Transition from a Linear Economy toward a Circular Economy in the Ramsey Model

Kiyoka Akimoto[*]   Koichi Futagami[†]

Abstract

We construct a Ramsey-type model where households recycle waste generated by consumption and the not recycled waste has negative externality. The aim of this paper is the following two points. First, we examine a structural change process from "a linear economy" based on consumption-disposal toward "a sound material-cycle economy" or "a circular economy" based on consumption-recycling; Second, we examine dynamics of optimal consumption tax and recycling subsidy. In addition, we discuss the Environmental Kuznets Curve(EKC). Previous theoretical literature explains mechanism of EKC by changes in production sectors: such as an abatement technology or technological change. In contrast, this paper tries to show EKC through the aforementioned structural change process.

Keywords: Household waste recycling; Economic growth; Environmental policies

JEL Classification: O44; Q53; Q58

---

[*]Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, JAPAN; e-mail: akimoto1015@gmail.com.

[†]Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, JAPAN; e-mail: futagami@econ.osaka-u.ac.jp

1 Introduction

Many countries try to transit from a linear economy based on consumption-disposal toward a sound material-cycle economy or a circular economy based on consumption-recycling (hereinafter, referred to as a circular economy). In a circular economy, the value of products, materials and resources are maintained in the economy for as long as possible, and waste generation is minimized. Thus, recycling activity is significant to convert used materials and resources into input factors for production and to decrease the amount of waste.

Sustainable Development Goals and the G7 Alliance state the necessity of a circular economy for sustainable world development. The European Union presented the Closing the loop An EU action plan for the circular Economy(the Circular Economy Package) in 2015. The Circular Economy package includes some proposals such as recycling 65% of municipal waste by 2030 and recycling 75% of packaging waste by 2030.

The economic literature has constructed dynamic models with recycling (e.g. Smith (1972), Lusky (1976), Ayres (1999), Di Vita (2001), Di Vita (2007), Pittel et al. (2010)). Most part of the literature examine recycling activities by firms on production process. For example, in the model of Pittel et al. (2010), the recycling firms mine the stock of waste generated by households' consumption. The firms recycle the waste and supply it for the final goods sector. Thus, materials circulate in the economy through recycling by firms. In contrast to the previous studies, we approach the topic of the economy based on recycling activities by households[1].

The aim of this paper is the following two points: (i) to examine a structural change process from a linear economy based on consumption-disposal toward a circular economy based on consumption-recycling; furthermore, (ii) to examine a dynamics of optimal consumption tax and recycling subsidy. To analyze the transition path toward a circular economy and the dynamics of optimal tax-subsidy policy, we construct a simple Ramsey type model. In our model, households recycle waste generated by consumption. The waste that is not recycled has negative externality on the household utility. In production sector, there are two types of goods. One is virgin goods produced by capital and labor. The other is recycled goods

---

[1]See Fullerton and Kinnaman (1995) for households recycling in static model.

produced by the recycled waste and virgin goods. We note that virgin goods and recycled goods are homogeneous. In this paper, a circular economy is an economy where the households recycle as possible as. Firms produce goods, the households consume the goods and recycle the waste. Then, the firms produce goods by using the recycled waste: that is, an economy based on consumption-recycling. A linear economy is an economy where the households recycle the waste a little. Firms produce goods, the households consume the goods and dispose the waste: that is, an economy based on consumption-disposal.

Firstly, we examine the market equilibrium path. The market equilibrium path is as follows. An economy where initial capital stock is low starts as a circular economy. As capital accumulation proceeds, the economic structure changes from a circular economy to a linear economy. The linear economy approaches the steady state. Thus, the recycling is low and waste is large. On the other hand, in the social optimal path, through capital accumulation, the structure can change from a circular economy to a linear economy, and to a circular economy. Thus, the economy in the steady state is circulated where recycling is large and waste is small. In the market equilibrium, the households maximize the utility without considering negative externality by the waste. Since recycling in the market economy is smaller than that in the social optimum even if capital accumulates and consumption increases, the market equilibrium path approaches a linear economy. Secondly, we investigate the optimal tax-subsidy policy along the transition path. The market economy needs to gradually increase the recycling subsidy along capital accumulation for realizing a circular economy. By stimulates recycling, the policy plan raises the market equilibrium level of recycling up to the social optimal level.

We moreover discuss the Environmental Kuznets Curve(EKC). The previous literature shows various mechanism based on production sectors: such as technological progress from dirty to clean, or increasing return to abatement technology[2]. On the other hand, we try to explain the EKC through the structural change process of households recycling rather than production sector. In the social optimal path, that is, when the households consider the negative externality by the waste, the relation between the amounts of waste and per

---

[2]Copeland and Taylor (2004) categorizes the literature into four groups: (1) sources of growth; (2) income effects; (3) threshold effects; and (4) increasing returns to abatement.

capita capital is an inverted U shape, like the EKC. In the state with low per capita capital, a circular economy realizes where consumption is small and recycling is large, and thus the waste is small. Through capital accumulation, the structure changes to a linear economy where consumption is large and recycling is low, and thus the waste becomes large. Along the social optimal path, a circular economy realizes again in the stage of enough capital accumulation. In this stage, consumption is large and also recycling level is high, and thus the waste decreases.

The remainder of the paper is organized as follows. Section 2 considers the market economy and structural change path. Section 3 considers the social optimum and derive the transition toward the circular economy. Section 4 derives the dynamic schedule of tax-subsidy policy. Section 5 discusses the EKC through transition path. Our conclusions are summarized in Section 6.

2   Market economy

We consider the simple Ramsey-type model with recycling by households and negative externality by waste. The population is constant over time and its size is normalized to be unity. The economy consists of firms, households and a government. In this section, we consider the market economy and derive its equilibrium path.

2.1   Production sectors

There are two types of goods. The one is virgin goods produced by capital and labor according to the neoclassical production function per capita, $f(k_t)$, where k_t stands for the capital stock per capita. The other is recycled goods produced by virgin goods and recycled waste according to the production function, $M(N_t, E_t)$ which is homogeneous with degree one. The recycled goods per capita are represented by $m(n_t, e_t)$, where n_t and e_t stand for the virgin goods per capita used in the production sector and recycled waste supplied by households, respectively. The characteristics of $m(\cdot)$ are as follows: $m' > 0$, $m'' < 0$, and $m_{ne} = m_{en} > 0$. We note that these goods are homogeneous.

Profit maximization of the virgin goods firms yields:

$$f'(k_t) = r_t, \tag{1}$$

$$f(k_t) - f'(k_t)k_t = w_t. \tag{2}$$

r_t and w_t represent the interest rate and wage rate, respectively. The recycled goods firms maximize the profit, $m(n_t, e_t) - n_t - p_t e_t$, where the price of virgin goods is normalized to one and the price of recycled waste is p_t. It yields:

$$m_n(n_t, e_t) = 1, \tag{3}$$

$$m_e(n_t, e_t) = p_t. \tag{4}$$

By solving (3) for the virgin goods used by the recycled goods firms, we obtain:

$$n_t = n(e_t), \tag{5}$$

where the derivative is given by:

$$\frac{dn_t}{de_t} = -\frac{m_{ne}}{m_{nn}} > 0.$$

Substituting (5) into (4) and solving for the price of the recycled waste, we obtain:

$$p_t = p(e_t),$$

where the derivative is given by

$$\frac{dp_t}{de_t} = \frac{-m_{en}^2 + m_{ee} \cdot m_{nn}}{m_{nn}} = 0.$$

Thus, the price of recycled waste becomes constant. The level of p depends on the function m. We assume that $m_n(n_t, e_t) > m_e(n_t, e_t)$. The assumption ensures $1 > p$, which means that the price of virgin goods is higher than that of recycled goods produced by the recycled

waste.

2.2 Household

The representative household consumes goods, generates waste, and recycles the waste. Recycling activity makes the household feel psychological cost because it is troublesome for the household rather than disposing the waste. The recycled waste can be sold to firms producing recycled goods at the price p. On the other hand, unrecycled waste are just disposed and affect on the utility of the household negatively. Then, the preference of household is expressed as follows:

$$U[0] = \int_0^\infty e^{-\rho t}[u(c_t) - v(e_t) - z(o_t)]dt, \tag{6}$$

where $u(\cdot)$, $v(\cdot)$, and $z(\cdot)$ represent the instantaneous functions of consumption c_t, recycling e_t, and the unrecycled waste o_t, respectively, and ρ is the rate of time preference. We note that $o_t = c_t - e_t > 0$. The instantaneous utility functions, $u(\cdot)$, $v(\cdot)$, and $z(\cdot)$ are twice differentiable. The characteristics of $u(\cdot)$ are as follows:

$$u' > 0, \quad u'' < 0, \quad \lim_{c_t \to 0} u' = \infty, \quad \lim_{c_t \to \infty} u' = 0.$$

The characteristics of $v(\cdot)$ are as follows:

$$v' > 0, \quad v'' > 0, \quad \lim_{e_t \to 0} v' = 0, \quad \lim_{e_t \to \infty} v' = \infty.$$

Lastly, the function, $z(\cdot)$ is $z' > 0$, $z'' > 0$.

The budget constraint is

$$\dot{k}_t = r_t k_t + w_t + (1 + \tau_t^e)pe_t - (1 + \tau_t^c)c_t + G_t. \tag{7}$$

The first term in the right hand side (RHS) of (7) is interest income from capital, the second is wage income, the third is income from selling the recycled waste, and the forth is consumption expenditure. In addition, G_t, the fifth term in the RHS, represents income transfer or lump-

sum tax by the government.

$$G_t = \tau_t^c c_t - \tau_t^e p e_t, \tag{8}$$

where τ_t^c and τ_t^e represent consumption tax and recycling subsidy, respectively. The Government budget constraint is balanced.

We assume that there is an upper limit on the recycling waste:

$$e_t \leq \gamma c_t. \tag{9}$$

The constant parameter $\gamma \in (0, 1]$ can be regarded as technology level of recycling in an economy[3]. If γ is small, the household recycles only the small amount of waste because the technology in the economy cannot convert waste into materials for production.

In the market economy, the representative household disregard the externality of unrecycled waste, $z(o_t)$. Then, the household maximizes its lifelong utility:

$$U[0] = \int_0^\infty e^{-\rho t}[u(c_t) - v(e_t)]dt, \tag{10}$$

subject to the budget constraint (7) and the technological limitation on recycling (9). To solve the maximization problem, we define the current value Hamiltonian as follows:

$$H \equiv u(c_t) - v(e_t) + \lambda_t[r_t k_t + w_t + (1 + \tau_t^e)p e_t - (1 + \tau_t^c)c_t + G_t] + \mu_t(\gamma c_t - e_t). \tag{11}$$

---

[3]The purpose of this paper is to examine recycling activity by households. Thus, we omit the case of $\gamma = 0$.

The first order conditions, the Kuhn-Tucker condition, and the transversality condition are:

$$u'(c_t) - (1 + \tau_t^c)\lambda_t + \gamma\mu_t = 0, \tag{12a}$$

$$-v'(e_t) + (1 + \tau_t^e)p\lambda_t - \mu_t = 0, \tag{12b}$$

$$r_t\lambda_t = -\dot{\lambda}_t + \rho\lambda_t, \tag{12c}$$

$$r_t k_t + w_t + (1 + \tau_t^e)pe_t - (1 + \tau_t^c)c_t = \dot{k}_t, \tag{12d}$$

$$\mu_t(\gamma c_t - e_t) = 0, \quad \gamma c_t - e_t \geq 0, \quad \mu_t \geq 0, \tag{12e}$$

$$\lim_{t \to \infty} e^{-\rho t}\lambda_t k_t = 0. \tag{12f}$$

We consider the following two cases. Th first case is the level of the household's recycling is less than the maximum level; that is, $e_t < \gamma c_t$. The other case is the household recycles as possible as; that is, $e_t = \gamma c_t$. Thus, we call the former a linear economy and the latter a circular economy. A linear economy consists of production-consumption-disposal structure. Most of waste generated by consumption is not repossessed for production since the household recycles the small amount of waste. On the other hand, a circular economy is based on production-consumption-recycling structure. In a circular economy, a lot of recycled waste reworks as the input factor for production. Then, the materials are circulating through recycling activity. In the following two subsections, we consider dynamics of these economy, respectively.

In addition, we suppose that the government does not take any tax-subsidy policy in this section. That is, $\tau_t^c = \tau_t^e = 0$. Section 4 derives dynamic schedule of tax-subsidy policy.

2.3 A linear economy

We derive the dynamic equations of the linear economy in a market competitive equilibrium. In the linear economy, $e_t < \gamma c_t$. Substituting $\mu_t = 0$ into FOCs, we rewrite (12a) and (12b) as follows:

$$u'(c_t) - \lambda_t = 0, \tag{13a}$$

$$-v'(e_t) + p\lambda_t = 0. \tag{13b}$$

Firstly, from (12c) and (13a), we obtain the Euler equation:

$$\dot{c}_t = \sigma_l^m(r_t - \rho), \quad \text{where} \quad \sigma_l^m \equiv -\frac{u'(c_t)}{u''(c_t)} > 0. \tag{14}$$

Next, (13a) and (13b) yield the rate of marginal utility from consumption and recycling:

$$\frac{v'(e_t)}{u'(c_t)} = p. \tag{15}$$

By solving this equation for recycling, we can show that:

$$e_t = e_l^m(c_t), \tag{16}$$

where the subscript represent, l, and superscript, m, represent the linear structure in the market economy. The derivative is given by:

$$\frac{de_t}{dc_t} = \frac{pu''(c_t)}{v''(e_t)} < 0. \tag{17}$$

Thus, the level of recycling is decreasing in c_t. Furthermore, $e_l^m(c_t)$ has the following characteristics:

$$\lim_{c_t \to 0} e_t = \infty, \qquad \lim_{c_t \to \infty} e_t = 0.$$

Figure 1 shows the relationship between recycling and consumption in the linear economy. The equation (15), which shows that the rate of marginal utility is equal to the rate of price of consumption and recycling, provides intuitions of the form of $e_l^m(c_t)$. When consumption is small, the marginal utility from consumption is large. To hold (15), the household recycles a lot. On the other hand, when consumption is large, the marginal utility is small. Then, the household recycles a little.

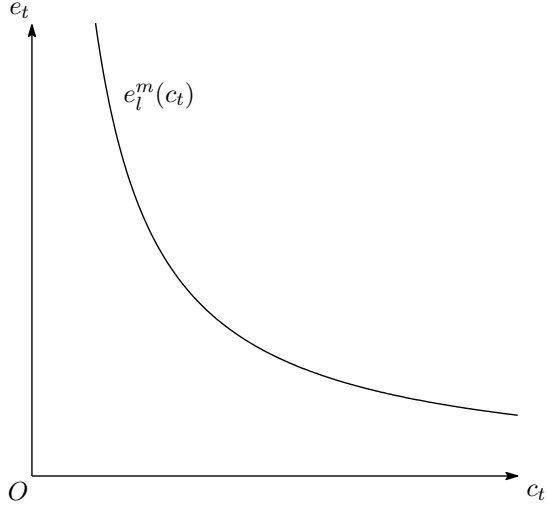Substituting (1), (2), and (16) into (7), we obtain the dynamic equation of the capital

8

Figure 1: recycling and consumption in the linear economy

stock per capita as follows:

$$\dot{k}_t = f(k_t) + pe_l^m(c_t) - c_t. \tag{18}$$

The equations (14) and (18) constitute the dynamic system of the linear economy in the market competitive equilibrium.

2.4 A circular economy

We derive the dynamic equations of the circular economy where $e_t = \gamma c_t$ in a market competitive equilibrium. Since $\mu_t > 0$, from (12a), (12b), and $\tau_t^c = \tau_t^e = 0$, we obtain λ_t and μ_t as follows:

$$\lambda_t = \frac{u'(c_t) - \gamma v'(e_t)}{1 - \gamma p}, \quad \mu_t = \frac{pu'(c_t) - v'(e_t)}{1 - \gamma p}. \tag{19}$$

Using (12a), (12b), (12c), and (19) yields the dynamic equation of consumption:

$$\dot{c}_t = \sigma_c^m(r_t - \rho) \quad \text{where} \quad \sigma_c^m \equiv -\frac{u'(c_t) - \gamma v'(e_t)}{u''(c_t) - \gamma^2 v''(e_t)}. \tag{20}$$

9

See Appendix A for derivation of the equation.

We assume that $\sigma_c^m > 0$. Since the sign of the denominator of σ_c^m is negative, we need the following assumption.

Assumption 1. $u'(c_t) - \gamma v'(e_t) > 0$.

From the equations, $e_t = e_c^m(c_t) = \gamma c_t$, (1), (2), and (7), we obtain the dynamic equation of capital[4]:

$$\dot{k}_t = f(k_t) - c_t(1 - \gamma p). \tag{21}$$

The equations (20) and (21) constitute the dynamics system of the circular economy in the market competitive equilibrium.

2.5 Transition of the market economy and steady state

This subsection examines the transition of the market economy and the steady state. Firstly, we derive the $\dot{c}_t = 0$ locus and the $\dot{k}_t = 0$ locus. The dynamics of consumption (14) in the linear economy and (20) in the circular economy yield the identical $\dot{c}_t = 0$ locus:

$$f'(k_t) = \rho. \tag{22}$$

Next, to derive the $\dot{k}_t = 0$ locus in the market economy, we consider the cutoff at which the economic structure changes. Figure 2 shows e_t in the linear economy and that in the circular economy. Since e_t must satisfy $e_t < \gamma c_t$, the feasible level of e_t is represented by the solid line:

$$e^m(c_t) = \begin{cases} \gamma c_t & \text{if} \quad c_t \le \tilde{c}^m, \\ e_l^m(c_t) & \text{if} \quad c_t > \tilde{c}^m. \end{cases} \tag{23}$$

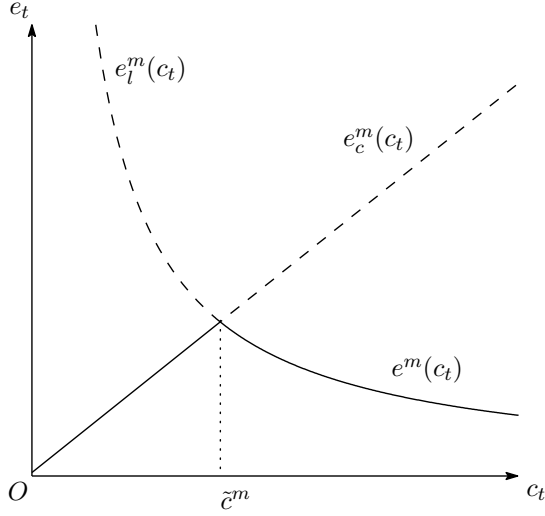[4]Since $0 < \gamma < 1$ and $p < 1$, we have $1 - \gamma p > 0$.

Figure 2: The cutoff of economic structural change

Then, we denote the cutoff $\tilde{c}^m$ that satisfies:

$$e_l^m(\tilde{c}^m) = \gamma\tilde{c}^m = e_c^m(\tilde{c}^m).$$

If the level of consumption in period t is less that $\tilde{c}^m$, the structure of a economy is a circular economy where the household recycles waste as much as possible, $e_t = \gamma c_t$. If the consumption level exceeds the cutoff, the economic structure changes from a circular economy toward a linear economy. Then, the level of recycling is decreasing in a linear economy. In addition, at the cutoff $\tilde{c}^m$, the $\dot{k}_t = 0$ locus of each structure crosses. We can write the $\dot{k}_t = 0$ locus in the economy from (18) and (21):

$$c_t = \begin{cases} \dfrac{f(k_t)}{1 - \gamma p}, & \text{if} \quad c_t \leq \tilde{c}^m, \\[2ex] f(k_t) + pe_l^m(c_t), & \text{if} \quad c_t > \tilde{c}^m. \end{cases} \tag{24}$$

We note that $c_t = f(k_t) + pe_l^m(c_t)$ does not cross the origin while $c_t = f(k_t)/(1 - \gamma p)$ does. We denote $\hat{c}^m$ that satisfies $\hat{c}^m = pe_l^m(\hat{c}^m)$ when $k_t = 0$. In this case, only recycled goods are produced and the household consumes the recycled goods. Thus, $\hat{c}^m > 0$.

Secondly, by drawing the $\dot{c}_t = 0$ locus (22) and the $\dot{k}_t = 0$ locus (24) as in Figure 3, we

examine the characteristics of the structural change and the steady state. Thus, we focus on the path on which structural change occurs.
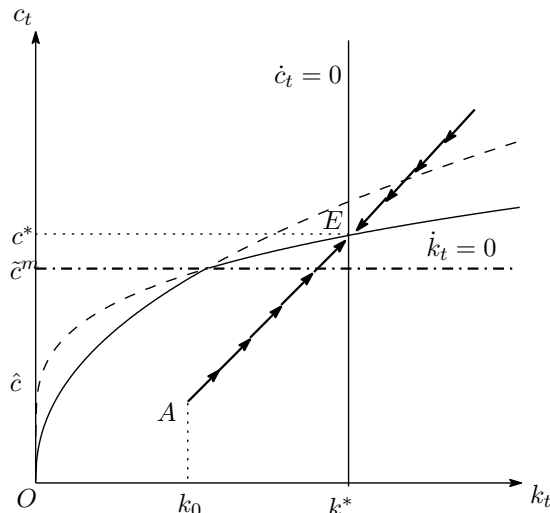

Figure 3: The transition and steady state in the market economy

The steady state E can exist that is upper the cutoff $\tilde{c}^{m5}$. Thus, the structure of the economy is the linear economy where the household dispose the large share of waste. Then, according to (22), (23) and (24), we obtain the steady state level of the variables as follows:

$$f'(k^*) = \rho, \qquad c^* = f(k^*) + pe_l^m(c^*), \qquad e^* = e_l^m(c^*).$$

In addition, under the Assumption 1, the economy satisfies the saddle-path stability. In order to get on the saddle path toward the steady state E, the economy where the initial stock of per capita capital is k_0 starts at point A. Along the saddle path, the structural change occurs as follows. When the level of capital stock is low, the structure of the economy is the circular economy based on consumption and recycling. As capital accumulation proceeds, the structure changes from a circular economy to a linear economy at the cutoff $\tilde{c}^m$. Furthermore, the linear economy reaches the steady state E. Thus, the market economy cannot realize the circular economy.

Lastly, we examine the amount of unrecycled waste along the saddle path. Figure 4 shows

---

[5]If the $\dot{c}_t = 0$ locus and the $\dot{k}_t = 0$ locus cross at a point under the cutoff $\tilde{c}^m$, structural change does not happen and then the circular economy realizes in the steady state.

the relationship between consumption, recycling, and unrecycled waste. If the structure of
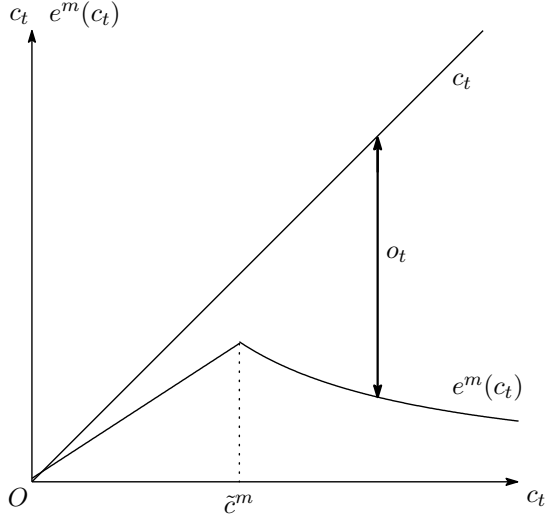

Figure 4: The unrecycled waste in the market economy

the economy is the circular economy where consumption is small and recycling is large, the amount of unrecycled waste is small. On the other hand, if the level of consumption exceeds the cutoff, the linear economy realizes where the unrecycled waste is large. Moreover, the unrecycled waste increases with consumption.

3   Social optimum

In this section, we consider the socially optimal economy where the social planner decides resource allocation. Taking account of the negative externality from the unrecycled waste, the social planner maximizes (6). The constraints for the social planner include the limitation on recycling (9) and the resource constraint:

$$\dot{k}_t = f(k_t) + m(n_t, e_t) - n_t - c_t. \tag{25}$$

To solve the maximization problem, we define the current value Hamiltonian as follows:

$$H \equiv u(c_t) - v(e_t) - z(o_t) + \lambda_t[f(k_t) + m(n_t, e_t) - n_t - c_t] + \mu_t(\gamma c_t - e_t) \tag{26}$$

13

Differentiating (26) with respect c_t, e_t, n_t, k_t, and λ_t, we obtain FOCs:

$$u'(c_t) - z'(o_t) - \lambda_t + \gamma\mu_t = 0, \tag{27a}$$

$$-v'(e_t) + z'(o_t) + \lambda_t m_e(n_t, e_t) - \mu_t = 0, \tag{27b}$$

$$\lambda_t[m_n(n_t, e_t) - 1] = 0, \tag{27c}$$

$$f'(k_t)\lambda_t = -\dot{\lambda}_t + \rho\lambda_t, \tag{27d}$$

$$f(k_t) + m(n_t, e_t) - n_t - c_t = \dot{k}_t. \tag{27e}$$

$\lambda_t > 0$ yields $m_n(n_t, e_t) = 1$ from (27c). Thus, $m_e(n_t, e_t)$ becomes constant. We denote $m_e(n_t, e_t) = \bar{m}_e$.

In the following two subsections, we consider the dynamics of variables in each economic structures.

3.1 A linear economy

We derive the dynamic equations of the linear economy in the social optimum. Substituting $\mu_t = 0$ into FOCs derived above, we rewrite (27a) and (27b) as follows:

$$u'(c_t) - z'(o_t) - \lambda_t = 0, \tag{28a}$$

$$-v'(e_t) + z'(o_t) + \lambda_t m_e(n_t, e_t) = 0. \tag{28b}$$

Before deriving the dynamic equations of variables, we consider the relationship between recycling and consumption in the linear economy of social optimum. From (28a) and (28b), the ratio of marginal consumption and recycling is given by:

$$\frac{v'(e_t) - z'(o_t)}{u'(c_t) - z'(o_t)} = \bar{m}_e. \tag{29}$$

Solving the equation for recycling, we obtain:

$$e_t = e_l^s(c_t), \tag{30}$$

14

where the derivative is:

$$\frac{de_t}{dc_t} = \frac{\bar{m}_e u''(c_t) + z''(o_t)(1 - \bar{m}_e)}{v''(e_t) + z''(o_t)(1 - \bar{m}_e)}.$$

While the sign of de_t/dc_t is not determined with the general form functions, $e_l^s(c_t)$ has the following characteristics:

$$\lim_{c_t \to 0} e_t = \infty, \quad \lim_{c_t \to \infty} e_t = \infty.$$

Here, we suppose that the unique and constant $\bar{c}$ exists that satisfies:

$$\left. \frac{de_t}{dc_t} \right|_{c_t = \bar{c}} = 0.$$

In addition, we impose the following assumption that as c_t goes to infinity, the slope of the function $e_l^s(c_t)$ is larger than γ:

$$\lim_{c_t \to \infty} \frac{de_t}{dc_t} > \gamma.$$

Under these assumptions, the relationship between e_t and c_t becomes U-shaped.

Next, we derive the dynamic equations of consumption, recycling, and capital. Firstly, we can show the dynamic equation of consumption:

$$\dot{c}_t = \sigma_l^s [f'(k_t) - \rho], \tag{31}$$
$$\text{where} \quad \sigma_l^s \equiv -\frac{[u'(c_t) - v'(e_t)]z''(o_t) + [u'(c_t) - z'(o_t)]v''(e_t)}{[u''(c_t) - v''(e_t)]z''(o_t) + u''(c_t)v''(e_t)}$$

We assume that $\sigma_l^s > 0$. Since the sign of the denominator of σ_l^s is negative, we need the following assumption.

Assumption 2. $[u'(c_t) - v'(e_t)]z''(o_t) + [u'(c_t) - z'(o_t)]v''(e_t) > 0.$

Secondly, the dynamics equation of recycling is given by:

$$\dot{e}_t = \hat{\sigma}_l^s[f'(k_t) - \rho], \tag{32}$$

$$\text{where} \quad \hat{\sigma}_l^s \equiv -\frac{[u'(c_t) - v'(e_t)]z''(o_t) + [v'(e_t) - z'(o_t)]u''(c_t)}{[u''(c_t) - v''(e_t)]z''(o_t) + u''(c_t)v''(e_t)}$$

See Appendix B for derivation of equations (31) and (32).

Lastly, the resource constraint (25) yields the dynamic equation of capital:

$$\dot{k}_t = f(k_t) + \bar{m}_e e_l^s(c_t) - c_t. \tag{33}$$

The equations (31) and (33) constitute the dynamic system of the linear economy in the social optimum.

3.2 A circular economy

We derive the dynamic equations of the circular economy of social optimum. From (27a) and (27b), we can calculate λ_t and μ_t:

$$\lambda_t = \frac{u'(c_t) - z'(o_t) - \gamma[v'(e_t) - z'(e_t)]}{1 - \gamma\bar{m}_e}, \tag{34a}$$

$$\mu_t = \frac{\bar{m}_e[u'(c_t) - z'(o_t)] - [v'(e_t) - z'(o_t)]}{1 - \gamma\bar{m}_e}. \tag{34b}$$

Firstly, we can show the dynamic equation of consumption as follows:

$$\dot{c}_t = \sigma_c^s[f'(k_t) - \rho], \tag{35}$$

$$\text{where} \quad \sigma_c^s \equiv -\frac{u'(c_t) - \gamma v'(e_t) - (1-\gamma)z'(o_t)}{u''(c_t) - \gamma^2 v''(o_t) - (1-\gamma)^2 z''(o_t)}$$

See Appendix C for derivation of this equation.

As $\sigma_l^s > 0$, we assume that $\sigma_c^s > 0$. Since the sign of the denominator of σ_c^s is negative, we need the following assumption.

Assumption 3. $u'(c_t) - \gamma v'(e_t) - (1-\gamma)z'(o_t) > 0$.

16

Secondly, we obtain the dynamic equation of capital:

$$\dot{k}_t = f(k_t) - c_t(1 - \gamma \bar{m}_e). \tag{36}$$

Thus, the dynamics (35) and (36) constitute the dynamic system of the circular economy of the social optimum.

3.3 Transition toward a circular economy

We derive a cutoff of economic structure in the social optimum. Figure 5 shows the relationship between e_t and c_t in each structures. Thus, there exists two cutoffs; $\tilde{c}_1^s$ and $\tilde{c}_2^s$. If the level of
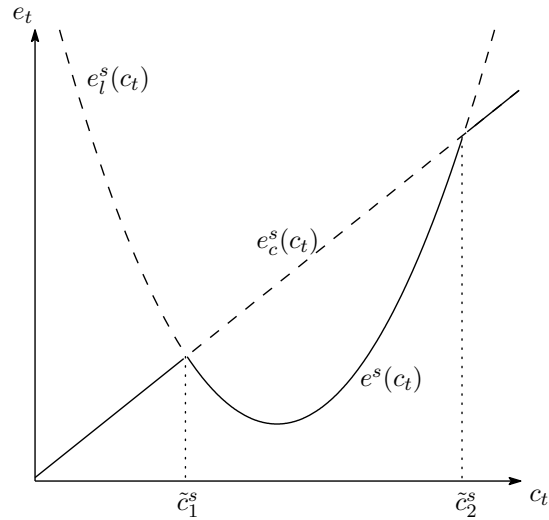

Figure 5: The cutoffs in the social optimum

consumption is less than $\tilde{c}_1^s$, the economic structure is a circular economy. If the consumption is in the interval between $\tilde{c}_1^s$ and $\tilde{c}_2^s$, the linear economy realizes. Finally, if the economy exceeds the cutoff $\tilde{c}_2^s$, the structure changes from a linear economy toward a circular economy.

17

Then, the level of recycling in social optimum is given by:

$$
e^s(c_t) = \begin{cases} \gamma c_t & \text{if} \quad c_t \leq \tilde{c}_1^s, \\ e_l^s(c_t) & \text{if} \quad \tilde{c}_1^s < c_t < \tilde{c}_2^s, \\ \gamma c_t & \text{if} \quad c_t \geq \tilde{c}_2^s. \end{cases} \tag{37}
$$

As the $\dot{c}_t = 0$ locus in the competitive market economy, from (31) and (35), the dynamics of consumption in each structures yields the identical $\dot{c}_t = 0$ locus in the social optimum:

$$
f'(k_t) = \rho \tag{38}
$$

Lastly, from (33) and (35), the dynamics of capital in the social optimum economy is

$$
c_t = \begin{cases} \dfrac{f(k_t)}{1 - \gamma \bar{m}_e}, & \text{if} \quad c_t \leq \tilde{c}_1^s \\ f(k_t) + \bar{m}_e e_l^s(c_t), & \text{if} \quad \tilde{c}_1^s < c_t < \tilde{c}_2^s \\ \dfrac{f(k_t)}{1 - \gamma \bar{m}_e}, & \text{if} \quad c_t \geq \tilde{c}_2^s \end{cases} \tag{39}
$$

When $k_t = 0$, the equation $c_t = f(k_t) + \bar{m}_e e_l^s(c_t)$ does not cross the origin. We denote $\hat{c}^s$ that satisfies $\hat{c}^s = \bar{m}_e e_l^s(\hat{c}^s)$.

Writing the $\dot{c}_t = 0$ locus (38) and the $\dot{k}_t = 0$ locus (39), we examine the transition of social optimum. Here, we also focus on the path toward a circular economy. There can exist the steady state E upper the cutoff $\tilde{c}_2^s$ where the circular economy realizes[6]. Then, according to (37) and (39), the steady state level of capital, consumption and recycling are:

$$
f'(k^*) = \rho, \qquad c^* = \frac{f(k^*)}{1 - \gamma \bar{m}_e}, \qquad e^* = \gamma c^*.
$$

---

[6]There can be other two cases. The one is that a steady state exists within the range between $\tilde{c}_1^s$ and $\tilde{c}_2^s$. In this case, the structure of the economy changes from circular to linear and then the linear economy realizes in the steady state. The other is that a steady state exists under the cutoff $\tilde{c}_1^s$, in which case the structural change does not occur and then the economy keeps the circular economy in the steady state.
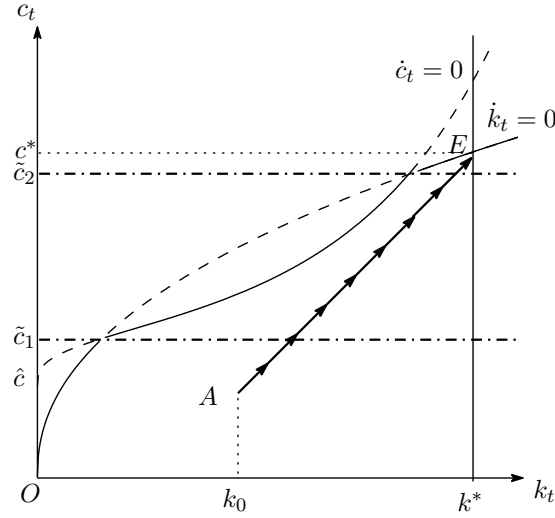
18

Figure 6: The transition and steady state in the social optimum

Under the Assumption 2 and 3, the economy satisfies the saddle-path stability. Thus, the economy with an initial capital of k_0 starts at point A and gets on the saddle path toward the steady state E. The structural changes along the path are as follows. Firstly, if the level of capital stock is small, the structure is a circular economy. Secondly, as capital accumulation proceeds and the economy crossed the cutoff, the structure changes from a circular economy to a linear economy. Lastly, if capital accumulation is sufficiently large, the circular economy realizes and reaches the steady state E. Along the structural change process, the level of recycling changes. Firstly, the recycling is increasing until the first cutoff $\tilde{c}_1$. Crossing the first cutoff, the recycling is decreasing and then increasing. However, the increasing level is still less than the maximum level of recycling. Lastly, if the economy becomes circulated with large capital accumulation, the level of recycling is the maximum level.

4 Optimal tax-subsidy

In this section, the government decides the schedule of consumption tax and recycling subsidy in order to take the social optimal path toward the circular economy. To obtain the dynamic equation of tax and subsidy, we return to the model of the market economy and derive the dynamics of variables in each structures. Firstly, we consider the linear economy. The dynamic

equations of consumption and recycling are given by:

$$\dot{c}_t = \sigma_l^m \left[(r_t - \rho) - \frac{\dot{\tau}_t^c}{1 + \tau_t^c} \right] \tag{40}$$

$$\dot{e}_t = \hat{\sigma}_l^m \left[(r_t - \rho) - \frac{\dot{\tau}_t^e}{1 + \tau_t^e} \right], \quad \text{where} \quad \hat{\sigma}_l^m \equiv -\frac{v'(e_t)}{v''(e_t)} \tag{41}$$

See Appendix D for derivation of equations (40) and (41).

Next, we consider the circular economy. In the circular economy, the dynamics of recycling is $\dot{e}_t = \gamma \dot{c}_t$ and the dynamic equation of consumption is as follows:

$$\dot{c}_t = \sigma_c^m \left[(r_t - \rho) - \frac{\dot{\tau}_t^c - \gamma p \dot{\tau}_t^e}{(1 + \tau_t^c) - \gamma p (1 + \tau_t^e)} \right] \tag{42}$$

See Appendix E for derivation of equation (42).

The government decides the dynamic schedule of τ_t^c and τ_t^e by making the dynamics of consumption and recycling equal to the dynamics in the social optimum. Thus, in the linear economy, from (31) and (40), the dynamics of τ_t^c satisfies:

$$\sigma_l^m \left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^c}{1 + \tau_t^c} \right] = \sigma_l^s [f'(k_t) - \rho].$$

Then, we obtain the dynamic schedule of consumption tax as follows:

$$\dot{\tau}_t^c = \frac{\sigma_l^m - \sigma_l^s}{\sigma_l^m} [f'(k_t) - \rho](1 + \tau_t^c) \tag{43}$$

On the other hand, from (32) and (41), the recycling subsidy are determined to satisfy the following equation:

$$\hat{\sigma}_l^m \left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^e}{1 + \tau_t^e} \right] = \hat{\sigma}_l^s [f'(k_t) - \rho],$$

20

and it yields the dynamic equation of recycling subsidy:

$$\dot{\tau}_t^e = \frac{\hat{\sigma}_l^m - \hat{\sigma}_l^s}{\hat{\sigma}_l^m}[f'(k_t) - \rho](1 + \tau_t^e) \tag{44}$$

Along the social optimum path, if the circular economy realizes, from (35) and (42), the government decides the dynamic schedule of tax and subsidy as follows:

$$\sigma_c^m\left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^c - \gamma p\dot{\tau}_t^e}{(1 + \tau_t^c) - \gamma p(1 + \tau_t^e)}\right] = \sigma_c^s[f'(k_t) - \rho]$$

$$\Leftrightarrow \quad \dot{\tau}_t^c - \gamma p\dot{\tau}_t^e = \frac{\sigma_c^m - \sigma_c^s}{\sigma_c^m}[f'(k_t) - \rho][(1 + \tau_t^c) - \gamma p(1 + \tau_t^e)] \tag{45}$$

Therefore, along the social optimal transition path, the government decides the tax-subsidy policy following (45) if the structure is the circular economy with small capital accumulation, (43) and (44) if the circular economy changes to the linear economy with the middle level of capital, and finally (45) if the circular economy realizes with large capital accumulation.

5 Discussion about the Environmental Kuznets Curve

We discuss the Environmental Kuznets Curve(EKC). The EKC is the inverted-U shaped relationship between environmental quality and economic development. We try to explain the inverted-U shape through the transitional path of optimal economy. In this paper, the stock of capital per capita, k_t, represents development. Along the transitional path of optimal economy, the level of consumption is increasing as capital accumulation proceeds. Thus, we examine the relationship between the unrecycled waste and the level of consumption. Firstly, we draw the level of recycling (37) and consumption in Figure 7. Then, the unrecycled waste in the optimum economy is given by:

$$o_t = c_t - e^s(c_t)$$

Figure 8 shows the relationship between the level of consumption and the unrecycled waste. The shape of this graph is inverted-U, like the EKC. In this paper, the inverted-U shaped
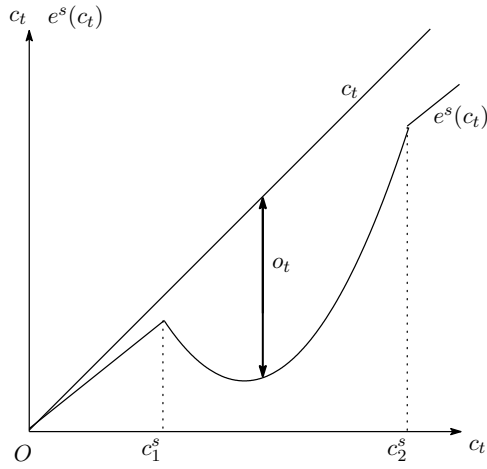
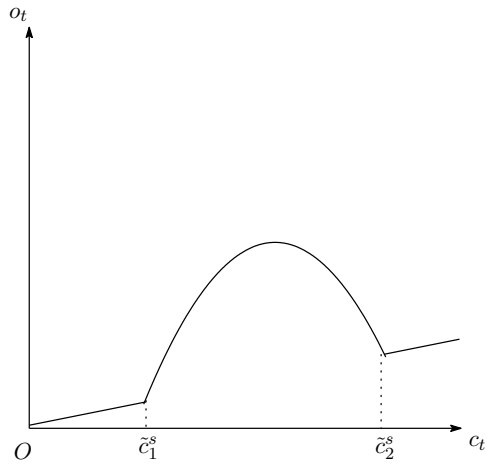Figure 7: The unrecycled waste in the optimal economy

Figure 8: The Environmental Kuznets Curve

relationship between economic development and the unrecycled waste appears through the economic structural change process: the structure changes from ciruculated to linear, and finally to circulated. When the economy is less developed, the household recycles waste as possible as because the stock of capital is small. Then, the unrecycled waste is small. As the economy develops, the level of recycling is decreasing and then the unrecycled waste is increasing. After the linear economy loads a lot of waste on the environment, the level of recycling rises because the negative externality becomes large. Finally, if the economy realizes the circulated structure, the household starts to recycle a lot of waste and dispose less. Therefore, we can explain the EKC along the optimum path.

6 Conclusion

We construct a Ramsey-type model with recycling activities by households. In the model, households recycles waste generated by consumption. The recycled waste is repossessed for production. On the other hand, the unrecycled waste is disposed and affects the household utility negatively. Using the model, we consider transition path toward a circular economy and dynamic schedule of tax-subsidy policy.

Firstly, along the transition path of the market equilibrium, the structure of the economy changes from a circular economy to a linear economy. Thus, the market economy cannot realize the circular economy. Since the household ignores the negative externality from the unrecycled waste, the household recycles waste a little. Secondly, in the social optimal path, the structure can change from a circular economy to a linear economy, and finally to a circular economy. We show that the optimal tax-subsidy policy for the market economy to realize the circular economy. The policy raises the market equilibrium level of recycling up to the social optimal level.

Appendix

A. The dynamics of the circular-competitive economy

Differentiating (12a) and (12b) with time yields:

$$u''(c_t)\dot{c}_t - \dot{\lambda}_t + \gamma\dot{\mu}_t = 0, \tag{A.1}$$

$$-v''(e_t)\dot{e}_t + p\dot{\lambda}_t - \dot{\mu}_t = 0. \tag{A.2}$$

By substituting (12c), (19), and (A.2) into (A.1) and using the equation $\dot{e}_t = \gamma\dot{c}_t$, we obtain the dynamic equation of consumption (20):

$$\dot{c}_t = -\frac{u'(c_t) - \gamma v'(e_t)}{u''(c_t) - \gamma^2 v''(e_t)}(r_t - \rho).$$

B. The dynamics of the linear-social optimal economy

Firstly, by taking log of both sides of (28a) and (28b) and differentiating the equations with time, we obtain:

$$\dot{c}_t = \frac{u'(c_t) - z'(o_t)}{u''(c_t) - z''(o_t)} \left[\frac{\dot{\lambda}_t}{\lambda_t} - \frac{z''(o_t)}{u'(c_t) - z'(o_t)} \dot{e}_t \right], \tag{A.3}$$

$$\dot{e}_t = \frac{v'(e_t) - z'(o_t)}{v''(e_t) + z''(o_t)} \left[\frac{\dot{\lambda}_t}{\lambda_t} + \frac{z''(o_t)}{v'(c_t) - z'(o_t)} \dot{c}_t \right]. \tag{A.4}$$

Substituting (27d) and (A.4) into (A.3) yields the dynamic equation of consumption (31):

$$\dot{c}_t = -\frac{[u'(c_t) - v'(e_t)]z''(o_t) + [u'(c_t) - z'(o_t)]v''(e_t)}{[u''(c_t) - v''(e_t)]z''(o_t) + u''(c_t)v''(e_t)}[f'(k_t) - \rho].$$

On the other hand, by substituting (27d) and (A.3) into (A.4), the dynamic equation of recycling (32) is given by:

$$\dot{e}_t = -\frac{[u'(c_t) - v'(e_t)]z''(o_t) + [v'(e_t) - z'(o_t)]u''(c_t)}{[u''(c_t) - v''(e_t)]z''(o_t) + u''(c_t)v''(e_t)}[f'(k_t) - \rho].$$

C. The dynamics of the circular-social optimal economy

Differentiating (27a) and (27b) with time yields:

$$u''(c_t)\dot{c}_t - z''(\dot{c}_t - \dot{e}_t) - \dot{\lambda}_t + \gamma\dot{\mu}_t = 0, \tag{A.5}$$

$$-v''(e_t)\dot{e}_t + z''(\dot{c}_t - \dot{e}_t) + \dot{\lambda}_t m_e(n_t, e_t) - \dot{\mu}_t = 0. \tag{A.6}$$

By substituting (27d), (34a), and (A.6) and using the equation $\dot{e}_t = \gamma\dot{c}_t$, we obtain the dynamic equation of consumption (35):

$$\dot{c}_t = -\frac{u'(c_t) - \gamma v'(e_t) - (1 - \gamma)z'(o_t)}{u''(c_t) - \gamma^2 v''(o_t) - (1 - \gamma)^2 z''(o_t)}[f'(k_t) - \rho].$$

D. The dynamics of the linear economy with the tax-subsidy policy

We rewrite the FOCs, (12a) and (12b) since $\mu = 0$ in the linear economy:

$$u'(c_t) - (1 + \tau_t^c)\lambda_t = 0, \tag{A.7}$$

$$-v'(e_t) + (1 + \tau_t^e)p\lambda_t = 0. \tag{A.8}$$

By taking log of both sides of (A.7) and (A.8) and differentiating with the equations with time, we obtain:

$$\dot{c}_t = \frac{u'(c_t)}{u''(c_t)}\left(\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{\tau}_t^c}{1 + \tau_t^c}\right),$$

$$\dot{e}_t = \frac{v'(e_t)}{v''(e_t)}\left(\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{\tau}_t^e}{1 + \tau_t^e}\right).$$

Substituting (12c) into these equations yields the dynamic equation of consumption and recycling with the tax-subsidy policy, (40) and (41), respectively:

$$\dot{c}_t = -\frac{u'(c_t)}{u''(c_t)}\left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^c}{1 + \tau_t^c}\right],$$

$$\dot{e}_t = -\frac{v'(e_t)}{v''(e_t)}\left[(f'(k_t) - \rho) - \frac{\dot{\tau}_t^e}{1 + \tau_t^e}\right].$$

E. The dynamics of the circular economy with the tax-subsidy policy

By differentiating (12a) and (12b) with time, we obtain:

$$u''(c_t)\dot{c}_t - \dot{\lambda}_t(1 + \tau_t^c) - \lambda_t\dot{\tau}_t^c + \gamma\dot{\mu}_t = 0, \tag{A.9}$$

$$-v''(e_t)\dot{e}_t + \dot{\lambda}_t p(1 + \tau_t^e) - \lambda_t p\dot{\tau}_t^e - \dot{\mu}_t = 0. \tag{A.10}$$

Substituting (12c), (34a), and (A.10) into (A.9) and using the equation $\dot{e}_t = \gamma\dot{c}_t$ yields the dynamic equation of consumption with tax-subsidy policy (42):

$$\dot{c}_t = -\frac{u'(c_t) - \gamma v'(e_t)}{u''(c_t) - \gamma v''(e_t)}\left[(r_t - \rho) - \frac{\dot{\tau}_t^c - \gamma p\dot{\tau}_t^e}{(1 + \tau_t^c) - \gamma p(1 + \tau_t^e)}\right].$$

References

André, Francisco J and Sjak Smulders, "Fueling growth when oil peaks: Directed technological change and the limits to efficiency," *European Economic Review*, 2014, *69*, 18–39.

Ayres, Robert U, "The second law, the fourth law, recycling and limits to growth," *Ecological Economics*, 1999, *29* (3), 473–483.

Cassou, Steven P and Stephen F Hamilton, "The transition from dirty to clean industries: optimal fiscal policy and the environmental Kuznets curve," *Journal of Environmental Economics and Management*, 2004, *48* (3), 1050–1077.

Copeland, Brian R and M Scott Taylor, "Trade, growth, and the environment," *Journal of Economic literature*, 2004, *42* (1), 7–71.

Fullerton, Don and Thomas C Kinnaman, "Garbage, Recycling, and Illicit Burning or Dumping," *Journal of Environmental Economics and Management*, 1995, *1* (29), 78–91.

Grimaud, André and Luc Rougé, "Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policies," *Journal of Environmental Economics and Management*, 2003, *45* (2), 433–453.

Lusky, Rafael, "A model of recycling and pollution control," *Canadian Journal of Economics*, 1976, pp. 91–101.

Nakamoto, Yasuhiro and Koichi Futagami, "Dynamic Analysis of a Renewable Resource in a Small Open Economy: The Role of Environmental Policies for the Environment," *Environmental and Resource Economics*, 2016, *64* (3), 373–399.

Ouchida, Yasunori and Daisaku Goto, "Environmental research joint ventures and time-consistent emission tax," 2014.

Pittel, Karen, Jean-Pierre Amigues, and Thomas Kuhn, "Recycling under a material balance constraint," *Resource and Energy Economics*, 2010, *32* (3), 379 – 394.

Ricci, Francesco, "Channels of transmission of environmental policy to economic growth: A survey of the theory," *Ecological Economics*, 2007, *60* (4), 688–699.

Smith, Vernon L, "Dynamics of waste accumulation: disposal versus recycling," *The Quarterly Journal of Economics*, 1972, *86* (4), 600–616.

Sugeta, Hajime and Takayoshi Shinkuma, "International trade in recycled materials in vertically related markets," *Environmental Economics and Policy Studies*, 2012, *14* (4), 357–382.

Sugiyama, Yasuyuki and Patcharin Koonsed, "Environmental R&D, Monopoly Recycling Markets, and Recycled Content Standards," 2016.

Vita, Giuseppe Di, "Technological change, growth and waste recycling," *Energy Economics*, 2001, *23* (5), 549–567.

_ , "Exhaustible resources and secondary materials: A macroeconomic analysis," *Ecological Economics*, 2007, *63* (1), 138–148.