Stability of Cartels in Multi-market Cournot Oligopolies

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That cartel formation among producers in a Cournot oligopoly may not be sustainable has been discussed seminally by [5], coining the term *merger paradox*. Various authors have investigated how this result depends on the assumptions concerning the functional form of demand and production cost. With sufficiently convex costs, for example, [4] show that the paradox disappears.

Our focus here is on the stability of cartel formation in a multi-market setting. We apply convex production costs, creating both strategic substitutes and diseconomies of scope across markets—using the terminology of [2].[1] In a recent work, [1] study conditions for sustainable cooperation between two firms in a symmetric two-market setting. Our work differs from theirs in that we allow for cartel formation among *three* firms in *asymmetric* markets, i.e., the two markets may differ in size. This allows us to provide further insights on the degree of cooperation and the location of cartel based on the relative market size.

We choose to employ the notion of the Core and focus on long-term stability of cooperation. We demonstrate that under certain conditions that ensure relative markets' size is similar, the unique Core element of the underlying game is one of cartel formation between the same two firms on both markets.[2]

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[1] Assuming linear costs in a multi-market environment does not add to the analysis of cartel stability since there are no linkages across markets, so the multi-market oligopoly is simply the original single-market oligopoly replicated twice independently. [6] provide conditions for the existence of stable Cournot-Nash equilibrium in multi-market environments. These authors, however, do not allow for the possibility of collusive behavior.

[2] The underlying environment is akin to a partition function form game. It is more general in the sense that different coalitions may form in each market.

1 The Model

We explore a setting where three firms are selling an identical product in two separate markets. The three firms are labelled 1, 2, and 3, and compete in quantities on markets A and B. Denote by q_i and Q_i, the quantities sold by firm $i = \{1, 2, 3\}$ in markets A and B, respectively. Similarly, let p and P stand for the market prices emerging on markets A and B, respectively.

We assume that competitors' products are strategic substitutes and there are diseconomies of scope across these markets. More specifically, markets A and B are characterized by linear demand with

$$p = \alpha - \sum_{i=1}^{3} q_i \quad \text{and} \quad P = \beta - \sum_{i=1}^{3} Q_i, \tag{1}$$

where $0 < \alpha < \beta$ are demand parameters describing A's and B's market size, respectively.

Firms produce under the same quadratic cost function given by

$$C(q_i, Q_i) = \frac{1}{2} (q_i + Q_i)^2 . \tag{2}$$

Therefore, profits of firm $i = \{1, 2, 3\}$ are determined as

$$\pi_i = pq_i + PQ_i - C(q_i, Q_i). \tag{3}$$

For this linear-quadratic formulation, the second order conditions for a maximum are always satisfied if α and β are not too different. Hence, one obtains a unique interior maximum through consideration of the first order conditions.

Cartel formation. We consider that any of the three firms may choose to coordinate their market strategies and form a cartel. A *cartel* is a coalition of at least two firms. Every two-firm cartel competes à la Cournot with the outsider firm. We further assume that members of a cartel in a certain market decide their production jointly in order to maximize their joint profits subject to the quantity choice of the firm outside of the cartel (if any) on this market and the decisions made by each firm in the other market.

Even in this simplified setting, there emerge various non-trivial cartel formation scenarios regarding the degree of cooperation and its location. Exploiting the homogeneity of firms, we note that agreements are distinguishable along

Case	**Market A**	**Market B**
$\mathbf{\Omega_1}$	$\Omega^A = \{1\}, \{2\}, \{3\}$	$\Omega^B = \{1\}, \{2\}, \{3\}$
$\mathbf{\Omega_2}$	$\Omega^A = \{1, 2, 3\}$	$\Omega^B = \{1, 2, 3\}$
$\mathbf{\Omega_3}$	$\Omega^A = \{1, 2, 3\}$	$\Omega^B = \{1\}, \{2\}, \{3\}$
$\mathbf{\Omega_4}$	$\Omega^A = \{1\}, \{2\}, \{3\}$	$\Omega^B = \{1, 2, 3\}$
$\mathbf{\Omega_5}$	$\Omega^A = \{\phi(1), \phi(2)\}, \{\phi(3)\}$	$\Omega^B = \{1\}, \{2\}, \{3\}$
$\mathbf{\Omega_6}$	$\Omega^A = \{1\}, \{2\}, \{3\}$	$\Omega^B = \{\phi(1), \phi(2)\}, \{\phi(3)\}$
$\mathbf{\Omega_7}$	$\Omega^A = \{\phi(1), \phi(2)\}, \{\phi(3)\}$	$\Omega^B = \{\phi(1), \phi(2)\}, \{\phi(3)\}$
$\mathbf{\Omega_8}$	$\Omega^A = \{\phi(1), \phi(2)\}, \{\phi(3)\}$	$\Omega^B = \{\phi(1), \phi(3)\}, \{\phi(2)\}$
$\mathbf{\Omega_9}$	$\Omega^A = \{1, 2, 3\}$	$\Omega^B = \{\{\phi(1), \phi(2)\}, \{\phi(3)\}$
$\mathbf{\Omega_{10}}$	$\Omega^A = \{\phi(1), \phi(2)\}, \{\phi(3)\}$	$\Omega^B = \{1, 2, 3\}$

Table 1: Possible competitive market configurations

two dimensions—the size of the cooperating group and the market on which cooperation occurs.[3]

Now, Ω^A and Ω^B refer to a bijective function $\phi\colon \{1, 2, 3\} \to \{1, 2, 3\}$, representing the competitive structure in either market. In particular, there are three possible market structures on each market: $\Omega = \{1\}, \{2\}, \{3\}$ denotes a market where all firms compete in quantity, i.e., no cartel is formed; $\Omega = \{\phi(1), \phi(2)\}, \{\phi(3)\}$ denotes a market where exactly two firms form a cartel and the cartel competes with the outsider firm à la Cournot; and, lastly, $\Omega = \{1, 2, 3\}$ denotes the case where all market participants form a single, monopolistic cartel.

Combining these possible market structures, we arrive at ten possible two-market configurations, denoted by $\mathbf{\Omega_k}$ with $k \in \{1, \ldots, 10\}$. These configurations are collected in Table 1.

Payoff structures. We assume that a firm's decision on whether to enter into a cartel agreement with other firms, where to locate the agreed cartel, and whether to leave a cartel is based solely on that firm's profits. Straightforward though tedious computations result in all profits for cartel members and outsiders in each scenario in Table 1 under Cournot competition. These payoff structures are

[3] In the case of two markets and three firms, we arrive at 64 possible cartel agreements.

collected in Table 2.[4]

Case[5]	Cartel Member[6]	Outsider
Ω_1		$\pi_N = \frac{17\alpha^2+17\beta^2-2\alpha\beta}{288}$
Ω_2	$\pi_C^{A,B} = \frac{7\alpha^2+7\beta^2-2\alpha\beta}{96}$	
Ω_3	$\pi_C^{A} = \frac{35\beta^2+42\alpha^2-10\alpha\beta}{578}$	
Ω_4	$\pi_C^{B} = \frac{35\alpha^2+42\beta^2-10\alpha\beta}{578}$	
Ω_5	$\pi_C^{A} = \frac{3(359\alpha^2-26\alpha\beta+375\beta^2)}{19208}$	$\pi_N = \frac{418\alpha^2-128\alpha\beta+297\beta^2}{4802}$
Ω_6	$\pi_C^{B} = \frac{3(359\beta^2-26\alpha\beta+375\alpha^2)}{19208}$	$\pi_N = \frac{418\beta^2-128\alpha\beta+297\alpha^2}{4802}$
Ω_7	$\pi_C^{A,B} = \frac{485\alpha^2+2\alpha\beta+485\beta^2}{8712}$	$\pi_N = \frac{193\alpha^2-98\alpha\beta+193\beta^2}{2178}$
Ω_8	$\pi_C^{A} = \frac{110\alpha^2-79\alpha\beta+149\beta^2}{1521}$ $\pi_C^{B} = \frac{149\alpha^2-79\alpha\beta+110\beta^2}{1521}$ $\pi_C^{A,B} = \frac{5(\alpha+\beta)^2}{169}$	
Ω_9	$\pi_C^{A,B} = \frac{87\alpha^2+63\beta^2+7\alpha\beta}{1250}$ $\pi_C^{A} = \frac{99\alpha^2+126\beta^2-86\alpha\beta}{1250}$	
Ω_{10}	$\pi_C^{A,B} = \frac{63\alpha^2+87\beta^2+7\alpha\beta}{1250}$ $\pi_C^{B} = \frac{126\alpha^2+99\beta^2-86\alpha\beta}{1250}$	

Table 2: Cournot equilibrium profits in each configuration

Some configurations accommodate multiple Cournot equilibria. Here, in the interest of comparability across configurations and following the adopted convention in the literature, we focus on symmetric equilibria only. This leads to a unique payoff for each player in the game for each configuration. Due to the symmetry of the firms, we only make a distinction between the profits of cartel members and outsiders. For the sake of clarity we use a subscript C to denote the profits of a cartel member and a subscript N to denote the profits of an outsider. Furthermore,

[4]The supporting computations of the payoff structures presented in Table 2 are available from the authors upon request.

[5]Index k in Ω_k refers to the market configuration listed in Table 1.

[6]Subscript C indicates a firm that is a member of at least one cartel; subscript N identifies a firm that is not a member of cartel in any market; and superscripts A and B indicate the market location of the cartel.

we use superscripts to denote the location of the cartel in the reported equilibrium. Thus, A or B, or A, B refer to the market in which the cartel operates in the equilibrium.

2 On the stability of cartels

We note first that the underlying cartel formation game is akin to a partition function form game as the profit of firms depends on the competitive structure on both markets. Various stability notions have been proposed for partition function games in the literature. For the purpose of our analysis we adopt the notion of the *Core* as it best captures the deviation possibilities of the firms. Some modification of the standard definition of the Core, however, is required, since our cartel formation game is more general than a partition function game due to the distinction of each competitive market configuration.

Before we present a formal definition of our modified Core concept, we introduce some auxiliary notions that are used to clarify possible deviations by firms. Given configuration Ω^k in market $k \in \{A, B\}$, we denote by $i(\Omega^k)$ the coalition of which firm i is a member. We say that, given configuration Ω^k, a coalition $S \subseteq \{1, 2, 3\}$ can *establish* configuration $\widehat{\Omega}^k$ on market $k \in \{A, B\}$ if for all $i \in S\colon i(\widehat{\Omega}^k) \subseteq S$ and for all $j \notin S\colon j(\widehat{\Omega}^k) = j(\Omega^k) \setminus S$.

Definition. *A market configuration $\Omega = (\Omega^A, \Omega^B)$ is **Core stable** if there does not exist a coalition $S \subseteq \{1, 2, 3\}$ and an alternative market configuration $\widehat{\Omega} = \left(\widehat{\Omega}^A, \widehat{\Omega}^B\right)$ such that*

(i) *for any $k \in \{A, B\}$ with $\widehat{\Omega}^k \neq \Omega^k$, coalition S can establish $\widehat{\Omega}^k$ from Ω^k, and;*

(ii) *$\pi_i(\widehat{\Omega}) \geqslant \pi_i(\Omega)$ for all $i \in S$ and $\pi_j(\widehat{\Omega}) > \pi_j(\Omega)$ at least one $j \in S$.*

Our Core notion presumes that a deviating coalition amends its existing cartel arrangements in any way it deems fit and can form any arbitrary coalitional structure *among its own members.* If, by doing so, it can make one of its members strictly better off and the other members no worse off, the original market configuration is not Core stable. We remark that this definition of Core stability is stronger than the established notion of the Core for partition function form games introduced by [3]; not only because the game in question is more general than a partition function game, but also because here a blocking coalition can form an arbitrary partitioning among its members.

Proposition. *For* $\frac{379}{523}\beta < \alpha < \beta$ *there exists a unique Core stable market configuration given by*

$$\Omega_7^* = (\{\{\phi(1), \phi(2)\}, \{\phi(3)\}\}, \{\{\phi(1), \phi(2)\}, \{\phi(3)\}\}) \, .$$

Sketch of a proof: To prove this result we go through all ten cases of possible market structure and demonstrate that in all but Ω_7^* there exists a coalition of firms that can establish a profitable deviation. As the supporting computations, though tedious, are quite straightforward, we only outline the main arguments here.

The grand coalition $\{1, 2, 3\}$ can use configuration Ω_2 to profitably deviate from configurations Ω_1, Ω_3, and Ω_4. Clearly, firm $\phi(3)$ can establish configuration Ω_7 from both Ω_2 and Ω_{10}. In both these cases such a deviation will give $\phi(3)$ higher profits.

Configuration Ω_6 is blocked by firms $\phi(1)$ or $\phi(2)$ which can earn higher profits in Ω_1 and either of these firms can establish this configuration by leaving the bilateral cartel in Market B.

For configuration Ω_8 we note that firms $\phi(1)$ or $\phi(3)$ can establish configuration Ω_5 by leaving the cartel in Market B. Calculations show that if such a deviation is not profitable for one of these firms, then it must be profitable for the other.[7]

For configuration Ω_5, one can show that depending on the relative market size, either the grand coalition $\{1, 2, 3\}$ can profitably deviate by establishing full cooperation on both markets, Ω_2 when $\alpha < \beta < \frac{3883\alpha}{559}$; or firms $\phi(1)$ or $\phi(2)$ find it profitable to establish configuration Ω_1 if $\beta > \frac{3883\alpha}{559}$.[8]

Similarly, configuration Ω_9 is not Core-stable as it can be blocked by firm $\phi(1)$ (or, equivalently, by $\phi(2)$) establishing configuration Ω_3; alternatively, the firm $\phi(3)$ can establish Ω_7 where it earns higher profits.

Finally, to see that configuration Ω^* is Core stable, consider a deviation by the firm $\phi(1)$ (or $\phi(2)$) to Ω_5. Such a deviation is profitable, if the relative market size is such that $\alpha < \frac{379\beta}{523}$. Any other possible deviation can be ruled out if this inequality is not satisfied. □

[7] For neither of these firms to find it profitable to deviate to Ω_5, it must hold that for firm $\phi(1)$: $\pi_c^{A,B}(\Omega_8) > \pi_C^A(\Omega_5)$ and $\pi_c^B(\Omega_8) > \pi_N(\Omega_5)$ is satisfied for firm $\phi(3)$ where the respective expressions for profits are reported in Table 2. Straightforward algebraic manipulation shows that these two inequalities cannot hold simultaneously.

[8] The threshold for the relative market size is derived by analyzing the two inequalities $\pi_C^{A,B}(\Omega_2) - \pi_C^A(\Omega_5)$ and $\pi_N(\Omega_1) - \pi_C^A(\Omega_5)$.

We can contrast our results with that on a single market. In the single market case with linear demand and quadratic cost functions, one can show that in the Cournot model, there are no Core stable market configurations. In contrast, in the multi-market setting that we explore here, a stable cartel agreement exists provided that markets are sufficiently similar in size. Interestingly, that agreement is unique and hinges upon the existence of cartels with the same membership in both markets. This result is achieved without the need to rely on punishment strategies, entry deterrence, or intertemporal consideration discussed in the literature.

References

[1] Belleflamme, P., and F. Bloch (2008) "Sustainable Collusion in Separate Markets", *Economics Letters*, 99, 384–386.

[2] Bulow, J.I., J.D. Geanakoplos and P.D. Klemperer, (1985) "Multimarket Oligopolies: Strategic Substitutes and Complements", *The Journal of Political Economy*, 93, 488–511.

[3] Chander and Tulkens, (1997) "The Core of an Economy with Multilateral Environmental Externalities", *International Journal of Game Theory*, 26, 379–401.

[4] Perry, M.K. and R.H. Porter (1985), "Oligopoly and Incentives for Horizontal Merger", *American Economic Review*, 75, 219–227.

[5] Salant, S.W., S. Switzer and R.J. Reynolds (1983), "Losses from Horizontal Merger: The Effects of Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", *Quarterly Journal of Economics*, 98, 185–199.

[6] Zhang, A. and Y. Zhang (1996), "Stability of Cournot-Nash Equilibrium: The Multiproduct case", *Journal of Mathematical Economics*, 24, 441–462.