

Efficiency Gains from Tagging

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Abstract

The paper deals with the efficiency gains from tagging, that is a policy where separate income tax schedules are designed for different groups of the population. We first show that tagging can make income taxation more efficient if either the initial tax rates are close to the Rawlsian tax rates or if the distribution of income in one of the groups differs substantially from that in the population. Second, we show that there is a trade-off between efficiency and horizontal equity if and only if the skill-distribution has the same support in all groups. Third, we establish that a Pareto-superior outcome cannot be implemented by increasing the tax burden of high-income groups, but can involve higher tax payments for low-income groups.

Keywords: nonlinear income taxation, tagging, Pareto efficiency, horizontal equity, redistributive taxation

JEL-Classification: H21, H24, D61, D63

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1 Introduction

- Main findings of the paper: see abstract and Sections 3-5
- Setup: nonlinear income taxation, see Section 2 for details
- Related Literature:
 - Pareto-efficient nonlinear income taxation: Guesnerie and Seade (1982), Stiglitz (1982, 1987), Brito et al. (1990), Werning (2007)
 - welfare effects from tagging and age-dependent income taxation: e.g. Akerlof (1978), Immonen et al. (1998), Boadway and Pestieau (2006), Werning (2007), Blomquist and Micheletto (2008), Cremer et al. (2010), Mankiw and Weinzierl (2010), Weinzierl (2011), Pestieau and Racionero (2015), Kanbur and Tuomala (2016), Saez and Stantcheva (2016)

2 The model

We consider the general discrete version of the optimum income tax model first introduced by Guesnerie and Seade (1982). There are I types of individuals, $i = 1, 2, \dots, I$, which differ with respect to their wage rates w_i . Without loss of generality, we assume $0 < w_1 < w_2 < \dots < w_I$. It is assumed that the individuals' preferences can be represented by a strictly quasiconcave and twice continuously differentiable utility function $u(c_i, l_i)$ which is increasing in consumption c_i and decreasing in labor supply l_i . These preferences can alternatively be expressed in terms of the functions $v_i(c_i, y_i) = u(c_i, y_i/w_i)$, where $y_i = w_i l_i$ is the pretax income. In addition to monotonicity, quasiconcavity and differentiability, we also assume that the single-crossing property holds.

It is assumed that the government cannot observe the type of an individual, but knows the frequency $f_i > 0$ of each type in the economy. The concept of tagging is based on the additional assumption that the unobservable type of an individual is correlated with another, observable characteristic. Therefore, we assume that the population can be divided into several groups, indexed by $g = 1, \dots, G$. Let $f_g > 0$ denote the percentage of individuals in group g and $f_{ig} \geq 0$ the frequency of type i within group g . These definitions imply

$$\sum_{i=1}^I f_i = 1, \quad \sum_{g=1}^G f_g = 1, \quad \sum_{i=1}^I f_{ig} = 1, \quad \text{and} \quad \sum_{g=1}^G f_g f_{ig} = f_i.$$

The information about the group to which the individuals belong can only be informative about the individuals' type if the wage distribution is not identical

in each group. Therefore, tagging becomes interesting only if $f_{ig} \neq f_i$ holds for at least one i and g .

The government observes the pretax income y_i of each individual and the group g to which the individual belongs. We will distinguish between a no-tagging situation where the information about group-membership is ignored and a tagging-situation where it is used. In the no-tagging situation, tax policy is about the design of a single, possibly non-linear income tax function $T(y)$ that applies to all individuals. Each of them then chooses y_i and c_i by maximizing utility $v_i(c_i, y_i)$ subject to the budget constraint $c_i = y_i - T(y_i)$. This choice then determines the individuals' tax payment $T_i := T(y_i) = y_i - c_i$. Without loss of generality, we consider the case of a purely redistributive income tax where the government's tax revenue must be nonnegative:

$$\sum_{i=1}^I f_i(y_i - c_i) \geq 0. \quad (1)$$

It is well-known that an allocation $(c_1, y_1), \dots, (c_I, y_I)$ can be implemented by a tax function $T(y)$ if the budget constraint and the self-selection constraints

$$v_i(c_i, y_i) \geq v_i(c_j, y_j), \quad \forall i, j \in \{1, \dots, I\} \quad (2)$$

are satisfied. Hence, (1) and (2) are the relevant constraints for tax policy in a situation without tagging.

In a situation with tagging, the information about group membership allows to impose group-specific income-tax functions $T_g(y)$ such that each group may end up with a different allocation $(c_{1g}, y_{1g}), \dots, (c_{Ig}, y_{Ig})$. A single group can have a budget surplus or deficit, but on average the same constraint as above, namely

$$\sum_{g=1}^G f_g \left(\sum_{i=1}^I f_{ig}(y_{ig} - c_{ig}) \right) \geq 0 \quad (3)$$

must hold. Since each group has a separate tax function $T_g(y)$, we also have separate self-selection constraints

$$v_i(c_{ig}, y_{ig}) \geq v_i(c_{jg}, y_{jg}), \quad \forall i, j \in \{1, \dots, I\} \quad (4)$$

for each group g . Any allocation $((c_{1g}, y_{1g}), \dots, (c_{Ig}, y_{Ig}))_{g=1}^G$ that does not violate (3) and (4) can be implemented if tagging is employed. Note that (3) and (4) become equivalent to (1) and (2) if one adds the restriction $(c_{ig}, y_{ig}) = (c_{ih}, y_{ih}), \forall g, h$. This is the only difference between the model with and the model without tagging.

The subsequent analysis deals with the question whether tagging can make an initial allocation without tagging more efficient. Some of the findings rely on the assumption that the no-tagging allocation (i) is Pareto-efficient subject to the

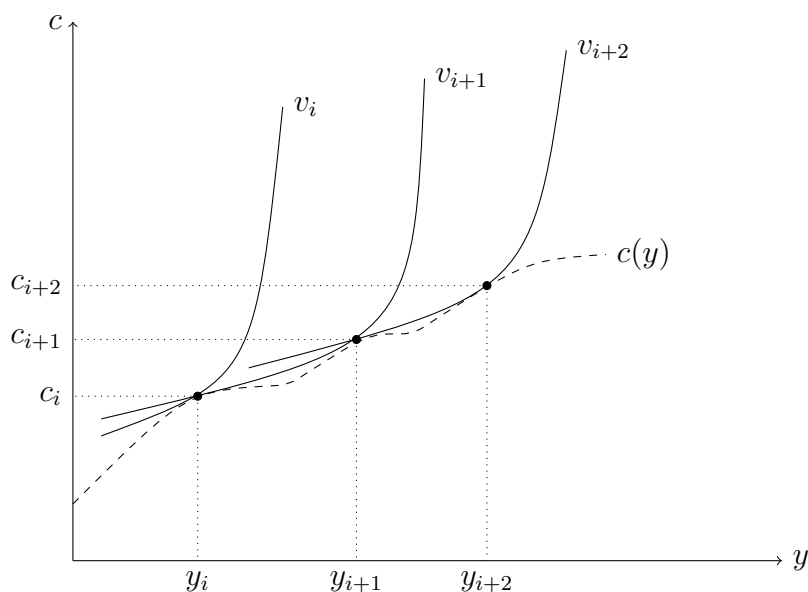


Figure 1

constraints (1) and (2), has (ii) binding downward incentive-compatibility constraints, (iii) positive marginal tax rates for all types $i < n$, and (iv) no bunching. We will refer to this assumption as Assumption R. Such a non-tagging equilibrium is illustrated in Figure 1. Tax policy, that is the design of a function $c(y) := y - T(y)$, implements an allocation where the slope of the individuals' indifference curves is below unity and where further redistribution from high-income to low-income earners is restricted by the downward self-selection constraints.

We say that tagging allows for a *Pareto-improvement* if a tagging allocation exists such that $v_{ig} \geq v_i$ holds for all i and g with at least one of these inequalities strict. We say that a tagging allocation does not violate the principle of *horizontal equity* if the individuals of each type i are equally well-off in all groups g , that is if v_{ig} is the same in all groups even though the allocations (c_{ig}, y_{ig}) may not.

3 Efficiency gains from tagging

It is obvious that tagging must allow for a Pareto superior outcome at least in some situations, for example if the exogenous characteristic is perfectly correlated with the individuals' wage rates. However, little is known about whether this special case is relevant also in a more general setting. So far, the issue of a Pareto improvement has been discussed by Blomquist and Micheletto (2008), Werning (2007), Mankiw and Weinzierl (2010), and Weinzierl (2011). Blomquist and Micheletto (2008) consider a model with two types – low wage rate and high wage rate – and two groups – young and old – and the assumption that all young workers have low wage rates. This is basically the same set-up as in

Akerlof (1978). Blomquist and Micheletto (2008) then explain why age-dependent taxation allows for a Pareto-improvement and their argument is similar to that of Proposition 3.1 below. Werning (2007) primarily deals with Pareto-efficient tax schedules in general, but notes that tagging has positive efficiency effects if the initial tax schedule is efficient for the whole population, but not for each of the groups. This argument will also be used below. Mankiw and Weinzierl (2010) and Weinzierl (2011) primarily deal with utilitarian welfare but report that they have also calculated a Pareto superior outcome. Their analysis is based on data that fulfill the assumptions of Proposition 3.1. In the following, we will consider a more general model than in the previous literature and provide two sufficiency conditions for a Pareto improvement - one depending on the skill distribution and the other on the tax schedule in the initial equilibrium.

Lemma 3.1: *Assume that the non-tagging allocation with tax schedule $T(y)$ is constrained Pareto-efficient. Then the following two statements are equivalent.*

- (i) *Tagging allows for a Pareto-improvement.*
- (ii) *For at least one group \hat{g} , there exists another tax schedule $T_{\hat{g}}(y)$ such that $v_{i\hat{g}} \geq v_i$ for all i and $T_{\hat{g}} := \sum_i f_{i\hat{g}}T_{i\hat{g}} \geq \sum_i f_{i\hat{g}}T_i =: T_g$ with at least one inequality strict.*

Proof: (ii) \Rightarrow (i): If one of the inequalities $v_{i\hat{g}} \geq v_i$ is strict, a Pareto improvement is obtained by taxing group \hat{g} by means of schedule $T_{\hat{g}}(y)$ and all other groups with schedule $T(y)$. If the inequality $\sum_i f_{i\hat{g}}T_{i\hat{g}} \geq \sum_i f_{i\hat{g}}T_i$ is strict, group \hat{g} can be taxed with schedule $T_{\hat{g}}(y)$ and the surplus can be used for making some other group better off.

(i) \Rightarrow (ii): Because of (i) there must exist some group \hat{g} such that $v_{i\hat{g}} \geq v_i$ for all i with at least one inequality strict. If $T_{\hat{g}} \geq T_g$ holds as well, one has (ii). Otherwise, we have $T_{\hat{g}} < T_g$. In this case, however, there must exist some other group \tilde{g} where $T_{\tilde{g}} > T_g$. Because $v_{g\tilde{g}} \geq v_i$ is assumed to hold for that group as well, (ii) follows again. ■

Lemma 3.1 basically says that tagging allows for a Pareto improvement if the initial tax schedule is Pareto efficient for the whole population, but not separately for each single group. This is not very surprising, but has some important consequences: First, the question whether a Pareto improvement is possible can be answered by investigating each group separately and without changing their tax payment T_g . Hence, one can ignore sidepayments between the groups even if they may play some role when tagging is implemented. The question of efficiency can thus be separated from the question of intergroup redistribution similar to the two-stage method that Immonen et al. (1998) have proposed for solving the welfare-maximization problem under tagging. Second, if a Pareto improvement is possible, it can easily be implemented as follows: Those groups for which (ii) does not hold stay with the initial tax schedule which preserves their status quo. The other groups go through a tax reform which makes them better off, but without reducing their aggregate tax payment. Third, the lemma gives a hint,

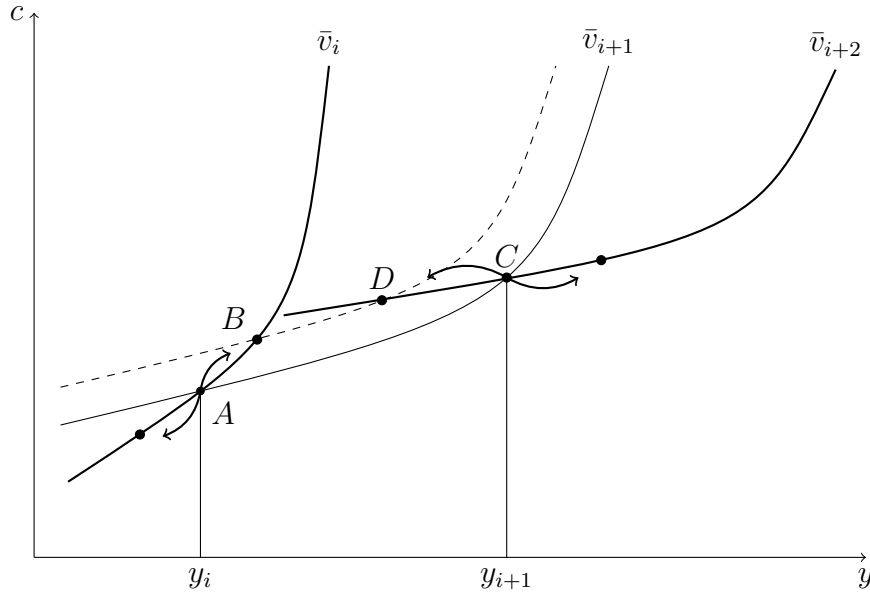


Figure 2

how a reform must be designed in order to implement a more efficient allocation: Condition (ii) implies that one type of individuals must pay a strictly higher tax T_{ig} under tagging than in the initial equilibrium even though he or she does not become worse off. This is only possible if the marginal tax rate of that type decreases.

The last point is illustrated in Figure 2. The tax payment T_i that individuals of type i pay in A can be increased without reducing their utility if one shifts y_i and c_i along the indifference curve \bar{v}_i towards B . This amounts to a reduction of their marginal tax rate. However, B becomes also attractive for individuals who initially have earned higher income like those of type $i + 1$. Their utility must increase and this goes along with a reduction of their tax payment, either by accepting that they also move to B or by inducing them to choose another allocation D that also has a lower tax payment than the initial allocation C . In any case, there is a tradeoff between a gain $\Delta T_i > 0$ in tax payment of type i and a loss $\Delta T_j < 0$ of one or several types $j > i$. The total effect on tax payment then amounts to

$$\Delta T = f_i \Delta T_i + \sum_{j=i+1}^I f_j \Delta T_j < 0. \quad (5)$$

Note that the reform would make type $i + 1$ better off and no one worse off. Since we have assumed that the initial allocation is Pareto-efficient (among those allocations that can be implemented without tagging), we must have $\Delta T < 0$.

The situation with tagging differs from that without tagging only to the extent that a tax reform can now be implemented separately for each of the groups. In

particular, it is possible to refrain from the reform in those groups where the budgetary effect is negative and implement it in those groups where it is positive. The latter holds as long as

$$\Delta T_g = f_{ig}\Delta T_i + \sum_{j=i+1}^I f_{jg}\Delta T_j \geq 0. \quad (6)$$

According to Lemma 3.1, tagging allows for a Pareto improvement if ΔT_g is nonnegative for some group even though ΔT is negative. There are basically two reasons why this may be the case:

- (i) The skill distribution in one of the groups may differ substantially from the skill-distribution of the whole population such that f_{ig}/f_{jg} is much larger than f_i/f_j for some or all $j > i$.
- (ii) The initial tax schedule has efficient, but very high marginal tax rates such that ΔT is negative, but close to zero. Then a small deviation of f_{ig}/f_{jg} from f_i/f_j is sufficient for $\Delta T_g \geq 0$.

The following two results confirm this reasoning.

Proposition 3.1: *If $f_{ig} = 0$ holds for some $g \geq 1$ and $i \geq 2$, then any non-tagging allocation under Assumption R is Pareto-dominated by a tagging-allocation.*

Proof: The assumption $f_{ig} = 0$ for some $i \geq 2$ is equivalent to the assumption $f_{i+1g} = 0$ for some $i \geq 1$. Hence, we have a situation like in Figure 2, but with no individual of type $i + 1$ in one of the groups. The corresponding incentive-compatibility constraint (comparison of C and A) is thus irrelevant for that group. Therefore, an increase in T_{ig} can be obtained for constant utility v_{ig} . ■

Proposition 3.2: *If $f_{ig} \neq f_i$ holds for at least some type i and group g , then at least some non-tagging allocation, the Rawlsian one, is Pareto-dominated by a tagging allocation.*

Proof: The Rawlsian tax schedule implements the allocation that maximizes utility $v_1(c_1, y_1)$ subject to the feasibility constraint (1) and the incentive compatibility constraints (2). Let v_1^R denote the maximal utility that results from solving this problem. It is well-known that the Rawlsian allocation coincides with the allocation that maximizes tax revenue $T = \sum_{i=1}^I f_i(y_i - c_i)$ subject to incentive compatibility (2) and the participation constraint $v_1(c_1, y_1) \geq v_1^R$. In the following, we will refer to the latter problem for proving the result.

Consider first the bundle (y_i, c_i) of some type $i < n$ and (y_{i+1}, c_{i+1}) of type $i+1$ in the optimum without tagging. Let $T_i = y_i - c_i$ and $T_{i+1} = y_{i+1} - c_{i+1}$ denote the corresponding tax payments. It is well-known that that the marginal tax rates are positive for all types $i < I$ and that the downward incentive-compatibility

constraints are binding in the Rawlsian optimum. The initial allocation for type i and type $i + 1$ thus looks like in Figure 2, where the slope of indifference curve \bar{v}_i is below unity at y_i and the slope of \bar{v}_{i+1} is also below unity at y_{i+1} if $i + 1 < n$ and equal to unity if $i + 1 = n$. In the following, we will restrict attention to the case $i + 1 < n$. The proof for the case $i + 1 = n$ is analogous.

Consider now a small (positive or negative) change dy_i of income y_i and an accompanying change of c_i such that utility $v_i(c_i, y_i)$ remains constant (see Figure 2). Since the slope of \bar{v}_i is below unity at y_i , these changes imply $dT_i/dy_i = d(c_i - y_i)/dy_i > 0$. In addition to the perturbation of (c_i, y_i) , we also modify (c_{i+1}, y_{i+1}) in such a way that the incentive constraint $v_{i+1}(c_{i+1}, y_{i+1}) \geq v_{i+1}(c_i, y_i)$ remains binding and utility v_{i+2} remains constant. Since the slope of indifference curve \bar{v}_{i+2} is also below unity at income y_{i+1} , we get $dT_{i+1}/dy_i < 0$. Hence, the effect of dy_i on aggregate tax revenue T equals

$$\frac{dT}{dy_i} = f_i \frac{dT_i}{dy_i} + f_{i+1} \frac{dT_{i+1}}{dy_i}. \quad (7)$$

Note that tagging allows one to undertake the same perturbation separately for each group. The effect on tax revenue T_g of such a group is

$$\frac{dT_g}{dy_i} = f_{ig} \frac{dT_i}{dy_i} + f_{i+1g} \frac{dT_{i+1}}{dy_i}. \quad (8)$$

Since the Rawlsian optimum maximizes T subject to the participation constraint $v_1(c_1, y_1) \geq v_1^R$, the right hand side of (7) must be zero. Because the first term is positive and the second term is negative, the right hand side of (8) is positive provided that $f_{ig}/f_{i+1g} > f_i/f_{i+1}$. Since this inequality must hold for some type in some group, the above-mentioned reform with $dy_{ig} > 0$ must be self-financing for some y_{ig} . The reform makes individuals of type $i + 1$ better off and no one worse off. Therefore, a Pareto-improvement is obtained. ■

4 Horizontal equity

Violation of horizontal equity is probably the most important critique against the concept of tagging. Akerlof (1978) is very clear about the issue by noting that the tagged unskilled individuals are better off than the untagged unskilled individuals in his utilitarian optimum and that the corresponding inequity is an important disadvantage of tagging. Later contributions confirmed this view: The welfare optima that have been documented in the literature all suggest that there is a general trade-off between a welfare gain from tagging and the discrimination that comes with it. Accordingly, support for the concept seems to depend very much on whether horizontal equity is seen as a major, minor, or irrelevant criterion in tax policy.

This section is not about whether horizontal equity should be taken seriously, but whether there is indeed a conflict between efficiency gains from tagging and horizontal equity. We will first point out that positive welfare effects from tagging do not automatically rule out horizontal equity. Interestingly, this holds under the assumptions of Proposition 3.1, which includes Akerlof's (1978) canonical example. Second, we will show that the special assumption of Proposition 3.1 are also necessary for the result: If the skill distribution has the same support in all groups and if horizontal equity has to be accepted, then then tagging can only have *negative* welfare effects.

Consider the set of allocations that can be implemented under tagging, that is all allocations that are feasible as in (3) and incentive-compatible as in (4). Consider next the subset of these allocations in which each type is indifferent between what she gets in group g and what she would get in some other group h , that is where

$$v_i(c_{ig}, y_{ig}) = v_i(c_{ih}, y_{ih}) \quad \text{for all groups } g, h \text{ and types } i. \quad (9)$$

It is clear that (9) is less restrictive than the no-tagging constraint

$$(c_{ig}, y_{ig}) = (c_{ih}, y_{ih}) \quad \text{for all groups } g, h \text{ and types } i. \quad (10)$$

In the last section, we have asked whether tax policy can become more efficient if constraint (10) is discarded. Now we ask whether Pareto-dominance still holds if (10) is replaced by (9), that is if attention is restricted to those tagging allocations that do not violate the principle of horizontal equity.

At first sight, it seems to be obvious that tagging is in conflict with horizontal equity. The reason is that if the tax schedule for one group differs from the tax schedule of another group, the tax burden must be lower for some income level in one group than in the other. This point is illustrated in Figure 3 which shows two functions $c_1(y) = y - T_1(y)$ and $c_2(y) = y - T_2(y)$ that are identical for incomes below A and above C . In the interval AB , the tax burden is lower under $c_1(y)$, whereas in the interval BC , the schedule $c_2(y)$ is more attractive. Does this imply that horizontal equity in (9) is violated? Not necessarily because the second group may have no earners in the interval AB and the first group no earners in the interval BC . This may happen for two reasons: First, the skill distribution may not have the same support in all groups and this may actually be the reason why the planner finds it optimal to 'discriminate' between the groups. Second, the tax schedule might be constructed in such a way such that the members of the second group self-select themselves out of the unattractive interval AB and similarly for the members of the first group with respect to the interval BC .

In the following, we will show that the first reason is valid whereas the second is not. We will first show that tagging allows for a Pareto-improvement without sacrificing horizontal equity as long as the skill-distribution does not have the

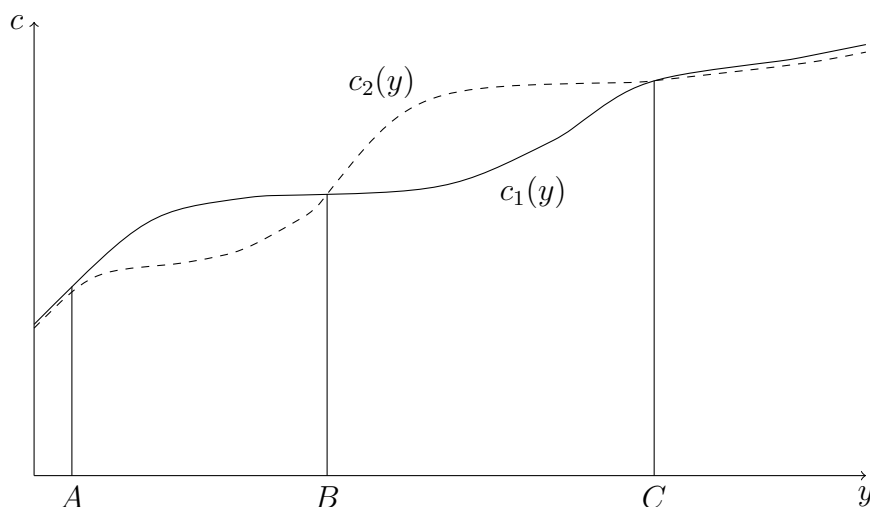


Figure 3

same support in all groups. Second, we will show that the opposite holds if one assumes the same support of the skill distribution in all groups. In that case, tagging might be implemented without violating horizontal equity but then it is always Pareto-inferior to a non-tagging equilibrium.

Proposition 4.1: *Under the assumptions of Proposition 3.1, tagging allows for a Pareto-improvement even without violating the principle of horizontal equity.*

Proof: Consider the shift of the individuals of type i in Figure 2 from A to B . The increase in tax payment T_{ig} can be distributed equally to all individuals of type I in all groups by reducing their tax burden without changing their income y_I . This makes them better off without violating incentive constraints or the constraint (9). ■

Proposition 4.1 shows that tagging does not automatically imply that some individuals are better off or worse off depending on which group they belong. The reason can be seen from Akerlof's (1978) example: If one group has only individuals of type 1, then it is efficient give them a lump-sum transfer and not tax their income at all. Whether this is more or less attractive than their initial situation depends on the size of the transfer relative to the initial (negative) income tax. Hence, from the individuals' perspective, there is a choice between two outcomes, one with low-income and low-consumption and the other with high income and high consumption. Depending on the individuals' preferences, these outcomes can be equally attractive.

Proposition 4.2: *Assume that the skill-distribution of each group has the same support. Then any tagging-allocation that respects the principle of horizontal equity is Pareto-dominated by a non-tagging allocation.*

Proof: Consider an allocation with tagging that does not violate the principle

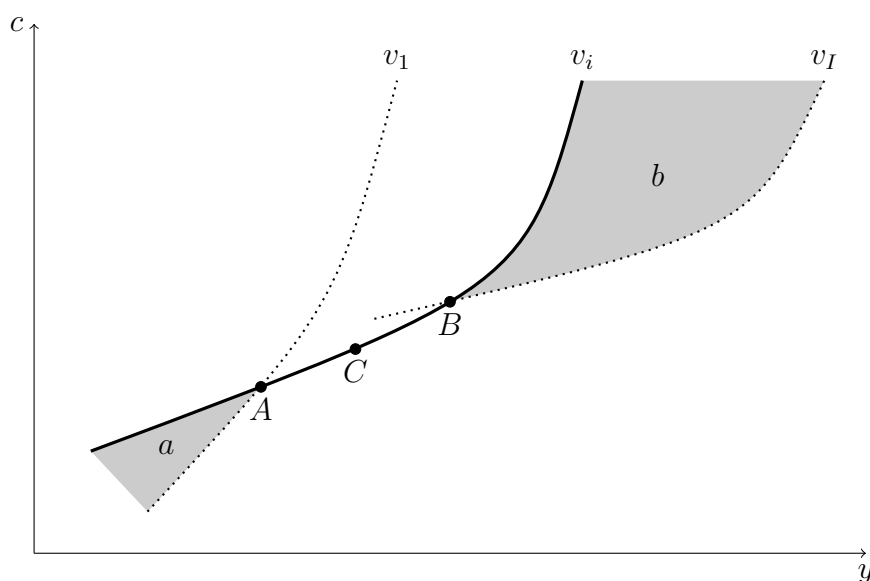


Figure 4

of horizontal equity. Then there must exist some type i and two groups g, h such that $u_i(c_{ig}, y_{ig}) = u_i(c_{ih}, y_{ih})$ and $(c_{ig}, y_{ig}) \neq (c_{ih}, y_{ih})$. These allocations are illustrated by means of points A and B in Figure 4. Since $u_i(c_{ig}, y_{ig}) = u_i(c_{ih}, y_{ih})$ is assumed to hold for all i , individuals of type $j > i$ who belong to group g or h cannot be worse off than in B . Similarly, individuals of type $j < i$ cannot be worse off than in A . Hence all allocations (c_{jg}, y_{jg}) and (c_{jh}, y_{jh}) for types $j > i$ must lie in area b (which includes B) and all allocations for types $j < i$ must lie in area a (which includes A).

The tax payment in A can be larger, equal to, or smaller than in B . If it is larger in A than in B , tax revenue could be increased by making B slightly less attractive such that all individuals in B move to A . Since no self-selection constraint of any type in any of the two groups is affected, the additional tax revenue can be used for implementing a Pareto-superior allocation. A similar argument holds if the tax payment in A is smaller than in B . If taxes are the same in A and B , then a point C between A and B (and on the indifference curve v_i) exists where the tax is higher than in A and B . ■

5 Redistribution of income within and between groups

Another important aspect of tagging is its effect on the distribution of income between groups with many and groups with few low-income households. This is the second main issue in Akerlof's (1978) analysis. He argued that tagging

allows for an indirect channel of redistributing income, namely by decreasing the average tax burden of low-income groups and increasing it for high income groups. While such a policy is detrimental for the low-income individuals in the high-income group, it is nevertheless beneficial for the low-income individuals on average. Indeed, Akerlof showed for his example, that his low-income group (the tagged group) pays lower taxes in the utilitarian optimum with tagging than in the utilitarian optimum without tagging. Hence, tagging might also be used for changing the distribution of income between groups.

Boadway and Pestieau (2006) and Cremer et al. (2010) make a similar point: They show that the high-income group pays higher taxes in the welfare optimum with tagging than the low-income group. These findings confirm Akerlof's argument under the considerably weaker assumption that a low-income group just has a higher proportion of low-income individuals than the population average.

Our analysis differs from the previous work in some or the other way. First, following Akerlof (1978), we concentrate on the change of a group's tax payment due to the introduction of tagging. Hence, we do not ask whether T_g of a particular group is positive or negative, but whether it is higher or lower than in the initial non-tagging situation. Second, we concentrate on Pareto-superior allocations. Hence, we want to know how the realization of potential efficiency gains from tagging affects intergroup redistribution.

We show that tagging results in a different pattern of intergroup redistribution when it is carried out as a Pareto improvement than in the utilitarian approach of Akerlof (1978). The main point of our analysis is that a group can only pay more taxes in a Pareto-superior allocations when the initial tax schedule was inefficient for that group. As argued above, this can only hold for groups that have a high likelihood ratio f_{ig}/f_{jg} for some type i and $j > i$. Hence, groups that have more weight on low-income households than the population average are candidates for paying additional taxes, but groups with a strong dominance of high-income individuals are not.

In a two-class economy with proportions f_1 and f_2 for low- and high-income earners, a group with $f_{2g}/f_{1g} > f_2/f_1$ must be a high-income group and a group with $f_{2g}/f_{1g} < f_2/f_1$ must be a low-income group. The two-class economy was investigated in Akerlof (1978) and Boadway and Pestieau (2006). Cremer et al. (2010) consider the general case with many classes (a continuum) and define high- and low-income groups by means of first-order stochastic dominance. Our findings are based on the stronger concept of likelihood ratio dominance: A group is called a *low-income group* if

$$\frac{f_{ig}}{f_{i+1g}} \geq \frac{f_i}{f_{i+1}}, \quad \forall i \in \{1, \dots, I-1\} \quad (11)$$

holds with at least one inequality strict. Conversely, we refer to a group as *high-income group* if the inequalities in (11) are reversed, also with at least one of them strict.

The budget constraints (1) and (3) can both be written in the form

$$\sum_{i=1}^I f_i T_i = \sum_{g=1}^G f_g \left(\sum_{i=1}^I f_{ig} T_i \right) = \sum_{g=1}^G f_g T_g \geq 0.$$

The subsequent analysis deals with the question how tagging affects the distribution of the average tax rates T_g in each group. Note first that $T(y)$ is increasing under Assumption R. Therefore, high-income groups must pay higher taxes in the no-tagging equilibrium than low-income groups.

Remark: Consider a non-tagging allocation under Assumption R. Then the taxes $T_g > 0$ of high-income groups are used for financing the transfers $T_g < 0$ to low-income groups.

The following result shows that high income groups necessarily become worse off if tagging is accompanied with a higher tax burden T_g . The finding does not extend to low-income groups.

Proposition 5.1 *Assume R. Then Pareto-superior tagging never affects intergroup-redistribution at the expense of high-income groups.*

Proof: Let $\Delta T_{ig} := T_{ig} - T_i$ denote the changes in tax payment that the members of group g encounter when tagging is introduced. The tax payment of the group then increases if

$$A := \sum_{i=1}^I f_{ig} \Delta T_{ig} > 0. \quad (12)$$

In the following, we will show that no Pareto-superior tagging allocation exists in which a high-income group pays more taxes than in the initial equilibrium without tagging.

(Step 1) Note first that Pareto-superiority of the tagging allocation $(c_{ig}, y_{ig})_{i=1}^I$ and Assumption R imply that the inequalities

$$\sum_{i=j}^I f_i \Delta T_{ig} \leq 0, \quad \forall j \in \{1, \dots, I\} \quad (13)$$

must hold. The first of these inequalities ($j = 1$) means that the allocation $(c_{ig}, y_{ig})_{i=1}^I$ would not generate a budget surplus if it were applied to the whole population. Otherwise, one could implement $(c_{ig}, y_{ig})_{i=1}^I$ in the initial situation without tagging and then use the surplus for reducing the tax payments of all individuals. This would make them better off which contradicts Assumption R.

The other inequalities in (13) with index $j > 1$ are based on a similar reasoning: If one of them would not hold, one could implement the following non-tagging allocation $(\hat{c}_i, \hat{y}_i)_{i=1}^I$ as an alternative to the status quo $(\tilde{c}_i, \tilde{y}_i)_{i=1}^I$: The individuals of type $i < j$ obtain the same allocation as before, that is $(\hat{c}_i, \hat{y}_i) = (\tilde{c}_i, \tilde{y}_i)$

and all other types with index $i \geq j$ obtain the allocation (c_{ig}, y_{ig}) , hence $(\hat{c}_i, \hat{y}_i) = (c_{ig}, y_{ig})$. Since $(\tilde{c}_i, \tilde{y}_i)_{i=1}^I$ and $(c_{ig}, y_{ig})_{i=1}^I$ are incentive compatible and since $v_{jg} \geq \tilde{v}_j$, the allocation $(\hat{c}_i, \hat{y}_i)_{i=1}^I$ is incentive compatible as well. If (13) does not hold for j , $(\hat{c}_i, \hat{y}_i)_{i=1}^I$ also generates a budget surplus, which in turn could be used for implementing an allocation which is strictly Pareto-superior to $(\tilde{c}_i, \tilde{y}_i)_{i=1}^I$.

(Step 2) In the following, we will show that (12), (13), and the assumption of a high-income group, that is

$$\frac{f_{ig}}{f_{i+1g}} \leq \frac{f_i}{f_{i+1}}, \quad \forall i \in \{1, \dots, I-1\} \quad (14)$$

cannot hold simultaneously. Note first that (12) and (13) can only be true if there exists at least one positive ΔT_{ig} and at least one negative ΔT_{ig} . Let m denote the largest element of $\{1, \dots, I\}$ with $\Delta T_{ig} > 0$. Then we can express A in the form

$$A = \sum_{i=1}^{m-1} f_{ig} \Delta T_{ig} + \frac{f_{mg}}{f_m} \left(f_m \Delta T_{mg} + \sum_{i=m+1}^I \underbrace{\frac{f_m}{f_{mg}} \frac{f_{ig}}{f_i}}_{\geq 1} \underbrace{(f_i \Delta T_{ig})}_{\leq 0} \right),$$

where the inequalities follow from (14) and the definition of m . They imply

$$A \leq \sum_{i=1}^{m-1} f_{ig} \Delta T_{ig} + \frac{f_{mg}}{f_m} \left(f_m \Delta T_{mg} + \sum_{i=m+1}^I f_i \Delta T_{ig} \right). \quad (15)$$

Next, let l denote the second largest element of $\{1, \dots, I\}$ with $\Delta T_{ig} > 0$. Then (15) can be written in the form

$$A \leq \sum_{i=1}^{l-1} f_{ig} \Delta T_{ig} + \frac{f_{lg}}{f_l} \left(f_l \Delta T_{lg} + \sum_{i=l+1}^{m-1} \underbrace{\frac{f_l}{f_{lg}} \frac{f_{ig}}{f_i}}_{\geq 1} \underbrace{(f_i \Delta T_{ig})}_{\leq 0} + \underbrace{\frac{f_l}{f_{lg}} \frac{f_{mg}}{f_m}}_{\geq 1} \sum_{i=m}^I \underbrace{f_i \Delta T_{ig}}_{\leq 0} \right)$$

where the inequalities follow from (14), the definition of l , and (13). They imply

$$A \leq \sum_{i=1}^{l-1} f_{ig} \Delta T_{ig} + \frac{f_{lg}}{f_l} \left(\sum_{i=l}^I f_i \Delta T_{ig} \right)$$

Next proceed in the same way for all elements of $\{1, \dots, I\}$ with $\Delta T_{ig} > 0$. This implies

$$A \leq \sum_{i=1}^{k-1} \underbrace{f_{ig} \Delta T_{ig}}_{\leq 0} + \frac{f_{kg}}{f_k} \left(\sum_{i=k}^I f_i \Delta T_{ig} \right) \quad (16)$$

where k denotes the smallest element of $\{1, \dots, I\}$ with $\Delta T_{ig} > 0$. Since A is positive in (12), the second sum on the right hand side of (16) must also be positive. This, however, contradicts (13). ■

The intuition for Proposition 5.1 can easily be explained by means of the inequalities (5) and (6) in Section 3 that take the form

$$\Delta T = f_i \Delta T_i + \sum_{j=i+1}^I f_j \Delta T_j < 0 \quad \text{and} \quad \Delta T_g = f_{ig} \Delta T_i + \sum_{j=i+1}^I f_{jg} \Delta T_j \geq 0.$$

Assume that a Pareto-superior allocation exists in which one group pays more taxes than before. This implies that at least one type i in that group must pay more taxes without being worse off. As argued before, this can only happen if the marginal tax rate of that type has been decreased. As a consequence of the decreasing marginal tax rate at income y_i , however, the tax burden of some type with income $y_j > y_i$ must be reduced. Hence, for any tax-increase at income y_i , there will be a tax-reduction at some higher income $y_j > y_i$. The total effect on tax revenue then depends on the proportion of individuals with income y_i and y_j . Since the total effect is negative for the whole population, the group that generates the additional revenue must have more taxpayers at y_i relative to y_j than the population average. Clearly, this cannot hold for a high-income group which has systematically fewer people at lower levels of incomes than at higher levels of income.

The argument of the preceding paragraph is also relevant for the redistribution of income within a group. An efficiency gain can only occur if the marginal tax rate of some individuals is reduced such that they earn more income and pay more taxes without being worse off. The additional tax from these individuals is the only source that one has for financing both the income transfer to other groups and the lower tax for those people who are in the same group but have higher income. This means that there is always a redistribution of income from some income level y_i – which can be at the lower end of the income distribution, in the middle, or close to the top – to some higher level of income $y_j > y_i$.

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