Patent Examination Duration in an Endogenous Growth Model

Kiyoka Akimoto*   Takaaki Morimoto†

Abstract

We introduce patent examination into a standard "variety expansion" and "lab-equipment" type R&D-based growth model. Patent examination takes both time and cost. We examine the effects of reducing the patent examination duration on patent pending varieties, economic growth, and welfare. The relation between the number of patent pending varieties and patent examination duration is inverted-U shaped, because reduction in the patent examination duration both decreases the patent pending varieties by granting patents and increases same by promoting R&D. The reduction in examination duration promotes economic growth. However, the relationship between examination duration and social welfare is also inverted-U shaped because extremely short patent examination requires massive resources.

Keywords: Patent office; Patent examination; Endogenous growth; Innovation.
JEL Classification: O31; O34; O38.

1 Introduction

Innovation and patenting activity have been growing all over the world. This growth contributes to world economic development. On the other hand, it causes some problems at patent offices through patent backlogs. Patent backlogs refer to the number of pending patent applications[1]. Firms need to take the following steps in the process of obtaining a patent: filing the application, requesting for research, taking the examination, receiving results and decisions from the patent officer, and finally obtaining the patent. Thus, the larger the patent backlogs, the longer the duration from filing to obtaining the patent.

---

*Graduate student; Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, JAPAN; e-mail: akimoto1015@gmail.com.

†Graduate student; Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, JAPAN; e-mail: qge020mt@student.econ.osaka-u.ac.jp.

[1]The backlogs are not well defined. Other definitions include unexamined applications and excess applications beyond the capacity of the patent office.

Patent offices cope with the backlogs, including the delay in obtaining patents. Some offices set the first action (FA) targets and try to reduce the actual the FA pendency. FA pendency is the average time lapse between the request for research and the preliminary decision by a patent office, which is used as an indicator for backlogs. The annual FA target of the United States Patent and Trademark Office (USPTO) is 10 months by FY2019[2]. The USPTO achieved 16.4 months of FA pendency in 2015. Between the end of FY2014 and that of FY2015, FA pendency decreased by 1.7 months. The Japan Patent Office (JPO) achieved a long-term FA goal proposed in 2004 that it would reduce FA pendency to 11 months by FY2013 (the goal is called FA11). After achieving FA11, the JPO set a new target to reduce FA pendency to less than 10 months by FY2023.

However, it remains unclear whether the policy for reducing the patent examination duration to deal with the backlogs is better for economic growth and social welfare, because less attention has been paid to the patent examination duration. Thus, this study investigates the effects of reducing the duration of patent examination and derives the optimum duration. We use the Romer (1987)'s standard "variety expansion" and "lab-equipment" type research and development (R&D) based growth model. This is because final goods are used for R&D and the production of intermediate goods; furthermore, the variety of products is expanding through R&D. We introduce the patent examination duration into the standard model. Our model makes two assumptions. First, a patent office exists. The patent office examines each application by firms who develop new varieties of goods. The amount of final goods used for patent examination depends on the duration of the examination. That is, reducing the patent examination duration requires more resources. Second, new entrant firms must pass patent examination by the patent office to obtain the patent. After that, they start producing the patented products. Thus, there is a lag between invention and production. In the dynamic general equilibrium models of previous literature on innovation or patent policies, firms start production as soon as they succeed in innovation. That is, there is no lag because previous studies do not focus on patent examination. Our model leads to the following results. The number of patent pending varieties (or, patent backlogs) is an inverted U-shaped function of the patent examination duration. That is, when the patent examination duration is sufficiently long, reduction of the duration increases patent backlogs. The shorter the patent examination duration, the faster the economic growth. If the examination duration is short, the expected profits for new entrants are high; furthermore, the high expected profits accelerate R&D. On the other hand, it takes considerable resources and costs to realize short examination durations. Thus, extremely short durations hurt the welfare of households. Moreover, we show the inverted-U shaped relationship between the patent examination duration and social welfare through numerical analysis.

This study relates to the literature on patent policies, especially macroeconomic literature based on R&D-based growth models. We can divide the macroeconomic literature into three groups based on policies: patent length, patent breadth, and patentability. On patent length,

---

[2]The fiscal year refers to the duration from October 1 to September 30 in the United States and from April 1 to March 31 in Japan.

for example, see Judd (1985), Iwaisako and Futagami (2003), and Futagami and Iwaisako (2007). On patent breadth, see Li (2001), Goh and Olivier (2002), and Chu and Furukawa (2011). There exist a large number of other seminal studies on patent length and patent breadth. On the other hand, there are only a few studies on patentability, such as Hunt (1999), Koléda (2004), O'donoghue and Zweimüller (2004), Chu and Furukawa (2013), and Kishi (2014). However, as far as we know, no study has been conducted on patent examination duration. In addition, our study relates to works on intellectual property rights protection in a North-South model. See Helpman (1992), Lai (1998), Glass and Saggi (2002), Kwan and Lai (2003), Tanaka et al. (2007), and Tanaka and Iwaisako (2014).

The remainder of this paper is organized as follows. Section 2 presents the basic model, mainly based on Romer (1987). Section 3 describes the dynamics of the model and steady state. Section 4 investigates the effect of reducing the patent examination duration on welfare. The results are confirmed in section 5. Section 6 provides concluding remarks.

2   Model

Consider a variety-expansion and lab-equipment type R&D-based growth model as in Romer (1987). The model includes a representative household, a representative final good firm, a continuum of intermediate good firms, potential entrants to the intermediate goods sector, and a patent office. A representative final good firm produces final goods by using intermediate goods and labor in a competitive market, whereas intermediate goods are produced by using final goods in a monopolistically competitive market. New varieties are developed by R&D using final goods. Most of the settings of the model follow Romer (1987). In contrast to the standard variety-expansion and lab-equipment type endogenous growth model, a newly developed variety cannot be produced in our model until the variety obtains a patent. It takes time and cost for a patent office to grant a patent, and the cost is met by income tax. For simplicity, patents are assumed to not expire.

2.1   Households

Households live infinitely, supply L units of labor inelastically, and have a constant relative risk aversion utility function:

$$U = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} dt, \tag{1}$$

where c is consumption, $\rho > 0$ is the discount factor, and $\sigma > 0$ is the rate of relative risk aversion or inverse of the intertemporal elasticity of substitution. Assume that the discount rate is high enough to satisfy $\rho > (1 - \sigma)r$ at the equilibrium so that the utility will be well-defined. The budget constraint is

$$\dot{W}_t = r_t W_t + (1 - \tau_t) w_t L - c_t L, \tag{2}$$

3

where W is the asset, r is the interest rate, and w is the wage rate. Maximizing (1) with respect to (2) yields the following Euler equation:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho).\tag{3}$$

2.2 Final Good

A representative firm produces final goods Y by using labor L and intermediate goods x. The production technology is specified as

$$Y_t = L^{1-\alpha} \int_0^{A_t} x_t(j)^\alpha dj,\tag{4}$$

where A is the number of varieties produced, that is, the number of patented varieties. Profit maximization yields the following first order conditions:

$$w_t = (1 - \alpha)\frac{Y_t}{L},\tag{5}$$

$$p_t(j) = \alpha \left[\frac{x_t(j)}{L}\right]^{\alpha-1},\tag{6}$$

where p is the price of intermediate goods.

2.3 Intermediate Good

Each patented intermediate good firm translates γ units of final goods into one unit of differentiated intermediate goods. The profit function of the intermediate good firm is

$$\pi_t(j) = \max\left\{p_t(j)x_t(j) - \gamma x_t(j)\right\},\tag{7}$$

subject to (6). The optimal prices, quantities, and profits are

$$p_t(j) = p \equiv \frac{\gamma}{\alpha},\tag{8}$$

$$x_t(j) = x \equiv \left(\frac{\alpha^2}{\gamma}\right)^{\frac{1}{1-\alpha}} L,\tag{9}$$

$$\pi_t(j) = \pi \equiv (1 - \alpha)\alpha \left(\frac{\alpha^2}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} L.\tag{10}$$

The optimal prices (8), quantities (9), and profits (10) are the same for all $j \in [0, A_t]$ and time invariant.

4

2.4 R&D and Patent Claims

New entrants to the intermediate goods sector should first develop new varieties and, then, apply patents. In order to develop one variety, firms need to invest η units of final goods in R&D activities. An entrant that has developed a new variety applies a patent claim to a patent office. The entrant obtains a value V_t in exchange for the R&D costs. Therefore, the profits of new entrant firms are $V_t - \eta$. As long as the profits are positive, new entrants enter the intermediate goods sector, and thus the number of developed varieties, D_t, increases. In contrast, when the profits are negative, there are no entrants. Therefore,

$$\dot{D}_t \begin{cases} = \infty, & \text{if } V_t > \eta, \\ \in [0, \infty), & \text{if } V_t = \eta, \\ = 0, & \text{if } V_t < \eta. \end{cases}$$

In equilibrium, $V_t \le \eta$, $\dot{D}_t \ge 0$, and $(V_t - \eta)\dot{D}_t = 0$.

2.5 Patent Examination

We assume that the patent examination duration is stochastically determined according to the Poisson process, and that the instantaneous probability is μ. In other words, at each point of time, the patent office awards patents for a fraction of μ of the patent pending varieties, $D_t - A_t$, that is,

$$\dot{A}_t = \mu(D_t - A_t). \tag{11}$$

The expected patent examination duration is $1/\mu$. It takes $\delta(\mu)$ units of final goods to patent one variety. Since the shorter patent examination duration takes more costs, $\delta(\mu)$ is increasing in μ, that is, $\delta'(\mu) \ge 0$. The cost of the patent office is paid by labor income tax. Therefore the government budget constraint is

$$\delta(\mu)\mu(D_t - A_t) = \tau_t w_t L. \tag{12}$$

2.6 No Arbitrage Condition

For simplicity, assume that the patent is not expired. Since the expected patent examination duration is independent of when a variety was developed, the discounted sum of expected profits is the same for all patent pending (i.e., unpatented) varieties, $D_t - A_t$. Therefore, the value of patent pending varieties V_t is also the same for all patent pending varieties. The no arbitrage condition about patent pending varieties requires that their value is just the discounted sum of expected profits, that is,

$$V_t = \int_t^\infty \mu e^{-\mu(s-t)} \int_s^\infty e^{-\int_t^u r_z dz} \pi_u \, du \, ds. \tag{13}$$

The term $\int_s^{\infty} e^{-\int_t^u r_z dz} \pi_u du$ in (13) is the discounted sum of profits when a firm obtains a patent at $s > t$, and the term $\mu e^{-\mu(s-t)}$ is the probability that a firm obtains a patent at $s > t$, given that it does not do so at t.

The patented varieties, A_t, do not face any uncertainty, so the no arbitrage condition about patented varieties requires that the value of patented varieties are the discounted sum of profits.

$$V_{At} = \int_t^{\infty} e^{-\int_t^u r_z dz} \pi_u du. \tag{14}$$

2.7 Market Clearing

In this model, there is a final goods market, a continuum of intermediate goods markets of $[0, A_t)$, a labor market, and an asset market. The intermediate goods market clears since each intermediate good firm behaves while considering the demand (6). The labor market also clears. The clearing condition of the final goods market is

$$Y_t = c_t L + \int_0^{A_t} \gamma x dj + \eta \dot{D}_t + \delta(\mu)\mu(D_t - A_t). \tag{15}$$

The clearing condition of the asset market is

$$W_t = V_{At} A_t + V_t(D_t - A_t), \tag{16}$$

which is cleared using Walras' law focusing on the final goods market (15).

2.8 Equilibrium

In equilibrium, x is symmetric for all $j \in [0, A_t)$ from (9), and thus the final good production can be written as

$$Y_t = A_t \left(\frac{\alpha^2}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} L. \tag{17}$$

The national income is distributed to the input for the intermediate good production, labor income, and profit of intermediate good firms as follows:

$$\int_0^{A_t} \gamma x dj = \alpha^2 Y_t \tag{18}$$

$$w_t L = (1 - \alpha)Y_t$$

$$A_t \pi = (1 - \alpha)\alpha Y_t \tag{19}$$

6

The no arbitrage condition about patent pending varieties (13) yields the dynamics of the interest rate as follows (see Appendix):

$$\dot{r}_t = r_t(\mu + r_t) - \frac{\mu\pi}{\eta}. \tag{20}$$

Since $\dot{r}_t$ is increasing in r_t, r_t decreases with time when r_t is small and *vice versa*; that is, the dynamics of the interest rate (20) are unstable. Since the interest rate is a jumpable variable, it initially jumps to the steady state defined as $r(\mu + r)/\mu = \pi/\eta$, or

$$r = r(\mu) \equiv \frac{\mu}{2}\left[\left(1 + 4\frac{\pi}{\mu\eta}\right)^{1/2} - 1\right]. \tag{21}$$

Moreover, note that $\frac{dr(\mu)}{d\mu} = \frac{r^2}{\mu(\mu+2r)} > 0$ and $\lim_{\mu\to\infty} r(\mu) = \pi/\eta$. Thus, the interest rate r is increasing in μ and converges to π/η as μ goes to infinity.

3 Dynamics and Steady State

Intermediate good prices (8), intermediate good production (9), and profits for intermediate good firms (10) are constant over time. The value of patent pending varieties is also constant as long as R&D occurs. The interest rate initially jumps to its steady state value (21). Wage (5) depends on final good production (17), which depends on the number of patented varieties. The value of patented varieties is determined to meet the households' budget constraint (2) and to clear the asset market (16), depending on the interest rate (see Appendix). The dynamics of the rest of the variables, number of developed varieties D_t, number of patented varieties A_t, and consumption c_t are investigated in this section.

These three variables, D_t, A_t, and c_t, and their differential equations are translated into two new variables and their differential equations. Then, a steady state of the new variables is investigated. This steady state corresponds to a balanced growth path on which the original three variables, D_t, A_t, and c_t, grow at the same constant growth rate. Therefore, the growth rate at the steady state is calculated.

3.1 Dynamics

Substituting (17) and (18) into (15) yields

$$(1 - \alpha^2)A_t\left(\frac{\alpha^2}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} L = c_t L + \eta\dot{D}_t + \delta(\mu)\mu(D_t - A_t). \tag{22}$$

The left hand side of (22) is the net output of the final goods, that is, the output of final goods minus the input for the intermediate good production. The right hand side is the demand of the final goods, that is, consumption, the input for R&D, and the input for patent examination.

Equation (22) implies that several patent pending varieties (or patent backlogs) $D_t - A_t$ use a great amount of resources that can be used for consumption and R&D. Together with (3) and (11), equation (22) describes the dynamics in case of positive R&D. In addition to these three differential equations, (3), (11), and (22), the following feasibility condition must hold:

$$(1 - \alpha^2)A_t \left(\frac{\alpha^2}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} L \geq c_t L + \delta(\mu)\mu(D_t - A_t), \tag{23}$$

where equality holds when $\dot{D}_t = 0$. Feasibility condition (23) requires $\dot{D}_t$ to be non-negative[3].

To reduce the number of variables, define

$$\omega_t = \frac{D_t}{A_t}, \qquad\qquad \chi_t = \frac{c_t L}{A_t}. \tag{24}$$

Because $\omega_t - 1 = (D_t - A_t)/A_t$ is the ratio of patent pending varieties to patented varieties, ω_t represents a measure of the number of patent pending varieties (or patent backlogs). Substituting (11), (17), and (19) into (22) yields the dynamics of ω as

$$\dot{\omega}_t = \frac{1}{\eta}\left\{-\mu[\eta\omega_t + \delta(\mu)](\omega_t - 1) + \frac{1+\alpha}{\alpha}\pi - \chi_t\right\}. \tag{25}$$

Note that the term $(1 - \alpha^2)(\alpha^2/\gamma)^{\frac{\alpha}{1-\alpha}} L$ is rewritten as $\frac{1+\alpha}{\alpha}\pi$ in (25) by using (10). The dynamics of χ can be obtained from (3) and (11) as

$$\dot{\chi}_t = \chi_t \left\{\frac{1}{\sigma}[r(\mu) - \rho] + \mu - \mu\omega_t\right\}. \tag{26}$$

Feasibility condition (23) is rewritten as

$$\chi_t \leq \frac{1+\alpha}{\alpha}\pi - \delta(\mu)\mu(\omega_t - 1). \tag{27}$$

In addition, $\omega_t \geq 1$ must hold for all t since $D_t \geq A_t$ for all t. Gathering (25), (26), (27), and $\omega_t \geq 1$, a phase diagram of this economy can be depicted as in Figure 1. The diagonal area in Figure 1 is not feasible.

3.2 Steady State

In this model, the steady states of ω_t and χ_t correspond to the balanced growth paths on which c_t, A_t, D_t, and Y_t grow at the same rate. The point E in Figure 1 represents the steady state (or the balanced growth path), which is saddle path stable (see Appendix).
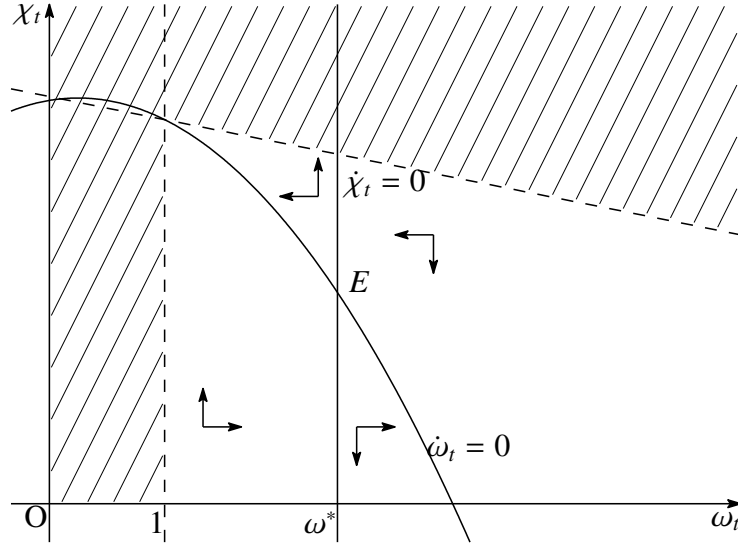
---

[3]See Appendix for the dynamics in case of no R&D.

Figure 1: Phase diagram

From (25) and (26), the steady state values of ω and χ are

$$\omega^* = 1 + \frac{1}{\sigma\mu}[r(\mu) - \rho], \tag{28}$$

$$\chi^* = -\mu[\eta\omega^* + \delta(\mu)](\omega^* - 1) + \frac{1+\alpha}{\alpha}\pi. \tag{29}$$

From (3), the growth rate at the steady state is

$$g \equiv \frac{\dot{A}_t}{A_t} = \frac{\dot{D}_t}{D_t} = \frac{\dot{c}_t}{c_t}$$

$$= \frac{1}{\sigma}[r(\mu) - \rho]. \tag{30}$$

Since $r(\mu)$ is increasing in μ, the growth rate at the steady state is also increasing in μ, that is,

$$\frac{dg}{d\mu} = \frac{1}{\sigma}\frac{dr(\mu)}{d\mu} > 0. \tag{31}$$

Therefore, the shorter the patent examination duration, the faster the growth because of the higher expected return from R&D.

To ensure the positive steady state growth, first assume that

$$\frac{\pi}{\eta} > \rho. \tag{A1}$$

This assumption (A1) ensures positive steady state growth when μ goes to infinity; that is, it ensures positive steady state growth in the standard variety expansion type endogenous

growth model. Then, there is a lower bound of μ, above which the positive steady state growth is ensured since the steady state growth rate is increasing in μ. Solving $r(\mu) > \rho$ for μ and applying (A1) yields

$$\mu > \mu_0 \equiv \frac{\rho}{\frac{\pi}{\rho\eta} - 1} > 0. \tag{32}$$

The inequality (32) specifies a lower bound of Poisson arrival rate, μ_0, over which there exists a balanced growth path. In other words, there is no balanced growth path if the expected patent examination duration is too long.

4 Comparative Statics

This section investigates the effects of reducing the patent examination duration, or increasing μ.

4.1 Effects on Steady State Values

Differentiating (28) with respect to μ yields

$$\frac{d\omega^*}{d\mu} = \frac{1}{\sigma\mu^2} \left[\rho - \frac{\pi/\eta}{\left(1 + 4\frac{\pi}{\mu\eta}\right)^{1/2}} \right] \begin{cases} \geq 0, \text{ if } \mu \leq \mu_1, \\ < 0, \text{ if } \mu > \mu_1, \end{cases} \tag{33}$$

where $\mu_1 \equiv \frac{4\frac{\pi}{\eta}}{\left(\frac{\pi}{\rho\eta}\right)^2 - 1} > \mu_0$ since $\mu_1 - \mu_0 = \frac{3\frac{\pi}{\eta+\phi} - \rho}{\left(\frac{\pi}{\rho(\eta+\phi)}\right)^2 - 1} > 0$. Then, the following proposition can be stated.

Proposition 1 (The steady state ratio of developed varieties to patented varieties)**.** *The steady state ratio of developed varieties to patented varieties, ω^*, is an inverted U-shaped function of the Poisson arrival rate of patent examination, μ, and maximized at μ_1, which is greater than the lower bound of the Poisson arrival rate, μ_0.*

Proposition 1 implies that when the patent examination duration is sufficiently long ($\mu < \mu_1$), a reduction in the duration (or an increase in μ) increases the measure of patent backlogs, ω^*. In contrast, when the patent examination duration is sufficiently short ($\mu > \mu_1$), the shorter the duration (or higher the μ), the lesser will be the patent backlog, ω^*. Therefore, reducing the patent examination duration does not always reduce patent backlogs. This is because reducing the duration accelerates both R&D activities, $\dot{D}_t$, and the patent examination, $\dot{A}_t$. When the patent examination duration is sufficiently long ($\mu < \mu_1$), R&D becomes more active than patent examination ($\dot{A}_t < \dot{D}_t$), and thus ω^* increases and *vice versa*.

Differentiating (29) with respect to μ yields

$$\frac{d\chi^*}{d\mu} = -[\eta\omega^* + \delta(\mu) + \mu\delta'(\mu)](\omega^* - 1) - \mu\left[2\eta(\omega^* - 1) + \eta + \delta(\mu)\right] \frac{d\omega^*}{d\mu}. \tag{34}$$

10

Since $d\omega^*/d\mu \geq 0$ for $\mu \leq \mu_1$ and $\omega^* \geq 1$ for $\mu \geq \mu_0$, $d\chi^*/d\mu$ is negative at least for $\mu \in [\mu_0, \mu_1]$ and ambiguous for $\mu > \mu_1$.

Summarizing the above, when μ increases, in Figure 1, the steady state shifts to the lower right for $\mu \in [\mu_0, \mu_1]$, and it shifts to the upper or lower left for $\mu > \mu_1$. The new steady state is also saddle path stable, and the economy converges to it from the lower left or upper right. Since ω_t is a state variable and χ_t is a jump variable, when the steady state shifts to the lower right, χ_t jumps to the lower level. That is, a reduction in the patent examination duration makes χ_t jump to the lower level for $\mu \in [\mu_0, \mu_1]$, implying that consumption level at that point decreases. In contrast, for $\mu > \mu_1$, it is not obvious whether χ_t jumps lower or higher.

4.1.1   Effect on Welfare

Before investigating the effect of reducing the patent examination duration on welfare, it is convenient to note that there exists the upper bound of μ, or the minimum patent examination duration. Since χ_t should be non-negative for all t, the feasibility condition (27) requires

$$\delta(\mu)\mu \leq \frac{1+\alpha}{\alpha} \frac{\pi}{\omega_t - 1}. \tag{35}$$

Since the left hand side of (35) is increasing in μ and there is ω on the right hand side, there is an upper bound of μ depending on ω_t defined as

$$\delta(\mu_2(\omega_t))\mu_2(\omega_t) = \frac{1+\alpha}{\alpha} \frac{\pi}{\omega_t - 1}, \tag{36}$$

and $d\mu_2/d\omega_t < 0$. When μ goes to $\mu_2(\omega_t)$, χ_t goes to 0 because there are no resources to consume. This upper bound is decreasing in ω_t. This means that when there are a lot of patent pending varieties, the patent examination duration cannot be reduced too much; however, when there are only a few patent pending varieties, the duration can be reduced substantially.

Then, we define welfare as the indirect utility of the household. Substituting (3) and (21) into (1) yields the equilibrium (not only at the steady state) welfare level with positive R&D as follows:

$$U = W(c_0, r(\mu)) \equiv \frac{\sigma c_0^{1-\sigma}}{(1-\sigma)\rho - (1-\sigma)^2 r(\mu)} - \frac{1}{\rho(1-\sigma)}. \tag{37}$$

Assume first that the economy initially is in the steady state E, and, second, that the patent examination duration is reduced at time 0. The effect of reducing this duration on welfare is

$$\frac{dW(c_0, r(\mu))}{d\mu} = \frac{dW(c_0, r(\mu))}{dc_0} \frac{dc_0}{d\mu} + \frac{dW(c_0, r(\mu))}{dr(\mu)} \frac{dr(\mu)}{d\mu}. \tag{38}$$

11

Table 1: Baseline parameter values

μ^{US}	L	σ	ρ	α	η	δ	γ
0.42	1	3	0.02	1/3	1	0.0634	0.605

As discussed above, at least for $\mu \in [\mu_0, \mu_1]$, $dc_0/d\mu < 0$. Therefore, the first term of (38) is negative at least for $\mu \in [\mu_0, \mu_1]$, whereas the second term is positive. Although it seems that $dc_0/d\mu$ becomes positive for higher μ, extremely high μ or extremely short patent examination duration hurt the welfare. This is because faster patent examination takes more cost. When μ goes to $\mu_2(\omega_0)$, χ_0 goes to 0 because there are no resources to consume, that is,

$$\left. \frac{dc_0}{d\mu} \right|_{\mu=\mu_2(\omega_0)} < 0, \qquad\qquad \lim_{\mu \to \mu_2(\omega_0)} c_0 = 0.$$

Then, as μ goes to $\mu_2(\omega_0)$, the effect on the welfare (38) goes to negative infinity, because consumption goes to 0 and the instantaneous utility function satisfies Inada conditions. Therefore, there is an optimal μ between μ_0 and $\mu_2(\omega_0)$ in terms of welfare. In addition, the initial jumps of χ_0 depend on initial state variable, and thus the optimal μ depends on ω_0.

5   Numerical Analysis

To illustrate the result of the model regarding the optimal μ, this section conducts a numerical analysis. Although the model is highly stylized, these numerical results help to understand the results of the model.

In order to calculate the initial jump of χ_0, we use the Relaxation Algorithm of Trimborn et al. (2008). In time 0, the economy is at the steady state and μ changes. For each change of μ, we calculate the time path of χ_t, which converges to new steady state given the initial state variable ω_0. From those paths, we obtain the first jump of χ_0 and calculate welfare (37). Then, the optimal change of μ can be obtained. For calculation, $\delta(\mu)$ is specified as $\delta\mu^\varepsilon$.

Baseline parameters are given by Table 1. The population size is normalized to 1. σ is chosen to ensure 2% steady state growth rate. ρ is 0.02, and α is 1/3. A parameter of R&D technology, η, is normalized to 1. The baseline inverse of patent examination duration, μ^{US}, is suited to the average US final action pendency from 2013 (2.42 years) to 2014 (2.28 years). A parameter of patent examination cost, δ, is chosen to make $\delta(\mu)$ the average ratio of expenditure of the USPTO to the US R&D expenditure from 2010 to 2013 when $\mu = \mu^{US}$ and $\varepsilon = 1$. A parameter of intermediate good production is set to ensure $r = 0.08$.

We investigate four versions of parameters and the initial state. The results are given by Table 2. Columns (1)-(3) give the optimal μ for different ε, whereas other parameters and initial state variable are the same. The elasticity of patent examination cost on the duration is ε, and higher ε implies that more costs are need to reduce the patent examination duration.

Table 2: Optimal inverse of the patent examination duration

	(1)$\varepsilon = 1$	(2)$\varepsilon = 3$	(3)$\varepsilon = 5$	(4)$\varepsilon = 3, \mu = 0.788$
Optimal μ	7.59	3.24	2.03	3.31

Therefore, for higher ε, a longer patent examination duration is desirable. In contrast, column (4) presents a result for when the initial μ differs, and thus the initial state variables also differ, whereas other parameters are the same as in column (2). The initial μ in column (4) is set to the Japanese final action pendency at 2013. In column (4), the initial patent examination duration is shorter than in column (2). Therefore, the initial ω in column (4) is less than that in column (2), and thus a shorter patent examination duration is desirable.

6   Conclusion

We construct a variety expansion and lab-equipment type R&D-based growth model in which patent examination takes time and cost. Then, we investigate the effect of reducing the patent examination duration on the number of patent pending varieties (or patent backlogs), economic growth on a balanced growth path, and welfare. We consider the ratio of the number of patent pending varieties to the number of patented varieties as a measure of patent backlogs. Then, the relation between this measure of patent backlogs and patent examination duration was found to be inverted U-shaped; that is, a reduced patent examination duration increases patent backlogs when patent examination is sufficiently long. The shorter patent examination, the faster is the economic growth on the balanced growth path because there is greater incentive for R&D. However, shorter examination is not better from the view-point of welfare. It takes more cost to examine patents sooner. Therefore, extremely short patent examination requires enormous costs and hurts the welfare. Moreover, the optimal patent examination duration depends on the initial state. This is because when there are several patent pending varieties a reducing in the duration requires more resources to grant these patents, and thus reduces initial consumption. These mechanisms of the model are confirmed by a numerical analysis, and we show that there exists an optimal duration between the upper and lower bounds of the patent examination duration.

Appendix

Dynamics of Interest Rate with Positive R&D

The value of one variety that is developed at t and patented at s is

$$V_{t,s} = \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du$$

The patent examination duration is stochastically determined according to the Poisson process with the instantaneous probability μ. Therefore, the probability that a patent claim has not been accepted during the time period $(s - t)$ is $e^{-\mu(s-t)}$, and that it is accepted at the end of this period is $\mu e^{-\mu(s-t)}$. Therefore, the expected value of one variety that is developed at t is

$$V_t = \int_t^\infty \mu e^{-\mu(s-t)} V_{t,s} ds = \int_t^\infty \mu e^{-\mu(s-t)} \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du ds.$$

Differentiating this with respect to time, we obtain

$$\dot{V}_t = -\mu \int_t^\infty e^{-\int_t^u r_z dz} \pi_u du + \int_t^\infty \mu^2 e^{-\mu(s-t)} \int_s^\infty e^{-\int_t^u r_z dz} \pi_u du ds$$
$$+ \int_t^\infty \mu e^{-\mu(s-t)} \int_s^\infty r_t e^{-\int_t^u r_z dz} \pi_u du ds$$
$$= -\mu \int_t^\infty e^{-\int_t^u r_z dz} \pi_u du + (\mu + r_t) V_t.$$

The profit, π, is constant and the firm value, V, is constant η, as long as R&D occurs. Therefore, the non-arbitrage condition is

$$(\mu + r_t)\eta = \mu \int_t^\infty e^{-\int_t^u r_z dz} \pi du.$$

Differentiating both sides with respect to t results in

$$\dot{r}_t \eta = -\mu\pi + r_t \mu \int_t^\infty e^{-\int_t^u r_z dz} \pi du$$
$$= -\mu\pi + r_t(\mu + r_t)\eta$$
$$\dot{r}_t = r_t(\mu + r_t) - \frac{\mu\pi}{\eta}$$

The dynamics are unstable. Therefore, the interest rate initially jumps to the steady state defined as

$$\frac{r(\mu + r)}{\mu} = \frac{\pi}{\eta}.$$

The Value of Patented Varieties with Positive R&D

Differentiating (14) with respect to time yields

$$\dot{V}_{At} = -\pi_t + r_t \int_t^\infty e^{-\int_t^u r_z dz} \pi_u du$$
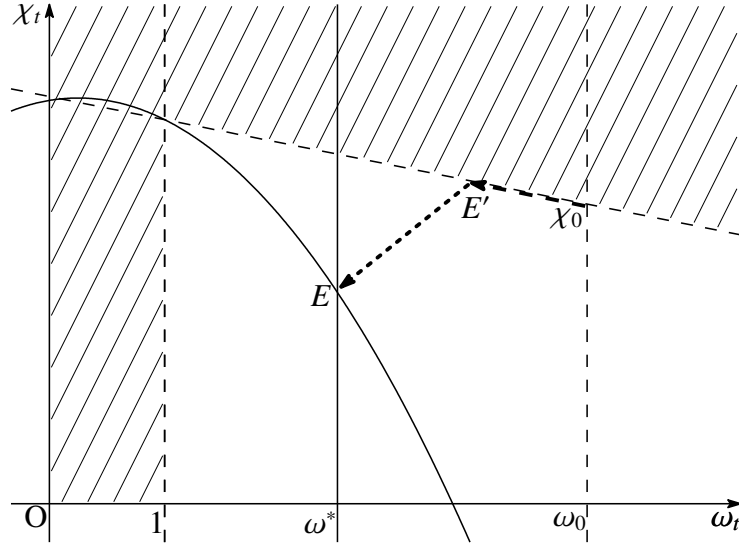$$= -\pi_t + r_t V_{At}. \tag{39}$$

14

Figure 2: Phase diagram with high initial ω

Substituting (5), (12), (15), and (16) into (2) yields

$$\dot{V}_{At}A_t + V_{At}\dot{A}_t + V(\dot{D}_t - \dot{A}_t) = r_t[V_{At}A_t + V(D_t - A_t)] + (1 - \alpha)Y_t - \delta\mu(D_t - A_t)$$
$$- [Y_t - \alpha^2 Y_t - \eta\dot{D}_t - \delta\mu(D_t - A_t)],$$
$$\dot{V}_{At}A_t + (V_{At} - V)\dot{A}_t = r_t[V_{At}A_t + V(D_t - A_t)] - (1 - \alpha)\alpha Y_t.$$

Using (11), (39), and (19), this can be rewritten as

$$(r_t V_{At} - \pi)A_t + (V_{At} - V)\mu(D_t - A_t) = r_t[V_{At}A_t + V(D_t - A_t)] - (1 - \alpha)\alpha Y_t,$$

$$V_{At} = \left(1 + \frac{r_t}{\mu}\right)\eta.$$

This determines the value of patented varieties.

Case of No R&D

Since the economy converges to the steady state E from the lower left or upper right in Figure 1, the economy cannot take a saddle path converging to the steady state E when the initial ω is large enough. Figure 2 illustrates such a case. A dotted arrow represents the saddle path converging to the steady state E from the upper right. In Figure 2, the initial ω is too large to initially take the saddle path. In order to reach the steady state, the economy should start without R&D, and initial χ satisfies the feasible condition (27) with equality, that is, the following no R&D condition:

$$\chi_t = \frac{1 + \alpha}{\alpha}\pi - \delta(\mu)\mu(\omega_t - 1). \tag{40}$$

15

The economy goes to the upper left along the no R&D condition (40) (a dashed arrow) until it reaches point E'. After reaching E', R&D becomes positive, and the economy starts for the steady state taking on the saddle path (a dotted arrow). That is, when the initial ω is sufficiently large, there is no R&D.

When there is no R&D, $\dot{D}_t = 0$ and $V_t < \eta$. Therefore, r_t, V_t, and V_{At} are different from those in case of positive R&D. Since π_t is constant regardless of whether R&D is positive or not, V_t and V_{At} are determined by r_t from (13) and (14). Therefore, it is necessary to investigate how r_t is determined in case of no R&D.

In order to reach point E', the economy should move along the no R&D condition (27). Therefore, the ratio of the change of χ_t to the change of ω_t should be the same as the slope of the no R&D condition (40):

$$\left. \frac{\dot{\chi}_t}{\dot{\omega}_t} \right|_{\dot{D}_t=0} = \left. \frac{d\chi_t}{d\omega_t} \right|_{\dot{D}_t=0} = -\delta(\mu)\mu. \tag{41}$$

Substituting (40) into (25) and (26) yields

$$\dot{\omega}_t|_{\dot{D}_t=0} = -\mu\omega_t(\omega_t - 1), \tag{42}$$

$$\dot{\chi}_t|_{\dot{D}_t=0} = \left[\frac{1+\alpha}{\alpha}\pi - \delta(\mu)\mu(\omega_t - 1) \right]\left[\frac{1}{\sigma}(r_t - \rho) - \mu(\omega_t - 1) \right]. \tag{43}$$

Substituting (42) and (43) into (41) and solving for r_t yields

$$r_t = \frac{\sigma\left[\frac{1+\alpha}{\alpha}\pi + \mu\delta(\mu) \right]\mu(\omega_t - 1)}{\frac{1+\alpha}{\alpha}\pi - \delta(\mu)\mu(\omega_t - 1)} + \rho. \tag{44}$$

Equations (42), (43), and (44) describe the dynamics in case of no R&D.

Since r_t is not constant in case of no R&D, welfare is also different from (37). Let $\tilde{t}$ denote the period when the economy reaches point E' in Figure 2. Substituting (3), (21), and (44) into (1) yields the welfare level when there is initially no R&D, as follows:

$$W \equiv \left\{ \int_0^{\tilde{t}} e^{\frac{1}{\sigma}\int_0^t[(1-\sigma)r_t-\rho]ds}dt + \frac{\sigma e^{\frac{1}{\sigma}[(1-\sigma)r(\mu)-\rho]\tilde{t}}}{\rho - (1 - \sigma)r(\mu)} \right\} \frac{c_0^{1-\sigma}}{1 - \sigma} - \frac{1}{\rho(1 - \sigma)}.$$

Stability

The linearized dynamic system in the neighborhood of steady state E is

$$\dot{z}_t = Hz_t,$$

where

$$z_t = \begin{pmatrix} \omega_t - \omega^* \\ \chi_t - \chi^* \end{pmatrix}, \qquad\qquad H = \begin{pmatrix} -2\mu\omega^* + \mu\left(1 - \frac{\delta}{\eta}\right) & -\frac{1}{\eta} \\ -\mu\chi^* & 0 \end{pmatrix}.$$

The matrix H has one positive characteristic root and one negative characteristic root since

$$\det H = -\frac{\mu}{\eta}\chi^* < 0$$

Therefore, the steady state E is a saddle point.

Acknowledgements

References

Chu, Angus C and Yuichi Furukawa, "On the optimal mix of patent instruments," *Journal of Economic Dynamics and Control*, 2011, *35* (11), 1964–1975.

_ **and** _ , "Patentability and knowledge spillovers of basic R&D," *Southern Economic Journal*, 2013, *79* (4), 928–945.

Futagami, Koichi and Tatsuro Iwaisako, "Dynamic analysis of patent policy in an endogenous growth model," *Journal of Economic Theory*, 2007, *132* (1), 306–334.

Glass, Amy Jocelyn and Kamal Saggi, "Intellectual property rights and foreign direct investment," *Journal of International economics*, 2002, *56* (2), 387–410.

Goh, Ai-Ting and Jacques Olivier, "Optimal Patent Protection in a Two-Sector Economy," *International Economic Review*, 2002, *43* (4), 1191–1214.

Helpman, Elhanan, "Innovation, imitation, and intellectual property rights," Technical Report, National Bureau of Economic Research 1992.

Hunt, Robert, "Nonobviousness and the incentive to innovate: an economic analysis of intellectual property reform," Technical Report, Federal Reserve Bank of Philadelphia 1999.

Iwaisako, Tatsuro and Koichi Futagami, "Patent policy in an endogenous growth model," *Journal of Economics*, 2003, *78* (3), 239–258.

Judd, Kenneth L, "On the performance of patents," *Econometrica: Journal of the Econometric Society*, 1985, pp. 567–585.

Kishi, Keiichi, "A patentability requirement and industries targeted by R&D," Technical Report 2014.

Koléda, Gilles, "Patents' novelty requirement and endogenous growth," *Revue d'économie politique*, 2004, *114* (2), 201–221.

Kwan, Yum K and Edwin L-C Lai, "Intellectual property rights protection and endogenous economic growth," *Journal of Economic Dynamics and Control*, 2003, *27* (5), 853–873.

Lai, Edwin L-C, "International intellectual property rights protection and the rate of product innovation," *Journal of Development economics*, 1998, *55* (1), 133–153.

Li, Chol-Won, "On the policy implications of endogenous technological progress," *The Economic Journal*, 2001, *111* (471), 164–179.

O'donoghue, Ted and Josef Zweimüller, "Patents in a model of endogenous growth," *Journal of Economic Growth*, 2004, *9* (1), 81–123.

Romer, Paul M, "Growth based on increasing returns due to specialization," *The American Economic Review*, 1987, *77* (2), 56–62.

Tanaka, Hitoshi and Tatsuro Iwaisako, "Intellectual property rights and foreign direct investment: A welfare analysis," *European Economic Review*, 2014, *67*, 107–124.

_ , _ , **and Koichi Futagami**, "Dynamic analysis of innovation and international transfer of technology through licensing," *Journal of International Economics*, 2007, *73* (1), 189–212.

Trimborn, Timo, Karl-Josef Koch, and Thomas M Steger, "Multidimensional transitional dynamics: a simple numerical procedure," *Macroeconomic Dynamics*, 2008, *12* (03), 301–319.