Tax reform in a two-sector model with endogenous health

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Abstract

This paper explores how taxes on unhealthy commodities impact consumer behaviour. The two sectors employed in the model are the goods sector, which produces consumption commodities, and the health sector, which provides the agent with health. Health produces so-called 'healthy time' to replace unproductive 'sick time'. The role of healthy time is twofold: first, it affects the level of utility by enhancing leisure time; second, it allows the agent to have more time available for work. Although unhealthy commodities provide the agent with utility, they also pose detrimental effects to the accumulation of health. The analytical results show that the implementation of taxes on unhealthy commodities does not have direct effects on health in the steady state; however, in a revenue-neutral tax reform with adjustments of taxes on labour income and those on capital income, the implementation of taxes on unhealthy commodities can improve health through the channel of income effect. The simulation results show that the tax reform of replacing taxes on capital income would contribute to better welfare in the long run with improved levels of health and leisure. These findings have important consequences on, for example, how the UK sugar tax reform should be implemented.

Keywords: Tax reform, two-sector model, endogenous health.

JEL classification: D91, E20, H20, I18.

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1 Introduction

The rising global burden of non-communicable diseases has driven policy makers to explore approaches to improve population health. Since many major health problems are due to individual behaviours (Lim et al., 2013), it is possible to address the issue of the consumption of certain 'unhealthy commodities' (some energy-dense yet low-nutrient food for instance). Changing the relative prices of these unhealthy commodities is probably one of the policies which has been proposed and explored the most. Real examples include the public health product tax in Hungary, taxes on saturated fat in Denmark, and the recently passed Soft Drink Industry Levy (also called the sugar tax) which is expected to be implemented in 2018 in the UK.

Intuitively, taxes on unhealthy commodities such as sugar and fat should discourage consumption and thus contribute to better population health. However, empirical studies do not find consensus in this logic: Although Nordström and Thunström (2009) agree that taxes do reduce targeted consumption, they pointed out that the reduction would not be significant without earmarking the tax revenue to subsidise some healthy foods. Allais et al. (2010) suggested that such a tax in France would only have a small and ambiguous effect on population health. On the other hand, a study of the Danish fat tax conducted by Jensen and Smed (2013) shows that the introduction of the tax significantly reduced the level of consumption of fat by about 10-15%. Finkelstein et al. (2013) even conclude that decreased consumption induced by taxes on unhealthy commodities in the U.S. would translate into significant health benefits. One reason for this gap could be the relatively short period used in the analysis which prevents some empirical studies from showing the long term trend of health. This is an inevitable flaw since certain taxes on unhealthy commodities have received more attention somewhat recently, whilst the evolution of health generally follows a much longer process. Another reason is that, even though the taxes are effective in improving population health, some investments in health could be directed away to other investments. Investment in health, such as exercise and health care, could improve the level of health even in the long run(e.g. Lucas et al., 2003; Oja et al., 2016). However, decreasing investments in health can offset the contribution from the reduced detrimental effect (Dalgaard and Strulik, 2014). Therefore, if governments do not provide more comprehensive policies along with the new taxes, the agent's behaviour might be distorted toward non-optimal directions.

This paper proposes a general theory to examine the impacts of taxes on unhealthy commodities and to form a more complete tax reform for a dynamic economy. To better present the impacts, the model is founded upon the agent's trade-offs between the investments in the goods sector and the health sector, and between the utilities from leisure and consumption of both unhealthy commodities and other commodities. The idea of health in this model is set to follow that in Grossman (1972): health is not only a consumption good (the good which provides the agent with utility), but also an investment good (the good which produces more 'healthy time' to replace the unproductive 'sick time'). One of the novelties of this model is found in how it embeds the agent's preference for health in that of leisure. More specifically, in this model, the agent allocates healthy time into leisure for higher utility or into labour supply for higher income. During sick time, the agent can neither provide labour supply nor attain utility from leisure. In other words, the

lack of monetary and mental benefit during sick time provides the incentive to accumulate more health. However, unhealthy commodities, which provide the agent with utility, can pose a detrimental effect on the accumulation of health. The agent has to find the balance between the partial utility and the detrimental effect of unhealthy commodities. This paper adopts a two-sector framework which specifies both income and health as in Dalgaard and Strulik (2014), but this paper addresses a more general health problems in the economy rather than focusing on ageing. Moreover, this paper also emphasises on the agent's trade-offs between the investments, so that one can have a clearer view on how the investment in health exactly influence the economy.

   This paper constructs tax reforms based on the macroeconomic influence of health. In addition to the optimal taxation to the balance the dynamic inefficiency induced by income taxes (e.g. Lucas, 1990; Chen and Lu, 2013), this paper introduces taxes on unhealthy commodities as an extra tool to balance the inefficiency. The results show that health would converge to a certain optimum rather than growing without limit as proposed by Van Zon and Muysken (2001). The results also indicate that the implementation of taxes on unhealthy commodities does not improve the level of health directly in the steady state. Nevertheless, with a revenue-neutral tax reforms which raise taxes on unhealthy commodities but lower those on income, the implementation can change the steady state level of health.

   The paper proceeds as follows. A two-sector model with endogenous health is introduced in Section 2. First of all, the model will be presented in a centralised economy in order to provide a more general and clearer view of the system. The agent's utility and the resource constraints for the goods sector and the health sector will be carefully examined. The model will then be applied in a decentralised economy in Section 2.1, and the taxes on unhealthy commodities, numeraire commodities, capital income, and labour income will also be introduced into the model. The optimisation conditions for the problem will be discussed in Section 2.2. Following those conditions, Section 2.3 will then explore the steady state solutions. In Section 3, a tax analysis will then be performed based on the steady state solutions solved in the previous sections. The analytical work will start from the comparative static analysis with respect to the taxes in Section 3.1, and then explore revenue-neutral tax reforms and their impacts on the economic variables in Section 3.3. Finally, conclusions will be offered in Section 4.

2   The model

   This section presents the model for the analysis. The basic framework borrows the form of two sectors from Lucas (1988), but replaces the human capital section with a health section to highlight the trade-off between goods and health. The utility function is extended to include leisure and unhealthy commodities.

   The agent can freely allocate healthy time into leisure or labour supply. Healthy time can be attained through the accumulation of health, h, with a decreasing marginal return (Grossman, 1972). The agent determines l fraction of healthy time spent on labour supply and leaves $(1 - l)$ for leisure. Along with leisure, the numeraire commodities, c, and unhealthy commodities, x, also bring the agent with utility. The agent's lifetime utility is

as follows:

$$U = \int_0^\infty u(c, x, (1-l)h^\mu) e^{-\rho t} dt, \tag{1}$$

where

$$u(c, x, (1-l)h^\mu) = \ln c + \theta \ln x + \psi \ln[(1-l)h^\mu], 0 < \mu < 1,$$

where h^μ is the healthy time, θ and ψ are measures of preference toward unhealthy commodities and leisure, and $\rho > 0$ is rate of time preference. The utility is separable between leisure and both commodities as in Benhabib and Perli (1994).

The economy is constituted by the goods sector y and the health sector m. The production functions in both sectors are similar because both products require the same inputs of physical capital and labour supply as in Azariadis et al. (2013). The production functions apply the Cobb-Douglas form for simplicity:

$$y = A(sk)^\alpha (vlh^\mu)^{1-\alpha},$$
$$m = B((1-s)k)^\beta ((1-v)lh^\mu)^{1-\beta},$$

where A and B are the production efficiency in the goods sector and the health sector; s and v are the fractions of physical capital and labour supply devoted into the the goods sector; and α and β represent the shares of physical capital in the goods sector and those in the health sector.

In the scenario where the taxes on unhealthy commodities have not been implemented, the prices of the two commodities are identical and are standardised to unity for simplicity. The law of motion in the physical capital thus is as follows:

$$\dot{k} = y - c - x, \tag{2}$$

where the variable with a dot on the top hereafter represents the growth of that variable.

x should enter the law of motion of h because it poses a detrimental effect on the accumulation of health. With the lack of either theoretical backup or empirical evidence, the model utilises a linear form for simplicity. The law of motion in the health sector is as follows:

$$\dot{h} = m - \eta x - \delta h, \tag{3}$$

where η is the measure of detrimental effect of unhealthy commodities to health. η is set to be greater than zero to ensure that the detrimental effect offsets the accumulation of health. δ is the natural depreciation of health.

2.1 The decentralised economy

In this section, the model is extended to consider the role of government. The government aims to maximise social welfare by implementing taxes properly. At every point of

time, the government receives tax revenue from taxes on capital income, labour income, and commodities. Assuming the government finances a lump-sum transfer, G, with a fixed budget:

$$G = \tau_k(sr_y + (1-s)r_m)k + \tau_l(vw_y + (1-v)w_m)lh^\mu + \tau_c c + \tau_x x, \tag{4}$$

where τ_k, τ_l, τ_c, and τ_x are the taxes on capital income, labour income, numeraire commodities, and unhealthy commodities; r_y and w_y are the factor prices on physical capital and labour supply in the goods sector; and r_m and w_m are the factor prices in the health sector. The conditions for the firm in the goods sector to equate factor prices to marginal products are:

$$r_y = \alpha \frac{y}{sk}, \tag{5a}$$

$$w_y = (1-\alpha)\frac{y}{vlh^\mu}. \tag{5b}$$

The conditions for the health sectors are:

$$r_m = p_m\beta\frac{m}{(1-s)k}, \tag{6a}$$

$$w_m = p_m(1-\beta)\frac{m}{(1-v)lh^\mu}, \tag{6b}$$

where p_m is the price of the health services m.

In a decentralised economy, the firms in both sectors seek to maximise their own profits. With the factor prices from (5a) and (5b), the maximisation problem for the firm in the goods sector is as follows:

$$\text{max. } y - r_y sk - w_y vlh^\mu. \tag{7}$$

This problem shows that the firm in the goods sector sells its production y and pays the rental prices of r_y and w_y for the physical capital and labour supply that it uses.

With (6a) and (6b), the optimisation problem for the firm in the health sector can be written as:

$$\text{max.} p_m m - r_m(1-s)k - w_m(1-v)lh^\mu. \tag{8}$$

This problem shows that the firm in the health sector sells the health services m with the price of p_m, and pays the rental prices of r_m and w_m for the physical capital and labour supply that it uses.

Incorporating (8) into the agent's maximisation problem, one can view $p_m m$ and the taxes paid in the health sector as forgone opportunities to accumulate more physical capital. Along with the taxes and the factor prices from (5a)-(6b), equation (2) can then be transformed into:

$$\begin{aligned}\dot{k} =&(1-\tau_k)(r_y sk + r_m(1-s)k) + (1-\tau_l)(w_y vlh^\mu + w_m(1-v)lh^\mu) - (1+\tau_c)c \\ &- (1+\tau_x)x - p_m m + G.\end{aligned} \tag{9}$$

The resource constraint presented in (3) for a centralised economy remains the same in the decentralised economy.

4

2.2 The optimisation problem

The agent seeks to maximise the utility under the constraints in (9) and (3). One can form a Hamiltonian function as follows:

$$
\begin{aligned}
\mathcal{H} &= \ln c + \theta \ln x + \psi \ln\left[(1-l)h^{\mu}\right] + \lambda\left[(1-\tau_k)(r_y s k + r_m(1-s)k)\right. \\
&\quad + (1-\tau_l)(w_y v l h^{\mu} + w_m(1-v)l h^{\mu}) - (1+\tau_c)c - (1+\tau_x)x - p_m m + G\right] \\
&\quad + q[m - \eta x - \delta h],
\end{aligned}
$$

where λ is the shadow price of physical capital, and q is the shadow price of health. The first-order conditions for this optimisation problem are:

$$
\frac{1}{c} = \lambda(1+\tau_c), \tag{10a}
$$

$$
\frac{\theta}{x} = \lambda(1+\tau_x) + q\eta, \tag{10b}
$$

$$
\frac{\psi}{1-l} = \lambda(1-\tau_l)(w_y v h^{\mu} + w_m(1-v)h^{\mu}), \tag{10c}
$$

$$
r_y = r_m, \tag{10d}
$$

$$
w_y = w_m, \tag{10e}
$$

$$
\lambda(1-\tau_k)(r_y s + r_m(1-s)) = \lambda\rho - \dot{\lambda}, \tag{10f}
$$

$$
\frac{\mu\psi}{h} + \lambda\mu(1-\tau_l)(w_y v l h^{\mu-1} + w_m(1-v)l h^{\mu-1}) - q\delta = q\rho - \dot{q}, \tag{10g}
$$

along with the transversality conditions,

$$
\lim_{t\to\infty} e^{-\rho t}\lambda(t)k(t) = 0, \tag{10h}
$$

$$
\lim_{t\to\infty} e^{-\rho t}q(t)h(t) = 0. \tag{10i}
$$

(10a) equates the optimal consumption of numeraire commodities to the product of shadow price of k and $(1+\tau_c)$; (10b) shows that the optimal consumption of unhealthy commodities partly depends on both the shadow price of k and that of h; (10c) equates the marginal utility of leisure to the marginal costs of labour supply in both sectors; (10d) and (10e) describe the optimal allocation of inputs between the two sectors; (10f) and (10g) are the Euler equations; and (10h) and (10i) are the conditions to exclude any Ponzi game in the economy.

In the long run, m will be pinned down by the ratio of $(1-s)$ to $(1-v)$ and that of labour supply to physical capital. To simplify the derivation, one can directly attain an additional first-order condition with respect to m as follows:

$$
\lambda p_m = q, \tag{10j}
$$

which can be transformed into:

$$
\frac{q}{\lambda} = p_m.
$$

This additional condition implies that the relative value of the two shadow prices depends on p_m.

Incorporating the information from (5a)-(6b), the conditions in (10d) and (10e) can be specified as:

$$\alpha \frac{y}{sk} = p_m \beta \frac{m}{(1-s)k},$$ (11)

and

$$(1-\alpha)\frac{y}{vlh^\mu} = p_m(1-\beta)\frac{m}{(1-v)lh^\mu}.$$ (12)

To clarify the relationship between s and v, one can divide (11) by (12) and attain:

$$v = \frac{\beta(1-\alpha)s}{\alpha(1-\beta)-(\alpha-\beta)s},$$ (13)

which implies that v is increasing in s if $\alpha \geq \beta$. If the physical capital and labour supply are complements in both sectors, then the allocation of physical capital in one sector should be positively related to the allocation of labour supply in the same sector. Note that, when $\alpha = \beta$, $v = s$, indicating that the agent invests the two sectors with identical amounts of inputs.

Together with (10d), one can derive the evolution of λ from (10f):

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1-\tau_k)r,$$ (14)

where $r \equiv r_y = r_m$. The evolution of q can be derived from (10g), together with (10d) and (10c):

$$\frac{\dot{q}}{q} = \rho + \delta - \frac{\mu}{p_m}(1-\tau_l)wh^{\mu-1},$$ (15)

where $w \equiv w_y = w_m$.

2.3 The equilibrium

In this subsection, the steady state solutions will be explored by using the first-order conditions presented in Section 2.2.

Considering the evolution of λ will stop in the steady state, one can derive the following condition from (14) together with (5a):

$$\frac{lh^\mu}{k} = \frac{s}{v}\left(\frac{\rho}{\alpha A(1-\tau_k)}\right)^{\frac{1}{1-\alpha}}.$$ (16)

The left-hand side of the equation is actually the ratio of labour supply to physical capital in the economy.

q should also stop evolving in the steady state, so, with (15) and (5b), one can derive that:

$$\frac{lh^\mu}{k} = \frac{s}{v}\left(\frac{\mu A(1-\tau_l)(1-\alpha)}{p_m(\delta+\rho)}\right)^{\frac{1}{\alpha}} h^{\frac{-(1-\mu)}{\alpha}}, \tag{17}$$

which, just as (16), gives a value for the ratio between labour supply and physical capital. Utilising this relation, one can easily find the steady state value of h:

$$h^* = \left(\frac{\alpha A(1-\tau_k)}{\rho}\right)^{\frac{\alpha}{(1-\alpha)(1-\mu)}}\left(\frac{\mu A(1-\tau_l)(1-\alpha)}{p_m(\delta+\rho)}\right)^{\frac{1}{1-\mu}}. \tag{18}$$

The above equation for h^* might be somewhat counter-intuitive because τ_x does not appear. The main reason for this result is because m has to decrease in response to the decrease in x for the steady state condition in (3) to hold. The story behind this logic is that the agent finds it beneficial to decrease the consumption of health services, m, when the detrimental effect for the accumulation of h decreases. Therefore, the two effects offset each other, leaving h^* unchanged. However, (18) only implies that the implementation of τ_x would have no direct impacts on the level of health. τ_x can still affect the level of health through indirect channels with more complete tax reforms. Detailed information of the tax reform will be provided in Section 3.3.

The steady state of k can then be obtained by using both (16) and (18):

$$k^* = \frac{v^*}{s^*}\left(\frac{\alpha A(1-\tau_k)}{\rho}\right)^{\frac{1}{1-\alpha}} l^*(h^*)^\mu, \tag{19}$$

where k^* has to increase as the labour supply increases, lh^μ, so that the ratio of labour to capital is fixed in equilibrium.

Moreover, from (10f), (6a), and (16), one can attain the equation for p_m:

$$p_m = \frac{\alpha^{\frac{\alpha(1-\beta)}{1-\alpha}}}{\beta^\beta}\left(\frac{1-\alpha}{1-\beta}\right)^{1-\beta}\left(\frac{A^{\frac{1-\beta}{1-\alpha}}}{B}\right)\left(\frac{1-\tau_k}{\rho}\right)^{\frac{\alpha-\beta}{1-\alpha}}. \tag{20}$$

This equation implies that the relative price of health services is affected by the ratio of the production efficiency factors in both sectors. Given the inputs in both sector, a higher A contributes a more efficient production in the goods sector compared to that in the health sector, and thus decrease the prices of both c and x in the equilibrium. Therefore, p_m would increase as A increases. On the other hand, if the production in the health sector is more efficient than the production in the goods sector, one could expect lower prices of both c and x in the equilibrium. Therefore, p_m would decrease as B raises the relative productivity in the health sector. Equation (20) also implies a negative relationship between τ_k and p_m. The reason behind this negative relationship is that larger taxes on capital income reduce the after-tax marginal product of physical capital (as in (10f)) and thus decrease the relative shadow price of q to λ. According to the positive relationship between p_m and the ratio of q to λ shown in (10j), an increase in τ_k thus decreases p_m in the long run.

To simplify the calculation, one can derive x as a function of c by rewriting (10b) as:

$$x^* = \frac{\theta(1 + \tau_c)}{1 + \tau_x + p_m \eta} c^*.$$

(21)

This equation shows that the consumption choice of x over c is positively affected by τ_c and the measure of the agent's preference to unhealthy commodities, θ. Given c^*, equation (21) also indicates that $\frac{\partial x^*}{\partial \tau_x} < 0$, $\frac{\partial x^*}{\partial \eta} < 0$, and $\frac{\partial x^*}{\partial p_m} < 0$. In accordance to the prevailing hypothesis of the supporters of taxes on unhealthy commodities, τ_x deters the consumption of unhealthy commodities because it raises the relative prices of unhealthy commodities. To understand the negative effects of η and p_m, one can view these factors as the elements of the cost of consuming unhealthy commodities: η is the marginal detrimental effect to the agent by consuming one unit of x (as in (3)), and the agent would have to pay the marginal cost of p_m for the health services to compensate for the lose in health.

Next, one can combine (18) and (10c) to make:

$$c^* = \frac{p_m(\rho + \delta)}{\mu \psi(1 + \tau_c)}(1 - l^*)h^*.$$

(22)

In addition to the zero evolutions of λ and q, the evolutions of the state variables, k and h should also be zero. Therefore, forcing $\dot{h} = 0$ in (3), one attain:

$$m = \eta x + \delta h.$$

(23)

Together with (10g) and (6b), the left-hand side of the above equation, m, can be rewritten as follows:

$$
\begin{aligned}
m &= B((1 - s)k)^\beta ((1 - v)lh^\mu)^{1 - \beta}, \\
&= \frac{\rho + \delta}{\mu(1 - \beta)(1 - \tau_l)}(1 - v)lh.
\end{aligned}
$$

By incorporating (21), the right-hand side of (23) can be written as:

$$\eta x + \delta h = \frac{\eta \theta(1 + \tau_c)}{1 + \tau_x + p_m \eta} c + \delta h.$$

Therefore, equation (23) can be:

$$\frac{\rho + \delta}{\mu(1 - \beta)(1 - \tau_l)}(1 - v)lh = \frac{\eta \theta(1 + \tau_c)}{1 + \tau_x + p_m \eta} c + \delta h.$$

(24)

The optimality condition of (8) and that of (10f) implies that:

$$y = x + c.$$

(25)

With (16) and (21), the above equation can be reformed into:

$$\frac{p_m(\rho + \delta)vlh}{\mu(1 - \tau_l)(1 - \alpha)} = \frac{(\pi + \theta(1 + \tau_c))c}{\pi}.$$

(26)

8

Equations (18), (22), (24), and (26) form a system which could be used to solve for the steady state of c, l, and v:

$$c^* = \frac{\omega(1 - \tau_l)(1 - l^*)h^*}{\psi(1 + \tau_c)}, \tag{27}$$

$$l^* = \frac{p_m(1 - \beta)(\delta\pi h^* + \eta\theta(1 + \tau_c)c^*)}{\omega(1 - v^*)\pi h^*}, \tag{28}$$

$$v^* = \frac{(1 - \alpha)(\pi + \theta(1 + \tau_c))c^*}{\pi\omega l^* h^*}, \tag{29}$$

where $\omega \equiv \frac{p_m(\rho+\delta)}{\mu(1-\tau_l)}$ and $\pi \equiv 1 + \tau_x + p_m\eta$. With (13), one can also attain a function for s^*:

$$s^* = \frac{\alpha(1 - \beta)(\pi + \theta(1 + \tau_c))c^*}{(\alpha - \beta)(\pi + \theta(1 + \tau_c))c^* + \beta\pi\omega l^* h^*} \tag{30}$$

Combining the expressions of l^* and v^* into (27), one will find that c^* can be expressed into a closed form solution:

$$c^* = \frac{(\omega - (1 - \beta)\delta p_m)(1 - \tau_l)\pi h^*}{\theta(1 - \tau_l)(1 + \tau_c)((1 - \alpha) + \eta p_m(1 - \beta)) + \pi((1 - \alpha)(1 - \tau_l) + \psi(1 + \tau_c))},$$

where the term $\omega - (1 - \beta)\delta p_m$ would have to be non-negative to maintain a non-negative c^*. This restriction requires τ_l to satisfy the condition of:

$$\tau_l \geq 1 - \frac{\rho + \delta}{\mu\delta(1 - \beta)}. \tag{31}$$

3 Tax analysis

This section first conducts a comparative static analysis to examine the impacts of taxes on unhealthy commodities. Next, it calibrates the model on the UK economy to find out the implications for the economy and to clarify the complicated interrelationships characterised in the previous section. Moreover, this section also proposes a revenue-neutral tax reform regarding taxes on labour income, capital income, and those on unhealthy commodities.

3.1 Comparative static analysis

To investigate the impacts of τ_x on the variables in steady state, one can perform comparative static analysis based on the steady state solutions. Referring to (27), τ_x affects c^* through the following channel:

$$\frac{dc^*}{d\tau_x} = \frac{\partial c^*}{\partial l^*}\frac{dl^*}{d\tau_x},$$

$$= \frac{\theta(\omega - (1 - \beta)\delta p_m)(1 - \alpha + (1 - \beta)\eta p_m)(1 + \tau_c)(1 - \tau_l)^2 h^*}{((1 - \alpha)(1 - \tau_l)(\pi + \theta(1 + \tau_c)) + \psi\pi(1 + \tau_c) + \theta\eta p_m((1 - \tau_l)(1 - \beta)(1 + \tau_c)))^2} > 0.$$

9

Since $\frac{\partial c^*}{\partial l^*} = \frac{-c^*}{1-l^*} < 0$, the full derivative of c^* with respect to τ_x then implies that $\frac{dl^*}{d\tau_x} < 0$. According to the equation above, the changes in τ_x do not affect c^* directly; instead, they affect c^* via their impacts on l^*. Therefore, the agent has to increase the consumption of c in response to the increase in leisure, $(1-l)h^\mu$, with a constant marginal rate of substitution (MRS) between and the utility function concave in both inputs.

As stated in Section 2.3, s and v move in the same direction as long as $\alpha \geq \beta$. Considering the complementarity between physical capital and labour supply, the parameters should not violate this condition. According to (29), the changes in τ_x should affect v^*, and thus s^*, both directly and indirectly:

$$
\begin{aligned}
\frac{dv^*}{d\tau_x} &= \frac{\partial v^*}{\partial l^*}\frac{\partial l^*}{\partial \tau_x} + \frac{\partial v^*}{\partial c^*}\frac{dc^*}{d\tau_x} + \frac{\partial v^*}{\partial \tau_x}, \\
&= \frac{-\theta p_m(1-\alpha)(1-\beta)(\omega - (1-\beta)\delta p_m)(1-\tau_l)(1+\tau_c)(\delta\psi(1+\tau_c) - \eta\omega(1-\tau_l))}{(p_m(1-\beta)(\eta\theta\omega(1-\tau_l) + \delta\psi\pi)(1+\tau_c) + \omega(1-\alpha)(1-\tau_l)(\pi + \theta(1+\tau_c)))^2} \gtrless 0,
\end{aligned}
$$

where $\frac{\partial v^*}{\partial l^*} = \frac{-v^*}{l^*} < 0$, $\frac{\partial v^*}{\partial c^*} = \frac{v^*}{c^*} > 0$, and $\frac{\partial v^*}{\partial \tau_x} = \frac{-\theta(1+\tau_c)v^*}{\pi(\pi + \theta(1+\tau_c))} < 0$. Whilst l^* decreases with larger τ_x, the agent has to increase v^* so that the ratio of labour supply to physical capital remains constant (as in (16) and (17)). The decreases in c^* induced by τ_x as discussed earlier would force v^* to decrease so that the goods market clearing condition in (25) can be satisfied. In addition to the indirect effects, τ_x also affects v^* directly. The illustration for the direct effect is that larger τ_x increase the accumulation of k as shown in the resource constraint of (9). Therefore, the investment in the goods sector has to be reduced in order to conform to the goods market clearance condition in (25). It should be noted that the direct effect of τ_x and the the indirect effect through c^* are opposed to the effect through l^*, and one cannot determine which effect dominate the others. Therefore, one has to know the values of the parameters to solve the complex interrelationships between variables. The calibration will be performed in Section 3.2, and a more specific analysis will be presented with specific parameters.

Referring to (28), the negative relationship between τ_x and l^* as discussed earlier can be disentangeled into the following form:

$$
\begin{aligned}
\frac{dl^*}{d\tau_x} &= \frac{\partial l^*}{\partial v^*}\frac{dv^*}{d\tau_x} + \frac{\partial l^*}{\partial \tau_x}, \\
&= \frac{-\theta\psi(\omega - (1-\beta)\delta p_m)(1-\alpha + (1-\beta)\eta p_m)(1-\tau_l)(1+\tau_c)^2}{\omega((1-\alpha)(1-\tau_l)(\pi + \theta(1+\tau_c)) + \psi\pi(1+\tau_c) + \theta\eta p_m((1-\tau_l)(1-\beta)(1+\tau_c)))^2} < 0,
\end{aligned}
$$

where $\frac{\partial l^*}{\partial v^*} = \frac{l^*}{1-v^*} > 0$ and $\frac{\partial l^*}{\partial \tau_x} = \frac{-\eta\theta(1+\tau_c)c^*l^*}{\pi(\delta\pi h^* + \eta\theta(1+\tau_c)c^*)} < 0$. Due to the ambiguous effect of τ_x on v^*, one cannot determine whether the indirect effect is positive or negative. On the other hand, the direct effect of τ_x on x^* is negative in that an increased τ_x would violate the goods market condition in (9). To maintain the condition, the agent has to decrease the input in the production function y. Although one of the effects is ambiguous in a general case, the full derivative of l^* with respect to τ_x appears to be negative as shown in the second line of the equation.

The changes in τ_x would change x^*:

$$\frac{dx^*}{d\tau_x} = \frac{\partial x^*}{\partial \tau_x} + \frac{\partial x^*}{\partial c^*}\frac{dc^*}{d\tau_x}$$

$$= \frac{-\theta(\omega - (1-\beta)\delta p_m)(1-\tau_l)(1+\tau_c)((1-\alpha)(1-\tau_l) + \psi(1+\tau_c))h^*}{((1-\alpha)(1-\tau_l)(\pi + \theta(1+\tau_c)) + (1+\tau_c)(\psi\pi + (1-\beta)\eta\theta p_m(1-\tau_l)))^2} < 0,$$

where $\frac{\partial x^*}{\partial \tau_x} = \frac{-x^*}{\pi} < 0$ and $\frac{\partial x^*}{\partial c^*} = \frac{x^*}{c^*} > 0$. The illustration of the direct relationship is straight-forward in that higher prices on x deter the consumption of x. Referring to the positive relationship between c^* and τ_x discussed earlier, the indirect effect is opposed to the direct effect. The illustration of $\frac{\partial x^*}{\partial c^*}$ is that the agent has to consume more x with increased c due to larger τ_x, so that the MRS between x and c remains constant (as in (10b)). Nevertheless, based on the second line of the full derivative, the direct effect of τ_x on x^* outweighs the indirect effect. This negative relationship between x^* and τ_x is in line with the proposition of most of the supporters for the taxes on unhealthy commodities.

Intuitively, since τ_x reduces the consumption of x, τ_x should be beneficial to the accumulation of h because the detrimental effect decreases. However, as shown in (18), it is surprising that τ_x plays no role in h^*. As explained in Section 2.3, although τ_x reduces the consumption of x, the agent has to reduce the investment in the health sector in order to hold the optimality condition in (3). The decreased investment in the health sector offsets the positive force from the reduced consumption of x. On the other hand, h^* can be affected by the implementations of τ_k and τ_l:

$$\frac{dh^*}{d\tau_k} = \frac{\partial h^*}{\partial \tau_k} + \frac{\partial h^*}{\partial p_m}\frac{\partial p_m}{\partial \tau_k}$$

$$= \frac{-\beta h^*}{(1-\alpha)(1-\mu)(1-\tau_k)} < 0,$$

$$\frac{dh^*}{d\tau_l} = \frac{\partial h^*}{\partial \tau_l}$$

$$= \frac{-h^*}{(1-\mu)(1-\tau_l)} < 0,$$

where $\frac{\partial h^*}{\partial \tau_k} < 0$, $\frac{\partial h^*}{\partial p_m} < 0$, and $\frac{\partial h^*}{\partial \tau_l} < 0$. It is worth noting that both taxes have negative direct effects on h^*, because the implementation of either taxes would immediately crowd out the resources available for the health sector. In addition to the direct effect, τ_k would also affect h^* through the channel of p_m. h^* is negative in p_m because a higher prices on health services would deter the investment in the health sector. Considering the negative relationship between p_m and τ_k, the indirect effect of τ_k on h^* should be positive. Nevertheless, the direct effect of τ_k outweighs the indirect effect, so the relationship between h^* and τ_k appears to be negative.

3.2 Calibration

In order to clarify the complexity and to provide a quantitatively meaningful result, this subsection calibrates the model in steady state to reproduce the features of the UK

economy. According to McDaniel (2007) and Dhont and Heylen (2009), the average tax rates on labour income and capital income in the UK are around 0.2 and 0.3 respectively. Therefore, the initial tax rates on income are set to be $\tau_l = 0.2$ and $\tau_k = 0.3$. In line with the standard VAT rate in the UK, τ_c and τ_x are both selected to be 0.2.

Although the model is calibrated on the UK economy, the calibration has to rely on US and Canadian data for some observations. Accordingly, the natural force of depreciation of the health is chosen to be $\delta = 0.043$ as documented in Mitnitski et al. (2002) and Dalgaard and Strulik (2014). The time preference of ρ is selected to be 0.04 following Kydland and Prescott (1991), Chen and Lu (2013), and Azariadis et al. (2013). The share of capital in the goods sector, α, is set to be 0.3 as in Chen and Lu (2013). Assuming the structure of the health sector is similar across countries, one can employ the data of $\beta = 0.22$ in the health industry as documented in Acemoglu and Guerrieri (2008). It is worth noting that, in order not to violate the condition of $v'(s) \geq 0$, β should never be larger than α (as shown in (13)), and the observation of $\alpha = 0.3$ and $\beta = 0.22$ complies with this condition.

Ragan (2013) pointed out that the fraction of time allocated to labour market in the UK is around 25 percent, so l^* is chosen to be 0.25. The initial output in the goods sector, y, is normalised into 1, so that all the other economic variables can be easily presented as a fraction of y. According to the data presented by Euromonitor (2016), the contribution of the fast food industry to GDP in the UK is around 0.85% in 2014 and 2015. As shown in (9), the GDP in this model should be $y + p_m m$, so the consumption of unhealthy commodities (in the fast food industry) as a ratio of GDP can be calibrated in the following form:

$$\frac{x}{y + p_m m} = 0.0085.$$

According to the World Bank Database, the ratio of health expenditure to GDP in the UK in 2014 is around 9%, this information can be employed into the calibration in the form of:

$$\frac{p_m m}{y + p_m m} = 0.09.$$

With the normalisation of y, one knows that the ratio of $p_m m$ to y is 0.00989. This information, together with the ratio of x to GDP, refers to the fraction x to y being 0.0093. (25) then indicates that the fraction of c to y should be 0.9907.

From (10d), one can attain:

$$\frac{1-s}{s} = \frac{\beta}{\alpha}\left(\frac{p_m m}{y}\right).$$

By using the same logic, one can rewrite (10e) into the form of:

$$\frac{1-v}{v} = \frac{1-\beta}{1-\alpha}\left(\frac{p_m m}{y}\right).$$

With the specified parameters, one can calibrate $s = 0.9324$ and $v = 0.9007$. Using the information in (5b) and (6b), one can rewrite (10c) in to the form of:

$$
\begin{aligned}
\psi &= \frac{(1-\tau_l)(1-l)}{(1+\tau_c)l}\left(\frac{1}{c}\right)(w_y v l h^\mu + w_m(1-v)l h^\mu) \\
&= \frac{(1-\tau_l)(1-l)}{(1+\tau_c)l}\left(\frac{y}{c}\right)\left((1-\alpha) + (1-\beta)\frac{p_m m}{y}\right).
\end{aligned}
$$

Inserting the specified parameters into the above equation, one can calibrate that $\psi = 1.5689$. With (10f), (5a), and (6a), one can attain:

$$\frac{k}{y} = \frac{(1 - \tau_k)(\alpha + \beta\frac{p_m m}{y})}{\rho}.$$

In this calibration, the initial ratio of k to y is 5.6308.

With (10g), (10j), and the condition of $\dot{h} = 0$ in steady state, (10b) can be transformed into:

$$
\begin{aligned}
\theta &= \frac{(1 + \tau_x)x}{(1 + \tau_c)c} + \frac{p_m m}{(1 + \tau_c)c} - \frac{\psi\delta\mu}{(1 - l)(\rho + \delta)} \\
&= \frac{(1 + \tau_x)}{(1 + \tau_c)}\left(\frac{x}{y}\right)\left(\frac{y}{c}\right) + \frac{1}{(1 + \tau_c)}\left(\frac{p_m m}{y}\right)\left(\frac{y}{c}\right) - \frac{\psi\delta\mu}{(1 - l)(\rho + \delta)}.
\end{aligned}
$$

With the specified parameters, the above equation implies the condition of $\mu \leq 0.0854$ for a non-negative θ. In this paper, θ is selected to be 0.09, and it simultaneously indicates that μ is 0.0024.

To calibrate the initial h, one can combine (10c) and (10j):

$$\frac{m}{h} = \left(\frac{\rho + \delta}{1 + \tau_c}\right)\left(\frac{p_m m}{y}\right)\left(\frac{y}{c}\right)\left(\frac{1 - l}{\mu\psi}\right).$$

Together with the previous selection of the parameters, the above equation implies that $\frac{m}{h} = 1.3753$. With this value, the calculation for the initial h would be straightforward as long as the value of p_m is chosen. The determination of p_m is relatively flexible, because a different p_m could be the result of health services being calculated in different units. In this paper, p_m is normalised to be 1 for simplicity. With the normalisation of p_m, h is then calibrated as 0.0719. A can thus be calibrated as 1.7342 with the production functions of y. Referring to the normalisation of p_m, B can then be calibrated as 2.1964. Moreover, η can also be calibrated as 10.3003 by employing the condition of $\dot{h} = 0$. The benchmark parameters and the calibrated parameters are listed in the table in Appendix A.

With exact parameters, one can avoid ambiguous effects in comparative static analysis. The results of the comparative static analysis with the parameters of the UK economy are listed in the Table 1.

Table 1: Changes in tax rates with government revenue being inconsistent

	τ_l	τ_k	τ_c	τ_x
c^*	$-$	$-$	$-$	$+$
x^*	$-$	$-$	$+$	$-$
l^*	$-$	$-$	$-$	$-$
v^*	$+$	$+$	$-$	$+$
h^*	$-$	$-$	0	0
k^*	$-$	$-$	$-$	$-$

It should be noted that the changes of the variables are discussed in the case with an inconsistent government revenue. The effects of τ_l, τ_k, and τ_c on the variables in steady state are disentangled in Appendix B.

3.3 Tax reform

This subsection focuses on revenue-neutral tax reforms. The derivation starts from the simplification of the government budget (4).

$$F \equiv \tau_k rk + \tau_l wlh^\mu + \tau_x x - G = 0. \tag{32}$$

Due to the complex interrelationship between variables, changes in taxes could result in ambiguous effects on government budget and complicate the tax reform analysis. To solve this problem, one can apply the benchmark and calibrated parameters listed in Appendix A. Along with the parameters, it is clear that $\frac{dF}{d\tau_k} > 0$, $\frac{dF}{d\tau_l} > 0$, $\frac{dF}{d\tau_x} > 0$, $\frac{dF}{d\tau_c} > 0$. Employing the implicit function theorem, the balanced government budget leads to:

$$\frac{d\tau_k}{d\tau_x} = -\frac{dF/d\tau_x}{dF/d\tau_k} < 0, \quad \frac{d\tau_k}{d\tau_c} = -\frac{dF/d\tau_c}{dF/d\tau_k} < 0, \quad \frac{d\tau_l}{d\tau_x} = -\frac{dF/d\tau_x}{dF/d\tau_l} < 0, \quad \frac{d\tau_l}{d\tau_c} = -\frac{dF/d\tau_c}{dF/d\tau_l} < 0.$$

These negative relationships confirm that the government can raise one of the taxes and reduce the other one whilst maintaining a fixed government revenue simultaneously. This subsection proposes two of the potential policies: First, reducing τ_l whilst raising τ_x; second, reducing τ_k whilst raising τ_x. Referring to (32), one would find that the τ_k and τ_l would range from 36.97% to 21.04% and 20.48% to 18.30% respectively in response to the changes of τ_x from 0 to 100%. The changes in tax rates are plotted in Figure 1 by using MATLAB.
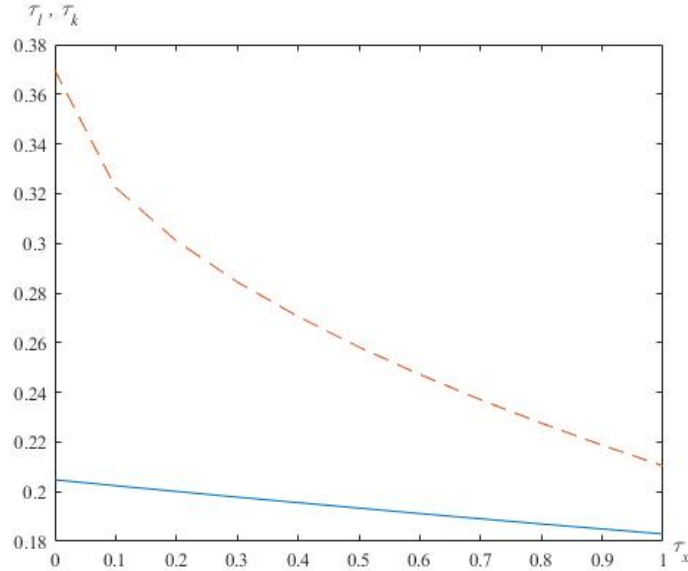

Figure 1: The replacement of τ_l and that of τ_k with τ_x in a revenue-neutral budget

In Figure 1, the solid curve represents the replacements of τ_l with τ_x, and the dashed curve represents the replacements of τ_k with τ_x. It is obvious that the τ_k curve is generally steeper than the τ_l curve. One potential reason for this difference is that the tax bases are relatively more sensitive to the changes in τ_l than those in τ_k; therefore, to maintain a

14

constant government revenue, one has to decrease τ_k more than τ_l respectively with an identical increase in τ_x. To verify this proposition, one can examine the elasticities of the steady state variables to τ_k and τ_l. The results of the elasticity analysis are listed in Table 2.

Table 2: The effects of decreasing τ_k and τ_l on the tax bases

τ_k	τ_l	c^*	x^*	rk^*	$wl^*(h^*)^\mu$
0.3	0.2	0.9907	0.0093	0.3218	0.7772

Decreasing one of the taxes by one percent

0.29	0.2	0.9967(18.17%)	0.0093(0%)	0.3237(17.71%)	0.7819(18.14%)
0.3	0.19	1(18.77%)	0.0094(21.51%)	0.3248(18.65%)	0.7845(18.79%)

The absolute point-elasticities are shown in the parenthesis in Table 2. It should be noted that the elasticities of the tax bases with respect to τ_k are lower than those with respect to $z\tau_l$. This fact confirms the proposition that the τ_k curve is steeper due to the relative insensitivity of the tax bases to τ_k.

In the following analysis, the model is applied to the simulation of the UK economy by changing τ_x from 0% to 100% to quantify the long run effects of the tax reforms. Figure 2 shows the variations of the variables in steady state due to the replacement of τ_l with τ_x.
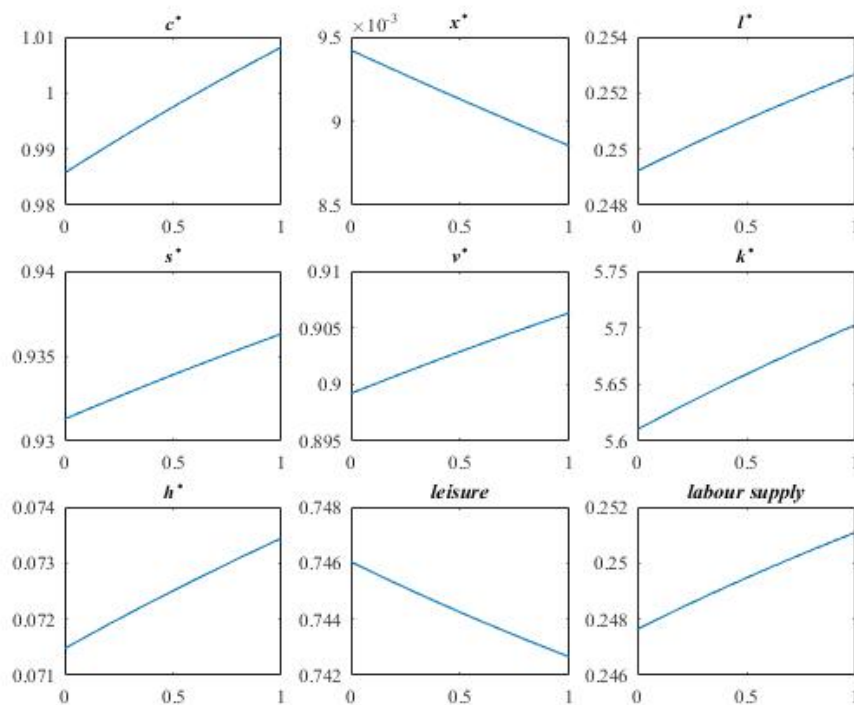

Figure 2: The effects of replacing τ_l with τ_x

15

The variations of the variables at the steady state due to the replacement of τ_k with τ_x is presented in Figure 3.
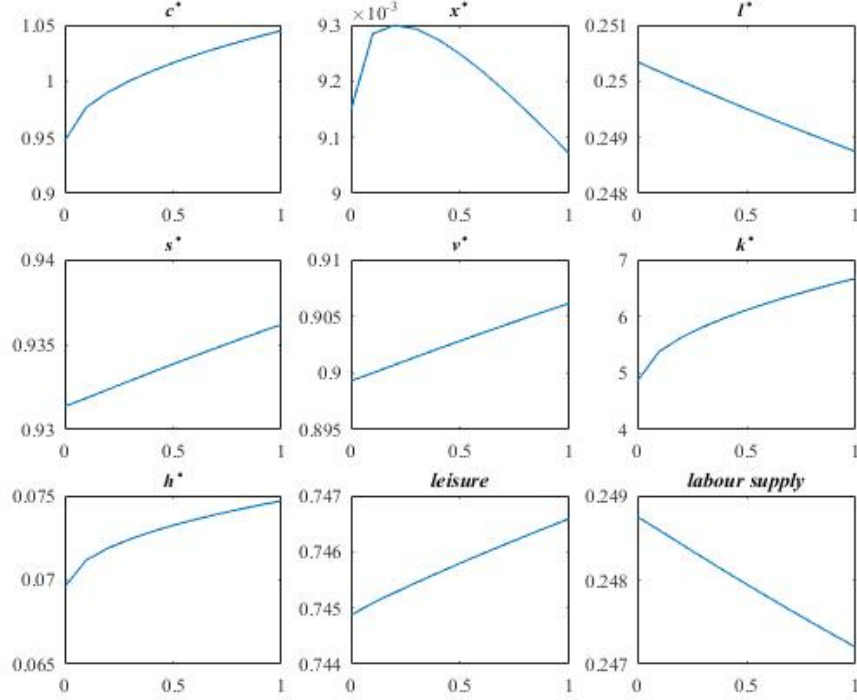
Figure 3: The effects of replacing τ_k with τ_x

The effects of the tax reforms of replacing τ_l or τ_k with τ_x on c^* can be separated into two parts:

$$\frac{dc^*}{d\tau_x} = \frac{\partial c^*}{\partial \tau_x} + \frac{\partial c^*}{\partial \tau_i}\frac{d\tau_i}{d\tau_x}, \text{ where } i = l,k.$$

As shown in Table 1, it is known that $\frac{\partial c^*}{\partial \tau_x} > 0$, $\frac{\partial c^*}{\partial \tau_l} < 0$, and $\frac{\partial c^*}{\partial \tau_k} = \frac{\partial c^*}{\partial p_m}\frac{\partial p_m}{\partial \tau_k} < 0$. Based on the analysis in Section 3.1, larger τ_x would contribute to a higher c^* through the channel of l^*. In addition to this effect, an increase in τ_x would also affect c^* through decreasing τ_l or τ_k in a revenue-neutral tax reform. To illustrate the channel of decreasing τ_l: a decrease in τ_l raises the marginal cost of leisure (as in (10c)). This effect would encourage the agent to provide more labour supply, resulting in a higher output in the goods sector. To hold the goods market clearing condition in (25), the consumption of c (and also x) has to increase in the long run. To examine the channel of decreasing τ_k, one could examine (10f): a decrease in τ_k raises the after-tax marginal product of physical capital, so an increase in k^* is needed to reduce the pre-tax marginal product of physical capital. As a result, the after-tax marginal product of physical capital can then return to ρ. To sump up, c^* unambiguously increases in both tax reforms.

Figures 2 and 3 show that x^* decreases in τ_x for $\tau_x \geq 0.2$ in both reforms. The only difference is that, x^* is monotonously decreasing in τ_x with the tax reform of replacing

τ_l; on the other hand, x^* experiences an increase before τ_x reaches the 0.2 threshold with the tax reform of replacing τ_k. To explore this difference, one can examine the total differentiations of (21) with respect to τ_l and τ_k with the two reforms:

$$\frac{dx^*}{d\tau_x} = \frac{\partial x^*}{\partial \tau_x} + \frac{\partial x^*}{\partial \tau_i}\frac{d\tau_i}{d\tau_x},$$

where $\frac{\partial x^*}{\partial \tau_x} < 0$, $\frac{\partial x^*}{\partial \tau_l} < 0$, and $\frac{\partial x^*}{\partial \tau_k} = \frac{\partial x^*}{\partial p_m}\frac{\partial p_m}{\partial \tau_k} < 0$. The implementation of τ_x would result in a lower x^* as discussed in Section 3.1. To understand the effects through the channels of decreasing income taxes, one should note that any changes in τ_l or τ_k which affects c would require x to change so as to restore the MRS as shown in (10b). Therefore, an increase in c^* induced by smaller τ_l would prompt the agent to raise x^* proportionally; likewise, an increase in c^* due to smaller τ_k would also make the agent increase x^* proportionally. Contrary to the effect of directly implementing τ_x, the effects of τ_x through the channels of decreasing income taxes are positive to x^*. In the case of replacing τ_l, the direct effect of increasing τ_x dominate at all times, so x^* is monotonously decreasing in τ_x. In the case of replacing τ_k, the positive effect of decreasing τ_k outweighs the negative effect of increasing τ_x for $\tau_x < 0.2$; however, the negative effect dominates the positive one after $\tau_x \geq 0.2$, so the direction of the curve changes beyond the 0.2 threshold.

To explain the difference between the impacts of the two tax reforms on l^* as shown in Figures 2 and 3, one should look at the total derivatives of l^* with respect to τ_x:

$$\frac{dl^*}{d\tau_x} = \frac{\partial l^*}{\partial \tau_x} + \frac{\partial x^*}{\partial \tau_i}\frac{d\tau_i}{d\tau_x},$$

where $\frac{\partial l^*}{\partial \tau_x} < 0$, $\frac{\partial l^*}{\partial \tau_l} < 0$, and $\frac{\partial l^*}{\partial \tau_k} = \frac{\partial l^*}{\partial p_m}\frac{\partial p_m}{\partial \tau_k} < 0$. A decrease in τ_l raises the marginal cost of leisure (as in (10c)). Therefore, the agent would find it optimal to decrease leisure in the steady state (l^* increases). The simulation of the tax reform shows that the positive effect through decreasing τ_l dominates the negative effect of τ_x on l^*, so l^* increases in τ_x in the reform of replacing τ_l. The impact of τ_k on l^* can be separated into two oppositional effects. The first effect can be examined from (10c) and (10f): smaller τ_k encourage the accumulation of k and thus increases the level of c in the long run; to maintain the MRS between c and $(1-l)h^\mu$, the agent has to decrease l^*. The second effect can be viewed in (16): smaller τ_k result in a lower ratio of labour supply to physical capital, which increases the marginal product of labour. To restore marginal product of labour, l is required to increase in the long run. With the parameters specified in Section 3.2, the second effect of τ_k dominates, so l^* increases with reduced τ_k. The simulation result shows that the negative effect of τ_x outweighs the positive effect through decreasing τ_k, so l^* decreases in τ_x in the reform of replacing τ_k.

The total differentiations of v^* with respect to τ_x in the two tax reforms yield:

$$\frac{dv^*}{d\tau_x} = \frac{\partial v^*}{\partial \tau_x} + \frac{\partial v^*}{\partial \tau_i}\frac{d\tau_i}{d\tau_x},$$

where $\frac{\partial v^*}{\partial \tau_x} > 0$, $\frac{\partial v^*}{\partial \tau_l} > 0$, and $\frac{\partial v^*}{\partial \tau_k} = \frac{\partial v^*}{\partial p_m}\frac{\partial p_m}{\partial \tau_k} > 0$. As shown in Table 1, an increase in τ_x would have a positive effect on v^* with the parameters calibrated on the UK economy. To understand

17

the effects of decreasing the income taxes, one should note that decreases in either income taxes would encourage the accumulation of k as shown in (9). To maintain $\dot{k} = 0$ in the steady state, the investment must shifts from the goods sector to the health sector in response in both tax reforms. However, the simulation results show that the positive effects of τ_x dominate. Therefore, v^* (and thus s^*) increases in both reforms.

It should be noted that h^* is increasing in τ_x in both cases. To understand the mechanisms behind them, one can take total derivations of h^* as below:

$$\frac{dh^*}{d\tau_x} = \frac{\partial h^*}{\partial \tau_i} \frac{\partial \tau_i}{\partial \tau_x},$$

where $\frac{\partial h^*}{\partial \tau_l} < 0$ and $\frac{\partial h^*}{\partial \tau_k} < 0$ as mentioned in Section 3.1. It should be noted that the increases in h^* are not because of the reduced detrimental effects of decreased x, but because of the income effects from the decreased τ_l and τ_k.

The comparative-static effects on k^* with the two types of tax reforms can be disentangled into two parts as below:

$$\frac{dk^*}{d\tau_x} = \frac{\partial k^*}{\partial \tau_x} + \frac{\partial k^*}{\partial \tau_i} \frac{d\tau_i}{d\tau_x},$$

where $\frac{\partial k^*}{\partial \tau_x} < 0$, $\frac{\partial k^*}{\partial \tau_l} < 0$, and $\frac{\partial k^*}{\partial \tau_k} < 0$. The implementation of τ_x should have a negative effect on k^* as it crowds out the resource available for the accumulation of k (as in (9)). To hold the ratio of labour supply to physical capital constant in the steady state, k^* has to increase (decrease) as $l^*(h^*)^\mu$ increases (decreases). Therefore, the comparative-static effects on k^* can be separated into two parts: the effects of the variations in l^* and the effects of the variations in h^*. In the tax reform where τ_l is replaced by τ_x, the agent would find it optimal to raise l^* in response to the increased marginal labour productivity and the marginal cost of leisure (as in (10c)). Following this increase in l^*, k^* has to increase in order to fix the ratio of labour supply to physical capital. Since both h^* and l^* increase in τ_x, the replacement of τ_l with τ_x creates a positive effect on k^* through the indirect effect of decreasing τ_l. In the reform of replacing τ_k with τ_x, the decreased τ_k would encourage the agent to accumulate more k. Therefore, the effect through the channel of decreasing τ_k appears to be positive when τ_k is replaced by τ_x. Nevertheless, the negative effects are overshadowed by the positive effects of decreasing income taxes in both cases. Therefore, k^* is increasing in τ_x with either reform.

With the quantitative results of the two reforms, one can then simulate the effects on welfare in the economy by taking the quantitative results into the utility function (1). The changes in welfare are plotted in Figure 4.
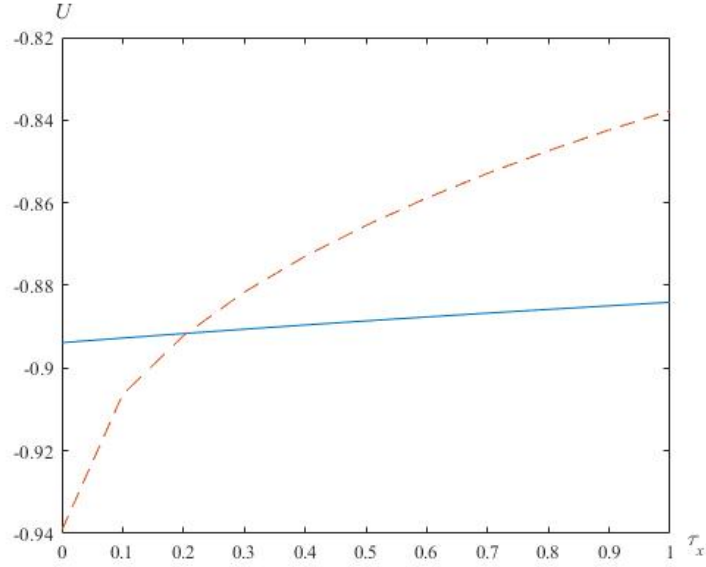
Figure 4: The changes in welfare in response to the tax reforms

In Figure 4, the solid curve represents the economic welfare in the case of replacing τ_l with τ_x, and the dashed curve represents the economic welfare in the case of replacing τ_k with τ_x. The tax reform where τ_l is replaced by τ_x results in decreases in both leisure and x; nevertheless, due to its contribution to the increase in c, this tax reform still contributes to better welfare in the long run. Figure 4 also indicates that a switch from τ_k to τ_x would result in even better welfare in the economy. Compared to the former reform, the replacement of τ_k with τ_x not only increase c but also leisure in the long run. The analysis implies that the replacement of τ_k with τ_x could be a more effective reform in terms of long-run welfare.

4 Conclusion

This paper constructs a two-sector model with endogenous health to study how taxes on unhealthy commodities affect the economy. The two sectors employed in the model are the goods sector, which produces consumption commodities, and the health sector, which provides the agent with health. Health produces so-called 'healthy time' to replace unproductive 'sick time'. The role of healthy time is twofold: first, it affects the level of utility by enhancing leisure time; second, it allows the agent to have more time available for work. In this model, the agent has to make trade-offs between investments in both sectors, and the utilities from leisure, consumption of numeraire commodities, and that of unhealthy commodities. Although unhealthy commodities provide the agent with utility, they are simultaneously detrimental to the accumulation of health. Intuitively, taxes on unhealthy commodities should directly affect the level of health in the steady state as long as the taxes are effective in reducing the consumption of unhealthy commodities. However, the steady state solution shows that, although the taxes can decrease the consumption of unhealthy commodities, the implementation of the taxes does not directly affect the level

19

of health in the long run. The reason is that, as the detrimental effect decreases, the agent would find it beneficial to cut down the investment in the health sector. Nevertheless, with the adjustments of taxes on labour income or those on capital income in a revenue-neutral reform, the implementation of taxes on unhealthy commodities can improve the level of health through the channel of income effect. The results offer important implications to policy makers: the introduction of, for example, sugar tax should always be coupled with subsidies or reductions in other tax burdens so that the level of health can be improved. The simulation results of this paper show that both tax reform result in higher consumptions of the numeraire commodities and improved levels of health. However, the reform which decreases taxes on labour income contributes to less leisure time whilst the reform which decreases taxes on capital income contributes to more leisure time. The quantitative results indicate that both reforms raise welfare in the long run, but the reform which decreases taxes on capital income could be a more effective reform in terms of welfare.

Appendix A   Benchmark parameters and calibration

Benchmark parameters		
Share of physical capital in the good sector	α	0.3
Share of physical capital in the health sector	β	0.22
Initial production in the good sector	y	1
Relative price of health services	p_m	1
Fraction of labour supply	l	0.25
Ratio of numeraire commodities to output production	$\frac{c}{y}$	0.9907
Ratio of unhealthy commodities to output production	$\frac{x}{y}$	0.0093
Health expenditure as a fraction of GDP	$\frac{p_m m}{y + p_m m}$	0.0900
Rate of time preference	ρ	0.04
Natural depreciation of health	δ	0.043
Tax rate on capital income	τ_k	0.3
Tax rate on labour income	τ_l	0.2
VAT rate	τ_c and τ_x	0.2
Calibration		
Ratio of physical capital to output production	$\frac{k}{y}$	5.6308
Ratio of the level of health to output production	$\frac{h}{y}$	0.0719
Fraction of physical capital in the goods sector	s	0.9324
Fraction of labour supply in the goods sector	v	0.9007
Production efficiency of healthy time	μ	0.0024
Relative preference to unhealthy commodities	θ	0.0900
Relative preference to leisure	ψ	1.5689
Detrimental effect from unhealthy commodities	η	10.3003
Total factor productivity	A	1.7342
Productivity in the health sector	B	2.1964

Appendix B Comparative-static effects

By differentiating (27) with respect to τ_l and τ_k, one can attain:

$$\frac{dc^*}{d\tau_l} = \frac{\partial c^*}{\partial l^*}\frac{dl^*}{d\tau_l} + \frac{\partial c^*}{\partial h^*}\frac{dh^*}{d\tau_l},$$

$$\frac{dc^*}{d\tau_l} = \frac{\partial c^*}{\partial l^*}\frac{dl^*}{d\tau_k} + \frac{\partial c^*}{\partial p_m}\frac{dp_m}{d\tau_k} + \frac{\partial c^*}{\partial h^*}\frac{dh^*}{d\tau_k},$$

where

$$\frac{\partial c^*}{\partial l^*} = \frac{-c^*}{1 - l^*} < 0,$$

$$\frac{\partial c^*}{\partial p_m} = \frac{c^*}{p_m} > 0,$$

$$\frac{dp_m}{d\tau_k} = \frac{-p_m(\alpha - \beta)}{(1 - \alpha)(1 - \tau_k)} < 0,$$

$$\frac{\partial c^*}{\partial h^*} = \frac{c^*}{h^*} > 0.$$

By differentiating (21) with respect to τ_l and τ_k, one can attain:

$$\frac{dx^*}{d\tau_l} = \frac{\partial x^*}{\partial c^*}\frac{dc^*}{d\tau_l},$$

$$\frac{dx^*}{d\tau_k} = \frac{\partial x^*}{\partial c^*}\frac{dc^*}{d\tau_k} + \frac{\partial x^*}{\partial p_m}\frac{dp_m}{d\tau_k},$$

where

$$\frac{\partial x^*}{\partial c^*} = \frac{x^*}{c^*} > 0,$$

$$\frac{\partial x^*}{\partial p_m} = \frac{\eta x^*}{\pi} > 0.$$

By differentiating (28) with respect to τ_l and τ_k, one can attain:

$$\frac{dl^*}{d\tau_l} = \frac{\partial l^*}{\partial c^*}\frac{dc^*}{d\tau_l} + \frac{\partial l^*}{\partial v^*}\frac{dv^*}{d\tau_l} + \frac{\partial l^*}{\partial h^*}\frac{dh^*}{d\tau_l} + \frac{\partial l^*}{\partial \tau_l},$$

$$\frac{dl^*}{d\tau_k} = \frac{\partial l^*}{\partial c^*}\frac{dc^*}{d\tau_k} + \frac{\partial l^*}{\partial v^*}\frac{dv^*}{d\tau_k} + \frac{\partial l^*}{\partial p_m}\frac{dp_m}{d\tau_k} + \frac{\partial l^*}{\partial h^*}\frac{dh^*}{d\tau_k},$$

where

$$\frac{\partial l^*}{\partial c^*} = \frac{\eta\theta(1+\tau_c)l^*}{\delta\pi h^* + (\eta\theta(1+\tau_c))c^*} > 0,$$

$$\frac{\partial l^*}{\partial v^*} = \frac{l^*}{1-v^*} > 0,$$

$$\frac{\partial l^*}{\partial p_m} = \frac{-\eta^2\theta(1+\tau_c)c^*l^*}{\pi(\delta\pi h^* + \eta\theta(1+\tau_c)c^*)} < 0,$$

$$\frac{\partial l^*}{\partial h^*} = \frac{-\eta\theta(1+\tau_c)c^*l^*}{(\delta\pi h^* + \eta\theta(1+\tau_c)c^*)h^*} < 0,$$

$$\frac{\partial l^*}{\partial \tau_l} = \frac{-l^*}{1-\tau_l} < 0.$$

By differentiating (29) with respect to τ_l and τ_k, one can attain:

$$\frac{dv^*}{d\tau_l} = \frac{\partial v^*}{\partial c^*}\frac{dc^*}{d\tau_l} + \frac{\partial v^*}{\partial l^*}\frac{dl^*}{d\tau_l} + \frac{\partial v^*}{\partial h^*}\frac{dh^*}{d\tau_l} + \frac{\partial v^*}{\partial \tau_l},$$

$$\frac{dv^*}{d\tau_k} = \frac{\partial v^*}{\partial c^*}\frac{dc^*}{d\tau_k} + \frac{\partial v^*}{\partial l^*}\frac{dl^*}{d\tau_k} + \frac{\partial v^*}{\partial h^*}\frac{dh^*}{d\tau_k} + \frac{\partial v^*}{\partial p_m}\frac{dp_m}{d\tau_k},$$

where

$$\frac{\partial v^*}{\partial c^*} = \frac{v^*}{c^*} > 0,$$

$$\frac{\partial v^*}{\partial l^*} = \frac{-v^*}{l^*} < 0,$$

$$\frac{\partial v^*}{\partial h^*} = \frac{-v^*}{h^*} < 0,$$

$$\frac{\partial v^*}{\partial p_m} = \frac{-\eta^2 p_m^2 + (\pi + \eta p_m)(1+\tau_x+\theta(1+\tau_c))v^*}{p_m\pi(\pi+\theta(1+\tau_c))} < 0,$$

$$\frac{\partial v^*}{\partial \tau_l} = \frac{-v^*}{1-\tau_l} < 0.$$

By differentiating (19) with respect to τ_l and τ_k, one can attain:

$$\frac{dk^*}{d\tau_l} = \frac{\partial k^*}{\partial v^*}\frac{dv^*}{d\tau_l} + \frac{\partial k^*}{\partial l^*}\frac{dl^*}{d\tau_l} + \frac{\partial k^*}{\partial h^*}\frac{dh^*}{d\tau_l},$$

$$\frac{dk^*}{d\tau_k} = \frac{\partial k^*}{\partial v^*}\frac{dv^*}{d\tau_k} + \frac{\partial k^*}{\partial l^*}\frac{dl^*}{d\tau_k} + \frac{\partial k^*}{\partial h^*}\frac{dh^*}{d\tau_k} + \frac{\partial k^*}{\partial \tau_k},$$

where

$$\frac{\partial k^*}{\partial l^*} = \frac{k^*}{l^*} > 0,$$

$$\frac{\partial k^*}{\partial v^*} = \frac{(\alpha - \beta)k^*}{\beta(1-\alpha) + (\alpha - \beta)v^*} > 0,$$

$$\frac{\partial k^*}{\partial h^*} = \frac{\mu k^*}{h^*} > 0,$$

$$\frac{\partial k^*}{\partial \tau_k} = \frac{-k^*}{(1-\alpha)(1-\tau_k)} < 0.$$

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