The good MOOC and the universities [*]

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Abstract

We propose a model to analyze competition between an on–line course and a traditional brick-and-mortar supply for higher education. The brick and mortar supplier is located at some address in the linear city and students pay a transportation cost to attend the traditional course. This transportation cost increases linearly with the distance to the university. The traditional supplier charges a fee for the course. On the contrary, the on–line course is free, without transportation cost but students incurred a fixed cost when choosing the on–line course. This fixed cost captures an homogeneous disutility related to the loss of face to face relationship with teachers and peers, possible haircut in the value of the diploma,...

We derive the optimal fee policy of a single university as a function of its address and the fixed cost associated with the on–line course. We discuss market sharing in this competitive setting and compare the outcome with the socially optimal provision of distant learning.

We study the impact of distant learning on the competition between two brick and mortar universities. One university is assumed to enjoy a central position, whereas the other one is located at the extreme left of the town. We discuss equilibria and market sharing for non–regulated (*i.e* pure fee competition) and regulated (*i.e.* quantity competition).

Keywords: On–line learning, spatial competition.

JEL Classification: I21, I23, L13, L17, R12, R39.

1 Introduction

Massive On–line Open Courses recently emerged as one of the most promising pedagogical innovations.

True, the initial goal is waning. Potential students in the developing countries or more generally people left without higher education solely for financial, cultural or physical reasons; do not massively follow on–line courses. On the contrary some skeptical views have emerged. Some fear the ultimate goal of MOOC is to replace (arguably good and costly) professors with (supposedly cheap but poor) technology. Teaching pundits rose concerns about the loss of social and personal aspects of learning. Many stress the very high dropout rates witnessed by most MOOC.

Nevertheless, the shift from a costly, centralized, hierarchical management of teaching to on–line, mainly free, round the clock, peer to peer collaborative supply for knowledge and known–how represents a formidable challenge to traditional learning sector. MOOC's advantages shall not be underestimated. First, according to the Telecommunication Union, 40% of the World Population now enjoys access to Internet connection.[1] Potential access to higher education offered by MOOC is much larger than physical supply for higher education. Second, on–line learning allows flexibility. People can study at their own pace. Third, most MOOC are free of charge whereas higher education is not affordable for many potential students (in particular in developing countries).

Major difficulties still entails that distant learning is a second best alternative. The poor feedback –both from students' and teachers' viewpoints– offered by video-lecture is a common explanation for the formidable dropout rates witnessed. The "connectivist" paradigm (Siemens [2005]) seems an interesting way to address this problem, but such MOOC are time consuming – both from students' and teachers' viewpoints. Certification also raises numerous difficulties. Certificates for on–line diploma may have lower job market value (see, for instance, Rumble [1997] and Chi Wai Chan [2009] for a reassessment).[2] Finally, the collapse of the dot–com bubble has taught us that the resilience of brick and mortar supply technologies shall not be underestimated.[3]

From an economic perspective, occurrences of the word 'MOOC' in academic journals are scare. Few articles list possible ways to tackle "economics of MOOC" (Belleflamme and Jacqmin [2014a]) or question the existence of a business model for MOOC (see Belleflamme and Jacqmin [2014a] and Saltzman [2014]).

On a more quantitative standpoint, several authors focused on comparison of rates of success but evidences are often mixed. For instance Xu and Jaggars [2013] report "negative estimates for on–line learning in terms of both course persistence and course grade, contradicting the notion that there is no significant

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[1] The penetration rate of internet connection in developing countries has grown from 8.1% in 2005 to 27.7% in 2013. Over the same period, the WHO target for access to drinkable water was planned to grow at 5% per year and this goal will not been reached.

[2] Hoxby [2014] argues it is not the case, but her rough test based on Mincer equation may lack of power and endogenity issues are not tackled.

[3] The largest on–line world retailer –namely Amazon– has not been profitable until 2009 and its shares under–performed NASDAQ index by 26% over last year. Over the same period, the largest brick and mortar retailer –namely Wallmart– out–performed S&P 500 by 9%. As for the year 2014, the turnovers were 22,72 bn USD for Amazon and 479 bn USD for Wallmart and the net results were 255 millions for Amazon and 16,37 bn for Wallmart.

difference between on–line and face to face student outcomes –at least within the community college setting" whereas experimental evidence by Figlio, Rush and Yin [2010] suggests that this fact may be due to selection effects.[4] Barnejee and Duflo [2014] suggest that high dropout rates reflect "learning about what the MOOC are " both from users and suppliers viewpoints. They also stress the potential importance of MOOC for the supply of higher education in developing countries.

In a stimulating article, Hoxby [2014] sketches ways to address the competitive issues raised by MOOC. She recalls that "[d]istance learning was prevalent well before the introduction of modern on–line courses". Among US students who began to take their higher education courses in 2004, 13% report to have taken part of their course in a distance form according to the Beginning Postsecondary Student survey. This rate is the same as in 1990 and 1996. She then argues that the development of MOOC may change this status quo. She concludes that on–line learning alternative could shake the current equilibrium between non– and highly selective suppliers of post–secondary education. In particular, MOOC may be a threat to both current US business models, albeit more serious for highly selective institutions.

The current French post–secondary education system offers another clear qualitative distinction between universities. Financial costs directly supported by students in French universities are notoriously low, but indirect costs –in particular due to housing problems– may be large, notably in cities where universities are present for centuries. The program "Universités 2000" endowed by 40 billions of French francs (approx. 7.59 billions of current euros) lead to 8 new universities, 24 new Instituts Universitaires de Technologie and 196 new departments.[5] As part of the French program explicitly aimed at a finer spatial coverage, some students end up in modest towns. [6] Not surprisingly, these remote structures propose a very small variety of diplomas, sometimes a single one. In most cases, only undergraduate diploma are delivered. Lately, major French universities are strongly required to cooperate. Some of the smallest places have been closed (as Bethune –25.430 inhabitants– Cambrai –32.770 inhabitants– or the IUTJOB in Clermont-Ferrand). It could be claimed that MOOC is a much cheaper and efficient way to reach distant students. In any case, the introduction of MOOC may change the current equilibrium between old universities located in major towns and new, remote ones.

In this article, we propose a model to study market sharing between a distant access and traditional universities. The approach rests on a spatial competition model *à la* Hotelling. The spatial dimension is of course one major aspect of the difference between on–line and traditional supply. It is also relevant since

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[4]Xu and Jaggars [2013] control this potential selection bias using panel data and instrumental variables estimates.

[5]Spain also witnessed a large growth in the number of universities. Between 1987 and 1998, 19 new universities have been created. For a comparison between France and Spain, see Losego and Milard [2003].

[6]Let us briefly describe one particular example. The university of Caen – 109.899 inhabitants – goes back to the XV-th century. It received in 2004 two new IUTs. The first is located in Lisieux –21.391 inhabitants– and the other one in Vire –11.936 inhabitants –. They are currently 172 students in Vire and 227 in Lisieux. The travel time between the Caen and Vire is 48 minutes by car. As there is no direct train from Caen to Vire the minimal travel time by train is 98 minutes. The maximum distance and travel time seems to be reached by the Mende IUT which is part of the Perpignan University. The distance is 299 kms and the travel time is 172 minutes by car – 329 minutes by train–).

according to the latest Beginning Post-secondary Survey the two leading reasons that guide choice of a university are 'location' and 'financial constraints', before 'reputation'. Finally, this kind of approach is useful to study the resilience of brick and mortar equivalent to on–line retailers (see Foncel, Jouneau-Sion and Longuepee [2013]). The 'Open' nature of the course is simply captured by setting the price of the MOOC to zero.[7] The costs structures also differ since variable costs are only supported by the traditional university.[8]

Clearly, if the traditional university offers a second best alternative, the free, on–line course covers all the market. Thus, as in Foncel, Jouneau-Sion and Longuepee [2013] the student choosing MOOC faces a fixed disutility F^*.

This parameter F^* encapsulates several aspects

- following MOOC may be more engaging –due to lack of motivation, poor contact with pairs,...;

- poor certification raises stronger problems of asymmetrical information, notably on the job market;

- costs incur to cope with technological and pedagogical requirements following Banerjee and Duflo [2014].

Of course, F^* also incorporates differences between course quality. Hence, if the traditional course is very bad (and the on–line relatively good), this difference may offset the above disadvantage. Again, the MOOC would then enjoy a monopoly position and market sharing is trivial. We shall then assume that the net value of F^* is positive. Crucially, this disutility is assumed to be the same whatever the location of the student.

We first set up the model in the context of a single face to face supplier. We study a short term scenario in which the location of the traditional university is fixed and it sets the price charged per student, then a second scenario in which the university may choose its location. We also address the question of controlled prices by the central government. This issue is particularly discussed among French scholars since fees paid by the students are fixed at a national level. As a consequence universities can only set a maximal number of students. This number is set for each national diploma. Allocations are made by competitive examinations and by centralized matching. For beginners in Post-Secondary studies, this matching is currently realized thanks to the 'Admission Post- Baccalauréat' platform. This platform operates at the national level, but in case of excess demand by the students, local ones are served first.

In a second part of the paper, we study the competition between two brick and mortar universities. The first one is located in the city center whereas the other one lies at the extreme left part of the Hotelling line. This second model is used to evaluate the impact of the MOOC on the competition between a 'central' and 'periphery' university. In this part of the paper, we examine two competitive frameworks. When universities may freely fix their fees, we are close to the US case. In a second setting –closer to the French case– universities engage

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[7]This may be viewed as unsatisfactory since openness also stems from the fact that student may –and do– interact to improve the content of the course. However, such interactions are also a vital part of *any* good teaching practice. So it is not clear whether this specifically favors the on–line supplier. In quite the opposite way, it could be claimed that special pedagogical efforts are needed to compensate distant students for the lost of face to face interactions.

[8]The issue of fixed –more precisely sunk– costs is briefly discussed.

in a Cournot competition whereas physical students are charged a common fee whatever the university they choose.

The major findings are as follows.

The results crucially depend on the **relative** disutility of the MOOC. The MOOC enjoys a largest market share when F^* decreases and/or when the linear cost per student and transportation costs increase. Hence questions about market power or sustainability of whatever business model for the MOOC are linked to supply and transportation costs in brick and mortar universities.

In the single university case, we obtain the optimal pricing rule as a function of location and relative disutility. If the relative disutility is too large, no student choose the MOOC. The more central university's location, the more difficult it is for the MOOC to enter the market. At the other extreme, if the MOOC offers a relative *utility*, the student located at the university's address chooses the MOOC and there is no brick and mortar university. This argument does not depends on the university's location. In intermediate situations, three different market sharing rules arise as a function of both relative disutility and location. If the price is set by a central government and the university chooses a physical capacity constraint, the university may choose not to cover all the market in order to select its most valuable part, leaving the rest of the students with an on–line course.

In the central/periphery case, we characterize Bertrand equilibria depending on the value of the relative disutility. For very low quality MOOC, the pricing rules do not depend on the MOOC and no student chooses to study on–line. For very high quality MOOC, the two universities are no longer in direct competition, are we are back to the single case studied before. Intermediate situations reveal complex combinations of pricing/ market share policies. In particular, it may be that the periphery university charges the highest price. Also, the market share of the central university may increase with the quality of the MOOC. Actually, this is the first manifestation of the MOOC : it forces the central university to lower its fees to keep the MOOC out of the market (this is reminiscent to the "deterred entry" in Bain's typology).

We also characterize equilibria in the Cournot setting. The common price rule puts the periphery university in a more difficult position, since it cannot lower the price to attract more students. As a consequence, its market share cannot be larger than 1/4. On the other hand, compared to the Bertrand case, its resilience is larger, in the sense that the MOOC needs to be much better to affect the market share of the periphery.

In both cases, several conclusions emerge that are fairly independent from the type of competition (either quantity or price driven) as the (relative) quality of the MOOC increases –provided it is sufficiently good to influence the market–:

- the profits are always strictly decreasing;

- the prices are decreasing;

- the profit of the periphery university is always the lowest.

In particular, this last result implies that in case of equal sunk costs, the periphery university will leave the market first.

The rest of the paper is organized as follows : Section 2 sets the basic model in the single brick and mortar case. In Section 3 we study the two competitive

cases. We conclude in Section 4. An appendix section contains the proofs and computations.

2 the Single brick and mortar university case

Our model follows Foncel, Jouneau-Sion, Longuepee [2013] which is a variant of the celebrated Hotelling [1923] linear city. We consider a uniform distribution of potential students on the real interval $[0, 1]$. This interval is meant to represent either a physical space (*e.g.* a country or more abstractly the whole World) or a symbolic space (see below for details). Each potential student has a given address on this interval. The traditional university is located at some address $x_a \in [0, 1]$ on this interval. The student located at address x incurs a linear transportation cost $t|x - x_a|$ when (s)he chooses to attend the traditional course. (Sh/H)e can avoid this cost by choosing the on–line course. In this case the student incurs a fix loss $F^* \geq 0$. This loss captures the adaptation to the technological and pedagogical requirements of the MOOC (very few students are actually able to succeed in MOOC courses). It could also incorporate a relative defiance regarding the quality of the diploma and/or the qualifications' reliability acquired by on–line students (a primary source for this defiance could be the job market if on–line and traditional diplomas are distinct). We assume this loss is given and that nobody can control it.[9]

The traditional university charges a fee p and faces a cost c per student. It could also incur a sunk cost, but this will play no role in this section. Indeed, to capture the "Open" feature of the MOOC we assume that the MOOC is free of entry for the student. So the MOOC has no strategic role. Hence the presence of the sunk cost could only drive the university out of the market if the profit can no longer compensate for it.

It is assumed that the MOOC and the university propose the same kind of course. We eschew from 'qualitative' considerations to focus on the geographical impact of the MOOC. Recall also, as we argue in the introduction Section, that the location and the price are the two main considerations that matter for the choice of post–secondary studies in the US (for similar considerations in the French case, in the particular context of local universities, see Felouzis G. [2001]). Finally, without loss of generality, the utility of the course itself is set to zero.[10]

2.1 Short run

In the short run, we assume the address of the university is fixed. Without loss of generality we assume $x_a \in [0, 1/2]$ (the other case follows by symmetry). Each student chooses either the MOOC or the university. This is the usual "covered market" assumption. This is harmless in this case, since the MOOC provides the same utility level $-F^*$ to each potential student. Thus the MOOC offers a lower bound to the attainable utility. As this lower bound is exogenously fixed, the 'strategic space' of the MOOC is empty and we simply have to compute

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[9]This is strong an hypothesis, but we also consider exogenous variations of this parameter, see below for details.

[10]The parameter F^* can also capture issues regarding the relative pedagogical quality of the MOOC compared to its brick and mortar counterpart.

the optimal pricing strategy of the traditional university. Computations are detailed in Appendix 1. The following rules emerge:

1. If $\frac{F^*-c}{t} \geq 2 - x_a$ it is optimal to set $p = F^* - t(1 - x_a)$. In this case, the market share of the university is 1 and its profit is $F^* - t(1 - x_a) - c$ (a strictly positive quantity under the above condition).

2. If $3x_a \leq \frac{F^*-c}{t} \leq 2 - x_a$ then $p = \frac{F^*+tx_a+c}{2}$, the market share of the traditional university is $x_a + \frac{F^*-c}{2t}$ and its profit is $\frac{(F^*+tx_a-c)^2}{4t}$.

3. $2x_a \leq \frac{F^*-c}{t} \leq 3x_a$ then $p = \frac{F^*+c}{2}$, the market share is $2x_a$ and the profit is $\left(\frac{F^*-c}{2t}\right)^2$

4. If $0 \leq \frac{F^*-c}{t} \leq 2x_a$ then $p = \frac{F^*+c}{2}$, the market share is $\frac{F^*-c}{t}$ and the profit is $\left(\frac{F^*-c}{2t}\right)^2$

5. If $\frac{F^*-c}{t} \leq 0$ the maximum is reached for any price larger or equal to F^* in this case the market share is zero and so is the profit.

The short run situation is summarized in figure 1.

Figure 1: Short run optimal solutions

(dashed line : equal split of the market)
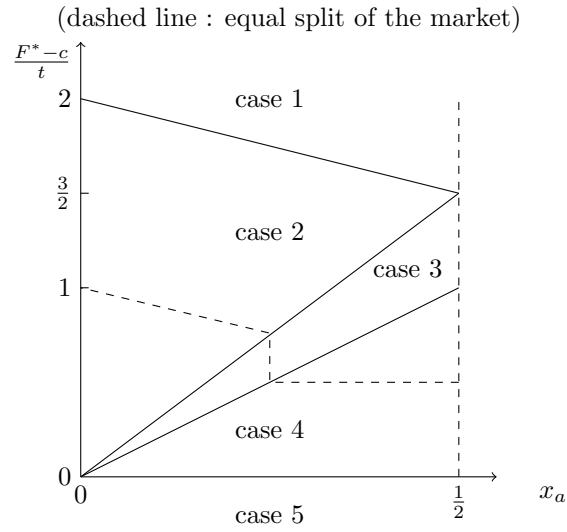
The analysis reveals several interesting features. First, the relative disutility of the MOOC may be captured by the quantity $(F^* - c)/t$. When it is negative, the face to face course is no longer viable since even the student located at x_a prefers the MOOC. Apart from this case, the resilience of the university is quite strong. For the market share of the MOOC to be larger than 1/2 the couple of parameters $(x_a, (F^* - c)/t)$ must lie below the dashed line.

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2.2 Social optimum and provision of MOOC by the university

We now compare the competitive allocation with optimal choice of technology from the central planner viewpoint. Let $0 \leq \Delta \leq 1$ be the market share of the university, the global surplus is equal to[11]

$$
\begin{array}{ll}
t\frac{\Delta^2}{4} + c\Delta + F^*(1-\Delta) & \text{if} \quad 0 \leq \Delta \leq 2x_a \\
t\frac{x_a^2}{2} + t\frac{(\Delta-x_a)^2}{2} + c\Delta + F^*(1-\Delta) & \text{if} \quad 2x_a \leq \Delta \leq 1 - x_a
\end{array}
$$

As straightforward computations reveal, the maximization of the global surplus is obtained when

$$
\begin{array}{ll}
\Delta = 1 & \text{if} \quad \frac{F^*-c}{t} \geq 1 - x_a \\
\Delta = x_a + \frac{F^*-c}{t} & \text{if} \quad 1 - x_a \geq \frac{F^*-c}{t} \geq x_a \\
\Delta = 2\frac{F^*-c}{t} & \text{if} \quad x_a \geq \frac{F^*-c}{t} \geq 0
\end{array}
$$

Figure 2 identifies the situations in which the competitive allocation offers too few or too large MOOC supply (we consider only the case $F^* > c$, otherwise competitive and socially optimal solutions –specifically $\Delta = 0$– clearly coincide.)

Figure 2: Competitive vs socially optimal allocations

(dashed line : actual US split of the market according to the BPS)
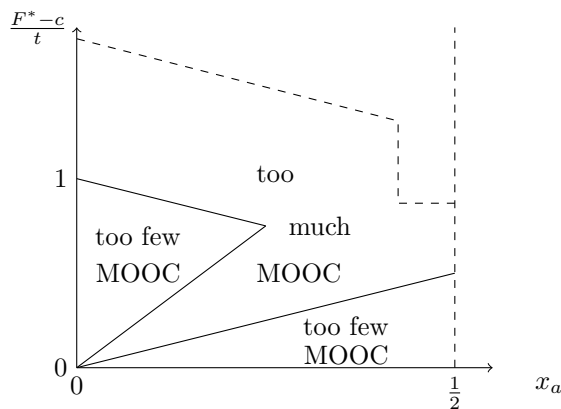
If we consider, as a rough estimate, the BPS figure (13% of students following distant courses) the competitive allocation is consistent with a link between location and MOOC's disutility described by the dashed broken lines in figure 2. From this perspective, the provision of MOOC in the competitive case is currently too large. More precisely, the BPS figure and the competitive allocation are consistent with such a high level of disutility, that the MOOC shall not produce from a central planner viewpoint.

The current result may also shed light on the incentives underlying the current provision of MOOC. Many on–line courses are currently proposed directly

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[11]Recall we set the utility of the course to zero, so that only the costs enter the surplus.

by universities or through of platforms (Coursera, edX and Udacity being the three largest ones). In France, 'France Université Numérique' proposes 78 different courses, all of which have been designed by major French scholars. Assume that the university is forced to enlarge considerably its audience.[12] Then providing a MOOC may be used as a device for the university to cope with the "massification" constraint.

It should be stressed that this holds true even if the MOOC provides no extra revenue for the university (as we assumed in the model, consistently with the current situation). It could even be the case that in order to cope with the coverage constraint the university prefers *free* provision of *costly* MOOC to price cut.

2.3 Long run

In the long run, let us now assume that the university may change its location x_a. As explained above, there are at least two different interpretations for varying x_a. In a geographical sense, universities may change their location. In France for instance, the periods 1970-1980 and 2000-2010 witnessed two opposite movements. During the first period, several universities and 'Grandes Ecoles' have left downtown historical buildings for modern locations in the suburb (*e.g* Ecole Polytechnique moved from Paris student center 'quartier Latin' to the 'Plateau de Saclay' in 1976, Lille universities went to Villeneuve d'Ascq, Lyon 2 moved to Bron,...). Also as we said in the introduction, several local universities have been created. Lately, several universities went back to downtown areas (Lille 2, Lyon 2,...) and many local universities witnessed dramatic shortage of central public subsidies.

From a more pedagogical standpoint, we may interpret the location as a distance between student's capabilities and universities knowledge requirement. It may be stressed that several universities have put large efforts to adapt their requirement to the so-called 'massification' shock (*i.e* the dramatic increase of the population and the recent evolution of student's quality).

Under the above "pedagogical" interpretation of the space, the MOOC is assumed to provide a "premium" version of a product with no particular effort toward more demanding audiences. Such an interpretation is backed by the relatively high failure rate in current MOOC (see Perna and *al.* [2013] for detailed figures of failure rates by MOOC applicants).

The derivation of long run solution follows from maximization of short run profits as function of x_a for all levels of $(F^* - c)/t$.

First remark that the short run profit function are continuous in x_a for every values of $\frac{F^* - c}{t}$.

It is clear that in cases 4 and 5 the profit function does not (locally) depend on x_a. In cases 1 and 2 the profit is an increasing function of x_a. So in both these cases, the symmetric location is optimal. In such cases, the profits are respectively $F^* - t/2 - c$ and $(F^* + t/2 - c)^2/4t$. Finally in case 3, we have

$$\frac{\partial}{\partial x_a} \left\{ \frac{2(F^* - tx_a - c)x_a}{t} \right\} = 2\frac{-2tx_a + F^* - c}{t}$$

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[12]This is consistent with the French case in which students may apply to most universities cursus at the post-secondary level, see Section 3.2 below for details.

and the First Order Condition provides $x_a = \frac{F^*-c}{2t}$. Now this is possible only if $\frac{F^*-c}{2t} \leq \frac{1}{2}$ otherwise the best choice is to set $x_a = \frac{1}{2}$. As it is clear from figure 1, two cases emerges. If $(F^* - c)/t \geq 1$ then it is optimal to choose a central location, otherwise it is optimal to move toward the center whenever $x_a < 2(F^* - c)/t$ and the location has no effect on the profit for any value of $x_a \in [2(F^* - c)/t, 1/2]$.

We then distinguish two cases. If $\frac{F^*-c}{t} \geq \frac{3}{2}$ only cases 1 and 2 are possible. Assume that the best choice is to choose a value of x_a compatible with case 2. Then increasing x_a improves the profit, but we eventually reach case 1, and in this case, the university is better off when x_a increases more. So by continuity of the profit function, the best choice is always $x_a = 1/2$, the market share is 1 and the profit is $F^* - t/2 - c$.

The best choice in the long run is then to block the entry of the MOOC using a central location as long as $(F^* - c)/t \leq 1$. In the opposite case, the entry is no longer avoidable and any choice of location that is not too far from the center is equally good. The smaller the relative disutility of the MOOC the broader the range of possible locations. So the emergence of MOOC may lead the brick and mortar university to modify its location, but this is not necessary.

Finally, we may remark that for any value of $(F^* - c)/t$ there exists a long run competitive solution which coincides with the social optimum. This is trivial if $(F^* - c)/t = 1$ and if $(F^* - c)/t < 1$ it coincides with the long run solution in which the MOOC has the largest possible market share.

3   MOOC and competition between universities

In the previous section, we consider the viewpoint of a single university. This may be –at best– viewed as a description of the university community (as a whole) in front of the rise of on–line teaching technologies. But the brick and mortar supply for higher education is competitive regardless the availability of on–line alternative.[13] As we recalled in the introduction, the emergence of MOOC may shake the current state of equilibrium reached by the traditional providers.

We now consider two universities. The first one is located in the center of the town whereas the second one is located at address $x = 0$. This case is meant to study the effects of MOOC on the spatial competition between two universities in a core/periphery context.

Indeed, as already mentioned in the introduction, some universities recently created in France (Cergy, Evry, Marne-la-Vallée, UVSQ) are located quite far from downtown area and/or in moderately large French cities, sometimes nearby the sea (Littoral, la Rochelle, Bretagne-Sud). It may be argued that it is more challenging for such universities to attract students and/or to hire best scholars.[14]

We consider two types of competition : price and quantity. The first is meant to capture a non regulated market, whereas the second situation follows

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[13]We do not claim, of course, that this competition is a perfect one. Reputation effects together with asymmetric information issues are well-documented.

[14]To this end, we may exploit the data on the "Prime d'Excellence Scientifique". If we control for the size and the type of university, the distance to the major local city has a negative effect on the number of PES allotted to the university.

closely the rules of the current French case.

3.1 Price competition

The effect of MOOC on the competition between universities is not trivial to guess. The situation is related to Hinloopen and Maarewijk [1999] except that we study fixed but asymmetric locations.[15] They identify several regimes of competitions as both firms move closer to each other when the usual full coverage assumption is removed. On the one hand, if the MOOC is able to capture some market share between the two universities then it could potentially soften the competition. This is alike the "mind your own business" case identified by Hinloopen and Maarewijk [1999]. On the other hand, the MOOC is at least a potential newcomer with a very aggressive pricing rule. As such, it may lead to more competition among physical incumbents. This is more alike "threat of war" case in Hinloopen and Maarewijk [1999].

In Appendix 2, we show that six cases emerge depending on $\frac{F^*-c}{t}$:

- $\frac{F^*-c}{t} \leq 0$

- $0 \leq \frac{F^*-c}{t} \leq \frac{1}{2}$

- $\frac{1}{2} \leq \frac{F^*-c}{t} \leq \frac{21}{32}$

- $\frac{21}{32} \leq \frac{F^*-c}{t} \leq 1$

- $1 \leq \frac{F^*-c}{t} \leq \frac{5}{3}$

- $\frac{5}{3} \leq \frac{F^*-c}{t}$

The first case clearly corresponds, as previously mentioned to a monopoly position for the MOOC.

In the second case, the MOOC does supersedes any physical universities but it is good enough to attract some students located between the two physical universities. In this case, the two universities are no longer directly competing since the MOOC is the most serious competitor for both of them.

The case $\frac{1}{2} \leq \frac{F^*-c}{t} \leq \frac{21}{32}$ is special for it leads to a multiplicity of equilibria. This situation arises since the two physical universities deter the entry of the MOOC for student located between them. A multiplicity arises since the burden of this deterrence (that is lowering prices) may fall on either of the two universities. The students choosing the MOOC are all located on the right sides on the market.

In the next case, the situation is very similar (in the sense the market share of the MOOC is limited to the right part of the market) but the burden of the deterrence is supported by the central university alone. It shall be notice that the pricing rule of the periphery university no longer depends directly on the quality of the MOOC. But the dependence is indirect since the periphery adapts its price to the fee chosen by the central university which is a decrease function of the relative quality of the MOOC.

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[15]Another technical difference may be mentioned. Hinloopen and Maarewijk [1999] study a fixed relative reservation price and consider smaller distance between the competitors, whereas we fix the distance and consider various levels of reservation utility.

If the relative quality of the MOOC is even worse, the central university deters entry of the right side of the market.

Finally if $(F^* - c)/t > 5/3$ the price competition between the two physical universities is sufficient to drive the MOOC out of the market. In this case, the pricing equilibrium no longer depends on the relative quality of the MOOC.

Let the index $_p$ (resp. $_c$) refer to periphery (resp. central) university, the Figures 3 to 5 describe the consequences of these regimes for the pricing rules, the market shares and the profits.

Figure 3: Pricing policies (dashed parts correspond to average over possible multiple equilibria)
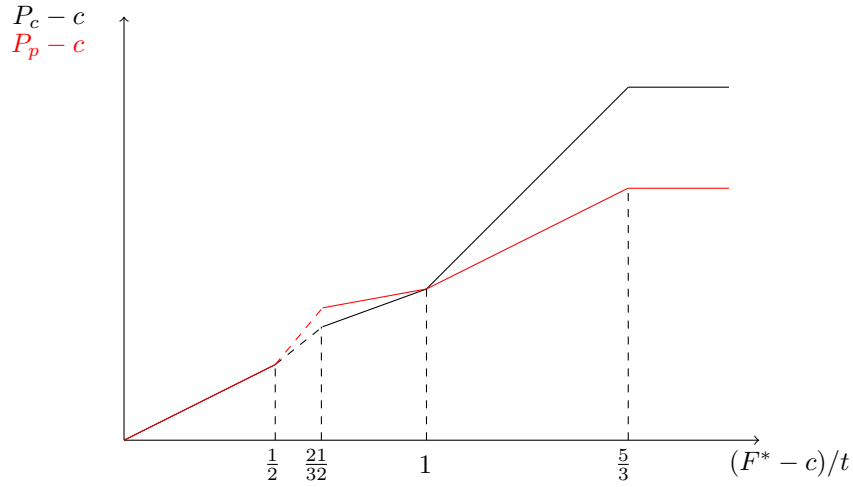
Several interesting features arise. First, provided $0 \leq \frac{F^*-c}{t} \leq \frac{5}{3}$ the profits are strictly decreasing functions of the relative quality of the MOOC. As a consequence, there is no direct incentive for a university to provide a non profitable MOOC. As most higher education on–line courses are currently provided by universities, it must be that they capture some indirect revenues. The model does not allow for such extra revenues, but it helps to measures the amount they much reach to keep universities afloat.[16] What the model does explain, however, is that providing non profitable on–line course to attract more student may not be a profitable strategy in short run, at least. The graphs also make clear that the reaction to an increasing relative quality for on–line course is rather complex. For instance, it may lead the central university to undercut the price charged by the periphery. Also, it may be that an increasing quality for the MOOC implies a decreasing on–line market share. These surprising phenomena are due to the competition between the periphery and the central university to keep the MOOC out of the part market corresponding to student located between 0 and 1/2.

---

[16]It is not difficult to suggest some. For instance, on–line massification may help to discover some extremely talented people, leaving in remote areas of poor countries.
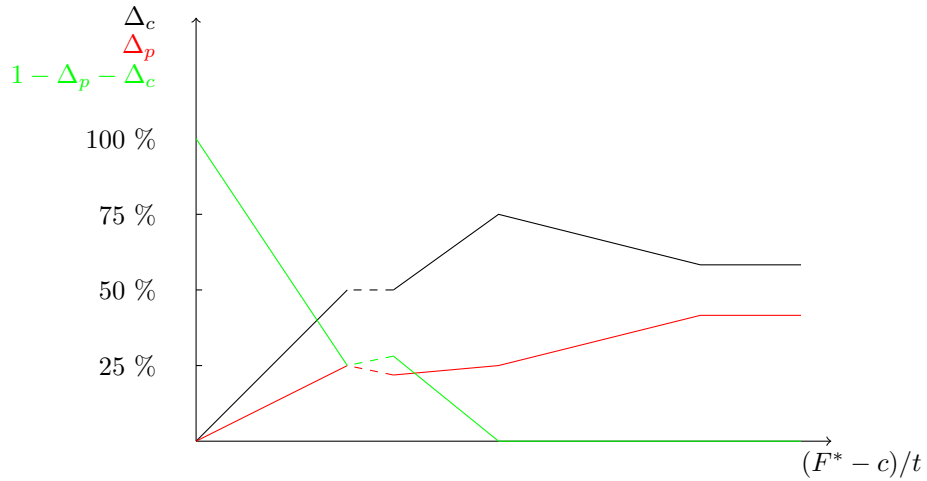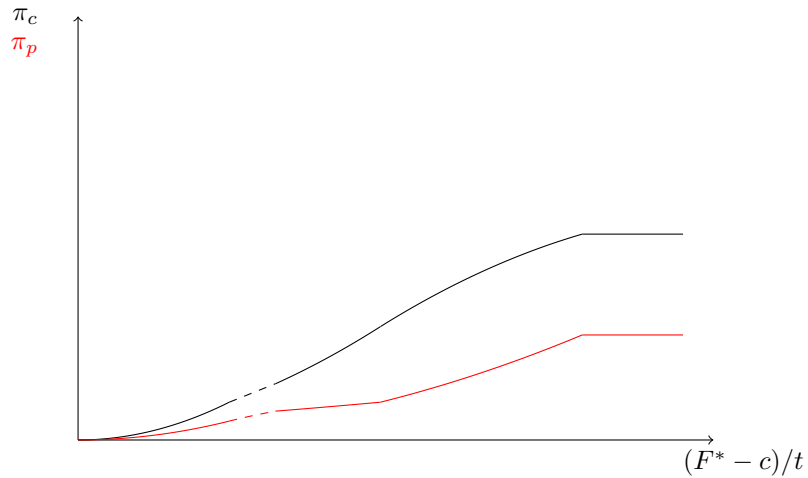
Figure 4: market shares

Figure 5: profits

3.2 Quantity competition

In France post-secondary students may apply to any university provided they succeed to the 'Baccalauréat'. For most diplomas the universities cannot select the applicants. Also, a common national fee is fixed by the ministry of Education (Article 48 loi de finances du 24 mai 1951) for all types of state diplomas. As for 2014, the yearly fees are 181, 250, and 380 euros per student for the three diplomas (licence, master, doctorat). Students must also pay 211 euros for the compulsory health insurance. Universities are often sued when trying to charge

extra fees, and cases are regularly lost by universities.

Each year, the universities fix capacity constraints for each diploma. This is their only strategic instrument (at least in short run). The matching is realized by an automatic procedure (APB for 'Admission Post-Bac').

Clearly, the model proposed in the previous section cannot account for the French case. We now propose a model that captures the most striking features of this regulated market. Again, we consider two universities : a 'core' one located in the middle of the line, and a 'periphery' one located at one edge of the city. The game is a single step one. Both universities set up their capacity constraints simultaneously. If at least one of the capacity constraint is not zero, the government then sets a common fee for each physical student. The price is set according to the following rules (in order).

1. No university can be forced to enlist student when its capacity is reached.

2. When one of the two universities sets zero capacity constraint, the fee is set considering the remaining university only.

3. The government maximizes the fee over feasible opportunities taking into account the possible alternative provided by the MOOC.

This last requirement implies that the government tries to favors incumbents. This may also be justified since according to Section 2.2 above, the governement has no clear incentive to favor the MOOC.

We show in Appendix 3 that five different equilibria corresponding to various values of $(F^* - c)/t$ emerge

- if $(F^* - c)/t \geq 5/4$ then $\Delta_c = 3/4$ and $\Delta_p = 1/4$

- if $5/4 \geq (F^* - c)/t \geq 3/4$ then $\Delta_c = 1/8 + (F^* - c)/2t$ and $\Delta_p = 1/4$

- if $3/4 \geq (F^* - c)/t \geq 1/2$ then $\Delta_c = 1/2$ and $\Delta_p = 1/4$

- if $1/2 \geq (F^* - c)/t \geq 0$ then $\Delta_c = (F^* - c)/t$ and $\Delta_p = (F^* - c)/(2t)$

- if $0 \geq (F^* - c)/t$ then $\Delta_c = 0$ and $\Delta_p = 0$

Compared to the price competition, the regulation implies that the periphery university keeps its initial market share (namely 1/4) except when the MOOC is very good. But, compared to the Bertrand case, it is easier for the MOOC to enter the market, since $\Delta^c + \Delta^p < 1$ requires $(F^* - c)/t < \frac{5}{4}$ in the regulated market and $(F^* - c)/t < 1$ in the Bertand case.

The comparison between regulated and 'Bertrand' fees (averaged over physical students) shows that the regulated system is not necessarily cheaper. When the market share of the MOOC is strictly positive in both systems, the regulated one is cheaper only if the relative quality of the MOOC is low enough.

Finally as in the Bertrand case, the profit of the remote university is always smaller. Yet, the slope of the profit curve is always higher for the central university. Hence at the intensive margin, it is the central university that suffers the most from the introduction of MOOC.
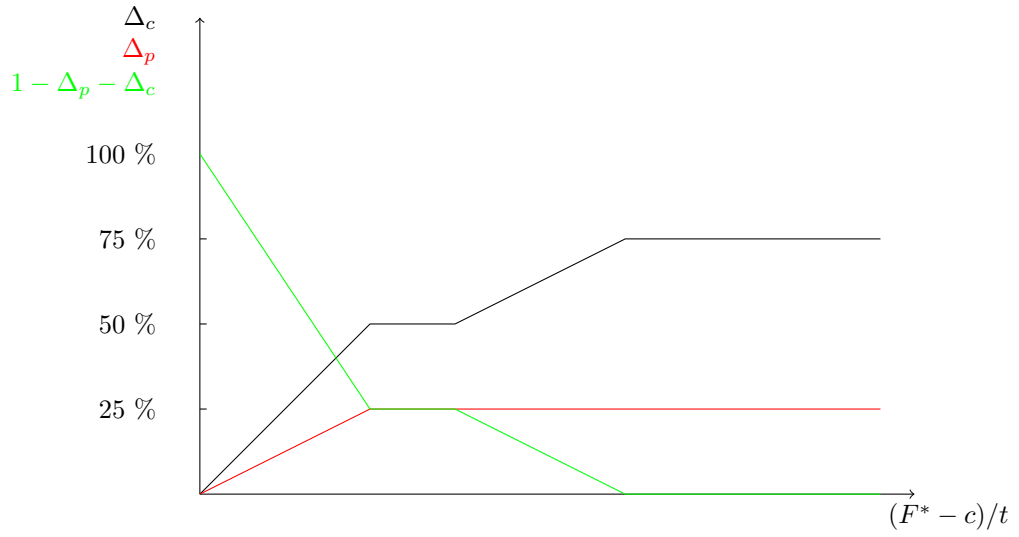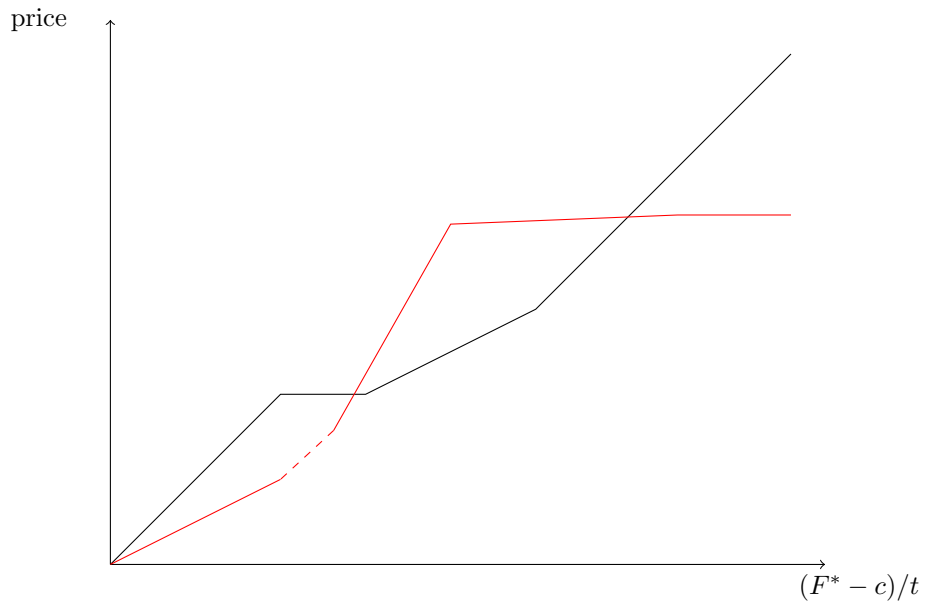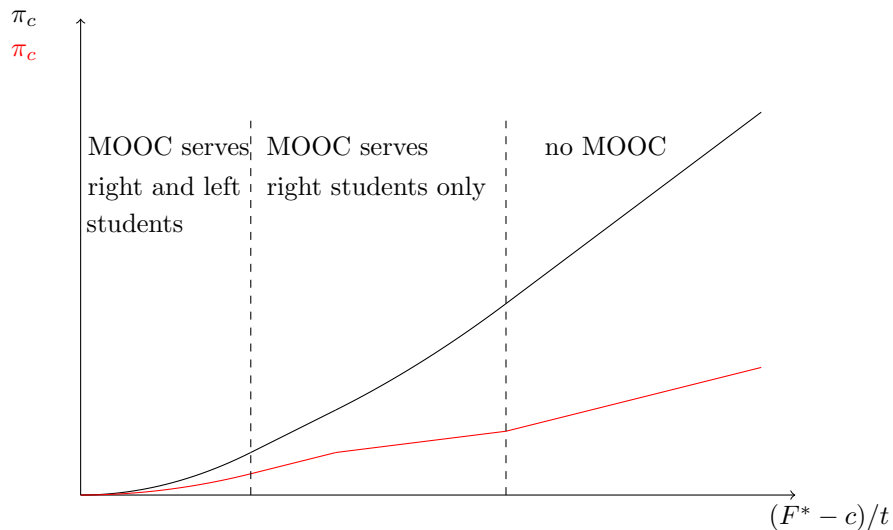
Figure 6: Market shares

Figure 7: Regulated price (average price of physical students in Bertrand competition in red)

4 Conclusion

The emergence and development of on–line teaching technologies may potentially shake the current status of higher education market. In this paper, we

Figure 8: Profits vs comparative disadvantage of the MOOC

propose a model which aims at identifying the key features of this competitive framework. In our model, the main advantage of MOOC is to avoid transportation cost. First, we show that the MOOC should be judged on a relative level, which involves transportation and production costs. More precisely, we show how to define a *relative* quality of the MOOC. Second, we examine the optimal choice of a single university facing a given MOOC. Third, we consider the impact of a MOOC on the equilibrium between a central and a remote university.

As a whole, our results show that the direct impact of MOOC on the current state of the higher education market shall not be overestimated. Our model together with the current, rather stable frequency of students attending distance courses is compatible with a relatively low quality for actual on–line teaching alternatives to traditional ones. In particular, such a quality is much too low to justify provision of on–line course from the central planner viewpoint.

The most convincing case for the introduction for provision of on–line teaching in our model seems to be a "massification" constraint. In order to cope with a coverage goal, the university prefers providing a MOOC, even a low–quality one, to price cutting.

However, the effects of the introduction of MOOC on competition between universities are much stronger. Regardless the type of competition, remote universities face lower profits so in presence of given sunk costs they are the most likely to be kicked out of the market if the quality of the MOOC increases. But the marginal impact of increasingly good MOOC is stronger for central university.

This model may be improved on several grounds. For instance, potential students may show different interests regarding flexibility. Also, the quality of on–line taught courses may be strategic. Finally, on–line and traditional learning could be seen as complementary. We intend to pursue this research in order to address some of theses issues in a future work.

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5 Appendix 1 : single university

A student located at address x chooses the MOOC whenever

$$-F^* > -p - t|x - x_a|$$

Let $\Delta(p, c, F^*, x_a)$ be the market share of the university, it is clear that $\Delta(p, c, F^*, x_a) = \Delta(p, c, F^*, 1 - x_a)$. So we may consider without loss of generality $0 \leq x_a \leq 1/2$.

We can distinguish four different cases.

1. The student located at address $x = 1$ (the most distant form the university) prefers the university. In this case $\Delta(p, c, F^*, x_a) = 1$ and this occurs whenever
$$F^* > p + t(1 - x_a)$$

2. The student located at address $x = 1$ prefers the MOOC but the student located at $x = 0$ prefers the university. This case occurs whenever $p + tx_a < F^* < p + t(1 - x_a)$. In this case, the university is chosen by each student whose address is smaller than x_a. There is only one indifferent student whose address is $x_a + (F^* - p)/t$. The market share $\Delta(p, c, F^*, x_a)$ equals $x_a + (F^* - p)/t$

3. The student located at address $x = 0$ prefers the MOOC but the student located at $x = x_a$ prefers the university. This occurs whenever $p < F^* < p + tx_a$. In this case there are two indifferent students symmetrically located to the right and to the left of the university. The address of the most left one is $x_a + (F^* - p)/t$. The market share $\Delta(p, c, F^*, x_a)$ equals $2(F^* - p)/t$.

4. All students prefer the MOOC. This happens when $F^* < p$ and then $\Delta(p, c, F^*, x_a) = 0$

Computation of the profit function of the university follows directly

$$
\begin{array}{ll}
\pi(p, c, F^*, x_a) = p - c & F^* \geq p + t(1 - x_a) \\
\pi(p, c, F^*, x_a) = (p - c)\left(x_a + \frac{F^* - p}{t}\right) & p + tx_a \leq F^* \leq p + t(1 - x_a) \\
\pi(p, c, F^*, x_a) = 2(p - c)\frac{F^* - p}{t} & p \leq F^* \leq p + tx_a \\
\pi(p, c, F^*, x_a) = 0 & F^* \leq p
\end{array}
$$

As this continuous function is quadratic by parts, the First Order Condition may not be necessary for an optimum. Moreover it may neither be a sufficient condition for a global optimum. Let us first check when the quadratic parts of the above function reach their maximums inside the relevant intervals. Consider the first one. We have

$$\frac{\partial}{\partial p}\left\{(p - c)\left(x_a + \frac{F^* - p}{t}\right)\right\} = x_a + \frac{F^* - p}{t} - \frac{p - c}{t}$$

and

$$\frac{\partial}{\partial p}\left\{(p - c)\left(x_a + \frac{F^* - p}{t}\right)\right\} = 0 \Leftrightarrow p = \frac{F^* + tx_a + c}{2}$$

Now the condition $\frac{F^*+tx_a+c}{2} + tx_a \le F^* \le \frac{F^*+tx_a+c}{2} + t(1-x_a)$ is equivalent to

$$c + 3tx_a \le F^* \le c + t(2-x_a).$$

Since we assumed $x_a \le 1/2$ we always have $c + 3tx_a \le c + t(2-x_a)$.

For the second part we have

$$\frac{\partial}{\partial p} \left\{ 2(p-c)\frac{F^*-p}{t} \right\} = 2\frac{F^*-p}{t} - \frac{p-c}{t}$$

and

$$\frac{\partial}{\partial p} \left\{ 2(p-c)\frac{F^*-p}{t} \right\} = 0 \Leftrightarrow p = \frac{F^*+c}{2}$$

Now the condition $\frac{F^*+c}{2} \le F^* \le \frac{F^*+c}{2} + tx_a$ is equivalent to

$$c \le F^* \le c + 2tx_a.$$

Finally it is clear that the maximum in the monopoly case is reached when $p = F^* - t(1-x_a)$.

The optimal pricing for the university follows.

1. If $F^* \ge c + t(2-x_a)$ then

$$\frac{\partial}{\partial p} \left\{ (p-c)\left(x_a + \frac{F^*-p}{t} \right) \right\} > 0 \text{ whenever } p + tx_a \le F^* \le p + t(1-x_a)$$

   and
$$\frac{\partial}{\partial p} \left\{ 2(p-c)\frac{F^*-p}{t} \right\} > 0 \text{ whenever } p \le F^* \le p + tx_a$$

   so the maximum is reached when $p = F^* - t(1-x_a)$. In this case, the market share is 1 and the profit is $F^* - t(1-x_a) - c$ (a strictly positive quantity under the above condition).

2. If $c + 3tx_a \le F^* \le c + t(2-x_a)$ then

$$\frac{\partial}{\partial p} \left\{ 2(p-c)\frac{F^*-p}{t} \right\} > 0 \text{ whenever } p \le F^* \le p + tx_a$$

   and the maximum is reached when $p = \frac{F^*+tx_a+c}{2}$. In this case, the market share is $x_a + \frac{F^*-c}{2t}$ and the profit is $\frac{(F^*+tx_a-c)^2}{4t}$.

3. $c + 2tx_a \le F^* \le c + 3tx_a$

$$\frac{\partial}{\partial p} \left\{ (p-c)\left(x_a + \frac{F^*-p}{t} \right) \right\} < 0 \text{ whenever } p + tx_a \le F^* \le p + t(1-x_a)$$

   and
$$\frac{\partial}{\partial p} \left\{ 2(p-c)\frac{F^*-p}{t} \right\} > 0 \text{ whenever } p \le F^* \le p + tx_a$$

   so the maximum is reached when $F^* - tx_a$. In this case the market share is $2x_a$ and the profit is $\frac{2(F^*-tx_a-c)x_a}{t}$ (a strictly positive quantity under the above condition)

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4. If $c \leq F^* \leq c + 2tx_a$ then

$$\frac{\partial}{\partial p}\left\{(p-c)\left(x_a + \frac{F^* - p}{t}\right)\right\} < 0 \text{ whenever } p + tx_a \leq F^* \leq p + t(1 - x_a)$$

and the maximum is reached when $\frac{F^* + c}{2}$. In this case, the market share is $\frac{F^* - c}{t}$ and the profit is $\frac{(F^* - c)^2}{4t}$

5. If $F^* \leq c$ the maximum is reached for any price larger or equal to F^*. In this case the market share is zero and so is the profit.

6 Appendix 2: Bertrand–like competition

In this subsection, we consider without loss of generality the case $c = 0$ and $t = 1$.

It is clear that if $F^* < 0$ no university can survive (for it must charge a price so low it looses money on every student otherwise its market share is zero). Hence we consider the case $F^* \geq 0$.

We also consider P_c positive (for otherwise the profit is strictly negative and this may be avoided by setting $P_c > F^*$). Similarly, we consider $P_p \geq 0$.

6.1 Optimal response of the central university

6.1.1 Case $F^* \geq P_p + 1$

The profit of the central university is

$$
\begin{array}{lll}
\text{if} & P_c \leq P_p - 1/2 & \pi_c = P_c \\
\text{if} & P_p + 1/2 \geq P_c \geq P_p - 1/2 & \pi_c = \frac{P_c}{2}\left(P_p + \frac{3}{2} - P_c\right) \\
\text{if} & P_c > P_p + 1/2 & \pi_c = 0
\end{array}
$$

Conditions for interior optima are

$$P_p + 1/2 \geq \frac{P_p}{2} + \frac{3}{4} \geq P_p - 1/2 \Leftrightarrow \frac{1}{2} \leq P_p \leq \frac{5}{2}$$

6.1.2 Case $P_p + 1 \geq F^* \geq P_p + 1/2$

The profit of the central university is

$$
\begin{array}{lll}
\text{if} & P_c \leq P_p - 1/2 & \pi_c = P_c \\
\text{if} & F^* - 1/2 \geq P_c \geq P_p - 1/2 & \pi_c = \frac{P_c}{2}\left(P_p + \frac{3}{2} - P_c\right) \\
\text{if} & P_p + 1/2 \geq P_c \geq F^* - 1/2 & \pi_c = \frac{3P_c}{2}\left(\frac{2F^* + P_p + \frac{1}{2}}{3} - P_c\right) \\
\text{if} & P_c > P_p + 1/2 & \pi_c = 0
\end{array}
$$

Conditions for interior optima are

$$F^* - 1/2 \geq \frac{P_p}{2} + \frac{3}{4} \geq P_p - 1/2 \Leftrightarrow F^* \geq \frac{P_p + \frac{5}{2}}{2} \text{ and } P_p \leq \frac{5}{2}$$

and

$$P_p + \frac{1}{2} \geq \frac{2F^* + P_p + \frac{1}{2}}{6} \geq F^* - 1/2 \Leftrightarrow F^* \leq \min\left\{\frac{1}{4}\left(P_p + \frac{7}{2}\right), \frac{5}{2}\left(P_p + \frac{1}{2}\right)\right\}$$

Now we have $\frac{5}{2}\left(P_p + \frac{1}{2}\right) \geq \frac{1}{4}\left(P_p + \frac{7}{2}\right)$ when $P_p \geq 0$ so that the condition is

$$F^* \leq \frac{1}{4}\left(P_p + \frac{7}{2}\right).$$

6.1.3 Case $P_p + 1/2 \geq F^* \geq P_p$

The profit of the central university is

$$
\begin{array}{lll}
\text{if} & P_c \leq P_p - 1/2 & \pi_c = P_c \\
\text{if} & F^* - 1/2 \geq P_c \geq P_p - 1/2 & \pi_c = \frac{P_c}{2}\left(P_p + \frac{3}{2} - P_c\right) \\
\text{if} & 2F^* - P_p - 1/2 \geq P_c \geq F^* - 1/2 & \pi_c = \frac{3P_c}{2}\left(\frac{2F^* + P_p + \frac{1}{2}}{3} - P_c\right) \\
\text{if} & F^* \geq P_c \geq 2F^* - P_p - 1/2 & \pi_c = 2P_c(F^* - P_c) \\
\text{if} & P_c > F^* & \pi_c = 0
\end{array}
$$

Conditions for interior optima are

$$F^* - 1/2 \geq \frac{P_p}{2} + \frac{3}{4} \geq P_p - 1/2 \Leftrightarrow F^* \geq \frac{P_p + \frac{5}{2}}{2} \text{ and } P_p \leq \frac{5}{2}$$

and

$$2F^* - P_p - \frac{1}{2} \geq \frac{2F^* + P_p + \frac{1}{2}}{6} \geq F^* - 1/2 \Leftrightarrow \frac{7}{10}\left(P_p + \frac{1}{2}\right) \leq F^* \leq \frac{1}{4}\left(P_p + \frac{7}{2}\right)$$

and finally

$$F^* \geq \frac{F^*}{2} \geq 2F^* - P_p - \frac{1}{2} \Leftrightarrow 0 \leq F^* \leq \frac{2}{3}\left(P_p + \frac{1}{2}\right)$$

6.1.4 Case $P_p \geq F^*$

The profit of the central university is

$$
\begin{array}{lll}
\text{if} & P_c \leq F^* - 1/2 & \pi_c = P_c \\
\text{if} & F^* \geq P_c \geq F^* - 1/2 & \pi_c = 2P_c(F^* - P_c) \\
\text{if} & P_c > F^* & \pi_c = 0
\end{array}
$$

Conditions for interior optima are

$$F^* \geq \frac{F^*}{2} \geq F^* - 1/2 \Leftrightarrow 1 \geq F^* \geq 0$$

6.1.5 Best responses

The best responses of the central university are represented in Figure 9.

Figure 9: Central optimal pricing rules as a function of F^* and P_p (Nash equilibria in red)

F^*

$P_p - \frac{1}{2}$

$\frac{P_p + \frac{3}{2}}{2}$

$\geq P_p + \frac{1}{2}$

$F^* - \frac{1}{2}$

$\frac{2F^* + P_p + \frac{1}{2}}{6}$

$\frac{F^*}{2}$

P_p

$\boxed{2F^* - P_p - \frac{1}{2}}$

20
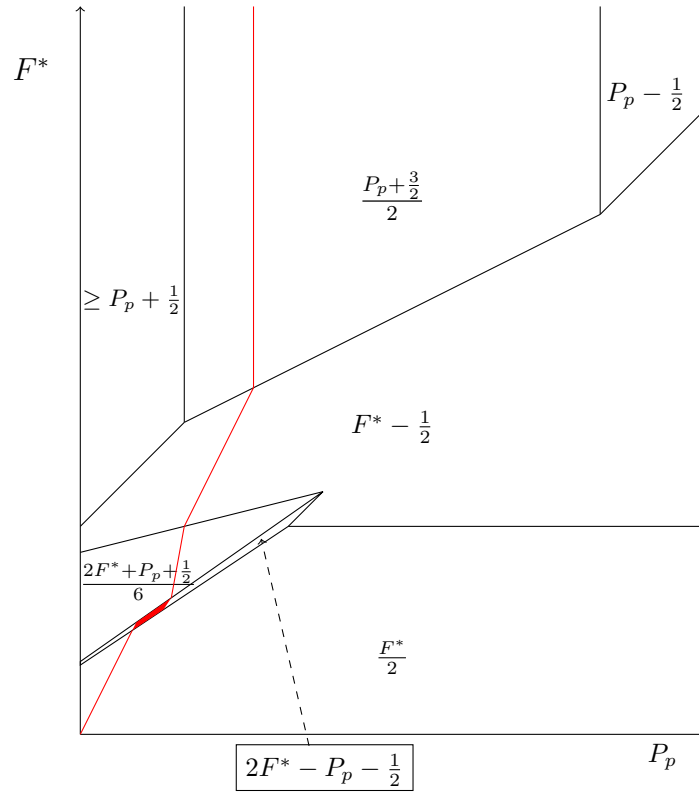
6.2 Optimal response of the periphery university

6.2.1 Case $F^* > P_c + 1/2$

The profit of the periphery university is

$$
\begin{array}{lll}
\text{if} & P_p \leq P_c - 1/2 & \pi_p = P_p \\
\text{if} & P_c + 1/2 \geq P_p > P_c - 1/2 & \pi_p = \frac{P_p}{2}\left(P_c + \frac{1}{2} - P_p\right) \\
\text{if} & P_p > P_c + 1/2 & \pi_p = 0
\end{array}
$$

The conditions for an interior optimal are given by

$$
P_c + \frac{1}{2} \geq \frac{P_c + \frac{1}{2}}{2} > P_c - \frac{1}{2}
$$

Which is equivalent to $-\frac{1}{2} < P_c \leq 3/2$.

6.2.2 Case $P_c + 1/2 \geq F^* \geq P_c$

The profit of the periphery university is

$$
\begin{array}{lll}
\text{if} & P_p \leq F^* - 1 & \pi_p = P_p \\
\text{if} & P_c - 1/2 \geq P_p \geq F^* - 1 & \pi_p = P_p(F^* - P_p) \\
\text{if} & 2F^* - P_c - 1/2 \geq P_p > P_c - 1/2 & \pi_p = \frac{P_p}{2}\left(P_c + \frac{1}{2} - P_p\right) \\
\text{if} & F^* \geq P_p \geq 2F^* - P_c - 1/2 & \pi_p = P_p(F^* - P_p) \\
\text{if} & P_p \geq F^* & \pi_p = 0
\end{array}
$$

This case is more demanding since the profit function may not be quasi concave. Observe first that when $2F^* - P_c - 1/2 \geq P_p > P_c - 1/2$ the profit is always smaller than $P_p(F^* - P_p)$.

Consider first the case in which the function $P_p \to P_p(\min\{1, F^* - P_p\})$ defined over the interval $[F^* - 1, P_c - 1/2]$ admits a unique interior maximum. This interior maximum is reached when $P_p = F^*/2$ if and only if $F^* - 1 \leq F^*/2 \leq P_c - 1/2$ that is $F^* \leq \min\{2, 2P_c - 1\}$. If this holds true, $P_c = F^*/2$ also provides a global maximum of the profit, according to the remark made in the previous paragraph. Similarly, if $P_c + 1/2 \geq F^* \geq 2$ the global maximum of the profit is reached $P_p = F^* - 1$.

From now, let us now assume we do not have $P_c \leq F^* \leq \min\{2, 2P_c - 1\}$ nor $P_c + 1/2 \geq F^* \geq 2$.

The condition for an interior maximum of the function $P_p(F^* - P_p)$ over the interval $[2F^* - P_c - 1/2, F^*]$ is $2F^* - P_c - 1/2 \leq F^*/2 \leq F^*$ which is equivalent to $0 \leq F^* \leq \frac{2}{3}\left(P_c + \frac{1}{2}\right)$. This implies that if $P_c \leq F^* \leq \frac{2}{3}\left(P_c + \frac{1}{2}\right)$ the global maximum is reached when $P_p = F^*/2$ for the same reason as above.

Consider then from now on that the profit function is strictly increasing over the interval $\left[0, P_c - \frac{1}{2}\right]$ which happens whenever

$$
P_c + 1/2 \geq F^* \geq \max\left\{\frac{2}{3}\left(P_c + \frac{1}{2}\right), 2P_c - 1\right\}.
$$

The condition of interior maximum of the function $P_p \to \frac{P_p}{2}\left(P_c + \frac{1}{2} - P_p\right)$ over the interval $]P_c - 1/2, 2F^* - P_c - 1/2]$ is $F^* \geq \frac{3}{4}\left(P_c + \frac{1}{2}\right)$ and $P_c < 3/2$ but this last condition is always fulfilled in the relevant case.

First if $P_c + \frac{1}{2} \geq F^* \geq \max\left\{\frac{3}{4}\left(P_c + \frac{1}{2}\right), 2P_c - 1\right\}$ we must compare the profits realized when $P_p = P_c - \frac{1}{2}$ and $P_p = \frac{P_c + \frac{1}{2}}{2}$. That is $\frac{\left(P_c + \frac{1}{2}\right)^2}{8}$ and $\left(P_c - \frac{1}{2}\right)\left(F^* - P_c + \frac{1}{2}\right)$. The best choice is then $P_p = P_c - \frac{1}{2}$ whenever

$$F^* \geq P_c - \frac{1}{2} + \frac{\left(P_c + \frac{1}{2}\right)^2}{8\left(P_c - \frac{1}{2}\right)}$$

and $P_p = \frac{P_c + \frac{1}{2}}{2}$ in the opposite case.

Finally, if $\frac{3}{4}\left(P_c + \frac{1}{2}\right) \geq F^* \geq \max\left\{2P_c - 1, \frac{2}{3}\left(P_c + \frac{1}{2}\right)\right\}$ we must compare the profits reached when $P_p = P_c - \frac{1}{2}$ and $P_p = 2F^* - P_c - \frac{1}{2}$. As, in both these cases, the profit of the periphery function is also equal to $P_p(F^* - P_p)$, the maximum of the profit is reached for the choice of P_p which is closest to $F^*/2$. We then deduce the best choice is $P_p = P_c - \frac{1}{2}$ if and only if $F^* \geq 1$ and $P_p = 2F^* - P_c - \frac{1}{2}$ otherwise.

6.2.3 Case $P_c > F^*$

The profit of the periphery university is

$$\begin{array}{lll}
\text{if} & P_p \leq F^* - 1 & \pi_p = P_p \\
\text{if} & F^* - 1 \geq P_p \geq F^* & \pi_p = P_p(F^* - P_p) \\
\text{if} & P_p \geq F^* & \pi_p = 0
\end{array}$$

The condition for interior optimal is

$$F^* - 1 \geq F^*/2 \geq F^*$$

Which is equivalent to $0 \leq F^* \leq 2$.

6.2.4 Best responses

The best responses of the periphery university are represented in Figure 10.

6.3 Equilibria

We now investigate the possible equilibria by considering all possible intersections of best responses. We organize this investigation by considering all possible best responses for the central university for each possible best response of the periphery university.

6.3.1 $P_p = (P_c + 1/2)/2$

- $P_c \geq P_p + 1/2$: no equilibrium

  In this case, we have $P_c \geq (P_c + 1/2)/2$ which implies $P_c \geq 1/2$ and hence $P_p \geq 1/2$. But, considering the domain in which $P_c \geq P_p + 1/2$ is a best response, we must have $P_p = 1/2$. But then $P_p = (P_c + 1/2)/2$ implies $P_c = 1/2$ in contradiction with $P_c \geq P_p + 1/2$.

- $P_c = (P_p + 3/2)/2$: equilibrium if $F^* \geq 5/3$

  This implies $P_c = 5/6, P_p = 7/6$. The intersections of this lines with the regimes $P_c = (P_p + 3/2)/2$ and $P_p = (P_c + 1/2)/2$ requires $F^* \geq 5/3$.

Figure 10: Periphery optimal pricing rules as a function of F^* and P_p (Nash equilibria in red)

F^*

$P_c - \frac{1}{2}$

$F^* - 1$

$\frac{P_c + \frac{1}{2}}{2}$

$P_c - \frac{1}{2}$

$\frac{F^*}{2}$

$2F^* - P_c - 1/2$

P_c

23
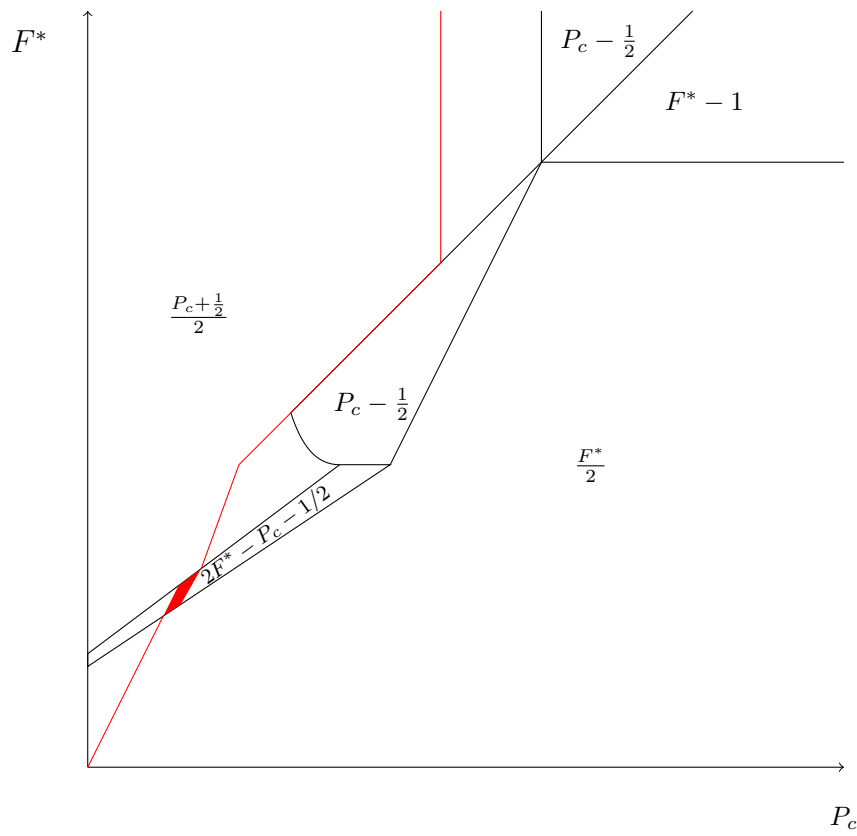
- $P_c = P_p - 1/2$: no equilibrium

  These implies $P_p = 0$, which is clearly not compatible with the domain on which $P_c = P_p - 1/2$ is a best response.

- $P_c = F^* - 1/2$: equilibrium if $5/3 \geq F^* \geq 1$

  This implies $P_p = F^*/2$ and $P_c = F^* - 1/2$. The intersections of this lines with the relevant regimes requires $5/3 \geq F^* \geq 1$.

- $P_c = (2F^* + P_p + 1/2)/6$: equilibrium if $1 \geq F^* \geq 21/32$

  This implies $P_c = (8F^* + 3)/22$ and $P_p = (4F^* + 7)/22$ This equilibrium exists when $1 \geq F^* \geq 21/32$. The intersections of this lines with the relevant regimes requires $1 \geq F^* \geq 21/32$.

- $P_c = 2F^* - P_p - 1/2$: no equilibrium

  This gives $P_c = 4F*/3 - 1/2$ and $P_p = 2F^*/3$. But this last condition is incompatible with the fact that $P_c = 2F^* - P_p - 1/2$ is a best response for the central university.

- $P_c = F^*/2$: equilibrium $F^* = 3/5, P_p = 2/5, P_c = 3/10$

  This implies $P_p = (F^* + 1)/4$. Now for the line $P_c = F^*/2$ to intersect the regime $P_p = (P_c + 1/2)/2$ we need $F^* \geq 3/5$. On the other hand, for the line $P_p = (F^* + 1)/4$ to intersect the regime $P_c = F^*/2$ we need $F^* \leq 3/5$. We then have a single equilibrium in which $F^* = 3/5, P_p = 2/5, P_c = 3/10$.

6.3.2 $\quad P_p = P_c - 1/2$

- $P_c \geq P_p + 1/2$: no equilibrium

  For $P_c \geq P_p + 1/2$ to be a best response we need $P_p \leq 1/2$. But this implies $P_c \leq 1$, which together with $P_c \geq P_p + 1/2$ implies $P_p = 1/2$ and $P_c = 1$. But for the line $P_c = 1$ to intersect the regime $P_p = P_c - 1/2$ we need $1 \leq F^* \leq 3/2$ and for the line $P_p = 1/2$ to intersect the regime $P_c \geq P_p + 1/2$ we need $F^* \geq 3/2$. Hence $F^* = 3/2$ and we derive $F^* = 3/2, P_p = 1/2, P_c = 1$. But this is not a equilibrium for without changing the decision of the central university we already found for the same value of F^* another equilibrium witha a higher value of P_p and hence (since P_c and F^* are unchanged) a higher profit for the periphery university.

- $P_c = (P_p + 3/2)/2$: no equilibrium

  This implies $P_p = -1/6$ which clearly not an equilibrium.

- $P_c = P_p - 1/2$: no equilibrium

  This system has no solution.

- $P_c = F^* - 1/2$: equilibrium if $F^* \in [1, 3/2]$ This requires $P_p = F^* - 1$. Now the intersection of this line with the regime $P_c = F^* - 1/2$ requires $F^* \in [1, 3/2]$. That is a particular case of the equilibrium we already described for $F^* \in [1, 5/3]$.

- $P_c = (2F^* + P_p + 1/2)/6$: no equilibrium

  This requires $P_p = (2F^* - 5/2)/6$ and this line has no intersection with the regime $P_c = (2F^* + P_p + 1/2)/6$.

- $P_c = 2F^* - P_p - 1/2$: no equilibrium

  This requires $P_p = F^* - 1/2$ and this line has no intersection with the regime $P_c = 2F^* - P_p - 1/2$.

- $P_c = F^*/2$: no equilibrium

  The line $P_c = F^*/2$ has no intersection with the regime $P_p = P_c - 1/2$.

6.3.3 $P_p = F^*/2$

- $P_c \geq P_p + 1/2$: no equilibrium

  There is no intersection between the line $P_p = F^*/2$ and the regime $P_c \geq P_p + 1/2$.

- $P_c = (P_p + 3/2)/2$: no equilibrium

  This requires $P_c = (F^* + 3)/4$. Now the intersection of this line with the regime $P_p = F^*/2$ requires $F^* \leq 1$. But the intersection between the line $P_p = F^*/2$ with the regime $P_c = (P_p + 3/2)/2$ requires $F^* \geq 5/3$ a contradiction.

- $P_c = P_p - 1/2$: no equilibrium

  The intersection of the line $P_p = F^*/2$ with the regime $P_c = P_p - 1/2$ requires $F^* \geq 5$ whereas the regime $P_p = F^*/2$ requires $F^* \leq 2$ a contradiction.

- $P_c = F^* - 1/2$: equilibrium if $F^* \in [1, 5/3]$

  Remark that in this case we have $P_c = (F^* - 1)/2$ so that also have $(P_c + 1/2)/2 = F^*/2 = P_p$. We are then back to a case previously studied.

- $P_c = (2F^* + P_p + 1/2)/6$: equilibrium if $F^* = 7/13$, $P_p = 7/26$ $P_c = 4/13$.

  This requires $P_c = (5F^* + 1)/12$ and the intersection of this line with the regime $P_p = F^*/2$ requires $F^* \leq 7/13$. But the intersection of the line $P_p = F^*/2$ with the regime $P_c = (2F^* + P_p + 1/2)/6$ requires $F^* \geq 7/13$. Hence we get $F^* = 7/13$, $P_p = 7/26$ $P_c = 4/13$.

- $P_c = 2F^* - P_p - 1/2$: equilibrium if $1/2 \leq F^* \leq 7/13$.

  This implies $P_c = (3F^* - 1)/2$. The intersection of this line with the regime $P_p = F^*/2$ requires $1/3 \leq F^* \leq 2$. On the other hand, the intersection of the regime $P_c = 2F^* - P_p - 1/2$ with the line $P_p = F^/2$ requires $1/2 \leq F^* \leq 7/13$.

- $P_c = F^*/2$ $P_p = F^*/2$: equilibrium if $0 \leq F^* \leq 1/2$

  By mere inspection of the intersection of the lines $P_c = F^*/2$ $P_p = F^*/2$ with the relevant regime, we obtain an equilibrium whenever $0 \leq F^* \leq 1/2$.

6.3.4 $P_p = F^* - 1$

- $P_c \geq P_p + 1/2$: no equilibrium

  The regime $P_p = F^* - 1$ imposes $F^* \geq 2$ whereas the intersection of the line $P_p = F^* - 1$ with the regime $P_c \geq P_p + 1/2$ imposes $1 \leq F^* \leq 3/2$, a contradiction.

- $P_c = (P_p + 3/2)/2$: no equilibrium

  This requires $P_c = (F^* + 1/2)/2$ and this line does not intersect with the regime $P_p = F^* - 1$.

- $P_c = P_p - 1/2$: no equilibrium

  This requires $P_c = F^* - 3/2$ and this line does not intersect with the regime $P_p = F^* - 1$.

- $P_c = F^* - 1/2$: no equilibrium

  The intersection between the line $P_p = F^* - 1$ and the regime $P_c = F^* - 1/2$ is the same as the intersection of this line with the regime $P_c \geq P_p + 1/2$. Again, we obtain a contradiction since this intersection requires $F^* \leq 3/2$ whereas the regime $P_p = F^* - 1$ requires $F^* \geq 2$.

- $P_c = (2F^* + P_p + 1/2)/6$: no equilibrium

  The line $P_p = F^* - 1$ does not intersect the regime $P_c = (2F^* + P_p + 1/2)/6$.

- $P_c = 2F^* - P_p - 1/2$: no equilibrium

  The line $P_p = F^* - 1$ does not intersect the regime $P_c = 2F^* - P_p - 1/2$.

- $P_c = F^*/2$: no equilibrium

  The line $P_p = F^* - 1$ does not intersect the regime $P_c = F^*/2$.

6.3.5 $P_p = 2F^* - P_c - 1/2$

- $P_c \geq P_p + 1/2$: no equilibrium

  The regime $P_p = 2F^* - P_c - 1/2$ requires $F^* \leq 1$ whereas the regime $P_c \geq P_p + 1/2$ requires $F^* \geq 1$. Hence $F^* = 1$ but in this case the regime $P_c \geq P_p + 1/2$ imposes $P_p = 0$ and then $P_p = 2F^* - P_c - 1/2$ requires $P_c = 3/2$ but this point does not belong to the regime $P_p = 2F^* - P_c - 1/2$.

- $P_c = (P_p + 3/2)/2$: no equilibrium

  The regime $P_p = 2F^* - P_c - 1/2$ requires $F^* \leq 1$ whereas the regime $P_c \geq P_p + 1/2$ requires $F^* \geq 3/2$, a contradiction.

- $P_c = P_p - 1/2$: no equilibrium

  The regime $P_p = 2F^* - P_c - 1/2$ requires $F^* \leq 1$ whereas the regime $P_c = P_p - 1/2$ requires $F^* \geq 5/2$, a contradiction.

- $P_c = F^* - 1/2$: no equilibrium

  This implies $P_p = F^*$ and the intersection of this line with the regime $P_c = F^* - 1/2$ together with the condition $F^* \leq 1$ implies $F^* = 1$ hence $P_c = 1/2$ but this point does not belong to the regime $P_p = 2F^* - P_c - 1/2$.

- $P_c = (2F^* + P_p + 1/2)/6$: no equilibrium

  This requires $P_c = 4F^*/7$ and $P_p = 10F^*/7 - 1/2$. But the intersection of this lines with the relevant regimes provides incompatible conditions for F^*.

- $P_c = 2F^* - P_p - 1/2$: multiple equilibria for $F^* \in [1/2, 21/32]$

  The conditions for this equilibrium to exists are

  $$\frac{1}{3} + \frac{2}{3}P_p \leq F^* \leq \frac{7}{20} + \frac{7}{10}P_p$$
  $$\frac{1}{3} + \frac{2}{3}P_c \leq F^* \leq \frac{3}{8} + \frac{3}{4}P_p$$
  $$P_p = 2F^* - P_c - 1/2$$
  $$P_c \geq 0$$
  $$F^* \leq 1$$

  after elimination of P_p (say) we get

  $$\frac{7}{4}P_c \leq F^* \leq 2P_p$$
  $$\frac{1}{3} + \frac{2}{3}P_c \leq F^* \leq \frac{3}{8} + \frac{3}{4}P_p$$
  $$P_c \geq 0$$
  $$F^* \leq 1$$

  which provides a multiplicity of equilibria.

- $P_c = F^*/2$: equilibrium if $F^* \in [1/2, 3/5]$.

  This requires $P_p = (3F^* - 1)/2$ and the intersection of this line with the regime $P_c = F^*/2$ together with the intersection of the regime $P_p = 2F^* - P_c - 1/2$ with the line $P_c = F^*/2$ provides $F^* \in [1/2, 3/5]$.

7 Appendix 3 : Cournot–like competition

7.1 Rules of the game

The game is a single step one. Both universities set up their capacity constraints simultaneously. If at least one of the capacity constraint is not zero, the government then set a common fee for each physical student. The price is set according to the following rules (in order).

1. No university can be forced to enlist students when its capacity is reached.

2. When one of the two universities set zero capacity constraint, the fee is set considering the remaining university only.

3. The government maximizes the fee over feasible opportunities

7.2 Payoffs

Let Δ_p^n be the notional market share chosen by the periphery and Δ_c^n the notional market share chosen by the central university. Accordingly, Δ_i where $i \in \{p, c\}$ is the actual market share after the fee has been fixed and students made their choices.

7.3 case $\min\{\Delta_p^n, \Delta_c^n\} = 0$

If $\Delta_p^n = \Delta_c^n = 0$ then no fee is fixed, and payoffs are both zero.

Otherwise if $\Delta_p^n = 0$ then we have $\pi_p = 0$ and $\Delta_c = \Delta_c^n$. Moreover if $\Delta_c^n = 1$ the price is set so that $F^* = p + \frac{t}{2}$ and $\pi_c = F^* - \frac{t}{2} - c$, if $0 < \Delta_c^n < 1$ then price is set so that $F^* = p + t\frac{\Delta_c^n}{2}$ and $\pi_c = \Delta_c^n\left(F^* - t\frac{\Delta_c^n}{2} - c\right)$.

In the opposite case, we have $\pi_c = 0$ and $\Delta_p = \Delta_p^n$. Moreover if $\Delta_p^n = 1$ the price is set so that $F^* = p + t$ and $\pi_p = F^* - t - c$, if $0 < \Delta_p^n < 1$ then price is set so that $F^* = p + t\Delta_p^n$ and $\pi_p = \Delta_p^n\left(F^* - t\Delta_p^n - c\right)$.

7.4 case $0 < \Delta_p^n < 1/4$ and $0 < \Delta_c^n$

If $\Delta_c^n \geq 2\Delta_p^n$ we then have $\Delta_c = 2\Delta_p^n$, $\Delta_p = \Delta_p^n$, $F^* = p + t\Delta_p^n$ and the payoffs are

$$
\begin{aligned}
\pi_p &= \Delta_p^n(F^* - t\Delta_p^n - c) \\
\pi_c &= 2\Delta_p^n(F^* - t\Delta_p^n - c)
\end{aligned}
$$

Otherwise we have
$\Delta_p = \Delta_c^n/2$, $\Delta_c = \Delta_c^n$, $F^* = p + t\Delta_c^n/2$ and the payoffs are

$$
\begin{aligned}
\pi_p &= \frac{\Delta_c^n}{2}\left(F^* - t\frac{\Delta_c^n}{2} - c\right) \\
\pi_c &= \Delta_c^n\left(F^* - t\frac{\Delta_c^n}{2} - c\right)
\end{aligned}
$$

7.5 case $\Delta_p^n \geq 1/4$ and $0 < \Delta_c^n$

If $\Delta_c^n \geq 3/4$ then

$$
\begin{aligned}
\Delta_p &= \tfrac{1}{4}, \Delta_c = \tfrac{3}{4} \\
p &= F^* - \tfrac{t}{2} \\
\pi_p &= \tfrac{1}{4}\left(F^* - \tfrac{t}{2} - c\right) \\
\pi_c &= \tfrac{3}{4}\left(F^* - \tfrac{t}{2} - c\right)
\end{aligned}
$$

If $3/4 > \Delta_c^n \geq 1/2$
then

$$
\begin{aligned}
\Delta_p &= \tfrac{1}{4}, \Delta_c = \Delta_c^n \\
p &= F^* - t\left(\Delta_n^c - \tfrac{1}{4}\right) \\
\pi_p &= \tfrac{1}{4}\left(F^* - t\left(\Delta_n^c - \tfrac{1}{4}\right) - c\right) \\
\pi_c &= \Delta_c^n\left(F^* - t\left(\Delta_n^c - \tfrac{1}{4}\right) - c\right)
\end{aligned}
$$

Finally if
If $1/2 > \Delta_c^n > 0$
then

$$
\begin{aligned}
\Delta_p &= \tfrac{\Delta_c}{2}, \Delta_c = \Delta_c^n \\
p &= F^* - t\tfrac{\Delta_n^c}{2} \\
\pi_p &= \tfrac{\Delta_c}{2}\left(F^* - t\tfrac{\Delta_n^c}{2} - c\right) \\
\pi_c &= \Delta_c\left(F^* - t\tfrac{\Delta_n^c}{2} - c\right)
\end{aligned}
$$

7.6 Best responses : central university

It is clear from the above derivation that we may always consider the case $c = 0$ and $t = 1$ provided we consider the change in variables $F^* \to (F^* - c)/t$. We distinguish three sub–cases.

7.6.1 $\Delta_p^n \geq 1/4$

If the payoff of the central university is $\Delta_c^n \left(F^* - \left(\Delta_n^c - \frac{1}{4} \right) \right)$ then it is optimal to choose $\Delta_c^n = \frac{F^* + 1/4}{2}$ provided this choice belong to $[1/2, 3/4]$ which entails

$$\frac{3}{4} \leq F^* \leq \frac{5}{4}$$

Also, if $F^* \geq \frac{5}{4}$ then it is optimal to set $\Delta_c^n = \frac{3}{4}$. If the payoff of the central university is $\Delta_c^n \left(F^* - \frac{\Delta_n^c}{2} \right)$ then it is optimal to choose $\Delta_c^n = F^*$ provided this choice belong to $]0, 1/2]$. Otherwise, if $\frac{3}{4} \geq F^* \geq \frac{1}{2}$ the best choice is $\Delta_c^n = \frac{1}{2}$ and if $F^* < 0$ then $\Delta_c^n = 0$.

7.6.2 $0 < \Delta_p^n \leq 1/4$

The only case in which the payoff of the central university depends on its own choice is when $\Delta_c^n \leq 2\Delta_p^n$ in which case, it is best to set $\Delta_c^n = F^*$ provided this choice is compatible with the constraint. Hence we have

$$\begin{aligned} F^* > 2\Delta_p & \qquad \Delta_c^n \geq 2\Delta_p^n \\ 0 < F^* \leq 2\Delta_p & \qquad \Delta_c^n = F^* \\ F^* \leq 0 & \qquad \Delta_c^n = 0 \end{aligned}$$

7.6.3 $\Delta_p^n = 0$

In this case, it easy to show that the optimal choices are

$$\begin{aligned} F^* \geq 1 & \qquad \Delta_c^n = 1 \\ 0 \leq F^* < 1 & \qquad \Delta_c^n = F^* \\ F^* < 0 & \qquad \Delta_c^n = 0 \end{aligned}$$

7.7 Best responses : periphery university

First , if $\Delta_c^n = 0$ then the best choices are given by

$$\begin{aligned} F^* \geq 2 & \qquad \Delta_p^n = 1 \\ 0 \leq F^* < 2 & \qquad \Delta_p^n = F^*/2 \\ F^* < 0 & \qquad \Delta_p^n = 0 \end{aligned}$$

Next, if $\Delta_c^n > 0$ the only case in which the periphery university influences the price is when $\Delta_p^n \leq \Delta_p^n/2$ and $0 < \Delta_p^n \leq 1/4$. In this case, the unique interior optimum is $\Delta_p^n = F^*/2$ which requires $0 < F^* \leq 1/2$. Hence if $F^* > 1/2$ it is optimal to set $\Delta_p^n = 1/4$, otherwise we have

$$F^* \geq \Delta_c^n \qquad \Delta_p^n = \Delta_c/2$$
$$0 \geq F^* > 2 \quad \Delta_p^n = F^*/2$$
$$F^* < 0 \qquad \Delta_p^n = 0$$

7.8 Equilibria

From the above derivations, we obtained five different equilibria corresponding to various values of $(F^* - c)/t$

1. if $(F^* - c)/t \geq 5/4$ then $\Delta_c^n = 3/4$ and $\Delta_p^n = 1/4$

2. if $5/4 \geq (F^* - c)/t \geq 3/4$ then $\Delta_c^n = 1/8 + (F^* - c)/2t$ and $\Delta_p^n = 1/4$

3. if $3/4 \geq (F^* - c)/t \geq 1/2$ then $\Delta_c^n = 1/2$ and $\Delta_p^n = 1/4$

4. if $1/2 \geq (F^* - c)/t \geq 0$ then $\Delta_c^n = (F^* - c)/t$ and $\Delta_p^n = (F^* - c)/(2t)$

5. if $0 \geq (F^* - c)/t$ then $\Delta_c^n = 0$ and $\Delta_p^n = 0$