

Inheritance taxation in a model with intergenerational time transfers*

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Abstract

We consider a two-period overlapping generation model with rational altruism *à la* Barro, where time transfers and bequests are available to parents. Starting from a steady state where public spendings are financed through taxation on saving income, labor and consumption, we analyze a tax reform that consists in a shift of the tax burden from saving income tax towards inheritance tax, leaving the capital-labor ratio unchanged. In the standard Barro model with no time transfer and inelastic labor supply, such a policy decreases steady-state welfare. We assume the young have elastic labor supply and can receive time transfers from their parents. Then inheritance tax modifies the trade-off parents make between both kind of private transfers, and may increase steady-state welfare.

Keywords: family transfers, altruism, bequests, time transfers, inheritance tax.

JEL Classifications: H22, H24, J22.

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1 Introduction

Intergenerational family relationships have profoundly changed thanks to social protection establishment. Grandparents who were previously a heavy burden for parents, are now a financial, social and domestic support for them. Family transfers moved towards the next generation and not to the earlier one. Furthermore, improvement of living standards and life expectancy in OECD countries have generalized financial support, through inheritance and gift, to the entire population and increased the family time transfers.

Theoretical literature about intergenerational transfers within family has essentially focused on monetary transfers, and in the case of transfers from parents to children, on bequests. Nevertheless, a number of empirical studies suggest that time transfers from parents to their children are substantial and on average almost as important as monetary transfers (see for example Albertini et al. (2007) and Schoeni et al. (1997)). Some studies based on the SHARE survey¹, like Attias-Donfut et al. (2005), show that parent's time transfers to children consists mainly in childcare. According to Wolff and Attias-Donfut (2007), two-fifth of grandparents keep their grandchildren every week. A common finding is that grandparents still support parents' home production with household tasks for instance.

Thanks to the intergenerational transfers of time in the form of grandparenting, parents free up more time for working and taking care of their children. Labor supply of the heirs as well as life cycle resources are affected differently by time transfers compared to inheritances as shown by Cardia and Michel (2004). Taking into account time transfers lead parents to make a trade-off between both types of transfers. Therefore, time transfers have some macroeconomic implications other than bequest (see for example Cardia and Ng (2003) or also Belan et al. (2010)).

Despite the importance of time transfers and their macroeconomic implications, the theoretical literature about taxation has not devoted much attention to its fiscal incidence. In this paper, we intend to focus on inheritance taxation impact. Indeed, with time transfers, inheritance taxation is not only a tax on the wealth accumulated by parents for their children. It also modifies the trade-off between two ways of giving to heirs: giving time or giving money. Inheritance tax reduces incentives for leaving bequests, and reduces the relative cost of time transfers.

We consider for this purpose a two overlapping generation model with rational altruism and taking into account both types of family transfers: inheritance and time transfers. In OLG-model with altruism is usually assumed an inelastic labor supply. In our model, every household consumes a composite good that aggregate market good and home production. Agents' labor supply decisions depend on their trade-off between formal work and home production. Then, parents and grand parents both contribute to home production of parents. Furthermore, the government finances public spending using taxation on saving income, labor income and consumption.

¹The Survey of Health, Ageing and Retirement in Europe is conducted in 2004 in ten Western European countries.

The objective of this paper is to analyze, at the steady state equilibrium, the households' welfare impact of a shift from saving income tax towards inheritance tax, leaving the capital-labor ratio constant. In the standard model *à la* Barro (1974) with no time transfer and inelastic labor supply, such a policy decreases steady-state welfare. When both private transfers are positive, we find that this tax reform may increase the life cycle households' utility by affecting the grandparents' trade-off between bequests and time transfers. This trade-off relies on elasticities of substitutions between intertemporal home productions and between time devoted to home production and market good by grandparents. Three different cases arise depending on elasticities of substitutions. In each situation, the tax reform affects differently the agents' welfare. Our results show that tax reform encourages the grandparents' consumption, whereas the effect on time transfers and on parents' consumption changes with respect to the case considered.

The paper is organized as follows. In Section 2, the model is presented. Section 3 studies the steady state equilibrium with operative bequest and positive time transfers. In this section, the non-negativity constraints in the Cobb-Douglas case is analyzed. Section 4 presents the impact of tax reform without time transfer on the steady state agents' utility. In section 5, we consider the tax reform at the steady state when both transfers are positive.

2 The model

We consider a two-period overlapping generation model. Time is discrete. Population is constant and normalized to unity. It consists in one dynasty where the representative agent of generation t has one child, born in $t + 1$. We consider dynastic altruism *à la* Barro (1974) from parents to children.

2.1 Households

The representative household of generation t works during his first period of life and then retires. Labor supply when young is elastic and depends on the allocation of a one-unit time endowment between formal work and home production. In both periods, the household consumes a composite good that aggregates market good and home production. Life-cycle utility writes

$$u(f^y(c_t, T_t^y)) + v(f^o(d_{t+1}, T_{t+1}^o))$$

where u and v are increasing and strictly concave. Function $f^y(c_t, T_t^y)$, respectively $f^o(d_{t+1}, T_{t+1}^o)$, is the quantity of composite good when young, resp. when old. The former is obtained with market good expenditures c_t and time devoted to home production T_t^y . In the latter, d_{t+1} represents market good expenditures when old, while T_{t+1}^o is time spent in home production. Home production functions f^y et f^o are assumed to be linear homogeneous and concave. Marginal products are strictly positive and strictly decreasing.

Let ℓ_t denote labor supply of the young in the formal sector. Time devoted to home production when young, T_t^y , aggregates time for home production of the household and time transfer from his parent (denoted by λ_t):

$$T_t^y = 1 - \ell_t + \mu\lambda_t \quad (1)$$

where $\mu > 0$ represents the relative efficiency of time transfer of the parent. Since the parent is retired, time spent in home production when old is the fraction of the one-unit endowment that is not transferred to his child

$$T_t^o = 1 - \lambda_t \quad (2)$$

In the following, τ_t^c , τ_t^w , τ_t^x and τ_t^R are the respective period- t tax rates on consumption, wages, bequests and saving income. R_t and w_t denote the gross interest rate and the wage rate. When young, a household born in t receives after-tax wage income $(1 - \tau_t^w) w_t \ell_t$ and after-tax bequest $(1 - \tau_t^x) x_t$. These resources are allocated between consumption spendings $(1 + \tau_t^c) c_t$ and saving s_t :

$$(1 + \tau_t^c) c_t + s_t = (1 - \tau_t^w) w_t \ell_t + (1 - \tau_t^x) x_t \quad (3)$$

When old, the same household allocates after-tax saving income $(1 - \tau_{t+1}^R) R_{t+1} s_t$ between consumption spendings $(1 + \tau_{t+1}^c) d_{t+1}$ and bequest x_{t+1} :

$$(1 + \tau_{t+1}^c) d_{t+1} + x_{t+1} = (1 - \tau_{t+1}^R) R_{t+1} s_t \quad (4)$$

Following Barro (1974), households are altruistic in the sense that they enjoy utility of their children. Utility of the household born in t , U_t , depends on his consumptions in composite goods in both periods and utility of his child U_{t+1} :

$$U_t = u(f^y(c_t, T_t^y)) + v(f^o(d_{t+1}, T_{t+1}^o)) + \beta U_{t+1} \quad (5)$$

where β denotes the degree of altruism, $0 < \beta < 1$.

Using equations (1)-(4), both consumptions in market goods rewrite

$$c_t = \frac{(1 - \tau_t^w) w_t}{1 + \tau_t^c} [1 - T_t^y + \mu(1 - T_t^o)] + \frac{1 - \tau_t^x}{1 + \tau_t^c} x_t - \frac{s_t}{1 + \tau_t^c} \quad (6)$$

$$d_{t+1} = \frac{(1 - \tau_{t+1}^R) R_{t+1}}{1 + \tau_{t+1}^c} s_t - \frac{x_{t+1}}{1 + \tau_{t+1}^c} \quad (7)$$

Replacing consumptions in U_t gives household's utility as a function of s_t , x_{t+1} , T_t^y and T_{t+1}^o . The representative household maximizes U_t with respect to these four variables. For an interior solution, this leads to the following first-order conditions:

- with respect to s_t

$$-\frac{1}{1+\tau_t^c} u'_t f_{c_t}^y + \frac{(1-\tau_{t+1}^R) R_{t+1}}{1+\tau_{t+1}^c} v'_{t+1} f_{d_{t+1}}^o = 0 \quad (8)$$

where u'_t , $f_{c_t}^y$, v'_{t+1} and $f_{d_{t+1}}^o$ respectively stand for $\frac{\partial u_t}{\partial f_t^y}$, $\frac{\partial f_t^y}{\partial c_t}$, $\frac{\partial v_{t+1}}{\partial f_{t+1}^o}$ and $\frac{\partial f_{t+1}^o}{\partial d_{t+1}}$.

- with respect to T_t^y

$$-\frac{(1-\tau_t^w) w_t}{1+\tau_t^c} f_{c_t}^y + f_{T_t^y}^y = 0, \text{ if } 0 < T_t^y < 1 + \mu(1 - T_t^v) \quad (9)$$

where $f_{T_t^y}^y$ stands for $\frac{\partial f_t^y}{\partial T_t^y}$.

- with respect to x_{t+1}

$$-\frac{1}{1+\tau_{t+1}^c} v'_{t+1} f_{d_{t+1}}^o + \beta \frac{1-\tau_{t+1}^x}{1+\tau_{t+1}^c} u'_{t+1} f_{c_{t+1}}^y = 0, \text{ if } x_{t+1} > 0 \quad (10)$$

- with respect to T_{t+1}^o

$$v'_{t+1} f_{T_{t+1}^o}^o - \beta \mu \frac{(1-\tau_{t+1}^w) w_{t+1}}{1+\tau_{t+1}^c} u'_{t+1} f_{c_{t+1}}^y = 0, \text{ if } 0 < T_{t+1}^o < 1 \quad (11)$$

where $f_{T_{t+1}^o}^o$ stands for $\frac{\partial f_{t+1}^o}{\partial T_{t+1}^o}$.

All the constraints for an interior solution are not necessarily satisfied at equilibrium. The less critical one is the constraint $T_t^y < 1 + \mu(1 - T_t^o)$, which is equivalent to $\ell_t > 0$. Assuming that it is satisfied remains to consider equilibria where the production sector use labor. Two other constraints should be satisfied with small additional assumptions: $T_t^y \geq 0$ and $T_{t+1}^o \geq 0$. Time spent in home production remains positive if substitutability with market goods is not too strong.

Finally, non-negativity constraints on bequests and time transfers deserve some discussion. As shown by Weil (1987), in the standard Barro framework without time transfers, positive bequests are obtained at steady state if the steady-state capital-labor ratio in the corresponding Diamond economy is below the modified Golden-rule. With time transfers, Cardia and Michel (2004) have shown that there also exist cases where bequests are zero and time transfers are positive at steady state. Of course, both can also be positive. It depends on the relative returns that parents may expect with both kinds of transfers. Even if our framework is not exactly the same, these results still holds.

2.2 Equilibrium

The production sector consists in a representative firm that behaves competitively, and produces output with labor and capital. The production function $F(k, \ell)$ is linear homogenous and concave, and includes capital depreciation. Marginal products are strictly positive and strictly decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

$$w_t = F_L(k_t, \ell_t) \quad (12)$$

$$R_t = F_K(k_t, \ell_t) \quad (13)$$

where k_t is quantity of capital. F_L and F_K stand for the partial derivative of F with respect to labor and capital.

At equilibrium, household savings s_t split into private capital that will be used in $t + 1$ and public debt Δ_t

$$k_{t+1} + \Delta_t = s_t$$

In each period, government spendings amounts to a fraction Γ of total production. Government resources come from taxation on labor income, saving income, bequests and consumption. Then, public debt accumulates according to the following equation:

$$\Delta_t = R_t \Delta_{t-1} + \Gamma F(k_t, \ell_t) - \tau_t^w \ell_t w_t - \tau_t^x x_t - \tau_t^R R_t s_{t-1} - \tau_t^c (c_t + d_t)$$

3 Steady state with positive transfers

From now on, tax rates and public debt are assumed to be constant. Additionally, we consider steady states with operative bequests and positive time transfers. From the marginal conditions (8) and (10), the gross interest rate satisfies the modified Golden-rule, and is equal to R_M defined as

$$\beta (1 - \tau^x) (1 - \tau^R) R_M = 1 \quad (14)$$

which characterizes the capital-labor ratio $k/\ell = z_M$ and the wage rate $w_M = F_L(z_M, 1)$.

Other marginal conditions of the household problem can be rewritten as

$$\frac{v' f_d^o}{u' f_c^y} = \beta (1 - \tau^x) \equiv P^R \quad (15)$$

$$\frac{f_{T^y}^y}{f_c^y} = \frac{(1 - \tau^w) w_M}{1 + \tau^c} \equiv P^y \quad (16)$$

$$\frac{f_{T^o}^o}{f_d^o} = \mu \frac{(1 - \tau^w) w_M}{(1 - \tau^x) (1 + \tau^c)} \equiv P^o \quad (17)$$

where P^R is the relative price of market good consumed when old (d) in terms of market good consumed when young (c), P^y (resp. P^o) is the relative price of time devoted to home production in terms of market good when young (resp. when old).

With the household time constraint when young

$$\ell = 1 - T^y + \mu(1 - T^o), \quad (18)$$

the resource constraint

$$c + d = (1 - \Gamma) F(k, \ell) - k$$

can be rewritten as

$$c + d = C_M [1 - T^y + \mu(1 - T^o)] \quad (19)$$

where C_M denotes aggregate consumption per labor unit

$$C_M = (1 - \Gamma) F(z_M, 1) - z_M \quad (20)$$

Consequently, for given tax rates $(\tau^w, \tau^R, \tau^x, \tau^c)$, equations (15)-(17) and the resource constraint (19) characterize household's choice in terms of consumption in market goods, c and d , and time devoted to home production, T^y and T^o .

The household's intertemporal budget constraint (obtained by eliminating s_t from (6) and (7)) allows to compute steady-state bequest. Indeed, using the time constraint (18) and the relation between relative prices $P^R P^o = \beta \mu P^y$, one gets

$$c + P^y T^y + P^R (d + \beta^{-1} P^o T^o) = P^y (1 + \mu) + \frac{1 - \tau^x}{1 + \tau^c} (1 - \beta) x \quad (21)$$

Bequests are positive if the present value of market goods spendings $(1 + \tau^c) (c + P^R d)$ is higher than net wage income $(1 - \tau^w) w_M \ell$.

Finally, public debt is deduced from the budget constraint of the government

$$\Delta = ((1 - \tau^R) R_M - 1)^{-1} (\tau^x x + \tau^c (c + d) + [\tau^w w_M + \tau^R R_M z_M - \Gamma F(z_M, 1)] \ell)$$

One may notice that public debt results from the tax policy. For instance, if all tax rates are zero, steady-state public debt is negative, equal to $-(R_M - 1)^{-1} \Gamma F(z_M, 1) \ell$. This means that, at each period, the government uses interests on public capital to finance public spendings $\Gamma F(z_M, 1) \ell$. Of course, this is possible either if there is initial public capital that allows to finance the whole sequence of public spendings, or if the government has taxed households in order to accumulate some public capital amount to this end. In the following, we analyze situations where the whole sequence of public spendings has not been financed yet, giving rise to positive tax rates at steady state.

3.1 Non-negativity constraints in the Cobb-Douglas case

We consider Cobb-Douglas home production functions

$$f^y(c, T^y) = c^{\alpha^y} (T^y)^{1-\alpha^y} \quad \text{and} \quad f^o(d, T^o) = d^{\alpha^o} (T^o)^{1-\alpha^o} \quad (22)$$

where $0 < \alpha^y < 1$ and $0 < \alpha^o < 1$. Assuming logarithmic form for u and v , that is $u(f^y) = \ln f^y$ and $v(f^o) = \gamma \ln f^o$, life-cycle utility writes

$$\alpha^y \ln c + (1 - \alpha^y) \ln T^y + \gamma [\alpha^o \ln d + (1 - \alpha^o) \ln T^o] \quad (23)$$

where $\gamma > 0$. Marginal conditions of the household problem then rewrite

$$\frac{c}{T^y} = P^y \frac{\alpha^y}{1 - \alpha^y}, \quad \frac{d}{T^o} = P^o \frac{\alpha^o}{1 - \alpha^o} \quad \text{and} \quad \frac{T^y}{T^o} = \frac{\beta \mu}{\gamma} \frac{1 - \alpha^y}{1 - \alpha^o} \quad (24)$$

Substitutions in the resource constraint (19) leads to the following expression for T^o

$$T^o = \frac{\gamma (1 - \alpha^o) C_M (1 + \mu)}{\beta \mu P^y \alpha^y + \gamma P^o \alpha^o + [\beta (1 - \alpha^y) + \gamma (1 - \alpha^o)] \mu C_M} \quad (25)$$

that allow to compute c , d and T^y from (24). Let us also assume a Cobb-Douglas production function

$$F(k, \ell) = Ak^\alpha \ell^{1-\alpha} \quad (26)$$

with $A > 0$ and $0 < \alpha < 1$. Then

$$C_M = [1 - \Gamma - \alpha\beta(1 - \tau^x)(1 - \tau^R)] \frac{w_M}{1 - \alpha}$$

Lemma 1. *Time transfer from old to young is positive, i.e. $T^o < 1$, iff*

$$\mu > \frac{\gamma (1 - \alpha^o)}{\beta (1 - \alpha^y) + \frac{(1-\alpha)(1-\tau^w)}{(1-\tau^x)(1+\tau^c)} \frac{\beta(1-\tau^x)\alpha^y + \gamma\alpha^o}{1-\Gamma-\alpha\beta(1-\tau^x)(1-\tau^R)}} \equiv \bar{\mu}(\beta)$$

where $\bar{\mu}$ is a decreasing function of β .

Proof. The inequality is obtained from $T^o < 1$ using (25). To show that function $\bar{\mu}(\beta)$ is decreasing, notice that $1 - \Gamma > \alpha\beta(1 - \tau^x)(1 - \tau^R)$ since aggregate consumption per unit of labor C_M must be positive in equilibrium. ■

To analyze the non-negativity constraint on bequest, we compute x from the intertemporal budget

constraint (21), (24) and (25), and get the following necessary and sufficient condition for $x > 0$,

$$\Psi \left(\beta (1 - \tau^x), \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)}, \frac{1 - \tau^R}{1 - \Gamma} \right) < 0$$

where

$$\Psi (X_1, X_2, X_3) \equiv \left(\frac{1}{X_1} - 1 \right) \left(\frac{(1 - \alpha) \gamma \alpha^o}{\alpha^y + \gamma \alpha^o} X_2 - \alpha X_1 \right) + (1 - \alpha) X_2 + \alpha X_3 - 1$$

One gets the following Lemma.

Lemma 2. *The condition for positive bequests, i.e. $x > 0$, does not depend on the relative efficiency μ of the time transfer. For given tax rates, bequests are positive for values of the degree of altruism β that belongs to some subset of $(0, 1)$ iff one of the two following sets of conditions are satisfied:*

(i) *either Ψ is negative at $\beta = 1$:*

$$\Psi \left(1 - \tau^x, \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)}, \frac{1 - \tau^R}{1 - \Gamma} \right) < 0,$$

and there exists some positive threshold on β above which bequests are positive;

(ii) *or Ψ is positive at $\beta = 1$, and tax rates satisfy*

$$\frac{\gamma \alpha^o}{\alpha^y + \gamma \alpha^o} \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)} < \frac{\alpha}{1 - \alpha} (1 - \tau^x)$$

and

$$\Psi \left(\left[\frac{(1 - \alpha) \gamma \alpha^o}{\alpha (\alpha^y + \gamma \alpha^o)} \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)} \right]^{1/2}, \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)}, \frac{1 - \tau^R}{1 - \Gamma} \right) < 0.$$

In this case, bequests are positive if β belongs to some subset of the interval $(0, 1)$.

Proof. Function Ψ is infinite as X_1 tends to 0. Its derivative with respect to X_1 has the same sign as

$$\alpha (X_1)^2 - \frac{(1 - \alpha) \gamma \alpha^o}{\alpha^y + \gamma \alpha^o} X_2$$

which implies that Ψ is decreasing with respect to X_1 until some positive threshold and then increasing. This U-shape form leads to the two sets of conditions given in Lemma 2: case (i) corresponds to the situation where Ψ is negative at $\beta = 1$, while case (ii) is the situation where Ψ is positive at $\beta = 1$ and the minimum of the curve is reached for value of β that belongs to $(0, 1)$ and leads to a negative value of Ψ . ■

If one assumes $\tau^w = \tau^R = \Gamma$ and $\tau^x = \tau^c = 0$, public spendings are entirely financed by taxes on wage and savings, and public debt is zero. In this case, the condition for positive bequests is equivalent to

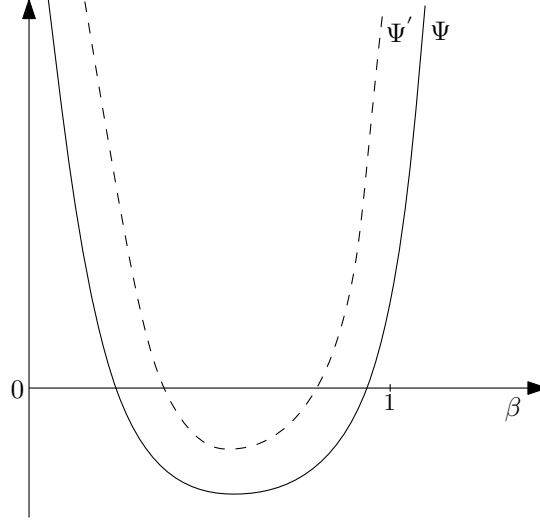
$$(1 - \beta) \left[(1 - \alpha) \frac{\gamma \alpha^o}{\beta (\alpha^y + \gamma \alpha^o)} - \alpha \right] < 0$$

Lemma 2 shows that positive bequests are then obtained for values of β higher than $\frac{1-\alpha}{\alpha} \frac{\gamma \alpha^o}{\alpha^y + \gamma \alpha^o}$, provided that this threshold is lower than one. Notice that, with $\alpha^y = \alpha^o$, the condition for positive bequests is the same as the condition obtained in the standard Barro model without home production. With home production, the threshold increases with the ratio α^o/α^y of the market good weights in the household utility function: the more households prefer to consume market goods when old rather than when young, the lower the bequest.

The Cobb-Douglas case allows to stress the possibility of an interval of values of β for operative bequests. This means that, for high values of β , desired bequests can be negative. This is the consequence of the two opposite effects involved by an increase in β . First, parents are more altruistic and therefore want to leave higher resources to their children. The second effect comes from the intertemporal budget constraint of the consumers. Indeed, at steady state, children receive $(1 - \tau^x) x$ while parents spend in present value $\frac{x}{(1 - \tau^R)R} = \beta (1 - \tau^x) x$. Thus, since $\beta < 1$, positive bequests increase the life-cycle income of the consumers, as shown by the intertemporal budget constraint (21). But, as β increases, the interest rate becomes lower as well as the household life-cycle income. This second effect vanishes when β tends to 1. By the first effect, an increase in β means that parents wish to increase utility of their children, but, with the second effect, the increase in β lowers the positive impact of bequest on life-cycle utility. With the policy $\tau^w = \tau^R = \Gamma$ and $\tau^x = \tau^c = 0$, as β tends to one, parents would like to transfer more resources to their children, but the effect of bequests on children utility vanishes. For other values of the tax instruments, the upper bound on β for positive bequests may be lower than 1. For instance, starting with $\tau^w = \tau^R = \Gamma$ and $\tau^x = \tau^c = 0$, either an increase in τ^R , or a fall in one of the other tax rates (τ^x, τ^w, τ^c) shifts Ψ upward as shown on Figure 1. As a result, bequest would be operative for a smaller range on values of β , with an upper bound lower than one.

Let us consider for instance a fall in τ^x , that is, the government chooses to subsidize bequests. Assuming β close to one, the fall in τ^x leads consumers to prefer negative desired bequests. Indeed, the fall in τ^x increases the return of bequests on life-cycle income, leading to an increase in life-cycle utility. Parents react by leaving less resources to their wealthier children.

Figure 1: Operative bequests in the Cobb-Douglas case



Note: an increase in τ^x moves the Φ curve upward, i.e. from the bold curve to the dashed curve. Bequests are operative when Φ is negative.

Remark. The condition for positive bequests ($x > 0$)

$$\Psi \left(\beta (1 - \tau^x), \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)}, \frac{1 - \tau^R}{1 - \Gamma} \right) < 0$$

can be considered as a condition on τ^x . Consider a steady-state equilibrium with positive bequests. An increase in τ^x makes the economy closer to the lower bound of the interval of values of β such that bequests are positive. As τ^x increases the positive effect of x on life-cycle utility decreases.

4 Tax reform without time transfer

To decompose the different effects of a tax reform, we first analyze a shift from saving taxation toward inheritance taxation in an economy where time transfer are inoperative. We thus leave aside the fact that inheritance taxation modifies the trade-off between both parental transfers.

We focus on steady-state utility:

$$V = u(f^y(c, T^y)) + v(f^o(d, T^o)) \quad (27)$$

At a steady-state equilibrium with positive bequests and zero time transfer, i.e. $x > 0$ and $T^o = 1$, the capital-labor ratio, the gross interest rate and the wage rate are at the modified Golden-rule levels. Market goods consumptions (c and d) and time spent to home production when young T^y

are characterized by equations (15), (16) and (19).

Let us consider a fiscal reform that consists in a shift from saving income taxation toward inheritance taxation, such that the capital-labor ratio is leaved unchanged. From equation (14), the marginal changes in τ^R and τ^x then satisfies

$$\frac{d\tau^R}{1 - \tau^R} = -\frac{d\tau^x}{1 - \tau^x}$$

Let us denote by σ^u , the (absolute value of) intertemporal elasticity of substitution between f^y and f^o

$$\frac{df^y}{f^y} - \frac{df^o}{f^o} = \sigma^u \frac{d\left(\frac{f_c^y P^R}{f_d^o}\right)}{\frac{f_c^y P^R}{f_d^o}} = \sigma^u \left(\frac{df_c^y}{f_c^y} - \frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R} \right) \quad (28)$$

We first show the tax reform in the standard Barro model with inelastic labor supply has a negative effect on welfare.

Proposition 1. *At a steady-state equilibrium with no time transfer and inelastic labor supply, consider a switch from saving income tax towards inheritance tax leaving the capital labor ratio constant. Then, first period consumption in the market good c decreases, while the second period consumption d increases. Moreover, steady-state utility V decreases.*

Proof. Differentiating steady-state life-cycle utility (27), and using marginal condition (15), dV has the same sign as

$$dc + P^R dd$$

Moreover, differentiating the resource constraint (19), one gets

$$c \frac{dc}{c} + d \frac{dd}{d} = 0 \quad (29)$$

Thus dV has the same sign as

$$(P^R - 1) \frac{dd}{d}.$$

We now need to state the sign of dd . Equation (28) rewrites as

$$\alpha^y \frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(\frac{f_{cc}^y(c, T^y)}{f_c^y(c, T^y)} \frac{d dc}{c} - \frac{f_{dd}^o(d, 1)}{f_d^o(d, 1)} \frac{d dd}{d} + \frac{dP^R}{P^R} \right)$$

since

$$\frac{df_c^y}{f_c^y} = \frac{f_{cc}^y(c, T^y)}{f_c^y(c, T^y)} \frac{d dc}{c} \quad \text{and} \quad \frac{df_d^o}{f_d^o} = \frac{f_{dd}^o(d, 1)}{f_d^o(d, 1)} \frac{d dd}{d}$$

and

$$\frac{df^y}{f^y} = \alpha^y \frac{dc}{c} \quad \text{and} \quad \frac{df^o}{f^o} = \alpha^o \frac{dd}{d}$$

where $\alpha^y = f_c^y c / f^y$ and $\alpha^o = f_d^o d / f^o$. Then, using (29), one easily checks that dd has an opposite sign to dP^R . Since the tax reform considered implies a fall in $P^R = \beta(1 - \tau^x)$, one gets $dV < 0$, which concludes the proof. ■

Extending the model to elastic labor supply when young modifies the effect of the tax reform. So doing, we introduce the following notation. From the linear homogeneity of home production functions, equation (16) allows to write the ratio c/T^y as a function of P^y : $c/T^y = \phi^y(P^y)$, where ϕ^y is increasing.

Proposition 2. *At a steady-state equilibrium with no time transfer, let us consider a switch from saving income tax towards inheritance tax leaving the capital labor ratio constant. Then, first period consumption in the market good (c) and time spent in home production (T^y) decrease, while the second period consumption (d) increases. As a result, steady-state utility increases iff*

$$P^R > \frac{\phi^y(P^y) + P^y}{\phi^y(P^y) + C_M}. \quad (30)$$

Proof. From the linear homogeneity of the home production function when young and since $dP^y = 0$, we deduce from (16) that

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df_c^y = 0$$

Then, equation (28) rewrites as

$$\frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(-\frac{f_{dd}^o(d, 1)}{f_d^o(d, 1)} \frac{d}{d} \frac{dd}{d} + \frac{dP^R}{P^R} \right)$$

Differentiating the resource constraint (19), one gets

$$(c + C_M T^y) \frac{dc}{c} = -d \frac{dd}{d} \quad (31)$$

Thus, straightforward computations lead to

$$\left[\frac{-f_{dd}^o(d, 1)}{f_d^o(d, 1)} \sigma^u + \frac{d}{c + C_M T^y} + \alpha^o \right] \frac{dd}{d} = -\sigma^u \frac{dP^R}{P^R}$$

which shows that the sign of dd is opposite to dP^R , while dc and dT^y have the same sign as dP^R . Moreover, the sign of dV is the same as

$$dc + P^y dT^y + P^R dd$$

Using (31), $dV > 0$ is equivalent to condition (30), since the tax reform considered implies a fall in

$$P^R = \beta(1 - \tau^x). \blacksquare$$

Condition (30) is not necessarily satisfied at a steady state equilibrium with operative bequests. For instance, let us consider the Cobb-Douglas case: condition (30) rewrites

$$(1 - \alpha X_1 X_3) X_1 > (1 - \alpha) X_2 \left[1 + (1 - X_1) \frac{\alpha^y}{1 - \alpha^y} \right]$$

where $X_1 = \beta(1 - \tau^x)$, $X_2 = \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)}$ and $X_3 = \frac{1 - \tau^R}{1 - \Gamma}$. In Appendix A, we state the condition for operative bequests in the Cobb-Douglas case, that can be rewritten as

$$(1 - \alpha X_1 X_3) X_1 > (1 - \alpha) X_2 \left[1 - (1 - X_1) \frac{\alpha^y}{\alpha^y + \gamma \alpha^o} \right]$$

This implies that, for steady-state equilibria with operative bequest, the marginal effect on utility of the tax reform can be positive or negative.

To interpret results in Proposition 2, notice that steady-state equilibrium values of c , d and T^y correspond to the solution of the following maximization problem

$$\max_{c, T^y, d} u(f^y(c, T^y)) + v(f^o(d, 1))$$

subject to

$$c + P^y T^y + P^R d = \Omega$$

where $\Omega \equiv P^y + \frac{1 - \tau^x}{1 + \tau^c} (1 - \beta) x$, setting x at its steady-state equilibrium value. Thus marginal change in utility writes

$$dV = u' f_c^y [dc + P^y dT^y + P^R dd]$$

Differentiating the constraint, one gets

$$dV = u' f_c^y [-d \cdot dP^R + d\Omega]$$

since P^y remains constant with the tax reform considered. Thus, the effect of the tax reform on utility can be decomposed into a price effect ($-d \cdot dP^R$) and an income effect ($d\Omega$). Indeed, the price effect is positive and consists in the fall in second-period consumption price P^R , while the income effect depends on the marginal change in the after-tax bequest $(1 - \tau^x) x$. The latter effect has the same sign as

$$-P^R + \frac{\sigma^u \left[P^R - \frac{c + P^y T^y}{c + C_M T^y} \right]}{\frac{-f_{dd}^o(d, 1) d}{f_d^o(d, 1)} \sigma^u + \frac{d}{c + C_M T^y} + \alpha^o}$$

and is ambiguous. Nevertheless, considering the Cobb-Douglas case, the preceding expression

reduces to

$$\frac{-(1 + \gamma\alpha^o)}{1 + \frac{C_M}{P^y}(1 - \alpha^y) + \frac{\gamma\alpha^o}{P^R}}$$

which is negative.

5 Tax reform when both transfers are positive

Let us now introduce time transfer by considering the tax reform at a steady state where both private transfers are positive: $x > 0$ and $T^o < 1$. Compared with the preceding section where private time transfers were zero, the marginal shift from saving taxation towards inheritance taxation now modifies the parent's trade-off between bequests and time transfers. One key element in this trade-off is the elasticity of substitution between time devoted to home production and market good when old. Let σ^o be the elasticity of substitution associated with home production technology f^o . By definition:

$$\frac{dd}{d} - \frac{dT^o}{T^o} = \sigma^o \frac{dP^o}{P^o} = -\sigma^o \frac{dP^R}{P^R} \quad (32)$$

We first state that the marginal effect on second-period consumption of market good d is positive.

Lemma 3. *At a steady-state equilibrium with positive bequests and positive time transfer, consider a marginal switch from saving income tax towards inheritance tax leaving the capital labor ratio constant. Then, marginal effect on second-period consumption d is positive and given by*

$$\frac{dd}{d} = - \left[\left(1 - \frac{d}{(1 + \mu) C_M} \right) \sigma^o + \frac{c + C_M T^y}{(1 + \mu) C_M} \alpha^o (\sigma^u - \sigma^o) \right] \frac{dP^R}{P^R} > 0 \quad (33)$$

Proof. Since P^y remains constant with the tax reform

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y}$$

Moreover (32) and linear homogeneity of the home production functions,

$$\frac{df^o}{f^o} = \frac{dd}{d} + (1 - \alpha^o) \sigma^o \frac{dP^R}{P^R}$$

where $\alpha^o = f_d^o d / f^o$. From the definition of elasticity of substitution between both composite goods σ^u (28),

$$\frac{dc}{c} - \frac{dd}{d} - (1 - \alpha^o) \sigma^o \frac{dP^R}{P^R} = \sigma^u \left(\frac{df_c^y}{f_c^y} - \frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R} \right)$$

where $df_c^y = 0$, since P^y is constant. We now need to compute $\frac{df_d^o}{f_d^o}$:

$$\frac{df_d^o}{f_d^o} = \frac{f_{dd}^o dd + f_{dT^o}^o dT^o}{f_d^o}$$

Therefore, using $T^o f_{dT^o}^o(d, T^o) = -df_{dd}^o(d, T^o)$,

$$\frac{df_d^o}{f_d^o} = \frac{-f_{dd}^o d}{f_d^o} \left(\frac{dT^o}{T^o} - \frac{dd}{d} \right) = \frac{-f_{dd}^o d}{f_d^o} \sigma^o \frac{dP^R}{P^R}$$

Moreover, since f^o is linear homogenous

$$\frac{-f_{dd}^o d}{f_d^o} \sigma^o = 1 - \alpha^o$$

Thus

$$\frac{df_d^o}{f_d^o} = (1 - \alpha^o) \frac{dP^R}{P^R}$$

Consequently, the relation between $\frac{dc}{c}$ and $\frac{dd}{d}$ is

$$\frac{dc}{c} - \frac{dd}{d} = [\sigma^u \alpha^o + (1 - \alpha^o) \sigma^o] \frac{dP^R}{P^R} \quad (34)$$

Differentiating the resource constraint (19), one gets

$$c \frac{dc}{c} + d \frac{dd}{d} + C_M \left(T^y \frac{dT^y}{T^y} + \mu T^o \frac{dT^o}{T^o} \right) = 0$$

and deduces

$$\frac{dd}{d} = - \frac{(c + C_M T^y) [\sigma^u \alpha^o + (1 - \alpha^o) \sigma^o] + C_M \mu T^o \sigma^o}{c + d + C_M (T^y + \mu T^o)} \frac{dP^R}{P^R} > 0$$

which concludes the proof. ■

Increase in the inheritance tax τ^x lowers the relative price P^R of second-period consumption in market good d . Lemma 3 shows that this results in an increase in d . Then, the sign of the effect on time devoted to home production T^o depends on the elasticity of substitution between T^o and d . Since the relative price between T^o and d ($P^o = \beta \mu P^y / P^R$) increases, high σ^o leads to a fall in home production time T^o , and so, to higher time transfer to the young. With inheritance tax, parents are incited to transfer time rather than money.

Inversely, for a low elasticity of substitution σ^o , the tax reform has a negative effect on intergenerational time transfer. Indeed, the increase in d associated with strong complementarity between d and T^o results in an increase in T^o .

The following Lemma states the effect of the elasticity of substitution σ^o .

Lemma 4. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from saving income tax towards inheritance tax leaving the capital labor ratio constant.*

- (i) *If $\sigma^o \geq \sigma^u$, marginal effect on time devoted to home production T^o in second period is negative.*
- (ii) *If $\sigma^u \geq \sigma^o$, marginal effects on first-period consumption in market good c and time devoted to home production T^y are negative.*
- (iii) *If the ratio σ^o/σ^u is close to zero, c and T^y decrease, while T^o increases.*
- (iv) *If σ^o/σ^u is close to unity, then c , T^y and T^o decrease.*
- (v) *If σ^o/σ^u tends to infinity and $C_M > \frac{\beta P^y}{PR}$, c and T^y increase, while T^o decreases.*

Proof. Marginal effects on c , T^y and T^o can be computed from (32), (33) and (34):

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \sigma^o \frac{d + C_M \mu T^o}{(1 + \mu) C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1 \right) + \frac{d}{d + C_M \mu T^o} \right] \frac{dP^R}{P^R}$$

$$\frac{dT^o}{T^o} = - \sigma^o \frac{c + C_M T^y}{(1 + \mu) C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1 \right) - \frac{d}{c + C_M T^y} \right] \frac{dP^R}{P^R}$$

which proves result (i), (ii) and (iii). To complete the proof, one needs to notice that, for σ^u close to zero, dc and dT^y are positive iff

$$\alpha^o > \frac{d}{d + C_M \mu T^o}$$

which is equivalent to $C_M > \frac{\beta P^y}{PR}$, since $\alpha^o = d / \left(d + \frac{\beta \mu P^y}{PR} T^o \right)$. ■

From the proof of the preceding Lemma, one may notice that increases in both consumptions c and d , and both home production times T^y and T^o cannot arise simultaneously, since $dc > 0$ requires $\sigma^u < \sigma^o$, which implies $dT^o < 0$. Therefore, only three cases can arise:

- $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o > 0$. This case is likely to happen when σ^o/σ^u is close to zero. Intergenerational time transfers have been reduced by the increase in the inheritance tax.
- $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u is close to one, as in the Cobb-Douglas example we have studied before. It induces a rise in intergenerational time transfers, and should result in a fall in after-tax bequests.

- $dc > 0$, $dT^y > 0$, $dd > 0$ and $dT^o < 0$. This case may happen when σ^o/σ^u tends to infinity, but, in this case, it also requires that $C_M > \frac{\beta P^y}{P^R}$ (which will be true if the policy before the reform is, for instance: $\tau^w = \tau^R = \Gamma$ and $\tau^x = \tau^c = 0$). Here, intergenerational time transfers increase with the inheritance tax.

In the following Proposition, we establish the condition for the tax reform to be welfare improving in the general case.

Proposition 3. *The marginal effect on utility dV has the same sign as*

$$P^R - \Theta - \frac{c}{d} \left[1 - \Theta + (P^y - \Theta C_M) \frac{T^y}{c} \right] \alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1 \right) \quad (35)$$

where

$$\Theta \equiv \frac{c + P^y T^y + P^R d + \mu P^y T^o}{(1 + \mu) C_M}$$

Proof. Using the marginal conditions of the household problem (15)-(17), dV has the same sign as

$$dc + P^y dT^y + P^R dd + \mu P^y dT^o \quad (36)$$

Since $dP^y = 0$, relative changes dc/c and dT^y/T^y are equal. Consequently, replacing (32) and (34) in (36) and using (33) in Lemma 3, one obtains that dV has the same sign as

$$-\sigma^o [P^R - \Theta] d \frac{dP^R}{P^R} + \alpha^o (\sigma^u - \sigma^o) [(c + P^y T^y) - \Theta (c + C_M T^y)] \frac{dP^R}{P^R}$$

which concludes the proof. ■

In the Cobb-Douglas case where $\sigma^u = \sigma^o = 1$, condition (35) reduces to

$$dV > 0 \Leftrightarrow P^R > \Theta$$

where

$$\Theta = \frac{(1 + \gamma) P^R}{P^R \alpha^y + \gamma \alpha^o + \left[1 - \alpha^y + \frac{\gamma}{\beta} (1 - \alpha^o) \right] P^R \frac{C_M}{P^y}}$$

Corollary 1. Consider the initial policy: $\tau^R = \tau^w = \Gamma$ et $\tau^x = \tau^c = 0$. There exists a threshold on β above which dV is positive iff

$$\alpha [2 - \alpha^y + \gamma (1 - \alpha^o)] > 1$$

Proof. With the tax rates $\tau^R = \tau^w = \Gamma$ et $\tau^x = \tau^c = 0$, condition (35) rewrites as

$$P(\beta) \equiv \alpha(1 - \alpha^y)\beta^2 - [1 - \alpha[\alpha^y + \gamma(1 - \alpha^o)]]\beta + 1 - \alpha[1 + \gamma(1 - \alpha^o)] < 0$$

where $P(1) = 0$. Thus:

- If $\alpha[1 + \gamma(1 - \alpha^o)] \geq 1$, then $P(\beta) < 0$ for any value of β in $(0; 1)$.
- If $\alpha[1 + \gamma(1 - \alpha^o)] < 1$, a necessary and sufficient condition for the existence of a threshold on β , above which $P(\beta) < 0$, is

$$P'(1) \equiv 2\alpha(1 - \alpha^y) - 1 + \alpha[\alpha^y + \gamma(1 - \alpha^o)] > 0$$

or equivalently

$$\alpha[2 - \alpha^y + \gamma(1 - \alpha^o)] > 1$$

This concludes the proof. ■

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A Cobb-Douglas case: Non-negativity constraint on bequests without time transfers

Without time transfers and positive bequests, marginal conditions of the household problem write

$$\frac{c}{T^y} = P^y \frac{\alpha^y}{1 - \alpha^y}, \text{ and } \frac{c}{d} = P^R \frac{\alpha^y}{\gamma \alpha^o}. \quad (37)$$

From the resource constraint (19) with $T^o = 1$, we deduce first-period consumption

$$c = \left(1 + \frac{\gamma \alpha^o}{\alpha^y P^R} + \frac{1 - \alpha^y}{\alpha^y P^y} C_M \right)^{-1} C_M$$

Then, from the intertemporal budget constraint (21), one gets

$$\frac{1 - \tau^x}{1 + \tau^c} (1 - \beta) x = c + P^y T^y + P^R d - P^y$$

Using (37), $x > 0$ is equivalent to

$$\left(1 + \frac{\gamma \alpha^o}{\alpha^y} + \frac{1 - \alpha^y}{\alpha^y} \right) c > P^y$$

or

$$\Psi_0 \left(\beta(1 - \tau^x), \frac{1 - \tau^w}{(1 - \Gamma)(1 + \tau^c)}, \frac{1 - \tau^R}{1 - \Gamma} \right) < 0$$

where

$$\begin{aligned} \Psi_0(X_1, X_2, X_3) &= \left(\frac{1}{X_1} - 1 \right) \left(\frac{(1 - \alpha) \gamma \alpha^o}{\alpha^y + \gamma \alpha^o} X_2 - \alpha X_1 \right) + (1 - \alpha)(X_2 - 1) + \alpha(X_3 - 1) X_1 \\ &= \Psi(X_1, X_2, X_3) + \alpha(X_3 - 1)(X_1 - 1) \end{aligned}$$