Cournot duopolies with investment in R&D: regions of Nash investment equilibria

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Abstract

We study an economy with a single sector under Cournot competition with complete information, in a game with two subgames. In the first subgame, the firms can make investments in R&D to reduce their production costs. In the second subgame, after the cost reduction, the firms choose their optimal output quantities in the usual Cournot competition. The second subgame has a unique perfect Nash equilibrium. Depending on the parameters and on the final costs, after investment, a firm may be in Monopoly, in Duopoly or out of the market (corresponding to a Monopoly of the other firm). Furthermore, the investment is also dependent on the parameters and on the initial costs. It can be categorized as follows: nil investment, when neither firm invests; single investment, when only one firm invests; competitive investment, when both firms invest. Depending on the parameters, we have found regions in the parameter space with one, two or three Nash investment equilibria. We study the effect of the parameters in these regions, in particular we study the effect of product differentiation, giving special attention to regions with multiple Nash equilibria.

Keywords: Nash equilibria; Cournot duopoly model; multiple equilibria; R&D investment

1 Introduction

This working paper is based on our work in [6]. We consider a Cournot duopoly competition model where two firms invest in R&D projects to reduce their production costs. This competition is modeled, as usual, by a two stages game (see [1, 8]). In the first subgame, two firms choose, simultaneously, R&D investment strategies to reduce their initial production costs. In the second subgame, the two firms are involved in a Cournot competition with production costs equal to the reduced cost determined by the R&D investment strategies chosen in the first stage. We use an R&D cost reduction function inspired in the logistic equation that was first introduced in [8].

We consider two firms that are identical except, at most, in their production costs. We present the Perfect Nash equilibria of this two stages game and we study the economical effects of these equlibria. The second subgame, consisting of a Cournot competition, has a unique perfect Nash equilibrium. For the first subgame, consisting of an R&D cost reduction investment program, we exhibit four different regions of Nash investment equilibria that we characterize as follows: a competitive Nash investment region C where both firms invest, a single Nash investment region S_1 for firm F_1, where just firm F_1 invests, a single Nash investment region S_2 for firm F_2, where just firm F_2 invests, and a nil Nash investment region N, where neither of the firms invest (see [8, 9]).

The Nash investment equilibria are not necessarily unique. The non uniqueness leads to an economical complexity in the choice of the best R&D investment strategies by the firms. For high production costs, that can correspond to the production of new technologies, there are subregions of production costs where there are multiple Nash investment equilibria: a region $R_{S_i \cap C}$ where the intersection between the single Nash investment region S_i and the competitive Nash investment region C is non-empty; a region $R_{S_1 \cap S_2}$ where the intersection between the single Nash investment regions S_1 and S_2 is non-empty; a region $R_{S_1 \cap C \cap S_2}$ where the intersection between the single Nash investment regions S_1 and S_2 and the competitive Nash investment region C is non-empty. When we compare the cases symmetric efficient (SE) and symmetric inefficient (SI), we observe that, in the SI-scenario, the single Nash investment regions S_1 and S_2 increase in size and so the competitive Nash investment region C becomes smaller. In the asymmetric case (A), we observe that the single Nash investment region S_2 of firm F_2 is considerably bigger due to its advantage in the R&D cost reduction program efficiency.

2 R&D investments on costs

The Cournot duopoly competition with R&D investments on the reduction of the initial production costs consists of two subgames in one period of time. The first subgame is an R&D investment program, where both firms have initial production costs and choose, simultaneously, their R&D investment strategies to obtain new production costs. The second subgame is a Cournot competition with production costs equal to the reduced cost determined by the R&D investment program. As it is well known, the second subgame has a unique perfect Nash equilibrium.

2.1 The R&D program

We consider an economy with a monopolistic sector with two firms, F_1 and F_2, each one producing a differentiated good. The inverse demands p_i are linear:

$$p_i = \alpha - \beta q_i - \gamma q_j, \tag{1}$$

with parameters $\alpha > 0$, $\beta > 0$ and γ. We assume that $\gamma > 0$ and thus the goods are substitutes. The firm F_i invests an amount v_i in an R&D program $a_i : \mathbb{R}_0^+ \to [b_i, c_i]$ that reduces its production cost to

$$a_i(v_i) = c_i - \frac{\epsilon(c_i - c_L)v_i}{\lambda_i + v_i}. \tag{2}$$

Next, we explain the parameters of the R&D program: (i) the parameter c_i is the unitary production cost of firm F_i at the beginning of the period satisfying $c_L \leq c_i \leq \alpha$; (ii) the parameter c_L is the minimum attainable production cost; (iii) the parameter $0 < \epsilon < 1$ has the following meaning: since $b_i = a_i(+\infty) = c_i - \epsilon(c_i - c_L)$, the maximum reduction $\eta_i = \epsilon(c_i - c_L)$ of the production cost is a percentage $0 < \epsilon < 1$ of the difference between the current cost c_i and the lowest possible production cost c_L; (iv) the parameter $\lambda_i > 0$ can be seen as a measure of the inverse of the quality of the R&D investment program for firm F_i and is directly related to what we call efficiency of the R&D investment program that we define next (a smaller λ_i will result in a bigger reduction of the production costs for the same investment). The R&D investment program of firm F_1 is more efficient than the R&D investment program of firm F_2 if and only if with the same investment $v_1 = v_2 = v$, the new cost obtained by firm F_1, a_1, is smaller or equal to the new cost obtained by firm F_2, a_2, i.e $a_1(v) \leq a_2(v)$. This R&D program was first introduced in [8].

2.2 Optimal output levels

The profit $\pi_i(q_i, q_j)$ of firm F_i is given by

$$\pi_i(q_i, q_j) = q_i(\alpha - \beta q_i - \gamma q_j - a_i) - v_i, \tag{3}$$

for $i, j \in \{1, 2\}$ and $i \neq j$. The Nash equilibrium output (q_1^*, q_2^*) is given by

$$q_i^* = \begin{cases} 0, \ if \ R_i \leq 0 \\ R_i, \ if \ 0 < R_i < \frac{\alpha - a_j}{\gamma} \\ \frac{\alpha - a_i}{2\beta}, \ if \ R_i \geq \frac{\alpha - a_j}{\gamma} \end{cases}, \tag{4}$$

where

$$R_i = \frac{2\beta(\alpha - a_i) - \gamma(\alpha - a_j)}{4\beta^2 - \gamma^2},$$

with $i, j \in \{1, 2\}$ and $i \neq j$. Hence, if $R_i \leq 0$ the firm F_j is at monopoly output level and, conversely, if $R_i \geq (\alpha - a_j)/\gamma$ the firm F_i is at monopoly output level and for intermediate values $0 \leq R_i < (\alpha - a_j)/\gamma$, both firms have positive optimal output levels and so we are in the presence of duopoly competition. From now on, we will always consider that both firms choose their Nash equilibrium output (q_1^*, q_2^*).

2.3 New production costs

The sets of possible new production costs for firms F_1 and F_2, given initial production costs c_1 and c_2 are, respectively,

$$A_1 = A_1(c_1, c_2) = [b_1, c_1] \quad \text{and} \quad A_2 = A_2(c_1, c_2) = [b_2, c_2],$$

where $b_i = c_i - \epsilon(c_i - c_L)$, for $i \in \{1, 2\}$. The R&D programs a_1 and a_2 of the firms determine a bijection between the investment region $\mathbb{R}_0^+ \times \mathbb{R}_0^+$ of both firms and the new production costs region $A_1 \times A_2$, given by the map

$$\begin{array}{rcccc} \mathbf{a} = (a_1, a_2) & : & \mathbb{R}_0^+ \times \mathbb{R}_0^+ & \rightarrow & A_1 \times A_2 \\ & & (v_1, v_2) & \mapsto & (a_1(v_1), a_2(v_2)) \end{array}$$

where

$$a_i(v_i) = c_i - \frac{\eta_i v_i}{\lambda_i + v_i}.$$

We denote by $W = (W_1, W_2) : \mathbf{a}(\mathbb{R}_0^+ \times \mathbb{R}_0^+) \rightarrow \mathbb{R}_0^+ \times \mathbb{R}_0^+$

$$W_i(a_i) = \frac{\lambda_i(c_i - a_i)}{\eta_i - (c_i - a_i)}$$

the inverse map of $\mathbf{a}$.

The new production costs region can be decomposed, at most, in three disconnected economical regions characterized by the optimal output level of the firms:

M_1 The *monopoly region* M_1 of firm F_1 that is characterized by the optimal output level of firm F_1 being the monopoly output and, so, the optimal output level of firm F_2 is zero;

D The *duopoly region* D that is characterized by the optimal output levels of both firms being non-zero and, so, below their monopoly output levels;

M_2 The *monopoly region* M_2 of firm F_2 that is characterized by the optimal output level of firm F_2 being the monopoly output and, so, the optimal output level of firm F_1 is zero.

The boundaries between the duopoly region D and the monopoly region M_i are l_{M_i} with $i \in \{1, 2\}$ and are presented, explicitly in [8, 9, 10].

In equilibrium, i.e. when both firms choose their optimal output levels, the profit function $\pi_i : A_i \times A_j \to \mathbb{R}$ of firm F_i, in terms of its new production costs (a_1, a_2), is a piecewise smooth continuous function given by

$$\pi_i(a_1, a_2) = \begin{cases} \pi_{i,M_i}, & \text{if} \quad (a_1, a_2) \in M_i \\ \\ \pi_{i,D}, & \text{if} \quad (a_1, a_2) \in D \\ \\ -W_i(a_1, a_2), & \text{if} \quad (a_1, a_2) \in M_j \end{cases},$$

where

$$\begin{aligned} \pi_{i,M_i} &= \pi_{i,M_i}(a_1, a_2; c_1, c_2) = \frac{(\alpha - a_i)^2}{4\beta} - W_i(a_1, a_2), \\ \pi_{i,D} &= \pi_{i,D}(a_1, a_2; c_1, c_2) = \beta \left(\frac{2\beta(\alpha - a_i) - \gamma(\alpha - a_j)}{4\beta^2 - \gamma^2} \right)^2 - W_i(a_1, a_2). \end{aligned}$$

2.4 Nash investment regions

Let $V_i(v_j)$ be the best investment response function of firm F_i to a given investment v_j of firm F_j. The best investment response function $V_i : \mathbb{R}_0^+ \to$

$\mathbb{R}_0^+$ of firm F_i is explicitly computed in [8]. Note that the best investment response function $V_i : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ can be a multi-valued function.

Let c_L be the minimum attainable production cost and α the value to buyers. Given production costs $(c_1, c_2) \in [c_L, \alpha] \times [c_L, \alpha]$, the *Nash investment equilibria* $(v_1, v_2) \in \mathbb{R}_0^+ \times \mathbb{R}_0^+$ are the solutions of the system

$$\begin{cases} v_1 = V_1(v_2) \\ v_2 = V_2(v_1) \end{cases}$$

where V_1 and V_2 are the best investment response functions computed in the previous sections.

All the results presented, hold in an open region of parameters $(c_L, \epsilon, \alpha, \beta, \gamma)$ containing the point $(4, 0.2, 10, 0.013, 0.013)$. The parameter λ_i that measures the efficiency of the R&D investment program, i.e the smaller the λ_i, the more efficient the R&D investment program, is the parameter we are interested in studying. In the case we referred to as symmetric efficient, λ_i is equal to 10; in symmetric inefficient case, λ_i is equal to 20; in the asymmetric case, λ_1 is equal to 30 and λ_2 is equal to 10.

We observe that the Nash investment equilibria consists of a unique, or two, or three points depending upon the pair of initial production costs, as we will explain throughout the chapter. The set of all Nash investment equilibria form the *Nash investment equilibrium set*. We discuss the Nash investment equilibria by considering the following three regions of production costs:

C the *competitive Nash investment region* C that is characterized by both firms investing;

S_i the *single Nash investment region* S_i that is characterized by only one of the firms investing;

N the *nil Nash investment region* N that is characterized by neither of the firms investing.

3 Nash investment equilibria

In this section we compare the Nash investment equilibria dependency on the parameters β, ϵ and γ . We observe the existence, in the three distinct cases, of four different regions of Nash investment equilibria: a competitive Nash investment region C where both firms invest, a single Nash investment region S_1 for firm F_1, where just firm F_1 invests, a single Nash investment region S_2 for firm F_2, where just firm F_2 invests, and a nil Nash investment region N, where neither of the firms invest.

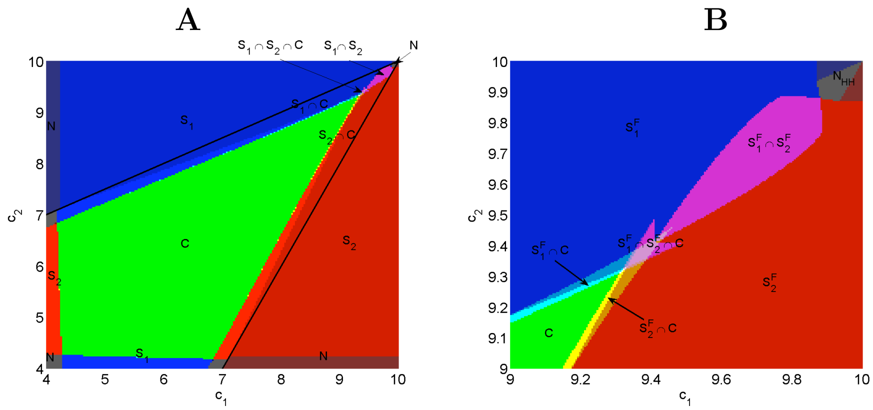

Figure 1: **A**: Full characterization of the Nash investment regions in terms of the firms' initial production costs (c_1, c_2). The monopoly lines l_{M_i} are colored black. The nil Nash investment region N is colored grey. The single Nash investment regions S_1 and S_2 are colored blue and red, respectively. The competitive Nash investment region C is colored green. The region where S_1 and S_2 intersect are colored pink, the region where S_1 and C intersect are colored light blue and the region where S_2 and C intersect are colored yellow. The region where the regions S_1, S_2 and C intersect are colored light grey. **B**: Nash investment regions in the high production costs region, $c_i \in [9, 10]$.

Let $R = [c_L, \alpha] \times [c_L, \alpha]$ be the region of all possible pairs of productions costs (c_1, c_2). Let $A^c = R - A$ be the complementary of A in R. The intersection between different Nash investment regions can be non-empty: (i) the intersection $R_{S_1 \cap S_2} = S_1 \cap S_2 \cap C^c$ between the *single Nash investments regions* S_1 and S_2 can be non empty; (ii) the intersection $R_{C \cap S_i} = C \cap S_i \cap S_j^c$ with $i \neq j$ between the *competitive Nash investment region* C and the *single Nash investment region* S_i can be non-empty; (iii) the intersection $R_{S_1 \cap C \cap S_2} = S_1 \cap C \cap S_2$ between the *competitive Nash investment region* C and the *single Nash investment regions* S_1 and S_2 can be non-empty.

Let us consider the region of high production costs, that can correspond to the production of new technologies, where there are multiple Nash investment equilibria. In this section, we exhibit the production costs that correspond to the existence of multiple Nash investment equilibria. We observe that the intersection $R_{S_1 \cap S_2} = S_1 \cap S_2 \cap C^c$ between the *single Nash investments regions* S_1 and S_2 is non empty. Thus, in this region we have two equilibria: a single Nash investment equilibrium to firm F_1 and a single Nash investment equilibrium to firm F_2. We also observe that the intersection $R_{C \cap S_i} = C \cap S_i \cap S_j^c$ with $i \neq j$ between the *competitive Nash investment region* C and the *single Nash investment region* S_i is non-empty. Therefore, in this region we have two Nash investment equilibria, one single Nash investment equilibrium for firm F_1 and a competitive Nash investment equilibrium. Finally, we see that the intersection $R_{S_1 \cap C \cap S_2} = S_1 \cap C \cap S_2$ between the *competitive Nash investment region* C and the *single Nash investment regions* S_1 and S_2 is non-empty. Thus, we have, simultaneously, a competitive equilibrium, a single favorable Nash investment equilibrium for firm F_1 and a single Nash investment equilibrium for firm F_2. This aspect enhances the high complexity of the R&D strategies of the firms, for high values of initial production costs.

We will observe the effect on the regions when we change the following parameters: $\beta = \beta_1 = \beta_2$, (lower values of β correspond to higher quantities being produced); $\epsilon = \epsilon_1 = \epsilon_2$, (higher values of ϵ correspond to more efficient R&D projects to reduce the production costs); and $\hat{\gamma} = \frac{\gamma}{\beta_1 \beta_2}$, (lower values of $\hat{\gamma}$ correspond to higher product differentiation). Their default values will be $\beta = 0.0013$, $\epsilon = 0.2$ and $\hat{\gamma} = 1$.

We observe in Figure 2 that when β increases, the region of competitive investment decreases. The regions with multiple Nash Equilibria are present for intermediate values of β.

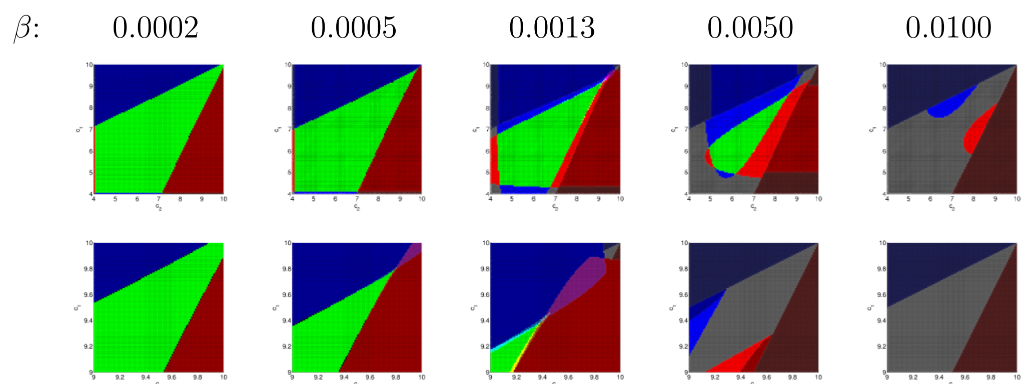

Figure 2: Effect of β in the regions of Nash investment equilibria. Top row: production costs in $[4, 10]$. Bottom row: zoom with production costs in $[9, 10]$. $\epsilon = 0.2$ and $\hat{\gamma} = 1$.

We observe in Figure 3 that when ϵ decreases, the region of competitive investment decreases. The regions with multiple Nash Equilibria are present for intermediate values of ϵ.

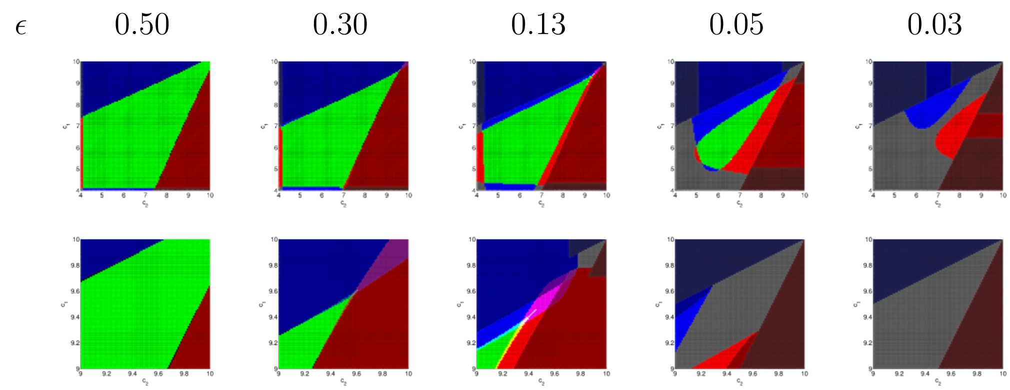

Figure 3: Effect of ϵ in the regions of Nash investment equilibria. Top row: production costs in $[4, 10]$. Bottom row: zoom with production costs in $[9, 10]$. $\beta = 0.0013$ and $\hat{\gamma} = 1$.

We observe in Figure 4 that when γ decreases, the region of competitive investment increases and the regions with multiple Nash Equilibria shrink.

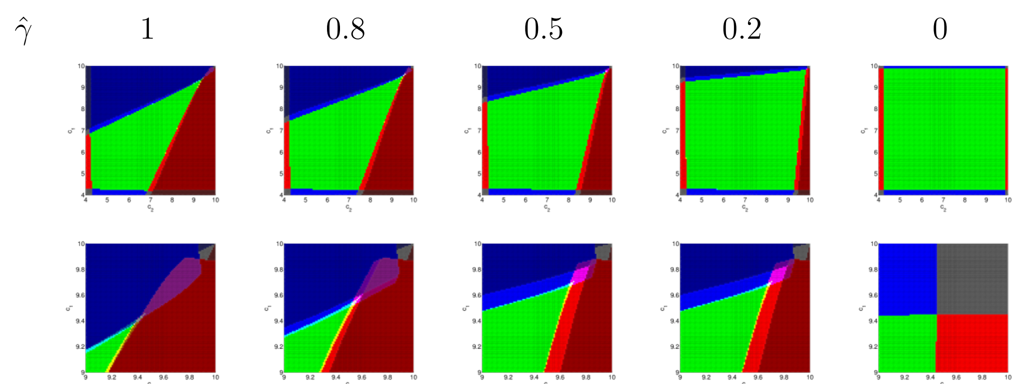

Figure 4: Effect of γ in the regions of Nash investment equilibria. Top row: production costs in $[4, 10]$. Bottom row: zoom with production costs in $[9, 10]$. $\beta = 0.0013$ and $\epsilon = 0.2$.

4 Conclusions

We used an R&D investment function inspired in the logistic equation introduced in [8] and found all Perfect Nash investment equilibria of the Cournot competition model with R&D programs. We described four main economic regions for the R&D deterministic dynamics corresponding to distinct perfect Nash equilibria: a competitive Nash investment region C where both firms invest, a single Nash investment region for firm F_1, S_1, where just firm F_1 invests, a single Nash investment region for firm F_2, S_2, where just firm F_2 invests, and a nil Nash investment region N where neither of the firms invest. The following conclusions are valid in some parameter region of our model. We showed, following [8], the existence of regions where the Nash investment equilibrium are not unique: the intersection $R_{S_1 \cap S_2}$ between the single Nash investment region S_1 and the single Nash investment region S_2 is non empty; the intersection $R_{S_i \cap C}$, with between the single Nash investment region S_i and the competitive Nash investment region C is non empty; the intersection $R_{S_1 \cap C \cap S_2}$ between the single Nash investment region S_1, the single Nash investment region S_2 and the competitive Nash investment region C is non empty. In this article we observed the persistence of these regions and described how these regions change as we change the parameters β, ϵ and γ of the R&D programs of both firms.

Acknowledgments

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