

# Listing renewal with phantom vacancies

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March 2017 - preliminary

## Abstract

On job boards, obsolete information leads job seekers to apply for young vacancies and employers to renew their listings. This paper provides a search equilibrium model of unemployment that rationalizes this claim. Setting up a job comes at a fixed cost and renewing the ad is cheaper. Match formation creates phantom vacancies and workers condition their search on the listing age. We calibrate the model on US aggregate data and examine its (steady-state) response to productivity shocks. In bad times, firms create few new jobs, renew their ads more frequently and job seekers concentrate their search on young listings. There are many phantom vacancies as a result, which contributes to matching frictions and unemployment.

**JEL codes:** J60, D83

**Keywords:** Directed search; equilibrium unemployment; obsolete information

# 1 Introduction

Why do firms renew their listings on job boards? In this paper, we explain this behavior by the presence of phantom vacancies, i.e., ads for already filled jobs. Older listings are more likely obsolete, which implies that job seekers prefer younger ads. Therefore applications per listing fall with listing age. This leads employers to renew their listings after the vacancy spent some time unfilled in the market. We argue that this behavior is stronger in bad economic times. When vacancies are scarce, workers are especially willing to consider new listings. In turn firms have incentive to renew their listings faster. This mechanism favors phantom formation, thereby contributing to informational frictions and induced unemployment.

Most job boards offer employers the possibility to repost or renew their listings. This is generally costly and comes with limits. For instance on Craigslist, a classified website with a section for jobs, reposting involves paying the fee for a new ad. Employers cannot repost before 48 hours have elapsed since the previous posting. From a cross-section of all job listings on the US part of Craigslist’s website, we can estimate that between 30% and 50% of the listings have a twin advertising for the same job. Though a number of these twins actually advertise for similar but different jobs, the phenomenon of reposting is arguably a serious one.

Meanwhile there is evidence that job seekers target their search towards recently posted job listings. The analysis of a 2012 cross-section of applications to vacancies in the DHI Vacancy and Flow Applications Database reveals that “job seekers exhibit a striking propensity to target new and recently posted vacancies: 39 percent of applications flow to vacancies posted in the last 48 hours, and 59% go to those posted in the last 96 hours. Older postings attract relatively few applications.” (DHI Hiring Indicators, October 2016). Using older data, Faberman and Menzio (2017) show the cross-sectional distribution of vacancy duration across all hires in 1982. They find that 35% of all hires are from vacancies aged one week at most, 20% from vacancies aged between one and two weeks, 5% between two and three weeks. They also report that the number of applications per week decreases from 22.3 in the first week to 1.5 after one month.

To analyze this market situation, we enrich our model of directed search with phantom vacancies (Albrecht et al, 2017). In this previous paper there is a fixed number of jobs, match formation creates phantom vacancies and job seekers condition their search on the listing age. In equilibrium, the job-finding rate is the same for all possible listing age. As the proportion of obsolete ads rises with listing age, the job queue, the ratio of seekers to listings, falls with listing age. In the current paper, we focus on the supply of vacancies. Namely we consider firms’ incentive to create jobs and renew their listings.

Our model distinguishes the cost of setting-up a job from the (smaller) cost of renewing a listing. In exchange for the cost of vacancy creation, the firm holds a new listing that ages with calendar time. The listing can be renewed at any time provided the firm pays the renewal cost. In the absence of phantoms, firms would have no interest in renewing their listing. Time spent in the market would not affect the odds of filling the job and paying the renewal cost would be useless. Things are different with phantoms. The density of applications per listing decreases with listing age. At some point, firms are willing to pay the cost to benefit from a better chance of recruiting an employee.

Therefore our model predicts the inflow of newly created vacancies, the limit age at which firms renew

their listing, and the distribution of workers' applications by listing age. We focus on the sensitivity of these variables to macroeconomic shocks likely to affect the supply of vacancies. In practice we follow the rest of the literature by calibrating our model and examining its reactions to unexpected permanent productivity shocks.

Our calibrations deliver a key lesson: the interaction between job seekers' behavior, vacancy creation and listing renewal significantly contribute to amplifying the model responses to aggregate productivity shocks. In bad economic times, there are few new listings and employers quickly fill in their vacancies. Phantoms accumulate at a high pace, which leads the job seekers to concentrate their search on young listings. Applications per vacancy rapidly fall with listing age, which, in turn, pushes employers to renew their vacancies earlier. Overall, there are fewer jobs, workers especially search for young listings, employers frequently renew their vacancies and the share of obsolete ads is particularly large. This combination is detrimental to the matching efficiency, thereby increasing unemployment.

Quantitatively, the aggregate state of activity has very large effects on the job-seeking behavior and listing renewal age. We compare three scenarios, each characterized by a particular value of output per worker,  $y$ : normal times where  $y$  is normalized to one, a recession where  $y$  is decreased by 2.5% and a boom where it is symmetrically increased by 2.5%. In our calibration, the renewal age is one month in normal times. This age increases by 25% during a recession and decreases by 20% during a boom. Meanwhile, about 67% of applications go to listings aged less than 48 hours in normal times, against 82% in a recession and 48% during a boom. These figures have strong implications for the magnitude of obsolete information. After 48 hours, the probability that a given listing is still available is 0.38 in a recession, 0.56 in normal times and 0.73 in a boom.

This paper relates to different strands of literature. First, we are indebted to Chéron and Decreuse (2017) and Albrecht et al (2017). Chéron and Decreuse introduce the notion of phantom vacancies and show that this consists of an intertemporal form of matching frictions. Albrecht et al examine how phantom vacancies lead the job seekers to condition their search effort to the listing age. We pursue this line of research by studying the role played by the supply of vacancies. In particular, we show that vacancy creation and listing renewal alter job seekers' search by listing age, thereby affecting the efficiency of the matching process in the labor market.

Second, there is a growing body of research devoted to the analysis of the supply of vacancies. Davis et al (2013) estimate the hazard rate of filling vacancies and examine recruitment spending during the lifetime of vacancies (see also Faberman and Menzio, 2017). The main findings are that vacancies tend to last short, typically less than a month, recruitment spendings decline with time in the market and the job-filling rate also decreases with vacancy age. We study a complementary margin that is, to our knowledge, still unexplored, i.e., the decision to renew listings. We offer a possible reason why listing renewal occurs, insert it in the standard equilibrium search model of unemployment and study its potential contribution to aggregate volatility.

Third, our paper is linked to the literature on the determinants of matching efficiency. Barnichon and Figura (2015) argue that the matching efficiency is procyclical. The matching process works better in good economic times where the job-finding rate is large than in bad economic times when it is low.

Our work provides a possible rationale to this outcome.<sup>1</sup> In booms, job seekers spread their effort more equally across listings of different ages and firms renew their vacancies less frequently. Phantoms are relatively scarce and informational frictions are reduced. More generally, our paper opens the black box of the matching function. The elasticity of the aggregate matching function is endogenous and responds to changes in the environment through the distribution of workers' search efforts by listing age, firms' entry and listing renewal decisions.

Lastly, our paper relates to the literature on stock-flow matching initiated by Coles and Smith (1998). In these models new job seekers initially try to match with the stock of vacancies, as well as new vacancies try to match with the stock of unemployed. In case this first stage is ineffective, agents on one side of the market have to wait for new agents on the other side because the older ones do not suit them. In other words, flows match with stocks. As our model based on obsolete information, stock-flow models also predict that the rate of filling jobs goes down with the duration spent in the market. However, employers have no incentive to renew their listings. Duration dependence is due to technological reasons; therefore renewing the listing does not affect the chance of filling the job.

The paper proceeds as follows. We first focus on the job creation margin (Section 2). We discuss equilibrium existence and characteristics, study the (constrained) efficient allocation and calibrate our model to quantify the effects of parameter shocks on equilibrium unemployment and vacancies, the duration of vacancies, and the distribution of applications per vacancy by listing duration. We then examine the listing renewal decision (Section 3). We introduce costs of renewing listings and then discuss their impacts on equilibrium outcomes. Lastly we revisit our calibrations. Section 4 concludes.

## 2 The model without listing renewal

In this section we develop an equilibrium search model of unemployment with directed search by listing age, phantom vacancies and endogenous job creation. However, firms cannot renew their listing. We set the model, solve it and then calibrate it to emphasize the role played by phantoms and firms' entry in the magnitude of unemployment and the distribution of job-seekers by listing age.

### 2.1 The model

The model closely follows Albrecht et al (2017). There are two main changes: first, the wage is set by Nash bargaining; second, the supply of new jobs responds to a free-entry condition.

We focus on the stationary state of a continuous time model. The unit of time is the month. There is a continuum of workers of unit mass. Each worker can be either employed or unemployed. The mass of unemployed is  $u$ . There is also a continuum of vacancies of (endogenous) mass  $v$ . Each vacancy is associated to a listing. These listings differ in age  $a \geq 0$ .

Creating a new vacancy comes at cost  $c$ . In exchange the firms owns a listing that gradually ages with calendar time. This listing is destroyed after the termination age  $A$ .

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<sup>1</sup>There are alternative explanations. See, e.g., Galenianos (2014) who examine the role played by social networks and Barnichon and Figura (2015) who focus on worker heterogeneity.

All jobs produce  $y$  and pay the bargained wage  $w$ . Employed workers and jobs separate with Poisson rate  $\lambda$ . Newly separated workers join the pool of unemployed, whereas jobs are destroyed.

The search market is segmented by listing age. In sub-market  $a$ ,  $u(a)$  unemployed try to match with  $v(a)$  vacancies. The search process is frictional. Each time there is a match, a phantom vacancy is created and persists in the market.

The flow of new matches in sub-market  $a$  is

$$M(a) = \pi(a)m(u(a), v(a) + p(a)),$$

where  $p(a)$  is the phantom flow and  $\pi(a) = \frac{v(a)}{v(a)+p(a)}$  is the nonphantom proportion. Workers cannot distinguish between phantoms and nonphantoms. Therefore the number of contacts  $m(\cdot)$  depends on the number of listings  $v(a) + p(a)$ . As none can match with a phantom,  $m$  is multiplied by the fraction of contacts with unfilled jobs. Hereafter, the variable  $\theta(a) = (v(a) + p(a))/u(a)$  is the market tightness for listings of age  $a$ . Moreover, we simply write  $m(\theta)$  for  $m(1, \theta)$  when there is no risk of confusion. Thus the job-finding rate is  $\mu(a) = m(\theta(a))\pi(a)$  and the job-filling rate  $\eta(a) = m(\theta(a))/\theta(a)$ .

For all  $a \in [0, A]$ , phantoms and vacancies evolve as follows:

$$\begin{aligned}\dot{v} &= -\eta v, \\ \dot{p} &= \eta v,\end{aligned}$$

where a dot over a variable denotes its derivative with respect to the listing age. These motions imply the number of listings stays constant across age, i.e.,  $v(a) + p(a) = v(0)$ .

The resulting nonphantom proportion has the following motion  $\dot{\pi} = -\pi(m(\theta)/\theta)$ . It decreases with age because phantoms accumulate as employers gradually fill in their job.

Workers direct their search by vacancy age. The values of being unemployed,  $U$ , and employed,  $W$ , are defined as follows:

$$rU = \max_{a \in [0, A]} \{b + \mu(a)[W - U]\}, \quad (1)$$

$$rW = w + \lambda[U - W]. \quad (2)$$

In equilibrium, job-seekers self allocate across the different listing ages so that the job-finding rate  $\mu(a) = \pi(a)m(\theta(a))$  stays constant, i.e.,  $\pi(a)m(\theta(a)) = m(\theta(0))$ . As the nonphantom proportion decreases with listing age, this condition implies that tightness increases with listing age.

Let  $V(a)$  be the value of a listing of age  $a$ . Let also  $J$  be the value of a filled job. We have

$$rV(a) = \eta(a)[J - V(a)] + V'(a), \quad (3)$$

$$rJ = y - w - \lambda J. \quad (4)$$

The value of a listing changes with the listing age, reflecting the rate of applications by vacancy age and the termination age of the listing. Solving for this differential equation with the boundary constraint  $V(A) = 0$  gives

$$V(a) = \frac{y - w}{r + \lambda} \int_a^A \eta(s) \exp \left[ \int_a^s (r + \eta(b)) db \right] ds.$$

Realizing that  $r$  must be very small before  $\eta(a)$  when  $A$  takes reasonable values, we can finally write

$$V(0) = \frac{y - w}{r + \lambda} [1 - \pi(A)],$$

where  $\pi(A)$  is the probability that the vacancy reaches age  $A$  and, therefore, coincides with the nonphantom proportion in the listing cohort of age  $A$ .

The wage is determined by Nash bargaining over the match surplus. We assume this wage can be renegotiated at any time, which explains why it is not conditional on the listing age. If  $\beta \in (0, 1)$  denotes workers' bargaining power, then we have  $(1 - \beta)(W - U) = \beta J$ . Using equations (1) to (4) and solving for the wage gives

$$\begin{aligned} w &= \beta y \frac{r + \lambda + m(\theta(0))}{r + \lambda + \beta m(\theta(0))} + (1 - \beta)b \frac{r + \lambda}{r + \lambda + \beta m(\theta(0))}, \\ J &= \frac{(1 - \beta)(y - b)}{r + \lambda + \beta m(\theta(0))}. \end{aligned}$$

To close the model, we suppose there is a free-entry condition stating that firms create new jobs until the exhaustion of all rents. Therefore  $V(0) = c$ .

## 2.2 Equilibrium

We list the different equilibrium conditions:

$$m(\theta(0)) = \pi(a)m(\theta(a)), \tag{5}$$

$$\pi'(a) = -\frac{m(\theta(a))}{\theta(a)}\pi(a), \tag{6}$$

$$c = \frac{(1 - \beta)(y - b)}{r + \lambda + \beta m(\theta(0))} [1 - \pi(A)], \tag{7}$$

$$u = \lambda / (\lambda + m(\theta(0))) \tag{8}$$

From the arbitrage condition (5), we can express tightness as a function of initial tightness and nonphantom proportion, i.e.,  $\theta(a) = m^{-1}(m(\theta(0))/\pi(a))$ . We then insert it in the motion of the nonphantom proportion (6) to obtain the following Cauchy problem  $\pi'(a) = -\frac{m(\theta(0))/\pi(a)}{m^{-1}(m(\theta(0))/\pi(a))}\pi(a)$ ,  $\pi(0) = 1$ . The solution of this problem is  $\pi(a, \theta(0))$  to highlight the dependence vis-à-vis  $\theta(0)$ . Solving reduces to find initial tightness  $\theta(0)$  such that the free-entry condition (7) holds, i.e.,  $c = \frac{(1 - \beta)(y - b)}{r + \lambda + \beta m(\theta(0))} [1 - \pi(A, \theta(0))]$ . Lastly unemployment is determined by the Beveridge curve (8).

There exists a unique equilibrium provided  $(1 - \beta)(y - b)/(r + \lambda) > c$ . The heuristic proof is as follows. The value of a filled job  $J$  decreases with initial tightness because the bargained wage increases with it. Meanwhile the nonphantom proportion  $\pi(A)$  at the termination age increases with  $\theta(0)$ , which slows the pace of vacancy transformation into filled jobs. Therefore the right-hand side of the free-entry condition is strictly decreasing in  $\theta(0)$ , implying uniqueness. Existence results from the fact that the right-hand side tends to  $(1 - \beta)(y - b)/(r + \lambda)$  when  $\theta(0)$  tends to 0.

Initial tightness qualitatively responds to changes in the economic environment in a standard way. It decreases with workers' bargaining power,  $\beta$ , unemployment income,  $b$ , rate of discount,  $r$ , job separation

rate,  $\lambda$ , and listing creation cost,  $c$ . It increases with output per worker,  $y$ . Unemployment varies accordingly.

The density function of applications per vacancy by listing age is  $\phi_u(a) = u(a)/\mathbf{u} = v(0)/(\theta(a)\mathbf{u})$  and the associated cumulative distribution function (cdf) is  $\Phi_u(a) = v(0)\mathbf{u}^{-1} \int_0^a \theta(s)^{-1} ds$ . The motion of the aggregate number of vacancies is  $dv/dt = v(0) - \int_0^A \eta(a)v(a)da - v(A)$ . In steady state,  $dv/dt = 0$  and  $v(A) = \pi(A)v(0)$ . Moreover,  $\eta(a)v(a) = m(\theta(0))u(a)$ . Therefore  $v(0) = m(\theta(0))\mathbf{u}/[1 - \pi(A)]$ . Thus  $\Phi_u(a) = [1 - \pi(A)]^{-1} \int_0^a m(\theta(0))\theta(s)^{-1} ds$ . But  $m(\theta(0))/\theta(a) = \eta(a)\pi(a)$  by the arbitrage condition (5). It follows that  $\int_0^a m(\theta(0))\theta(s)^{-1} ds = - \int_0^a \pi'(s) ds = 1 - \pi(a)$ . Therefore,

$$\Phi_u(a) = \frac{1 - \pi(a)}{1 - \pi(A)}.$$

The cdf of applications per vacancy by age is simply the ratio of the phantom proportion at age  $a$  to the phantom proportion at age  $A$ .

To conclude this section, we briefly discuss the case without phantoms. The nonphantom proportion is just one at all ages, therefore the arbitrage condition becomes  $m(\theta(a)) = m(\theta(0))$ . Thus tightness does not change across submarkets. The equilibrium reduces to the free-entry condition (7) and the Beveridge curve (8). In the free-entry condition,  $\pi(A)$  must be replaced by  $\exp(-Am(\theta(0))/\theta(0))$ . For a given  $\theta(0)$ , this quantity is smaller than with phantoms. Phantoms imply that firms are less likely to fill in their vacancies before the termination age  $A$  is reached. This implies that initial tightness  $\theta(0)$  is lower with phantoms than without them.

### 2.3 Constrained efficiency

We here discuss the social efficiency of private job creation decisions. We let  $r \rightarrow 0$  and consider a social planner who would control the inflow of new jobs, taking as given the different sources of frictions and the fact that agents spread over the different sub-markets so as to equalize job-finding rates. In our previous paper, we show that workers search too much for young vacancies. Based on calibrations we argue that the magnitude of this externality is likely small. Here we neglect it to focus on the job creation margin.

The goal of the planner is to maximize aggregate output  $(1 - \mathbf{u})y + \mathbf{u}b$  net of job listing costs  $c v(0)$ . Using the fact that the flow of newly created vacancies is  $v(0) = m(\theta(0))\mathbf{u}/[1 - \pi(A)]$ , the maximization program is the following

$$\max_{\theta(0)} \left\{ \frac{b - y - cm(\theta(0))[1 - \pi(A, \theta(0))]^{-1}}{\lambda + m(\theta(0))} \right\}. \quad (*)$$

Let  $\alpha_0 \equiv \alpha(\theta(0))$  be the elasticity of the meeting technology with respect to initial tightness and  $\gamma_0 \equiv -d \ln[1 - \pi(A, \theta(0))]/d \ln \theta(0)$  be the (opposite of the) elasticity of the phantom proportion at age  $A$  with respect to initial tightness. The first-order condition of optimality is necessary. This gives

$$\frac{\varepsilon_0(y - b)(1 - \pi(A))}{\lambda + (1 - \varepsilon_0)m(\theta(0))} = c, \quad (9)$$

where  $\varepsilon_0 \equiv \alpha_0/(\alpha_0 + \gamma_0)$ .

This expression must be compared to its decentralized counterpart:

$$\frac{(1 - \beta)(y - b)(1 - \pi(A))}{\lambda + \beta m(\theta(0))} = c. \quad (10)$$

It follows that the decentralized allocation is constrained efficient when  $\beta = 1 - \varepsilon_0$ .

There is a modified Hosios rule in our environment characterized by directed search and phantom vacancies. Nash bargaining is efficient provided it compensates firms for their overall impact on match formation. The usual elasticity of the matching function must be replaced by  $\alpha_0/(\alpha_0 + \gamma_0)$ , a modified elasticity taking into account intra and intertemporal frictions induced by phantom formation and persistence.

## 2.4 Calibrations

We solve the model when the meeting technology is Cobb-Douglas, i.e.,  $m(\theta) = B\theta^\alpha$ ,  $0 < \alpha < 1$ . Then we set the different parameters to reproduce various US labor market outcomes.

Conditional on  $\theta(0)$ , the age patterns of the nonphantom proportion and tightness can be solved explicitly. This gives

$$\pi(a) = \left[ \frac{1-\alpha}{\alpha} B\theta(0)^{\alpha-1} a + 1 \right]^{\frac{\alpha}{\alpha-1}}, \quad (11)$$

$$\theta(a) = \theta(0) \left[ \frac{1-\alpha}{\alpha} B\theta(0)^{\alpha-1} a + 1 \right]^{\frac{1}{1-\alpha}}. \quad (12)$$

The nonphantom proportion decreases with the listing age  $a$  and increases with initial tightness  $\theta(0)$ , whereas  $\theta(a)$  increases with  $a$  and  $\theta(0)$ .

Solving the equilibrium consists of finding the root  $\theta(0) = x$  of

$$\psi(x) = [(1-\beta)(y-b)/c - (r+\lambda)] - (1-\beta)(y-b)/c \left[ \frac{1-\alpha}{\alpha} Bx^{\alpha-1} A + 1 \right]^{\frac{\alpha}{\alpha-1}} - \beta Bx^\alpha.$$

The equilibrium number of vacancies is  $v = v(0) \int_0^A \pi(a) da$ . Therefore we have

$$\begin{aligned} u &= \frac{\lambda}{\lambda + B\theta(0)^\alpha}, \\ v &= \theta(0)u \frac{\alpha}{2\alpha-1} \frac{1 - \pi(A)^{\frac{2\alpha-1}{\alpha}}}{1 - \pi(A)}. \end{aligned}$$

The latter equation generically holds for  $\alpha \neq 1/2$ .

We set the termination age  $A$  to one month, i.e.,  $A = 1$ . This corresponds to the max ad duration on a job board like Craigslist. We then choose the search parameters  $B$ ,  $\alpha$ ,  $\lambda$ . Using Shimer's methodology, we compute the mean job-finding rate and the mean job separation rate for the period 2000-2008. This gives  $\mu = 0.5$  and  $\lambda = 0.03$ . The corresponding unemployment rate is  $u = \lambda/(\lambda + \mu) = 0.0563$ . Moreover,  $B\theta(0)^\alpha = \mu$ , which gives a first relationship between  $B$ ,  $\theta(0)$  and  $\alpha$ .

The second relationship follows from Albrecht et al (2017) who match the average duration of a vacancy. Davis et al (2013) estimate that this duration is between 14 and 25 days. We target the mean of this interval, i.e., 19.5 days. To compute this duration in our model, we suppose that all unfilled jobs are reposted (and so firms repay the vacancy creation cost  $c$ ). Then the average duration of a vacancy is  $d = \int_0^A \pi(a)(1 - \pi(A))^{-1} da = (v/u)/\mu$ , where

$$v/u = \theta(0) \frac{\alpha}{2\alpha-1} \frac{1 - \pi(A)^{\frac{2\alpha-1}{\alpha}}}{1 - \pi(A)}.$$



The third relationship derives from the elasticity of the job-finding rate with respect to  $v/u$ . Using monthly JOLTS vacancy data for the period 2000-2008, we regress the log job-finding rate on the log vacancy-to-unemployed ratio and obtain an elasticity slightly below 0.4 (see also Shimer, 2005), so we say 0.4. This target may be misleading because of various endogeneity issues. However, in the absence of a clear model of such sources of endogeneity, we stick to 0.4 so as to broadly fit the empirical Beveridge curve on the 2000-2008 period (see Figure 1 below and the subsequent discussion).

The corresponding theoretical elasticity is

$$\begin{aligned} \frac{d \ln \mu}{d \ln(v/u)} &= \frac{d \ln \mu / d \ln \theta(0)}{d \ln(v/u) / d \ln \theta(0)} \\ &= \frac{\alpha}{1 + (1 - \alpha) \frac{\mu}{\theta(0)} A \left\{ \frac{2\alpha - 1}{\alpha} \frac{\pi(A)}{\pi(A)^{\frac{2\alpha - 1}{\alpha} - 1}} + \frac{\pi(A)^{1/\alpha}}{1 - \pi(A)} \right\}}. \end{aligned}$$

We numerically solve the conditions  $v/u = 19.5$  and  $\frac{d \ln \mu}{d \ln(v/u)} = 0.4$  to find  $\theta(0)$  and  $\alpha$ . Then we set  $B = \mu\theta(0)^{-\alpha}$ .

We turn to parameters  $y$ ,  $b$ ,  $r$  and  $\beta$ . We normalize  $y = 1$  and set  $r = 0.003$ , which corresponds to an annual rate of 3.5%. We set  $\beta$  to the modified Hosios value, i.e.,  $\beta = 1 - \varepsilon_0$ . As

$$\gamma_0 = \alpha \frac{(1 - \alpha)\mu A / \theta(0)}{(1 - \alpha)\mu A / \theta(0) + \alpha},$$

we have

$$\beta = \frac{\gamma_0}{\gamma_0 + \alpha} = \frac{(1 - \alpha)\mu A / \theta(0)}{2(1 - \alpha)\mu A / \theta(0) + \alpha}.$$

To fix  $b$  we match the elasticity of the unemployment rate with respect to productivity shocks. The relative volatility of unemployment with respect to output per worker is about 10. The corresponding theoretical elasticity is

$$\frac{d \ln u}{d \ln y} = -\alpha(1 - u) \frac{y}{y - b} \frac{r + \lambda + \beta\mu}{\gamma_0(r + \lambda + \beta\mu) + \alpha\beta\mu}.$$

We invert it and this gives  $b$ .

Lastly, we derive the listing cost from the free-entry condition. Namely,

$$c = (1 - \beta) \frac{y - b}{r + \lambda + \beta B x^\alpha} [1 - \pi(A)].$$

Table 1 shows the parameter set.

$A$	$\lambda$	$B$	$\alpha$	$y$	$b$	$\beta$	$r$	$c$
1	0.03	1.07	0.16	1	0.93	0.35	0.003	0.14

Table 1: Parameters of the baseline calibration

The elasticity  $\alpha$  is lower than the elasticity of the job-finding rate,  $\mu$ , with respect to the vacancy-to-unemployed ratio,  $v/u$ . An increase in  $v/u$  not only raises the rate of contacts at all ages, which  $\alpha$  captures well, but also the nonphantom proportion at all ages. It also implies a reallocation of workers across listing ages. Overall the elasticity of  $\mu$  with respect to  $v/u$  is larger than  $\alpha$ .

Unemployment income is close to output per worker. This is the small-surplus calibration initially suggested by Hagedorn and Manovski (2008). Lastly, the listing cost is about 15% of monthly output per worker. This is reasonable once accounting for all hiring and job creation costs.

We then investigate the model behavior. We suppose that output per worker  $y$  varies from -3% to +3%. This implies changes in initial tightness  $\theta(0)$ , and corresponding impacts on  $u$  and  $v$ , as well as on the distribution of applications per vacancy by vacancy duration and the job queue by vacancy duration.

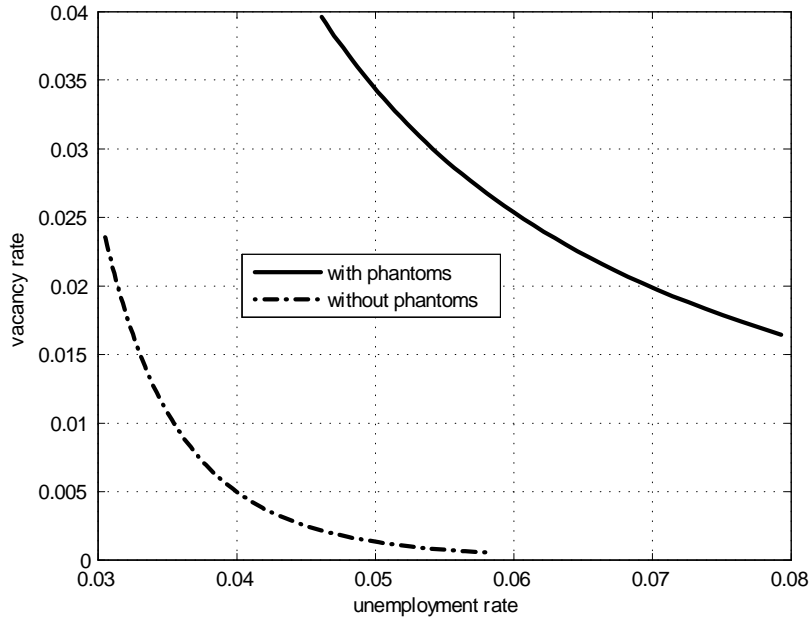


Figure 1: Beveridge curve. The curve with phantoms corresponds to the baseline calibration in which we let  $y$  varies from 0.97 to 1.03. The curve without phantoms involves the same parameters, but obsolete ads do not persist. The vacancy rate is rescaled so that the mean vacancy-to-unemployed ratio is equal to one half.

Figure 1 shows the Beveridge curve in the  $(u, v)$  plane. Each point corresponds to a steady state of our model. We also plot the counterfactual Beveridge curve that would obtain without phantoms. The Beveridge curve with phantoms is roughly consistent with the empirical Beveridge curve. The main reason is that we broadly reproduce the empirical elasticity of the job-finding rate with respect to tightness. While regressing the simulated log job-finding rate on simulated log tightness, we obtain  $d \ln \mu / d \ln (v/u) \approx 0.4$ .

The Beveridge curve without phantoms stands on the left of the Beveridge curve with phantoms. It is also more convex. Both properties are due to phantom vacancies: they reduce the matching efficiency and increase the elasticity of the matching technology with respect to vacancies.

From this set of simulations, we extract three scenarios: *normal times* correspond to the baseline calibration where  $y = 1$ . In a *recession*,  $y = 0.975$ , a decrease by 2.5%. In a *boom*,  $y = 1.025$ , a symmetric increase by 2.5%. Table 2 shows the steady-state aggregate variables in the three cases.

	$u$	$v/u$	$\mu$	$d$	$\bar{\pi}$
recession	7.26%	0.26	0.39	0.43	0.32
normal	5.63%	0.5	0.51	0.64	0.42
boom	4.77%	0.78	0.61	0.84	0.50

Table 2: Aggregate labor market variables in the three scenarios

In a recession  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom. The  $v/u$  ratio has been rescaled so that it is equal to 1/2 in normal times. The mean duration of a vacancy,  $d$ , is given in months.

The elasticity of the unemployment rate to output per worker is stronger in bad times than in good times. It is  $(7.26 - 5.63)/5.63 \approx 30\%$  when getting from normal times to a recession and  $(4.77 - 5.63)/5.63 \approx -15\%$  when getting to a boom. Another striking feature shown by Table 2 is the response of the mean nonphantom proportion. It is 50% higher in a recession than in a boom. These figures suggest that phantoms are very widespread, as they compose 58% of the ads in the baseline calibration.

Figure 2 shows the distribution of job seekers by listing age under the three scenarios. It reveals a strong preference for recently created jobs, a property already emphasized in Albrecht et al (2017). However, this behavior is stronger in bad times. Table 3 shows the probability mass of job-seekers at different listing age intervals. Job-seekers send their applications to listings aged less than 48 hours in 58% of the cases during booms. This proportion increases to 67% in normal times and 78% in recessions.

	density of job-seekers at				
	day 1	day 2	day 3	day 4	week 1
recession	0.70	0.08	0.04	0.03	0.89
normal	0.56	0.11	0.06	0.04	0.84
boom	0.45	0.13	0.07	0.05	0.79

Table 3: Probability mass of job seekers at various listing ages in the three scenarios

In a recession,  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom. Reading: the probability mass of job-seekers who search for job listings aged between 24 and 48 hours is 0.11 in normal times.

Figure 3 shows the nonphantom proportion by listing age. As evidenced by Table 2, it is much larger in a boom than in a recession. Lack of job creation implies that vacancies last shorter. Therefore phantoms contaminate all listing cohorts, thereby contributing to overall frictions and raising unemployment.

### 3 Accounting for vacancy renewal

The purpose of this section is to endogenize the termination age  $A$ . We first modify the previous model to allow for listing renewal and then turn to calibrations.

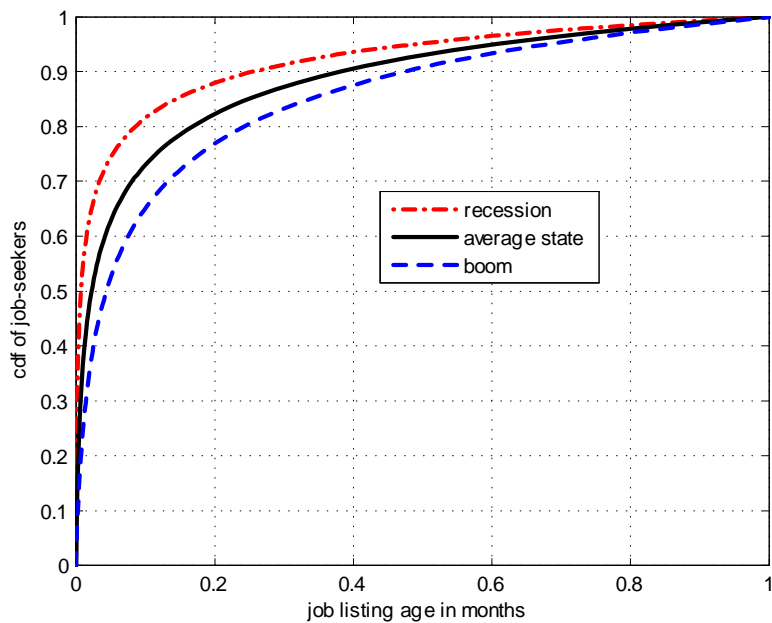


Figure 2: Distribution of job-seekers by listing age. In a recession,  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom.

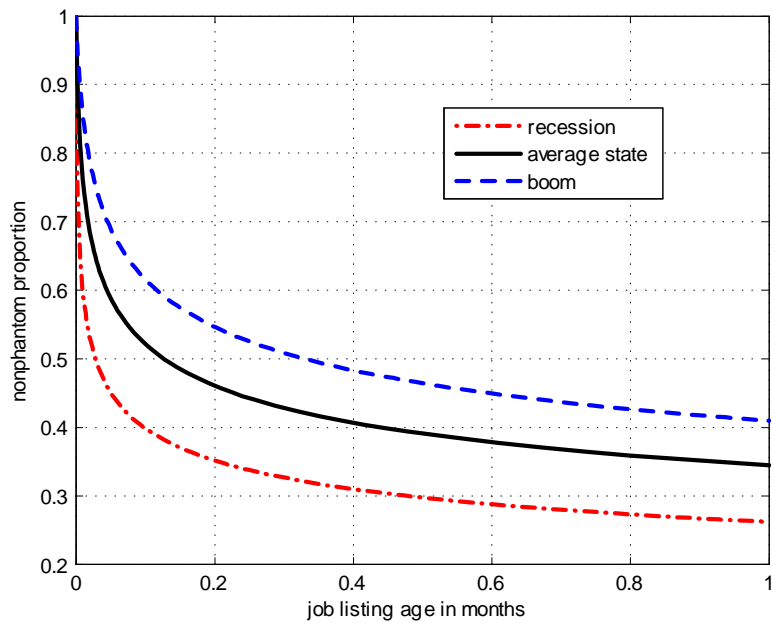


Figure 3: Nonphantom proportion by listing age. In a recession,  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom.

### 3.1 Modified model

We distinguish the cost of setting up a job  $c$ , which we have emphasized up to now, from the cost of renewing a listing  $k < c$ . In exchange for the renewal cost, firms have a new listing of age 0, whereas the previous ad disappears. On websites this corresponds to cases where the listing becomes ranked first right after renewal.

In the standard model without phantom vacancies, or when workers search randomly among the different listings, the rate of applications to vacancies does not change with the listing age. Therefore the value of the vacancy does not change with age and firms have no reason to renew their listing. With phantoms and directed search, tightness increases with age. Thus the value of a listing decreases with the listing age, reflecting the fall in the rate of applications. This provides incentive to renew the listing.

Right after renewal, the listing age is reset to 0. By continuity of the value function, we have  $V(A) = V(0) - k$ . Integrating forward equation (3) with this new boundary condition gives:

$$V(0) = \frac{J \int_0^A \eta(s) \exp \left[ - \int_0^s (r + \eta(b)) db \right] ds - k \exp \left[ - \int_0^A (r + \eta(b)) db \right]}{1 - \exp \left[ - \int_0^A (r + \eta(b)) db \right]}. \quad (13)$$

To find the optimal renewal age, we have to discuss firms' coordination problem. Consider a firm believing that all other firms set the renewal age  $\tilde{A}$ . Then it must be that workers also hold this belief. Therefore they do not consider job listings older than  $\tilde{A}$ : they must be phantoms. It follows that our firm must either set a renewal age lower than  $\tilde{A}$  or equal to it. Keeping the listing after  $\tilde{A}$  does not make sense because the job-filling rate is 0 after this age.

Suppose first that  $\tilde{A}$  is large. The optimal renewal age  $\hat{A}$ , the firm's best-response to  $\tilde{A}$ , results from the condition  $V'(\hat{A}) = 0$ . We obtain

$$V(\hat{A}) = V(0) - k = \frac{\eta(\hat{A})}{r + \eta(\hat{A})} J. \quad (14)$$

Firms renew their listing when the rate of applications to their vacancies becomes sufficiently small. This rate is all the higher than the value of a filled job is small. Therefore parameters increasing the value of a filled job tend to reduce the threshold rate  $\eta(\hat{A})$ .

Now suppose that  $\tilde{A}$  is small, i.e., smaller than the optimal  $A$  found previously. Then the firm is forced to set  $A = \tilde{A}$ . It follows that any  $\tilde{A}$  belonging to  $[0, \hat{A}]$  can be an equilibrium of the renewal game. Hereafter, we only focus on equilibria such that  $\tilde{A} = \hat{A}$ .<sup>2</sup>

<sup>2</sup>Note that such equilibria are symmetric. There cannot be asymmetric equilibria. In an asymmetric equilibria, there are at least two distinct  $\hat{A}$  solving the first-order condition (14), say  $A_1$  and  $A_2$  with  $A_1 < A_2$  without loss of generality. This implies that  $\theta(A_1) = \theta(A_2)$ , with  $\theta(a) > 0$  for  $a \in [A_1, A_2]$ . This is impossible because the non-arbitrage condition states  $\pi(a)m(\theta(a)) = m(\theta(0))$ , whereas  $\pi(A_1) > \pi(A_2)$ .

Under free entry,  $V(0) = c$ . The equilibrium is composed of the following system of equations:

$$m(\theta(0)) = \pi(a)m(\theta(a)), \quad (15)$$

$$\pi'(a) = -\frac{m(\theta(a))}{\theta(a)}\pi(a), \quad (16)$$

$$c = \frac{\frac{(1-\beta)(y-b)}{r+\lambda+\beta m(\theta(0))} \int_0^A \eta(a) \exp\left[-\int_0^a (r+\eta(s))ds\right] da - k \exp\left[-\int_0^A (r+\eta(s))ds\right]}{1 - \exp\left[-\int_0^A (r+\eta(s))ds\right]}, \quad (17)$$

$$c - k = \frac{\eta(A)}{r + \eta(A)} \frac{(1-\beta)(y-b)}{r + \lambda + \beta m(\theta(0))}, \quad (18)$$

$$u = \lambda/(\lambda + m(\theta(0))), \quad (19)$$

with  $\pi(0) = 1$  and  $\eta(a) = m(\theta(a))/\theta(a)$ .

Before turning to the model calibration, we emphasize two extreme cases. In the first case,  $c = k$ . There renewing a listing is as costly as creating an entirely new vacancy. The free entry condition gets back to its earlier formulation, whereas  $\eta(A)$  must tend to 0. This implies that  $\theta(A)$  tends to infinity. Thus  $A$  tends to infinity as well.

In the second case,  $k = 0$ . Then  $A$  tends to 0. The distribution of listings by listing age is concentrated in 0 and job seekers only search for those jobs. Phantoms disappear from the matching scene and the only frictions are captured by the function  $m$  that governs meeting between listings and job seekers.

### 3.2 Calibrations

We focus on the general case where  $c > k > 0$ . We modify as few parameters as possible to emphasize the role played by the renewal age  $A$ . We start with  $A = 1$  in the baseline calibration. Therefore parameters adjust so that  $A = 1$  is an equilibrium outcome of the model. Having this in mind, we derive  $\theta(0)$ ,  $B$  and  $\alpha$  in exactly the same way as we did in the previous section. Whether  $A$  is endogenous or exogenous does not affect the computation of the vacancy-to-unemployed ratio, the job-finding rate or its elasticity.

Once done, we turn to parameters  $b$  and  $\beta$  and fix them as in the previous section. Lastly we set parameters  $c$  and  $k$  so that the free-entry condition holds and  $A$  is optimally chosen. We obtain:

$$c = \frac{(1-\beta)(y-b)}{r + \lambda + \beta m(\theta(0))} \left\{ \int_0^A \eta(a) \exp\left[-\int_0^a (r+\eta(s))ds\right] da + \frac{\eta(A)}{r + \eta(A)} \exp\left[-\int_0^A (r+\eta(s))ds\right] \right\},$$

$$k = \frac{(1-\beta)(y-b)}{r + \lambda + \beta m(\theta(0))} \left\{ \int_0^A \eta(a) \exp\left[-\int_0^a (r+\eta(s))ds\right] da - \frac{\eta(A)}{r + \eta(A)} \left(1 - \exp\left[-\int_0^A (r+\eta(s))ds\right]\right) \right\}.$$

Table 4 provides the parameter set.

$\lambda$	$B$	$\alpha$	$y$	$b$	$\beta$	$r$	$c$	$k$
0.03	1.07	0.16	1	0.93	0.35	0.003	0.21	0.002

Table 4: Parameters of the baseline calibration with vacancy renewal

The cost of creating a new job is 50% larger than in the previous calibration. In the model with renewal the vacancy does not disappear when the termination age is reached. The creation cost must be increased to compensate for the longer lifetime expectancy of the vacancy. The renewal cost is very small, 1% of the job creation cost.

We then reexamine the three scenarios studied in the previous section. We stick to the same parameters but  $y$ , which is increased by 2.5% to simulate a boom and decreased by 2.5% to simulate a recession. We especially focus on changes in the termination age.

Table 5 shows the main aggregate variables in the three cases.

	$u$	$v/u$	$\mu$	$d$	$\bar{\pi}$	$A$
recession	8.25%	0.16	0.34	0.31	0.29	0.82
normal	5.63%	0.5	0.51	0.64	0.42	1.00
boom	4.28%	1.23	0.68	1.18	0.53	1.24

Table 5: Aggregate labor market variables with endogenous vacancy renewal

In a recession  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom. The  $v/u$  ratio has been rescaled so that it is equal to 1/2 in normal times. The mean duration of a vacancy,  $d$ , is given in months.

Comparison with Table 2 reveals that the consideration of endogenous vacancy renewal contributes to increasing predicted aggregate volatility. When getting to a recession, the elasticity of unemployment vis-à-vis output per worker is  $(8.25 - 5.63)/5.63 \simeq 45\%$ , 50% larger than in the model without listing renewal (see Table 2). When getting to a boom, the elasticity is  $(4.28 - 5.63)/5.63 \simeq -0.25\%$ , and, here again, this elasticity is 50% larger than in the basic model.

Extra volatility is due to the reaction of the renewal age,  $A$ , to productivity shocks. The renewal age increases with output per worker. Optimal  $A$  responds to  $c - k = \frac{\eta(A)}{r + \eta(A)} J$ . Ex-post rent-sharing implies that the value of a filled job  $J$  increases with output per worker  $y$ . It follows that  $\eta(A)$  must decrease to compensate for this increase. The behavior of job seekers implies that  $\theta$  increases with the listing age. Therefore the renewal age rises as a response to the increase in  $J$ .

When firms renew their listings later, job seekers are more willing to consider older listings. Figure 4 suggests that this is indeed the case. The distribution of job seekers seems less (more) concentrated on young listing ages in booms (recessions) than in Figure 2. The comparison of Table 6 with Table 2 confirms this visual impression. In normal times, the density of job seekers by listing age is the same in the two tables. In Table 6, 82% of applications go to listings aged less than 48 hours in a recession, against 48% during a boom. The corresponding figures in Table 2 are, respectively, 78% and 58%.

	density of job-seekers at				
	day 1	day 2	day 3	day 4	week 1
recession	0.75	0.07	0.03	0.02	0.92
normal	0.56	0.11	0.06	0.04	0.84
boom	0.35	0.13	0.08	0.06	0.73

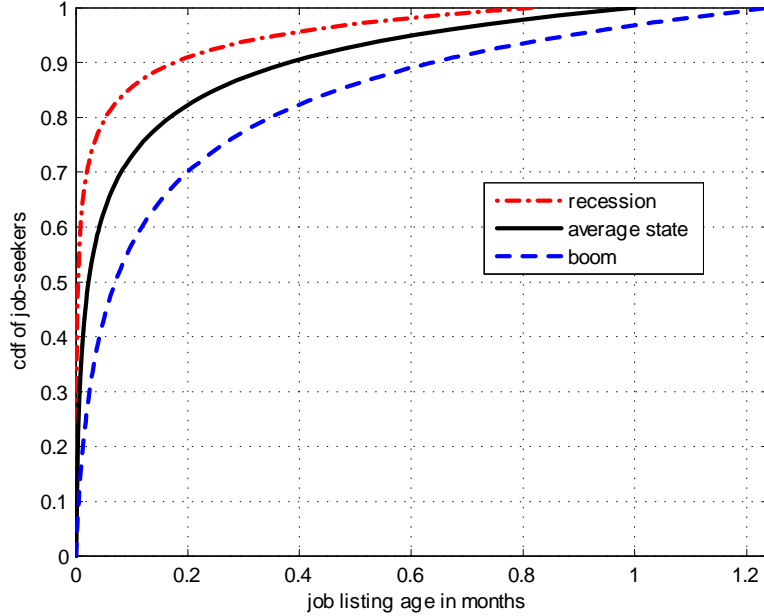


Figure 4: Distribution of job-seekers by listing age with endogenous vacancy renewal. In a recession,  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom.

Table 6: Probability mass of job seekers at various listing ages with endogenous renewal

In a recession,  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom. Reading: the probability mass of job-seekers who search for job listings aged between 24 and 48 hours is 0.11 in normal times.

Figure 5 shows the nonphantom proportion by listing age in the three scenarios. The pattern seems more responsive to productivity shocks than in Figure 3. In Figure 5, after 48 hours of a listing existence the nonphantom proportion is equal to 0.38 in a recession, 0.56 in normal times and 0.73 in a boom. In Figure 3, the corresponding numbers are 0.43 in a recession, 0.56 in normal times and 0.66 in a boom.

To summarize, the key message of this section is that in bad times firms renew their listings more often. This leads the job seekers to concentrate their search on recent vacancies, thereby justifying frequent listing renewal. The consequence is that the nonphantom proportion falls very fast with the listing age. This contributes to worsen matching frictions, further increasing unemployment.

## 4 Conclusion

We study firms' incentive to renew their listings on job boards. In our theory, the key determinant of listing renewal is the presence of obsolete ads. Match formation generates phantom vacancies that affect the job seekers who spend time trying to contact already filled jobs. Workers condition their search effort on the listing age as a result, expecting that older listings are more likely obsolete. Therefore the number



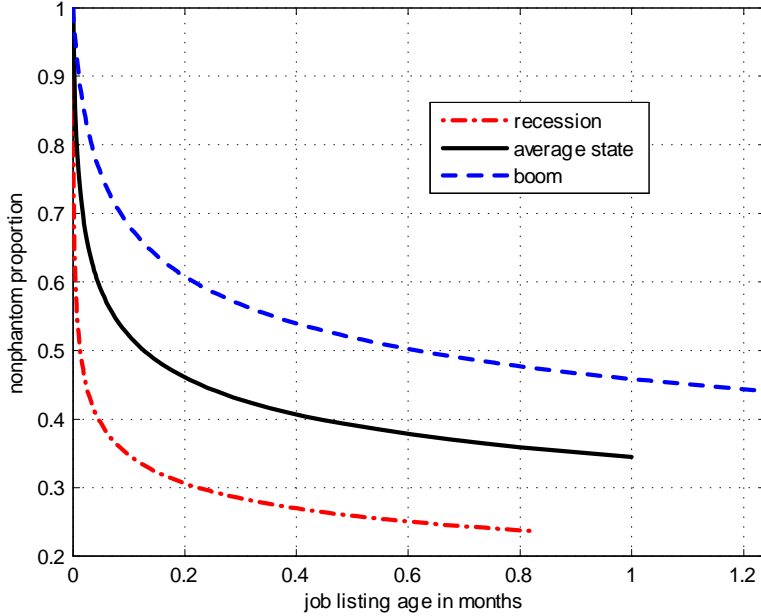


Figure 5: Nonphantom proportion by listing age with endogenous vacancy renewal. In a recession,  $y$  is decreased by 2.5%, whereas it is increased by 2.5% in a boom.

applications per vacancy falls with listing age. In turn employers are willing to renew their listing when the likelihood of filling the job becomes too low.

We complete the standard equilibrium search model of unemployment by adding an equation featuring the motion of the phantom proportion across listing age, a non-arbitrage condition stating that the job-finding rate is the same at all ages and an equation setting the optimal age of listing renewal. The consideration of vacancy renewal contributes to increasing the model sensitivity to aggregate productivity shocks. In bad economic times, vacancies get rapidly filled and the phantom proportion is large. This leads the job seekers to concentrate their search on recent listings. Firms react to this behavior by renewing their listing faster, further increasing workers' preference for new listings. Overall, workers mostly apply for the most recent listings, firms renew their ads very frequently and the proportion of obsolete ads grows very rapidly with listing age.

In our construction with homogenous jobs, all employers set the same renewal age. We do not pretend this is realistic. Instead, we call for additional work considering vacancy heterogeneity, at least with respect to the determinants of vacancy renewal. Enriched models could, for instance, predict a non-degenerate distribution for the renewal age. Alternatively, one could introduce some random component in the vacancy renewal decision. Though all firms would behave similarly in expectation, their effective behavior would differ due to heterogeneous realizations of this random component.

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