How Socialization and Family Structure Affect Crime*

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Abstract

We develop a two-period overlapping generation model where, in the first period, children are socialized to either an honest or dishonest trait with respect to criminality while, in the second period, when adults, they have to decide whether or not committing crime. The latter affects whether or not a single-mother or a biparental family is formed, which has a key impact on the transmission of the honest trait. We analyze the impact of the structure of the family on criminal behaviors and socialization patterns. We show that the steady-state fraction of honest individuals and crime rates depends on the interplay between the deterrence effect, since an increase in the probability of being caught reduces crime, and the social disorganization effect, since an increase in incarceration disrupts the family structure, which has a negative impact of the transmission of the honest trait. We are also able to explain the emergence of criminal gangs and the existence and persistence of neighborhoods characterized by high crime rates and a large fraction of single-mother families.

JEL Classification: K42, J15, Z13.

Keywords: Crime, social interactions, cultural transmission, social disorganization theory, gangs, location.

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1 Introduction

In the United States, nearly four in ten births are to unmarried women (Ventura, 2009) and the fraction of children under age 18 living in mother-only families has risen from 8 percent in 1960 to 23 percent in 2010 (U.S. Census Bureau 2010). Overall, 30 percent of U.S. children are estimated to spend some time living in stepfamilies (Bumpass et al., 1995). This dramatic trend toward father-absent families is similar in most countries around the world and has focused the attention of policy makers and researchers alike on the important role that fathers play in child and adolescent development.

The aim of this paper is to investigate the impact of the family structure (single-mother versus biparental families) on the criminal behavior of the children.

We develop a two-period overlapping generation model where one half of the population are male while the other half are female. Each individual lives two periods. In the first period, which corresponds to childhood, the individuals do not make economic choices but are subject to socialization. They belong to either single-mother or biparental families and can inherited (through both vertical and horizontal transmission) either the "honest" trait or the "dishonest" trait. Being "dishonest" means that a person is more incline to commit crime in the next period. At the beginning of the second period, each child becomes an adult and the parents are dead. Males and females are matched to form a household. After matching with a female, given the trait he has inherited in the first period, each male has to decide whether or not becoming a criminal. If a male is not a criminal or if he is a criminal and is not caught, then he forms a biparental family. If he is a criminal and is arrested, then he spends some time in prison and therefore his wife raises their offsprings alone as a single mother. Then, each family exerts a socialization effort in order to influence their offsprings to adopt the honest trait.

We analyze the dynamics of the fraction of honest individuals in the population. We show that, when the probability of being arrested takes intermediary values, the unique stable steady-state equilibrium is such that there are no-honest individuals in the population in the long run. When the probability of being arrested takes either low or high values, then the steady-state fraction of honest individuals will depend on initial conditions. We then analyze a policy that aims at reducing total crime. We show that the effectiveness of this policy depends on When ψ is high enough, then the opposite is true since a high p means that few people choose to be criminals (deterrence effect) and those who are criminals are

more likely to arrested (incapacitation effect). Interestingly, the effectiveness of this policy depends on the interaction between the probability of being arrested and the cost of the policy. Furthermore, we show that it also depends on the initial conditions.

We then extend our model to encompass criminal gangs. Individuals can now choose between committing crime alone, or within a gang, or not committing crime. We show that the dynamics of the fraction of honest individuals in the population changes when a criminal gang is introduced in the model. In particular, this dynamics depends on the different relative remunerations of the gang members and on the relative payoffs of the gang itself.

Finally, we endogeneize the location choices in our model. Indeed, each individual has to reside in one of two neighborhoods in the city. All individuals bid for land and we analyze the resulting urban equilibrium. We show that there are two possible urban equilibria. In the segregated equilibrium, all honest families live in one neighborhood while all the dishonest families reside in the other neighborhood. In the integrated equilibrium, half of the honest families reside in one neighborhood while the other half reside in the other neighborhood. We show how spatial segregation strengthens social disorganization and vice versa. In particular, we show that, depending on the initial conditions, we can end up in the long run with a segregated equilibrium where in one neighborhood crime rate is high, most families are dishonest and are single mothers while the opposite is true in the other neighborhood.

The rest of the paper unfolds as follows. In the next section, we discuss our contribution with respect to the related literature. In Section 3, we present our benchmark model, determine the long-run equilibrium and analyze a policy aiming at reducing total crime. In Section 4, we extend our model to introduce criminal gangs. In Section 5, we extend our model to introduce location choices. Finally, Section 6 concludes.

2 Related literature

Our paper is related to different literatures.

2.1 Social interactions and crime

There is a growing empirical literature in economics suggesting that peer effects are important in criminal activities. In the economic literature, Glaeser et al. (1996) show that the amount of social interactions is highest in petty crimes and moderate in more serious crimes. Ludwig

et al. (2001) and Kling et al. (2005) study the relocation of families from high- to low-poverty neighborhoods using data from the Moving to Opportunity (MTO) experiment. They find that this policy reduces juvenile arrests for violent offences by 30 to 50 percent, relative to a control group. Bayer et al. (2009) consider the influence that juvenile offenders serving time in the same correctional facility have on each other's subsequent criminal behavior. They also find strong evidence of learning effects in criminal activities since exposure to peers with a history of committing a particular crime increases the probability that an individual who has already committed the same type of crime recidivates that crime.

More recently, Damm and Dustmann (2014) and Corneo (2017) investigate the influence of friends on crime. The former exploit a Danish natural experiment that randomly allocates parents of young children to neighborhoods with different shares of youth criminals while the latter uses data collected among the homeless. Both find strong peer effects in crime.

Using a more structural approach, Patacchini and Zenou (2012) and Liu et al. (2012) also test peer effects in crime using the National Longitudinal Survey of Adolescent Health (AddHealth) where students in schools between grades in grades 7-12 have friendship networks and self-report different types of crimes they have committed. The authors find that, for an average group of 4 best friends, a standard deviation increase in the level of delinquent activity of each of the peers translates into a roughly 17 percent increase of a standard deviation in the individual level of activity.

From a theoretical viewpoint, Glaeser et al. (1996) were among the first to develop a crime social interaction model in which criminals are located on a circle where some of them are conformists (i.e. copy what their neighbors do) while others decide their criminal activities by themselves. They show that criminal interconnections act as a social multiplier on aggregate crime. Calvó-Armengol and Zenou (2004), Ballester et al. (2006, 2010), Patacchini and Zenou (2012) were the first to embed criminal activities in a general social network. They study the effect of the structure of the network on crime. They show that the location in the social network of each criminal not only affects her direct friends but also friends of friends of friends, etc.

In our paper, peers play an important role since they determine whether or not someone adopts the honest trait and thus whether he is more likely to commit crime and forms a biparental family. Compared to this literature, we add two other dimensions of criminal activities: the cultural transmission of crime (more exactly the honest trait that affects

crime) and the structure of the family.

2.2 Family structure and crime

The increased rates of mother-headed households are well documented (Bureau of the Census, 1994). There is substantial evidence that children growing up in single-mother families are at greater risk for behavior problems (Barber and Eccles, 1992; Dornbusch et al., 1985; Kellam et al., 1977), and for engaging in a variety of high-risk behaviors such as crime (Stern et al., 1984; Turner et al., 1991; Florsheim et al., 1998). For example, using data from the National Longitudinal Study of Adolescent Health, Cobb-Clark and Tekin (2014) find that adolescent boys engage in more delinquent behavior if there is *no* father figure in their lives. Adolescent girls' behavior is largely independent of the presence (or absence) of their fathers.

In sociology, the *social disorganization* theory aims at explaining these facts (for a recent overview, see Porter et al., 2016). This theory explains the variations in criminal offending and delinquency, across both time and space, by the differences in institutions (family, school, church, friendship, etc.). Indeed, according to the social disorganization theory, these institutions are historically responsible to the establishment of organized and cooperative relationships among groups within the local community. This organization is then linked to the bond or "sense of belonging" one might feel in regards to their community, which decreases the likelihood of their involvement in criminal or delinquent behaviors that might negatively affect that community.

In this paper, we propose a new mechanism that explains how the structure of the family affects crime. In our dynamic model, male individuals first decide whether or not to become criminal based on the benefits and costs of crime as well as their degree of honesty that they adopt from their parents and peers (cultural transmission). Then, two types of families emerge (single-mother or biparental families) depending on whether or not the father is criminal and has been arrested. This structure of the family, has, in turn, an impact on the transmission of the honesty trait to their offsprings since biparental families have more time to spend with their children than single-mother families (this is well-documented; see e.g. Florsheim et al., 1998). This, in turn, affect the honesty trait for the next generation male individuals, who will then decide whether or not becoming a criminal. And so forth. We believe that this is the first paper that explicitly models the main aspect of the *social disorganization* theory within in an economic model, which explains why the structure of

the family is a key determinant of criminal decisions.

2.3 Transmission of crime

Based on some works on anthropology and sociology (see, in particular, Boyd and Richerson, 1985 and Cavalli-Sforza and Feldman, 1981), there is a theoretical literature initiated by Bisin and Verdier (2000, 2001) arguing that the transmission of a particular trait (religion, ethnicity, social status, etc.) is the outcome of a socialization *inside* (parents) or/and *outside* the family (peers or role models). There is also large body of empirical research[1] that provides evidence of substantial intergenerational associations in criminal behavior. The key findings from this literature are that family background and parental criminality are among the strongest predictors of criminal activity, stronger even than one's own income or employment status. For a review of this literature, see Rowe and Farrington (1997), Thornberry (2009), and Hjalmarsson and Lindquist (2012).

Compared to the cultural transmission literature a la Bisin-Verdier, we have a different transmission mechanism. First, all agents agree that one trait (honesty) is better than the other (dishonesty). Second, for a child to be socialized to one trait, the parent's vertical transmission through his effort is not enough. For example, to become honest, both the parent's socialization effort must succeed *and* the role model met randomly by the child must be honest. When parents and the society send conflicting messages about honesty, then the child is matched a second time with a role model (also met randomly) and adopts her trait. This implies a different dynamic of the fraction of honest individuals in the population than in the Bisin-Verdier framework. Furthermore, our model is able to predict the empirical facts described above, therefore giving an exact mechanism of why there is a positive correlation in crime between fathers and sons. Indeed, the more criminal is the father, the more likely he will be arrested so that the son will grow up in a single-mother family, which, in turn, implies that the son is more likely to have the dishonest trait, which makes him more likely to be criminal.

[1] See, in particular, Case and Katz (1991), Williams and Sickles (2002), Duncan et al. (2005) and Hjalmarsson and Lindquist (2010, 2012, 2013).

2.4 Criminal gangs

We have seen that social interactions between criminals are an important part of criminal activities. Indeed, in his very influential theory of differential association, Sutherland (1947) locates the source of crime and delinquency in the intimate social networks of individuals. Emphasizing that criminal behavior is *learned* behavior, Sutherland (1947) argued that persons who are selectively or differentially exposed to delinquent associates are likely to acquire that trait as well.[2] In particular, one of his main propositions states that when criminal behavior is learned, the learning includes (i) techniques of committing the crime, which are sometimes very complicated, sometimes very simple, (ii) the specific direction of motives, drives, rationalization and attitudes. Interestingly, the positive correlation between self-reported delinquency and the number of delinquent friends reported by adolescents has proven to be among the strongest and one of the most consistently reported findings in the delinquency literature (for surveys, see War, 1996 and Matsueda and Anderson, 1998).

One natural way of interpreting the social connections between criminals is through a *gang* since the latter is in general viewed as a specific type of criminal network (Sarnecki, 2001). Indeed, when individuals belong to the same gang, they learn from each other. Using data from the Rochester Youth Development study, which followed 1,000 adolescents through their early adult years, Thornberry *et al.* (1993) find that once individuals become members of a gang, their rates of delinquency increase substantially compared to their behavior before entering the gang. In other words, networks of criminals or gangs amplify delinquent behaviors. In the sociological literature, this is referred to as the *social facilitation* model, where gang members are intrinsically no different from nongang members in terms of delinquency or drug use. If they do join a gang, however, the normative structure and group processes of the gang (network) are likely to bring about high rates of delinquency and drug use. Gang membership is thus viewed as a major cause of deviant behavior. This is also what is found by Thornberry *et al.* (2003).

In Section 4, we extend our model to incorporate gangs so that individuals may either commit crime by themselves or within a gang. Compared to this literature, we are able to show under which condition a gang emerges and how it affects total the crime rate in the population.

[2] Sutherland (1947) and Akers (1998) expressly argue that criminal behavior is learned from others in the same way that *all* human behavior is learned. Indeed, young people may be influenced by their peers in all categories of behavior - music, speech, dress, sports, and *delinquency*.

2.5 Residential mobility and social disorganization

It is well documented that, within cities, crime is highly concentrated in a limited number of areas. For instance, in U.S. metropolitan areas, after controlling for education, crime rates are much higher in central cities than in suburbs. Between 1985 and 1992, crime victimizations averaged 0.409 per household in central cities, while they averaged 0.306 per household in suburbs (Bearse, 1996, Figure 1).[3] More generally, U.S. *central cities* have higher crime and unemployment rates, higher population densities and larger relative black populations than their corresponding *suburban rings* (South and Crowder, 1997, Table 2). There is also strong evidence that central cities have more single-mother families than suburbs. Indeed, between 1970 and 1994, the percentage of children living in single-mother households rose from 12.8% to 30.8%. The rate of increase has been particularly sharp among *inner-city minority families* (Bureau of the Census, 1994) for whom the scarcity of employment opportunities has made it more difficult for men to fulfill their expected roles as fathers and husbands (Wilson, 1987). For example, Florsheim et al. (1998) show that inner-city boys in single-mother families are at greater risk for developing behavior problems than boys in two-parent families.

From a theoretical viewpoint, there are different papers that have explain the variation in crime within cities (Freeman et al., 1996; Glaeser and Sacerdote, 1999; Zenou, 2003; Verdier and Zenou, 2004; Decreuse et al., 2015).[4]

In Section 5, we extend our model to incorporate location choices. We give an exact mechanism extend our model to incorporate location choices between two neighborhoods. Compared to this literature, we are able to show under which condition there is a steady-state equilibrium with segregation where one neighborhood (city center) is characterized by high-levels of crime, no "honest" families and more single-mother families and another neighborhood (suburbs) where the opposite is true. In other words, we explain under which condition the emergence of an *urban ghetto* emerges.

[3] Grogger and Willis (2000, Table 2) also show that central cities are more crime-ridden than suburbs for most crimes. For instance, the mean murder rate in central cities is five times greater than that in the suburbs and for property crimes they differ by a factor of two or three.

[4] For an overview, see O'Flaherty and Sethi (2015).

3 The benchmark model

Consider a two-period overlapping generation model populated by a continuum of agents with mass equal to two. One half of the population are male while the other half are female. Each individual lives two periods. In the first period, which corresponds to childhood, the individuals do not make economic choices but are subject to socialization. They belong to either single-mother or biparental families and can inherited (through both vertical and horizontal transmission) either the "honest" trait or the "dishonest" trait. Being "dishonest" means that a person is more incline to commit crime in the next period. At the beginning of the second period, each child becomes an adult and the parents are dead. Males and females are matched to form a household. To keep the population of males and females constant, we assume that each household has two children, a boy and a girl. After matching with a female, given the trait he has inherited in the first period, each male has to decide whether or not becoming a criminal (illegal activities). For simplicity, we assume that female workers always choose legal activities. If a male is not a criminal or if he is a criminal and is not caught, then he forms a biparental family. If he is a criminal and is arrested, then he spends some time in prison and therefore his wife raises their offsprings alone as a single mother. Clearly, the structure of the family depends on the male criminal behaviors. Indeed, when the father is incarcerated, then the family turns into a single-mother family.[5] This simple mechanism implies that family disruption increases with criminality.[6] Then, each family exerts a socialization effort in order to influence their offsprings to adopt the honest trait.

Let us first analyze the decision of male individuals in the second period.

3.1 Criminal decision

Consider the male population. We assume that they can be of two types: they can either be "honest" (type h) and bear a psychological cost K when they engage in criminal activities or "dishonest" (type d) and bear no psychological cost when committing crime. Let β be the proceeds from crime, p, the probability of being arrested, σ, the cost of punishment, w

[5] The father, who is arrested, does not have to spend all his time in prison in period 2. It suffices that he spends a sufficient long time in prison during the socialization period of his children. Empirical evidence suggests that adult males spend a discontinuous time in prison, first, in different local prisons, and then, over time, in federal prisons because they tend to commit more serious crimes. See, e.g. Goffman (2014).

[6] For evidence on this issue, see Wildeman (2010), Geller et al. (2011), Geller et al. (2012) and Geller (2013).

the earnings in the legal labor market and θ an individual idiosyncratic component, which captures the individual ability of exerting criminal activities. The variable θ is *uniformly* distributed on the support $[0,1]$. An individual of type d with a given θ chooses to engage in criminal activities if and only if:

$$(1-p)\beta - p\sigma - \theta > w.$$

Therefore, θ^d, the fraction of type$-d$ individuals who engage in criminal activities, is given by:

$$\theta^d = (1-p)\beta - p\sigma - w. \tag{1}$$

Similarly, an individual of h with a given θ chooses to engage in criminal activities if and only if:

$$(1-p)\beta - p\sigma - K - \theta > w.$$

Thus, θ^h, the fraction of type$-h$ individuals who engage in criminal activities, is equal to:

$$\theta^h = (1-p)\beta - p\sigma - w - K. \tag{2}$$

We have $\theta^d > \theta^h$, which means that honest individuals have a lower probability to engage in criminal activities. We assume throughout that $\beta - w - K > 0$ so that $\theta^h > 0$ and $\theta^d > 0$ for some $p \in [0,1]$. For any $p > \frac{\beta-w-K}{\beta+\sigma} \equiv \overline{p}_1$ (resp. $p > \frac{\beta-w}{\beta+\sigma} \equiv \overline{p}_2$), no honest (dishonest) individual engages in criminal activities.

Let q_t be the fraction of honest individuals in the economy at time t. Then, $C_t = C(q_t)$, the fraction of criminal individuals at time t, is given by:

$$C(q_t) = \begin{cases} q_t\theta^h + (1-q_t)\theta^d, & \forall p \in [0, \overline{p}_1] \\ (1-q_t)\theta^d, & \forall p \in [\overline{p}_1, \overline{p}_2] \\ 0, & \forall p \geq \overline{p}_2 \end{cases} \tag{3}$$

Quite naturally, $C(q_t)$ is decreasing in q_t since the higher is the fraction of honest individuals in the population, the lower is the level of crime in the economy. Using the values of θ^d and

θ^h defined in (1) and (2), we obtain:

$$C(q_t) = \begin{cases} -q_t K + (1-p)\beta - p\sigma - w, & \forall p \in [0, \overline{p}_1] \\ (1-q_t)\left[(1-p)\beta - p\sigma - w\right], & \forall p \in [\overline{p}_1, \overline{p}_2] \\ 0, & \forall p \geq \overline{p}_2 \end{cases} \tag{4}$$

where $\overline{p}_1 = (\beta - w - K) / (\beta + \sigma)$ and $\overline{p}_2 = (\beta - w) / (\beta + \sigma)$.

3.2 Dynamics of traits and crime

As stated above there are two "types" or two "traits" in the population, namely h and d. The way individuals adopt one of these traits is modeled as follows. In our model, there are two different traits related to crime: honest or dishonest. The child becomes honest if both parents' socialization effort succeeds *and* the role model met randomly by the child is honest. Symmetrically, the child becomes dishonest if both socialization within the family fails *and* the role model is dishonest. However, if parents and the society send conflicting messages about honesty, e.g., socialization by parents succeeds but the model met is dishonest, then the child is matched a second time with a role model (also met randomly) and adopts her trait. This is different from the Bisin-Verdier framework where the parents by themselves were enough to transmit a trait. Here, if the parent transmits one trait while the child meets a role with another trait, then it is the second meeting with a role model that will determine which trait will be adopted. Remember that there are two types of families, $k = S, B$, where $k = S$ stands for a single-mother family and $k = B$ stands for a biparental family. Therefore, the probability P_t^{hk} for a child of a parent from a type$-k$ family ($k = S, B$) to become of type h (i.e. honest) at time t is given by:

$$\begin{aligned} P_t^{hk} &= 1 - P_t^{dk} \\ &= \tau_t^k q_t + \tau_t^k (1-q_t) q_t + (1-\tau_t^k) q_t^2 \\ &= q_t \left[2\tau_t^k (1-q_t) + q_t\right]. \end{aligned} \tag{5}$$

where τ_t^k is the socialization effort of a type$-k$ parent and P_t^{dk} is the probability for a child of a parent from a type$-k$ family ($k = S, B$) to become of type d (i.e. dishonest) at time t. Indeed, a child can become of type h if either (i) the parent of type k is successful in transmitting the honesty trait (which occurs with a probability equal to the socialization

effort τ_t^k) *and* the child (randomly) meets in the population a role model who is honest (which occurs with a probability q_t) or (ii) the parent of type k is successful in transmitting the honesty trait and the child meets first a dishonest role model, which occurs with a probability $1 - q_t$, (conflicting messages about honesty) but then the child is matched a second time with an honest role model or (iii) the parent of type k is unsuccessful in transmitting the honesty trait (which occurs with probability $1 - \tau_t^k$) and the child meets first an honest role model (conflicting messages about honesty) and then meets again an honest role model.

Let us now model the parents' choice. All parents (of both types) value the honesty trait for their children. Let V^h (resp. V^d) be the gain of having a child of type h (resp. d) with $V^h > V^d$, $V^i < 1$, $\forall i \in \{h, d\}$. Observe that we do not have the superscript k in V^i because we assume that $V^{hS} = V^{hB} = V^h$ and $V^{dB} = V^{hS} = V^d$. In other words, the utility (disutility) of having a child of type h (type d) is the same for both types of parents. This assumption is made for simplicity and does not affect any of our results. Motivated by empirical evidence (see e.g. Wildeman, 2010; Geller, 2013), we assume that the structure of the family has an impact on children socialization into values that influence criminal behaviors. In particular, single-mother families bear a higher socialization cost than biparental families because, for example, of time constraints. Let $c(\tau_t^S) = c^S \left(\tau_t^S\right)^2 /2$ be the individual socialization cost of a *single-mother* family exerting effort τ_t^S and $c(\tau_t^B) = c^B \left(\tau_t^B\right)^2 /2$ be the individual socialization cost of a *biparental* family exerting effort τ_t^B. We assume that $c^S > c^B > 1$.

A parent from a type$-k$ family chooses his/her socialization effort τ_t^k at time t to maximize

$$P_t^{hk} V^h + P_t^{dk} V^d - c^k \frac{\left(\tau_t^k\right)^2}{2}$$

Using (5), it is easily verified that the optimal socialization effort of a type-k family is given by:

$$\tau_t^k = 2q_t(1 - q_t)\Delta^k, \tag{6}$$

where $\Delta^k = \left(V^h - V^d\right)/c^k$. Observe that

$$\frac{\partial \tau_t^k}{\partial q_t} \gtreqqless 0 \Leftrightarrow q_t \lesseqqgtr \frac{1}{2} \tag{7}$$

This implies that, if, at time t, the majority of the people in the male population are

dishonest (honest), then an increase in q_t, the fraction of honest individuals, leads to an increase (a decrease) in the parent's socialization effort. In other words, when $q_t < 1/2$, socialization inside (parents) and outside the family (peers) are *cultural complements* while, when $q_t > 1/2$, they are *cultural substitutes*. Indeed, when $q_t < 1/2$ ($q_t > 1/2$), parents have more (less) incentive to socialize their children to the honest trait, the more widely dominant is this trait in the population. This is due to the fact that, contrary to the standard Bisin-Verdier cultural transmission model where *either* the parent's effort *or* the peers' influence is enough for the successful transmission of a trait, here, we assume that we need *both* vertical and horizontal transmissions for a trait to be successfully transmitted (see (5)). When the two send contradictory messages, then the individual needs to meet other role models to determine which trait he will adopt. Therefore, when most people in the population are dishonest, the parent increases his/her effort with q_t while the opposite is true when $q_t > 1/2$.

The dynamics of the honesty trait h is then described by the following equation:

$$q_{t+1} = \underbrace{[1 - C(q_t)]}_{\text{fraction of non-criminals}} P_t^{hB} + \underbrace{(1-p)\, C(q_t)}_{\text{fraction of non-caught criminals}} P_t^{hB} + \underbrace{pC(q_t)}_{\text{fraction of caught criminals}} P_t^{hS}$$

Indeed, there is a mass 1 of males in the population. Among them, $1 - C(q_t)$ are not criminals and $C(q_t)$ are criminals. Among the mass (or fraction) of criminals, $(1-p)\, C(q_t)$ of them are not arrested and $pC(q_t)$ are arrested. As a result, among the mass 1 of males, $1 - C(q_t) + (1-p)\, C(q_t)$ will form biparental families while $pC(q_t)$ will form single-mother families. This dynamic equation can thus be written as:

$$q_{t+1} = [1 - pC(q_t)]\, P_t^{hB} + pC(q_t) P_t^{hS}$$

where $C(q_t)$ is given by (4). Using (5) and (6) and denoting $\Delta q_t \equiv q_{t+1} - q_t$, we obtain:

$$\Delta q_t = q_t(1-q_t)\left[4q_t\,(1-q_t)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] - 1\right] \tag{8}$$

3.3 Steady-state equilibrium

Assumption 1: (i) $\Delta^B > 1$ and (ii)

$$\frac{(\beta+\sigma)\left[\Delta^B - \frac{(\beta-w-K)^2\left(\Delta^B-\Delta^S\right)}{4(\beta+\sigma)}\right]^2}{K\left(\beta-w-K\right)\left(\Delta^B-\Delta^S\right)} < \frac{1}{8}$$

In (i), we assume that $V^h - V^d > c^B$, which means that, for biparental families, the net benefit of having an honest child is larger than the unit effort cost of socialization. An implication of (i) is that, when p is low, so that the prevalence of single-mother families is low, socialization within family is effective enough to allow the survival of the honesty trait within the community (at least when the fraction of honest agents is initially high). Assumption (ii) implies that, when the rate of single-parent families is high, socialization within families is not sufficiently effective to maintain the honesty trait within the community.

Proposition 1 *Suppose that Assumption 1 holds.*

(i) *If* $p \in \left]\widehat{p}_1, \widehat{p}_2\right[$, *for any* $q_0 \in [0,1]$, *then, the sequence* q_t *converges to* $q^* = 0$.

(ii) *If* $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \overline{p}_2]$, *then, for any* $q_0 \in \left[0, \underline{q}\right[$, *the sequence* q_t *converges to* $q^* = 0$ *while, for any* $q_0 \in \left[\underline{q}, 1\right]$, *the sequence* q_t *converges to* $q^* = \overline{q}$, *where* $\overline{q} \in \left]0,1\right[$.

Proposition 1 totally characterizes the steady-state equilibrium q^*, the long-run fraction of individuals with the honest trait. This proposition puts forward the interplay between the *deterrence effect* since an increase in p reduces crime by decreasing the expected returns from criminal activities and the *social disorganization effect* since an increase in incarceration disrupts the family structure, which has a negative impact of the transmission of the honest trait.

Indeed, consider part (i) of Proposition 1. When p, the probability of being arrested, takes intermediary values, the unique stable steady-state equilibrium is such that there are no-honest individuals in the population ($q^* = 0$). Figure 1 illustrates this dynamics and shows that the other equilibrium for which $q^* = 1$ is unstable. Indeed, when $q^* = 1$, all males are honest and crime is quite low. However, because p is relatively high, the men who are committing crime will end up in prison and, therefore, many single-mother families will be formed. This, with the fact that parents will not put too much effort in socializing their

offsprings to the honest trait (see (7)), will result in the fact that the fraction of honest people in the population will be reduced so that $q^* = 1$ is unstable. Consider now the equilibrium $q^* = 0$. In that case, no male is honest and thus most of them commit crime and, because p is relatively high, many single-mother families will be formed, which reinforce the fact that the dishonest trait will be adopted.

[*Insert Figure* 1 *here*]

Consider now part (ii) of Proposition 1 where p can take small and high values. We show that the steady-state value of q^* will depend on initial conditions as illustrated in Figure 2. Indeed, if, at time $t = 0$, the fraction of honest individuals is small, then the unique stable equilibrium is such that $q^* = 0$. The argument is similar to the one described above for case (i) of Proposition 1. When q starts at a high value, then the economy converges to an interior solution $q^* = \overline{q}$. Indeed, when $q_t < \overline{q}$, then if one slightly increases q_t, less people are committing crime and more biparental families are formed, which means that the honest trait is more likely to emerge. This increases q_t up to $q^* = \overline{q}$. This result is also due to the fact that the probability of being arrested is both high and low so there is a "neutral" effect of p on the structure of families. The same type of reasoning applies when we start at $q_t > \overline{q}$.

More generally, this proposition highlights the importance of the interaction between the deterrence effect of p and its impact on the structure of the family. When p is low, there will be few single-mother families so the transmission of the honest is more likely to occur. When p is high, there are two effects. On the one hand, few individuals will decide to commit crime (deterrence effect) but, on the other hand, those who decide to commit crime are more likely to be arrested. The impact on the structure of the family is therefore unclear and thus the transmission of the honest trait will depend on the initial fraction of honest individuals in the population.

[*Insert Figure* 2 *here*]

Proposition 2 *Suppose that Assumption 1 holds and that* $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \overline{p}_2]$. *Then, an increase in* w *and* σ *or a decrease in* β, *increases* $\overline{q}$, *the long-run fraction of honest individuals in the population.*

Proposition 2 provides some comparative statics results of the interior equilibrium $\overline{q}$. We find that, when w, the outside opportunity in the legal market, or the cost of the punish-

ment σ, increases or β, the proceeds from crime decreases, then the steady-state fraction of honest individuals in the population increases. Indeed, when there are less incentives to commit crime, few individuals become criminals and, thus, more families are biparental. This facilitates the transmission of the honest trait because it is less costly for these families to exert socialization effort. At any moment of time, this increases q_t, which reinforces the transmission of the honest trait since both parents and peers are more likely to influence the child.

3.4 Public enforcement

We have seen how the steady-state fraction of honest individuals is determined in equilibrium. In this section, we analyze a policy that aims at reducing crime. Suppose that crime generates a social cost equal to the steady-state crime rate $C(q^*) = q^*\theta^h + (1-q^*)\theta^d$. The government can reduce $C(q^*)$ by choosing the optimal p but then bears a cost of $\Psi(p) = \frac{1}{2\psi}p^2$. For example, choosing p could be choosing the number of policemen or improving technology of detecting crime or any other policy that increases the probability of being caught for the criminals. The government solves the following program:

$$\min_p \Gamma(q,p) \text{ s.t. } q_{t+1} = q_t = q^* \tag{9}$$

where

$$\Gamma(q^*,p) \equiv q^*\theta^h + (1-q^*)\theta^d + \frac{p^2}{2\psi}$$

Proposition 3 *Consider a planner whose aim is to choose an optimal level of p, the probability of arresting criminals, in order to reduce the crime rate in the economy.*

(*i*) *When the (inverse) cost ψ of this policy is high, the optimal policy is to set a low level of p.*

(*ii*) *When ψ is low, the optimal policy is to set a high level of p.*

(*iii*) *When ψ takes intermediary values, the optimal policy is to set an intermediary level of p.*

The proof of this proposition is cumbersome because the planner solves the minimization problem (9), where p has a direct impact on the objective function but also an indirect one

through q^*, the long run fraction of honest individuals, which is determined in a non-trivial way by Proposition 1. When ψ is low enough, which means that the cost of the policy is high enough, the planner sets a low level of p because she wants to see more biparental families and more individuals with the honest trait. When ψ is high enough, then the opposite is true since a high p means that few people choose to be criminals (deterrence effect) and those who are criminals are more likely to arrested (incapacitation effect). Interestingly, the effectiveness of this policy depends on the initial conditions q_0. We show that, if q_0 is small enough, then the optimal policy is to set a low p if ψ is low enough and a high p if ψ is high enough. On the contrary, if q_0 is high enough, then the optimal policy is to set intermediary values of p depending on ψ.

Figure 3 illustrates the optimal $p-$policy when ψ is high. It displays the interaction between the long-run value of q and the optimal p. This figure shows that, when q^* reaches a low value, then it is optimal to set a low p (equal to $p_{\min}$) while, when q^* reaches a high value, then it is optimal to set a high p (equal to $\overline{p}$). This is because when p is set to a high value, few individuals become criminals and therefore there are plenty of biparental families who are more likely to transmit the honest trait. This implies that a high value of q^* will be reached in the long run. The reverse reasoning applies for a low p.

[*Insert Figure* 3 *here*]

Figure 4 illustrates the optimal $p-$policy when ψ is low. We see that we obtain a similar result but at a much higher level of p.

[*Insert Figure* 4 *here*]

4 Organized crime

We now extend our benchmark model to incorporate a gang of criminals in the economy so that some crimes are committed within a gang and some individually. There are plenty of evidence that gangs have a key impact on crime (see Section 2.4) and this is what we want to understand in this section.

The timing is exactly as before. The only difference is that, at each period t, a gang is formed and each male individual who is adult has to decide whether or not he wants to be

a criminal and if he wants to commit crime by himself or be part of a gang, which offers a fixed remuneration w^G to each of its members. We suppose that when individual i enters into the gang, his expected gains do not depend anymore on θ, his innate ability in crime. On the contrary, when he is committing crime by himself, it matters.

4.1 Crime decisions

An individual θ of type d chooses to commit crime if and only if:

$$\max\left\{(1-p)w^G - p\sigma,\, (1-p)\beta - p\sigma - \theta\right\} > w$$

Indeed, each individual may either join a criminal gang and his utility is then equal to: $(1-p)w^G - p\sigma$ or commit crime by himself and obtain: $(1-p)\beta - p\sigma - \theta$. If he does not commit crime, he gets the outside option w. For simplicity, we assume that p, the probability of being caught, is the same whether the individual commits crime by himself or he is a member of a gang.

Similarly, an individual θ of type h chooses to commit crime if and only if:

$$\max\left\{(1-p)w^G - p\sigma,\, (1-p)\beta - p\sigma - \theta\right\} - K > w.$$

Whatever his type $k = h, d$, an individual θ chooses whether or not to be a member of a gang if and only if:

$$\theta > (1-p)(\beta - w^G) \equiv \overline{\theta}\left(w^G\right). \tag{10}$$

We assume that $\beta > w^G$. It is well-documented that gang members are paid less than people committing crime by themselves (Venkatesh, 1997; Levitt and Venkatesh, 2000). Let us denote by $w^G \equiv \overline{w}_1^G$ (resp. $w^G \equiv \overline{w}_2^G$), the gang's remuneration that attracts all honest (resp. dishonest) agents who do not engage in individual crimes. We have:

$$\overline{w}_1^G = \frac{w + p\sigma + K}{(1-p)} \text{ and } \overline{w}_2^G = \frac{w + p\sigma}{(1-p)}$$

with $\overline{w}_1^G > \overline{w}_2^G$. Using (1) and (2), observe that $\overline{\theta}\left(w^G \equiv \overline{w}_1^G\right) = \theta^h$ and $\overline{\theta}\left(w^G \equiv \overline{w}_2^G\right) = \theta^d$. Therefore, when the gang sets a remuneration that is equal to $\overline{w}_1^G$ (resp. $\overline{w}_2^G$), all honest (resp. dishonest) individuals with $\theta > \theta^h$ (resp. $\theta > \theta^d$) will join the gang while those with

$\theta < \theta^h$ (resp. $\theta < \theta^d$) will commit crime by themselves. The resulting supply of crime (both individual and gang crime) is depicted in Figure 5.

[*Insert Figure* 5 *here*]

Therefore, the crime rate is now given by

$$C(q_t) = \begin{cases} q_t\theta^h + (1-q_t)\theta^d, & \forall w^G < \overline{w}_2^G \\ q_t\theta^h + (1-q_t)\theta^d, & \forall w^G \in \left[\overline{w}_2^G, \overline{w}_1^G\right[\\ 1, & \forall w^G \geq \overline{w}_1^G \end{cases} \tag{11}$$

Using the values of θ^d and θ^h defined in (1) and (2), we obtain:

$$C(q_t) = \begin{cases} -q_tK + (1-p)\beta - p\sigma - w, & \forall w^G < \overline{w}_2^G \\ -q_tK + (1-p)\beta - p\sigma - w, & \forall w^G \in \left[\overline{w}_2^G, \overline{w}_1^G\right[\\ 1, & \forall w^G \geq \overline{w}_1^G \end{cases} \tag{12}$$

4.2 The gang's remuneration decision

The gang chooses the remuneration w^G that maximizes its profit. Let denote by H, the gain per crime committed by each of its member. Then, the gang solves the following program:

$$\max_{w^G} \Pi(w^G) = (H - w^G)l(q_t),$$

where $l(q_t)$ is the mass of criminals in the gang, which is given by:

$$l(q_t) = \begin{cases} 0, & \forall w^G < \overline{w}_2^G \\ (1-q_t)\left[1 - \overline{\theta}(w^G)\right], & \forall w^G \in \left[\overline{w}_2^G, \overline{w}_1^G\right[\\ 1 - \overline{\theta}(w^G), & \forall w^G \geq \overline{w}_1^G \end{cases}$$

Let us define

$$\Pi^1(w^G) \equiv (H - w^G)\left[1 - \overline{\theta}\left(w^G\right)\right],$$
$$\Pi^2(w^G) \equiv (H - w^G)(1-q_t)\left[1 - \overline{\theta}\left(w^G\right)\right]$$

where $\overline{\theta}\left(w^G\right)$ is defined by (10). We denote by w^{G*}, the remuneration that maximizes both $\Pi^1(w^G)$ and $\Pi^2(w^G)$ on $\mathbb{R}^+$, i.e.

$$w^{G*} = \frac{1}{2}(\beta + H) - \frac{1}{2(1-p)} > 0 \tag{13}$$

Define

$$\widehat{q}_1 \equiv 1 - \frac{\Pi^1(\overline{w}_1^G)}{\Pi^1(w^{G*})} \text{ and } \widehat{q}_2 \equiv 1 - \frac{\Pi^1(\overline{w}_1^G)}{\Pi^1(\overline{w}_2^G)}$$

with $\widehat{q}_1 > \widehat{q}_2$.

Proposition 4 *The presence of a criminal organization leads to the following crime rates:*

(i) If $w^{G*} < \overline{w}_2^G$, then,

$$C(q_t) = \begin{cases} q_t\theta^h + (1-q_t)\theta^d, & \forall q_t \leq \widehat{q}_2 \\ 1, & \forall q_t > \widehat{q}_2 \end{cases}$$

(ii) If $\overline{w}_2^G < w^{G*} < \overline{w}_1^G$, then

$$C(q_t) = \begin{cases} q_t\theta^h + (1-q_t)\theta^d, & \forall q_t \leq \widehat{q}_1 \\ 1, & \forall q_t > \widehat{q}_1 \end{cases}$$

(iii) If $w^{G*} > \overline{w}_1^G$, then $C(q_t) = 1, \quad \forall q_t \in [0,1]$.

We obtain this proposition because the gang will not always choose w^{G*} since it depends on the impact on $l(q_t)$. If $w^{G*} \geq \overline{w}_1^G$ (case (iii)), then the gang will choose w^{G*} because it maximizes both profits $\Pi^1(w^G)$ and $\Pi^2(w^G)$. However, if $\overline{w}_2^G < w^{G*} < \overline{w}_1^G$ (case (ii)), this is not anymore true. Indeed, compared to $\overline{w}_1^G$, there is a trade-off. On the one hand, w^{G*} maximizes $\Pi^1(w^G)$. On the other hand, because $w^{G*} < \overline{w}_1^G$, it does not maximize $\Pi^2(w^G)$ because, at w^{G*}, only the "dishonest" criminals will join the gang while, at $\overline{w}_1^G$, all criminals will join the gang. The choice of the remuneration will then depend whether q_t is larger or smaller than $\widehat{q}_1$. If q, the fraction of dishonest individuals, is large enough, then it is optimal for the gang to set a remuneration of $\overline{w}_1^G$, which implies that all individuals are criminals and belong to the gang. In case (i) where w^{G*} is the lowest wage, the gang will never choose w^{G*}. The trade-off will be between $\overline{w}_1^G$ and $\overline{w}_2^G$, and this choice will depend on whether q_t is larger or smaller than $\widehat{q}_2$.

Not surprisingly, the crime rate varies according to q_t, the fraction of honest individuals in the population. In particular, when w^{G*} is large enough, all individuals (honest and dishonest) become criminals and all are gang members. This is true when $w^{G*} > \overline{w}_1^G$, which is equivalent to: $(\beta + H)(1-p) > 2(w + p\sigma + K) + 1$. Indeed, if β, H are high enough or p, w, σ, K are low enough, then this condition is always satisfied.

The dynamics of q_t is given by (8). Using the value of $C(q_t)$ in Proposition 4, we easily obtain:

Proposition 5 *With the presence of a criminal gang, the dynamics of q_t, the population with honest trait, is given by:*

(i) If $w^{G*} < \overline{w}_2^G < \overline{w}_1^G$, then

$$\Delta q_t = \begin{cases} q_t(1-q_t)\left[4q_t(1-q_t)\left[\Delta^B - p\left[q_t\theta^h + (1-q_t)\theta^d\right](\Delta^B - \Delta^S)\right] - 1\right], & \forall q_t \leq \widehat{q}_2 \\ q_t(1-q_t)\left[4q_t(1-q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] - 1\right], & \forall q_t > \widehat{q}_2 \end{cases} \tag{14}$$

(ii) If $\overline{w}_2^G < w^{G*} < \overline{w}_1^G$, then

$$\Delta q_t = \begin{cases} q_t(1-q_t)\left[4q_t(1-q_t)\left[\Delta^B - p\left[q_t\theta^h + (1-q_t)\theta^d\right](\Delta^B - \Delta^S)\right] - 1\right], & \forall q_t \leq \widehat{q}_1 \\ q_t(1-q_t)\left[4q_t(1-q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] - 1\right], & \forall q_t > \widehat{q}_1 \end{cases} \tag{15}$$

(iii) If $w^{G*} > \overline{w}_2^G > \overline{w}_1^G$, then

$$\Delta q_t = q_t(1-q_t)\left[4q_t(1-q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] - 1\right], \quad \forall q_t \in [0,1]\ . \tag{16}$$

This proposition shows how the dynamics of q_t changes when a criminal gang is introduced in the model. First, this dynamics depends on the different relative remunerations of the gang members, w^{G*}, $\overline{w}_2^G$, $\overline{w}_1^G$. Second, it also depends on the relative payoffs of the gang itself, as captured by $\widehat{q}_1$ and $\widehat{q}_2$, since, depending on the fraction of "dishonest" criminals who will join the gang, the dynamics is very different.

4.3 Policy implications

Let us show that the introduction of a gang in the economy implies that the negative impact of p on $\bar{q}$ (interior steady-state equilibrium when there is no gang) is weaker than the negative

impact of p on $\bar{q}^G$ (interior steady-state equilibrium when there is a gang), i.e.

$$\frac{d\bar{q}^G}{dp} < \frac{d\bar{q}}{dp}.$$

Proposition 6 *The presence of a criminal organization reduces the efficiency of the incarceration policy p. The criminal organization implies a stronger negative (resp. weaker positive) impact of weak (resp. high) incarceration policies on long-run honesty (quantitative impact), i.e.*

$$\frac{d\bar{q}^G}{dp} < \frac{d\bar{q}}{dp}.$$

When the return to organized criminal activities, H, is high, incarceration policies always have undesirable negative impact on long run honesty (qualitative impact).

5 Endogenous location

5.1 The model

Let us go back to the benchmark model with no gang and let us assume that there is city with two residential areas (or neighborhoods) indexed by $l = 1, 2$. The population of the city is a continuum of families of mass 2.

The timing is as follows. The first period is as above, i.e. the child is subject to socialization. At the end of the first period, each child has inherited a trait $i = h, d$. At the beginning of the second period, when the child is adult, he first matches with a female and then he (and his wife) has to decide in which neighborhood he wants to reside. We assume that each family lives in one house and the inelastic supply of houses within a residential area is normalized to 1. As in Verdier and Zenou (2004), we assume that each individual makes his location decision without knowing his θ, i.e. his ability (or degree of honesty) of committing crime. Then, types (or honesty parameters) are revealed and individuals decide to commit crime or not. The assumption that types are revealed only after location choices has been made to take into account the relative inertia of the land market compared to the crime market. Obviously, individuals make quicker decisions in terms of crime than in terms

of residential location. This assumption is made to simplify the analysis and relaxing it do not alter the main results of this paper. Then the structure of the family is determined ($k = S, B$). Finally, at the end of the second period, each family exerts a socialization effort in order to influence their offsprings to adopt the honest trait.

Let $Q_t = q_{1,t} + q_{2,t}$, be the mass of honest agents. The bid rent for a parent of type (i, n), i.e. a parent of type $i = h, d$ in neighborhood $n = 1, 2$, at time t is denoted by $\rho^i_{n,t}$. Without loss of generality, we impose that $q_{1,t} \geq q_{2,t}$ (the fraction of honest agents is higher in neighborhood 1) and $\rho^i_{2,t} = 0$ (the land rent is zero in neighborhood 2). For $k = S, B$, denote

$$u^k_n = P^{hk}_t V^h + P^{dk}_t V^d - c^k \frac{\left(\tau^k_t\right)^2}{2}$$

Using (5) and (6), we have:

$$u^k_n = 4q^2_{n,t}(1 - q_{n,t})^2 \frac{(\Delta V)^2}{c^k} + q^2_{n,t}\Delta V + V^d - c^k \frac{\left(\tau^k_t\right)^2}{2} \tag{17}$$

where $\Delta V = V^h - V^d$.

In order to analyze the land market, we can compute the expected utility of a worker of type (i, n) before the revelation of θ. For the ease of the presentation, we skip the time index. We have:

$$U^i_n = \int_0^{\theta^i} \left[(1-p)\beta - p\sigma - K\,\mathbf{1}_{i=h} - \theta\right] d\theta + \int_{\theta^i}^1 w d\theta - \rho^i_{n,t} + \int_0^{\theta^i} \left[p u^S_n + (1-p)\, u^B_n\right] d\theta + \int_{\theta^i}^1 u^B_n d\theta$$

where $\mathbf{1}_{i=h}$ is an indicator function equal to 1 if the parent is of type d and zero otherwise. This utility can be written as:

$$U^i_n = \left[(1-p)\beta - p\sigma - K\,\mathbf{1}_{i=h}\right]\theta^i - \frac{\theta^2}{2} + \left(1 - \theta^i\right) w - \rho^i_{n,t} + p\theta^i u^S_n + \left(1 - p\theta^i\right) u^B_n \tag{18}$$

We can now define the bid rent for a parent of type $i = h, d$ residing in neighborhood $n = 1, 2$. We have:

$$\rho^i_{n,t} = \left[(1-p)\beta - p\sigma - K\,\mathbf{1}_{i=h}\right]\theta^i - \frac{\theta^2}{2} + \left(1 - \theta^i\right) w + p\theta^i u^S_n + \left(1 - p\theta^i\right) u^S_n - U^i_n \tag{19}$$

where u^S_n and u^B_n are defined in (17).

5.2 Equilibrium

To obtain the urban equilibria, we need to know who is eager to bid more for land in a particular neighborhood. Following the literature (Fujita, 1989; Benabou, 1993), the urban equilibrium is defined as follows:

Definition 1 *At any date t and given Q_t, the urban configuration, characterized by $\rho_{1,t}^{i*}$, $q_{1,t}^{*}$, $\tau_{1,t}^{S*}$, $\tau_{1,t}^{B*}$, $\tau_{2,t}^{S*}$, $\tau_{2,t}^{B*}$, is an equilibrium if no one wants to move and change their location choice. The highest bidders for neighborhood 1 are individuals of trait h.*

For the ease of the presentation, we skip the time index. The bid rent $\rho^i = \rho_1^i$ that makes both neighborhoods equally attractive to a trait$-i$ parent is such that: $U_1^i = U_2^i$ for $i = h, d$. Given that $\rho_2^i = 0$, we obtain:

$$\rho^i = p\theta^i \left(u_1^S - u_2^S\right) + \left(1 - p\theta^i\right)\left(u_1^B - u_2^B\right)$$

To determine the urban equilibrium, we must study the bid rent differential $\Delta\rho \equiv \rho^h - \rho^d$. We have:

$$\Delta\rho = p\left(\theta^d - \theta^h\right)\left[\left(u_1^B - u_2^B\right) - \left(u_1^S - u_2^S\right)\right]$$

Using (17), it is easily verified that:

$$\left(u_1^B - u_2^B\right) - \left(u_1^S - u_2^S\right) = 4\left(\Delta V\right)^2\left(\frac{1}{c^B} - \frac{1}{c^S}\right)\left[q_1^2(1-q_1)^2 - q_2^2(1-q_2)^2\right]$$

As a result, using (1) and (2), we obtain:

$$\Delta\rho = 4pK\left(\Delta V\right)^2\left(\frac{1}{c^B} - \frac{1}{c^S}\right)\left[q_1^2(1-q_1)^2 - q_2^2(1-q_2)^2\right] \tag{20}$$

Since $q_2 = Q - q_1$, this equation can be written as:

$$\Delta\rho\left(q_1\right) = 4pK\left(\Delta V\right)^2\left(\frac{1}{c^B} - \frac{1}{c^S}\right)\left[q_1^2(1-q_1)^2 - \left(Q-q_1\right)^2\left(1-Q+q_1\right)^2\right] \tag{21}$$

There are two possible urban equilibria.

Definition 2

(*i*) *An urban equilibrium is* ***segregated*** *at time t if all "honest" families reside in neighborhood 1 and all "dishonest" families reside in neighborhood 2, i.e.* $q_{1,t} = Q_t$ *and* $q_{2,t} = 0$.

(*ii*) *An urban equilibrium is* ***integrated*** *at time t if half of the "honest" families reside in neighborhood 1 and the other half in neighborhood 2, i.e.* $q_{1,t} = q_{2,t} = Q_t/2$.

We have the following result:

Proposition 7 *When $Q_t < 1$, the unique stable urban equilibrium is segregated. When $Q_t > 1$, the unique stable urban equilibrium is integrated.*

What drives this result is $\Delta\rho(q_1)$, the difference in bid rents between "honest" and "dishonest" families. When $\Delta\rho(q_1) > 0$, for all $q_1 \geq q_2$ (i.e. $q_1 \geq Q/2$), then all "honest" families will bid away the "dishonest" families from neighborhood 1, and we obtain the segregated equilibrium. Using (21), given that $c^S > c^B$, $\Delta\rho(q_1) > 0$ when $q_1(1-q_1) > q_2(1-q_2)$ or equivalently, since $q_1 \geq q_2$, when $Q \equiv q_1 + q_2 < 1$. So basically, when, at time t, there is a higher fraction of "honest" families in neighborhood 1, then, if $Q < 1$, all them will bid away the other families. This is because $Q < 1$ is equivalent to $u_1^B - u_2^B > u_1^S - u_2^S$ so that the difference in expected utility between the two neighborhoods is higher for biparental families than single-mother families. Since "honest" families are more likely to become biparental than "dishonest" families, we obtain the segregated equilibrium. When $Q > 1$, which means that $u_1^B - u_2^B < u_1^S - u_2^S$, then we have an integrated equilibrium since "honest" families cannot bid away "dishonest" families from neighborhood 1 because there expected gain is not high enough.

Proposition 8 *Suppose that the probability of being apprehended, p, is such that $p \in [0, \widehat{p}_1[\cup]\widehat{p}_2, \overline{p}_2]$ as in part (ii) of Proposition 1.*

(*i*) *If $Q_0 \in [0, \underline{q}[$, then, in the long run, there is no spatial pattern of social disorganization and the crime rate is high in both neighborhoods, i.e.* $q_1^* = q_2^* = 0$ *and the crime rate is such that* $C_1 = C_2 = \theta^d$.

(*ii*) *If $Q_0 \in [\underline{q}, 2\underline{q}[$, then, in the long run, social disorganization is spatially differentiated and crime is concentrated in one neighborhood only, i.e.* $q_1^* = \overline{q}$, $q_2^* = 0$, *and*

$$C_1 = \overline{q}\theta^h + (1-\overline{q})\theta^d < C_2 = \theta^d$$

(*iii*) *If* $Q_0 \in \left[2\underline{q}, 2\right]$*, then, in the long run, there is no spatial pattern of social disorganization. The crime rate is low in both neighborhoods, i.e.* $q_1^* = q_2^* = \overline{q}$*, and*

$$C_1 = C_2 = \overline{q}\theta^h + (1 - \overline{q})\theta^d$$

In this proposition, we focus on the case when the probability of being apprehended, p, takes intermediary values, i.e. $p \in [0, \widehat{p}_1[\cup]\widehat{p}_2, \overline{p}_2]$, which corresponds to part (*ii*) of Proposition 1.[7] We have seen in the latter that, when there is no location choices, there are two stable steady-state equilibria depending on the initial conditions. If q_0 is low, then $q^* = 0$ while, if it is high enough, then, $q^* = \overline{q} < 1$. Now, when we introduce location choices, the results are different because crime is now spatially differentiated. In Proposition 8, we show that, when Q_0, the initial total fraction of honest families is very low, then there is a unique long-term equilibrium for which $q_1^* = q_2^* = 0$. The intuition is the same as in Proposition 1. The interesting results is when Q_0 takes higher values. When $Q_0 \in \left[\underline{q}, 2\underline{q}\right[$, we show that there is a unique steady-state equilibrium where, in neighborhood 1, all honest families live there and where the crime rate is low while, in neighborhood 2, no honest families reside there and the crime rate is much higher. This is because we have spatial segregation so that honest and dishonest families reside in different neighborhoods. In neighborhood 1, honest families have a higher chance to transmit the honest trait, which implies that individuals are less likely to be criminals and therefore families are more likely to be biparental. This, in turn, implies that the honest trait is more likely to be transmitted to children born in neighborhood 1. And so forth. The opposite is true in neighborhood 2. This results thus shows how *spatial segregation strengthens social disorganization* and vice versa. Indeed, even when $Q_0 \in \left[\underline{q}, 2\underline{q}\right[$, it is possible that $Q_0 > 1$ so that we have start with spatial integration. However, because of the socialization process, eventually, $Q_t < 1$ and we end up with a segregated equilibrium because of different family structures. Finally, when Q_0 is large enough ($Q_0 \in \left[2\underline{q}, 2\right]$), we show that the spatial equilibrium is integrated and the crime rate and the fraction of honest families are the same in both neighborhood. In other words, when the culture of honesty is sufficiently widespread at the beginning, there is no spatial disorganization.

[7] The other case in Proposition 1 is uninteresting since the unique stable steady-state equilibrium is such that $q^* = 0$, i.e. no honest families in the long run.

6 Concluding remarks

Summary and policy implications.

References

[1] Akers, R.L. (1998), *Social Learning and Social Structure: A General Theory of Crime and Deviance*, Boston: Northeastern University Press.

[2] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006), "Who's who in networks. Wanted: The key player," *Econometrica* 74, 1403-1417.

[3] Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou (2010), "Delinquent Networks," *Journal of the European Economic Association* 8(1), 34-61.

[4] Barber, B.L. and J.S. Eccles (1992), "Long-term influence of divorce and single-parenting on adolescent family- and work-related values, behaviors, and aspirations," *Psychological Bulletin* 222, 108-126.

[5] Bayer, P., Hjalmarsson, R. and D. Pozen (2009), "Building criminal capital behind bars: Peer effects in juvenile corrections," *Quarterly Journal of Economics* 124(1), 105-147.

[6] Bearse, P.M. (1996), "Community, youth crime and deterrence: A quantitative theoric analysis," Unpusblished manuscript, University of North Carolina at Greensboro.

[7] Grogger, J. and M. Willis (2000), "The emergence of crack cocaine and the rise in urban crime rates," *Review of Economics and Statistics* 82, 519-529.

[8] Becker, Gary S. (1968) "Crime and Punishment: An Economic Approach," *Journal of Political Economy* 76(2), 169-217.

[9] Benabou, R. (1993), "Workings of a city: Location, education, and production," *Quarterly Journal of Economics* 108, 619-652.

[10] Bisin, A. and T. Verdier (2000), "Beyond the melting pot: Cultural transmission, marriage, and the evolution of ethnic and religious traits," *Quarterly Journal of Economics* 115, 955-988.

[11] Bisin, A. and T. Verdier (2001), "The economics of cultural transmission and the dynamics of preferences," *Journal of Economic Theory* 97, 298-319.

[12] Bisin, A. and T. Verdier (2011), "The economics of cultural transmission and socialization," In J. Benhabib, A. Bisin, and M.O. Jackson (Eds.), *Handbook of Social Economics*, Amsterdam: Elsevier Science, pp. 339-416.

[13] Boyd, R. and P.J. Richerson (1985), *Culture and the Evolutionary Process*, Chicago: University of Chicago Press.

[14] Bumpass, L.L., Raley, R.K. and J.A. Sweet (1995), "The changing character of stepfamilies: Implications of cohabitation and nonmarital childbearing," *Demography* 32, 425-436.

[15] Bureau of the Census (1994), "Marriage and family statistics," Washington, DC: U.S. Department of Commerce.

[16] Calvó-Armengol, A. and Y. Zenou (2004), "Social networks and crime decisions. The role of social structure in facilitating delinquent behavior," *International Economic Review* 45(3), 939-958.

[17] Case, A. and L.F. Katz (1991), "The company you keep: the effects of family and neighborhood on disadvantaged youths," NBER Working Paper No. 3705.

[18] Cavalli-Sforza, L. and M. Feldman (1981), *Cultural Transmission and Evolution. A Quantitative Approach*, Princeton: Princeton University Press.

[19] Cobb-Clark, D.A. and E. Tekin (2014), "Fathers and youths' delinquent behavior," *Review of Economics of the Household* 12, 327-358.

[20] Corno, L. (2017), "Homelessness and crime: do your friends matter?" *Economic Journal*, forthcoming.

[21] Damm, A.P. and C. Dustmann (2014), "Does growing up in a high crime neighborhood affect youth criminal behavior?" *American Economic Review* 104(6), 1806-1832.

[22] Decreuse, B., Mongrain, S. and T. van Ypersele (2015), "Property crime and private protection allocation within cities: Theory and evidence," CEPR Discussion Paper No. 10707.

[23] Dornbusch, S.M., Cardsmith, J., Bushwall, S., Ritter, P., Leiderman, H., Hastorf, A. and R. Gross (1985), "Single parents, extended households, and control of adolescents," *Child Development* 56, 326-341.

[24] Duncan, G., Kalil, A., Mayer, S., Tepper, R. and M. Payne (2005), "The apple does not fall far from the tree," In: Bowles, S., Gintis, H., Osborne Groves, M. (Eds.), *Unequal Chances: Family Background and Economic Success*, Chapter 1, Princeton: Princeton University Press.

[25] Florsheim, P., Tolan, P. and D. Gorman-Smith (1998), "Family relationships, parenting practices, the availability of male family members, and the behavior of inner-city boys in single-mother and two-parent families," *Child Development* 69, 1437-1447.

[26] Freeman, R.B. (1996), "Why do so many young American men commit crimes and what might we do about it?" *Journal of Economic Perspectives* 10, 24-42.

[27] Fujita, M. (1989), *Urban Economic Theory*, Cambridge: Cambridge University Press.

[28] Geller, A. (2013), "Paternal incarceration and father-child contact in fragile families," *Journal of Marriage and Family* 75(5), 1288-1303.

[29] Geller, A., Cooper, C.E., Garfinkel, I., Schwartz-Soicher, O. and R.B. Mincy (2012), "Beyond absenteeism: Father incarceration and child development," *Demography* 49(1), 49-76.

[30] Geller, A., Garfinkel, I. and B. Western (2011), "Paternal incarceration and support for children in fragile families," *Demography* 48(1), 25-47.

[31] Glaeser, E., Sacerdote, B. and J.A. Scheinkman (1996), "Crime and social interactions," *Quarterly Journal of Economics* 111(2), 507-548.

[32] Goffman, A. (2014), *On the Run. Fugitive Life in an American City*, Chicago: The University of Chicago Press.

[33] Hjalmarsson, R. and M.J. Lindquist (2010), "Driving under the influence of our fathers," *The B.E. Journal of Economic Analysis & Policy* 10, 1-15.

[34] Hjalmarsson, R. and M.J. Lindquist (2012), "Like godfather, like son: Exploring the intergenerational nature of crime," *Journal of Human Resources* 47, 550-582.

[35] Hjalmarsson, R. and M.J. Lindquist (2013), "The origins of intergenerational associations in crime: Lessons from Swedish adoption data," *Labour Economics* 20, 68-81.

[36] Kellam, S.G., Ensminger, M.E. and R.J. Turner (1977), "Family structure and the mental health of children," *Archives of General Psychiatry* 34, 1012-22

[37] Kling, J.R., Ludwig, J. and L.F. Katz (2005), "Neighborhood effects on crime for female and male youth: Evidence from a randomized housing voucher experiment," *Quarterly Journal of Economics* 120(1), 87-130.

[38] Levitt, S.D. and S.A. Venkatesh (2000), "An analysis of a drug-selling gang's finances," *Quarterly Journal of Economics* 115(3), 755-789.

[39] Liu, X., Patacchini, E., Zenou, Y. and L.-F. Lee (2012), "Criminal networks: Who is the key player?" CEPR Discussion Paper No. 8772.

[40] Ludwig, J., Duncan, G.J. and P. Hirschfield (2001), "Urban poverty and juvenile crime: Evidence from a randomized housing-mobility experiment," *Quarterly Journal of Economics* 116(2), 655-679.

[41] Matsueda, R.L. and K. Anderson (1998), "The dynamics of delinquent peers and delinquent behavior," *Criminology* 36, 269-308.

[42] O'Flaherty, B. and R. Sethi (2015), "Urban crime," In: G. Duranton, V. Henderson and W. Strange (Eds.), *Handbook of Regional and Urban Economics, Vol. 5B*, Amsterdam: Elsevier Publisher, pp. 1519-1621.

[43] Patacchini, E. and Y. Zenou (2012), "Juvenile delinquency and conformism," *Journal of Law, Economic, and Organization* 28, 1-31.

[44] Porter, J.R., Capellan, J. and S. Chintakrindi (2016), "Social disorganization theory," In W.G. Jennings (Ed.), *The Encyclopedia of Crime and Punishment,* London: Wiley-Blackwell.

[45] Rowe, D.C. and D.P. Farrington (1997), "The familial transmission of criminal convictions," *Criminology* 35, 177-201.

[46] Sarnecki, J. (2001), *Delinquent Networks: Youth Co-Offending in Stockholm*, Cambridge: Cambridge University Press.

[47] South, S.J. and K.D. Crowder (1997), "Residential mobility between cities and suburbs: Race, suburbanization, and back-to-the-city moves," *Demography* 34, 525-538.

[48] Stern, M., Northman, J.E. and M.R. Van Slyck (1984), "Father absence and adolescent problem behavior: Alcohol consumption, drug use and sexual activity," *Adolescence* 19, 301-312.

[49] Turner, R.A., Irwin, C.E. and S.G. Millstein (1991), "Family structure, family processes, and experimenting with substances during adolescence," *Journal of Research on Adolescence* 1, 93-106.

[50] Sutherland, E.H. (1947), *Principles of Criminology*, fourth edition, Chicago: J.B. Lippincott.

[51] Thornberry, T.P. (2009), "The apple doesn't fall far from the tree (or does it?): Intergenerational patterns of anti-social behavior," *Criminology* 47, 297-325.

[52] Thornberry, T.P., Krohn, M.D., Lizotte, A.J. and D. Chard-Wierschem (1993), "The role of juvenile gangs in facilitating delinquent behavior," *Journal of Research in Crime and Delinquency* 30, 55-87.

[53] Thornberry, T.P., Krohn, M.D., Lizotte, A.J., Smith, C.A. and K. Tobin (2003), *Gangs and Delinquency in Developmental Perspective*, Cambridge: Cambridge University Press.

[54] U.S. Census Bureau (2010), "Current Population Survey 2009," Annual Social and Economic Supplement, Release date January 2010, Available at: www.census.gov.

[55] Venkatesh, S.A. (1997), "The social organization of street gang activity in an urban ghetto," *American Journal of Sociology* 103(1), 82-111.

[56] Ventura, S.J. (2009), "Changing patterns of nonmarital childbearing in the United States," NCHS Data Brief No. 18, May, Available at: www.ndc.gov.

[57] Verdier, T. and Y. Zenou (2004), "Racial beliefs, location and the causes of crime," *International Economic Review* 45, 731-760.

[58] Warr, M. (1996), "Organization and instigation in delinquent groups," *Criminology* 34, 11-37.

[59] Warr, M. (2002), *Companions in Crime: The Social Aspects of Criminal Conduct*, Cambridge: Cambridge University Press.

[60] Wildeman, C. (2010), "Paternal incarceration and children's physically aggressive behaviors: Evidence from the Fragile Families and Child Wellbeing Study," *Social Forces* 89, 285-309.

[61] Williams, J. and R. Sickles (2002), "An analysis of the crime as work model: Evidence from the 1958 Philadelphia Birth Cohort Study," *Journal of Human Resources* 37, 479-509.

[62] Wilson, W.J. (1987), *The Truly Disadvantaged: The Inner City, the Underclass, and Public Policy*, Chicago: University of Chicago Press.

[63] Zenou, Y. (2003), "The spatial aspects of crime," *Journal of the European Economic Association* 1, 459-467.

APPENDIX: PROOFS

Proof of Proposition 1: Let us denote by $f(q_t)$, the function defined on $[0,1] \to [0,1]$ and given by:

$$f(q_t) = q_t(1-q_t)\left[4q_t\left(1-q_t\right)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] - 1\right] \tag{22}$$

Stationary equilibria of the economy for which $q_t = q_{t+1} = q$ are such that $f(q) = 0$.

First, we have $f(0) = f(1) = 0$.

Second, if there exists some $q_t \neq 0, 1$, then, solving $f(q_t) = 0$ must lead to:

$$4q_t\left(1-q_t\right)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] - 1 = 0$$

which is equivalent to:

$$q_t\left(1-q_t\right)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] = \frac{1}{4}$$

Let us denote by $h(q_t)$ the function defined on $[0,1] \to [0,1]$ and given by:

$$h(q_t) = q_t(1-q_t)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] \tag{23}$$

We have:

$$h'(q_t) = (1-2q_t)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] - q_t(1-q_t)pC'(q_t)\left(\Delta^B - \Delta^S\right)$$

Using (4), we have:

$$h'(q_t) = (1-2q_t)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right] + q_t(1-q_t)pK\left(\Delta^B - \Delta^S\right)$$

The function $h'(q_t)$ is a polynomial of order two, which is concave. We have

$$h'(0) = \Delta^B - pC(0)\left(\Delta^B - \Delta^S\right) > 0$$

and

$$h'(1) = -\left[\Delta^B - pC(1)\left(\Delta^B - \Delta^S\right)\right] < 0$$

so that there exists a unique

$$q_m = \frac{-\left[\Delta^B - pK\left(\Delta^B - \Delta^S\right)\right] + \sqrt{D}}{3K\left(\Delta^B - \Delta^S\right)},$$

with

$$D = \left[\Delta^B - pK\left(\Delta^B - \Delta^S\right)\right]^2 + 3K\left(\Delta^B - \Delta^S\right)\left[\Delta^B - p\theta^d\left(\Delta^B - \Delta^S\right)\right]$$

such that $h'(q_m) = 0$. It implies that the function $h\left(q_t\right)$ reaches a global maximum at $q = q_m$. We deduce that there exists $\underline{q} \leq \overline{q} \neq 0, 1$ such that $f(\underline{q}) = f(\overline{q}) = 0$ if and only if $h(q_m) \geq \frac{1}{4}$. Note that this condition depends on the parameters of the function $h\left(q_t\right)$. In particular, let us focus on the parameter p (the probability of being arrested).

Denote by $h(q;p)$, the function $h\left(.\right)$ parametrized by p. We look for a p such that

$$h\left(q_m(p);p\right) \geq \frac{1}{4}.$$

First, let us differentiate $h\left(q_m(p);p\right)$ with respect to p. We obtain:

$$\begin{aligned}\frac{dh\left(q_m(p);p\right)}{dp} &= \frac{\partial h\left(q_m(p);p\right)}{\partial q_m}\frac{dq_m}{dp} + \frac{\partial h\left(q_m(p);p\right)}{\partial p} \\ &= q_m(p)\left[1 - q_m(p)\right]\left(\Delta^B - \Delta^S\right)\left[q_m(p)K - (\beta - 2p(\beta+\sigma) - w)\right].\end{aligned}$$

We have $\frac{dh(q_m(p);p)}{dp} > 0$ if and only if

$$q_m(p) > \frac{\beta - 2p(\beta+\sigma) - w}{K} \equiv \widetilde{q}.$$

Since q_m is the maximum of $h\left(.\right)$, we deduce that $q_m(p) > \widetilde{q}$ if and only if $h'(\widetilde{q}) < 0$. Let us study this condition. We have:

$$\begin{aligned}h'(\widetilde{q}) &= \frac{\left(K - 2\left[\beta - 2p(\beta+\sigma) - w\right]\right)\left[\Delta^B - p^2\left(\beta+\sigma\right)\left(\Delta^B - \Delta^S\right)\right]}{K} \\ &\quad + \frac{p\left(\Delta^B - \Delta^S\right)\left(\beta - 2p(\beta+\sigma) - w\right)^2}{K}\end{aligned}$$

This is a polynomial function of p with coefficients associated to the squared term p^2 equal to $-\left(\beta+\sigma\right)\left[2(\beta - w) + K\right]$, which is negative. Thus this function $h'(\widetilde{q})$ is concave. Furthermore, we know that at $p = \left(\beta - w - K\right)/\left[2\left(\beta+\sigma\right)\right]$, $\widetilde{q} = 1$ so that the polynomial is

negative. Also, at $p = (\beta - w) / [2(\beta + \sigma)]$, $\widetilde{q} = 0$ so that the polynomial is positive. As a result, there exists a unique $\widetilde{p}$ such that, for any $p \leq \widetilde{p}$, $q_m(p) \leq \widetilde{q}$, which implies that

$$\frac{dh\left(q_m(p); p\right)}{dp} \leq 0,$$

while, for any $p \geq \widetilde{p}$, $q_m(p) \geq \widetilde{q}$, which implies that

$$\frac{dh\left(q_m(p); p\right)}{dp} \geq 0.$$

At $p = 0$, we have: $h\left(\widetilde{q}(0); 0\right) = \frac{1}{4}\Delta^B$, which is higher than $1/4$ due to part (i) of Assumption 1. Since $h\left(q_m(p); p\right)$ reaches a minimum at $p = \widetilde{p}$, a necessary and sufficient condition for the equation $h\left(q_m(p); p\right) - 1/4 = 0$ to admit two solutions on $[0, \overline{p}_1]$ is: $h\left(q_m(\widetilde{p}); \widetilde{p}\right) < 1/4$. This is equivalent to: $h(\widetilde{q}; \widetilde{p}) < 1/4$, that is

$$\widetilde{q}(1 - \widetilde{q})\left[\Delta^B - \widetilde{p}^2(\beta + \sigma)\left(\Delta^B - \Delta^S\right)\right] < \frac{1}{4}.$$

Note that, at $p = \widetilde{p}$, we have:

$$\widetilde{q}(1 - \widetilde{q}) = \frac{(2\widetilde{q} - 1)\left[\Delta^B - \widetilde{p}^2(\beta + \sigma)\left(\Delta^B - \Delta^S\right)\right]}{K\,\widetilde{p}\,(\Delta^B - \Delta^S)}.$$

The condition above can thus be written as:

$$\frac{(2\widetilde{q} - 1)\left[\Delta^B - \widetilde{p}^2(\beta + \sigma)\left(\Delta^B - \Delta^S\right)\right]^2}{K\,\widetilde{p}\,(\Delta^B - \Delta^S)} < \frac{1}{4}.$$

We have:

$$\frac{(2\widetilde{q} - 1)\left[\Delta^B - \widetilde{p}^2(\beta + \sigma)\left(\Delta^B - \Delta^S\right)\right]^2}{K\,\widetilde{p}\,(\Delta^B - \Delta^S)} \leq \frac{\left[\Delta^B - \widetilde{p}^2(\beta + \sigma)\left(\Delta^B - \Delta^S\right)\right]^2}{K\,\widetilde{p}\,(\Delta^B - \Delta^S)}$$

so that the expression of the left-hand side of this inequality is bounded above by:

$$\frac{\left[\Delta^B - \widetilde{p}^2(\beta + \sigma)\left(\Delta^B - \Delta^S\right)\right]^2}{K\,\widetilde{p}\,(\Delta^B - \Delta^S)}.$$

We do not have an exact expression for $\widetilde{p}$ but we know that the above term is a decreasing function of $\widetilde{p}$. As $\widetilde{p} > \frac{\beta-w-K}{2(\beta+\sigma)}$, we have:

$$\frac{\left[\Delta^B - p^2(\beta+\sigma)\left(\Delta^B - \Delta^S\right)\right]^2}{Kp\left(\Delta^B - \Delta^S\right)} < \frac{2(\beta+\sigma)\left[\Delta^B - \frac{(\beta-w-K)^2}{4(\beta+\sigma)}\left(\Delta^B - \Delta^S\right)\right]^2}{K(\beta-w-K)\left(\Delta^B - \Delta^S\right)}$$

A sufficient condition for $h(\widetilde{q};\widetilde{p}) < \frac{1}{4}$ is then

$$\frac{2(\beta+\sigma)\left[\Delta^B - \frac{(\beta-w-K)^2\left(\Delta^B-\Delta^S\right)}{4(\beta+\sigma)}\right]^2}{K(\beta-w-K)\left(\Delta^B - \Delta^S\right)} < \frac{1}{4},$$

which is part (ii) of Assumption 1.

Hence, we can deduce that, for $p \in [0, \overline{p}_1[$, there exists $\widehat{p}_1$ such that, for any $p \leq \widehat{p}_1$, the equation $h(q;p) - \frac{1}{4}$ has two solutions.

Let us now study the function $h(.)$ on the interval $[\overline{p}_1, \overline{p}_2]$. First note that we have $h\left(q_m(\overline{p}_2);\overline{p}_2\right) = \frac{1}{4}\Delta^B$, which is higher than $\frac{1}{4}$ due to part (i) of Assumption 1. Also,

$$\frac{dh\left(q_m(p);p\right)}{dp} = q_m(p)\left[1 - q_m(p)\right]\left[-\beta + w + 2p(\beta+\sigma)\right]\left(\Delta^B - \Delta^S\right) > 0,$$

since $p > \overline{p}_1 = \frac{\beta-w}{(\beta+\sigma)}$. We deduce that there exists a unique $\widehat{p}_2$ (higher or lower than $\overline{p}_1$) such that, for any $p \in [\widehat{p}_2, \overline{p}_2]$, the equation $h(q;p) - \frac{1}{4}$ has two solutions. For any $p \in]\widehat{p}_1, \widehat{p}_2[$, $h(q;p) - \frac{1}{4}$ has one solution.

We conclude that for $p \in]\widehat{p}_1, \widehat{p}_2[$, the equation $f(q) = 0$ has two solutions $q = 0$ and $q = 1$ with $f'(0) < 0$ and $f'(1) > 0$ so that 0 is stable and 1 is unstable. By continuity of $f(.)$ (and because q is bounded), we deduce that, for any $q_0 \in [0,1]$, the dynamic system globally converges to 0.

For $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \bar{p}_2]$, the equation $f(q) = 0$ has four solutions: $q = 0$, $q = 1$ and two interior solutions $\underline{q}$ and $\overline{q}$ with $f'(0) < 0$, $f'(1) > 0$, $f'(\underline{q}) > 0$ and $f'(\overline{q}) < 0$. We conclude that, for any $q_0 \in \left[0, \underline{q}\right[$, the sequence q_t converges to 0 while, for any $q_0 \in [\underline{q}, 1]$, the sequence q_t converges to $\overline{q}$. ∎

Proof of Proposition 2: Let us focus on the interior equilibrium $\overline{q}$ of Proposition 1 and, therefore, let us assume that $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \overline{p}_2]$. In the proof of Proposition 1, we have shown that the steady-state fraction of honest individuals in the population, $\overline{q}$, is implicitly given by

$$f(\overline{q}) = 0,$$

where $f(.)$ is defined by (22). Hence,

$$\frac{d\overline{q}}{dw} = -\frac{\partial f(.)/\partial w}{\partial f(.)/\partial q}.$$

In the proof of Proposition 1, we have shown that: $\partial f(.)/\partial q < 0$, which means that so that the sign of $d\overline{q}/dw$ is the same as the sign of $\partial f(.)/\partial w$. Totally differentiating $f(.)$ in (22) and using (4) lead to:

$$\frac{\partial f(q_t)}{\partial w} \mid_{q_t=\overline{q}} = \overline{q}(1-\overline{q})4\overline{q}\,(1-\overline{q})\,p\left(\Delta^B - \Delta^S\right) > 0$$

As a result,

$$\frac{d\overline{q}}{dw} > 0$$

Using the same approach, we can show that: $\frac{d\overline{q}}{d\beta} < 0$ and $\frac{d\overline{q}}{d\sigma} > 0$. ▮

Proof of Proposition 3: In Proposition 1, especially part (ii), we have seen that, depending on q_0, different equilibrium fractions of honest people may emerge in the long run, which means that the planner cannot implement the same incarceration policy for different values of q^*.

Let us thus determine the optimal incarceration policies, independently of the initial condition q_0. Once the policy is identified, we can check whether this policy can be implemented for a given initial distribution of honest norms q_0.

For that, we need to determine the value of p that minimizes the social cost (i.e. crime rate) given the long run distribution of q. This is not trivial since the long run distribution depends on p, which is the choice variable. Several cases may arise.

We will consider four cases.

Case A corresponds to the situation when the long run distribution of q is equal to zero ($q^* = 0$) for any $p \in [0, \overline{p}_2]$.

Case B corresponds to the case when the long run distribution of q is equal to $\overline{q} \equiv \overline{q}(p) > 0$ for any $p \in [0, \widehat{p}_1]$ and is equal to zero for any $p \in]\widehat{p}_1, \overline{p}_2]$.

Case C corresponds to the case when the long run distribution of q is equal to $\overline{q} \equiv \overline{q}(p) > 0$ for any $p \in [\widehat{p}_2, \overline{p}_2]$ and is equal to zero for any $p \in [0, \widehat{p}_2[$.

Case D corresponds to the case when the long run distribution of q is equal to $\overline{q} \equiv \overline{q}(p) > 0$ for any $p \in [0, \widehat{p}_1] \cup [\widehat{p}_2, \overline{p}_2]$ and is equal to zero for any $p \in]\widehat{p}_1, \widehat{p}_2[$.

Let us now study each case separately.

Case A: The minimization problem can be written as:

$$\min_p \Gamma(q^*, p) \equiv q^* \theta^h + (1 - q^*)\theta^d + \frac{p^2}{2\psi} \text{ s.t. } q^* = 0$$

Using the values of θ^h and θ^d given in (2) and (1), the optimal solution $p_{\min}$ of this problem is such that

$$-(\beta + \sigma) + \frac{p_{\min}}{\psi} = 0,$$

that is

$$p_{\min} = (\beta + \sigma)\psi$$

We deduce that $p_{\min}$ minimizes the social cost (total crime) on the whole interval $[0, \overline{p}_2]$ if $p_{\min} \leq \overline{p}_2$, that is if $\psi \leq \overline{p}_2 / (\beta + \sigma)$ while $\overline{p}_2$ minimizes the social cost (total crime) if $\psi \geq \overline{p}_2 / (\beta + \sigma)$.

Case B: The minimization problem can be written as:

$$\begin{aligned} \min_p \Gamma(q^*, p) &\equiv q^* \theta^h + (1 - q^*)\theta^d + \frac{p^2}{2\psi} \\ \text{s.t. } q^* &= \begin{cases} \overline{q}(p) & \forall p \in [0, \widehat{p}_1] \\ 0 & \forall p \in]\widehat{p}_1, \overline{p}_2] \end{cases} \end{aligned}$$

To solve this problem, we proceed in the following way. First, we identify the optimal solution for each sub-interval (i.e. we decompose this discrete problem into two continous problems). Second, we compare these optimal solutions and select the one that globally minimizes the social cost $\Gamma(q^*, p)$.

(i) Consider the case when $p \in [0, \widehat{p}_1]$.

Let us define the function $q^I(p;\Gamma) : [0, \overline{p}_2] \to [0,1]$ given by:

$$q^I(p;\Gamma) = \frac{1}{K}\left[\beta - w - p(\beta+\sigma) + \frac{p^2}{2\psi} - \Gamma\right],$$

The function q^I captures the set of points q that gives the same level of social cost Γ. The point $\overline{p}$ solving the minimisation problem is such that

$$\frac{dq^I}{dp}|_{p=\overline{p}} - \frac{d\overline{q}}{dp}|_{p=\overline{p}} = 0,$$

which is equivalent to:

$$\frac{1}{K}\left[-(\beta+\sigma) + \frac{p}{\psi}\right] - \frac{-\overline{q}(p)\left[1-\overline{q}(p)\right](\Delta^B - \Delta^S)\left[\overline{q}(p)K - [\beta - 2p(\beta+\sigma) - w]\right]}{\left[1-2\overline{q}(p)\right]\left[\Delta^B - pC(p)(\Delta^B - \Delta^S)\right] + \overline{q}(p)\left[1-\overline{q}(p)\right]\Delta^S pK} = 0.$$

We know from the proof of Proposition 1 that the function $\overline{q}(p)$ is decreasing in p on the relevant interval, i.e. for any $p \in [0, \widehat{p}_1]$,

$$\overline{q}(p)K - [\beta - 2p(\beta+\sigma) - w] \leq 0$$

However, we do not know the sign of the seconde derivative. Let us assume that

$$\frac{dq^I}{dp}|_{p=0} - \frac{d\overline{q}}{dp}|_{p=0} < 0,$$

which is equivalent to:

$$\frac{1}{K}(\beta+\sigma) > \frac{\overline{q}(0)\left[1-\overline{q}(0)\right](\Delta^B - \Delta^S)\left[\overline{q}(0)K - (\beta - w)\right]}{(2\overline{q}(0)-1)\Delta^B}$$

If this assumption holds, then $p = 0$ minimizes the social cost on $p \in [0, \widehat{p}_1]$. We could make the reverse assumption and compare the welfare at 0 to the welfare obtained for the optimal solution on $]\widehat{p}_1, \overline{p}_2]$. We would obtained identical qualitative results. In order to simplify the exposition, we do not allow this case to arise (by restricting the different cases).

At $p = \widehat{p}_1$, we have:

$$\frac{d\overline{q}}{dp}|_{p=\widehat{p}_1} = 0 \quad \text{since } \bar{q}(\widehat{p}_1)K - [\beta - 2\widehat{p}_1(\beta+\sigma) - w] = 0,$$

and

$$\frac{dq^I}{dp}|_{p=\widehat{p}_1} = \frac{1}{K}\left[-(\beta+\sigma) + \widehat{p}_1/\psi\right].$$

First, if

$$\frac{dq^I}{dp}|_{p=\widehat{p}_1} > 0 \Leftrightarrow \widehat{p}_1 > p_{\min}$$

then there exists at least one $p \in [0, \widehat{p}_1]$ such that

$$\frac{dq^I}{dp}|_p - \frac{d\overline{q}}{dp}|_p = 0.$$

If there are more than one p solving this equation, then we select the lowest one since it necessarily provides the lowest social cost. We denote this solution $\overline{p}$, i.e. $\overline{p}$ minimizes the social cost on $[0, \widehat{p}_1]$.

Second, if

$$\frac{dq^I}{dp}|_{p=\widehat{p}_1} < 0 \Leftrightarrow \widehat{p}_1 < p_{\min},$$

then either the optimal solution on $[0, \widehat{p}_1]$ is $\widehat{p}_1$ (since $\frac{dq^I}{dp}|_p - \frac{d\bar{q}}{dp}|_p < 0$, $\forall [0, \widehat{p}_1]$) or there exists at least one $p \in [0, \widehat{p}_1]$ such that

$$\frac{dq^I}{dp}|_p - \frac{d\overline{q}}{dp}|_p = 0.$$

Again, in this case, we denote $\overline{p}$, the global minimum on $[0, \widehat{p}_1]$.

(ii) Consider the case when $p \in]\widehat{p}_1, \bar{p}_2]$. One easily shows that the optimal solution is given by:

$$\begin{array}{lll} \widehat{p}_1 & \text{if} & p_{\min} < \widehat{p}_1 \\ p_{\min} & \text{if} & p_{\min} \in [\widehat{p}_1, \overline{p}_2] \\ \overline{p}_2 & \text{if} & p_{\min} > \overline{p}_2 \end{array}$$

(iii) Comparison of optimal solutions.

We will show that the solution that globally minimizes the social cost (i.e. on the whole interval $[0, \overline{p}_2]$) depends on the technology of the incarceration captured by the parameter ψ.

$(iii1)$ Suppose that $p_{\min} \leq \widehat{p}_1$, which is equivalent to $\psi \leq \widehat{p}_1 / (\beta + \sigma)$. Here, we must compare the social cost at $p = \overline{p}$ to the social cost at $p = \widehat{p}_1$. The social cost at $p = \overline{p}$, which we denote by $\overline{\Gamma}$ is given by:

$$\overline{\Gamma} = \overline{q}(\overline{p})\theta^h(\overline{p}) + [1 - \overline{q}(\overline{p})]\, \theta^d(\overline{p}) + \frac{\overline{p}^2}{2\psi}.$$

The social cost at $p = \widehat{p}_1$ is given by:

$$\theta^d(\widehat{p}_1) + \frac{\widehat{p}_1^2}{2\psi}.$$

We examine the sign of the social cost differential. We have:

$$\overline{\Gamma} - \theta^d(\widehat{p}_1) - \frac{\widehat{p}_1^2}{2\psi}.$$

Note that $q^I(p, \overline{\Gamma})$ is the isoquant corresponding to a social cost $\overline{\Gamma}$. Thus, we know that:

$$\forall p \in [0, \overline{p}_2], \quad q^I(p, \overline{\Gamma})\theta^h(p) + \left[1 - q^I(p, \overline{\Gamma})\right] \theta^d(p) + \frac{p^2}{2\psi} = \overline{\Gamma}.$$

Then, in particular, at $p = p_{\min}$,

$$q^I(p_{\min}, \overline{\Gamma})\theta^h(p_{\min}) + \left[1 - q^I(p_{\min}, \overline{\Gamma})\right] \theta^d(p_{\min}) + \frac{p_{\min}^2}{2C} = \overline{\Gamma}.$$

The social cost differential then can be written as:

$$\begin{aligned} & q^I(p_{\min}, \overline{\Gamma})\theta^h(p_{\min}) + \left[1 - q^I(p_{\min}, \overline{\Gamma})\right] \theta^d(p_{\min}) + \frac{p_{\min}^2}{2\psi} - \theta^d(\widehat{p}_1) - \frac{\widehat{p}_1^2}{2\psi} \\ & = -q^I(p_{\min}; \overline{\Gamma})K + \theta^d(p_{\min}) + \frac{p_{\min}^2}{2\psi} - \theta^d(\widehat{p}_1) - \frac{\widehat{p}_1^2}{2\psi}. \end{aligned}$$

The social cost differential is a function of ψ. To see this, first, one can show that

$$\theta^d(p_{\min}) + \frac{p_{\min}^2}{2\psi} - \theta^d(\widehat{p}_1) - \frac{\widehat{p}_1^2}{2\psi}$$

is increasing in ψ. Indeed, by the envelop theorem, the derivative is equal to:

$$-\frac{p_{\min}^2}{2\psi^2} + \frac{\widehat{p}_1^2}{2\psi^2} > 0,$$

since $p_{\min} < \widehat{p}_1$.

Second, one can also show that $-q^I(p_{min};\overline{\Gamma})K$ is increasing in ψ. To see this, let us rewrite

$$-q^I(p_{\min};\overline{\Gamma})K = \overline{q}(\overline{p})\theta^h(\overline{p}) + [1-\overline{q}(\overline{p})]\,\theta^d(\overline{p}) + \frac{\overline{p}^2}{2\psi} - \theta^d(p_{\min}) - \frac{p_{\min}^2}{2\psi} \equiv G$$

Using the envelop theorem

$$\begin{aligned}\frac{dG}{D\psi} =& \frac{\partial G}{\partial \overline{p}}\frac{\partial \overline{p}}{\partial \psi} + \frac{\partial G}{\partial p_{\min}}\frac{\partial p_{\min}}{\partial \psi} + \frac{\partial G}{\partial \psi} \\ =& \frac{\partial G}{\partial \psi} = \frac{p_{\min}^2 - \overline{p}^2}{2\psi^2} > 0,\end{aligned}$$

since $p_{\min} > \overline{p}$.[8] We deduce that the social cost differential is increasing in ψ (meaning that the social cost at $\overline{p}$ increases as compared to the social cost at $\widehat{p}_1$).

[8] We know that $p_{\min} > \overline{p}$. Indeed for any $p > p_{\min}$,

$$\frac{dq^I}{dp} > 0$$

while $\overline{p}$ is such that

$$\frac{dq^I}{dp} = \frac{d\overline{q}}{dp}$$

with

$$\frac{d\overline{q}}{dp} < 0,\ \forall p \in [0, \widehat{p}_1].$$

When ψ tends to zero, $\overline{p}$ tends $p_{\min}$ and both $\overline{p}$ and $p_{\min}$ tends to zero. Then,

$$-q^I(p_{\min};\overline{\Gamma})K = -\overline{q}(\overline{p})K < 0.$$

At $\psi = \frac{\widehat{p}_1}{\beta+\sigma}$, the differential is equal to:

$$-q^I(\widehat{p}_1;\overline{\Gamma})K.$$

We deduce the following.

• If $q^I(\widehat{p}_1;\overline{\Gamma}) > 0$, then for any $\psi \in [0, \frac{\widehat{p}_1}{\beta+\sigma}]$, the differential is negative, which is equivalent to say that the social cost is minimized at $p = \overline{p}$.

• If $q^I(\widehat{p}_1;\overline{\Gamma}) < 0$, then there exists a threshold $\widetilde{\psi}$ such that: $\forall\psi \in [0, \widetilde{\psi}]$, the differential is negative, and $\forall\psi \in \left[\widetilde{\psi}, \frac{\widehat{p}_1}{\beta+\sigma}\right]$, the differential is positive and the social cost is minimized at $p = \widehat{p}_1$.

$(iii2)$ Suppose now that $p_{\min} > \widehat{p}_1$, which is equivalent to $\psi > \frac{\widehat{p}_1}{\beta+\sigma}$. Let compare the social cost at $p = \overline{p}$ to the social cost at $p = p_{\min}$.[9] We need to compare the social cost at $\overline{p}$, which is $\overline{\Gamma}$, to the social cost at $p_{\min}$, which is given by:

$$\theta^d(p_{\min}) + \frac{p_{\min}^2}{2\psi}.$$

We study the social cost differential $\overline{\Gamma} - \theta^d(p_{\min}) + \frac{p_{\min}^2}{2\psi}$, which, using the same reasoning as for the case $p_{\min} < \widehat{p}_1$, can be rewritten as

$$-q^I(p_{\min};\overline{\Gamma})K + \theta^d(p_{\min}) + \frac{p_{\min}^2}{2\psi} - \theta^d(p_{\min}) - \frac{p_{\min}^2}{2\psi} = -q^I(p_{\min};\overline{\Gamma})K.$$

From the previous analysis, we know that this is an increasing function of ψ, and we can deduce that:

• If $q^I(\widehat{p}_1;\overline{\Gamma}) < 0$, then for any $\psi \in \left[\frac{\widehat{p}_1}{\beta+\sigma}, \frac{\overline{p}_2}{\beta+\sigma}\right]$, the differential is positive, which is equivalent to say that the social cost is minimized at $p = p_{\min}$.

• If $q^I(\widehat{p}_1;\overline{\Gamma}) > 0$, then there are two cases:

[9] Remember that the optimal policy can also be $\widehat{p}_1$ but let us skip this case as it does not provide additional insights for our purpose.

either there exists a threshold $\widetilde{\psi}$ such that: $\forall\psi \in \left[\frac{\widehat{p}_1}{\beta+\sigma}, \widetilde{\psi}\right]$, the differential is negative, and $\forall\psi \in \left[\widetilde{\psi}, \frac{\overline{p}_2}{\beta+\sigma}\right]$, the differential is positive, or, the differential is negative $\forall\psi \in \left[\frac{\widehat{p}_1}{\beta+\sigma}, \frac{\overline{p}_2}{\beta+\sigma}\right]$. In this latter case, for any $\psi > \frac{\overline{p}_2}{\beta+\sigma}$, the optimal solution for the case $q = 0$ is $\overline{p}_2$. Then one can easily show that the social cost differential is positive when ψ tends to infinity so that again there exists a threshold $\widetilde{\psi}$ such that: $\forall\psi \in \left[\frac{\overline{p}_2}{\beta+\sigma}, \widetilde{\psi}\right]$, the differential is negative, and $\forall\psi > \widetilde{\psi}$, the differential is positive.

Finally, we can conclude that there exists some threshold $\widetilde{\psi} \in [0, +\infty[$ such that:

• $\forall\psi \leq \widetilde{\psi}$, the policy $\overline{p}$, implying a long run distribution of honesty $q = \overline{q}(\overline{p}) > 0$, minimizes the social cost,

• $\forall\psi \geq \widetilde{\psi}$, the policy $\widehat{p}_1$ or $p_{\min}$ or $\overline{p}_2$, implying a long run distribution of honesty $q = 0$, minimize the social cost.

Case C: The minimization problem can be written as:

$$\begin{aligned} \min_p \Gamma\left(q^*, p\right) &\equiv q^*\theta^h + (1-q^*)\theta^d + \frac{p^2}{2\psi} \\ \text{s.t. } q^* &= \begin{cases} 0 & \forall p \in [0, \widehat{p}_2] \\ \overline{q}(p) & \forall p \in]\widehat{p}_2, \overline{p}_2] \end{cases} \end{aligned}$$

As in the previous case, we can show that there exists $\widetilde{\psi}'$ such that:

• $\forall\psi \leq \widetilde{\psi}'$, the policy $p_{\min}$, or $\widehat{p}_2$, implying a long run distribution of honesty $q^* = 0$, minimize the social cost.

• $\forall\psi \geq \widetilde{\psi}'$, the policy $\overline{p}'$ or $\overline{p}_2$, implying a long run distribution of honesty $q = \overline{q}(\overline{p}) > 0$, minimizes the social cost, where $\overline{p}' \in [\widehat{p}_2, \overline{p}_2]$ is such that

$$\frac{dq^I}{dp}|_{p=\overline{p}'} - \frac{d\overline{q}}{dp}|_{p=\overline{p}'} = 0.$$

Case D: The minimization problem can be written as:

$$\begin{aligned} \min_p \Gamma\left(q^*, p\right) &\equiv q^*\theta^h + (1-q^*)\theta^d + \frac{p^2}{2\psi} \\ \text{s.t. } q^* &= \begin{cases} \overline{q}\left(p\right) & \forall p \in [0, \widehat{p}_1] \\ 0 & \forall p \in]\widehat{p}_1, \widehat{p}_2[\\ \overline{q}\left(p\right) & \forall p \in [\widehat{p}_2, \overline{p}_2] \end{cases} \end{aligned}$$

This case can be solved using the same techniques as above. In addition, here, we must impose further conditions allowing to analyze the different policies for any values of ψ. In particular we assume the following:

When $p_{\min} > \widehat{p}_2$, then $\overline{p}'$ minimizes the social cost, that is $\widetilde{\psi}' < \frac{\widehat{p}_2}{\beta+\sigma}$.

When $p_{\min} < \widehat{p}_1$, then $\bar{p}$ minimizes the social cost, that is $\widetilde{\psi}' > \frac{\widehat{p}_1}{\beta+\sigma}$.

Several cases may then arise.

(i) Either $\widetilde{\psi} > \widetilde{\psi}'$, then:
for any $\psi \leq \widetilde{\psi}$, $\overline{p}$ minimizes the social cost,
for any $\psi \geq \widetilde{\psi}'$, $\overline{p}'$ or $\overline{p}_2$ minimizes the social cost,
for $\psi \in \left[\widetilde{\psi}', \widetilde{\psi}\right]$, we can show the existence of a threshold $\widehat{\psi}$ such that:
• for any $\psi \in \left[\widetilde{\psi}', \widehat{\psi}\right]$, $\overline{p}$ minimizes the social cost,
• for any $\psi \in \left[\widehat{\psi}, \widetilde{\psi}\right]$, $\overline{p}'$ minimizes the social cost.

(ii) Or, $\widetilde{\psi} < \widetilde{\psi}'$, then:
for any $\psi \leq \widetilde{\psi}$, $\overline{p}$ minimizes the social cost,
for any $\psi \geq \widetilde{\psi}'$, $\overline{p}'$ or $\overline{p}_2$ minimizes the social cost,
for $\psi \in \left[\widetilde{\psi}, \widetilde{\psi}'\right]$, $p_{\min}$ minimizes the social cost.

Optimal policies depending on initial conditions: To complete the analysis, we need to determine which optimal policy arises depending on the initial fraction of honest agents q_0.

If $q_0 < \min\{\underline{q}(\overline{p}), \underline{q}(\overline{p}')\}$, the optimal policy is analyzed in Case A, which implies that $p = p_{\min}$ or $p = \overline{p}_2$.

If $\underline{q}(\overline{p}) < q_0 < \underline{q}(\overline{p}')$, the optimal policy is analyzed in Case B.

If $\underline{q}(\overline{p}') < q_0 < \underline{q}(\overline{p})$, the optimal policy is analyzed in Case C.

Suppose that $q_0 > \max\left\{\underline{q}(\bar{p}), \underline{q}(\overline{p}')\right\}$, the optimal policy is analyzed in Case D. ∎

Proof of Proposition 4: There are three cases to consider: (i) $w^{G*} < \overline{w}_2^G < \overline{w}_1^G$, (ii) $\overline{w}_2^G < w^{G*} < \overline{w}_1^G$, and (iii) $w^{G*} > \overline{w}_1^G > \overline{w}_2^G$.

In case (iii), we have $\Pi^1(w^{G*}) > \Pi^1(\overline{w}_1^G) > \Pi^2(\overline{w}_2^G)$ so that the gang fixes a remuneration equal to w^{G*}.

In case (ii), we have $\Pi^2(w^{G*}) > \Pi^2(\overline{w}_2^G)$ so that the gang never chooses $\overline{w}_2^G$. The gang chooses between $\overline{w}_1^G$ and w^{G*}. Since $\overline{w}_1^G$ maximises $\Pi^1(w^G)$, the gang chooses w^{G*} if and only if

$$\Pi^2(w^{G*}) \geq \Pi^1(\overline{w}_1^G) \Leftrightarrow (1-q)\Pi^1(w^{G*}) \geq \Pi^1(\overline{w}_1^G) \Leftrightarrow q_t \leq 1 - \frac{\Pi^1(\overline{w}_1^G)}{\Pi^1(w^{G*})} \equiv \widehat{q}_1.$$

In case (i), the gang chooses between $\overline{w}_1^G$ and $\overline{w}_2^G$. The gang chooses $\overline{w}_2^G$ if and only if

$$\Pi^2(\overline{w}_2^G) \geq \Pi^1(\overline{w}_1^G) \Leftrightarrow (1-q_t)\Pi^1(\overline{w}_2^G) \geq \Pi^1(\overline{w}_1^G) \Leftrightarrow q_t \leq 1 - \frac{\Pi^1(\overline{w}_1^G)}{\Pi^1(\overline{w}_2^G)} \equiv \widehat{q}_2.$$

with $\widehat{q}_1 > \widehat{q}_2$. We deduce the equilibrium crime rate in each case. Since $\overline{w}_2^G < \overline{w}_1^G$, we have:

Case (i): $w^{G*} < \overline{w}_2^G < \overline{w}_1^G$. Then,

$$C(q_t) = \begin{cases} q_t\theta^h + (1-q_t)\theta^d, & \forall q_t \leq \widehat{q}_2 \\ 1, & \forall q_t > \widehat{q}_2 \end{cases}$$

Case (ii): $\overline{w}_2^G < w^{G*} < \overline{w}_1^G$. Then,

$$C(q_t) = \begin{cases} q_t\theta^h + (1-q_t)\theta^d, & \forall q_t \leq \widehat{q}_1 \\ 1, & \forall q_t > \widehat{q}_1 \end{cases}$$

Case (iii): $w^{G*} > \overline{w}_1^G > \overline{w}_2^G$. Then,

$$C(q_t) = \ 1, \qquad \forall q_t \in [0,1]$$

This proves our result. ▮

Proof of Proposition 6:

Let us show that the introduction of a gang in the economy implies that the negative impact of p on $\bar{q}$ (interior steady-state equilibrium when there is no gang) is weaker than the negative impact of p on $\bar{q}^G$ (interior steady-state equilibrium when there is a gang), i.e.

$$\frac{d\overline{q}^G}{dp} < \frac{d\overline{q}}{dp}.$$

First, note that $w^{G*} > \overline{w}_1^G$ (i.e. case (iii) in Proposition 4) is equivalent to:

$$p < \frac{1/2(\beta + H) - (w + 1/2 + K)}{\sigma + 1/2(\beta + H)}.$$

Suppose that p is low enough so that we are in case (iii). The dynamics is given by (16), i.e.

$$\Delta q_t = q_t(1 - q_t)\left[4q_t(1 - q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] - 1\right] = q_t(1 - q_t)\left[4h^G(q_t) - 1\right]$$

where

$$h^G(q_t) \equiv q_t(1 - q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right]$$

First, it is easily shown that that $\overline{q}^G < \overline{q}$ where $\overline{q}$ is the interior solution for which $\Delta q_t(\overline{q}) = 0$, where Δq_t is defined by (8), and where $\overline{q}^G$ is such that $\Delta q_t(\overline{q}^G) = 0$ where Δq_t is defined by (16). Let us now show that:

$$\frac{d\overline{q}^G}{dp} < \frac{d\overline{q}}{dp}.$$

We will proceed in several steps.

1. Let consider the case *without* a gang. The dynamics of q_t is given by (8). Denote (see (23))

$$h(q_t) \equiv q_t(1 - q_t)\left[\Delta^B - pC(q_t)\left(\Delta^B - \Delta^S\right)\right]$$

where

$$C(q_t) = -q_t K + (1 - p)\beta - p\sigma - w$$

By differentiating (8), we obtain:

$$\frac{d\Delta q_t}{dq_t}|_{q_t=\overline{q}} = (1 - 2\overline{q})\left[4h(\overline{q}) - 1\right] + \overline{q}(1 - \overline{q})4h'(\overline{q}) = \overline{q}(1 - \overline{q})4h'(\overline{q}).$$

We also have:

$$\frac{d\Delta q_t}{dp}|_{q=\overline{q}} = 4\overline{q}(1 - \overline{q})\frac{dh}{dp}|_{q=\overline{q}}.$$

Therefore,

$$\frac{d\overline{q}}{dp} = -\frac{4\overline{q}(1 - \overline{q})\frac{dh}{dp}|_{q=\overline{q}}}{4\overline{q}(1 - \overline{q})h'(\overline{q})} = -\frac{\frac{dh}{dp}|_{q=\overline{q}}}{h'(\overline{q})}. \tag{24}$$

2. Let consider the case *with* the gang. We have:

$$\frac{d\overline{q}^G}{dp} = -\frac{\frac{dh^G}{dp}|_{q=\bar{q}^G}}{h^{G'}(\overline{q}^G)}. \tag{25}$$

3. Let us compare these two derivatives (24) and (25). We have:

$$\frac{dh^G}{dp}|_{q=\bar{q}^G} = -(\Delta^B - \Delta^S),$$

and, if we consider low values of p (see the proof of Proposition 1), then:

$$\frac{dh}{dp}|_{q=\bar{q}} = -\left[C\left(q_t\right) - p(\beta+\sigma)\right](\Delta^B - \Delta^S) < 0$$

Furthermore, since $C\left(q_t\right) - p(\beta+\sigma) < 1$, we have

$$\frac{dh}{dp}|_{q=\bar{q}} > \frac{dh^G}{dp}|_{q=\bar{q}^G}$$

In addition, we have:

$$h'(\overline{q}) = 4(1-2\overline{q})\left[\Delta^B - pC\left(q_t\right)(\Delta^B - \Delta^S)\right] + 4p\overline{q}(1-\overline{q})K(\Delta^B - \Delta^S)$$

$$h^{G'}(\overline{q}^G) = 4(1-2\overline{q}^G)\left[\Delta^B - p(\Delta^B - \Delta^S)\right]$$

One can show that when

$$(4\sqrt{5}-8)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] > 1$$

and, thus, we have $h'(\overline{q}) < h^{G'}(\overline{q}^G)$. Indeed, note that since $\overline{q} > \overline{q}^G$, we have $h'(\overline{q}) < h'(\overline{q}^G)$. Furthermore, we showed above that $h'(\overline{q}^G) < h^{G'}(\overline{q}^G)$ so that the first inequality follows.

The inequality $h'(\overline{q}^G) < h^{G'}(\overline{q}^G)$ holds if and only if

$$\begin{aligned}
&(1-2\overline{q}^G)\left[\Delta^B - pC\left(q_t\right)(\Delta^B - \Delta^S)\right] + p\overline{q}^G(1-\overline{q}^G)K(\Delta^B - \Delta^S) > (1-2\overline{q}^G)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] \\
&\Leftrightarrow -p(2\overline{q}^G - 1)\left[1 - C\left(q_t\right)\right](\Delta^B - \Delta^S) + p\overline{q}^G(1-\overline{q}^G)K(\Delta^B - \Delta^S) < 0 \\
&\Leftrightarrow \overline{q}^G(1-\overline{q}^G)K < (2\overline{q}^G - 1)\left[1 - C\left(q_t\right)\right].
\end{aligned}$$

Suppose that $\overline{q}^G > \frac{1-\sqrt{5}}{2}$, which is equivalent to:

$$(4\sqrt{5}-8)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] > 1,$$

then, we have $\overline{q}^G(1-\overline{q}^G) < 2\overline{q}^G - 1$ so that a sufficient condition for the later inequality is $K < 1 - C(q_t)$, that is:

$$1 - [\beta - p(\beta+\sigma) - w - (1-q_t)K] > 0$$

which is always true. The inequality implies

$$\frac{1}{-h^{G\prime}(\overline{q}^G)} > \frac{1}{-h'(\overline{q})}.$$

We deduce that:

$$\frac{\frac{dh^G}{dp}|_{q=\bar{q}^G}}{-h^{G\prime}(\overline{q}^G)} < \frac{\frac{dh}{dp}|_{q=\bar{q}}}{-h'(\overline{q})} \quad \text{when} \quad \frac{dh}{dp}|_{q=\bar{q}} < 0.$$

Hence weak incarceration policies have a higher negative impact on long run honesty.

4. Using similar arguments we can show that this result holds when the dynamics is given by (14), $\forall q_t \leq \widehat{q}_2$, i.e. case (i) when $w^{G*} < \overline{w}_2^G < \overline{w}_1^G$, that is:

$$\Delta q = q_t(1-q_t)\left[2q_t(1-q_t)\left[\Delta^B - \left[q_t\theta^h + (1-q_t)\right]p(\Delta^B - \Delta^S)\right] - 1\right]$$

or when the dynamics is given by (15), $\forall q_t \leq \widehat{q}_2$, i.e. case (ii) when $\overline{w}_2^G < w^{G*} < \overline{w}_1^G$, that is:

$$q_t(1-q_t)\left[2q_t(1-q_t)\left[\Delta^B - \left[q_t\theta^h + (1-q_t)\right]p(\Delta^B - \Delta^S)\right] - 1\right]$$

5. Using a similar reasoning, we can deduce that:

$$\frac{\frac{dh^G}{dp}|_{q=\bar{q}^G}}{-h^{G\prime}(\overline{q}^G)} < \frac{\frac{dh}{dp}|_{q=\bar{q}}}{-h'(\overline{q})} \quad \text{when} \quad \frac{dh}{dp}|_{q=\bar{q}} < 0.$$

for high values of p. That is, high incarceration policies have a lower positive impact on long run honesty when a gang has formed.

6. Also, one can show that a unique low steady-state q_t exists for a larger set of p in the presence of a gang.

7. When H is high, high incarceration policies have negative effects on the long-run q_t. To show this, simply note that:

$$\frac{dw^{G*}}{dH} > 0.$$

Hence, for H sufficiently high, we are in case (iii) for high values of p. This means that the dynamics is determined by:

$$\Delta q_t = q_t(1-q_t)\left[2q_t(1-q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] - 1\right]$$

Hence,

$$\frac{d\overline{q}^G}{dp} = -\frac{\partial \Delta q_t/\partial p}{\partial \Delta q_t/\partial q} < 0,$$

since we know that $\partial \Delta q/\partial q|_{q=\overline{q}^G} < 0$. Remember that, when there is no gang, for high values of p, the incareration policy had a positive impact on the long-run value of q_t. When H is high, the presence of the gang leads to a negative impact of such policy. The gang has thus a qualitative impact on the efficiency of incarceration policies.

8. When p is high (cases (i) and (ii)), whenever $q >$ the dynamics is given by... and no steady states???

Note that $w^{G*} > \overline{w}_1 < \overline{w}_2$ (i.e. case (i) in the text) is equivalent to

$$p < \frac{1/2(\beta + H) - (w + 1/2 + K)}{\sigma + 1/2(\beta + H)}.$$

When p is low the dynamics of honesty is determined by:

$$\Delta q_t = q_t(1-q_t)\left[2q_t(1-q_t)\left[\Delta^B - p(\Delta^B - \Delta^S)\right] - 1\right].$$

First, we know using the arguments developed in proof of Proposition 1 that this function has two steady states at $p = 0$.

When the economy converges to the high stationary state fraction of honest agents, the gang negatively affects the interior stationary fraction of honest agents. Whatever the

dynamics of q_t, we have:

$$\frac{d\overline{q}}{dp} = -\frac{\frac{\partial \Delta q_t}{\partial p}}{\frac{\partial \Delta q_t}{\partial q}}$$

Second, we know that at the set of the threshold value of p such that for any q_0 the economy converges to $q = 0$ is lower. ▮

Proof of Proposition 7: Remember that we assumed that $q_1 \geq q_2$ or $q_1 \geq Q/2$ (the fraction of honest individuals is higher in neighborhood 1) and $\rho^i_{2,t} = 0$ (the land rent is normalized to zero in neighborhood 2). As a result, there is complete segregation, i.e. all honest (type h) agents live in neighborhood 1 and all dishonest agents (type d) live in neighborhood 2, if and only if type$-h$ parents are willing to bid more than type$-d$ parents to live in neighborhood 1. Formally, using (21), the *segregated equilibrium* exists and is unique if and only if

$$\Delta\rho(q_1) > 0, \quad \forall q_1 \geq \frac{Q}{2}.$$

Let us now focus on the *symmetric equilibrium*, i.e. half of the "honest" families reside in neighborhood 1 and the other half in neighborhood 2. First, observe that the symmetric equilibrium always exists. One must check whether it is stable. Stability is defined as follows: for a small increase (resp. decrease) in the fraction of type$-h$ agents in neighborhood 1, agents of type d (resp. h) are willing to bid more than agents of type h (resp. d). Formally, the symmetric equilbrium is stable if and only if

$$\frac{d\Delta\rho(q_1)}{dq_1}|_{q_1=Q/2} < 0.$$

Note that the symmetric equilibrium is unique if and only if

$$\frac{d\Delta\rho(q_1)}{dq_1} \leq 0, \quad \forall q_1 \geq \frac{Q}{2}.$$

Using (20), the function $\Delta\rho(q_1)$ is positive for any q_1 if and only if: $q_1^2(1-q_1)^2 > q_2^2(1-q_2)^2$, which is equivalent to:

$$q_1(1-q_1) > q_2(1-q_2) \Leftrightarrow q_1 - q_2 > (q_1 - q_2)(q_1 + q_2)$$

Since $q_1 \geq q_2$, this is equivalent to: $q_1 + q_2 < 1$, that is $Q < 1$. We deduce that

$$\Delta\rho(q_1) > 0, \quad \forall q_1 \geq \frac{Q}{2} \quad \Leftrightarrow Q < 1,$$
$$\Delta\rho(q_1) < 0, \quad \forall q_1 \geq \frac{Q}{2} \quad \Leftrightarrow Q > 1.$$

This proves the result. ▮

Proof of Proposition 8:

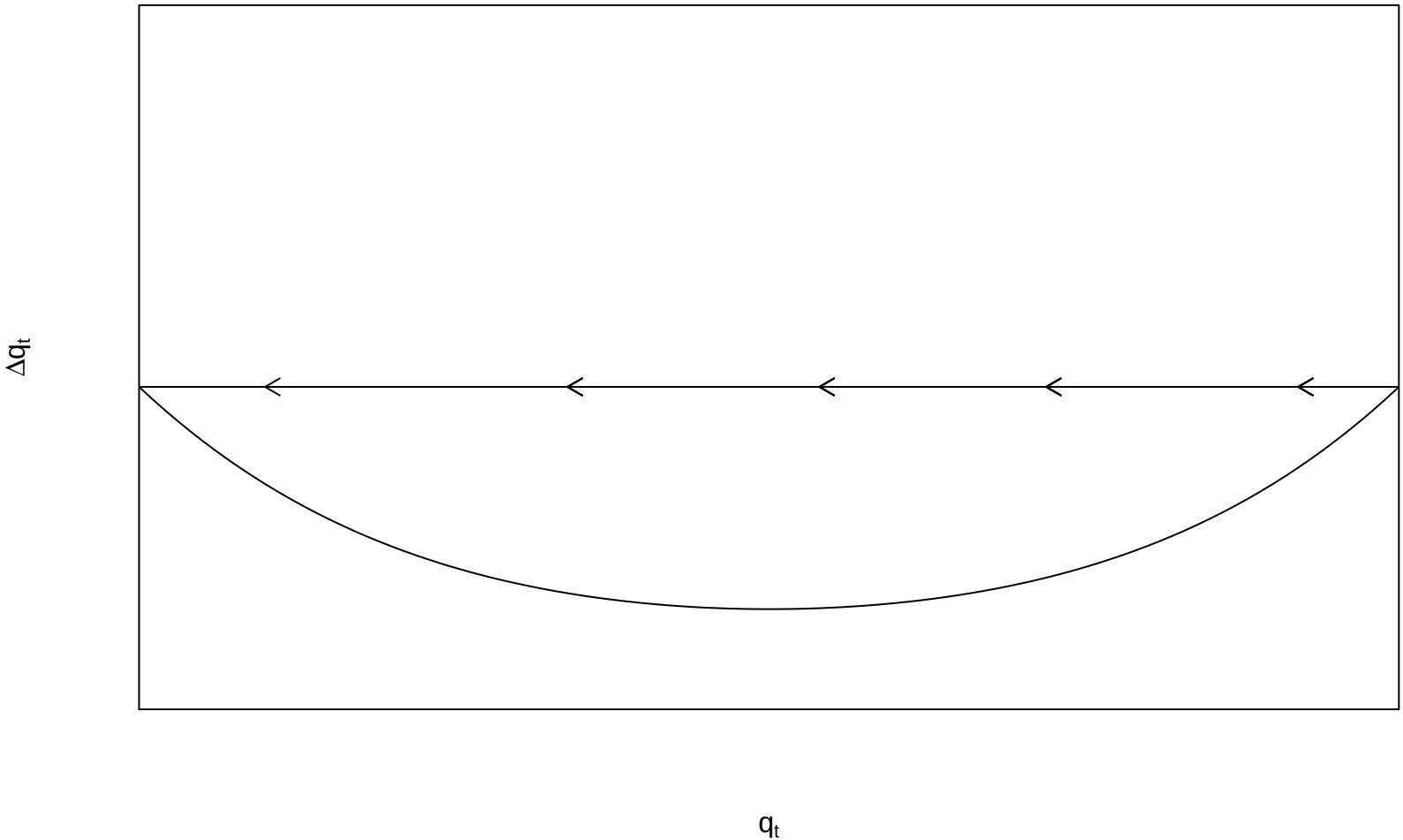
(i) Suppose that $Q_0 \in \left[0, \underline{q}\right[$, i.e. the initial fraction of honest families is low. First, by virtue of Proposition 7, at time $t = 0$, since $Q_0 < 1$, the unique urban equilibrium is *segregated* and we have $q_{1,0} \in \left[0, \underline{q}\right[$and $q_{2,0} = 0$. After socialization choices have been made, we have, $q_{2,1} = f(q_{2,0}) = 0$, and since $q_{1,0} < \underline{q}$, $q_{1,1} = f(q_{1,0}) < f(\underline{q}) = \underline{q}$. We deduce that, for all $t \geq 0$, $q_{1,t} < \underline{q}$, $q_{2,t} = 0$. For any $q_{1,t} \in \left[0, \underline{q}\right[$, the sequence $q_{1,t}$ is decreasing and converges to zero (see proof of Proposition 1). Hence, in the long run we have $q_1^* = 0$ and $q_2^* = 0$.

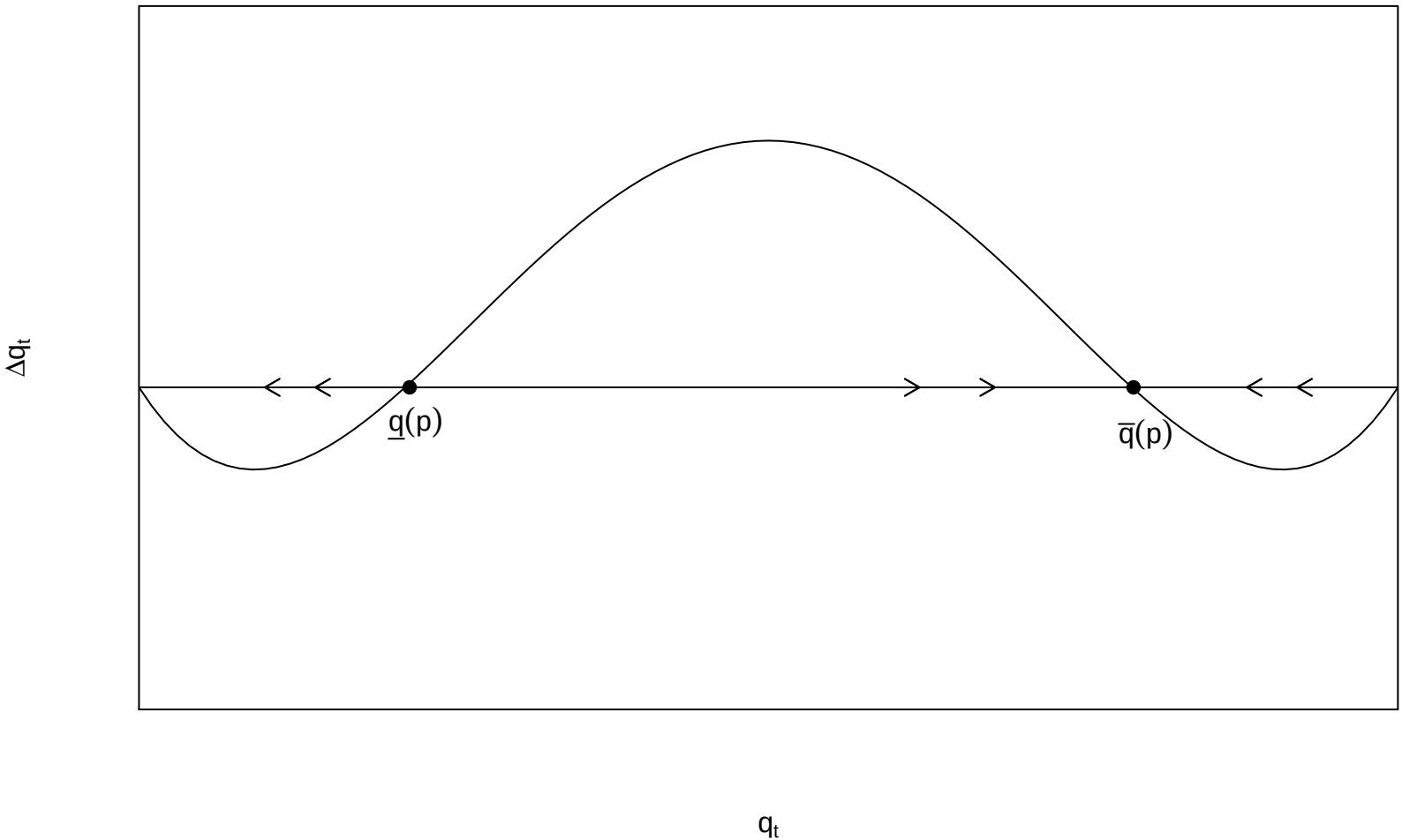
(ii) Suppose now that $Q_0 \in \left[\underline{q}, 2\underline{q}\right[$.

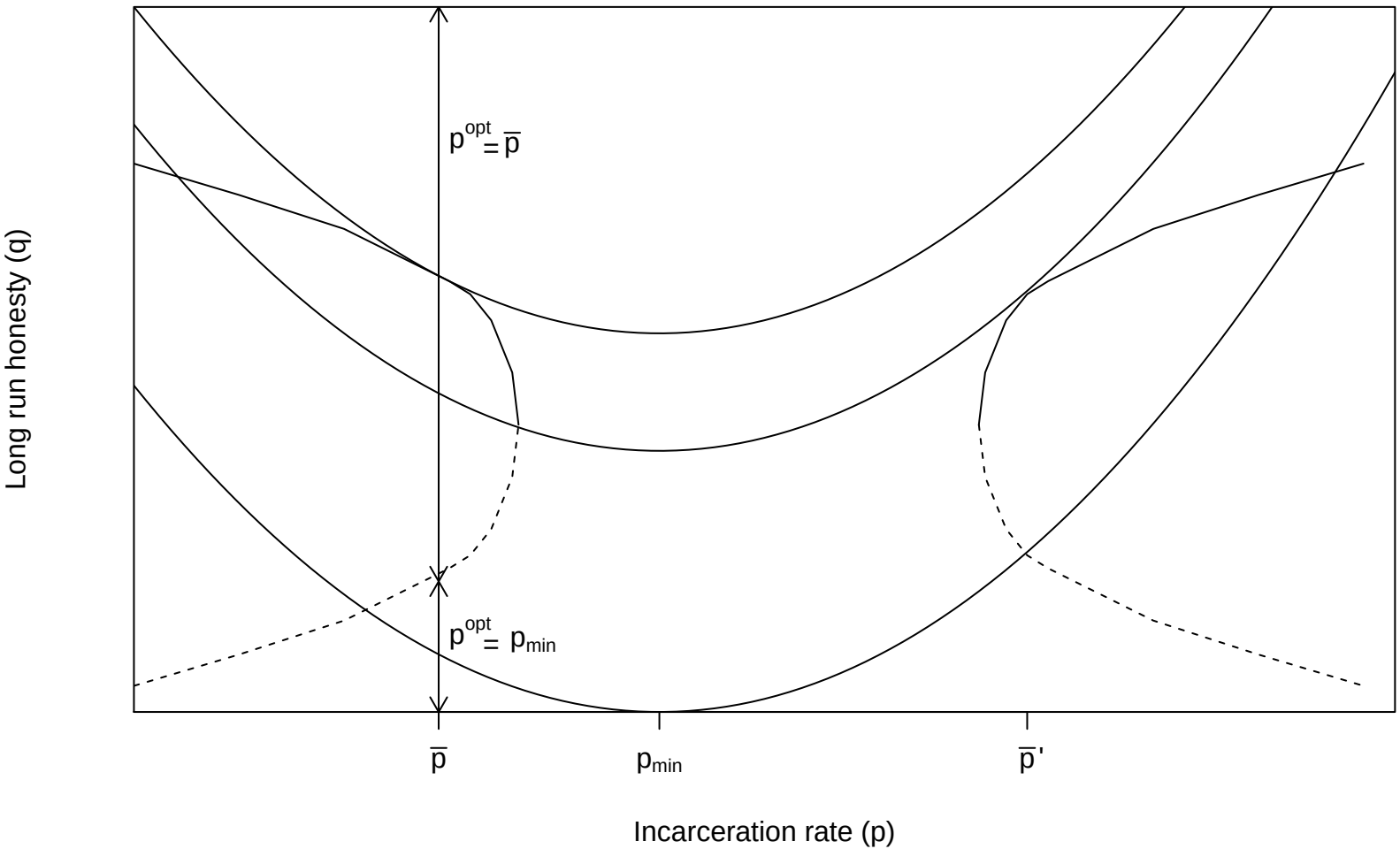
If $Q_0 < 1$, the unique urban equilibrium at time $t = 0$ is *segregated* so that $q_{1,0} \in \left[\underline{q}, 1\right[$ and $q_{2,0} = 0$. After socialization choices have been made, we have, $q_{2,1} = f(q_{2,0}) = 0$, and since $q_{1,0} \geq \underline{q}$, $q_{1,1} = f(q_{1,0}) \geq f(\underline{q}) = \underline{q}$. Using the arguments developed in the proof of Proposition 1, we have that, for any $q_{1,t} \in \left[\underline{q}, \overline{q}\right]$ (resp. $[\overline{q}, 1]$) the sequence $q_{1,t}$ is increasing (resp. decreasing) and converges to $\overline{q}$ (resp. $\overline{q}$). In the long run, we thus have $q_1^* = \overline{q}$ and $q_2^* = 0$.

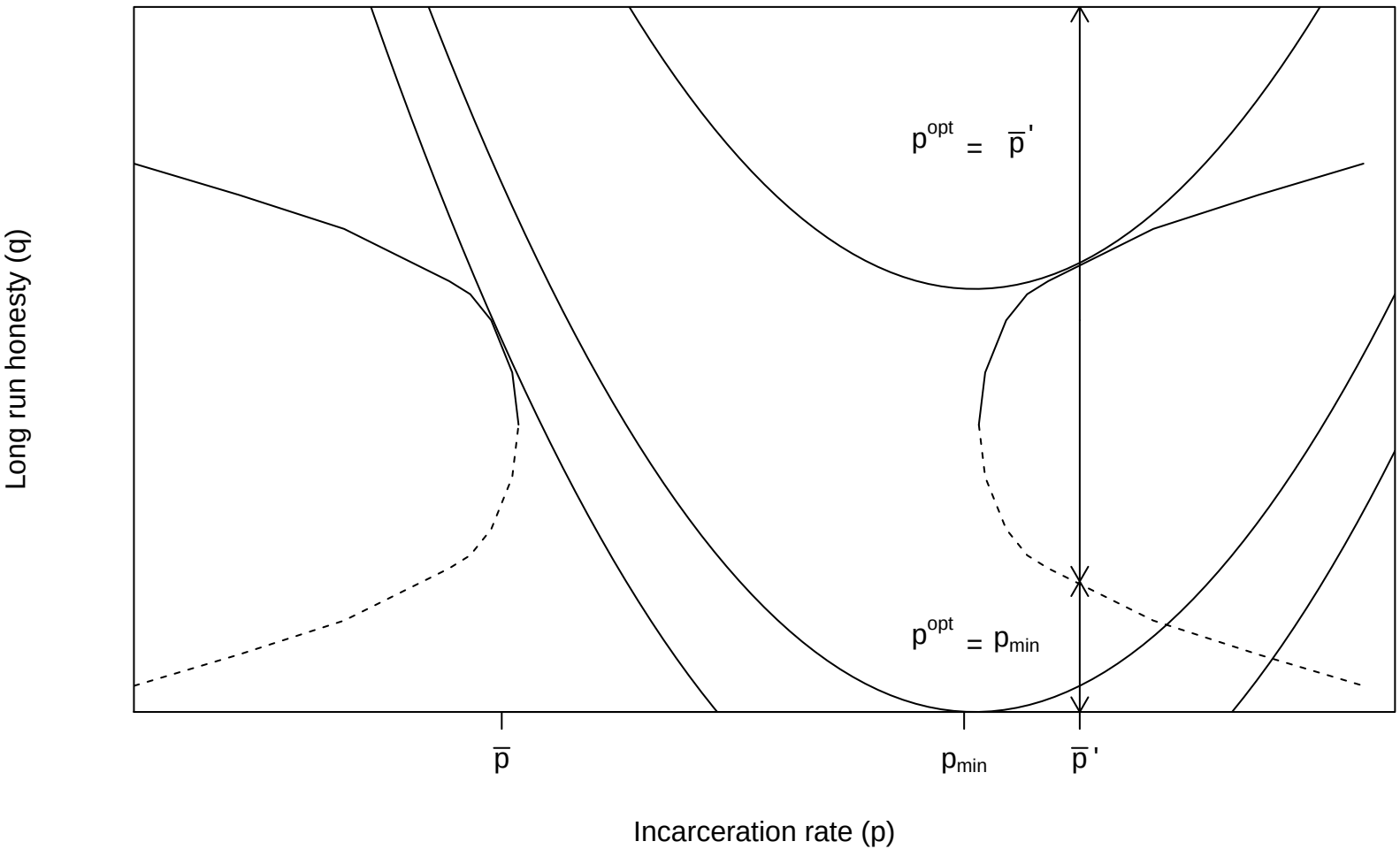
If $Q_0 > 1$, the unique urban equilibrium at time $t = 0$ is *integrated* so that $q_{1,0} = q_{2,0} = \frac{Q_0}{2}$. After socialization choices have been made, we have, $q_{1,1} = q_{2,1} = f(q_{2,0}) < f(\frac{Q_0}{2}) < \underline{q}$. Using the arguments developed in the proof of Proposition 1, we have that, for any $q_{1,t}, q_{2,t} \in \left[0, \underline{q}\right[$, the sequences $q_{1,t}$ and $q_{2,t}$ are decreasing. We deduce that there exists some t such that $q_{1,t} + q_{2,t} = Q_t < 1$ (segregated equilibrium) and we are exactly in the above case so that $q_1^* = \overline{q}$ and $q_2^* = 0$.

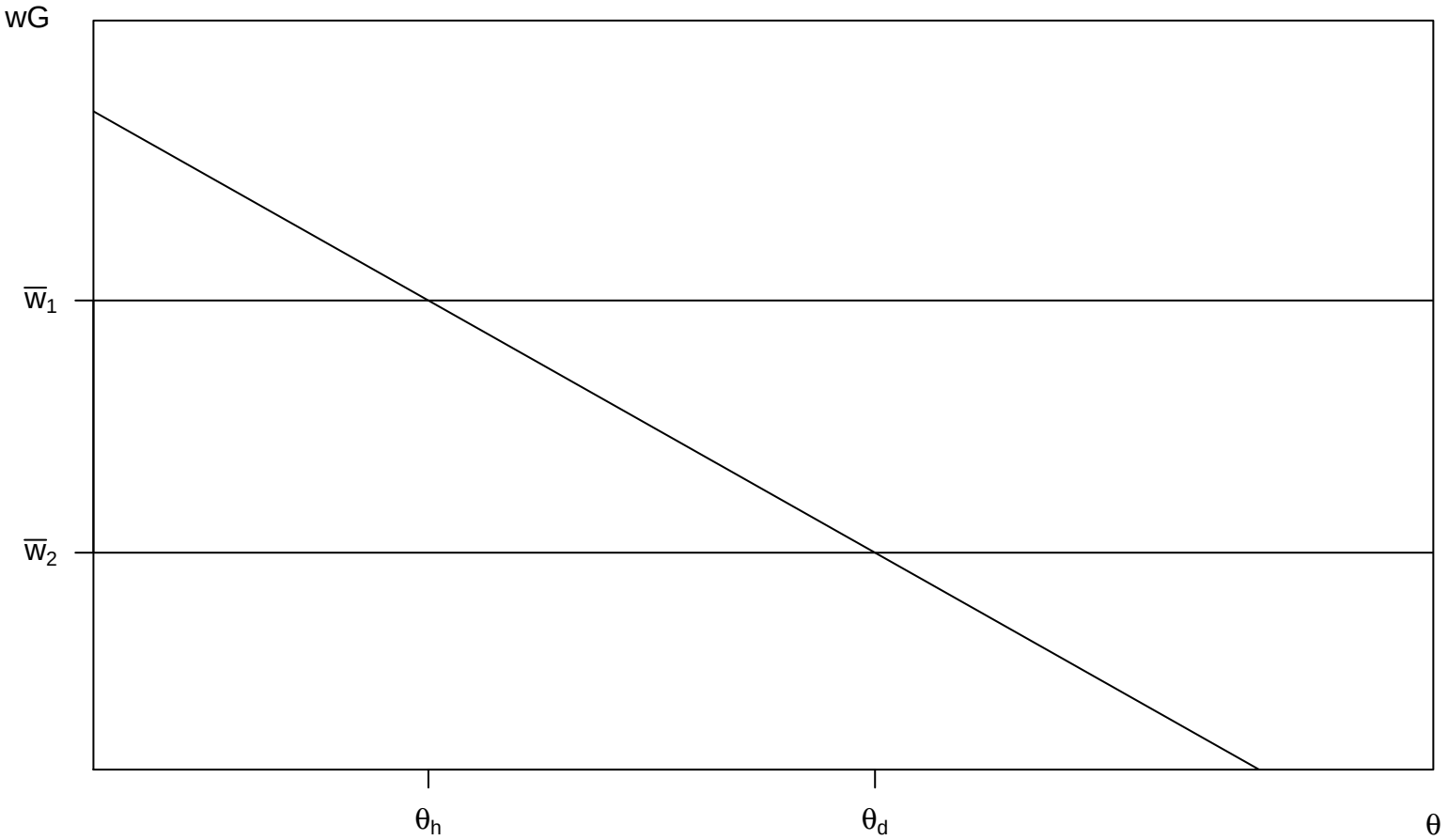
(iii) Suppose that $Q_0 \in \left[2\underline{q}, 2\right]$. Using a similar reasoning, we can deduce that the sequences $q_{1,t}$ and $q_{2,t}$ are increasing for $q_{1,t} = q_{2,t} \in \left[2\underline{q}, 2\overline{q}\right]$ and decreasing for any $q_{1,t} = q_{2,t} \in [2\overline{q}, 2]$. We deduce that, in the long run, we have $q_1^* = q_2^* = \overline{q}$. ▮


Δq_t
q_t


Δq_t
$\underline{q}(p)$
$\overline{q}(p)$
q_t


$p^{opt} = \overline{p}$
$p^{opt} = p_{min}$
Long run honesty (q)
$\overline{p}$
p_{min}
$\overline{p}'$
Incarceration rate (p)


$p^{opt} = \bar{p}'$
$p^{opt} = p_{min}$
Long run honesty (q)
$\bar{p}$
p_{min}
$\bar{p}'$
Incarceration rate (p)


wG
$\overline{w}_1$
$\overline{w}_2$
θ_h
θ_d
θ