

Inflow Independence in Transboundary River Problems¹

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Abstract

We consider the problem of sharing water among agents located along a river, who have quasi-linear preferences over water and money, as introduced by Ambec and Sprumont (2002). Given an efficient distribution of river water, where water can be sent from upstream agents to downstream agents but not the other way around, the question is what should be the monetary transfers that downstream agents have to pay to upstream agents as compensation for the upstream agents to abstain from water consumption. Under the quasi-linear utility functions, an efficient water allocation and a transfer scheme determine a welfare distribution. Under more general benefit functions, van den Brink, Estévez-Fernández, van der Laan and Moes (2014) consider three basic axioms for welfare distribution and, additionally, add an independence axiom with respect to benefit functions of upstream, respectively, downstream agents. Both independence axioms yield a unique welfare distribution. In this paper we investigate the impact of similar independence axioms but with respect to water inflows. Surprisingly, together with the three basic axioms, these do not characterize a welfare distribution. Moreover, independence of upstream inflows turns out to be incompatible with the three basic axioms, while independence of downstream inflows yields multiple solutions. We weaken one of the basic axioms to get compatibility with upstream independence, and then strengthen it to get uniqueness with downstream independence.

Keywords: Water allocation, international water law, independence of inflows, incompatibility, multiplicity, uniqueness.

JEL codes: C71, D62, Q25

1 Introduction

We consider the problem of sharing water among agents located along a international or transboundary river, who have quasi-linear preferences over water and money, as introduced by Ambec and Sprumont (2002). Given an efficient distribution of river water, where water can be sent from upstream agents to downstream agents but not the other way around, the question is what should be the monetary transfers that downstream agents have to pay to upstream agents as compensation for the upstream agents to abstain from water consumption. Under the quasi-linear utility functions, an efficient water allocation and a transfer scheme determine a welfare distribution. An efficient solution for river problems assigns such a welfare distribution to every river water allocation problem that can be obtained by an efficient allocation of water and transfers that add up to zero.

There is a recent literature that applies cooperative game theory to define solutions for river problems. As the number of agents, e.g. countries, cities, firms located, along a (international) river is usually small, and formal (international) water exchanges are scarce, trade in riverwater normally takes place by the signing of contracts between the parties involved. These contracts directly specify the amount of water to be delivered and the amount of money that has to be paid for this water, see Dinar et al. (1997) and Béal, Ghintran, Rémila and Solal (2013), and the references therein. For this reason, cooperative game theory is one of the main tools used in modeling (international) water resource issues, see Parrachino et al. (2006) for an overview. Ambec and Sprumont (2002) introduce a cooperative game model where the benefit functions are strictly increasing in the water consumption. Therefore, for every coalition of consecutive agents on the river, the problem to maximize the sum of benefits of the agents in this coalition under the assumption that they only allocate the water inflows on their territory respecting the direction of the water flow, gives a unique welfare maximizing water allocation over these agents. They determine the maximal welfare for every consecutive coalition, and introduce their *downstream incremental welfare distribution* based on this game. This downstream incremental welfare distribution assigns to every agent the increase in welfare it creates when it joins the coalition of its upstream agents.

Ambec and Sprumont (2002) characterize the downstream incremental solution by two axioms that are based on two (somewhat conflicting) water allocation principles. First, they consider the international water law principle of Absolute Territorial Sovereignty (shortly ATS), or the Harmon doctrine, which states that every country has the absolute sovereignty over the inflow of the river on its own territory. This principle is translated into (core) stability of the solution meaning that for every coalition of agents (countries) along the river, the total payoff is at least equal to the total benefit that they can obtain by allocating the water inflow on their own territory optimally amongst each other. This

principle is not (easily) compatible with other principles of international water law, such as the principle of unlimited territorial integrity (shortly UTI), saying that a state has the right to demand the natural flow of an international watercourse into its territory that is undiminished by its upstream states [stated in the rules of the Helsinki Convention on water rights of the International Law Association (1966)]. This is reflected in the *aspiration level property* requiring for an upstream coalition $\{1, \dots, j\}$, $j \in N$, that the total payoff to such a coalition is at most equal to the highest total benefit the agents can obtain from their own water inflows. Since the upper bound on the total payoff of an upstream coalition equals the lower bound required by stability, these bounds are the welfare levels of every upstream coalition and, consecutively starting with the most upstream agent, determines that every agent receives its contribution to the welfare of its upstream agents, i.e. the downstream incremental welfare distribution. Note that, when the downstream agents have higher marginal benefits of the use of water than the agents in an upstream coalition, the aspiration level axiom implies that the upstream coalition is at most compensated for its loss of welfare when it let pass some of its water to its downstream agents, and all the gain in benefits from optimally allocating the water over all agents goes to the downstream agents. So, an upstream agent is not rewarded for the positive externalities it generates for its downstream agents when letting pass some of its water inflow to the downstream agents in order to maximize the total welfare of the grand coalition of all agents.

Ambec and Ehlers (2008) weaken the assumption on the benefit functions by assuming them to be strictly concave, but not necessarily increasing. This allows for satiable agents whose benefit for water is decreasing as soon as they reach their satiation level. This gives rise to externalities for agents that are ‘between’ different nonconnected parts of a nonconsecutive coalition. If an agent is between two agents that ‘cooperate’ such that the upstream neighbour wants to send water to the downstream neighbour, an agent in the middle can take part of this water upto its satiation level. Although this cannot be modeled by a classic cooperative game, Ambec and Ehlers (2008) show that it can still be modeled by a *cooperative game in partition function form* which is a common model for situations with externalities. They characterize a generalization of the downstream incremental solution with similar axioms as Ambec and Sprumont (2002).

In van den Brink, Estévez-Fernández, van der Laan and Moes (2014), the assumption on the benefit functions is weakened further, by assuming them to be concave, and not necessarily strictly concave. Under this assumption the welfare maximization problem for coalitions need not have a unique solution. This is because agents that have a *satiation interval* can choose any consumption in this interval if enough water is available for them. In order to define a cooperative game, it is necessary to make additional assumptions, for example that an agent will never consume more water than its lowest satiation point.

However, also without defining a cooperative game, van den Brink, Estévez-Fernández, van der Laan and Moes (2014) characterize two solutions for river problems by an axiomatic approach.

First, they consider three basic axioms for solutions that weaken the two axioms that are used by Ambec and Sprumont (2002) and Ambec and Ehlers (2008) and are inspired by ATS and UTI. The lower bound property reflects ATS by requiring that every agent reaches a welfare level that is at least equal to what it reaches when it consumes no water. This is a weakening of Ambec and Sprumont (2002)'s stability which requires that *every* coalition of consecutive agents earns at least their maximal welfare when they allocate their own water inflow. This weakening is very useful since stability puts a severe requirement on the welfare distribution and so it might contradict other water principles such as the UTI or the weaker principle of Limited Territorial Sovereignty (UN Convention 1997), shortly LTS, saying that there exist legal restrictions on every state's use of the water. The lower bound property is a much weaker requirement of sovereignty and is more compatible with other water allocation principles.

Besides stability, van den Brink, Estévez-Fernández, van der Laan and Moes (2014) also weaken the aspiration level property into a weak aspiration level property which requires milder upper bounds to the welfare levels, in the sense that an agent at most earns the maximal benefit it can obtain when it can use all (upstream and downstream) water inflows. This respects the possible rights of agents in the welfare created by downstream water inflows as is reflected in the Principle of Territorial Integration of all Basin States (TIBS) which states that "The water of an international watercourse belongs to all basin states combined, no matter where it enters the watercourse. Each basin state is entitled to a reasonable and equitable share in the optimal use of the available water", see e.g. Lipper (@@) or McCaffrey (@@). This principle is also known as the principle of community (of interests) in the waters, the principle of common management or the drainage basin approach. The TIBS-principle does not make any country the legal owner of water. Instead, it states that the river water belongs to all the countries combined, no matter where it enters the river, and that each country has the right to a reasonable and equitable share in the optimal (efficient) use of the water.

Note that, although water cannot be allocated from downstream to upstream agents, welfare can be allocated upstream by monetary transfers. The weaker upper bound described by the weak aspiration level property allows an upstream coalition to be rewarded for positive externalities it might generate on its downstream agents.

Notice that TIBS explicitly requires efficiency of the water use and thus requires that the water is allocated in such a way that the total welfare is maximized. Besides, the efficiency principle is also mentioned as an explicit water allocation principle in itself, see

the Helsinki Document (1966). Efficiency is implied by core stability and the aspiration levels property together, but not by the weaker versions used in this paper. Therefore, we explicitly require efficiency stating that the total sum of payoffs equals the total welfare in an optimal water allocation.

Additionally to the three basic axioms discussed above, van den Brink, Estévez-Fernández, van der Laan and Moes (2014) add an independence axiom with respect to benefit functions. Two such independence axioms are distinguished. Independence of downstream benefit functions states that the welfare of an agent should not depend on the benefit functions of downstream agents.@@ Together with the three basic axioms this characterizes the downstream incremental solution of Ambec and Sprumont (2002) and mentioned above. Alternatively, independence of upstream benefit functions states that the welfare of an agent should not depend on the benefit functions of upstream agents.@@ Together with the three basic axioms this characterizes a new solution, called the UTI incremental solution which is based on the principle that for every downstream coalition (a coalition that for a particular agent consists of this agent and all its downstream agents) the total payoff is at least equal to the total benefit that they can obtain by allocating all water inflows (on their own territory and the territories of all other (upstream) agents) optimally amongst each other.

Since river water allocation problems on a particular river structure are defined by the benefit functions and the water inflows at the territories of the agents along the river, in this paper we want to explore the use of independence axioms with respect to the water inflows. That is, to the three basic axioms we add independence of upstream inflows, respectively independence of downstream inflows. Surprisingly, these independence axioms do not work as nice as the independence axioms with respect to benefit functions in the sense that they do not give a characterization of a unique solution. We will first show that independence of upstream inflows, stating that the welfare of an agent should not depend on the water inflows at territories of upstream agents, is incompatible with the three basic axioms. Therefore, we will modify one of the basic axioms to regain compatibility. We will weaken the weak aspiration level property further by requiring this only for agents for whom their water inflow as well as the water inflow at all upstream agents is zero. This *drought property* requires that a country such that the first positive water inflow occurs at one of its downstream countries, earns at most the welfare it obtains with zero consumption of water. Since it is a weak version of the aspiration level property, it reflects TIBS in a milder way, so that it is easily compatible, for example with ATS. In fact, under efficiency, it can be seen as a weak version of ATS in the sense that it respects the idea that welfare allocation on a river starts only at the most upstream country that has a positive water inflow. It turns out that this drought property is compatible with the three basic axioms,

and even characterizes the *upstream incremental solution* considered in van den Brink, van der Laan and Vasliév (2007) and Ambec and Ehlers (2008). This upstream incremental solution assigns to every agent its contribution to the welfare of all its downstream agents, assuming (under ATS) that these downstream countries only have access to their own water inflow. Ambec, Dinar, and McKinney (2013) show that this upstream incremental solution is doing well with respect to the vulnerability of such river water allocation agreements to reduced water flows, in the sense that among all such agreements that are acceptable to riparian countries, this is the one which is self-enforced under the most severe drought scenarios. This is related to independence of upstream inflows since so drought scenarios in upstream countries should not affect the welfare in a downstream country.

As mentioned, taking the ‘counterpart’ we introduce independence of downstream inflows, stating that the welfare of an agent should not depend on the water inflows at territories of downstream agents. This can be seen as reflecting ATS since the downstream countries are assumed to have the right on the inflow in their territories, and do not share the welfare with upstream countries. Combining this with the three basic axioms gives multiplicity of solutions. For example, the downstream incremental solution satisfies all these axioms, but also a new solution that we call upstream solution. We will show that a strengthening of the drought property, by requiring that an agent whose water inflow is zero earns at least the welfare of zero water consumption, characterizes this upstream solution. This upstream solution assigns to the most upstream agent, the total welfare among all countries, when they have access only to the water inflow of the most upstream agent. The second upstream agent receives the total welfare among all countries, when they have access to the water inflows of the two most upstream agents, minus the share that was already determined for the most upstream agent. Continuing in this way, we obtain an efficient water and welfare allocation.

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This paper is organized as follows. In Section 2 we review the river problem of Ambec and Sprumont (2002) and several generalizations, and introduce the three basic axioms mentioned above. In Section 3 we explore the possibilities of combining these with independence of upstream inflows, finding out that they are incompatible, and provide an axiomatization of the upstream incremental solution by weakening the aspiration level property to the drought property. In Section 4 we explore the possibilities of combining the three basic axioms with independence of downstream inflows, finding out that they give multiplicity, and by strengthening the drought property, obtain the new upstream solution.

2 Three basic axioms for river water allocation problems

Let $N = \{1, \dots, n\}$ be the set of agents along the river, in the sequel also called countries, numbered successively from upstream to downstream, and let $e_i \geq 0$ be the inflow of water on the territory of agent i , $i = 1, \dots, n$. Because of the one-directionality of the water flow from upstream to downstream, every agent can be assigned, at most, the water inflow on the territories of itself and its upstream agents, but the water inflow downstream of some agent cannot be allocated to this agent. Therefore, a *water allocation* $x \in \mathbb{R}_+^n$ assigns an amount of water x_i to agent i , $i \in N$, under the constraints

$$\sum_{i=1}^j x_i \leq \sum_{i=1}^j e_i, \quad j \in N,$$

i.e., $x \in \mathbb{R}_+^n$ is a water allocation if, for every agent j , the sum of the water assignments x_1, \dots, x_j is, at most, equal to the sum of the inflows e_1, \dots, e_j . Assuming that agents can make monetary compensations to other agents for receiving water, a *compensation scheme* $t \in \mathbb{R}^n$ gives a (possibly negative) monetary compensation t_i to agent i , $i \in N$, under the constraint $\sum_{i=1}^n t_i \leq 0$. Each agent i earns some benefit from consuming water that is given by a *benefit function* $b_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ yielding benefit $b_i(x_i)$ to agent i of the consumption x_i of water. Besides from consuming river water, the agents also derive utility from the (possibly negative) monetary compensations, and every agent is assumed to have a quasi-linear utility function which assigns to every pair (x_i, t_i) , with $x_i \in \mathbb{R}_+$ the amount of water allocated to i and $t_i \in \mathbb{R}$ the monetary compensation to i , the utility

$$\nu^i(x_i, t_i) = b_i(x_i) + t_i. \quad (2.1)$$

In the following, we denote a river situation by the triple (N, e, b) , where N is the set of agents, $e \in \mathbb{R}_+^n$ is the vector of nonnegative inflows, and $b = (b_i)_{i \in N}$ is the collection of benefit functions.

A pair (x, t) of a water allocation $x = (x_1, \dots, x_n)$ and compensation scheme $t = (t_1, \dots, t_n)$ is *Pareto efficient* if no water and no money is wasted, i.e., (x, t) is Pareto efficient if, and only if, $x \in \mathbb{R}_+^n$ maximizes the welfare maximization problem

$$\max_{x_1, \dots, x_n} \sum_{i=1}^n b_i(x_i) \quad \text{s.t.} \quad \sum_{i=1}^j x_i \leq \sum_{i=1}^j e_i, \quad j = 1, \dots, n, \quad \text{and } x_i \geq 0, \quad i \in N, \quad (2.2)$$

and the compensation scheme $t \in \mathbb{R}_+^n$ is *budget balanced*: $\sum_{i=1}^n t_i = 0$.

We say that $z \in \mathbb{R}^n$ is a *welfare distribution* if there exists a Pareto efficient pair (x^*, t) such that

$$z_i = b_i(x_i^*) + t_i, \quad i \in N.$$

Hence, a welfare distribution z shares the maximum attainable welfare $\sum_{i=1}^n b(x_i^*)$ amongst the agents by allocating x_i^* to agent i , $i \in N$, and implementing a budget balanced monetary compensation scheme t . Reversely, notice that for the optimal allocation x^* , every budget balanced compensation scheme t induces a welfare distribution.

Ambec and Sprumont (2002) assume that every benefit function $b_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ is an increasing and strictly concave function, which is differentiable at every $x_i > 0$ with derivative going to infinity as x_i tends to zero. Under this assumption, the maximization problem (2.2) has a unique solution x^* , and they introduced the *downstream incremental solution* assigns to every river problem (N, e, b) , the welfare distribution $d(N, e, b) \in \mathbb{R}^n$ given by

$$d_i(N, e, b) = v^i(e, b) - v^{i-1}(e, b), \quad (2.3)$$

with $v^0(e, b) = 0$ and for $j \in N$, $v^j(e, b) = \sum_{i=1}^j b_i(y_i^j)$, where $y^j = (y_1^j, \dots, y_j^j)$ is a solution of the welfare maximization problem

$$\max_{x_1, \dots, x_j} \sum_{i=1}^j b_i(x_i) \quad \text{s.t.} \quad \sum_{i=1}^k x_i \leq \sum_{i=1}^k e_i, \quad k = 1, \dots, j, \quad \text{and} \quad x_i \geq 0, \quad i = 1, \dots, j. \quad (2.4)$$

The welfare distribution $d(N, e, b)$ is called the *downstream incremental welfare distribution*. Note that $v^j(e, b)$ is the highest welfare that can be obtained by a coalition of upstream agents $\{1, \dots, j\}$ when the coalition of upstream agents $\{1, \dots, j\}$, $j \in N$, can allocate their own water amongst themselves.

Later, Ambec and Ehlers (2008) generalized the basic river game described above by allowing for satiable agents. They relax the requirement that the benefit function is strictly increasing, by assuming that every benefit function $b_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ is a strictly concave function, differentiable at every $x_i > 0$ with derivative going to infinity as x_i tends to zero. Under this assumption, it is possible that for some point $c_i > 0$, called the *satiation point* of agent i , the benefit is increasing from $x_i = 0$ to c_i , reaches its maximum value at c_i , and is decreasing for $x_i > c_i$. Ambec and Ehlers (2008) show that also for river situations with satiable agents, the downstream incremental solution $d(N, e, b)$ is still well-defined by (2.3)

In van den Brink, Estévez-Fernández, van der Laan and Moes (2014), the assumptions of Ambec and Ehlers (2008) and, therefore, also those of Ambec and Sprumont (2002), are weakened further by allowing the benefit functions to be concave instead of strictly concave. Moreover, differentiability is weakened to continuity. Since we make this assumption throughout the underlying paper, we state it explicitly.

Assumption 2.1 *In a river situation (N, e, b) , every benefit function $b_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ is concave and continuous for $x_i > 0$.*

According to this assumption, b_i may be nonincreasing. Moreover, it also allows for the existence of an interval $[c_i, c^i]$, $c^i \geq c_i$, such that b_i is increasing on $x_i < c_i$, constant on

$x_i \in [c_i, c^i]$, and decreasing when $x_i > c^i$. In the latter case, the point c_i is the *satiation point* of agent i . Therefore, agent i reaches its highest benefit at c_i and all water consumption levels between c_i and c^i also yield this maximal benefit. However, water consumption higher than c^i yields a lower benefit than the benefit obtained at c_i . We allow for $c_i = 0$ and $c^i = \infty$ (meaning that b_i is constant for $x_i \geq c_i \geq 0$). In particular, this allows for $b_i(x_i) = b_i(0)$ for every $x_i \geq 0$.

Ambec and Sprumont (2002) characterized the downstream incremental welfare distribution by the following two axioms that are derived from known water allocation principles from International Water Law. First, based on the ATS principle, they require a welfare distribution z to satisfy the stability condition that for every coalition of consecutive agents $\{i, i+1, \dots, j\}$, $i \leq j$, the total welfare to its agents is at least equal to the maximal (sum of) welfare these agents can obtain when they allocate only the water inflow on their territory.¹ Second, according to the UTI principle, a state has the right to demand that the natural flow is undiminished by its upstream states. Ambec and Sprumont (2002) use this as an upper bound on the welfare distribution, by requiring that for every coalition S of agents, the total welfare is bounded from above by its *aspiration level*, being the maximum welfare the agents in S can obtain by distributing optimally the water inflow on the territories of all agents $1, 2, \dots, s$, where s is the most downstream agent in S . For an upstream coalition $\{1, \dots, j\}$, $j \in N$, this requires that its total welfare is bounded from above by the maximum welfare that the coalition can obtain by optimally distributing their own water amongst themselves. Since for each j , the stability property requires that the total welfare $\sum_{i=1}^j z_i$ is at least equal to $v^j(e, b)$, and the aspiration level property requires that the total welfare $\sum_{i=1}^j z_i$ is at most equal to $v^j(e, b)$, it follows that these two properties together require that $\sum_{i=1}^j z_i = v^j(e, b)$ for every $j \in N$. Thus, the welfare distribution given by (2.3) is uniquely determined by these two properties. Ambec and Ehlers (2008) show that also for river situations with satiable agents, the downstream incremental solution $d(N, e, b)$ is not only well-defined by (2.3), but it is also uniquely determined by requiring for every upstream coalition both stability and the aspiration level property.

In van den Brink, Estévez-Fernández, van der Laan and Moes (2014), three basic axioms that weaken stability and the aspiration level property are used which, together with an independence of benefit functions, characterize solutions to distribution the welfare in river situations. From now on, in the sequel we assume that the benefit functions satisfy Assumption 2.1. Let \mathcal{W}^N denote the collection of all river situations (N, e, b) on N

¹For a single agent coalition, this stability notion reduces to *individual rationality*, saying that the payoff to agent i should be at least equal to $b_i(e_i)$. This notion of individual rationality based on the ATS principle cannot be used as a participation constraint in international agreements on water use, because also other water principles, as the UTI principle, have to be taken into account.

satisfying Assumption 2.1. Then, a *solution* is a function f assigning to every $(N, e, b) \in \mathcal{W}^N$ a welfare distribution $f(N, e, b) \in \mathbb{R}^n$. In the sequel, the component $f_i(N, e, b)$ is called the *payoff* of agent i , $i \in N$.

Under Assumption 2.1, the maximization problems (2.2), respectively (2.4), do not necessarily have a unique solution, but are still well-defined. Given a river problem (N, e, b) , we define

$$W(N, e, b) = \sum_{i=1}^n b_i(y_i),$$

with $y = (y_1, \dots, y_n)$ a solution of (2.2), as the total welfare that is obtained from an optimal allocation of the water.

Using principles from international watercourse law, van den Brink, Estévez-Fernández, van der Laan and Moes (2014) formulate the following three axioms.

Axiom 2.2 (Lower bound property) *For every river problem $(N, e, b) \in \mathcal{W}^N$, it holds that $f_i(N, e, b) \geq b_i(0)$ for all $i \in N$.*

Axiom 2.3 (Weak aspiration level property) *For every river problem $(N, e, b) \in \mathcal{W}^N$, it holds that $f_i(N, e, b) \leq \max_{x_i \leq \sum_{j \in N} e_j} b_i(x_i)$ for all $i \in N$.²*

Axiom 2.4 (Efficiency) *For every river problem $(N, e, b) \in \mathcal{W}^N$ it holds that $\sum_{i \in N} f_i(N, e, b) = W(N, e, b)$.*

The first axiom weakens the core stability requirement³ of Ambec and Sprumont (2002), and still reflects the ATS principle. Whereas stability requires that every consecutive coalition of agents have full authority over the watercourse in their territories, the lower bound property requires this only for individual agents.

The aspiration level axiom requires for an upstream coalition $\{1, \dots, j\}$, $j \in N$, that the total payoff to such a coalition is at most equal to the highest total benefit the agents can obtain from their own water inflows, and thus the upper bound on its total payoff equals the lower bound required by stability. Therefore, when the downstream agents have higher marginal benefits of the use of water than the agents in an upstream coalition, the aspiration level axiom implies that the upstream coalition is at most compensated for its loss of welfare when it let pass some of its water to its downstream agents, and all the gain in benefits from optimally allocating the water over all agents goes to the downstream

²Under increasing benefit functions, we can write this inequality as $f_i(N, e, b) \leq b_i(\sum_{i \in N} e_i)$ for all $i \in N$.

³The lower bound property is even weaker than individual rationality which requires stability only for the singletons.

agents. So, an upstream agent cannot be rewarded for the positive externalities it generates for its downstream agents when letting pass some of its water inflow to the downstream agents in order to maximize the total welfare of the grand coalition of all agents. The weak aspiration level property requires milder upper bounds to the welfare levels, in the sense that an agent at most earns the maximal benefit it can obtain when it can use all (upstream and downstream) water inflows. Note that, although water cannot be allocated from downstream to upstream agents, welfare can be allocated upstream by monetary transfers. The weaker upper bound described by the weak aspiration level property allows an upstream coalition to be rewarded for positive externalities it might generate on its downstream agents.

Finally, the efficiency axiom states that the total sum of payoffs equals the total welfare in an optimal water allocation.

3 Independence of upstream inflows: an incompatibility and a characterization of the upstream incremental solution

In van den Brink, Estévez-Fernández, van der Laan and Moes (2014) several motivations are given why the welfare payoff of an agent should not depend on the benefit functions of other agents. Some of these arguments also can be used to argue why the welfare payoff of an agent should not depend on the water inflows at the territory of other agents. For example, the ATS principle (or Harmone doctrine) states that each country has the right to use all its own water. According to this principle an agent should not be held responsible for a decrease in the inflows on the territories of other agents. In case the inflow in an upstream coalition increases, then even when this coalition does not consume all the water, they still have the authority over the water and can claim the benefit that is created downstream from ‘their’ water. Independence of upstream inflows states that the payoff of an agent does not depend on the inflows on the territories of its upstream agents. It reflects that a country has the right to use its own inflow and so a downstream country does not have any right to the water inflows upstream of its territory, but also is not responsible for decreasing upstream water inflows.

Axiom 3.1 (Independence of upstream inflows) *For every two river problems $(N, e, b), (N, e', b) \in \mathcal{W}^N$ such that $e_j = e'_j$ for all $j \geq i$, we have that $f_i(N, e, b) = f_i(N, e', b)$.*

Under the assumption of strictly increasing benefit functions, as done in Ambec and Sprumont (2002), this axiom directly reflects ATS, since upstream countries will and

can consume all their water inflow. In case of concave benefit functions, if upstream agents have satiation levels, then it might be that water flows to downstream agents. But notice that we consider solutions that allocate the welfare from water consumption, and this independence makes sense if there is uncertainty about the water flows. Further, the Prior Appropriation principle, being in the rules of the Helsinki Convention on water rights of the International Law Association (1966), states that a country that first makes use of some quantity of water from an international watercourse has the right to the continued use of the water. So, also when the demands of other countries@@

Whereas, independence of upstream benefits together with the three basic axioms of the previous section characterize a solution, called the UTI incremental solution, independence of upstream inflows is incompatible with the three basic axioms. Even stronger, independence of upstream inflows is incompatible with efficiency and the weak aspiration level property since the latter allows an agent to use all water inflows and, therefore, also depends on the inflows of its upstream agents.

Proposition 3.2 *There is no solution on the class \mathcal{W}^N of river problems that satisfies efficiency, the weak aspiration level property and independence of upstream inflows.*

Proof. Consider two river problems $(N, e, b), (N, e', b) \in \mathcal{W}^N$ that are given by $N = \{1, 2\}$, $b_i(x_i) = \sqrt{x_i}$ for $i \in \{1, 2\}$, $e_1 = 1$, $e'_1 = 8$ and $e_2 = e'_2 = 0$. So, the two river problems only differ in the water inflow at the upstream agent 1.⁴ Suppose that solution f satisfies the three axioms of the proposition. The weak aspiration level property requires that $f_2(N, e, b) \leq 1$ and $f_1(N, e', b) \leq \sqrt{8}$. Since an optimal water allocation in (N, e', b) assigns equal amounts of water to both agents, efficiency requires that $f_2(N, e', b) = 4 - f_1(N, e', b) \geq 4 - \sqrt{8} > 1$, and thus $f_2(N, e', b) \neq f_2(N, e, b)$, contradicting independence of upstream inflows. \square

Obviously, independence of upstream inflows is incompatible with the three basic axioms. Question is if there is an alternative weakening of the aspiration level property that is compatible with the other axioms. In order to make independence of upstream inflows compatible with the three basic axioms, we introduce another weakening of the aspiration level property, called the drought property. This axiom requires that the aspiration level upper bound holds for an upstream coalition of agents if the total water inflow of these agents is zero. It reflects the 1997 UN Convention rule of Limited Territorial Sovereignty in the sense that a state cannot claim any benefit from the water allocation when all inflows to this state and its upstream countries are zero.

Axiom 3.3 (Drought property) *For every river problem $(N, e, b) \in \mathcal{W}^N$ with $e_j = 0$ for all $j \leq i$, it holds that $f_i(N, e, b) \leq b_i(0)$.*

⁴Note that the benefit functions are strictly increasing and strictly concave.

The four axioms of efficiency, lower bound property, drought property and independence of upstream inflows characterize uniquely the *upstream incremental solution*, introduced in van den Brink, van der Laan and Vasil'ev (2007) for rivers with nonstatiabile agents, and in Ambec and Ehlers (2008) for rivers with satiable agents.

To define the upstream incremental solution, we consider, for every $j = 1, \dots, n$, the welfare maximization problem

$$\max_{x_j, \dots, x_n} \sum_{i=j}^n b_i(x_i) \quad \text{s.t.} \quad \sum_{i=j}^k x_i \leq \sum_{i=j}^k e_i, \quad k = j, \dots, n, \quad \text{and} \quad x_i \geq 0, \quad i = j, \dots, n, \quad (3.5)$$

i.e., for agent j , the maximization problem (3.5) optimally allocates the inflows e_j, \dots, e_n amongst the downstream agents $\{j, j+1, \dots, n\}$, given the uni-directionality of the water flow. Under Assumption 2.1, these maximization problems do not have a unique solution, but are well-defined. Given a solution $y^j = (y_j^j, \dots, y_n^j)$ of the maximization problem (3.5) for agent j , denote $w^j(e, b) = \sum_{i=j}^n b_i(y_i^j)$ as the maximum welfare that the agents in $\{j, \dots, n\}$ can obtain by distributing their own inflows. Notice that for $j = 1$, the maximization problem (3.5) is equal to problem (2.2), so that $w^1(e, b) = W(N, e, b)$.

The upstream incremental solution assigns to every agent its marginal contribution to the welfare when the agents enter subsequently from the most downstream agent to the most upstream agent. Hence, the upstream incremental solution assigns to every river situation (N, e, b) , the welfare distribution $u(N, e, b) \in \mathbb{R}^n$ given by

$$u_i(N, e, b) = w^i(e, b) - w^{i+1}(e, b), \quad i = 1, \dots, n, \quad (3.6)$$

with $w^{n+1}(e, b) = 0$. Notice that $\sum_{i=1}^n u_i(N, e, b) = w^1(e, b) = W(N, e, b)$ and thus, the upstream incremental solution is efficient.

Theorem 3.4 *A solution f on the class \mathcal{W}^N of river problems is equal to the upstream incremental solution u if, and only if, f satisfies efficiency, the lower bound property, the drought property and independence of upstream inflows.*

Proof. It is straightforward to show that the upstream incremental solution satisfies these four axioms. Therefore, it suffices to prove that the four axioms determine a unique solution.

Let $(N, e, b) \in \mathcal{W}^N$ be a river problem and suppose that solution f satisfies the four axioms. We apply induction on the labels of the agents, starting with the most downstream agent n . We first determine $f_n(N, e, b)$. Consider the modified river problem (N, e^n, b) with $(e^n)_n = e_n$ and $(e^n)_j = 0$ for all $j \in \{1, \dots, n-1\}$. The lower bound property requires that $f_j(N, e^n, b) \geq b_j(0)$ for all $j \in \{1, \dots, n-1\}$, while the drought property requires that

$f_j(N, e^n, b) \leq b_j(0)$ for all $j \in \{1, \dots, n-1\}$. Thus, we conclude that $f_j(N, e^n, b) = b_j(0)$ for all $j \in \{1, \dots, n-1\}$. By efficiency, we have that

$$f_n(N, e^n, b) = w^1(e^n, b) - \sum_{j=1}^{n-1} f_j(N, e, b) = w^1(e^n, b) - \sum_{j=1}^{n-1} b_j(0). \quad (3.7)$$

Since $e_j^n = 0$ for all $j \in \{1, \dots, n-1\}$ and $e_n^n = e_n$, it follows that $w^1(e^n, b) = \sum_{j=1}^{n-1} b_j(0) + w^n(e^n, b) = \sum_{j=1}^{n-1} b_j(0) + w^n(e, b)$ and, thus, with (3.7), we have $f_n(N, e^n, b) = w^n(e, b)$. Independence of upstream inflows implies that $f_n(N, e, b) = f_n(N, e^n, b) = w^n(e, b) = u_n(N, e, b)$.

Proceeding by induction, assume that $f_k(N, e, b) = u_k(N, e, b)$ is determined for all $k > i \geq 1$. Next, consider the modified river problem (N, e^i, b) with $(e^i)_j = e_j$ for all $j \in \{i, \dots, n\}$ and $(e^i)_j = 0$ for all $j \in \{1, \dots, i-1\}$. Similar as above, the lower bound property requires that $f_j(N, e^i, b) \geq b_j(0)$ for all $j \in \{1, \dots, i-1\}$, while the drought property requires that $f_j(N, e^i, b) \leq b_j(0)$ for all $j \in \{1, \dots, i-1\}$. Thus,

$$f_j(N, e^i, b) = b_j(0) \text{ for all } j \in \{1, \dots, i-1\}. \quad (3.8)$$

Independence of upstream inflows and the induction hypothesis imply that $f_j(N, e^i, b) = f_j(N, e, b) = u_j(N, e, b)$ for all $j \in \{i+1, \dots, n\}$. Hence,

$$\sum_{j=i+1}^n f_j(N, e^i, b) = \sum_{j=i+1}^n u_j(N, e, b) = w^{i+1}(e, b). \quad (3.9)$$

Efficiency, the induction hypothesis, (3.8) and (3.9) determine that

$$\begin{aligned} f_i(N, e^i, b) &= w^1(e^i, b) - \sum_{j=1}^{i-1} f_j(N, e^i, b) - \sum_{j=i+1}^n f_j(N, e^i, b) \\ &= w^1(e^i, b) - \sum_{j=1}^{i-1} b_j(0) - w^{i+1}(e, b). \end{aligned} \quad (3.10)$$

Since $e_j^i = 0$ for all $j \in \{1, \dots, i-1\}$ and $e_j^i = e_j$ for all $j \in \{i, \dots, n\}$, similar as above, it follows that $w^1(e^i, b) = \sum_{j=1}^{i-1} b_j(0) + w^i(e^i, b) = \sum_{j=1}^{i-1} b_j(0) + w^i(e, b)$. Thus, with (3.10), we have $f_i(N, e^i, b) = w^1(e^i, b) - \sum_{j=1}^{i-1} b_j(0) - w^{i+1}(e, b) = w^i(e, b) - w^{i+1}(e, b) = u_i(N, e, b)$. Finally, independence of downstream benefits implies that $f_i(N, e, b) = f_i(N, e^i, b) = u_i(N, e, b)$. \square

Whereas in the downstream incremental solution, every upstream coalition $\{1, \dots, j\}$ is exactly compensated for its loss of total benefit when part of its total inflow is allocated to its downstream agents and, thus, the downstream agents get all the gains in benefit from cooperation, according to the upstream incremental solution, the downstream agents

$\{j + 1, \dots, n\}$ get exactly the total benefit that they can achieve by optimally allocating their own inflow among themselves, while agent j gets all the gains in benefit from optimally allocating all inflows e_j, \dots, e_n among the agents $\{j, \dots, n\}$.

Notice that the welfare levels obtained by solving the welfare maximization problems (3.5) fully determine the solution. Hence, by definition, the upstream incremental solution satisfies stability for every downstream coalition $\{i, i + 1 \dots, n\}$. Like the downstream incremental solution, it also satisfies the stability requirement for every coalition of consecutive agents, see van den Brink, van der Laan and Vasil'ev (2007). In van den Brink, van der Laan and Moes (2012), a class of solutions satisfying a so-called TIBS-fairness axiom is introduced that, together with efficiency, yield the downstream incremental solution and the upstream incremental solution as extreme cases.

We show logical independence of the axioms in Theorem 3.4 by giving four alternative solutions, each of these solutions only satisfying three of the four axioms.

1. The solution $f_i(N, e, b) = b_i(0)$ for all $i \in N$ and all river problems (N, e, b) satisfies the lower bound property, the drought property, and independence of upstream inflows. It does not satisfy efficiency.
2. For some $\epsilon > 0$, define the solution f as $f(N, e, b) = u(N, e, b)$ if $e_n = 0$. Otherwise, define $f_1(N, e, b) = u_1(N, e, b) - \epsilon$, $f_i(N, e, b) = u_i(N, e, b)$ for $i = 2, \dots, n - 1$, and $f_n(N, e, b) = u_n(N, e, b) + \epsilon$. It is easily seen that f satisfies efficiency, the drought property, and independence of upstream inflows since u satisfies these properties. It does not satisfy the lower bound property.
3. The solution $f_1(N, e, b) = w^1(e, b) - \sum_{j=2}^n b_j(0)$ and $f_i(N, e, b) = b_i(0)$ for all $i \in N \setminus \{1\}$ satisfies efficiency, the lower bound property, and independence of upstream inflows. It does not satisfy the drought property.
4. The downstream incremental solution satisfies efficiency, the lower bound property, and the drought property. It does not satisfy independence of upstream inflows.

4 Independence of downstream inflows and the upstream solution

As a counterpart of independence of upstream inflows, we now consider independence of downstream inflows which states that the payoff of an agent does not depend on the inflows on the territories of its downstream agents.

Axiom 4.1 (Independence of downstream inflows) *For every two river problems $(N, e, b), (N, e', b) \in \mathcal{W}^N$ such that $e_j = e'_j$ for all $j \leq i$, we have that $f_i(N, e, b) = f_i(N, e', b)$.*

Whereas, in van den Brink, Estévez-Fernández, van der Laan and Moes (2014), independence of downstream benefits together with the three basic axioms of Section 3 characterized the downstream incremental solution of Ambec and Sprumont (2002), it turns out that independence of downstream inflows together with the three basic axioms do not give a unique solution. For example, the downstream incremental solution satisfies both downstream independence axioms. But also the following solution, which we call *upstream solution*, satisfies independence of downstream inflows and the three basic axioms.

To define the upstream solution, we start with agent 1. When all inflows are zero, every agent has payoff $b_i(0)$, $i = 1, \dots, n$. Now, let the most upstream inflow e_1 be optimally distributed amongst all agents. Then, agent 1 receives, in addition to $b_1(0)$, a payoff equal to the marginal contribution to the total benefit when optimally distributing its inflow e_1 amongst all agents, while assuming all other inflows to be equal to zero. Formally, the upstream solution, r , yields to agent 1 the payoff $r_1(N, e, b) = \widehat{v}^1(e, b)$, where

$$\widehat{v}^1(e, b) = b_1(0) + \sum_{j=1}^n (b_j(y_j^1) - b_j(0)) = b_1(y_1^1) + \sum_{j=2}^n (b_j(y_j^1) - b_j(0)),$$

with $y^1 = (y_1^1, \dots, y_n^1)$ a solution to the welfare maximization problem

$$\max_{x_1, \dots, x_n} \sum_{j=1}^n b_j(x_j) \quad \text{s.t.} \quad \sum_{j=1}^n x_j \leq e_1, \quad x_j \geq 0, \quad j = 1, \dots, n.$$

Next, the inflows e_1 and e_2 are optimally distributed over all agents, assuming all other inflows to be equal to zero. Then, agent 2 receives its initial payoff $b_2(0)$ plus the additional total benefit that the distribution of its inflow e_2 generates to the benefits obtained already from e_1 . Subsequently, for agent i , all inflows e_j , $j \leq i$, are optimally distributed over all agents, assuming all inflows of the downstream agents $j > i$ to be equal to zero. Then, agent i receives its initial payoff $b_i(0)$ plus the additional total benefit that the distribution of its inflow e_i generates to the benefit obtained already from e_1 to e_{i-1} . In general, the upstream solution assigns to a river situation $(N, e, b) \in \mathcal{W}^N$ the welfare distribution $r(N, e, b)$ given by

$$r_i(N, e, b) = \widehat{v}^i(e, b) - \widehat{v}^{i-1}(e, b), \quad i = 1, \dots, n, \quad (4.11)$$

where $\widehat{v}^0(e, b) = 0$ and $\widehat{v}^i(e, b) = \sum_{j=1}^i b_j(y_j^i) + \sum_{j=i+1}^n (b_j(y_j^i) - b_j(0))$, $i = 1, \dots, n$, with $y^i = (y_1^i, \dots, y_n^i)$ a solution to the welfare maximization problem

$$\max_{x_1, \dots, x_n} \sum_{j=1}^n b_j(x_j) \quad \text{s.t.} \quad \begin{cases} \sum_{j=1}^n x_j \leq \sum_{j=1}^i e_j, \\ \sum_{j=1}^k x_j \leq \sum_{j=1}^k e_j, \quad k = 1, \dots, i-1, \\ x_j \geq 0, \quad j = 1, \dots, n. \end{cases} \quad (4.12)$$

Observe that this maximization problem optimally distributes the water inflow of the agents in $\{1, \dots, i\}$ over all agents, taking into account that, for every agent $k < i$, the total consumption of the first k agents is, at most, equal to the sum of their own inflows. The payoffs of the welfare distribution $r(N, e, b)$ can also be written as

$$\begin{aligned} r_i(N, e, b) &= \widehat{v}^i(e, b) - \widehat{v}^{i-1}(e, b) \\ &= \sum_{j=1}^i b_j(y_j^i) + \sum_{j=i+1}^n (b_j(y_j^i) - b_j(0)) - \left(\sum_{j=1}^{i-1} b_j(y_j^{i-1}) + \sum_{j=i}^n (b_j(y_j^{i-1}) - b_j(0)) \right) \\ &= b_i(0) + \sum_{j=1}^n (b_j(y_j^i) - b_j(y_j^{i-1})), \quad i = 1, \dots, n. \end{aligned}$$

For $j = n$, the maximization problem (4.12) is again equal to problem (2.2), so that $\widehat{v}^n(e, b) = W(N, e, b)$. Since $\sum_{i=1}^n r_i(N, e, b) = \widehat{v}^n(e, b)$, also the upstream solution distributes the maximal attainable total welfare that the agents can achieve together, and, thus, also this solution is efficient.

To obtain an axiomatization of this upstream solution, we strengthen the drought property to a property, called the no contribution property, which states that an agent with zero inflow of water on its territory should get, at most, a payoff equal to its benefit of zero water consumption. It thus implies that a state cannot claim any benefit from the water allocation when its own inflow is zero.

Axiom 4.2 (No contribution property) *For every river problem $(N, e, b) \in \mathcal{W}^N$ and every $i \in N$ with $e_i = 0$, it holds that $f_i(N, e, b) \leq b_i(0)$.*

The four axioms of efficiency, lower bound property, no contribution property and independence of downstream inflows characterize uniquely the new *upstream solution*.

Theorem 4.3 *A solution f on the class \mathcal{W}^N of river problems is equal to the upstream solution r if, and only if, f satisfies efficiency, the lower bound property, the no contribution property and independence of downstream inflows.*

Proof. It is straightforward to show that the upstream solution satisfies these four axioms. Therefore, it suffices to prove that the four axioms determine a unique solution.

Let $(N, e, b) \in \mathcal{W}^N$ be a river problem and suppose that solution f satisfies the four axioms. Similar as in the proof of Theorem 3.4, we apply induction on the labels of the agents, but now starting with the most upstream agent 1. We first determine $f_1(N, e, b)$. Consider the modified river problem (N, e^1, b) with $(e^1)_1 = e_1$ and $(e^1)_j = 0$ for all $j \in \{2, \dots, n\}$. The lower bound property requires that $f_j(N, e^1, b) \geq b_j(0)$ for all $j \in \{2, \dots, n\}$, while

the no contribution property requires that $f_j(N, e^1, b) \leq b_j(0)$ for all $j \in \{2, \dots, n\}$. Thus, we conclude that $f_j(N, e^1, b) = b_j(0)$ for all $j \in \{2, \dots, n\}$. By efficiency, we have that $f_1(N, e^1, b) = v^n(e^1, b) - \sum_{j=2}^n f_j(N, e^1, b) = v^n(e^1, b) - \sum_{j=2}^n b_j(0)$. Since $e_j^1 = 0$ for all $j \in \{2, \dots, n\}$ and $e_1^1 = e_1$, it follows that $v^n(e^1, b) = \sum_{j=1}^n b_j(y_j^1) = \widehat{v}^1(e, b) + \sum_{j=2}^n b_j(0)$, and, thus, $f_1(N, e^1, b) = \widehat{v}^1(e, b) + \sum_{j=2}^n b_j(0) - \sum_{j=2}^n b_j(0) = \widehat{v}^1(e, b)$. Independence of downstream inflows implies that $f_1(N, e, b) = f_1(N, e^1, b) = \widehat{v}^1(e, b) = \widehat{v}^1(e, b) - \widehat{v}^0(e, b) = r_1(N, e, b)$.

Proceeding by induction, assume that $f_k(N, e, b) = r_k(N, e, b)$ is determined for all $k < i \leq n$. Next, consider the modified river problem (N, e^i, b) with $(e^i)_j = e_j$ for all $j \in \{1, \dots, i\}$ and $(e^i)_j = 0$ for all $j \in \{i+1, \dots, n\}$. Similar as above, the lower bound property requires that $f_j(N, e^i, b) \geq b_j(0)$ for all $j \in \{i+1, \dots, n\}$, while the no contribution property requires that $f_j(N, e^i, b) \leq b_j(0)$ for all $j \in \{i+1, \dots, n\}$. Thus,

$$f_j(N, e^i, b) = b_j(0) \text{ for all } j \in \{i+1, \dots, n\}. \quad (4.13)$$

Independence of downstream inflows and the induction hypothesis imply that $f_j(N, e^i, b) = f_j(N, e, b) = r_j(N, e, b)$ for all $j \in \{1, \dots, i-1\}$. Therefore,

$$\sum_{j=1}^{i-1} f_j(N, e^i, b) = \sum_{j=1}^{i-1} r_j(N, e, b) = \sum_{j=1}^{i-1} [\widehat{v}^j(e, b) - \widehat{v}^{j-1}(e, b)] = \widehat{v}^{i-1}(e, b). \quad (4.14)$$

Efficiency, the induction hypothesis, (4.13) and (4.14) determine that

$$\begin{aligned} f_i(N, e^i, b) &= v^n(e^i, b) - \sum_{j=1}^{i-1} f_j(N, e, b) - \sum_{j=i+1}^n f_j(N, e, b) \\ &= v^n(e^i, b) - \widehat{v}^{i-1}(e, b) - \sum_{j=i+1}^n b_j(0). \end{aligned} \quad (4.15)$$

Since $e_j^i = 0$ for all $j \in \{i+1, \dots, n\}$ and $e_j^i = e_j$ for all $j \in \{1, \dots, i\}$, similar as above, it follows that $v^n(e^i, b) = \sum_{j=1}^n b_j(y_j^i) = \widehat{v}^i(e, b) + \sum_{j=i+1}^n b_j(0)$. Thus, with (4.15), we have $f_i(N, e^i, b) = \widehat{v}^i(e, b) + \sum_{j=i+1}^n b_j(0) - \widehat{v}^{i-1}(e, b) - \sum_{j=i+1}^n b_j(0) = \widehat{v}^i(e, b) - \widehat{v}^{i-1}(e, b) = r_i(N, e, b)$. Finally, independence of downstream inflows implies that $f_i(N, e, b) = f_i(N, e^i, b) = r_i(N, e, b)$. \square

While the upstream incremental solution favors the upstream agents as much as possible under the restriction of stability for the downstream coalitions, the upstream solution favors the upstream agents as much as possible given the uni-directionality of the water flows. It thus requires that every agent receives the highest attainable additional benefit from its water inflow, given that the inflows of its upstream agents have been already distributed.

According to the upstream solution, every upstream coalition $\{1, \dots, j\}$, $j \in N$, receives the total welfare that can be attained by optimally allocating the water inflows of

such a coalition over all agents. Clearly, the maximal welfare corresponding to the welfare maximization problem (4.12) is, at least, as high as the maximal welfare corresponding to the welfare maximization problem (2.4), with $h = 1$, in which the inflows of the players in a coalition $\{1, \dots, j\}$ are optimally distributed amongst themselves. Therefore, the upstream solution certainly satisfies stability for the upstream coalitions and, thus, satisfies the Harmon principle for the upstream coalitions. However, the upstream solution does not satisfy stability in general. For example, agent n receives the marginal benefit $\widehat{v}^n(e, b) - \widehat{v}^{n-1}(e, b)$, being the difference between the total benefit of the water consumptions y^n and y^{n-1} . Nothing can be said about this difference and the benefit $b_n(e_n)$ that agent n can obtain by consuming its own water. Therefore, it might happen that $r_n(N, e, b) < b_n(e_n)$, violating the individual rationality constraint.

We show logical independence of the axioms in Theorem 4.3 by giving four alternative solutions.

1. The solution $f_i(N, e, b) = b_i(0)$ for all $i \in N$ and all river problems (N, e, b) satisfies the lower bound property, the no contribution property, and independence of downstream inflows. It does not satisfy efficiency.
2. For some $\epsilon > 0$, define the solution f by $f(N, e, b) = r(N, e, b)$ if $e_1 = 0$. Otherwise, define $f_1(N, e, b) = r_1(N, e, b) + \epsilon$, $f_i(N, e, b) = r_i(N, e, b)$ for $i = 2, \dots, n - 1$, and $f_n(N, e, b) = r_n(N, e, b) - \epsilon$. It is easily seen that f satisfies efficiency, the no contribution property, and independence of upstream inflows. It does not satisfy the lower bound property.
3. The solution $f_i(N, e, b) = b_i(0)$ for all $i \in N \setminus \{n\}$ and $f_n(N, e, b) = v^n(e, b) - \sum_{j=1}^{n-1} b_j(0)$ satisfies efficiency, the lower bound property, and independence of downstream inflows. It does not satisfy the no contribution property.
4. The upstream incremental solution u satisfies efficiency, the lower bound property, and the no contribution property. It does not satisfy independence of downstream inflows.

5 Concluding remarks

In this paper, we considered the problem of sharing water among agents located along a river. We used the same three basic axioms as we used in van den Brink, Estévez-Fernández, van der Laan and Moes (2014) but instead of independence of upstream, respectively downstream, benefit functions, here we considered independence of upstream, respectively downstream, *inflows*. Whereas, adding each of the two benefit independence axioms to

	Independence of Upstream	Independence of Downstream
Benefits	UTI incremental solution	Downstream Incremental solution
Inflows	Incompatibility	Multiplicity

Table 1:

the three basic axioms characterizes a unique solution, we saw that (i) independence of upstream inflows is incompatible with the three basic axioms, while (ii) independence of downstream inflows gives multiple solutions that satisfy this axiom and the three basic axioms, see Table 1. We considered another weakening of the aspiration level property to get an axiomatization of the upstream incremental solution which satisfies independence of upstream inflows. We strengthened this new axiom to get an axiomatization of the new upstream solution which satisfies independence of downstream inflows.

Although we stated the results under the assumptions on the benefit functions of van den Brink, Estévez-Fernández, van der Laan and Moes (2014), since we did not change the benefit functions in the proofs of uniqueness, the axiomatizations are also valid if we restrict ourselves to the class of continuous, strictly concave benefit functions (as in Ambec and Ehlers (2008)), or even the class of continuous, strictly increasing benefit functions (as in Ambec and Sprumont (2002)).

Finally, we briefly consider the special case of benefit functions where every agent has constant marginal benefit up to a satiation point and marginal benefit of zero thereafter. So, for some $b > 0$, for every agent i there is a $c_i > 0$ such that the benefit functions are given by $b_i(x_i) = bx_i$ if $x_i \leq c_i$, and $b_i(x_i) = bc_i$ if $x_i > c_i$. If $b = 1$ then this is similar to the *river claim problem* of Ansink and Weikard (2012). For this class of river problems, it is easy to verify that the upstream incremental solution and upstream solution are identical and, moreover, cannot be implemented without monetary transfers. On the contrary, in van den Brink, Estévez-Fernández, van der Laan and Moes (2014) it is shown that the downstream incremental and downstream solution can be implemented without monetary transfers.

References