**(Preliminary version)**

# Impacts of Misperceptions about Disastrous Events on International Security[†]

2 April 2017

Toshihiro Ihori[a] and Shintaro Nakagawa[b]

**Abstract**

When countries cope with the risk of disastrous events in a political economy, misperceptions of the damages from the events could occur. We incorporate misperceptions caused by interest groups that have special interests on security spending such as arm-producing companies. If there exists no spillover effect, such misperceptions distort the decision-making of the countries and deteriorate their welfares. However, when countries jointly cope with the risk of disastrous events through the voluntary provision of international public goods, such misperceptions could mitigate inefficiency caused by free-riding behaviors. We show that if the absolute risk aversion of an allied country satisfies certain conditions, the country increases its contribution to the international public good with the overestimation of the damage of a bad event. Furthermore, we show that there could be the optimal value of overestimation, which induces countries to provide the social-welfare-maximizing amounts of international public goods. We also show how the optimal overestimation varies with incomes of the allied countries and their losses in the disastrous events. Finally, we show that if interest groups endogenously determine the magnitude of overestimation, there exists the optimal marginal cost of lobbying inducing the socially optimal overestimation by the interest groups.

*Keywords: self-protection, international public goods, misperception.*

H41, H50, D81, F50.

---

[†]An earlier version of this paper was presented to the 53rd Annual Meetings of the Public Choice Society on March 12, 2016, at the Hyatt Regency Pier 66 in Ft. Lauderdale, Florida, and 2016 Australian Public Choice Conference on December 9, 2016, at the Australia National University, Canberra. We thank Alessandro Sontuoso and other conference participants for insightful comments on that earlier paper.

[a] National Graduate Institute for Policy Studies, 7-22-1 Roppongi, Minato-ku, Tokyo 106-8677, Japan. tel: +81-3-6439-6171 Email: ihori@grips.ac.jp.

[b] Faculty of Economics, Konan University; 8-9-1 Okamoto, Higashinada-ku, Kobe 658-8501, Japan.

## 1. Introduction

As more and more countries face a common risk of a disastrous event, they form an alliance and jointly avoid that risk. Examples of such events include invasion, terrorist attacks, environmental disaster, and the pandemic of infectious disease. When countries cope with the risk of disastrous events in a political economy, misperceptions by interest groups could occur. For example, arm-producing companies would have an incentive to overestimate the cost of emergency pessimistically. Such a political bias may affect the benevolent government in their choice of security spending. In the present paper, by explicitly incorporating misperceptions into the theory of security alliance, we investigate the positive and normative impacts of misperceptions caused by interest groups in a political economy framework.

Olson and Zeckhauser (“OZ,” 1966) proposed a now famous theory of security alliances. They considered “international security” as a public good, and did not explicitly formalize the risk management of the allied countries. Among the first to examine combinations of instruments open to individuals to manage risks to their well-being, Ehrlich and Becker (“EB,” 1972) identified several types of preparation available to expected utility maximizing agents faced with what we will call "costs of emergency." These costs consist of any mix of (a) probability of loss and (b) magnitude of loss (hereafter together referred to as "risk profile"). Among such preparations were (a) "self-insurance" to compensate for or reduce the magnitude of loss and (b) "self-protection" to reduce the probabilities of loss. Ihori and McGuire (2007) provided an analysis of the voluntary provision of self-protection as a public good in an alliance. Our previous paper investigated the burden sharing in NATO with a voluntary public goods model where EB’s self-protection and self-insurance are international public good within the alliance (Ihori et al. 2014). However, utilization of EB to model collective improvements to the entire "risk profiles" as international public goods is still sparse, except for some work such as Sandler (1992, 1997, and 2005), Lohse et al. (2006), and Muermann and Kunreuther (2007).

In the present paper, we consider a group of countries jointly cope with a risk event by providing self-protection as an international public good, and introduce misperceptions caused by interest groups that have special interests in security spending. In a one-country model, such misperceptions would not attain the optimal supply of security spending. However, in an alliance model, the nature of a public good itself has free riding problems and the voluntary provision of public good results in under-provision of the public good at the non-cooperative Nash solution. Then, if misperceptions stimulate the provision of the public good, they could improve the welfare of the allies. We construct a two-country model in which two countries constitute an alliance and voluntarily provide an intra-alliance public good to protect a loss from a common bad event. Our model consists of two periods. In the first period, the two countries independently determine their investments on self-protections. In the second period, a bad event stochastically occurs. The probability of the bad event depends on the sum of self-protection contributions made in the first period. Unlike conventional models, we assume that countries decide their self-protection contribution without knowing the true loss of the bad event. We assume that their estimation of the loss in the first stage is biased. Then, we show that there may exist a value of overestimation of the loss, which leads countries voluntarily contribute a socially optimal amount to the self-protection public good. Additionally, we conduct some comparative statics and show how incomes and losses of the allied countries influence the social-welfare-maximizing overestimation. Furthermore, we extend our model to allow interest groups endogenously determine the level of overestimation. Using numerical simulations, we present a case in which endogenous overestimation achieves socially optimal provision of the self-protection public good.

This paper consists of six sections. Section 2 formulates our two-country model. Section 3 investigates the Nash equilibrium of our model and discusses the level of overestimation that induces countries to purchase the socially optimal amounts of self-protection public good. Section 4 extends our model to a three-period model, in which the interest groups in the two countries endogenously determines the level of overestimation. Section 5 conducts several numerical simulations and explores whether there is the optimal overestimation of loss that induces countries to contribute the socially optimal amount of self-protection and whether there is the optimal marginal cost of lobbying that causes the profit-maximizing interest groups voluntarily chooses the optimal overestimation. Section 6 concludes.

## 2. Model

In this section, we construct a two-country model, in which two countries constitute an alliance to share a common risk of an emergency. This model is based on the two-country model investigated by Ihori and McGure (2007). We introduce misperceptions into their model. Although interest groups may cause the misperception, we initially assume that the misperception is exogenously determined. In the first subsection, we construct our model except for misperceptions. In the second subsection, we introduce misperceptions in the decision-making.

### 2.1 Model without misperception

We assume that there are two countries, indexed by countries A and B. Our model consists of two periods. In the first period, the two countries independently purchase self-protection. We also assume that the two countries non-cooperatively provide self-protection, as pure public goods for themselves and each other. In the second period, there are two states of the world: a bad event occurs in state 0, and no event happens in state 1. We refer to state 1 as a good state, and state 0 as a bad state. We assume that the two countries face the same state of the world. The provision of self-protection public good raises the probability of a good state. Then, we have the following two-period game.

Period 1: Government of country A (B) independently determines its purchase of self-protection to maximize its estimated expected welfare based on the estimation of loss, which is given as exogenous variables.

Period 2: The state of the world is stochastically determined based on the self-protection public good provided in Period 1. The consumptions of households in both countries are realized.

Let us consider the maximization problem for the government of country A. Replacing the superscript A in the following equations with B, we obtain the corresponding variables of country B. In the first period, the two countries finance their expenditures by issuing bonds in a financial market. We assume that the interest rates on the bonds are zero. The per-capita budget constraint of country A in the first period is given as the following:

$$b^A = m_1^A \tag{1}$$

where $b^A$ is the bond issued by country A, and $m_1^A$ is the amount of self-protection purchased by country A.

In the second period, the probability of a good state is denoted by $p$. We assume that the self-protection purchased by both countries raises the probability of a good state and that the benefit of increasing probability of a good state is nonrival and nonexcludable, which means that the self-protection is an international public good. The amount of the self-protection public good is given by the sum of self-protection contributions:

$$M_1 = m_1^A + m_1^B. \tag{2}$$

The probability of a good state is given by the following function:

$$p = p(M_1), \tag{3}$$

where

$$p''(M_1) > 0, p''(M_1) \le 0, p(M_1) \in (0,1) \quad \text{for any } M_1 \ge 0 . \tag{4}$$

The expected utility for the representative household in country A is given by the following:

$$W^A = pU^{1A} + (1-p)U^{0A}, \tag{5}$$

where $W^A$ is his/her expected utility, $U^{1A}$ is his/her realized utility if a good event happens, $U^{0A}$ is his/her realized utility if a bad event happens. The realized utility is given by a state-independent utility function, which is represented as the following:

$$U^{iA} = U(C^{iA}), \text{ for } i = 1, 0. \tag{6}$$

We assume that the utility function $U(.)$ satisfies the following inequalities:[1]

$$U_Y^{iA} \equiv \partial U(C^{iA}) / \partial C^{iA} > 0, U_{YY}^{iA} \equiv \partial^2 U(C^{iA}) / \partial (C^{iA})^2 < 0, (i = 0,1) . \tag{7}$$

In the second period, the budget constraints of the household in country A are given by the following:

$$C^{1A} = Y^A - b^A, \tag{8}$$

$$C^{0A} = Y^A - L^A - b^A, \tag{9}$$

where $C^{1A}$ is his/her consumption in the good state, $C^{0A}$ is his/her consumption in the bad state, $Y^A$ is his/her income in the second period and $L^A$ is his/her loss from the bad event. Substituting (1) in equations (8) and (9), we obtain the realized consumption in the second period as the following equations:

$$C^{1A} = Y^A - m_1^A, \tag{10}$$

$$C^{0A} = Y^A - L^A - m_1^A. \tag{11}$$

Similarly, country B's realized consumption is given as the following equations:

$$C^{1B} = Y^B - m_1^B, \tag{12}$$

$$C^{0B} = Y^B - L^B - m_1^B. \tag{13}$$

Using this model, we derive the socially optimum level of self-protection public good. We assume that a planner maximizes the utilitarian social welfare function as the following:

$$\max_{m_1^A, m_1^B} W^A + W^B$$

subject to (3), (2), (5), (6), (10), (11), (12), and (13).

The first order conditions are given as the following:

$$p'\left(U^{1A} + U^{1B} - U^{0A} - U^{1B}\right) - \left[ pU_Y^{1A} + (1-p)U_Y^{0A} \right] = 0, \tag{14}$$

[1] In the notation of the derivative of the utility function, we use subscript $Y$. As defined below, $Y$ represents the unconditional income of the country. The reason why we use $Y$ instead of C is that we focus on the income effect throughout this paper.

$$p'\left(U^{1A}+U^{1B}-U^{0A}-U^{1B}\right)-\left[pU_Y^{1B}+(1-p)U_Y^{0B}\right]=0. \tag{15}$$

We assume that the second order condition is satisfied.

Solving the first order conditions, we obtain the socially optimum levels of self-protection, $m_1^{A*}$ and $m_1^{B*}$. The first term of (14) represents the marginal benefit of self-protection, which is the gain in the social welfare multiplied by the increase in the probability of a good state. The second term represents the marginal costs of self-protection. Since one unit of self-protection reduces country A's consumptions in both states, the marginal cost is given as a probability-weighted sum of marginal utility of consumption.

**2.2 Misperception model**

Then, we introduce misperception on the magnitude of the loss. We assume that each government maximizes the expected welfare of its representative household. However, we assume that the interest groups may influence the government's estimation of the household's loss. For example, arm-producing companies may emphasize the threat of terrorist attacks and bias the government's estimation of the loss. In the following, we introduce overestimation of the magnitude of the loss in a bad state.

To begin with, we investigate the utility maximization of country A in the presence of misperceptions. The utility maximization of country B is similarly defined. Under the political biases in the first period, the government of country A estimates the consumption of the household in the second period as the following equations:

$$\tilde{C}^{1A}=Y^A-m_1^A, \tag{16}$$

$$\tilde{C}^{0A}=\tilde{Y}^A-\tilde{L}^A-m_1^A, \tag{17}$$

where $\tilde{C}^{1A},\tilde{C}^{0A}$ are the estimated consumptions in good and bad states, and $\tilde{L}^A$ is the estimated loss in the bad state. We assume that the estimated loss is given by the following equation:

$$\tilde{L}^A=L^A+\alpha^A, \tag{18}$$

where $\alpha^A$ represents the pessimistic bias, or the overestimation of the loss in a bad state. Although the government of country A is benevolent, it believes $\tilde{L}^A$ as the true estimations of the household's loss. The magnitudes of misperceptions, $\alpha^A$, are determined by the lobbying activities of the interest groups. They might put political pressures to make the benevolent government believe their estimates as the true one. The realized values of $\alpha^A$ is the outcome of such political efforts. In this section, we assume that the magnitude of overestimations is exogenously determined. In Section 4, we will extend our model to allow interest groups to determine the levels of overestimations to maximize their profits.

In the first period, country A maximizes the expected welfare on its purchase of self-protection, $m_1^A$. The maximization problem involving misperceptions is written as follows:

$$\max_{m_1^A,m_2^A}\tilde{W}^A=pU(\tilde{C}^{1A})+(1-p)U(\tilde{C}^{0A})$$

subject to (3), (2), (5), (6), (16), and (17) .

The first order conditions of the maximization problem for an interior solution is given as

$$p'\left(\tilde{U}^{1A}-\tilde{U}^{0A}\right)-\left[p\tilde{U}_Y^{1A}+(1-p)\tilde{U}_Y^{0A}\right]=0, \tag{19}$$

where $\tilde{U}^{iA} \equiv U(\tilde{C}^{iA}), \tilde{U}_Y^{iA} \equiv \partial U(\tilde{C}^{iA}) / \partial \tilde{C}^{iA}$ for $i = 0,1$ . Country A's contribution to self-protection raises the probability of a good state, which benefits not only country A but also B. However, equation (19) does not include the marginal benefit of self-protection for country B. In other words, country A ignores the benefit of country B. If there is no overestimation of loss, country A's contribution becomes less than the socially optimal level.

The second order condition is written as

$$\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2} < 0. \tag{20}$$

We assume that the second order condition is satisfied. Solving (19), we obtain country A's best response function, which depends on country B's contribution, country A's income, and country A's estimated loss:[2] $m_1^A(m_1^B, Y^A, \tilde{L}^A)$ .

Ihori and McGuire (2007) investigated the properties of the best response function in a model without misperceptions. The corresponding properties in our model are the same as in their model except for that the loss is not the true loss but a biased loss. Taking the total differentiation of (19), we obtain the properties of country A's best response function as follows:[3]

**Proposition 1**

*(i)Country A purchases a positive amount of self-protection if the following condition is satisfied:*

$$p'(m_1^B)\left(U(\tilde{Y}^A) - U(\tilde{Y}^A - \tilde{L}^A)\right) - \left[p(m_1^B)U_Y(\tilde{Y}^A) + (1 - p(m_1^B))U_Y(\tilde{Y}^A - \tilde{L}^A)\right] > 0. \tag{21}$$

*Otherwise, country A does not purchase any self-protection.*

*(ii)Suppose that equation (21) holds. Then, we have the following:*

$$\frac{\partial m_1^A}{\partial m_1^B} < 0 \Leftrightarrow p''(\tilde{U}^{1A} - \tilde{U}^{0A}) + p'(\tilde{U}_Y^{0A} - \tilde{U}_Y^{1A}) < 0, \tag{22}$$

$$\frac{\partial m_1^A}{\partial \tilde{L}^A} > 0 \Leftrightarrow p'\tilde{U}_Y^{0A} + (1-p)\tilde{U}_{YY}^{0A} > 0, \tag{23}$$

*where* $\tilde{U}_{YY}^{iA} \equiv \partial^2 U(\tilde{C}^{iA}) / \partial (\tilde{C}^{iA})^2$ .

The economic intuition of Proposition 1 is explained as follows. The first claim represents the condition for country A to make a positive contribution to the public good. The LHS of (21) is the marginal net benefit of self-protection for country A when country A does not purchase any self-protection, or $m_1^A = 0$ . If this condition is satisfied, freeriding in self-protection is not optimal for country A.

The second claim of Proposition 1 shows the comparative statics properties of the best response function. An increase in the loss in a bad state also has two effects on the marginal net benefit of self-protection. First, the increasing loss raises the marginal benefit of self-protection, which depends on the difference in utility in good and bad states. An increase in the loss reduces the utility in the bad state and enlarges the utility difference between the

---

[2] Since the estimated loss is biased by misperceptions, the best response does not maximize A's true expected welfare. In this sense, A's best response is not "best" in its literal meaning, but it maximizes A's estimated (and biased) expected welfare.

[3] The proof of Proposition 1 will be given in an updated version of this paper.

two states, which increases the marginal benefit of self-protection. Second, the increasing loss raises the marginal cost of self-protection, which is given as the probability-weighted sum of marginal utility in both states. The additional loss reduces the estimated consumption in the bad state, which raises the marginal utility of consumption in that state. Thus, the increasing loss raises the marginal cost of self-protection. The magnitude of the first effect depends on the marginal utility while the second effect depends on the second derivative of utility. Thus, if country A's absolute risk aversion is sufficiently small, the first effect dominates the second effect. Then, the increase in the loss from the bad event raises country A's purchase of self-protection.

## 3. Nash equilibrium and optimal overestimation

### 3.1 Nash equilibrium of misperception model

We define the Nash equilibrium of the two-country game in the first stage as the bundle of purchases of self-protection, $(m_1^{AN}, m_1^{BN})$, which are given as follows:

$$m_1^{AN} = m_1^A(m_1^{BN}, Y^A, \tilde{L}^A), \tag{24}$$

$$m_1^{BN} = m_1^B(m_1^{AN}, Y^B, \tilde{L}^B). \tag{25}$$

The Nash equilibrium levels of provision of self-protection, the realized consumption of household of country A are respectively given by

$$M_1^N = m_1^{AN} + m_1^{BN}\ . \tag{26}$$

$$C^{1AN} = Y^A - m_1^{AN}, \tag{27}$$

$$C^{0AN} = Y^A - L^A - m_1^{AN}. \tag{28}$$

As shown in (27) and (28), the realized consumption does not contain the bias on the loss. The utility of households in country A is given as follows:

$$U^{1AN} = U(C^{1AN}) \text{ and } U^{0AN} = U(C^{0AN})\ . \tag{29}$$

We refer to the probability weighted realized utility as the truly expected welfare:

$$W^{AN} = pU^{1AN} + (1-p)U^{0AN}. \tag{30}$$

Since misperceptions distort country A's welfare maximization, country A does not necessarily maximize its truly expected welfare.

In the absence of misperceptions, the Nash equilibrium level of self-protection is less than the socially optimum level as in ordinary voluntary provision models. However, Proposition 1 implies that if the absolute risk aversion of the representative household is sufficiently small, an increase in overestimation of loss may raise the provision of self-protection. In the following, we investigate whether an appropriate magnitude of overestimation of loss induces countries to contribute the socially optimal amounts to the self-protection public good.

### 3.2 Optimal overestimation of loss

Suppose that there is a vector of overestimation of loss, $(\alpha^{A*}, \alpha^{B*})$, such that when countries A and B consider their loss in a bad state as $L^A + \alpha^{A*}$ and $L^B + \alpha^{B*}$, they voluntarily contribute socially optimal amounts, $(m_1^{A*}, m_1^{B*})$, to the self-protection public good. Formally, we assume that the vector $(\alpha^{A*}, \alpha^{B*})$ satisfies the following system of equations:

$$p'^*\left[U(Y^A - m_1^{A*}) - U(Y^A - L^A - \alpha^{A*})\right] - \left[p^* U_Y(Y^A) + (1-p^*)U_Y(Y^A - L^A - \alpha^{A*})\right] = 0, \tag{31}$$

$$p'*\left[U(Y^B - m_1^{B*}) - U(Y^B - L^B - \alpha^{B*})\right] - \left[p*U_Y(Y^B) + (1-p*)U_Y(Y^B - L^B - \alpha^{B*})\right] = 0, \quad (32)$$

where $p^* \equiv p(m_1^{A*} + m_1^{B*}), p'^* \equiv p'(m_1^{A*} + m_1^{B*}).$ Equation (31) is the first order condition for country A to maximize its expected welfare, where countries A and B purchase the socially optimum levels of self-protection, $m_1^{A*}$ and $m_1^{B*}$. If there are no overestimations, or $\alpha^{A*} = 0$, the LHS of (31) is negative. However, there might exist a value of overestimation of loss, $\alpha^{A*}$, such that solves equation (31). Then, country A voluntarily contributes $m_1^{A*}$.

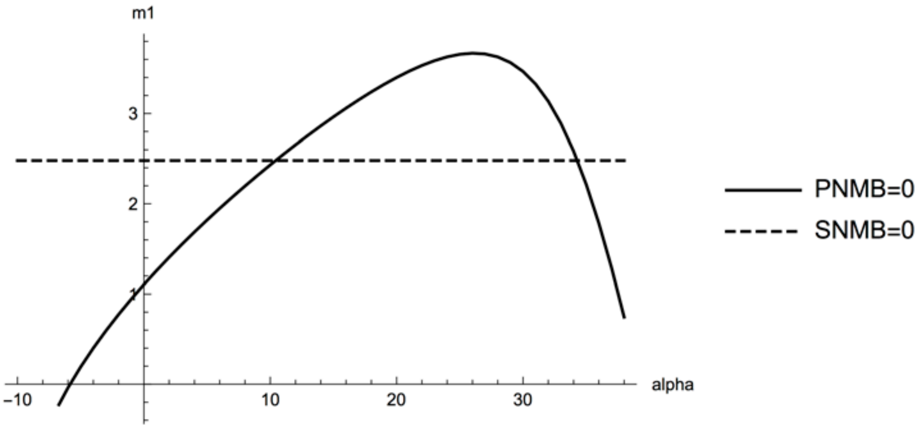
It is not straightforward to derive the condition of the existence of $(\alpha^{A*}, \alpha^{B*})$. Here, we limit ourselves to present some discussions. Let us consider a symmetric Nash equilibrium in a symmetric economy, where $Y^A = Y^B = Y, L^A = L^B = L, m_1^{A*} = m_1^{B*} = m_1^*,$ and $\alpha^{A*} = \alpha^{B*} = \alpha^*$. Then, the symmetric optimal self-protection and the optimal overestimation of the loss, $(m_1^*, \alpha^*)$, solve the following system of equations:

$$SNMB(m_1^*) \equiv 2p'(2m_1^*)\left[U(Y) - U(Y-L)\right] - \left[p(2m_1^*)U_Y(Y) + (1-p)U_Y(Y-L)\right] = 0, \quad (33)$$

$$PNMB(m_1^*, \alpha^*) \equiv p'*\left[U(Y - m_1^*) - U(Y - L - \alpha^*)\right] - \left[p*U_Y(Y) + (1-p*)U_Y(Y - L - \alpha^*)\right] = 0, \quad (34)$$

The LHS of (33) is the social net marginal benefit (or SNMB) of self-protection in this symmetric economy. If $m_1^*$ maximizes the utilitarian social welfare, the LHS must be zero. Equation (33) does not include misperceptions. The optimal self-protection, $m_1^*$, is independent of the overestimation. The LHS of (34) is the private net marginal benefit (or PNMB) of self-protection for each country. When $m_1^*$ and $\alpha^*$ satisfy equation (34), each country maximizes its biased expected welfare by choosing $m_1^*$ as the amount of self-protection contribution. Figure 1 represents how the value of $\alpha^*$ is determined. The horizontal axis of this figure represents the overestimation of loss, while its vertical axis represents the purchase of self-protection. This figure is drawn based on the specifications of functional forms and the settings of parameters in the numerical examples to be presented in Section 5. The dashed line represents the solution of (33). Since the socially optimal self-protection does not depend on overestimation, we have a horizontal line representing (33). The solid curve represents the locus of points satisfying equation (34) in a $(\alpha, m_1)$-plane. As shown in Figure 1, when there is no overestimation of loss ($\alpha = 0$), each country voluntarily purchases self-protection less than the socially optimal level. In Figure 1, the voluntary provision of self-protection increases with the overestimation of loss as far as the overestimation is small. Then, the solid curve intersects the dashed line, where the overestimation induces countries to purchase the socially optimal level of self-protection. Further overestimation of loss leads to overprovision of self-protection. When the solid curve passed its peak, additional overestimation reduces the voluntary purchase of self-protection. Then, the solid curve intersects the dashed line again. In Figure 1, there are two levels of overestimation of loss that make countries voluntarily purchase the socially optimal amount of self-protection.

Figure 1: Multiple optimal overestimations




However, that conclusion does not necessarily follow. Countries might not sufficiently increase their voluntary purchase in response to the overestimation of loss. Then, the solid curve may not intersect the dashed curve. No optimal overestimation of loss would induce countries to provide with the socially optimal amount of self-protection.

Finally, we assume that there is an optimal overestimation of loss, and investigate how increases in income and loss affect the optimal overestimation. For simplicity, we assume symmetric economy and symmetric Nash equilibrium. Taking the total differentiation of (33) and (34), we conduct some comparative statics analysis. Then, we have the following proposition.[4]

**Proposition 2**

*Let us consider a symmetric economy. We assume that there is a symmetric Nash equilibrium,* $m_1^* = m_1^{A*} = m_1^{B*}$, *and a value of overestimation,* $\alpha^* = \alpha^{A*} = \alpha^{B*}$, *such that countries voluntarily provide the social-welfare-maximizing amount of self-protection. Then, simultaneous increases in income of both countries,* $dY = dY^A = dY^B$, *and that in the loss in a bad state,* $dL = dL^A = dL^B$, *have the following impacts on the optimal overestimation.*
*If we obtain the following inequalities:*

$$p'(2m^*) > R(Y - L - m_1^*) \;\; and \;\; \frac{p'(2m^*)}{1 - p(2m^*)} < R(Y - L - \alpha - m_1^*)\,, \tag{35}$$

*we have the following:*

$$\frac{d\alpha^*}{dY} < 0 \;\; and \;\; \frac{d\alpha^*}{dL} > 0\,, \tag{36}$$

*where* $R(C) \equiv -U_{YY}(C)/U_Y(C)$

Proposition 2 claims that if the absolute risk aversions of the two countries satisfy expression (35), an increasing income reduces the optimal overestimation, while an increasing loss raises the optimal overestimation.

---

[4] The proof of Proposition 2 will be given in an updated version of this paper.

## 4. Endogenous overestimation

In this section, we extend our model to a three-period game. Here, we add a new period, or Period 0, before countries A and B independently determine their voluntary purchase of self-protection. In Period 0, interest groups in the two countries determine the magnitude of overestimation of loss to maximize their profit. To summarize, we investigate the following three-period game:

Period 0: Interest group in country A (B) independently determines it government's overestimation of loss to maximize its profit.

Period 1: Government of country A (B) independently determines its purchase of self-protection to maximize its estimated expected welfare based on the overestimation of loss.

Period 2: The state of the world is stochastically determined based on the self-protection public good provided in Period 1. The consumptions of households in both countries are realized.

We define the profit of interest group as the purchase of self-protection by the government of the country minus a lobbying cost. We assume that the lobbying cost is proportional to the level of overestimation. The profit of interest group in country A, $\Pi^A$, is defined as

$$\Pi^A = m_1^A - c\alpha^A, \tag{37}$$

where $c$ is the marginal cost of lobbying. The profit of interest group in country B is similarly defined. The interior first order conditions for the interest groups in the two countries are given as

$$\frac{\partial m_1^A}{\partial \alpha} - c = 0, \tag{38}$$

$$\frac{\partial m_1^B}{\partial \alpha} - c = 0. \tag{39}$$

The subgame perfect equilibrium of this model satisfies the system of equations (24), (25), (38), and (39).

We investigate whether there is an appropriate level of the marginal cost of lobbying, $c^*$, such that interest groups voluntarily choose the optimal overestimation. For simplicity, we consider the symmetric economy. If there is such a marginal cost, it must satisfy the following equations.

$$\left.\frac{\partial m_1^A}{\partial \alpha}\right|_{m_1^A = m_1^B = m_1^*, \alpha^A = \alpha^B = \alpha^*} = c^*, \tag{40}$$

$$SNMB(m_1^*) = 0, \tag{41}$$

$$PNMB(m^*, \alpha^*) = 0. \tag{42}$$

It is not clear whether such $c^*$ exists or not. In the next section, we specify the functional forms of the utility function and probability function and present some numerical examples.

## 5. Numerical examples

In this section, we conduct several numerical simulations to investigate whether there is the optimal overestimation of loss that induces countries to contribute the socially optimal amount of self-protection and whether there is the optimal marginal cost of lobbying that causes the profit-maximizing interest groups voluntarily choose the optimal overestimations.

To conduct numerical analysis, we specify the forms of the functions in our model. Following Ihori et al. (2014), we specify the utility function, $U(.)$, as a constant relative risk aversion (CRRA) function:

$$U(C) = \frac{C^{1-\theta}}{1-\theta}, \tag{43}$$

where $\theta$ is the parameter representing the relative risk aversion of the country. We also specify the function determining the probability of a good state as the following Tullock's contest success function:

$$p(M_1) = \frac{M_1}{M_1 + \gamma}, \tag{44}$$

where $\gamma$ is the effort made by the opponent of the alliance.

Table 1 Numerical simulations of exogenous overestimation

| Scenario | 1<br>Social optimum | 2<br>Nash equilibrium without misperceptions | 3<br>Nash equilibrium with optimal overestimation |
|---|---|---|---|
| $Y^A$ | 50 | 50 | 50 |
| $L^A$ | 10 | 10 | 10 |
| $Y^B$ | 50 | 50 | 50 |
| $L^B$ | 10 | 10 | 10 |
| $\alpha^A$ | | 0 | 10.429 |
| $\alpha^B$ | | 0 | 10.429 |
| $\gamma$ | 4 | 4 | 4 |
| $\theta$ | 0.9 | 0.9 | 0.9 |
| $m_1^A$ | 2.480 | 1.105 | 2.480 |
| $m_1^B$ | 2.480 | 1.105 | 2.480 |
| $M_1$ | 4.960 | 2.210 | 4.960 |
| $\tilde{W}^A$ | | 14.540 | 14.354 |
| $\tilde{W}^B$ | | 14.540 | 14.354 |
| $W^A$ | 14.559 | 14.540 | 14.559 |
| $W^B$ | 14.559 | 14.540 | 14.559 |

Table 1 presents three numerical examples of our two-period game, in which the overestimation of loss is exogenously determined. In this model, the loss in a bad state is 20% of the income in the second period. The first column of this table reports the solution of the utilitarian social welfare maximization problem. Under the settings of parameters in this table, the optimal purchase of self-protection is 2.480, which is 4.9% of the income. The second column represents the Nash equilibrium allocation of this model, where no overestimation of loss exists. In this case, each country voluntarily purchases 1.105 units of self-protection, which is less than the half of the optimal level. The third column reports the Nash equilibrium allocation in the presence of optimal overestimation of loss. In this case, the overestimation of loss is 10.429, which is slightly greater than the true loss. If this level of overestimation of loss is exogenously given, each country purchases 2.480 units of self-protection, which is the socially optimal level. The truly expected welfare in the third column is equal to those in the socially optimum scenario.

Figure 2a Income and optimal overestimation of loss in a symmetric economy

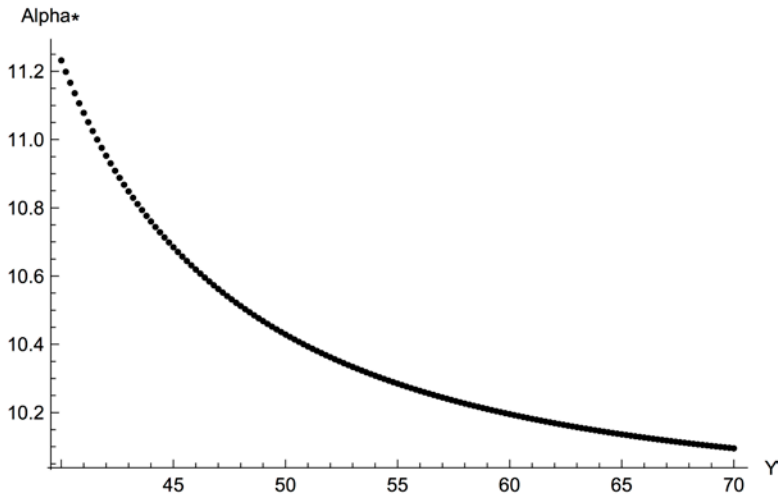
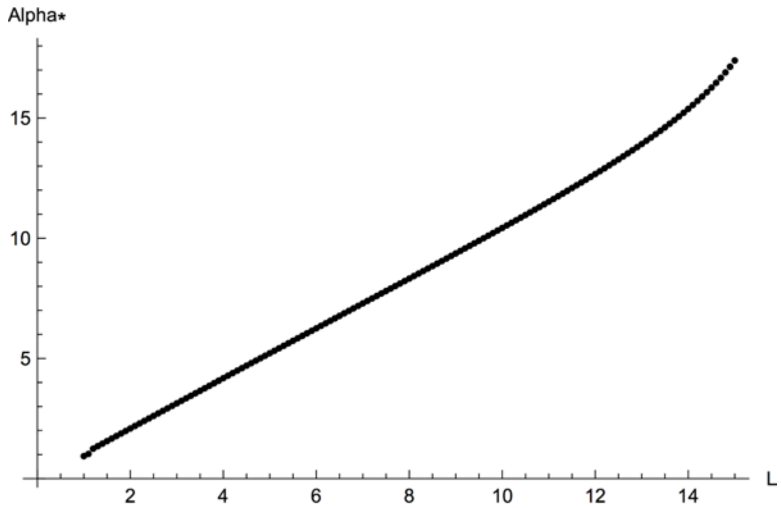



Figure 2b Loss in a bad state and optimal overestimation in a symmetric economy




Figures 2a and 2b illustrate how the optimal overestimation changes with income and loss in a bad state. In Figure 2a, the parameters are set as in Table 1 except for income. We assume that the incomes of countries A and B are identical, $Y^A = Y^B = Y$, and that $Y$ increases from 40 to 70 by 0.2. The horizontal axis represents the value of income, $Y$, while the vertical axis represents the optimal value of overestimation, $\alpha^{*A} = \alpha^{B*} = \alpha^*$. As shown in this figure, the optimal overestimation decreases with income. Figure 2b illustrates the relationship between the loss in a bad state and the optimal overestimation. We assume that the losses of the two countries are identical and that they increase from 1 to 15 by 1. The parameters except for the loss are identical to those in Table 1. As shown in this figure, the optimal overestimation increases with the loss.

Table 2 Numerical simulations of endogenous overestimation

| Scenario | 1<br>Social optimum | 4<br>Endogenous overestimation with non-optimal lobbying cost | 5<br>Endogenous overestimation with optimal lobbying cost |
|---|---|---|---|
| $Y^A$ | 50 | 50 | 50 |
| $L^A$ | 10 | 10 | 10 |
| $Y^B$ | 50 | 50 | 50 |
| $L^B$ | 10 | 10 | 10 |
| $\alpha^A$ | | 26.058 | 10.429 |
| $\alpha^B$ | | 26.058 | 10.429 |
| $\gamma$ | 4 | 4 | 4 |
| $\theta$ | 0.9 | 0.9 | 0.9 |
| $c$ | | 0.01 | 12.781 |
| $m_1^A$ | 2.480 | 3.669 | 2.480 |
| $m_1^B$ | 2.480 | 3.669 | 2.480 |
| $M_1$ | 4.960 | 7.338 | 4.960 |
| $\tilde{W}^A$ | | 13.951 | 14.354 |
| $\tilde{W}^B$ | | 13.951 | 14.354 |
| $W^A$ | 14.559 | 14.551 | 14.559 |
| $W^B$ | 14.559 | 14.551 | 14.559 |
| $m_1^A - c\alpha^A$ | | 3.409 | -130.811 |
| $m_1^B - c\alpha^B$ | | 3.409 | -130.811 |

Table 2 summarizes the results of numerical simulations of our three-period game, in which the interest groups endogenously determine the overestimation of loss. For comparison, the first column of the table represents the social optimum, which is identical to that in Table 1. The second column represents the result of numerical simulation in which the interest groups in the two countries independently and simultaneously choose the magnitude of overestimation to maximize their profit. In this column, we assume that the marginal cost of lobbying is 0.01. In Period 0, the interest group in country A chooses their lobbying activities to make its government estimate the loss in a bad state more than the true loss by 26.058 units. The interest group in B does the same. In Period 1, the government of country A purchases 3.669 units of self-protection. Since country B purchases the same amount of self-protection, the provision of self-protection public good becomes 7.338 units. Then, the truly expected welfares of the two countries are lower than the socially optimum levels. However, we notice that they are still higher than the Nash equilibrium level in the second column of Table 1. We also notice that the purchase of self-protection in the second column is greater than that in the first column, which implies that the self-protection is over-provided. The third column of Table 2 reports the results of the numerical simulation, in which the marginal cost of lobbying is sufficiently high that the interest groups choose the optimal overestimation. Then, the governments of the countries purchase the optimal amounts of self-protection. However, as shown in the last two rows of the third column, the profits of the interest groups are negative. In this sense, the optimal level of lobbying is not self-financing.

## 6. Conclusion

In the present paper, we have constructed a two-country model in which the countries constitute an alliance and share the common risk of an emergency. Our baseline model consists of two periods: in Period 1, the two countries non-cooperatively determine their investments on self-protection; in Period 2, the state of the world is stochastically determined. An interesting feature of our model is to incorporate misperceptions caused by interest groups such as arm-producing companies that have special interests in security spending. In this model, we assumed that their political efforts persuade the benevolent government to make misperception the magnitude of the loss in a bad state. We investigated how such misperceptions influence the non-cooperative Nash equilibrium allocation and welfare of the countries.

At first, we showed how the governments of the two countries are influenced by the misperception on the the loss in a bad state. We showed that if the absolute risk aversion of an allied country satisfies a certain condition, the country increases its contribution to the self-protection public good with the overestimation of the damage from a bad event.

Then, we investigated how much overestimation induces countries to provide the socially optimal amount of self-protection public good, and showed that there could be such an optimal value of overestimation. We conducted some comparative statics of the optimal overestimation and showed how the true incomes and losses of countries influence the optimal overestimation.

Finally, we extended our model to a three-period model, in which the interest groups in the two countries determine the magnitude of the overestimation of the loss. We showed that there exists the optimal marginal cost of lobbying inducing the profit-maximizing interest groups to choose the socially optimal overestimation.

## References

BERGSTROM, T., L. BLUME and H. VARIAN (1986) On the private provision of public goods, *Journal of Public Economics* 29:25-49.

BOADWAY, R. and M. HAYASHI, (1999) Country size and the voluntary provision of international public goods, *European Journal of Political Economy* 15: 619-38.

BUCHHOLZ, W. and K. A. KONRAD (1995) Strategic transfers and private provision of public goods. *Journal of Public Economics* 57(3), 489–505.

CORNES, R., and ITAYA, J. (2010) On the Private Provision of Two or More Public Goods, *Journal of Public Economic Theory* 12, , 363–385.

EHRLICH, I. and G. BECKER (1972) Market insurance, self-insurance, and self-protection, *Journal of Political Economy* 80: 623-648.

HIRSHLEIFER, J. (1989) Conflict and rent-seeking success functions: Ratio vs. difference models of relative success, *Public Choice* 63, 101-112.

IHORI, T. and M. C. MCGUIRE (2007), Collective risk control and group security: The unexpected consequences of differential risk aversion, *Journal of Public Economic Theory,* 9, 231-263.

IHORI, T. and M. C. MCGUIRE (2010), National self-insurance and self-protection against adversity: bureaucratic management of security and moral hazard, *Economics of Governance* 11:103–122.

IHORI, T., M. C. MCGUIRE and S. NAKAGAWA (2014), International Security, Multiple Public Good Provisions, and the Exploitation Hypothesis, *Defence and Peace Economics*, 25, 213-229.

IHORI, T., M. C. MCGUIRE and S. NAKAGAWA (2015) When Cornes and Itaya's (2010) claim about non-existence of Nash equilibrium in which simultaneous contribution to two public goods by two players does not hold?, mimeo.

KERSCHBAMER, R. and C. PUPPE (1998), Voluntary Contributions when the Public Good Is not Necessarily Normal, J*ournal of Economics* 68, 175-192.

LOHSE, T. J., ROBLEDO, and U. SCHMIDT (2006) Self-Insurance and Self-Protection as Public Goods, Leibniz Universität Hannover Discussion Paper No. 354, University of Hannover.

MUERMANN, A. and H. KUNREUTHER (2008) Self-protection and insurance with interdependences, Journal of Risk and Uncertainty, 36:103-123.

OLSON, M. and R. ZECKHAUSER (1966) An Economic Theory of Alliances, *Review of Economics and Statistics* 48, 266-279.

SANDLER, T. (1992) *Collective Action: The Theory and Application*, Ann Arbor: University of Michigan Press.

SANDLER, T. (1997) *Global Challenges: An approach to environmental, political, and economic problems.* New York: Cambridge University Press.

SANDLER, T., (2005) Collective versus unilateral responses to terrorism, *Public Choice* 124(1–2) 75–93.

SCHLESINGER, H. (2000) The Theory of Insurance Demand, Ch. 5 in Dionne ed., *Handbook of Insurance*, Kluwer.

TULLOCK, G. (1967) The Welfare Costs of Tariffs, Monopolies, and Theft, *Western Economic Journal* 5, 224-232.