Growth in overlapping generation economies with polluting non-renewable resources

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Abstract

In this paper, I develop the Ramsey program of an economy endowed with a polluting non renewable resource. I show that if pollution is a flow resulting from resource use or extraction, its adverse effects on current factor productivity provoke a diminution of the resource use and a higher long run rate of growth. Also, I show that an increase in resource dependence causes a lower long run growth while abundance in resources increases the level of income. These findings tend to confirm those that emerge from the recent resource curse literature. I then develop successively two cases of competitive equilibrium and I analyze their ability in decentralizing the Ramsey optimal balanced growth path. In the first case, the resource is initially owned by the first generation of agents. Each period, elderly agents sell their resource endowments to young households at an increased price. In such a framework, I characterize the tax that allows to decentralize the Ramsey allocation. In the second case, the management of the resource is given to an independent trust fund. I characterize the optimal policy that should be followed by this fund in order to decentralize the optimal equilibrium.

Keywords: Non-renewable Resources; Growth; Resource Curse; Pollution; Overlapping Generations

JEL Codes: Q32; Q38; Q53

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1 Introduction

Since the 70's, one of the most discussed question among economists is essentially: "Can economies endowed with a finite stock of an essential non-renewable resource may grow indefinitely ?". If the Club of Rome's answer was very pessimistic, this view has been challenged by neoclassical economists. Notably, Stiglitz (1974), Solow (1974), Dasgupta & Heal (1974, 1979) have highlighted the importance of (exogenous) technical progress, increasing returns and substitution possibilities between man-made and natural capital in order to surpass the problem of resource depletion. Those authors use the infinitely-lived agents (ILA) framework. This choice is not without consequences. In ILA models, agents are implicitly assumed to be intergenerationally altruistic. This makes a sustainable management more probabilistic: since current generations take care of future ones, they are more likely to preserve the resource stock or to invest the rent in order to promote a decent standard of living for future generations, following the Hartwick's rule.

This observation has led to the work of Agnani *et al.* (2005). Those authors use the overlapping generation (OLG) framework, in which each generation is supposed to be perfectly selfish, to study the sustainability of growth with natural resources. In an OLG framework, agents are prone to consume a larger share of the natural rent than in the ILA framework: the economy is more likely to contract. Formally, Agnani *et al.* (2005) show that the labor share has to be large enough to allow for a positive balanced growth rate. A high labor share allows savings to compensate for resource depletion and makes the economy sustainable. If the labor share is not high enough the economy will contract.

The present paper, reuses, extends and develops the model of Agnani *et al.* (2005) in order to answer three questions. *i*) What is the impact of a pollution resulting from resource extraction on the economy ? *ii*) Is this model consistent with the resource curse literature ? *iii*) Can we surpass the Pareto efficiency criterion proposed by Agnani *et al.* (2005) ? Notably, for a given social discount rate, can we propose some instruments able to decentralize the Ramsey optimal equilibrium? The contribution of this paper is thus threefold.

Firstly, an environmental externality is introduced. Human activities are sources of pollution which is usually introduced in economic models as a stock which increases proportionally with total output (Jouvet *et al.* , 2010), consumption (John & Pecchenino, 1994), or capital stock (Gradus & Smulders, 1993). In opposition, few papers have introduced pollution as resulting from resource extraction and/or resource use in the production process. Notable exception are Babu *et al.* (1997) and Schou (2000, 2002). Resource extraction is a polluting activity, especially for metals (gold, silver, nickel...), while it is the resource use in production which is polluting for other non-renewable resources (oil, coal...). While the literature concentrates on stock pollutants, some pollutants may be seen as a flows affecting uniquely current generations.[1] This is especially true in the overlapping generation framework where a period represents about 25-30 years. Examples are numerous: the lifetime of black carbon, fine air particulates or sulfur is less than few days; Nitrogen dioxide and tropospheric ozone disappear after few month; Methane has a lifetime of about 12 years. All those pollutants cause major local and global environmental problems: methane and black carbon are major contributors (after CO2) of climate change. Sulfur, black carbon, fine air particulate and tropospheric ozone have strongly negative impact on human health. Methane and tropospheric ozone decrease agricultural productivity. Flow pollutants should be modeled as pollutants affecting the economy when they are emitted. Since future generations are impacted by choices done

[1]Schou (2000, 2002) analyze the impact of flow pollution in endogenous growth models. Those papers aim to show that flow pollution, caused by natural resource use, does not cause any distortion in the economy. Thus, environmental policy is not needed in such a framework.

by present generations, it does not mean that pollution has no long run impacts. The long run balanced growth rate of an economy may thus be affected by pollution, even if only flow pollution is considered. In such a framework, it will be shown that the existence of a flow pollution resulting from resource extraction (or use) and which hurts current factor productivity may helps the economy to be sustainable, reducing the rate of resource exhaustion and preserving resources for the future. It should be noticed that the definition of sustainability which will be used along this paper is very simple: an economy which exhibits a positive balanced growth rate is defined as sustainable, because such a path doesn't restrain the welfare of future generations

Secondly, the impact of natural resources on the economy is analyzed according to the resource curse literature, natural resources may harm growth (or the income level) according to different channels. The most famous one is probably the dutch disease effect: the discovery and exploitation of new resource deposits imply an increase in the exchange rate which hurts the non-resource sectors (secondary and tertiary) which are the engine of growth (See for exemple Corden, 1984; Krugman, 1987; Bruno & Sachs, 1982; Torvik, 2001; Matsen & Torvik, 2005). A resource boom is also ofen followed by a reallocation of factor of productions from the secondary and the tertiary sectors to the primary one. Thus, the growth enhancing sectors are weakened (Sachs & Warner, 1995). Since the primary sector is not human capital intensive, a resource boom diminishes incentives to accumulates human capital, while human capital is one of the main determinant of growth (Gylfason, 2001; Sachs & Warner, 1999). The institutional channel is often supposed to be the most important channel: a resource boom increases rent seeking behavior, enhancing corruption and conflicts, which are known to be growth reducing (see for exemple Jensen & Wantchekon, 2004; Robinson *et al.* , 2006). Finally, resource rich economies are more sensitive to the resource price volatility which increases macroeconomic instability. This instability discourage investment, which in turns affects long run growth possibilities (Van der Ploeg & Poelhekke, 2009). Consistently with the recent resource curse literature, this paper differentiates resource dependence from resource abundance (Brunnschweiler & Bulte, 2008; Stijns, 2006). It appears that resource dependence has an adverse effect on the long run rate of growth while resource abundance has a positive impact on the level of income. Nevertheless, the underlying mechanism is somewhat different and rely on the exhaustibility of the resource stock.

Thirdly, this chapter aims to surpass the Pareto efficiency criterion used by Agnani *et al.* (2005). While in the ILA framework decentralization of the Ramsey allocation does not require public intervention (Schou, 2000), this is not ever true in the OLG framework: the discrete time formulation with finite lifetime exhibits limitations of the market. Indeed, in presence of rational selfish agents, the market is not able to preserve the welfare of unborn generations. We thus propose two alternative instruments which may be used in order to decentralize a Ramsey optimal equilibrium. If the economy follows the usual assumption that the resource initially belongs to the first generation of agents, the Ramsey optimal equilibrium may be decentralized using a tax which is characterized. If we assumes that the resource is shared by all generations of agents, we are able to characterize the optimal policy that should be followed by an independent trust fund.

The rest of the chapter is organized as follow: section 2 presents the Ramsey economy, section 3 develops the decentralized economy, section 4 is devoted to the decentralization of the Ramsey optimal allocation while section 5 concludes.

2 The Ramsey Economy

2.1 The Model

Let's consider a overlapping Ramsey economy which produce a representative good y_t thanks to a Cobb-Douglas combination of labor $l_t = 1$, capital k_t, a non renewable polluting resource x_t, and technological level A_t which increases at a rate a.[2] Each period, a quantity x_t of resources used in production generates a quantity e_t of flow emissions such that $e_t = \phi x_t$. Those emissions hurt negatively production with an elasticity $\theta < v$.[3] At each date t, the production is used for consumption of young c_t, consumption of old d_t and net investment in capital, which depreciates at a rate δ. Each generation is characterized by a log-linear utility function $U(c_t, d_{t+1})$ where the second period consumption d is discounted at a rate ρ. The economy is initially endowed by a quantity m_{-1} of a non-renewable resource. At each date a quantity $x_t = q_t m_{t-1}$ is used in the production with q the extraction rate of the resource.

Thus, a benevolent social planner is assumed to solve the following problem:

$$\max_{\{c_t;d_t;m_t;k_t;e_t\}_{t=0}^{\infty}} = \frac{1}{1+\rho} ln(d_0) + \sum_{t=0}^{\infty} \frac{1}{(1+R)^{t+1}} \left[ln(c_t) + \frac{1}{1+\rho} ln(d_{t+1}) \right]$$

[2]The labor share in the production is β, the capital share is α, and the resource share is v with $\alpha + \beta + v = 1$. That is, constant return to scale are assumed.

[3]It is thus assumed that the positive contribution of the resource to production overcomes its negative contribution in the form of increased pollution.

subject to

$$y_t = A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} \tag{1}$$

$$y_t = c_t + d_t + k_{t+1} - (1-\delta)k_t \tag{2}$$

$$A_{t+1} = (1+a)A_t \tag{3}$$

$$e_t = \phi x_t \tag{4}$$

$$m_t = (1-q_t)m_{t-1} \tag{5}$$

$$x_t = q_t m_{t-1} \tag{6}$$

$$m_{-1} = \sum_{t=0}^{\infty} q_t m_{t-1} \tag{7}$$

$$k_0,\ m_{-1},\ e_{-1},\ A_0 > 0 \,\text{given}, \tag{8}$$

where R is the social discount rate. (1) represents the production function. (2) establishes that the economy consumes or invests exactly its net production in each period. (3) represents the exogenous technological progress. (4) is the emissions implied by the resource use while (5) and (6) represents the dynamics of the resource. (7) is a total exhaustibility condition for the resource while (8) represents initial endowments.

The FOC of the previous problem may be reduced to:

$$\frac{1+R}{1+\rho} = \frac{d_t}{c_t} \tag{9}$$

$$(1+\rho)\frac{d_{t+1}}{c_t} = \alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^{v} e_{t+1}^{-\theta} + 1 - \delta \tag{10}$$

$$\frac{A_{t+1} k_{t+1}^{\alpha} x_{t+1}^{v} e_{t+1}^{-\theta}(v x_{t+1}^{-1} + \phi\theta e_{t+1}^{-1})}{A_t k_t^{\alpha} x_t^{v} e_t^{-\theta}(v x_t^{-1} + \phi\theta e_t^{-1})} = A_{t+1}\alpha k_{t+1}^{\alpha-1} x_{t+1}^{v} e_{t+1}^{-\theta} + 1 - \delta \tag{11}$$

$$\lim_{t\to\infty} \left(\frac{1}{1+R}\right)^t \frac{k_{t+1}}{c_t} = 0 \tag{12}$$

(9) is an intergenerational optimality condition establishing that the marginal rate of substitution between consumption of young and old has to be equal to one. (10) is an intragenerational optimality condition which states that the marginal rate of substitution between consumption while young and consumption while old has to be equal to the marginal product of physical capital net of depreciation. (11) characterizes the optimal inter-temporal resource allocation which indicates that the depletion of the resource stock implies a implicit return equal to the physical capital return. This condition implies that the economy should satisfy the Hotelling rule. (12) is the transversality condition associated with the planner problem.

Combining (1)-(12) we can define the balanced growth path of this Ramsey economy.

Proposition 1. *The optimal balanced growth path is defined by:*

$$\tilde{\mu}_k = \tilde{\mu}_y = \tilde{\mu}_c = \tilde{\mu} \tag{13}$$

$$\tilde{\mu}_x = \tilde{\mu}_e = \tilde{\mu}_m = 1 - \tilde{q} \tag{14}$$

$$\tilde{\mu}_a = 1 + a \tag{15}$$

$$\tilde{\mu} \ = (1+a)^{\frac{1}{1-\alpha}} (1-\tilde{q})^{\frac{v-\theta}{1-\alpha}} \tag{16}$$

Proof. Proof is reported in appendix A.1. □

Using (9), (10), (11), we have $(1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta}\tilde{\mu}_x^{-1} = (1+R)\tilde{\mu}_c$. Since $\tilde{\mu}_c = \tilde{\mu}_k$, it can be established that $\tilde{q} = \frac{R}{1+R}$. Thus, the optimal extraction rate only depends on the social rate of time preference. A higher social preference for the present implies a higher depletion rate of the resource stock, and a lower growth. To put it differently, a society which strongly cares about future generations is more conservative and achieve a larger rate of growth.

To decentralize the optimal balanced growth path, a government should find an instrument able to put the extraction rate at the optimal level $\frac{R}{1+R}$, keeping the rate of growth of other variables at their optimal level. In section 4, we will compare the ability of two scenarios of resource attribution in decentralizing the optimal allocation.

Before that, it may be interesting to do some comparative statics on important parameters. Moreover, since the optimal allocation depends crucially on the social rate of time preference, it seems useful to say some words on the debate on the *good* social discount rate.

2.2 Comparative Statics

Compared to the paper by Agnani *et al.* (2005), the most interesting parameter is θ which represents the detrimental effect of pollution on production.

$$\frac{\partial \tilde{\mu}}{\partial \theta} = -\frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}\log(1-q)}{1-\alpha} > 0 \tag{17}$$

Proposition 2. *The optimal long run rate of growth increases with the detrimental effect of pollution.*

θ diminishes the net resource contribution to production. When time goes by, the resource, which is not reproducible, will be used in ever small amount. Thus, when θ increases, it decreases the detrimental effect caused by the need to keep diminishing resource extraction. That leads to a higher growth rate of the economy. Nevertheless, the level of production may decrease for some instant of time when θ increases due to the contemporaneous adverse effect on production.

Let's consider now a variation of resource dependence. In such a model, resource dependence is characterized by the resource share of output v. In order to keep constant returns to scale, an increase in v should be compensated by an equivalent decrease of either β or α, or both.

Let's consider that an increase in v is compensated by a decrease in β such that returns to scale are kept constant.

$$\frac{\partial \tilde{\mu}}{\partial \beta} = 0 \tag{18}$$

$$\frac{\partial \tilde{\mu}}{\partial v} = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}\log(1-q)}{1-\alpha} < 0 \tag{19}$$

Now, Imagine that an increase dv of v is compensated by an equivalent decrease $d\alpha$ of α such that $dv = d\alpha$. The total impact on the balanced growth rate will be:

$$\begin{aligned}\frac{\partial \tilde{\mu}}{\partial v}dv - \frac{\partial \tilde{\mu}}{\partial \alpha}d\alpha = {} & \frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}\log(1-q)}{1-\alpha}dv \\ & - \frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}(v-\theta)\log(1-q)}{(1-\alpha)^2}d\alpha \\ & - \frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}\log(1+a)}{(1-\alpha)^2}d\alpha < 0\end{aligned} \tag{20}$$

Proposition 3. *The optimal long run rate of growth decreases with resource dependence.*

This result comes from the scarcity of the resource which imposes the economy to save more and more on the resource stock in order to avoid complete exhaustion in finite time. In itself, this give a negative resource contribution to growth. When v increases, the importance of the resource in the production increases, the negative contribution of the resource to growth is higher.

2.3 A discussion on the "good" social discount rate

Section 2.1 highlights the importance of the social discount rate. Indeed, the optimal extraction rate is only determined by the social rate of time preference R.[4] A competitive economy which aims to reach an optimal inter-temporal allocation should calibrate its instruments according to the social discount rate which is politically determined according to economical and ethical considerations.

In the literature, the amplitude and the existence of a positive social discount rate is discussed.

Ramsey (1928), Pigou (1932), Solow (1986) among others argue that while we know that individual agents discount there own future, there is no reason for a benevolent social planner to favor present generations. Nordhaus (1994), uses a annual discount rate of 3% by year. He wrote:

> "It is essential that the discount rate be based on actual behavior and returns on assets rather than on a hypothetical view of how society should behave or an idealized philosophy about the treatment of future generations..."

It makes no doubt that this approach is consistent while one consider the private discount rate. Nevertheless, the social discount rate, which is the discount rate used by a benevolent and infinitely lived dictator that does not exist, is a theoretical concept helping to analyze intergenerational welfare. Its definition in itself thus implies that it should be calibrated according to moral considerations. The last point does not necessarily mean that the policy maker should choose the zero discounting. There are still some arguments in favor of a positive discount rate.

[4] There are two discount rates in the model, a social discount rate R, and a private discount rate ρ. In this section, the focus is on the social one, simply because the private discount rate considered by the social planner is the individual agent's discount rate.

Since we focus on the social discount rate, arguments for its positivity based on evidences that agents discount their welfare in the future should not be taken into account because in the OLG framework the private discount rate is already considered by the social planner. The main argument one may argue for a positive discount rate is the uncertainty about future economic conditions. For example, our framework is not robust to the existence of a backstop technology that may appear in the future and which will modify the production function (resource may become unnecessary in the future). Since this is likely to happen in the long run, a positive social discount rate allow to avoid an overweighting of distant generations' welfare. Moreover, the existence of far distant generations is not guaranteed.

Even if one accepts the arguments for a positive discount rate, there exists a strong debate among economists about the value of that discount rate, with strong implications. A well-known example concerns climate change. Stern *et al.* (2006) uses a discount rate of 1.4% per year and estimates that the current social cost of carbon is around 85$ per ton of CO_2. Nordhaus (2008) uses an alternative discount rate of 5% per year and finds an estimation around 8$. Obviously, policy recommendations that emerge from those estimations are very different. Weitzman (1994), Gollier *et al.* (2008) and Gollier (2010) argue for declining discount rates because they allow to "correct the insufficient representation of future generations, without denying that current generations discount the future" (Gollier *et al.* , 2008).

In the Ramsey model presented in section 2.1, the discrete formulation implies that current generations discount their own future with a value ρ. The social discount rate, represented by R, discount the welfare of future generation. Considering arguments for a positive discount rate, one can accept a positive, but low, discount rate. Notably, this discount rate should be lower than the private discount rate. In following sections, we will consider $0 < R < \rho$.

3 The decentralized economy

In the present section, one analyses an economy where natural resources' property rights are initially held by the first generation of agents. This assumption will be released in section 4.2 which studies the ability of public property rights in decentralizing the Ramsey optimal equilibrium.

3.1 The Model

3.1.1 The Non-Renewable Resource

Following Agnani *et al.* (2005) the economy is initially endowed with a quantity m_{-1} of a necessary exhaustible resource held by the first generation of aged agents. At each date t, elderly agents sell their resource share to the young generation and a quantity x_t of the resource is used in the production process and generates an environmental externality. The resource stock in t is thus denoted by $m_t = m_{t-1} - x_t$ and it belongs to the generation t. The rate of exhaustion of the natural asset is

$$q_t = \frac{x_t}{m_{t-1}}. \tag{21}$$

The dynamics of the per worker resource stock is thus [5]

$$m_t = (1 - q_t)m_{t-1}. \tag{22}$$

[5]It may also be interpreted as a resource market clearing as in Agnani *et al.* (2005).

It leads, associated with the non renewability of the resource, to the exhaustibility condition

$$1 \geq \sum_{t=0}^{\infty} q_t \prod_{j=1}^{t} (1 - q_{j-1}) \tag{23}$$

3.1.2 Consumers

As in Agnani *et al.* (2005), agents are alive for two periods and maximize the following utility function

$$u_t(c_t; d_{t+1}) = ln(c_t) + \frac{1}{1+\rho} ln(d_{t+1}) \tag{24}$$

where c represents the consumption while young, d the consumption while old and ρ the individual rate of time preference.

In the first period of life, the representative agent works to earn a wage w_t, which may be consumed, saved as physical capital s_t, or used to buy rights on the resource stock m_t at a price p_t in terms of the representative good. His first period budget constraint is

$$w_t = c_t + s_t + p_t m_t \tag{25}$$

While old, he gets his savings increased at the interest rate, and he sells his resource rights at a price p_{t+1}. His second period budget constraint is

$$d_{t+1} = (1 + r_{t+1}) s_t + p_{t+1} m_t. \tag{26}$$

Combining (25) and (26), we obtain the following inter-temporal budget constraint (IBC hereafter):

$$w_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - \frac{p_{t+1} m_t}{1 + r_{t+1}} + p_t m_t \tag{27}$$

The maximization of utility with respect to m_t, c_t, d_{t+1} subject to the IBC leads to the following first order condition:

$$\frac{d_{t+1}}{c_t} = \frac{1 + r_{t+1}}{1 + \rho} \tag{28}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1} \tag{29}$$

(28) is the standard Euler equation while (29) is an arbitrage condition between the two assets in this economy, capital and resource.

3.1.3 Firms

Firms produce the representative good Y_t using a Cobb-Douglas technology. They use capital K_t, labor N_t, and resources X_t, with constant returns to scale for a given level of technology A_t which grows at a rate a such that:

$$A_{t+1} = (1 + a)A_t \tag{30}$$

The use of the resource in the production process generates a flow of pollution e_t such that:

$$e_t = \phi x_t \tag{31}$$

Pollution resulting from the use of the resource in the production process generates a productivity loss. θ captures the detrimental impact of pollution on the level of production. Nielsen *et al.* (1995) and Schou (2000) model the detrimental effect from pollution on productivity in the same way. The production function is thus:

$$y_t = A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} \tag{32}$$

with $\alpha+\beta+v=1$. Thus, the model assumes constant return to scale from the firm point of view. Indeed, the environmental externality is not taken into account by individual firms but they consider the aggregated level of pollution as given while they decide their production plan.

Capital is remunerated at the interest rate r_t and depreciates at a rate $0<\delta<1$. Firms pay a wage w_t to their workers and buy the natural input at its price p_t. We focus on the case $\theta<v$. That is, for an identical amount of resource and emissions, we assume that the positive impact of resources on income outweighs its negative one. A similar assumption may be found in Schou (2000), where pollution is also seen as a flow. The standard profits maximization leads to the following first order condition:

$$r_t = \alpha A_t k_t^{\alpha-1} x_t^{v} e_t^{-\theta} - \delta \tag{33}$$

$$w_t = \beta A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} \tag{34}$$

$$p_t = v A_t k_t^{\alpha} x_t^{v-1} e_t^{-\theta} \tag{35}$$

Each factor is thus paid at its marginal productivity.[6]

3.2 The Balanced Growth Path

The economy produces a representative good which may be consumed or saved as physical capital. Following Diamond (1965), the good market clearing condition is

$$s_t = k_{t+1} \tag{36}$$

Definition 1. *An inter-temporal competitive equilibrium is a solution of the system*

[6] This paper concentrates on interior solutions.

formed by equations (21-22;25-26;28-36). It is thus characterized by the following properties:

1. *Agents maximize their utility according to their inter-temporal budget constraint.*

2. *Firms maximize their profits.*

3. *The resource stock evolves according its law.*

4. *Markets clear.*

This paper focuses on balanced growth path because they constitute the only case where long-run positive growth is possible, as noted in Agnani *et al.* (2005).

Definition 2. *An inter-temporal equilibrium where all variables grow at a constant rate is defined as a balanced growth path (BGP hereafter).*

Let μ_h be the BGP notation of the ratio h_{t+1}/h_t. According to definition 2, μ_m should be constant. Thus (22) implies a constant rate of extraction along the BGP, i.e. $q_t = q_{t+1} = q$.

Proposition 4. *This overlapping economy is characterized by the following growth rates:*

$$\mu_k = \mu_y = \mu_s = \mu_w = \mu_c = \mu$$

$$\mu_x = \mu_e = \mu_m = 1 - q$$

$$\mu_p = \mu/\mu_x$$

$$\mu_a = 1 + a$$

$$\mu_r = 1$$

$$\mu \ = (1 + a)^{\frac{1}{1-\alpha}}(1 - q)^{\frac{v-\theta}{1-\alpha}}$$

Proof. Proof is reported in appendix A.2. □

From Proposition 4, it may be established that a necessary condition for a long run positive growth is $q < 1-(1+a)^{\frac{1}{\theta-v}}$. This threshold will be referred as a positive growth threshold (PGT hereafter). To analyze how the balanced growth path is affected by a change in θ, it is necessary to characterize the constant extraction rate. The market clearing condition (36) may be written, using (25), (26), (28), (34), (35), as

$$k_{t+1} = \left[\frac{\beta}{2+\rho} - \frac{(1-q)v}{q}\right] A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} \tag{37}$$

Evaluating this equation at the BGP, we can obtain:

$$\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}}\alpha(2+\rho)q}{\beta q - v(1-q)(2+\rho)} = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}}}{1-q} - (1-\delta) \tag{38}$$

It may now be established that q^* is solution to the preceding non linear equation. We denote LHS and RHS the left and right hand side of (38). $RHS(q)$ is defined on $[0;1[$ with $RHS(0) = (1+a)^{\frac{1}{1-\alpha}} - (1-\delta)$. Since $\lim_{q\to 1} RHS(q) = +\infty$, $RHS(q)$ admits a vertical asymptote in $q = 1$. Moreover, $\frac{\partial RHS(q)}{\partial q} > 0$ and $\frac{\partial RHS(q)^2}{\partial^2 q} > 0$ imply an increasing and convex function. $LHS(q)$ is defined on $[0;\hat{q}[U]\hat{q};1]$ with $\hat{q} = \frac{v(2+\rho)}{\beta+v(2+\rho)}$. $LHS(q) < 0 \,\forall\, q < \hat{q}$ which do not allow the possibility of an equilibrium extraction rate given that $RHS(q) > 0 \,\forall\, q \in [0;1[$. Since $lim_{q\to\hat{q}(+)} = +\infty$ and $lim_{q\to 1} = 0$ it exists a unique q^* such that (38) is satisfied. This situation is represented in Figure 1.

The threshold $\hat{q}$ is called the growth area threshold (GAT hereafter). Indeed, an increase in the GAT reduces the set of q such that the economy grows at the BGP, because the economy contracts if $PGT < \hat{q}$.

From Proposition 1 it appears that an higher extraction rate is associated with a lower

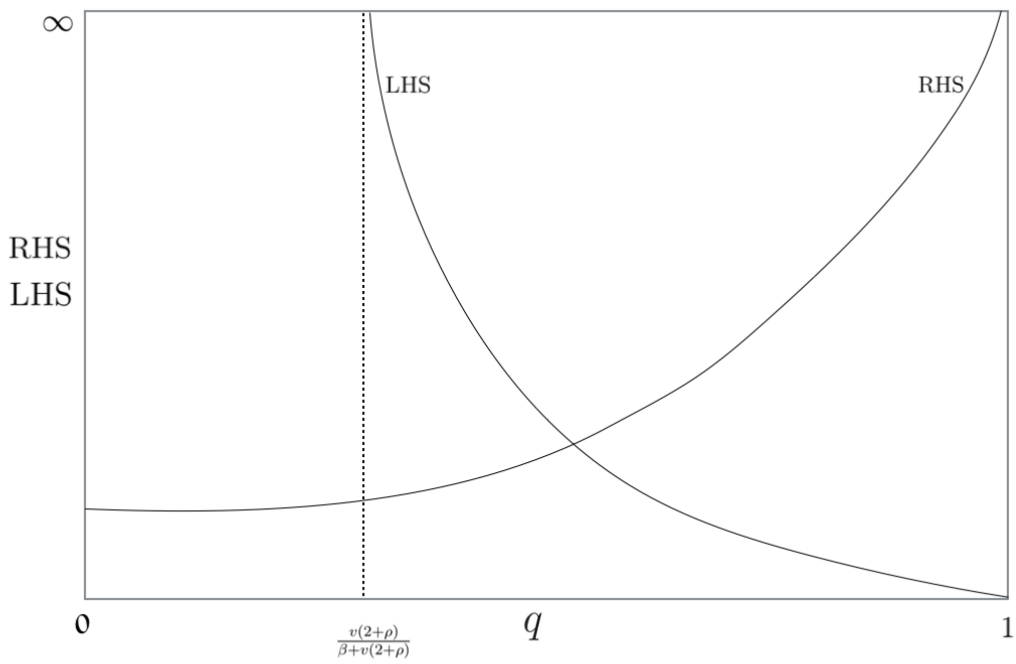

Figure 1: Characterization of the competitive equilibrium extraction rate

growth while looking at (21) and (32), an increase in q implies an higher income. It is thus necessary to distinguish between short and long run impacts of a higher extraction rate. In the short run, an increase in q, *ceteris paribus*, implies an increase of one input in the production process. Current production thus increases. Nevertheless, it will be harder to maintain this level of production, because less natural resources are available to produce. In the long run, the higher is the extraction, the more the economy needs capital to compensates for resource depletion. The pressure on natural resources thus limits future growth possibilities.

3.3 The impact of flow pollution on sustainability

In this work, we define sustainability as the ability of the economy to sustain a non declining balanced consumption path. The economy will contract if $PGT < GAT$ *i.e.* if $\beta < \frac{(1+a)^{\frac{1}{\theta-v}}(2+\rho)v}{1-(1+a)^{\frac{1}{\theta-v}}}$. This condition is less likely to be satisfied when θ increases. Figure 2 represents the results of a numerical simulation in order to see how the sustainability of the economy is affected when θ increases, keeping the constant return to scale assumption. The simulation is performed for the following annual values: $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$.[7] The space over the curve represents the set of capital, resource and labor shares such that the economy will contract.

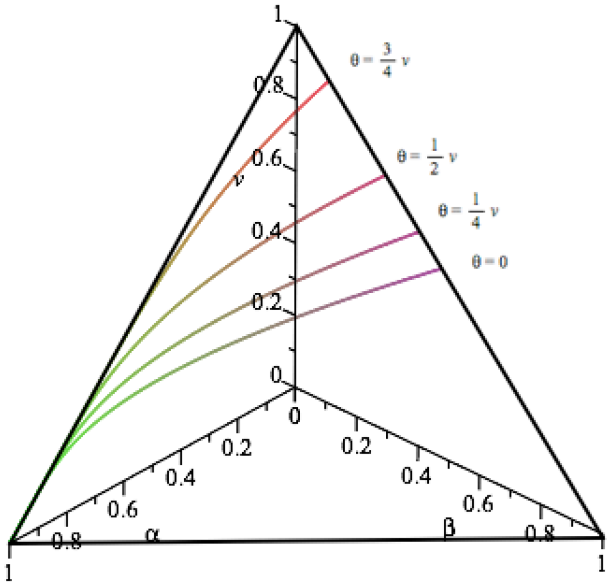

Figure 2: Effect of an increase of θ on sustainability

Proposition 5. *When pollution hurts severely productivity, the contraction area is reduced. The detrimental effect of pollution on production thus enhance sustainability.*

Proof.

$$\frac{\partial\left(\frac{(1+a)^{\frac{1}{\theta-v}}(2+\rho)v}{1-(1+a)^{\frac{1}{\theta-v}}}\right)}{\partial\theta} < 0$$

□

This effect may seems puzzling, it is nevertheless quite intuitive. θ diminishes the net resource's contribution to GDP. When θ increases, the BGP extraction rate is thus lower. It allows the economy to save some resources for the future. In the long run, a lower level of savings is needed to compensate for resource depletion. Thus, a lower labor share is needed to achieve a positive long run growth.

Using a numerical approach, appendix A.3 shows that an increase in θ increases the rate of growth for reasonable parameter values.

[7]The choice of parameter values comes from Agnani *et al.* (2005).

If sustainability is enhanced while the detrimental impact of pollution on production increases, this is due to the specificity of pollution which is considered here. Firstly, we consider the category of flow pollutants that hurts the current period productivity, and disappear in the next period. Secondly, we do not use the standard assumption that pollution is an externality resulting from production. Here, pollution is an externality coming from the extraction and/or the use of a non-renewable resource in the production. Along the balanced growth path, emissions thus evolve as the resource do, and disappear asymptotically. As previously exposed, those assumptions are realistic for some pollutants (tropospheric ozone, methane, black carbon...).

3.4 The impact of resource dependence on growth

The resource curse literature seems to show that a higher resource abundance is associated with a lower growth rate. Since the seminal work of Sachs & Warner (1995), there has been an huge literature on the resource curse, using different abundance indicators. Gylfason & Zoega (2006) among others, use the share of natural wealth in total wealth. Stijns (2006) argues that this is more a indicator of dependency. In that Cobb-Douglas economy, the resource dependence is captured by v. It constitutes the income share of natural resources. Indeed, $w_t/y_t = \beta$, $(r_t - \delta)k_t/y_t = \alpha$ and $p_t x_t/y_t = v$. The model developed here shows some ability in reproducing a resource curse. Indeed, we can calibrate the model in order to show that an increase in v is associated with a lower balanced growth rate. The derivative of growth with respect to the resource share is:

$$\frac{\partial \mu}{\partial v} = \mu \left[\frac{ln(1-q)}{1-\alpha} - \frac{\partial q}{\partial v} \frac{v-\theta}{(1-q)(1-\alpha)} \right] \tag{39}$$

Restricting our analysis to positive growth area, we can represent in the $(v, q, \frac{\partial \mu}{\partial v})$ space this derivative with respect to all possible v and q values.[8]

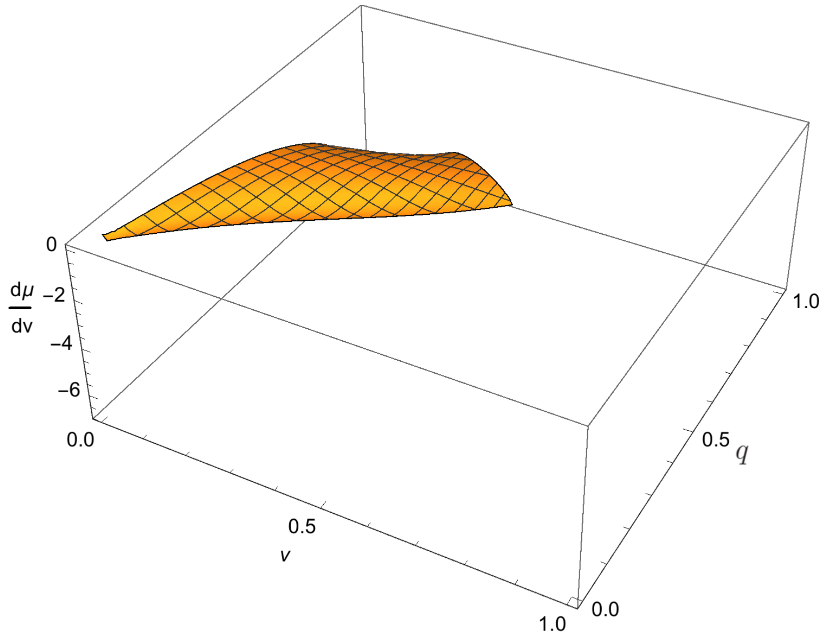

Figure 3: $\frac{\partial \mu}{\partial v}$ for $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $\theta = 0.01$, $\alpha = 0.3$

Proposition 6. *An increase in resource dependence is associated with a lower growth rate.*

This results is quite intuitive. Indeed, because of the exhaustibility of the resource stock, a given growth rate implies to save more and more on the resource. It is not surprising that when the importance of the resource increases growth is negatively affected because it necessitates more saving to be sustained. However, the constant returns to scale assumption has been lost.To keep the constant returns to scale assumption, an increase in v necessarily implies a decrease in either β or α by the same order (or both obviously). The effect of a change in either β or α are reported in appendix A.4. Results show that an increase in v compensated by a decrease in α or a decrease in β is associated

[8]A sensitivity analysis has been performed. This result is robust to variation of parameter in a reasonable range.

with a lower balanced growth for reasonable parameter values. Keeping constant returns to scale does not change the result.

The negative impact of resource dependence on growth is known as the resource curse. Here, an increase in resource abundance (i.e. higher initial endowment) affect positively the level of income reached at each date. Thus, in this model, dependence is a curse while abundance is a blessing. Nevertheless, the presented mechanism is new and doesn't echo the standard resource curse literature. The presented effect, which rely on the exhaustibility of the resource stock, appears in the very long run and may be avoided thanks to international trade. This lead to temper the conclusion that this model reproduces a resource curse. Nevertheless, it confirms results obtained by the resource curse literature: abundance is a blessing why dependence is a curse.

4 Decentralizing the Ramsey optimal balanced growth path

From this model it may be possible to do some policy recommendations. Imagine that a government wants to decentralize the optimal balanced growth path according to some decisions on the size of the optimal discount factor (see discussion in section 2.3). To decentralize the Ramsey path, the policy maker should use an instrument that put the market extraction rate q at its optimal level $\tilde{q}$, without modifying other determinants of growth (i.e. letting equations in proposition 4 unchanged). In this subsection, we propose and discuss two alternative strategies that may be implemented in order to decentralize the optimal BGP.

4.1 Tax on the private resource

A way that may be used to decentralize the optimal Ramsey equilibrium is to tax the resource used in the production, and to redistribute this income to young generations as a transfer. Since the resource is taxed, equation (35) becomes:

$$p_t + \tau_t = vA_t k_t^{\alpha} x_t^{v-1} e_t^{-\theta} \tag{40}$$

where τ_t is the tax rate set by the government in t. The tax is redistributed to the young generation as a transfer g_t such that the government budget is balanced[9]:

$$\tau_t x_t = g_t \tag{41}$$

The budgetary constraint of the young agent (25) is thus modified as follow:

$$g_t + w_t = c_t + s_t + p_t m_t \tag{42}$$

Proposition 7. *If the tax and the resource price increase at the same rate, the balanced growth path of the economy is not distorted by the tax.*

Proof. Proof is reported in Appendix A.5. □

Proceeding as in section 3.2 we can write the market clearing condition as

$$k_{t+1} = A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} \left[\frac{\beta}{2+\rho} - \frac{v(1-q)}{q} \right] + \frac{\tau_t x_t}{2+\rho} \tag{43}$$

[9] Considering that the tax is invested and then redistributed to old households leads to the same results.

Dividing both side by k_t, it leads to

$$\mu_k - \frac{\tau_t x_t}{(2+\rho)k_t} = A_t k_t^{\alpha-1} x_t^v e_t^{-\theta} \left[\frac{\beta}{2+\rho} - \frac{v(1-q)}{q}\right] \tag{44}$$

Since, $\mu_x \mu_\tau = \mu_k$, $\frac{\tau_t x_t}{k_t}$ is constant on the BGP. Let ξ denote this constant. Evaluating (44) on the BGP leads to

$$\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}}\alpha(2+\rho)q - \xi\alpha q}{\beta q - v(1-q)(2+\rho)} = (1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}-1} - (1-\delta) \tag{45}$$

Comparing (45) with its counterpart without tax (38) it appears that RHS has not been impacted by the tax. Nevertheless, the LHS of equation (38) has been modified by the existence of the resource tax. LHS of equation (45) is defined for $q \in [0, \hat{q}[U]\hat{q}, 1]$ with $\hat{q} = \frac{v(2+\rho)}{\beta+v(2+\rho)}$. Since $\tilde{q} = \frac{R}{1+R}$ the Ramsey equilibrium is not decentralizable for $R = \frac{v(2+\rho)}{\beta}$.

Below the threshold $\hat{q}$, the derivative of LHS(q)>0 for a sufficient level of resource taxation $\hat{\xi}$, $\lim_{q\to 0}(q) = 0$ and $\lim_{q\to\hat{q}}(q) = +\infty$, where

$$\hat{\xi} = v^{-1}(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}}[\beta q(v-\theta) - v(2+\rho)(1-q)(v-\theta) + v(2+\rho)] > 0 \tag{46}$$

Thus, if $0 < \tilde{q} < \hat{q}$ there exists a level $\xi > \hat{\xi}$ such that the Ramsey equilibrium is decentralizable.

Above the threshold $\hat{q}$, the derivative of LHS(q)<0 for a sufficiently low level of resource taxation $\hat{\xi}$, $\lim_{q\to\hat{q}}(q) = +\infty$ and $\lim_{q\to 1}(q) = -\frac{\xi\alpha}{\beta}$. Thus, if $\hat{q} < \tilde{q} < 1$, there exists a level of $\xi < \hat{\xi}$ which decentralizes the Ramsey optimal equilibrium.

From equation (45) the optimal taxation may be obtained:

$$\tilde{\xi} = -\frac{[(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}-1} - (1-\delta)][\beta\tilde{q} - v(2+\rho)(1-\tilde{q})] - (1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}\alpha(2+\rho)\tilde{q}}{\alpha\tilde{q}} \tag{47}$$

where it should be recalled that $\tilde{q} = \frac{R}{1+R}$. Calibrating the model with annual rates of $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $v = 0.05$, $\alpha = 0.3$, $\beta = 0.65$, and $\theta = 0.01$, the level of taxation ξ may be represented for all possible R values. For those parameter values,

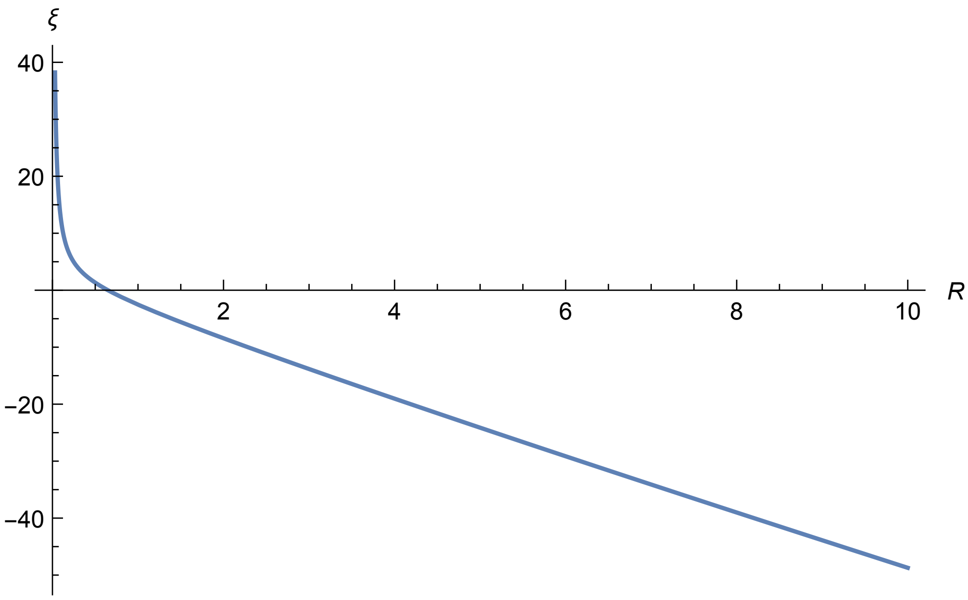

Figure 4: The optimal taxation for $R \in [0; 10]$

the numerical simulation shows that the level of the tax decreases with the social rate of time preference. With the above calibrated values, a subvention for resource extraction is needed for an annual discount rate of $R > 0.0202$. This value is high relatively to the private discount rate used for calibration ($\rho = 0.016$).[10]

[10]See subsection 2.3.

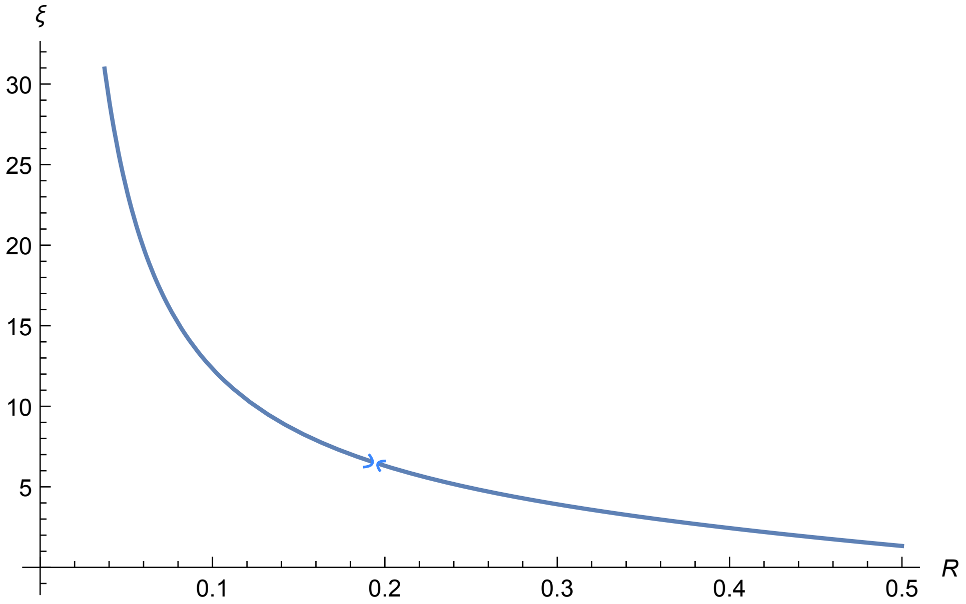

Figure 5: A zoom on $R \in [0; 0.5]$

4.2 Public property rights on the resource stock

Another way to decentralize the Ramsey optimal BGP is to modify the property rights on the resource stock. Considering that the resource is a common shared by all generations, one may decide that the resource should belong to a trust fund which cares about all generations. This fund invests the rent from resource extraction and gives a transfer to the old generation as a retirement pension. We assume that its budget is balanced.

In the first period of life, the household works and earn a wage which is used for savings and consumption. The first period budget constraint becomes

$$w_t = c_t + s_t \tag{48}$$

The second budget constraint becomes:

$$d_{t+1} = (1 + r_{t+1})s_t + (1 + r_{t+1})g_t \tag{49}$$

Maximization of utility with subject to the IBC leads to the Euler equation (28), which established that the marginal rate of substitution between the two consumptions is equal to their relative price.

Competitive productive firms are profit maximizing and their behavior is not modified with respect to previous sections.

The trust fund mission is to manage optimally the resource stock. Since it is the sole which can offer the non-renewable resource to firms, he knows the demand function that emerge from the firms. Solving the Ramsey model presented in section 2.1, the fund objective is thus to extract a quantity $\tilde{q}$ in each period. Thus it will sold to firms a quantity $\tilde{x}_t = \tilde{q}m_{t-1}$ at each date. Since it is a monopoly, the funds may choose a price allowing it to sold exactly the wanted quantity of resource. This price is

$$\tilde{p}_t = vA_t k_t^{\alpha} \tilde{x}_t^{v-1} e_t^{-\theta} \tag{50}$$

Since the budget of the fund should be balanced, we have

$$\tilde{p}_t \tilde{x}_t = \tilde{g}_t \tag{51}$$

The new market clearing condition is

$$\tilde{g}_t + s_t = k_{t+1} \tag{52}$$

Proposition 8. *The trust fund is able to decentralize the Ramsey optimal BGP.*

Proof. Proof is reported in appendix A.6 □

4.3 Discussion

I previously provide two solutions in order to decentralize the optimal balanced growth path. In the first case, one can see that a good tax implementation may allow to decentralize the optimal BGP. Nevertheless, implementation of such a tax may face a technical difficulty: if the social rate of time preference is $R = \frac{v(2+\rho)}{\beta}$, a market economy could never reach the optimal extraction rate $\tilde{q} = \frac{v(2+\rho)}{\beta+v(2+\rho)}$. Moreover, we propose to implement a tax by unit extracted. Of course, this proposition is not independent on the modeling framework and should be taken carefully. Smith (2013) remarks that there do not exist two countries which own similar resource tax regimes and that economists' conclusions on those regimes strongly depend on the modeling framework.

The other possibility that we propose is to declare public property rights on the resource stock and then to manage the latter optimally. This policy is likely to be not implementable if property rights on the resource stock are currently privately defined because it necessitates to expropriate one generation. Thus, depending on the current law, countries may choose to implement one or another policy. For example in United States, the owner of a land also own its subsoil assets. In France, the mining code declares that the state own property rights on the subsoil assets. Thus the trust fund policy is easier to implement in France than in United States where the tax policy seems to be a better solution.

The model established that investing the tax revenues and give it to old generations or give it directly to young agents is equivalent. Nevertheless, this is less likely to be true in the real world where young agents are more likely to consume a share of this rent. A safer policy would thus consist in investing the resource rent and give it to agents while retired. This policy is notably followed by The Government Pension Fund Global of Norway. Oil fiscal revenues as well as dividend from the national oil company are placed in the fund which invest them in order to save welfare for future generations.[11] The present work thus provides some theoretical insight that this policy is both ethical and fruitful since it preserves the welfare of unborn generations and promotes growth.

5 Conclusion and general discussion

In this chapter, the model of Agnani *et al.* (2005) is reused and extended. *i*) I show that a flow pollution resulting from resource extraction may help the economy to reach a non-declining consumption path, because it decreases the rate of resource extraction. Thus, pollution diminishes pressure on natural resources. This analytical result is obtained due to the quite restrictive but convenient assumption that pollutants have short lifetime. With stock pollutant coming from the resource use, pollution would follow an inverse U-shaped with respect to time, increasing in a first step due to resource extraction and then decreasing when the extraction rate is very low due to natural absorption. Thus, the answer of the economy to an increase in the elasticity of production with respect to pollution will be uncertain. Intuition suggests that our results stay true when natural absorption is sufficiently strong to prevent an increase of the stock of pollution. *ii*) Resource dependence reduces growth possibilities, because resource scarcity imposes the economy to save more and more on the resource stock. When resource dependence is high, saving on the resource is harder. Moreover, resource dependence increases the rate of resource depletion. One may be tempted to link this result with the resource curse literature. While there is no such thing as a resource curse in the paper, we may

[11]The funds declare on its homepage that "The Government Pension Fund Global is saving for future generations in Norway. One day the oil will run out, but the return on the fund will continue to benefit the Norwegian population." `https://www.nbim.no/en/the-fund/`

however argue that the obtained results confirm those of the resource curse literature that abundance is a blessing and dependence is a bad. *iii*) I analyze the ability of two resource allocation schemes in decentralizing the optimal equilibrium. If the resource is held by the first generation of agents, a resource tax redistributed to the young as a transfer generally allow to decentralize the Ramsey optimal equilibrium. If property rights on the resource stock are shared by all generations, an independent trust fund may be able to decentralize the optimal equilibrium. To summarize, the policy that should be implemented depends on the preexisting distribution of property rights in the economy.

Further research may include a model with stock pollution and a ceiling of emission in order to give more accurate policy recommendation. In first stages, pollution will increase due to high resource extraction. While extraction diminishes, the stock of pollution will begin to decrease in some point in time due to natural absorption. The aim of the work will thus be to keep pollution below the threshold. This project may be seen as an extension of Stern *et al.* (2006) where climate change is a byproduct of resource extraction and use only, while in the latter climate change is driven by population, production and consumption. If the present framework is conserved, the work should rely on simulation methods.

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A.1 Proof of Proposition 1

- (5) at the BGP leads to $\tilde{\mu}_m = 1 - \tilde{q}$.

- (6) at the BGP leads to $\tilde{\mu}_m = \tilde{\mu}_x$.

- (4) at the BGP leads to $\tilde{\mu}_e = \tilde{\mu}_x$.

- (9) at the BGP leads to $\tilde{\mu}_c = \tilde{\mu}_d$.

- (1) at the BGP implies that $\tilde{\mu}_y = (1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta}$.

- (10) at the BGP gives $(1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta} - (1-\delta) = A_{t+1}\alpha k_{t+1}^{\alpha-1}x_{t+1}^{v}e_{t+1}^{-\theta}$ At the BGP, it gives $1 = (1+a)\tilde{\mu}_k^{\alpha-1}\tilde{\mu}_x^{v}\tilde{\mu}_e^{-\theta}$ which implies that $\mu_k = \mu_y = \mu = (1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}$.

- $\mu_k = \mu_y$ in (2) at the BGP implies that $\mu_c = \mu$.

A.2 Proof of Proposition 4

The proof uses continuously definitions 1 and 2.

- (22) at the BGP leads to $\mu_m = 1 - q$

- (21) at the BGP leads to $\mu_m = \mu_x$

- (31) at the BGP leads to $\mu_e = \mu_x$.

- From (36) at the BGP $\mu_s = \mu_k$.

- (35) at the BGP leads to $\mu_p = (1+a)\mu_k^{\alpha}\mu_x^{v-1}\mu_e^{-\theta}$

- (34) at the BGP leads to $\mu_w = (1+a)\mu_k^{\alpha}\mu_x^{v}\mu_e^{-\theta}$

- (32) at the BGP leads to $\mu_y = (1+a)\mu_k^{\alpha}\mu_x^{v}\mu_e^{-\theta}$

- (30) at the BGP leads to $\mu_a = 1 + a$

- (29) may be written at the BGP as $\mu_p = 1 + r_{t+1} \Rightarrow \mu_r = 1$

- (28) at the BGP leads to $\mu_c = \mu_d$

- (33) at the BGP leads to $(1+a)\mu_k^{\alpha-1}\mu_x^v\mu_e^{-\theta} = 1 \Rightarrow \mu_k = (1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}}$

- (28) in the CBI implies

$$w_t = \frac{c_t(2+\rho)}{1+\rho} \Rightarrow \beta A_t k_t^\alpha x_t^v e_t^{-\theta} = \frac{c_t(2+\rho)}{1+\rho}$$

At the BGP

$$(1+a)\mu_k^\alpha \mu_x^v \mu_e^{-\theta} = \mu_c = \mu_w$$

A.3 Effect of an increase of θ on growth

The effect of an increase in θ is:

$$\frac{\partial \mu}{\partial \theta} = \mu \left[-\frac{ln(1-q)}{1-\alpha} - \frac{\partial q}{\partial \theta} \frac{v-\theta}{(1-q)(1-\alpha)} \right] \tag{53}$$

Restricting our analysis to the positive growth area, we can represent this derivative with respect to all possible values of θ and q in the space $(\theta, q, \frac{\partial \mu}{\partial \theta})$

When pollution hurts severely productivity, growth is higher. This effect may seems puzzling, it is nevertheless quite intuitive. θ diminishes the net resource contribution to GDP. The BGP extraction rate is thus lower and that implies that less savings are needed to reach a given growth rate.

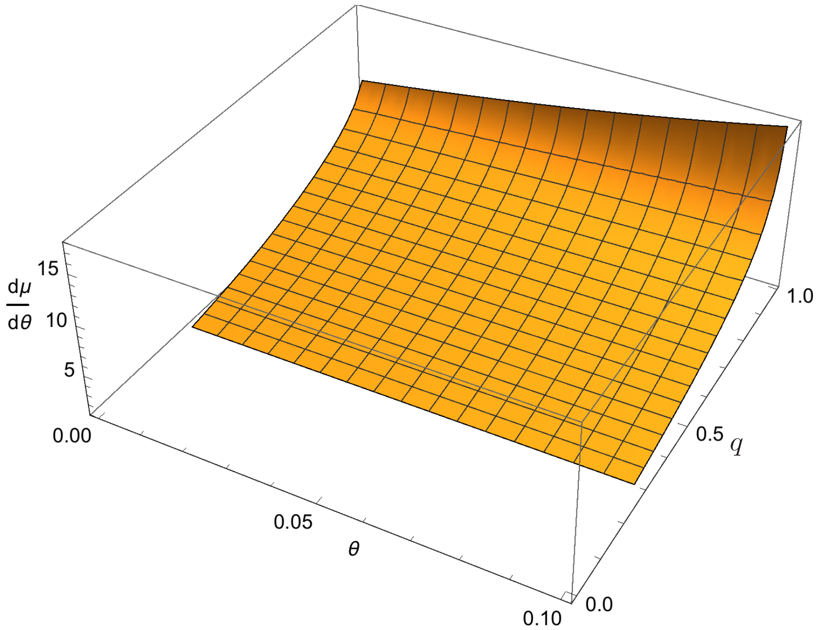

Figure 6: $\frac{\partial \mu}{\partial \theta}$ for $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $v = 0.1$, $\alpha = 0.3$

A.4 Keeping the constant return to scale hypothesis

The effect of a change in the labor share on growth is determined by:

$$\frac{\partial \mu}{\partial \beta} = \frac{-(1+a)^{\frac{1}{1-\alpha}}(v-\theta)(1-q)^{\frac{v-\theta}{1-\alpha}-1}}{1-\alpha}\frac{\partial q}{\partial \beta} \tag{54}$$

Since the sign of this expression appears ambiguous, we restrict the analysis on the positive growth area. We then calibrate the model and we show that all possible combinations (β,$q(\beta)$) on the space $GAT < q < PGT$ lead to a positive sign for $\frac{\partial \mu}{\partial \beta}$. The simulation is performed for a wide variety of parameter values and always lead to a positive sign for $\frac{\partial \mu}{\partial \beta}$. Figure 7, represents an example for given parameter values. Sensitivity analysis has been performed and the result is robust to variation in parameters values. The growth rate of the economy increases with the labor share. An increase in v compensated by a decrease in β leads to a lower growth rate. The effect of a change in the man-made capital share on growth is determined by:

$$\frac{\partial \mu}{\partial \alpha} = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{v-\theta}{1-\alpha}}}{(1-\alpha)^2}\left[ln(1+a)+(v-\theta)ln(1-q)-\frac{(v-\theta)(1-\alpha)\frac{\partial q}{\partial \alpha}}{1-q}\right] \quad (55)$$

Since the sign of this expression appears ambiguous, we restrict the analysis on the positive growth area. We then calibrate the model and we show that all possible combinations $(\alpha, q(\alpha))$ on the space $GAT < q < PGT$ lead to a positive sign for $\frac{\partial \mu}{\partial \alpha}$. The simulation is performed for a wide variety of parameter values and always lead to a positive sign for $\frac{\partial \mu}{\partial \alpha}$. Figures 7, represent an example for given parameter values. Sensitivity analysis has been performed and the result is robust to variation in parameters values. We have

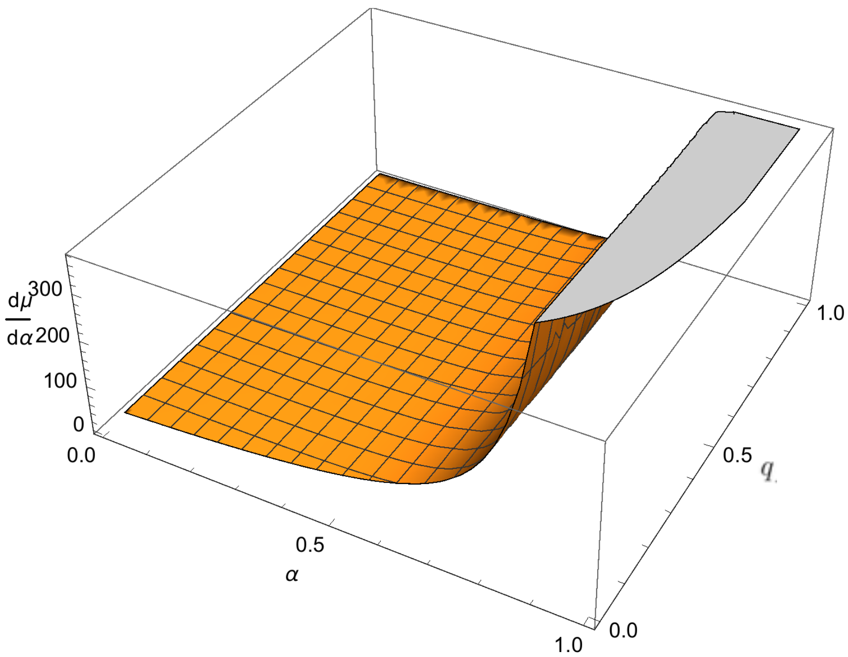

Figure 7: $\frac{\partial \mu}{\partial \alpha}$ for $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $\theta = 0.01$, $v = 0.02$

characterized possible derivative of μ with respect to α, for all feasible q. Nevertheless q is endogenously determined by α. Each possible level of α thus lead to one level of q. This combination (α, q) belongs to the plotted surface so it makes no doubt that for reasonable parameter values, growth increases with the capital share. Since an increase in v may imply a decrease in α to keep constant return, the proposition 2 remains true if the increase in v is compensated by a decrease in α.

A.5 Proof of proposition 7

The inter-temporal equilibrium of the market economy with a tax on the resource used in the production is a solution of the system formed by equations (21, 22, 26,28, 29, 30, 31, 32, 33, 34, 36, 40, 41, 42). Using definition 2, we have:

- (22) at the BGP $\Rightarrow$ $\mu_m = (1-q)$
- (21) at the BGP $\Rightarrow$ $\mu_m = \mu_x$
- (31) at the BGP $\Rightarrow$ $\mu_e = \mu_x$
- (36) at the BGP $\Rightarrow$ $\mu_s = \mu_k$
- (34) at the BGP $\Rightarrow$ $\mu_w = \mu_y$
- (30) at the BGP $\Rightarrow$ $\mu_a = (1+a)$
- (28) at the BGP $\Rightarrow$ $\mu_c = \mu_d$
- (32) at the BGP $\Rightarrow$ $\mu_y = (1+a)\mu_k^{\alpha}\mu_x^{v}\mu_e^{-\theta}$
- From (40), a BGP exists if $\mu_\tau = \mu_p = (1+a)\mu_k^{\alpha}\mu_x^{v-1}\mu_e^{-\theta}$
- (29) at the BGP $\Rightarrow \mu_p = 1 + r_{t+1} \Rightarrow \mu_r = 1$
- (33) at the BGP $\Rightarrow$ $1 = (1+a)\mu_k^{\alpha-1}\mu_x^{v}\mu_e^{-\theta} \Rightarrow \mu_k = (1+a)\mu_k^{\alpha}\mu_x^{v}\mu_e^{-\theta}$

- The IBC with the transfer is

$$w_t + g_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} - \frac{p_{t+1} m_t}{1 + r_{t+1}} + p_t m_t \tag{56}$$

Using (28), (41) and (34), we can write:

$$\beta A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} + \tau_t x_t = \frac{c_t (2 + \rho)}{1 + \rho} \tag{57}$$

On the BGP, it leads to

$$\mu_c = \frac{\mu_k \beta A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} + \mu_p \mu_x \tau_t x_t}{\beta A_t k_t^{\alpha} x_t^{v} e_t^{-\theta} + \tau_t x_t} \tag{58}$$

Since $\mu_p \mu_x = \mu_k$, $\mu_c = \mu_k$.

A.6 Proof of proposition 8

- $\mu_m = (1 - \tilde{q})$
- $\mu_x = \mu_m$
- $\mu_e = \mu_x$
- (50) $\Rightarrow \mu_{\tilde{p}} \mu_{\tilde{x}} = \mu_k$
- (51) $\Rightarrow \mu_g = \mu_k$
- (52) $\Rightarrow \mu_s = \mu_k$
- $\mu_w = \mu_y$
- $\mu_y = (1 + a) \mu_k^{\alpha} \mu_x^{v - \theta}$

- (48) $\Rightarrow \mu_c = \mu_k$

- From (28), $\mu_d = (\frac{1+r_{t+2}}{1+r_{t+1}})\mu_c$. A steady state exists only if $\mu_r = 1 \Rightarrow \mu_c = \mu_d$

- (33) $\Rightarrow \mu_k = (1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{v-\theta}{1-\alpha}}$