/\partial\gamma}=\frac{-(1+\rho)/\rho^2+\dfrac{A(1-\alpha)}{\left[A(\gamma^U-\lambda)(1-\alpha)+\rho(\chi^i-\alpha)\right]\rho}}{-(1+\rho)/\rho^2+\dfrac{A(1-\alpha)}{\left[A(\gamma^U-\lambda)(1-\alpha)+\rho(\chi^j-\alpha)\right]\rho}}=-\frac{1-\theta}{\theta}$$

Using (9a, b), this becomes:

$$\frac{dW_0^i}{dW_0^j}=\frac{(1+\rho)/\rho-\dfrac{1}{\left[(\gamma^U-\gamma_*^i)+\rho/(1+\rho)A\right]}}{(1+\rho)/\rho-\dfrac{1}{\left[(\gamma^U-\gamma_*^j)+\rho/(1+\rho)A\right]}}=-\frac{1-\theta}{\theta}$$

Or, after a couple more omitted tedious lines:

$$\frac{\gamma^U-\gamma_*^i}{\gamma_*^j-\gamma^U}=\frac{1-\theta}{\theta}\frac{\left[\gamma^U-\gamma_*^i+\rho/A(1+\rho)\right]}{\left[\gamma^U-\gamma_*^j+\rho/A(1+\rho)\right]}\equiv\frac{1-\theta}{\theta}\Gamma(\gamma^U) \tag{16''}$$

Noting the properties of Γ(.), namely $\Gamma(\gamma^U)>1$, $\Gamma'(\gamma^U)<0$ and

$$\Gamma(\gamma_*^i)=\frac{1-\theta}{\theta}\frac{\rho/A(1+\rho)}{\gamma_*^i-\gamma_*^j+\rho/A(1+\rho)}>\Gamma(\gamma_*^j)=\frac{1-\theta}{\theta}\frac{\gamma_*^j-\gamma_*^i+\rho/A(1+\rho)}{\rho/A(1+\rho)}>1,$$

comparing (12') with (16''), we readily have:

$$\frac{\gamma^U - \gamma_*^i}{\gamma_*^j - \gamma^U} > \frac{\gamma^{MV} - \gamma_*^i}{\gamma_*^j - \gamma^{MV}} \tag{17}$$

In words, the labour share of the utilitarian outcome is higher than the median-voter outcome; point U is to the right of MV in Figure 2.

Comparing the slopes of the 'contractarian' indifference curve, (14'), with that of the utilitarian one, (16'), we see that the former is flatter (steeper) if:

$$\left| -\frac{(1-\Theta)W_0^i}{\Theta W_0^j} \right| < (>) \left| -\frac{(1-\theta)}{\theta} \right| \tag{18}$$

Or, recalling $\Theta \equiv \varphi\theta$:

$$\frac{\varphi - \varphi\theta}{1-\varphi\theta} > (<) \frac{W_0^i}{W_0^j} \tag{18'}$$

To think through this condition, it helps to consider separately the cases where organisational power is tipped towards capitalists (workers), $\varphi > (<) 1$, respectively. In the former case, both ratios in (18') will be greater than one; the LHS will be higher if the effect of organisational power is not outweighed by the diminishing marginal utility of the capitalists; otherwise, the LHS is less than the RHS. If workers are more influential ($\varphi<1$), the LHS above will be less than unity while the RHS higher, so that LHS<RHS is immediately established. Thus, for a sufficiently high φ, the 'contractarian' indifference curve will be flatter (steeper) than the utilitarian and point C in Figure 2 will be on the left (right) of point U.

There remains the comparison between the median voter and the contractarian outcomes when the latter involves a lower labour share than utilitarian one. Taking the contractarian condition (14') and proceeding in the same way as with the utilitarian in the run-up to (16''), we get:

$$\frac{\gamma^C - \gamma_*^i}{\gamma_*^j - \gamma^C} = \frac{1-\Theta}{\Theta}\frac{W_0^i}{W_0^j}\Gamma(\gamma^C) \tag{14''}$$

Combination with (12') yields:

$$\frac{\gamma^C - \gamma_*^i}{\gamma_*^j - \gamma^C} = \frac{\gamma^{MV} - \gamma_i^*}{\gamma_j^* - \gamma^{MV}} \frac{(1-\Theta)\theta}{\Theta(1-\theta)} \frac{W_0^i}{W_0^j} \Gamma(\gamma^C) \tag{19}$$

Recalling that $\Theta \equiv \varphi\theta$, we readily see that conditions (18) and (18') strengthened to:

$$\frac{(1-\Theta)W_0^i}{\Theta W_0^j} \Gamma(\gamma^C) > \frac{(1-\theta)}{\theta} \tag{20}$$

or

$$\frac{\varphi - \varphi\theta}{1-\varphi\theta} > \frac{W_0^i}{W_0^j} \Gamma(\gamma^C) \tag{20'}$$

applied to (19) guarantee (sufficient) that: $\gamma^C < \gamma^{MV}$. In other words, with extreme capitalist power and influence, the labour share of the contractarian outcome will be less than that of the median voter. We bring all these results together in Proposition 2:

Proposition 2: Relation between labour shares in different outcomes or criteria:

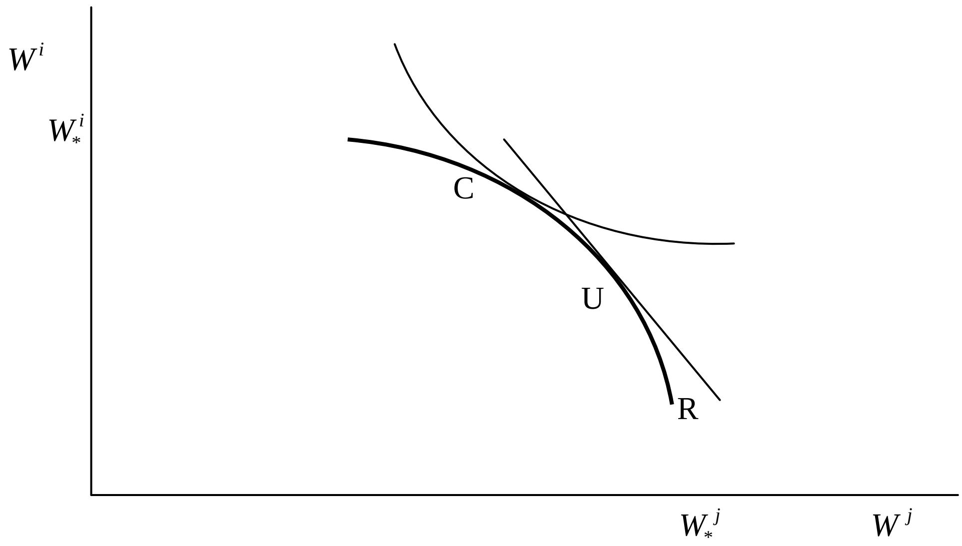
a) $\gamma^R > \gamma^U > \gamma^{MV}$

b) If the 'capitalists' influence is sufficiently high (low) in the sense that $\frac{\varphi - \varphi\theta}{1-\varphi\theta} > (<) \frac{W_0^i}{W_0^j}$, then $\gamma^U > (<) \gamma^C$.

c) If capitalist influence is very strong in the sense that $\frac{\varphi - \varphi\theta}{1-\varphi\theta} > \frac{W_0^i}{W_0^j} \Gamma(\gamma^C)$, then $\gamma^C < \gamma^{MV}$.

The corollary to Proposition 2 is that under a rather realistic condition, a high but not too high capitalists' influence, both our positive criteria deliver lower labour shares and are therefore less egalitarian than all the normative criteria we consider. Finally, there remain the growth rates to be determined according to these criteria. These can readily be given from the growth equation (6') once the labour shares are known.

Figure 2: Comparison of different outcomes and criteria under strong 'capitalist' influence



5. Justice as minimal social friction

We finally introduce a third normative criterion and concept of justice, based on minimizing 'social friction', defined below. To our knowledge, this concept has not been proposed before and is therefore another contribution of this paper. 'Social friction' may be defined as the degree of conflict divided by the degree of synergy existing in society. Conflict is captured by the (group size-weighted) difference in welfare between rich and poor; while synergy is the sum of welfares (as deviations from the competitive benchmark). In other words, synergy is the common improvement in the two classes' welfare from the competitive benchmark, while conflict is the remaining differentiation. Thus, social friction may be defined as:

$$F \equiv \frac{\left(\theta W_0^i - (1-\theta)W_0^j\right)^2}{\left(\theta W_0^i + (1-\theta)W_0^j - \Delta\right)^2} \tag{21}$$

Where $\Delta \equiv (1+\rho)[A(1-\lambda)-\rho]/\rho^2 + \theta \log\left[\rho(\chi_i - \alpha)\right]/\rho + (1-\theta)\log\left[\rho(\chi_j - \alpha)\right]/\rho$ is the average of welfares at the competitive outside option. The squares are taken in order to rule out negative quantities; and the denominator is squared for symmetry. So, there are two ways to reduce social friction: One is to reduce inequality, the other to increase the average compared to the competitive benchmark. The bottomline is that more unequal policies may be tolerated if they deliver a sufficiently strong increase in the societal average.

This definition of social friction has obvious affinities with the degree of non-cooperative vs cooperative games played by social actors, as exemplified in e.g. Bowles (20XX, Ch. YY). Based on this, distributive justice may be defined as,

$$\mathrm{Min}_{\gamma}\ F,$$

i.e., as that arrangement that minimises this social friction. This concept captures both the maximisation of joint welfare implicit in the utilitarian concept and the minimisation of difference (implying maximisation of the welfare of the poor) implicit in the Rawlsian outcome. By seeking maximum social harmony, the concept also has affinities with the contractarian notion of justice implicit in the 'mutual advantage' concept. The concept strives to minimise welfare differences whilst taking into account the implications of the solution for growth and therefore for welfare improvement which is of common interest.

Setting a labour share that minimises F implies:

$$\frac{dF}{d\gamma} = 2F\,\frac{\theta xUPFxdW_0^j - (1-\theta)dW_0^j}{\theta W_0^i - (1-\theta)W_0^j} - 2F\,\frac{\theta xUPFxdW_0^j + (1-\theta)dW_0^j}{\theta W_0^i + (1-\theta)W_0^j - \Delta} = 0$$

$$\rightarrow \theta xUPFxdW_0^j - (1-\theta)dW_0^j - \left(\theta xUPFxdW_0^j + (1-\theta)dW_0^j\right)\frac{\theta W_0^i - (1-\theta)W_0^j}{\theta W_0^i + (1-\theta)W_0^j - \Delta} = 0$$

$$\rightarrow (1-f)\theta xUPFxdW_0^j - (1+f)(1-\theta)dW_0^j = 0$$

Where $UPF \equiv dW_0^i / dW_0^j < 0$ is the co-movement of the two welfares along the UPF and f≡√F. Therefore, this criterion is satisfied on a point of the UPF where the following holds:

$$dW_0^i / dW_0^j = \frac{(1+f)(1-\theta)}{(1-f)\theta} < 0$$

Since the slope of the UPF is negative as shown, this is only defined for f high enough that 1-f<0 (otherwise the problem is trivial and this criterion collapses to the Rawlsian one). Since the

Utilitarian solution involves $\frac{dW_0^i}{dW_0^j} = -\frac{(1-\theta)}{\theta}$ and since (1+f)/(1-f)<-1, we have that the 'minimal social friction' criterion gives us a point with a steeper slope along the UPF than the utilitarian outcome, therefore the labour share thus determined will be between the utilitarian and Rawlsian criteria: To put it otherwise, the 'minimal friction' criterion, indicated my F, is more egalitarian than the utilitarian criterion, even though it also pays attention to the aggregate gains and incorporates the fact that a higher labour share hurts growth and welfare across the classes; but the minimal friction criterion pays more attention that the utilitarian to the differences between classes. Thus, this criterion may be seen as a half-way house between risk neutrality (utilitarian) and extreme risk aversion (Rawlsian) in institutional design by individuals behind the 'veil of ignorance'.

Proposition 3: The labour share according to the 'minimal friction' criterion:
$\gamma^R > \gamma^F > \gamma^U$

6. Conclusions

Distributive justice is a topic that is gaining attention as inequality is rising within most advanced economies. Despite the importance of the topic there is as yet rather little integration of it into standard macroeconomic theory; this paper takes a first step in that direction. It is often argued that there is a trade-off between equity and efficiency; this implies that distributive considerations should not be assessed in isolation from their implications on the total size of the economic pie. The model adopted here, a simple AK growth model, involves production and growth and therefore is in a position to assess the productive implications of various distributive scenarios.

There is a continuum of identical firms and of agents who are identical except their heterogeneous endowment of capital. There are two classes of capital-rich ('capitalists') and capital-poor ('workers') individuals; all supply one unit of labour inelastically. To cut through an embarrassing range of possibilities, we make the plausible assumption that 'workers' are more numerous. As is standard in the AK model (and is derived, not assumed), all individuals consume their wage (equal to the labour share) plus a fraction of their capital wealth; thus, capitalists consume more and have a higher period utility. All agents are subject to a status (or 'keeping up with the Joneses') consumption externality. The key point, and the main result of the capital heterogeneity, is the conflict in preferences of the two groups concerning the fraction of income that is awarded as a labour share (and therefore taken away from the capital share). A higher labour share fuels current consumption but also harms growth as it reduces the marginal profitability of capital (as stated in the Euler equation). As the capitalists' consumption is based relative more on wealth, they do like a labour share but are relatively more keen to safeguard growth, therefore prefer a lower (but non-zero) labour share; while the workers are keen to have a higher labour share (though not unity, as they are aware of the growth implications of that). Thus, both classes have non-trivial partisan preferences over the labour share, with the workers preferring more of it. We derive a Utility Possibility Frontier that shows the trade-off and conflict between the welfares of the two groups evaluated intertemporally (so that growth is taken into account).

Thus, our contribution is to incorporate a distributive social conflict in a simple production and growth setup. Our main advance is to consider alternative positive and normative scenarios as to how the labour share could or should be determined and compare their properties. Importantly, due to various externalities, the competitive outcome is Pareto-dominated and serves as a benchmark to which the various arrangements can be compared. We consider two positive outcomes, median voter and a 'social contractarian' outcome that is determined by a multi-faceted socio-political bargain between the two groups. In normative terms, we consider Rawls's (1971) minimax criterion of maximising the welfare of the weakest, and the utilitarian outcome. We also propose a novel normative criterion which is another contribution of our paper; namely the criterion of minimising of 'social friction', defined as the welfare difference among the groups divided by aggregate welfare gains from the competitive benchmark (as a measure of social 'synergy'). Thus, both distributive and 'common-good'/productive considerations are incorporated in this criterion, which may be further argued to be a combination between the extremely distribution-minded Rawlsian criterion and the 'common good' utilitarian approach.

We show that the highest labour share is proposed by the Rawlsian criterion, which is equal to the partisan workers' preference – almost by definition. The next lower labour share is given by our proposed 'minimum social friction' criterion, followed by the utilitarian one. The lowest labour share is given by the (quasi-) median voter arrangement while the 'contractarian' arrangement is between the median voter and the utilitarian criterion under the assumption that the 'capitalists'' overall influence (social and cultural as well as political) is relatively high; when it is very strong, then the order between the median voter and the contractarian arrangement may be reversed. Thus, under reasonable assumptions, all both positive criteria we consider deliver a lower share, and are therefore less egalitarian, than all our three normative criteria.

At the interface between economics philosophy and other social disciplines, the topic of distributive justice is bound to gain more attention. Our contribution is to introduce is into the simplest possible macro/growth model that is also meaningful. Future work will examine the issue under somewhat less restrictive assumptions, e.g. introducing a flexible labour supply.

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