Of Course Collusion Should be Prosecuted. But Maybe ... Or (The case for international antitrust agreements)

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Abstract

We study the incentives of competition authorities to prosecute collusive practices of domestic and foreign firms. For that purpose, we develop a model of multi-market contact between two firms that can engage in collusion in two countries. In each country, there is a competition authority with a mandate to maximize national welfare. Each competition authority decides its prosecution policy at the beginning of time and commits to it. In equilibrium, the ownership distribution of the firms (domestic versus foreign) affects prosecution policies. The country that does not own the firms prosecutes them as soon as information of collusion becomes available. On the contrary, the country that owns the firms has an incentive to protect their profits in foreign markets delaying prosecution. This strategic delay is valuable because it contains the information spreading that could trigger prosecution in the foreign country. Prosecution delays, however, are not optimal from the point of view of global welfare, something that could be solved through the integration of the competition authorities. The country of origin of the firms would nevertheless oppose integration. Finally, in a multi-industry setting, both countries delay prosecuting domestic firms, which again is not optimal from the point of view of global welfare. Moreover, in a multi-industry setting, both countries can be better off under integration.

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1 Introduction

The rise of globalization has increased the importance of anticompetitive conducts in internationals settings.[1] National competition authorities have reacted to this new environment paying more attention to collusive practices involving multinational companies. Specifically, large foreign companies seem to be a frequent target of antitrust enforcement. For example, Garrett (2014) documents that between 2001 and 2012, 45 percent of antitrust prosecutions carried out by the US Department of Justice were to foreign companies (namely 78 out of 175 companies). Data on Sherman Act violations yielding fines

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[1]See, for example, Connor (2004), Connor and Helmers (2007), and Barnett (2007).

over 10 million dollars also shows that, between 1995 and 2017, only 19 of the 139 firms fined were domestic companies.[2] The challenges associated with international antitrust enforcement have also lead to proposals for increasing international cooperation among competition authorities.[3]

Despite all these changes and the growing importance of international antitrust, most of the formal literature on the economics of antitrust has been focused on closed economies, that is, situations in which consumers, firms and competition authorities belong to and operate within a single country. In such environment, the objective of the antitrust authority is to promote domestic welfare, usually defined as the (weighted) sum of profits and consumer surpluses.[4] Unfortunately, this kind of analysis is incomplete in the context of open economies, in which firms operate in several countries. One immediate recognizable bias lies on the fact that competition authorities target national welfare, disregarding potential harm to foreign consumers stemming from the anticompetitive behavior of domestic firms. This paper studies the incentives of competition authorities to prosecute domestic and foreign firms involved in anticompetitive behavior; more precisely, in price-setting agreements. In particular, we show that a benevolent competition authority with a mandate to maximize national welfare might prefer to do not enforce antitrust policies, delaying the prosecution of domestic firms even after there is sufficient evidence of collusion.

The logic behind this result is as follows. Consider a group of firms owned by citizens of one country that is organizing collusion in domestic and foreign markets. Suppose that the domestic competition authority has collected the required information to file a collusion case against the firms. Moreover, assume that prosecution will lead to a conviction, plea deal or judicial agreement, forcing firms to pay a fine, stop collusive practices and start competing in the domestic market. There is no doubt that dismantling collusion will induce an increase in consumer surplus for domestic consumers that more than compensate the reduction in profits from domestic markets suffered by domestic firms. However, this does not exhaust the welfare implications of prosecution when firms are also operating in other countries. If foreign competition authorities learn from the domestic lawsuit how to prove collusion in their own territories, prosecution in the domestic country will trigger prosecution in foreign markets as well, hammering the profits of domestic firms in foreign markets. If collusive profits in foreign markets are large enough, a benevolent competition authority will prefer to turn a blind eye to domestic collusion, at least until firms are independently discovered and prosecuted in foreign markets.

In order to formally study the strategic enforcement of antitrust policies in a global economy, we develop a simple two-country model of collusion and antitrust policy. In our model there is an industry composed by two multinational firms which operate in both countries. In each country there is a competition authority in charge of enforcing antitrust laws. At the beginning of the game, each competition authority selects a prosecution policy to which it commits for the rest of the game. In every period each competition authority receives a signal on the conduct of the firms operating in its jurisdiction and, using this information, implements its prosecution policy. Given the prosecution policies, in each period firms decide whether to collude or compete. We characterize the equilibrium prosecution policies and collusion decisions for different scenarios, varying the information that competition authorities have and the collusive schemes of the firms.

In our baseline model we consider a scenario where country A owns multinational firms operating in both countries. We assume that country A's competition authority receives a signal that indicates

[2] See Antitrust Division (2017).

[3] See Barnett (2007).

[4] For discussions about welfare measures in the context of competition policy see Motta (2004) and Carlton (2007).

whether collusion is present in country A and, given this information, decides to prosecute (and fine) the firms or not. The competition authority of country B does not receive any independent signal of collusion in country B, but prosecution by country A is informative and can be used as a signal. In other words, the competition authority of country B must rely on the prosecution decision of country A to obtain information about collusion in its own market before it can decide whether or not to prosecute. Finally, we assume that country A implements its prosecution policy before country B. This Stackelberg structure implies that the competition authority of country A takes into account the reaction of the competition authority of country B. In this scenario, we prove that country B always prosecutes the firms as soon as there is information of prosecution in country A. The reason is that country B does not own the firms and, hence, it only takes into account the consumer surplus of country B's consumers. Regarding country A, we prove that it only prosecutes the firms when the gain in consumer surplus for country A's consumers exceeds the reduction in profits (including fines) from domestic and foreign markets for country A's firms. This condition can be expressed in as a discount factor higher than a certain threshold. The subgame perfect equilibrium of the game obtained by combining these two results, is such that, for low discount factors, neither competition authority prosecutes the companies: one due to lack of information and the other to protect the foreign profits of domestic firms.

We explore several variations of our baseline model including: the type of collusive schemes available to the firms (global versus market specific collusion); the punishment that firms can use to sustain collusion (in each market separately or in both markets); and the information prosecution spillover (prosecution in country A is indefinitely or only temporarily informative for country B). In all these variations of the baseline model, our main result does not change. For low discount factors (the threshold is always the same), the competition authority of country A never prosecutes the firms if they are colluding in both markets. For discount factors above the threshold, different specifications bring new results. Under market specific collusion, firms have the option to collude only in country B, avoiding prosecution indefinitely even when competition authority A prefers to prosecute collusion if firms are colluding only in country A or in both countries. This suggests that, when market specific collusion decisions are possible, multinational companies might be competing in their home-country (usually, a developed country with a professional and well-funded competition authority), but colluding in foreign markets in developing countries, which do not have a competition authority capable of independently detecting collusion. Regarding information prosecution spillovers, if prosecution in country A is only temporarily informative for country B, then firms have strong incentives to organize sequential collusion, first colluding in country A until they are detected and prosecuted and, afterward, begin colluding in country B.

We then proceed to study an extension of the baseline model in which we allow country B to detect collusion independently. In particular, in this setting country B has two channels to detect collusion: (i) through the prosecution decision of country A and (ii) through its own detection signal. This extension brings a more realistic scenario under which the competition authority of country B has some capacity to detect collusion on its own. It also allows us to formally compute equilibrium delays in prosecution. In this scenario, country A's decision of not prosecuting is less valuable, since foreign collusion profits of country A's firms cannot be indefinitely protected with a lenient prosecution decision in country A. Therefore, country A will be more willing to prosecute the firms as soon as collusion is detected, reducing the international bias of the prosecution decision. Furthermore, even if country A prefers to do not prosecute the firms when they are colluding in both countries, eventually, country B will detect and prosecute collusion on its own. When this happens, country A will prosecute collusion too, since there are no more collusive profits to protect in country B. In other words, while in the baseline model, when country A does not prosecute the firms, prosecution delays are infinite, in this extension, delays exist but are finite. To best of our knowledge, this is the first paper that formally models and obtains prosecution delays as an equilibrium.

We also extend the model to introduce endogenous detection probabilities. In particular, we do not only allow competition authorities to decide prosecution, but also to invest resources to determine the probability with which they detect collusion. As a consequence, each competition authority balances the marginal benefits and costs of detecting collusion. We show that, for a given marginal cost of detection, optimal probability of detection as chosen by the competition authorities depends on the ownership distribution of the firms. A competition authority with a mandate to maximize national welfare selects a higher detection probability if the market is served only by foreign firms than if it is served by purely domestic firms, and it selects an even lower probability if the market is served by domestic firms that also operate in foreign markets. Our model augmented with endogenous detection probabilities generates cross-sector implications on the enforcement of antitrust policies. Sectors operated by foreign companies are expected to be prime targets for antitrust cases, while sectors dominated by domestic companies with large foreign operations are expected to receive a more lenient treatment. To the best of our knowledge, this is the first model that generates predictions of antitrust enforcement based on the ownership origin of the firms and its participation in foreign markets.

Finally, in another extension, we incorporate a second industry to the model. We assume that country A owns the firms in industry x, country B owns the firms in industry y, and collusion detection works as in the one-industry model (with the roles of countries A and B reversed for industry y). In this scenario, a country can lead investigations in one industry and be a follower for the other. We can then study cross investigations, an interesting scenario for similar developed countries with multinational companies. More importantly, the multi-industry extension opens the door to mutual gains from the integration of competition authorities, something that cannot happen in the one-industry model. Indeed, in the one-industry model, while the equilibrium does not maximize the combined welfare of the countries (as it involves a strategic prosecution delay), there is not much room for an international agreement to fix the problem. Only integration of the competition authorities would work, but one country will be hurt by integration and, hence, will oppose it. This is not necessarily the case in a multi-industry world, in which both countries can be better off under an integrated competition authority that maximizes the combined welfare. The intuition is that, under integration, each country loses the collusion profits in the foreign market, but it gains the increase in consumer surpluses in both domestic markets. When countries are of different sizes, this might not be enough to avoid winners and losers from integration, but in the case of perfectly symmetric countries, we prove that both countries gain with an integrated competition authority.

There are several papers related to this work. Although most of the literature on competition policy is focused on closed economies, we are not the first to analyze international competition policy. For example, Barros and Cabral (1994), Head and Ries (1997) and Neven and Röller (2005) study international competition policy, but in the context of mergers and acquisitions. Our paper also belongs to the literature on multi-market collusion. Bernheim and Whinston (1990) were the first to formalize the idea of simultaneous collusion in multiple markets. They show that under asymmetric markets a larger set of collusive outcomes can be sustained in a multi-market setup. Bond (2004) and Bond and Syropoulos

(2008) study multi-market collusion in open economies. None of these papers, however, explore antitrust enforcement.

Closer to our work are Choi and Gerlach (2012a,b, 2013). Choi and Gerlach (2013) employ a multi-market model with symmetric countries to study the relationship between demand linkages and antitrust enforcement. They show that antitrust enforcement in one market spills over to other markets in a way that depends on whether the products are complements or substitutes. Choi and Gerlach (2013) serves as the basis for Choi and Gerlach (2012b), where they study the case of firms producing substitute products in different countries when there are trade frictions and local competition authorities have prosecution costs. They show that enforcement is non-monotonic with respect to trade integration. In particular, cartel enforcement is high if the economies are either closely integrated or trade costs are very high. Finally, they compare two regimes, one with local competition authorities and the other with a global competition authority, and identify two sources of inefficiency associated with local prosecution, namely, decentralized information and cross-market externalities. Finally, Choi and Gerlach (2012a) study multi-market collusion with leniency and different information sharing policies between competition authorities. In their model, competition authorities detect collusion and, once an investigation starts, firms are allowed to apply for leniency. If firms do not apply for leniency, competition authorities face a chance of unsuccessful prosecution. They show that a policy that involves (partial) information sharing between competition authorities increases the probability of detection and successful prosecution in each jurisdiction.

Our work departs from Choi and Gerlach (2012a) and Choi and Gerlach (2012b) in several important ways. First, we assume that competition authorities seek to maximize national welfare instead of the consumer surplus of domestic consumers. This is crucial when we are considering multinational firms because the ownership distribution of the firms matters for how profits are counted in national welfare. An antitrust policy that ignores the profits that national firms obtain in foreign markets prosecutes them more intensively than what is optimal for national welfare. Moreover, in our model, assuming national welfare as the measure of welfare employed by the competition authorities is key to induce strategic prosecution delays. Second, we employ an informational structure that allows us to model a leader and a follower in antitrust enforcement instead of having competition authorities deciding simultaneously. This is a relevant scenario to explore, given that the country of origin of the firms has more accessible information regarding the collusive behavior of the firms. It also captures the asymmetry in resources and capabilities between competition authorities in developed and developing countries.[5] More importantly, this information structure naturally pushes the leading country to internalize the potential information spillovers of its prosecution decisions. Third, in our model the nature of the relationships between products is not relevant for the sustainability of collusive agreements. The linkage in our setting is purely through an information channel. Prosecution in the country of origin of the firms reveals valuable information for the foreign competition authority to also build a solid antitrust case abroad. Finally, in our model integrating antitrust policy has cross-country distributive effects. In particular, we show that in a one-industry model, one country is always worse off under integration.

Antitrust enforcement in several jurisdictions has also been studied from a collective action approach. For example, Feinberg and Husted (2013) empirically explore free riding on antitrust enforcement between U.S. states. They find that the number of states participating in the litigation process promotes the free-riding behavior, while the resources available to the state government ameliorates it. Additionally,

[5] See, for example, Levenstein and Suslow (2003).

they use the number of horizontal conspiracies as a measurement of case complexity and find that it is associated with delays in prosecution efforts. Our model generates equilibrium strategic delays in prosecution through a completely different channel. In our model, it is not the case that a jurisdiction prefers to wait until another jurisdiction pays the cost of dissolving the cartels. The problem is that the jurisdiction of origin of the firms might prefer to delay prosecution to protect the profits of its firms in other jurisdictions.

Finally, the way we approach antitrust enforcement in an international setting has similarities with the literature on the terms of trade approach to international trade agreements (Bagwell and Staiger (1999)). In the context of trade policy, a country that seeks to maximize national welfare has an incentive to impose a tariff in order to improve its terms of trade at the expense of its trade partners. Analogously, the country of origin of the firms organizing collusion has an incentive to postpone prosecution in order to protect the market power of domestic firms in foreign markets. In the context of trade policy, trade agreements could make both countries better off inducing a mutual reduction in tariffs. In our two-industry model, the integration of antitrust policy in a single competition authority could make both countries better off eliminating prosecution delays in both industries.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 characterizes the equilibrium under our baseline scenario, that is, when country A owns both firms and country B cannot detect collusion on its own. Section 4 characterizes the equilibrium, when country B can also detect collusion on its own. Section 5 endogenizes detection probabilities. Section 6 extends the model to a multi-industry setting and explores the implications for international antitrust agreements and integration. Section 7 concludes. All proofs can be found in the Appendix.

2 A simple model of antitrust policy

This section develops a simple model of collusion and antitrust policy when the firms involved are multinational corporations operating in several countries. In particular, consider 2 multinational firms ($i \in \{1, 2\}$) which operate in 2 countries ($j \in \{A, B\}$) and must decide whether to collude or compete. In each country there is a competition authority which has the power to fine firms if collusion is detected. Time is infinite, discrete and indexed by $t \in \{0, 1, ...\}$. All agents have a common discount factor $\delta \in (0, 1)$.

At the beginning of period $t = 0$, the competition authorities simultaneously select their prosecution policies, to which they commit to for the remaining of the game. After observing the prosecution policies selected by the competition authorities, firms play an infinitely repeated game, where the stage game is given by:

1. Both firms simultaneously choose to collude or compete.
2. Nature sends a signal to the competition authority of country A regarding the behavior of the firms in country A. Upon observing the signal, the competition authority of country A implements its prosecution policy.
3. Nature sends a signal to the competition authority of country B regarding the behavior of the firms in country B. Upon observing the signal, the competition authority of country B implements its prosecution policy.

This timing is compatible with different signalling structures and contents. In particular, note that the signal received by the competition authority of country B could depend on the prosecution decision implemented by the competition authority of country A.

2.1 Firms, profits and ownership distribution

Let $\left(a_t^{i,A}, a_t^{i,B}\right)$ denote the decision of firm i in period t, where $a_t^{i,j} = 1$ indicates that firm i chooses to collude in country j and $a_t^{i,j} = 0$ indicates that firm i chooses to compete in country j. Let $\pi_t^{i,j}$ denote the profits earned by firm i from its operations in country j before paying any fine. Specifically, assume that profits in country j are given by:

$$\pi_t^{i,j} = \begin{cases} \pi^{c,j} & \textit{if } a_t^{i,j} = a_t^{-i,j} = 1, \\ \pi^{d,j} & \textit{if } a_t^{i,j} = 0 \textit{ and } a_t^{-i,j} = 1, \\ 0 & \textit{otherwise}, \end{cases}$$

where $\pi^{d,j} > \pi^{c,j} > 0$. Thus, if both firms collude in country j, each obtains collusion profits ($a_t^{i,j} = a_t^{-i,j} = 1$ implies $\pi_t^{i,j} = \pi_t^{-i,j} = \pi^{c,j} > 0$). However, each firm has a short term incentive to deviate from collusion, which leaves no profits for the rival ($a_t^{i,j} = 0$ and $a_t^{-i,j} = 1$ implies $\pi_t^{i,j} = \pi^{d,j} > \pi^{c,j}$ and $\pi_t^{-i,j} = 0$). Finally, competition fully dissipates profits ($a_t^{i,j} = a_t^{-i,j} = 0$ implies $\pi_t^{i,j} = \pi_t^{-i,j} = 0$).[6]

Each firm selects $\left\{\left(a_t^{i,A}, a_t^{i,B}\right)\right\}_{t=0}^{\infty}$ in order to maximize the expected discounted profits net of fines, i.e.:

$$\Pi_t^i = \mathbf{E}_t\left[\sum\nolimits_{\tau=t}^{\infty} \delta^{\tau-t}\left(\pi_\tau^{i,A} - f_\tau^{i,A} + \pi_\tau^{i,B} - f_\tau^{i,B}\right)\right],$$

where $f_t^{i,j}$ denote the fine imposed in period t to firm i by the competition authority of country j.

Finally, let $\sigma^{i,j} \in [0,1]$ be the share of firm i owned by citizens from country j. Naturally, $\sigma^{i,A} + \sigma^{i,B} = 1$ for $i = 1, 2$.

2.2 Consumers

The consumer surplus obtained by consumers from country j in period t is $S_t^j = S^{c,j}$ when $a_t^{i,j} = a_t^{-i,j} = 1$ and $S_t^j = S^{com,j}$, otherwise, where $S^{c,j}$ and $S^{com,j}$ are the consumer surpluses in country j under collusion and competition, respectively. We assume that $S^{com,j} > S^{c,j} + 2\pi^{c,j}$ for all j, i.e., in each country, the aggregate surplus under competition ($S^{com,j}$) is higher than under collusion ($S^{c,j} + 2\pi^{c,j}$).

2.3 Competition authorities, collusion detection and antitrust prosecution

In each period, immediately after firms make their decisions, the competition authority of each country receives a signal about the behavior of the firms. Let $c_t^j \in \{0,1\}$ denote the signal received by the competition authority of country j. $c_t^j = 1$ ($c_t^j = 0$) indicates that the competition authority of country

[6] The underlying assumption is that the firms sell homogeneous products and compete in prices. Hence, when reverting to competition upon either a choice of not colluding or a failure of collusion, the profits of the firms are zero. Alternatively, in the case that firms compete in quantities or in prices with differentiated products, we can assume that profits are normalized, i.e., all profits are relative to competitive profits. These alternative assumptions do not affect the analysis.

j detects (does not detect) collusion in period t. The signal conveys information about collusion only if firms are effectively colluding. Otherwise, the competition authority receives a no collusion signal.

After observing the signal, the competition authorities decide whether or not to prosecute the firms, following the policies that they establish in period $t = 0$. Let $p_t^j \in \{0, 1\}$ denote the prosecution action implemented by the competition authority j in period t, where $p_t^j = 1$ indicates prosecution and $p_t^j = 0$ indicates no prosecution. We assume that a competition authority cannot prosecute without detecting evidence of collusion. Thus, when $c_t^j = 0$, it must be the case that $p_t^j = 0$. If collusion is detected, i.e., $c_t^j = 1$, and the competition authority prosecutes the firms, i.e., $p_t^j = 1$, then, each firm must pay a fine $f_t^{i,j} = f^j > \pi^{c,j}$ and they are forced to compete in all subsequent periods.[7]

At the beginning of period $t = 0$, each competition authority chooses a prosecution policy in order to maximize the expected discounted welfare of the country. Thus, the competition authority of country j maximizes

$$W_0^j = \mathbf{E}_0 \left[\sum\nolimits_{\tau=0}^{\infty} \delta^\tau \left(S_\tau^j + f_\tau^{1,A} + f_\tau^{2,A} \right) \right] + \mathbf{E}_0 \left[\sigma^{1,j} \sum\nolimits_{\tau=0}^{\infty} \delta^\tau \left(\pi_\tau^{1,A} - f_\tau^{1,A} + \pi_\tau^{1,B} - f_\tau^{1,B} \right) \right] +$$
$$+ \mathbf{E}_0 \left[\sigma^{2,j} \sum\nolimits_{\tau=0}^{\infty} \delta^\tau \left(\pi_\tau^{2,A} - f_\tau^{2,A} + \pi_\tau^{2,B} - f_\tau^{2,B} \right) \right],$$

where the first term is the expected discounted consumer surplus in country j plus the expected discounted fines collected by the competition authority of country j, and the second and third terms are the shares of the expected discounted profits of firms 1 and 2 accruing to citizens of country j.

3 Equilibrium analysis I: A owns both firms and B cannot detect collusion on its own

This section studies the equilibrium of the model when both firms are owned by citizens of country A and the competition authority of country B is not able to detect collusion on its own, but it can learn from the prosecution process in country A. Formally, regarding the ownership distribution of the firms, we assume throughout this section that $\sigma^{1,A} = \sigma^{2,A} = 1$. This implies that the profits of both firms obtained in country B are accounted for in the welfare of country A, while country B's welfare only includes the consumer surplus in country B. With respect to collusion detection, consider the following assumption (which we relax later) regarding the signals received by each competition authority.

Assumption 1 *The signals received by the competition authorities are:*

$$c_t^A = \begin{cases} 1 \text{ with probability } \alpha^A & \text{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 \text{ with probability } \left(1 - \alpha^A\right) & \text{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 & \text{otherwise.} \end{cases}$$
$$c_t^B = \begin{cases} 1 & \text{if } a_t^{1,B} = a_t^{2,B} = 1 \text{ and } p_\tau^A = 1 \text{ for some } \tau \leq t, \\ 0 & \text{otherwise.} \end{cases}$$

[7] Note that we are implicitly assuming that the actions of the firms $a_t^{i,j}$ are public information, i.e., known by the firms and also by the competition authorities, but that knowing that there is collusion is not the same as having enough evidence to prosecute the firms. Only when $c_t^j = 1$, the competition authority of country j has the required information to successfully prosecute the firms.

Assumption 1 states that the competition authority of country A can detect collusion on its own, while the competition authority of country B must rely on observing that country A has prosecuted the firms in order to detect collusion in country B.[8]

The following lemmas formally characterize the prosecution policies of both countries.

Lemma 1 *Suppose that Assumption 1 holds. Then, the competition authority of country B prosecutes the firms as soon as collusion is detected in country B.* ■

The intuition behind Lemma 1 is very simple. Since the competition authority of country B only benefits from the consumer surplus in country B and the fines (which are paid by foreign firms), it immediately prosecutes the firms as soon as collusion is detected.

Lemma 2 *Suppose that Assumption 1 holds. Assume that the competition authority of country A detects collusion, i.e., $c_t^A = 1$.*

1. *If the firms are not colluding in country B, then A always prosecutes the firms.*
2. *If the firms are also colluding in country B, then A prosecutes the firms if and only if*

$$\delta \geq \bar{\bar{\delta}} = \frac{2f^B}{\Delta S^A - 2\pi^{c,A} + 2f^B - 2\pi^{c,B}}, \tag{1}$$

where $\Delta S^A = S^{com,A} - S^{c,A}$. Alternatively, (1) can be written as $\delta\left(\Delta S^A - 2\pi^{c,A}\right) \geq 2\left[\delta\pi^{c,B} + (1-\delta) f^B\right]$. ■

The intuition behind Lemma 2 is as follows. When firms are colluding in both countries, if the competition authority of country A prosecutes the firms, this will trigger prosecution in country B. As a consequence, firms owned by shareholders from country A will have to pay fines in country B and, in the future, they will be forced to compete, which eliminates their profits from collusion in country B. Thus, prosecution in country A increases the total surplus in country A, but it reduces the profits of the firms in country B. When firms are only colluding in country A, the second effect disappears and, hence, the best policy for the competition authority of country A is to prosecute the firms when collusion is detected.

We now proceed to study the equilibrium collusion decisions of the firms given the antitrust policies implemented by the competition authorities of both countries. The following propositions characterize firms' decisions when they can only make global collusion decisions as well as when collusion decisions are market specific. Since firms play a repeated game there might be multiple outcomes that can be sustained as a subgame perfect Nash equilibrium. To deal with this multiplicity, we assume that the firms always coordinate in their most preferred equilibrium, i.e., the one that generates the highest expected profits for each firm. Note that this does not mean that we ignore the other possible equilibria. On the contrary, for each set of parameters we deduce and compare all possible equilibria and select the equilibrium outcome with the highest expected profits.

[8]The assumed ownership distribution of the firms together with Assumption 1 could capture the following situation. Consider two multinational firms whose shareholders are from a developed country A, which counts with a professional competition authority with the capacity to detect collusion. The firms also operate in a developing country B, whose competition authority does not have the resources and/or the expertise to detect and prosecute collusion on its own. However, if the competition authority of country B observes country A prosecuting the firms in country A, it will learn how to detect and prosecute collusion in country B

3.1 Global collusion decisions

Proposition 1 explores a simple environment in which firms make global collusion decisions. That is, firms can collude in both countries or in none of them, but they cannot collude in one country and not in the other.

Proposition 1 *Suppose that Assumption 1 holds, firms can either collude in both countries or in none of them and they coordinate in their best equilibrium. Furthermore, assume that* $\pi^{c,A} + \pi^{c,B} > (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)$.

1. *Suppose that* $\delta < \bar{\delta}$. *Then, firms collude in both countries and they are never prosecuted.*
2. *Suppose that* $\delta \geq \bar{\delta}$. *Let* $\bar{\alpha}^1 = \frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{f^A+f^B+\delta\left(\pi^{d,A}+\pi^{d,B}\right)}$.
    (a) *If* $\alpha^A \leq \bar{\alpha}^1$, *then there is global collusion until the first time* $c_t^A = 1$, *when firms are prosecuted in both countries. Thereafter, there is competition.*
    (b) *If* $\alpha^A > \bar{\alpha}^1$, *then there is always competition.* ■

Proposition 1 Part 1 states that when the discount factor is bellow some threshold ($\delta < \bar{\delta}$), the expected discounted welfare of country A is higher if collusion in both countries is allowed, and, hence, the competition authority of country A never prosecutes the firms. Then, firms can safely collude in both countries without facing any risk of prosecution. Proposition 1 Part 2 studies the situation in which $\delta \geq \bar{\delta}$ and, hence, the expected discounted welfare of country A is higher if collusion is stopped. In such a situation the competition authority of country A prosecutes the firms as soon as collusion is detected, which also triggers prosecution in country B. This prosecution policy might not be enough to dissuade firms to collude when the detection probability is low. Indeed, for $\alpha^A \leq \bar{\alpha}^1$, the expected discounted profits from collusion are high enough to sustain collusion. If this is the case, in equilibrium, there will be collusion until the competition authority of country A detects it and prosecutes the firms. Thereafter, firms will be forced to compete. On the contrary, when $\alpha^A > \bar{\alpha}^1$, the expected discounted profits from collusion are not enough to sustain collusion. The antitrust policy effectively dissuades firms from colluding. Figure 1 illustrates Proposition 1.[9]

[9] Figures have been obtained assuming that in each country the industry is a symmetric Bertrand duopoly with linear demand $P^j = a - b\left(n^j\right)^{-1} Q^j$ and cost functions $C^{ij} = cq^{ij}$, where $a > c$ and n^j is a measure of the size of market j. Therefore, $\pi^{c,j} = \frac{n^j(a-c)^2}{8b}$, $\pi^{d,j} = \frac{n^j(a-c)^2}{4b}$, $S^{c,j} = \frac{n^j(a-c)^2}{8b}$ and $S^{com,j} = \frac{n^j(a-c)^2}{2b}$. Fines are selected such that $f^j > \pi^{c,j}$. Demand and cost parameters are $a = 2$, $b = 1$, $c = 1$, $n^A = 1$, $n^B = 0.30$, which implies $\pi^{c,A} = 1.00$, $\pi^{d,A} = 2.00$, $S^{com,A} = 4.00$, $S^{c,A} = 1.00$, $\pi^{c,B} = 0.30$, $\pi^{d,B} = 0.60$, $S^{com,B} = 1.20$, and $S^{c,B} = 0.30$. Fines: $f^A = 2.00$ and $f^B = 0.60$.

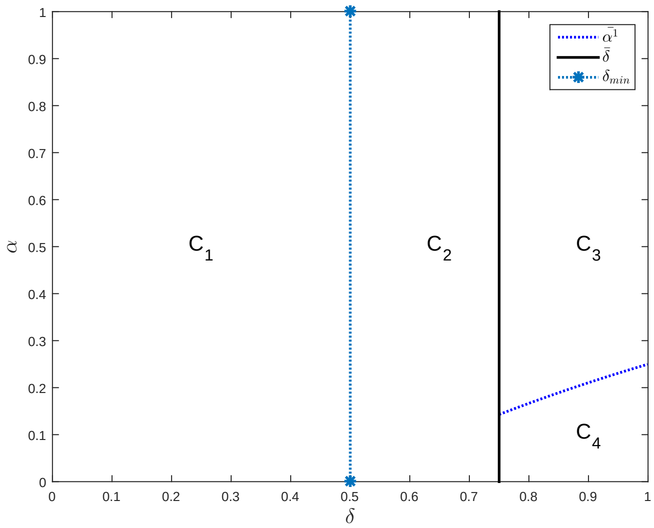

Figure 1: Proposition 1. C_1: Firms cannot collude in any country. C_2: Firms collude in both countries and they are never prosecuted. C_3: Prosecution dissuades collusion in both countries. C_4: Firms collude in both countries until the first time $c_t^A = 1$, then they are prosecuted.

3.2 Market-specific collusion decisions

Propositions 2 and 3 explore the more general case in which firms can make market-specific collusion decisions. Proposition 2 assumes that firms punish deviations from collusion in each market separately. Proposition 3 assumes that firms employ harsher punishment, stopping collusion in both countries even when the other firm deviates from collusion in only one market.

Proposition 2 *Suppose that Assumption 1 holds, firms punish deviations from collusion in each market separately and they coordinate in their best equilibrium. Furthermore, assume that* $\pi^{c,A} > (1-\delta)\,\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\,\pi^{d,B}$.

1. *Suppose that* $\delta < \bar{\delta}$. *Then firms collude in both countries and they are never prosecuted.*
2. *Suppose that* $\delta \geq \bar{\delta}$. *Let* $\bar{\alpha}_H^2 = \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ *and* $\bar{\alpha}_L^2 = \min\left\{\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\delta\frac{\pi^{c,B}}{1-\delta}+f^B}, \frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B}\right\}$.
   (a) *If* $\alpha^A > \bar{\alpha}_H^2$, *then firms collude in country* B *and they are never prosecuted.*
   (b) *Suppose that* $\bar{\alpha}_L^2 < \alpha^A \leq \bar{\alpha}_H^2$. *If* $\alpha^A < \frac{(1-\delta)\left(\pi^{c,A}-\pi^{c,B}\right)}{\delta\pi^{c,B}+(1-\delta)f^A}$, *firms only collude in country* A *until the first time* $c_t^A = 1$, *when they are prosecuted. Thereafter, there is competition. If* $\alpha^A > \frac{(1-\delta)\left(\pi^{c,A}-\pi^{c,B}\right)}{\delta\pi^{c,B}+(1-\delta)f^A}$, *firms only collude in country* B *and they are never prosecuted.*
   (c) *If* $\alpha^A \leq \bar{\alpha}_L^2$, *then firms collude in both countries until the first time* $c_t^A = 1$, *when they are prosecuted. Thereafter, there is competition in both countries.* ■

Proposition 2 Part 1 extends the result in Proposition 1 Part 1. Country A does not prosecute the firms when its expected discounted welfare is higher if firms collude in both countries. In that case, in equilibrium, there is collusion in both countries. Proposition 2 Part 2 is more involved than Proposition 1 Part 2. The reason is that now firms can decide to collude only in country B, in which case, they are never detected. In other words, firms can be dissuaded to collude in country A, but they will never be dissuaded to engage in collusion in country B. This generates three possible equilibrium paths.

When the detection probability is high ($\alpha^A > \bar{\alpha}_H^2$), the expected discounted profits from collusion in country A are not enough to sustain neither collusion in both countries nor collusion in country A as an equilibrium. Only collusion in country B can be sustained as an equilibrium. In that case, firms collude in country B and they are never detected. When the detection probability adopts intermediate values ($\bar{\alpha}_L^2 < \alpha^A \leq \bar{\alpha}_H^2$) either collusion in country A or collusion in country B can be sustained as an equilibrium, but collusion in both countries cannot. Then, firms opt for the type of collusion that generates higher expected discounted profits. Note that if they choose collusion in country A, eventually, they will be detected, forced to pay a fine and start competing. If they choose collusion in country B they will be never detected and collusion will last forever. Finally, when the detection probability is low ($\alpha^A \leq \bar{\alpha}_L^2$), the expected discounted profits from collusion in country A and B are both high enough to sustain every type of collusion as an equilibrium. In such situation, collusion in both countries generates the highest expected discounted profits and, hence, that is what firms choose to do. Eventually, the competition authority of country A detects and prosecutes the firms, the competition authority of country B follows the same course of action and, thereafter, firms are forced to compete in both countries. Figure 2 illustrates Proposition 2.

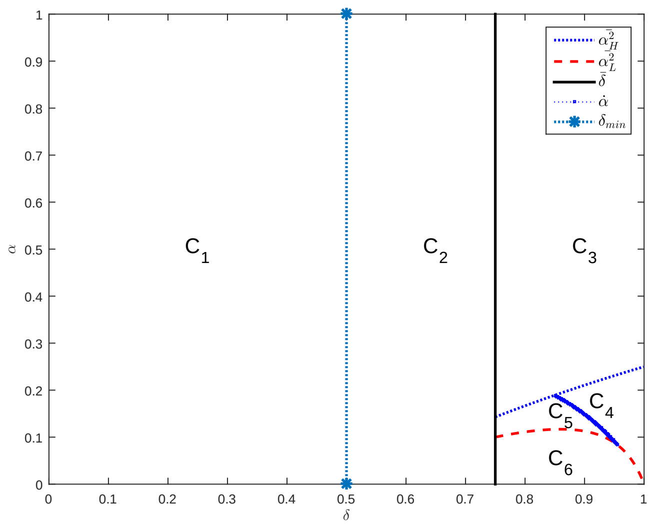

Figure 2: Proposition 2. C_1: Firms cannot collude in any country. C_2: Firms collude in both countries and they are never prosecuted. C_3 and C_4: Firms collude in country B and they are never prosecuted. C_5: Firms only collude in country A until the first time $c_t^A = 1$, then they are prosecuted. C_6: Firms collude in both countries until the first time $c_t^A = 1$, then they are prosecuted.

Proposition 3 *Suppose that Assumption 1 holds, firms punish deviations from collusion stopping collusion in both countries, and they coordinate in their best equilibrium. Furthermore, assume that* $\pi^{c,A} > (1-\delta)\,\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\,\pi^{d,B}$.

1. *Suppose that $\delta < \bar{\delta}$. Then firms collude in both countries and they are never prosecuted..*

2. *Suppose that $\delta \geq \bar{\delta}$. Let $\bar{\alpha}^3 = \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ and $\hat{\alpha}^3 = \frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B}$.*

   (a) *If $\alpha^A > \bar{\alpha}^3_H = \max\left\{\bar{\alpha}^3, \hat{\alpha}^3\right\}$, then firms collude in country B and they are never prosecuted.*

   (b) *Suppose that $\hat{\alpha}^3 < \alpha^A \leq \bar{\alpha}^3$. If $\alpha^A < \breve{\alpha}^3 = \frac{(1-\delta)\left(\pi^{c,A}-\pi^{c,B}\right)}{\delta\pi^{c,B}+(1-\delta)f^A}$, then firms only collude in country A until the first time $c^A_t = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A > \breve{\alpha}^3$, then firms only collude in country B and they are never prosecuted.*

   (c) *Suppose that $\bar{\alpha}^3 < \alpha^A \leq \hat{\alpha}^3$. If $\alpha^A < \tilde{\alpha}^3 = \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$, then firms collude in both countries until the first time $c^A_t = 1$, when they are prosecuted. Thereafter, there is competition in both countries. If $\alpha^A > \tilde{\alpha}^A$, then firms only collude in country B and they are never prosecuted.*

   (d) *Suppose that $\alpha^A \leq \bar{\alpha}^3_L = \min\left\{\bar{\alpha}^3, \hat{\alpha}^3\right\}$. If $\alpha^A < \min\left\{\frac{\pi^{c,B}}{f^B}, \tilde{\alpha}^3\right\}$, then firms collude in both countries until the first time $c^A_t = 1$, when they are prosecuted. Thereafter, there is competition in both countries. If $\frac{\pi^{c,B}}{f^B} < \alpha^A < \breve{\alpha}^3$, then firms only collude in country A until the first time $c^A_t = 1$, when they are prosecuted. Thereafter, there is competition. Finally, if $\alpha^A > \max\left\{\breve{\alpha}^3, \tilde{\alpha}^3\right\}$, then firms collude in country B and they are never prosecuted.* ■

Proposition 3 Part 1 extends the result in Proposition 1 Part 1 to a situation in which firms make market specific collusion decisions, but they employ more severe punishments than in Proposition 2, namely, deviations are punished by stopping collusion in all markets. The differences between Propositions 2 and 3 arise in Part 2. The reason is that the conditions required to support collusion in both countries as an equilibrium depend on the type of punishment employed by the firms, which affects the set of collusion agreements firms can sustain in equilibrium for a given value of α^A. In particular, while in Proposition 2 whenever collusion in both countries can be supported as an equilibrium, it automatically induces higher expected profits than collusion in country B, in Proposition 3 this is not necessarily the case. Figure 3 illustrates Proposition 3.

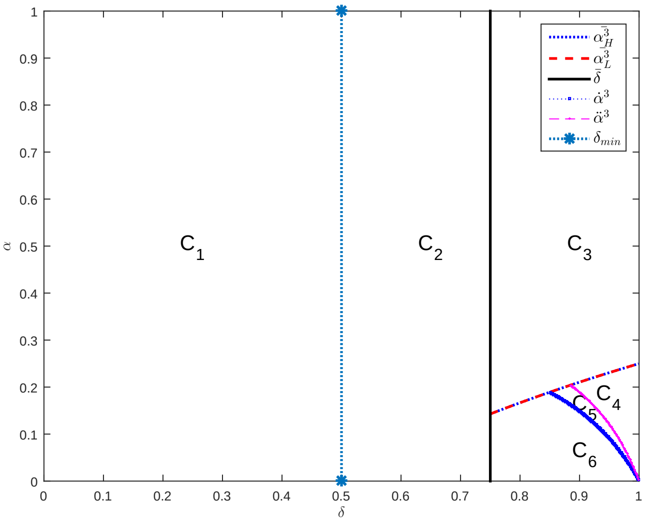

Figure 3: Proposition 3. C_1: Firms cannot collude in any country. C_2: Firms collude in both countries and they are never prosecuted. C_3 and C_4: Firms collude in country B and they are never prosecuted. C_5: Firms only collude in country A until the first time $c_t^A = 1$, then they are prosecuted. C_6: Firms collude in both countries until the first time $c_t^A = 1$, then they are prosecuted.

3.3 Prosecution in A is only temporarily informative for B

Assumption 1 states that when competition authority A prosecutes the firms in country A, competition authority B observes the process and learns how to detect and prosecute collusion in country B in all future periods. A weaker version of this assumption is to assume that prosecution in country A in period t only allows competition authority B to detect and prosecute collusion in period t, but it is not informative in future periods. Formally:

Assumption 2 *The signals received by the competition authorities are:*

$$c_t^A = \begin{cases} 1 \text{ with probability } \alpha^A & \text{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 \text{ with probability } \left(1 - \alpha^A\right) & \text{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$c_t^B = \begin{cases} 1 & \text{if } a_t^{1,B} = a_t^{2,B} = 1 \text{ and } p_t^A = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Note that while under Assumption 1, $c_t^B = 1$ when $a_t^{1,A} = a_t^{2,A} = 1$ and $p_\tau^A = 1$ for at least one $\tau \leq t$, under Assumption 2, $c_t^B = 1$ only when $a_t^{1,B} = a_t^{2,B} = 1$ and $p_t^A = 1$. Thus, under Assumption 1, prosecution in country A in one period, allows competition authority B to detect and prosecute collusion in every future period, while under Assumption 2, prosecution in country A is only temporarily informative for competition authority B. The main implication of replacing Assumption 1 by Assumption 2 is that now firms have access to new ways of organizing collusion. In particular, firms can collude only in country A until they are detected and, thereafter, start colluding in country B. Collusion in country

B will never be detected because it begins after $p_t^A = 1$ and, hence, there is no way that competition authority B detects it.[10] The following proposition fully characterizes the equilibrium.

Proposition 4 *Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that* $\pi^{c,A} > (1-\delta)\,\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\,\pi^{d,B}$.

1. *Suppose that* $\delta < \bar{\delta}$. *Then firms collude in both countries and they are never prosecuted.*
2. *Suppose that* $\delta \geq \bar{\delta}$. *Let* $\bar{\alpha}^4 = \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A-\frac{\delta\pi^{c,B}}{1-\delta}}$ *and* $\hat{\alpha}^4 = \frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B}$.
   - (a) *If* $\delta\pi^{d,A}+f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ *and* $\alpha^A > \bar{\alpha}_H^4 = \max\left\{\bar{\alpha}^4, \hat{\alpha}^4\right\}$, *then firms collude in country B and they are never prosecuted.*
   - (b) *Suppose that (i)* $\delta\pi^{d,A}+f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ *and* $\hat{\alpha}^4 < \alpha^A \leq \bar{\alpha}^4$ *or (ii)* $\delta\pi^{d,A}+f^A \leq \frac{\delta\pi^{c,B}}{1-\delta}$ *and* $\alpha^A > \hat{\alpha}^4$. *If* $\alpha^A < \breve{\alpha}^4 = \frac{\pi^{c,A}-\pi^{c,B}}{f^A}$, *then firms collude in country A until the first time* $c_t^A = 1$, *when they are prosecuted. Thereafter, they collude in country B, where they will never be prosecuted. If* $\alpha^A > \breve{\alpha}^4$, *then firms only collude in country B and they are never prosecuted.*
   - (c) *Suppose that* $\delta\pi^{d,A}+f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ *and* $\bar{\alpha}^4 < \alpha^A < \hat{\alpha}^4$. *If* $\alpha^A < \tilde{\alpha}^4 = \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$, *then firms collude in both countries until the first time* $c_t^A = 1$, *when they are prosecuted. Thereafter, there is competition in both countries. If* $\alpha^A > \tilde{\alpha}^4$, *then firms only collude in country B and they are never prosecuted.*
   - (d) *Suppose that (i)* $\delta\pi^{d,A}+f^A \leq \frac{\delta\pi^{c,B}}{1-\delta}$ *and* $\alpha^A \leq \hat{\alpha}^4$ *or (ii)* $\delta\pi^{d,A}+f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ *and* $\alpha^A \leq \min\left\{\bar{\alpha}^4, \hat{\alpha}^4\right\}$. *If* $\frac{\pi^{c,B}}{f^B+\delta\frac{\pi^{c,B}}{1-\delta}} < \alpha^A < \frac{\pi^{c,A}-\pi^{c,B}}{f^A}$, *then firms collude in country A until the first time* $c_t^A = 1$, *when they are prosecuted. Thereafter, they collude in country B where they will never be prosecuted. If* $\alpha^A < \min\left\{\frac{\pi^{c,B}}{f^B+\delta\frac{\pi^{c,B}}{1-\delta}}, \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}\right\}$, *then firms collude in both countries until the first time* $c_t^A = 1$, *when they are prosecuted. Thereafter, there is competition. If* $\alpha^A > \max\left\{\frac{\pi^{c,A}-\pi^{c,B}}{f^A}, \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}\right\}$, *then firms only collude in country B and they are never prosecuted.* ■

The novelty in Proposition 4 with respect to Proposition 3, is that the firms strategy of colluding in country A until detected and then switching to country B generates higher expected profits than colluding solely in country A. The reason is that collusion in country B cannot be detected once firms have been prosecuted in country A. This result points toward the idea that if firms are detected in their own market, they will start collusion in foreign markets. Figure 4 illustrates Proposition 4.

[10] In the Appendix we show that Lemmas 1 and 2 still hold (see the proof of Proposition 4). The reason Lemma 2 still applies is as follows. When competition authority A detects collusion in A, but there is no collusion in B, it could be that firms will never collude in B or they will start collusion in B once collusion in A is detected. The first case has been already considered in the proof of Lemma 2. Competition authority A always prefers to prosecute the firms. In the second case, prosecution in A triggers the start of collusion in B, which only increases the benefits of prosecuting firms in A.

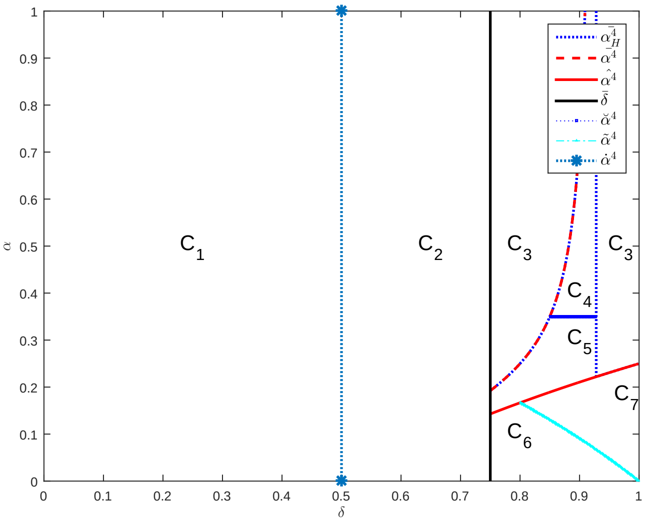

Figure 4: Proposition 4. C_1: Firms cannot collude in any country. C_2: Firms collude in both countries and they are never prosecuted. C_3 and C_4: Firms collude in country B and they are never prosecuted. C_5 and C_7: Firms only collude in country A until the first time $c_t^A = 1$, then they are prosecuted. C_6: Firms collude in both countries until the first time $c_t^A = 1$, then they are prosecuted.

4 Equilibrium analysis II: A owns both firms and B can also detect collusion on its own

This section studies the equilibrium of the model when both firms are owned by citizens of country A (formally, $\sigma^{1,A} = \sigma^{2,A} = 1$) and the competition authority of country B has two channels to detect collusion: observe the prosecution decisions of the competition authority of country A and detect collusion on its own. Thus,

Assumption 3 *The signals received by the competition authorities are:*

$$c_t^A = \begin{cases} 1 \textit{ with probability } \alpha^A & \textit{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 \textit{ with probability } \left(1-\alpha^A\right) & \textit{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 & \textit{otherwise.} \end{cases}$$

$$c_t^B = \begin{cases} 1 \textit{ with probability } \alpha^B & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_\tau^A = 0 \textit{ for all } \tau \le t, \\ 0 \textit{ with probability } \left(1-\alpha^B\right) & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_\tau^A = 0 \textit{ for all } \tau \le t, \\ 1 & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_\tau^A = 1 \textit{ for some } \tau \le t, \\ 0 & \textit{otherwise.} \end{cases}$$

Several remarks should be made about Assumption 3. First, as in the previous section, it is still the case that prosecution of the firms in country A is the most informative signal of collusion for competition authority B. Indeed, if competition authority A prosecutes the firms in country A, competition authority

B immediately learns how to detect collusion in B in the present as well as in all future periods (formally, if $a_t^{1,B} = a_t^{2,B} = 1$ and $p_\tau^A = 1$ for some $\tau \leq t$, then $c_t^B = 1$.) Second, even if competition authority A does not prosecute the firms, it is possible that competition authority B detects collusion in B (formally, if $a_t^{1,B} = a_t^{2,B} = 1$ and $p_\tau^A = 0$ for all $\tau \leq t$, then $c_t^B = 1$ with probability α^B).

It is easy to verify that Lemma 1 also holds under Assumption 3. Regardless of how competition authority B detects collusion, if $c_t^B = 1$, the expected discounted welfare of country B at period t if firms are prosecuted is $W_t^B(p_t^B = 1|c_t^B = 1) = S^{c,B} + 2f^B + \frac{\delta}{1-\delta}S^{com,B}$, while if they are not prosecuted it is $W_t^B(p_t^B = 0|c_t^B = 1) = \frac{S^{c,B}}{1-\delta}$. Since $S^{com,B} > S^{c,B}$, it is always the case that $W_t^B(p_t^B = 1|c_t^B = 1) > W_t^B(p_t^B = 0|c_t^B = 1)$.[11] Thus, as soon as $c_t^B = 1$, competition authority B immediately prosecutes the firms.

Next, we turn to the prosecution decision of competition authority A when it detects collusion.

Lemma 3 *Suppose that Assumption 3 holds. Assume that the competition authority of country A detects collusion, i.e.,* $c_t^A = 1$.

1. *If the firms are not colluding in country B, then A always prosecutes the firms.*
2. *If the firms are also colluding in country B, then A prosecutes the firms if and only if*

$$\Delta S^A - 2\pi^{c,A} \geq \frac{\left[1 - \left(1-\alpha^A\right)\delta\right] 2\left(1-\alpha^B\right)\left[\delta\pi^{c,B} + (1-\delta) f^B\right]}{\left[1-(1-\alpha^B)(1-\alpha^A)\delta\right]\delta}. \tag{2}$$

where $\Delta S^A = S^{com,A} - S^{c,A}$. ■

The intuition behind Lemma 3 is very similar to the one in Lemma 2. When firms are only colluding in country A, competition authority A prosecutes the firms as soon as collusion is detected because its prosecution decision does not have any impact on the profits of the firms in country B. When firms are colluding in both countries, prosecuting collusion in country A increases aggregate surplus in country A, but it reduces firms' profits in country B (forcing firms to compete in country B and making them pay fines). The main difference with Lemma 2 is that now competition authority B can detect collusion on its own and, hence, not prosecuting the firms is less valuable for country A. Formally, the right hand side of (2) is decreasing in α^B. Thus, as the probability that competition authority B detects collusion on its own increases, it is more likely that condition (2) holds. Finally, note that Lemma 3 is a generalization of Lemma 2. With $\alpha^B = 0$, the prosecution condition (2) becomes (1).

The following proposition studies the equilibrium decisions of the firms given the antitrust policies implemented by the competition authorities of both countries. We focus on the most interesting cases in which firms are willing to collude and the competition authority of country A does not prosecute the firms.

Proposition 5 *Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and* $\pi^{c,A} > (1-\delta)\pi^{d,A}$ *and*

[11] Note that since firms are owned by country A, market outcomes in country A are not relevant for the prosecution decision of country B.

$\pi^{c,B} > (1-\delta)\,\pi^{d,B}$. *Let*

$$\alpha^B > \frac{2\left[\delta\pi^{c,B} + (1-\delta)\,f^B\right] - \delta\left(\Delta S^A - 2\pi^{c,A}\right)}{2\left[\delta\pi^{c,B} + (1-\delta)\,f^B\right] + \frac{\delta(1-\alpha^A)}{1-(1-\alpha^A)\delta}\delta\left(\Delta S^A - 2\pi^{c,A}\right)}, \tag{3}$$

$$\alpha^A \leq \frac{\pi^{c,A} - (1-\delta)\,\pi^{d,A}}{\delta\pi^{d,A} + f^A}, \tag{4}$$

$$\alpha^B \leq \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) - \frac{\delta(\pi^{c,A} - \alpha^A f^A)}{1-(1-\alpha^A)\delta} + f^B}, \tag{5}$$

$$\alpha^B < \frac{(1-\delta)\,\pi^{c,B} + \alpha^A\left[\delta\left(\pi^{c,A} + \pi^{c,B}\right) + (1-\delta)\,f^A\right]}{\left[1-(1-\alpha^A)\,\delta\right] f^B}. \tag{6}$$

1. *Suppose that (3)-(6) hold. Then, firms collude in both countries until the first time $c_t^B = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $c_{t+\tau}^A = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries.*

2. *Suppose that (3)-(5) hold, but (6) does not hold. Then firms only collude in country A until the first time $c_t^A = 1$, when they are prosecuted. Thereafter, there is competition.* ■

In order to see the logic behind Proposition 5 it is useful to interpret conditions (3)-(6). (3) is the condition required for competition authority A to do not prosecute the firms when they are colluding in both countries. (4) states that $\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1-(1-\alpha^A)\delta} \geq \pi^{d,A}$, which means that firms are willing to collude in country A even if they know that as soon as collusion is detected, they will be prosecuted. (5) states that $\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^B f^B + \frac{\delta\alpha^B\left(\pi^{c,A} - \alpha^A f^A\right)}{1-(1-\alpha^A)\delta}}{1-(1-\alpha^B)\delta} \geq \pi^{d,A} + \pi^{d,B}$, which implies that firms are willing to collude in both countries if they know that competition authority A will prosecute them, but only after competition authority B detects and prosecutes collusion in country B. Finally, (6) means that $\Pi^{c,AB} > \Pi^{c,A}$, i.e., firms prefer to collude in both countries rather than only in country B.

Proposition 5 Part 1 describes an equilibrium in which firms start colluding in both countries, but they are not prosecuted by competition authority A, even when A detects collusion, i.e., even if $c_t^A = 1$. [12]. However, competition authority B eventually detects collusion on its own (the first time that $c_t^B = 1$), prosecutes the firms and forces them to compete in country B. This, of course, does not mean that firms stop colluding in country A. Indeed, since $\Pi^{c,A} \geq \pi^{d,A}$, firms will keep their collusive agreement in country A until they are detected by competition authority A. This time, competition authority A will prosecute the firms, because now there are no profits to protect in country B. Firms can also sustain only colluding in country A, but this collusive agreement generate lower expected profits when $\Pi^{c,AB} > \Pi^{c,A}$. Finally, note that Proposition 5 Part 1 is a generalization of Proposition 3 Part 1. If $\alpha^B = 0$, competition authority B cannot detect collusion on its own and, hence, if competition authority A does not prosecute the firms, there is collusion in both countries forever.

[12] This occurs because competition authority A accounts for the profits of the firms in country B and prosecution in country A will trigger prosecution in country B. Additionally, firms are able to collude in both countries because $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$

Proposition 5 Part 2 describes an equilibrium in which firms prefer to restrict collusion to country A because the fine that they will have to pay when collusion is detected in country B is too high. Note that this equilibrium is not very likely to occur. Indeed, if colluding in country B generates positive expected profits, firms will always prefer to start colluding in both countries. Formally, $\Pi^{c,B} = \frac{\pi^{c,B}-\alpha^B f^B}{1-(1-\alpha^B)\delta} > 0$ is sufficient for $\Pi^{c,AB} > \Pi^{c,A}$. However, Part 2 is useful to reveal a very interesting mechanism. Even when there is no intrinsic value in colluding in country B, firms might prefer to also collude in country B. Formally, even if $\Pi^{c,B} < 0$, it is possible that $\Pi^{c,AB} > \Pi^{c,A}$. The reason is that colluding in country B postpones prosecution in country A because when firms are colluding in both markets, competition authority A does not prosecute collusion in country A.

4.1 Prosecution Delays

The equilibrium in Proposition 5 involves a strategic prosecution delay. In order to see this, consider as a reference point a situation in which both competition authorities always prosecute collusion as soon as they detected it and firms are still willing to collude in both countries. In such environment, the expected duration of collusion will be given by:

$$\bar{d}^A = \sum_{t=0}^{\infty} t\left(1-\alpha^A\right)^t \alpha^A = \frac{1-\alpha^A}{\alpha^A}, \tag{7}$$

for country A and by:

$$\bar{d}^B = \sum_{k=0}^{\infty} k\left[\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]^k\left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right] = \frac{\left(1-\alpha^A\right)\left(1-\alpha^B\right)}{1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)} \tag{8}$$

for country B.[13]

The expected duration of collusion associated with the equilibrium in Proposition 5 is given by:

$$d^A = \sum_{k=0}^{\infty} k\left[\sum_{j=0}^{k-1}\left(1-\alpha^B\right)^j \alpha^B\left(1-\alpha^A\right)^{k-1-j}\alpha^A\right] = \frac{\alpha^A+\alpha^B-\alpha^A\alpha^B}{\alpha^B\alpha^A}, \tag{9}$$

for country A and by:

$$d^B = \sum_{k=0}^{\infty} k\left(1-\alpha^B\right)^k \alpha^B = \frac{1-\alpha^B}{\alpha^B}, \tag{10}$$

[13] The logic behind these formulas is straightforward. When competition authority A prosecutes the firms as soon as collusion in country A is detected, the probability that the firms are detected in period k is given by $\left(1-\alpha^A\right)^k\alpha^A$, i.e., the probability that firms are not detected from $t=0$ to $t=k-1$ times the probability that firms are detected in period $t=k$. Analogously, the probability that collusion in country B is detected in period k is given by $\left[\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]^k\left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]$, i.e., the probability that collusion is not detected neither in country A nor in country B from $t=0$ to $t=k-1$ times the probability that collusion is detected either in country A or in country B in period $t=k$. Finally, using the properties of the geometric distribution, it is easy to compute $\bar{d}^A$ and $\bar{d}^B$.

for country B.[14]

The following corollary summarizes the results on the expected duration of collusion and compares duration in equilibrium with an hypothetical situation in which both competition authorities prosecute collusion as soon as it is detected.

Corollary 1 *Expected Duration of Collusion. Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and* $\pi^{c,A} > (1-\delta)\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\pi^{d,B}$.

1. *Suppose that competition authority* $j \in \{A, B\}$ *prosecutes collusion the first time that* $c_t^j = 1$ *and, under such anti-trust policies, firms are still willing to collude in both countries. Then, the expected durations of collusion in countries* A *and* B *are given by (7) and (8), respectively.*

2. *Under the assumptions in Proposition 5 Part 1. The expected durations of collusion in countries* A *and* B *are given by (9) and (10), respectively.*

3. *Equilibrium prosecution delays in countries* A *and* B *are given by:*

$$d^A - \bar{d}^A = \frac{1}{\alpha^B} > 0,$$
$$d^B - \bar{d}^B = \frac{\alpha^A\left(1-\alpha^B\right)}{\alpha^B\left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]} > 0,$$

respectively. ■

Two remarks apply to Corollary 1. First, note that the equilibrium in Proposition 5 involves a strategic delay in the prosecution of collusion. On average, collusion lasts $\left(\alpha^B\right)^{-1}$ extra periods in country A and $\frac{\alpha^A\left(1-\alpha^B\right)}{\alpha^B\left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]}$ extra periods in country B. Second, $\lim_{\alpha^B\to 0}\left(d^A - \bar{d}^A\right) = \infty$ and $\lim_{\alpha^B\to 0}\left(d^B - \bar{d}^B\right) = \infty$. Thus, the equilibrium in Proposition 3 Part 1 generates an infinite prosecution delay.

5 Endogenous detection probabilities

Up until this point, we have assumed that the probabilities of detecting collusion are exogenously given. This section incorporates endogenous detection probabilities to the model. Suppose that in period $t = 0$

[14] The probability that competition authority A prosecutes collusion in period k is given by $\sum_{j=0}^{k-1}\left(1-\alpha^B\right)^j\alpha^B\left(1-\alpha^A\right)^{k-1-j}\alpha^A$, where $\left(1-\alpha^B\right)^j\alpha^B$ is the probability that competition authority B detects collusion in period $j \leq k-1$ and $\left(1-\alpha^A\right)^{k-1-j}\alpha^A$ is the probability that competition authority A detects collusion $k-j$ periods after collusion was detected and prosecuted in country B. Thus, $\left(1-\alpha^B\right)^j\alpha^B\left(1-\alpha^A\right)^{k-1-j}\alpha^A$ is the probability that B prosecutes in period j and A prosecutes $k-j$ periods after. (Recall that in the equilibrium described in Proposition 5, firms start colluding in both countries, but competition authority A only prosecutes the firms after competition authority B detects and prosecutes collusion in country B). Regarding country B, in equilibrium, the probability that competition authority B detects collusion in period k is $\left(1-\alpha^B\right)^k\alpha^B$. Again, using the properties of the geometric distribution, it is simple to compute d^A and d^B.

and, before the prosecution policies are selected, each competition authority chooses the probability of detecting collusion.[15] The cost associated to the probability of detection α is given by $C(\alpha)$, assumed smooth, increasing, convex, and satisfying $C(0) = 0$, $C'(0) = 0$ and $\lim_{\alpha \to 1} C'(\alpha) = \infty$.

We use the augmented model to perform two different exercises. First, taking as given the probability of detection of one competition authority, we study the incentives that the other competition authority has to prosecute collusion depending on the ownership structure and the international presence of the firms. Second, we characterize the Nash equilibrium levels of the probabilities of detection. While the first exercise provides prediction on cross-sector detection probabilities, the second one provides predictions on cross-country detection probabilities.

5.1 Exogenous α^B and endogenous α^A

Suppose that α^B is exogenously given and competition authority A selects α^A. Consider the following alternative scenarios: 1) The firms are owned by foreign citizens (formally, $\sigma^{1,B} = \sigma^{2,B} = 1$). 2) The firms are owned by local citizens and only operate in the domestic market (formally, $\sigma^{1,A} = \sigma^{2,A} = 1$ and $\pi_t^{1,B} = \pi_t^{2,B} = 0$ for all t). 3) The firms are owned by local citizens and they also operate in foreign markets (formally, $\sigma^{1,A} = \sigma^{2,A} = 1$ and $\pi_t^{1,B} = \pi_t^{2,B} \geq 0$ for all t, with strict inequality whenever $\left(a_t^{1,B}, a_t^{2,B}\right) \neq (0,0)$).

The following proposition characterizes the probability detection selected by competition authority A in each scenario. We restrict the analysis to the most interesting region of the parameter space in which firms have incentives to collude in both countries. From Proposition 5, this region is given by $R^j = \left\{\left(\alpha^j, \alpha^{-j}\right) \in [0,1] \times [0,1] : \textit{(11)-(14) hold}\right\}$, where $j \in \{A, B\}$ and

$$\alpha^{-j} > \frac{2\left[\delta \pi^{c,-j} + (1-\delta) f^{-j}\right] - \delta\left(\Delta S^j - 2\pi^{c,j}\right)}{2\left[\delta \pi^{c,-j} + (1-\delta) f^{-j}\right] + \frac{\delta(1-\alpha^j)}{1-(1-\alpha^j)\delta}\delta\left(\Delta S^j - 2\pi^{c,j}\right)}, \tag{11}$$

$$\alpha^j \leq \frac{\pi^{c,j} - (1-\delta)\pi^{d,j}}{\delta \pi^{d,j} + f^j}, \tag{12}$$

$$\alpha^{-j} \leq \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) - \frac{\delta(\pi^{c,j} - \alpha^j f^j)}{1-(1-\alpha^j)\delta} + f^{-j}}, \tag{13}$$

$$\alpha^{-j} < \frac{(1-\delta)\pi^{c,-j} + \alpha^j\left[\delta\left(\pi^{c,j} + \pi^{c,-j}\right) + (1-\delta) f^j\right]}{\left[1-(1-\alpha^j)\delta\right] f^{-j}}. \tag{14}$$

and $\Delta S^j = S^{com,j} - S^{c,j}$. Note that R^A is exactly the conditions in Proposition 5 Part 1, while R^B would be the conditions in Proposition 5 Part 1 when the roles of A and B are reversed.

Proposition 6 *For a given detection probability of competition authority B. Suppose that Assumption 3 holds (with the roles of A and B reversed in Part 1), that firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that $\pi^{c,A} > (1-\delta)\pi^{d,A}$ and $\pi^{c,B} > (1-\delta)\pi^{d,B}$. Then:*

[15] The timing is irrelevant because one of the competition authorities has a dominant strategy.

1. *Suppose that firms are owned by foreign citizens. Assume that* $\left(\alpha^B, \alpha_H\right) \in R^B$*, where* α^B *is the detection probability set by country* B *and* α_H *is the unique solution to*

$$\frac{\delta \Delta S^A + 2f^A (1-\delta)}{\left[1-(1-\alpha_H)\delta\right]^2} = C'(\alpha_H).$$

Then, the competition authority of country A *selects* $\alpha^A = \alpha_H$*.*

2. *Suppose that firms are owned by local citizens and only operate in the domestic market. Assume that* $\alpha_M \le \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$*, where* α_M *is the unique solution to*

$$\frac{\delta\left(\Delta S^A - 2\pi^{c,A}\right)}{\left[1-(1-\alpha_M)\delta\right]^2} = C'(\alpha_M).$$

Then, the competition authority of country A *selects* $\alpha^A = \alpha_M$*.*

3. *Suppose that firms are owned by local citizens and they also operate in foreign markets. Assume that* $\left(\alpha_L, \alpha^B\right) \in R^A$*, where* α^B *is the detection probability set by country* B *and* α_L *is the unique solution to*

$$\frac{\alpha^B \delta^2 \left(\Delta S^A - 2\pi^{c,A}\right)}{\left[1-(1-\alpha_L)\delta\right]^2 \left[1-(1-\alpha^B)\delta\right]} = C'(\alpha_L),$$

Then, the competition authority of country A *selects* $\alpha^A = \alpha_L$*. Moreover,* α_L *is increasing in* α^B*.*

4. *Suppose that* $\left(\alpha^B, \alpha_H\right) \in R^B$*,* $\alpha_M \le \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ *and* $\left(\alpha_L, \alpha^B\right) \in R^A$*. Then,* $\alpha_H > \alpha_M > \alpha_L$*, i.e., the competition authority of country* A *is tougher with foreign firms than with purely domestic firms and even less prone to prosecute collusion of domestic firms that operate in foreign markets.* ■

The intuition behind Proposition 6 is as follows. When firms are owned by foreign citizens, competition authority A always prosecutes collusion as soon as it is detected. (This is just Lemma 1 with A playing the role of B.) However, competition authority B will not prosecute the firms before they are prosecuted in country A. (This is Lemma 3 Part 2 with B playing the role of A). As a consequence, the only way in which A can detect and prosecute collusion in country A is relying on its own detection efforts. In the Appendix we prove that, including the costs of detection, the ex-ante expected welfare of country A is given by $W_0^A = \frac{S^{c,A}+2\alpha^A f^A + \frac{\alpha^A\delta}{1-\delta}S^{com,A}}{1-(1-\alpha^A)\delta} - C\left(\alpha^A\right)$, which has a maximum at $\alpha^A = \alpha_H$. Note that competition authority A simply equates the expected marginal benefit of detection ($\frac{\delta\Delta S^A+2f^A(1-\delta)}{[1-(1-\alpha_H)\delta]^2}$) with the marginal cost of detection ($C'(\alpha_H)$). Also note that α_H does not depend on α^B. The reason is that competition authority B refuses to prosecute the firms until they are prosecuted in country A.

When firms are owned by local citizens and only operate in the domestic market, competition authority A always prosecutes collusion as soon as it is detected. (This is Lemma 3 Part 1). In the Appendix we prove that, including the costs of detection, the ex-ante expected welfare of country A is given by $W_0^A = \frac{S^{c,A}+2\pi^{c,A} + \frac{\alpha^A\delta}{1-\delta}S^{com,A}}{1-(1-\alpha^A)\delta} - C\left(\alpha^A\right)$, which has a maximum at $\alpha^A = \alpha_M$. Since firms only operate in country A, α_M is independent of α^B.

When firms are owned by local citizens and they also operate in foreign markets, competition authority A will prosecute collusion in country A only after competition authority B prosecutes collusion in country B. In the Appendix we prove that, including the costs of detection, the ex-ante expected welfare of country A is given by $W_0^A = \frac{\left(S^{c,A}+2\pi^{c,A}\right)+\frac{\alpha^B\delta\left(S^{c,A}+2\pi^{c,A}\right)}{[1-(1-\alpha^A)\delta]}+2\left(\pi^{c,B}-\alpha^B f^B\right)+\frac{\alpha^A\alpha^B\delta^2 S^{com,A}}{[1-(1-\alpha^A)\delta](1-\delta)}}{[1-(1-\alpha^B)\delta]} - C\left(\alpha^A\right)$, which has a maximum at $\alpha^A = \alpha_L$. Note that α_L is a function of α^B. Indeed, α_L is increasing in α^B. The reason is that as competition authority B is more likely to detect collusion in country B, the sooner collusion will be prosecuted in country B and, hence, the higher the expected marginal benefit of detecting collusion sooner in country A.

Finally, Part 4 is the most interesting result in Proposition 6. It states that, ceteris paribus the market variables ΔS^A and $\pi^{c,A}$, the discount factor δ and the cost of detection C, competition authority A is more willing to invest in detection when firms are owned by foreigners than when they are owned by domestic citizens. Furthermore, competition authority A is less willing to invest in detection when firms owned by domestic citizens are colluding in foreign markets. Proposition 6 Part 4 can also be interpreted as a prediction of cross-sector detection probabilities. Ceteris paribus, ΔS^A, $\pi^{c,A}$, δ, and C, competition authorities will tend to invest more on detecting collusion in sectors dominated by foreign firms than in those dominated by firms owned by local citizens and, among the last group, competition authorities will tend to be even less harsh with firms that are colluding in foreign markets.

5.2 Nash equilibrium detection probabilities

The following proposition characterizes the Nash equilibrium levels of the detection probabilities.

Proposition 7 *Suppose that Assumption 3 holds, that firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Furthermore, assume that* $\pi^{c,A} > (1-\delta)\,\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\,\pi^{d,B}$. *Let* $\left(\alpha^{A,*},\alpha^{B,*}\right) \in R^A$, *where* $\left(\alpha^{A,*},\alpha^{B,*}\right)$ *is the unique solution to*

$$\frac{\alpha^{B,*}\delta^2\left(\Delta S^A - 2\pi^{c,A}\right)}{\left[1-\left(1-\alpha^{A,*}\right)\delta\right]^2\left[1-\left(1-\alpha^{B,*}\right)\delta\right]} = C'\left(\alpha^{A,*}\right),$$

$$\frac{\delta\Delta S^B + 2f^B(1-\delta)}{\left[1-\left(1-\alpha^{B,*}\right)\delta\right]^2} = C'\left(\alpha^{B,*}\right).$$

Then, the Nash equilibrium probabilities of detection selected by the competition authorities are $\left(\alpha^A,\alpha^B\right) = \left(\alpha^{A,*},\alpha^{B,*}\right)$. *Moreover, if* $\delta\Delta S^B + 2f^B(1-\delta) > \delta^2\left(\Delta S^A - 2\pi^{c,A}\right)$, *then* $\alpha^{B,*} > \alpha^{A,*}$. ■

Two remarks apply to Proposition 7. First, note that competition authority B has a dominant strategy ($\alpha^B = \alpha^{B,*}$ regardless of α^A), while the best response of competition authority A depends on α^B. Second, under a relatively mild condition, the country that owns the firms invest less on detection than the country that does not own the firms ($\alpha^{B,*} > \alpha^{A,*}$). The reason is that for the country that owns the firms, prosecuting collusion has the extra cost of losing collusion profits in the other country.

6 Antitrust policy, international agreements and integration

Up until this point we have studied the equilibrium prosecution policies implemented by two independent competition authorities. That is, the focus has been on the positive side of the problem. From a normative perspective, it is clear that in order to maximize the aggregate expected welfare of the world ($W_0^W = W_0^A + W_0^B$), both competition authorities should prosecute collusion as soon as it is detected. Indeed, this would be the policy selected by a globally integrated competition authority with a mandate to maximize W_0^W. Compared with the equilibrium prosecution policies, full prosecution increases the world's welfare as well as the welfare of the country that does not own the firms, but it hurts the country of origin of the firms, which would prefer to delay prosecution. Moreover, it would be very complicated to reach an international agreement that implements full prosecution, as the country of origin of the firms would not be willing to participate.[16] Note, however, that this logic might not apply if there are several industries and the firms of each country operate and try to organize collusion in different industries. In this section we argue that, by introducing multiple industries, with different countries of origin, there might be room for international agreements to be put in place. In order to formally explore this possibility, we incorporate a second industry to our model.

As in previous sections, assume there are two countries (A and B) with their respective competition authorities. In each country there are two industries, denoted by x and y. Only two companies operate in each industry: companies $1, x$ and $2, x$ in industry x and companies $1, y$ and $2, y$ in industry y. Let $\left(a_t^{i,z,A}, a_t^{i,z,B}\right)$ with $i \in \{1, 2\}$ and $z \in \{x, y\}$ denote the decision of firm i, z in period t, where $a_t^{i,z,j} = 1$ indicates that firm i, z chooses to collude in country j and $a_t^{i,z,j} = 0$ indicates that firm i, z chooses to compete in country j. Collusion can only occur within industry. Let $\left(\pi_t^{i,z,A}, \pi_t^{i,z,B}\right)$ denote the profits that company i, z obtains in period t from its operations in countries A and B. Assume that

$$\pi_t^{i,x,j} = \begin{cases} \pi^{c,x,j} & \textit{if } a_t^{i,x,j} = a_t^{-i,x,j} = 1, \\ \pi^{d,x,j} & \textit{if } a_t^{i,x,j} = 0 \textit{ and } a_t^{-i,x,j} = 1, \\ 0 & \textit{otherwise,} \end{cases}$$

where $\pi^{d,x,j} > \pi^{c,x,j} > 0$. Denote by $\sigma^{i,z,j}$ the share of firm i, z owned by citizens from country j. Let $S_t^{z,j}$ be the consumer surplus in industry z and country j. Assume that $S_t^{z,j} = S^{c,z,j}$ when $a_t^{i,z,j} = a_t^{-i,z,j} = 1$ and $S_t^{z,j} = S^{com,z,j}$, otherwise, where $S^{com,z,j} > S^{c,z,j} + 2\pi^{c,z,j}$ for all z, j. Finally, denote by $f^{z,j} > \pi^{c,z,j}$ the fine charged by the competition authority of country j if firms are found organizing collusion in industry z.

The timing is as follows. At the beginning of period $t = 0$, the competition authorities simultaneously select their prosecution policies. Then, firms play an infinitely repeated game, where the stage game is given by:

1. In each industry both firms simultaneously choose to collude or compete.
2. Nature sends a signal to the competition authority of country j (k) regarding the behavior of firms in industry x (y) in country j (k). Upon observing the signal, the competition authority of country j (k) implements its prosecution policy for industry x (y).

[16] If forced to do so, it will have a strong incentive to withhold any information of collusion.

3. Nature sends a signal to the competition authority of country $-j$ $(-k)$ regarding the behavior of firms in industry x (y) in country $-j$ $(-k)$. Upon observing the signal, the competition authority of country $-j$ $(-k)$ implements its prosecution policy for industry x (y).[17]

A natural generalization of the ownership distribution studied in Sections 3 and 4 is to assume that each country owns the two firms of one of the industries. Formally, let $\sigma^{1,x,A} = \sigma^{2,x,A} = 1$ and $\sigma^{1,y,B} = \sigma^{2,y,B} = 1$, meaning that only citizens of country A (B) own the firms in industry x (y). Regarding collusion detection, the following assumption is a generalization of Assumption 3 for an environment with two industries.

Assumption 4 *The signals received by the competition authorities are:*

1. *Industry x:*

$$c_t^{x,A} = \begin{cases} 1 \text{ with probability } \alpha^{x,A} & \text{if } a_t^{1,x,A} = a_t^{2,x,A} = 1, \\ 0 \text{ with probability } \left(1 - \alpha^{x,A}\right) & \text{if } a_t^{1,x,A} = a_t^{2,x,A} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$c_t^{x,B} = \begin{cases} 1 \text{ with probability } \alpha^{x,B} \text{ if } a_t^{1,x,B} = a_t^{2,x,B} = 1 \text{ and } p_\tau^{x,A} = 0 \text{ for all } \tau \leq t, \\ 1 \text{ if } a_t^{1,x,B} = a_t^{2,x,B} = 1 \text{ and } p_\tau^{x,A} = 1 \text{ for some } \tau \leq t, \\ 0 \text{ with probability } \left(1 - \alpha^{x,B}\right) \text{ if } a_t^{1,x,B} = a_t^{2,x,B} = 1 \text{ and } p_\tau^{x,A} = 0 \text{ for all } \tau \leq t, \\ 0 \text{ otherwise.} \end{cases}$$

2. *Industry y:*

$$c_t^{y,B} = \begin{cases} 1 \text{ with probability } \alpha^{y,B} & \text{if } a_t^{1,y,B} = a_t^{2,y,B} = 1, \\ 0 \text{ with probability } \left(1 - \alpha^{y,B}\right) & \text{if } a_t^{1,y,B} = a_t^{2,y,B} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$c_t^{y,A} = \begin{cases} 1 \text{ with probability } \alpha^{y,A} \text{ if } a_t^{1,y,A} = a_t^{2,y,A} = 1 \text{ and } p_\tau^{y,B} = 0 \text{ for all } \tau \leq t \\ 1 \text{ if } a_t^{1,y,A} = a_t^{2,y,A} = 1 \text{ and } p_\tau^{y,B} = 1 \text{ for some } \tau \leq t, \\ 0 \text{ with probability } \left(1 - \alpha^{y,A}\right) \text{ if } a_t^{1,y,A} = a_t^{2,y,A} = 1 \text{ and } p_\tau^{y,B} = 0 \text{ for all } \tau \leq t \\ 0 \text{ otherwise.} \end{cases}$$

6.1 Equilibrium under independent competition authorities

As in previous sections suppose that each country has a competition authority with a mandate to maximize the country's expected aggregate welfare. In order to characterize the equilibrium it is useful to define the following sets. Let $R^{z,j} = \left\{\left(\alpha^{z,j}, \alpha^{z,-j}\right) \in [0,1] \times [0,1] : \textit{(15)-(18) hold}\right\}$, where $z \in \{x, y\}$,

[17] Note that with this timing the signal received by the competition authority of country $-j$ $(-k)$ about industry x (y) could depend on the prosecution decision implemented by the competition authority of country $j(k)$.

$j \in \{A, B\}$ and

$$\alpha^{z,-j} > \frac{2\left[\delta\pi^{c,z,-j} + (1-\delta) f^{z,-j}\right] - \delta\left(\Delta S^{z,j} - 2\pi^{c,z,j}\right)}{2\left[\delta\pi^{c,z,-j} + (1-\delta) f^{z,-j}\right] + \frac{\delta(1-\alpha^{z,j})}{1-(1-\alpha^{z,j})\delta}\delta\left(\Delta S^{z,j} - 2\pi^{c,z,j}\right)}, \tag{15}$$

$$\alpha^{z,j} \leq \frac{\pi^{c,z,j} - (1-\delta)\pi^{d,z,j}}{\delta\pi^{d,z,j} + f^{z,j}}, \tag{16}$$

$$\alpha^{z,-j} \leq \frac{\left(\pi^{c,z,A} + \pi^{c,z,B}\right) - (1-\delta)\left(\pi^{d,z,A} + \pi^{d,z,B}\right)}{\delta\left(\pi^{d,z,A} + \pi^{d,z,B}\right) - \frac{\delta(\pi^{c,z,j} - \alpha^{z,j} f^{z,j})}{1-(1-\alpha^{z,j})\delta} + f^{z,-j}}, \tag{17}$$

$$\alpha^{z,-j} < \frac{(1-\delta)\pi^{c,z,-j} + \alpha^{z,j}\left[\delta\left(\pi^{c,z,j} + \pi^{c,z,-j}\right) + (1-\delta) f^{z,j}\right]}{\left[1-(1-\alpha^{z,j})\delta\right] f^{z,-j}}. \tag{18}$$

Proposition 8 *Suppose that Assumption 4 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and* $\pi^{c,z,j} > (1-\delta)\pi^{d,z,j}$ *for* $z \in \{x, y\}$ *and* $j \in \{A, B\}$*.*

1. *Industry* x*. Suppose that* $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A}$*. Then, in industry* x *firms collude in both countries until the first time* $c_t^{x,B} = 1$*, when they are prosecuted in country* B*. Thereafter, they collude in country* A *until the first time* $c_{t+\tau}^{x,A} = 1$ *with* $\tau \geq 1$*, when they are prosecuted in country* A*. Thereafter, there is competition in both countries.*

2. *Industry* y*. Suppose that* $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B}$*. Then, in industry* y *firms collude in both countries until the first time* $c_t^{y,A} = 1$*, when they are prosecuted in country* A*. Thereafter, they collude in country* B *until the first time* $c_{t+\tau}^{y,B} = 1$ *with* $\tau \geq 1$*, when they are prosecuted in country* B*. Thereafter, there is competition in both countries.* ■

Proposition 8 is a straightforward generalization of Proposition 5 Part 1. In industry x competition authority A delays prosecuting firms because it does not want to trigger prosecution in country B, while the opposite happens in industry y. More formally, condition (15) applied to $z = x$ and $j = A$ states that competition authority A does not prosecute collusion in industry x until collusion in industry x is not detected in country B. Conditions (16) and (17) mean that collusion in industry x can be sustained in country A as well as in both countries. Finally, condition (18) states that in industry x firms prefer to collude in both countries rather than only in country A. For industry y, the roles of countries A and B are reversed, but the interpretation is analogous. Here it is competition authority B the one that waits until firms are prosecuted in country A to prosecute collusion in country B.

6.2 Equilibrium under a globally integrated competition authority

The equilibrium in Proposition 8 assumes that competition authority j maximizes the expected welfare of country j. Next, we consider that competition authorities are integrated into one competition authority with the mandate to maximize the aggregate expected welfare of both countries. For example, this could capture the situation of the European Union if part of the integration process includes the consolidation of all the national competition authorities into one competition authority for the whole union. Alternatively, the situation under integration could better approximate the present anti-trust policy in the United States, while Proposition 8 could capture an alternative institutional arrangement in which anti-trust policy is fully delegated to the states.

Suppose that the competition authorities of both countries merge and form a unique globally integrated competition authority with a mandate to maximize the world's aggregate expected welfare. In such environment, the global competition authority will always prosecute collusion as soon as it is detected, which will change the incentives of the firms to organize collusion. In particular, to characterize the equilibrium under integration it is useful to define the following sets. Let $\bar{R}^{z,j} = \left\{ \left(\alpha^{z,j}, \alpha^{z,-j} \right) \in [0,1] \times [0,1] : \textit{(19)-(21) hold} \right\}$, where

$$\alpha^{z,j} \leq \frac{\pi^{c,z,j} - (1-\delta)\,\pi^{d,z,j}}{f^{z,j} + \delta\pi^{d,z,j}}, \tag{19}$$

$$\begin{gathered} \left[\alpha^{z,j} \leq \frac{\pi^{c,z,j} - \left(\frac{1-\delta}{\delta}\right) f^{z,j} - (1-\delta)\left(\pi^{d,z,A} + \pi^{d,z,B}\right)}{f^{z,A} + f^{z,B} + \delta\left(\pi^{d,z,A} + \pi^{d,z,B}\right)} \right] \;\; \textit{or} \\ \left[\frac{\pi^{c,z,j} - \left(\frac{1-\delta}{\delta}\right) f^{z,-j} - (1-\delta)\left(\pi^{d,z,A} + \pi^{d,z,B}\right)}{f^{z,A} + f^{z,B} + \delta\left(\pi^{d,z,A} + \pi^{d,z,B}\right)} < \alpha^{z,j} < \frac{\pi^{c,z,A} + \pi^{c,z,B} - (1-\delta)\left(\pi^{d,z,A} + \pi^{d,z,B}\right)}{f^{z,A} + f^{z,B} - \delta\left(\pi^{d,z,A} + \pi^{d,z,B}\right)} \;\; \textit{and} \right. \\ \left. \alpha^{z,-j} \leq \frac{\pi^{c,z,A} + \pi^{c,z,B} - \alpha^{z,j}\left(f^{z,A} + f^{z,B}\right) - \left[1 - \left(1 - \alpha^{z,j}\right)\delta\right]\left(\pi^{d,z,A} + \pi^{d,z,A}\right)}{\left(1 - \alpha^{z,j}\right)\left[\delta\pi^{d,z,-j} + f^{z,-j} - \frac{\delta\left(\pi^{c,z,j} - \alpha^{z,j} f^{z,j}\right)}{1 - \left(1 - \alpha^{z,j}\right)\delta} + \delta\pi^{d,z,j}\right]} \right], \end{gathered} \tag{20}$$

$$\begin{gathered} \left[\frac{\pi^{c,z,-j} - (1-\delta)\pi^{d,z,-j}}{f^{z,-j} + \delta\pi^{d,z,-j}} < \alpha^{z,-j} < \frac{\pi^{c,z,-j} - \alpha^{z,j} f^{z,-j}}{\left(1 - \alpha^{z,j}\right) f^{z,-j}} \right] \;\; \textit{or} \\ \left[\alpha^{z,-j} < \min\left\{ \frac{\pi^{c,z,-j} - (1-\delta)\pi^{d,z,-j}}{f^{z,-j} + \delta\pi^{d,z,-j}}, \frac{\pi^{c,z,-j} - \alpha^{z,j} f^{z,-j}}{\left(1 - \alpha^{z,j}\right) f^{z,-j}} \right\} \;\; \textit{and} \right. \\ \left. \frac{\pi^{c,z,j} - \alpha^{z,j} f^{z,j}}{1 - (1 - \alpha^{z,j})\delta} > \frac{\alpha^{z,j}\left(1 - \alpha^{z,-j}\right)\left[\delta\pi^{c,z,-j} + (1-\delta) f^{z,-j}\right]}{\left[1 - (1 - \alpha^{z,-j})\delta\right]\left[1 - (1 - \alpha^{z,j})(1 - \alpha^{z,-j})\delta\right]} \right] \end{gathered} \tag{21}$$

Proposition 9 *Suppose that Assumption 4 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and* $\pi^{c,z,j} > (1-\delta)\,\pi^{d,z,j}$ *for* $z \in \{x, y\}$ *and* $j \in \{A, B\}$*.*

1. *Industry x. Suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in \bar{R}^{x,A}$. Then, firms collude in both countries until the first time $c_t^{x,A} = 1$, when they are prosecuted in both countries, or until the first time $c_t^{x,A} = 0$ and $c_t^{x,B} = 1$, when they are prosecuted in country B. In the later case, firms keep colluding in country A until the first time $c_{t+\tau}^{x,A} = 1$ with $\tau \geq 1$.*

2. *Industry y. Suppose that $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in \bar{R}^{y,B}$. Then, firms collude in both countries until the first time $c_t^{y,B} = 1$, when they are prosecuted in both countries, or until the first time $c_t^{y,B} = 0$ and $c_t^{y,A} = 1$, when they are prosecuted in country A. In the later case, firms keep colluding in country B until the first time $c_{t+\tau}^{y,B} = 1$ with $\tau \geq 1$.* ■

Proposition 9 simply states the conditions under which firms are willing to collude in both countries until detected and, if they are only detected in one country, keep colluding in the other country, when they face an integrated competition authority. More formally, conditions (19) and (20) applied to industry x and country A state that collusion in industry x can be sustained in country A as well as in both countries. Condition (21) means that in industry x firms prefer to collude in both countries rather than only in one country. For industry y, the roles of countries A and B are reversed, but the interpretation is analogous.

The following corollary compares the expected welfare of each country when each competition authority maximizes the welfare of its country (Proposition 8) and under integration (Proposition 9).

Corollary 2 *Suppose that Assumption 4 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and* $\pi^{c,z,j} > (1-\delta)\,\pi^{d,z,j}$ *for* $z \in \{x,y\}$ *and* $j \in \{A,B\}$*. Assume that* $\left(\alpha^{x,A},\alpha^{x,B}\right) \in R^{x,A} \cap \bar{R}^{x,A}$ *and* $\left(\alpha^{y,B},\alpha^{y,A}\right) \in R^{y,B} \cap \bar{R}^{y,B}$*. Then:*

1. *Country A benefits from integration if and only if* $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$ *and (if* $j = B$*), where*

$$\Delta W_0^{x,A} = \frac{\alpha^{x,A}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]}\left\{\frac{\delta\left(\Delta S^{x,A}-2\pi^{c,x,A}\right)}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]} - \frac{2\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B}+(1-\delta)\,f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]}\right\},$$

$$\Delta W_0^{y,A} = \frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[\delta\Delta S^{y,A}+(1-\delta)\,2f^{y,A}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]},$$

2. *Country B benefits from integration if and only if* $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$*, where*

$$\Delta W_0^{x,B} = \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta\Delta S^{x,B}+(1-\delta)\,2f^{x,B}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B} = \frac{\alpha^{y,B}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\frac{\delta\left(\Delta S^{y,B}-2\pi^{c,y,B}\right)}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]} - \frac{2\left(1-\alpha^{y,A}\right)\left[\delta\pi^{c,y,A}+(1-\delta)\,f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\right\}.$$

3. *Moreover, if* $\pi^{c,z,j} = \pi^c$*,* $\pi^{d,z,j} = \pi^d$*,* $\Delta S^{z,j} = \Delta S$*,* $\alpha^{z,j} = \alpha$*, and* $f^{z,j} = f$ *for all* $z \in \{x,y\}$ *and* $j \in \{A,B\}$*, it is always the case that both countries are better off under integration.* ■

The intuition behind Corollary 2 is simple. On the one hand, country A obtains a lower aggregate welfare in industry x under integration ($\Delta W_0^{x,A} < 0$). The reason is that an independent competition authority will delay prosecution only when the profits from collusion in country B outweigh the benefit from stopping collusion in country A. On the other hand, country A obtains a higher aggregate welfare in industry y under integration ($\Delta W_0^{y,A} > 0$) because an integrated competition authority does not delay the prosecution of firms from country B organizing collusion in country A. Note that it is perfectly possible that $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$ and, hence, country A is better off under integration. The welfare comparisons for country B follow the same logic, except that we must reverse the industries. In other words, under integration, country B obtains a higher aggregate welfare in industry x, but a lower aggregate welfare in industry x ($\Delta W_0^{x,B} > 0$ and $\Delta W_0^{y,B} < 0$). Again, if $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$, country B is also better off under integration. Finally, Part 3 considers a particular case in which everything is symmetric (across industries and countries). In such a case, the extra profits that one country is obtaining from collusion in the other country in one industry are perfectly offset by the extra profits that foreign firms are obtaining from collusion in the domestic market in the other industry. Thus, once we aggregate both industries, integration is neutral with respect to collusive profits, but it eliminates the deadweight loss associated with collusion. Corollary 2 shows that, given the incentives that independent national authorities have to delay prosecution, both countries might be better off if they integrate their competition authorities.

7 Conclusions

This paper pushes the frontier of the analysis of antitrust policy in open economies. We develop a political economy model of antitrust enforcement in an open economy and characterize the equilibrium prosecution policies selected by benevolent national competition authorities. In the several scenarios studied in our model, we show that price fixing agreements reduce the world's aggregate welfare and, therefore, should be prevented. However, we also show that national competition authorities may have biased incentives towards the prosecution of collusive activities. In particular, the country of origin of the firms has weaker incentives to prosecute collusion because domestic prosecution spirals into foreign prosecution, which reduces the profits of domestic firms in foreign markets. This misalignment between the equilibrium prosecution policies and the global welfare maximizing solution could be solved by integrating the competition authorities. Integration would, nevertheless find resistance by the country of origin of the firms, undermining its efficacy. Such a solution is more likely to succeed in a multi-industry world where each country specializes in a different industry. Indeed, we have shown that in a multi-industry world each country could be better off if an internationally integrated competition authority decided on prosecution based on global welfare.

Our results have important implications for the design of antitrust enforcement institutions and agencies. First, although there might be benefits from decentralizing antitrust enforcement to subnational entities, our model suggests that countries should centralize antitrust enforcement in a national competition authority. Second, our results suggest that competition authorities should consider the origin of the firms and their foreign operations when they decide to initiate a collusion case. Political transparency could be a problem. How can the public distinguish a competition authority captured by the firms from one dedicated to maximize national welfare that does not prosecute some firms to protect their foreign profits? For some industries both could be observationally equivalent. There might also exist practical barriers to implement such policy. For example, firms can employ accounting tricks to assign profits to different countries in order to inflate their foreign operations and avoid prosecution.

Finally, there are several avenues to expand our analysis. For example, we have developed a two-country model, but the mechanism behind our results should also apply to a sitting with multiple countries. More importantly, in such setting there will be room for the strategic formation of coalitions of countries that decide to integrate their competition authorities.

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Appendix to "Of Course Collusion Should be Prosecuted. But Maybe ... or (The case for international antitrust agreements)"

This Appendix presents the proofs of all lemmas and propositions.

A.1 Proofs of Lemmas 1, 2, and Proposition 1

Proof of Lemma 1. Suppose that the competition authority of country B detects collusion in period t, i.e., $c_t^B = 1$. Note that this can only occur if $a_t^{1,B} = a_t^{2,B} = 1$ and $p_\tau^A = 1$ for some $\tau \leq t$. If the competition authority of country B prosecutes the firms, then the expected discounted welfare of country B at period t is

$$W_t^B\left(p_t^B = 1|c_t^B = 1\right) = S^{c,B} + 2f^B + \frac{\delta}{1-\delta}S^{com,B},$$

while if firms are not prosecuted, it is

$$W_t^B\left(p_t^B = 0|c_t^B = 1\right) = \frac{S^{c,B}}{1-\delta}.$$

Since $S^{com,B} > S^{c,B}$, it must be the case that $W_t^B\left(p_t^B = 1|c_t^B = 1\right) > W_t^B\left(p_t^B = 0|c_t^B = 1\right)$. Thus, whenever B detects collusion, it immediately prosecutes the firms. ■

Proof of Lemma 2. Suppose that firms are only colluding in country A while there is competition in country B and the competition authority of country A detects collusion in period t, i.e., $c_t^A = 1$. If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period t is[18]

$$W_t^A\left(p_t^A = 1|c_t^A = 1, A\right) = S^{c,A} + 2\left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta}S^{com,A},$$

while if firms are not prosecuted, it is

$$W_t^A\left(p_t^A = 0|c_t^A = 1, A\right) = \frac{S^{c,A} + 2\pi^{c,A}}{1-\delta}.$$

Since $S^{com,A} > S^{c,A} + 2\pi^{c,A}$, it must be the case that $W_t^A\left(p_t^A = 1|c_t^A = 1, A\right) > W_t^A\left(p_t^A = 0|c_t^A = 1, A\right)$.

Suppose that firms are colluding in both countries and the competition authority of country A detects collusion in period t, i.e., $c_t^A = 1$. If the competition authority of country A decides to prosecute the firms, then the competition authority of country B will detect collusion in country B as well, i.e., $c_t^B = 1$. Hence, firms will be also prosecuted in country B. Thus, if firms are prosecuted in country A, the expected discounted welfare of country A at period t will be

$$W_t^A\left(p_t^A = 1|c_t^A = 1, AB\right) = S^{c,A} + 2\left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta}S^{com,A}.$$

[18] Note that we employ the following notation. $W_t^A(p_t^A = 1|c_t^A = 1, A)$ is the discounted expected welfare of country A when firms are only colluding in country A, $c_t^A = 1$ and competition authority A choses to prosecute the firms. $W_t^A(p_t^A = 1|c_t^A = 1, AB)$ is the discounted expected welfare of country A when firms are colluding in both countries, $c_t^A = 1$ and competition authority A choses to prosecute the firms.

If the competition authority of country A does not prosecute the firms, then there is no way that the competition authority of country B finds that firms are also colluding in country B. Then, firms will continue colluding in both countries and the expected discounted welfare of country A at period t will be

$$W_t^A\left(p_t^A = 0 | c_t^A = 1, AB\right) = \frac{S^{c,A} + 2\left(\pi^{c,A} + \pi^{c,B}\right)}{1-\delta}.$$

$W_t^A\left(p_t^A = 1 | c_t^A = 1, AB\right) > W_t^A\left(p_t^A = 0 | c_t^A = 1, AB\right)$ if and only if $\delta\left(\Delta S^A - 2\pi^{c,A}\right) \geq 2\left[\delta\pi^{c,B} + (1-\delta) f^B\right]$ (equivalently, $\delta \geq \bar{\delta} = \frac{2f^B}{\Delta S^A - 2\pi^{c,A} + 2f^B - 2\pi^{c,B}}$), where $\Delta S^A = S^{com,A} - S^{c,A}$. ■

Proof of Proposition 1. Suppose that $\delta < \bar{\delta}$. Then, from Lemmas 1 and 2, if firms collude in both countries they will be never prosecuted. Therefore, the expected profits of a firm under collusion are given by:

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1-\delta}.$$

(Recall that firms can either collude in both countries or in none of them). Collusion can be sustained as a subgame perfect Nash equilibrium whenever $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which always holds since $\pi^{c,j} > (1-\delta)\pi^{d,j}$ for $j = A, B$.

Suppose $\delta \geq \bar{\delta}$. Then, from Lemmas 1 and 2, if firms collude in both countries they will be prosecuted the first time $c_t^A = 1$. Therefore, the expected profits of a firm under collusion are given by $\Pi^{c,AB} = \alpha^A\left(\pi^{c,A} + \pi^{c,B} - f^A - f^B\right) + \left(1 - \alpha^A\right)\left(\pi^{c,A} + \pi^{c,B} + \delta\Pi^{c,AB}\right)$, which implies

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1 - \left(1 - \alpha^A\right)\delta}.$$

Collusion can be sustained as a subgame perfect Nash equilibrium whenever $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. (Since firms are not allowed to collude in each market separately, when a firm violates the collusive agreement, it deviates in both countries). Therefore, if $\alpha^A \leq \bar{\alpha}^1 = \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{f^A + f^B + \delta\left(\pi^{d,A} + \pi^{d,B}\right)}$, firms collude until they are prosecuted, while if $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ (equivalently, if $\alpha^A > \bar{\alpha}^1$) firms do not collude at all. ■

A.2 Proof of Proposition 2

Suppose that $\delta < \bar{\delta}$. Then, from Lemmas 1 and 2, if firms collude in both countries or they only collude in country B, they will never be prosecuted, while if they only collude in country A, they will be prosecuted the first time $c_t^A = 1$. Therefore, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1 - \left(1 - \alpha^A\right)\delta}, \; \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta}, \; \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1-\delta},$$

respectively. In order to deduce $\Pi^{c,A}$ note that $\Pi^{c,A} = \alpha^A\left(\pi^{c,A} - f^A\right) + \left(1 - \alpha^A\right)\left(\pi^{c,A} + \delta\Pi^{c,A}\right)$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country A to be an equilibrium, it must be the case $\Pi^{c,A} \geq \pi^{d,A}$.

For collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in country A, in country B, or in both countries, which requires $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, respectively. Since $\pi^{c,j} > (1-\delta)\,\pi^{d,j}$ for $j = A, B$, all these conditions hold. Therefore, if $\Pi^{c,A} \geq \pi^{d,A}$, the three types of collusion can be sustained as an equilibrium, while if $\Pi^{c,A} < \pi^{d,A}$, only collusion in country B or collusion in both countries can be sustained as an equilibrium. Finally, note that $\Pi^{c,AB} > \Pi^{c,A}, \Pi^{c,B}$. Thus, firms prefer to coordinate in an equilibrium in which they collude in both countries.

Suppose that $\delta \geq \bar{\delta}$. Then, from Lemmas 1 and 2, if firms only collude in country B, they will never be prosecuted, while if they collude in country A or in both countries, they will be prosecuted the first time that $c_t^A = 1$. Therefore, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1 - (1-\alpha^A)\,\delta},\ \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta},\ \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1 - (1-\alpha^A)\,\delta},$$

respectively. In order to deduce $\Pi^{c,AB}$, note that $\Pi^{c,AB} = \alpha^A\left(\pi^{c,A} + \pi^{c,B} - f^A - f^B\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \pi^{c,B} + \delta\Pi^{c,AB}\right)$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in country A, in country B, or in both countries. No firm has an incentive to stop colluding in country A if $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$. No firm has an incentive to stop colluding in country B if $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$. No firm has an incentive to deviate in both countries if $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Therefore, we must consider four different cases.

1. Only collusion in country B can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and at least one of the following inequalities holds $\Pi^{c,AB} < \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} < \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$. These conditions hold if and only if $\Pi^{c,A} < \pi^{d,A}$ or, which is equivalent $\alpha^A > \bar{\alpha}_H^2 = \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}$. Therefore, if $\alpha^A > \bar{\alpha}_H^2$, firms collude in country B and they are never prosecuted.

2. Only collusion in country B or in both countries can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$, $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. These conditions lead to $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,A} \geq \pi^{d,A} + \frac{\alpha^A\left(\delta\Pi^{c,B} + f^B\right)}{1-(1-\alpha^A)\delta}$, a contradiction.

3. Only collusion in country B or collusion in country A can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and at least one of the following inequalities holds $\Pi^{c,AB} < \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} < \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$. These conditions simultaneously holds if and only if

$$\pi^{d,A} \leq \Pi^{c,A} < \pi^{d,A} + \frac{\alpha^A\left(\delta\Pi^{c,B} + f^B\right)}{1 - (1-\alpha^A)\,\delta}$$

or

$$\Pi^{c,A} \geq \pi^{d,A} + \frac{\alpha^A\left(\delta\Pi^{c,B} + f^B\right)}{1 - (1-\alpha^A)\,\delta} \text{ and } \Pi^{c,B} < \frac{\left[1 - \left(1-\alpha^A\right)\delta\right]\pi^{d,B} + \alpha^A f^B}{1-\delta}.$$

The first inequality is equivalent to $\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\delta\Pi^{c,B}+f^B} < \alpha^A \leq \bar{\alpha}_H^2$, while the second and third inequalities are equivalent to $\frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B} < \alpha^A \leq \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\delta\Pi^{c,B}+f^B}$. Then, only collusion in country B or collusion in country A can be sustained as an equilibrium when $\bar{\alpha}_L^2 < \alpha^A \leq \bar{\alpha}_H^2$, where $\bar{\alpha}_L^2 = \min\left\{\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\delta\Pi^{c,B}+f^B}, \frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B}\right\}$. Finally, firms prefer to collude in A when $\Pi^{c,A} > \Pi^{c,B}$ or, which is equivalent, $\alpha^A < \frac{(1-\delta)\left(\pi^{c,A}-\pi^{c,B}\right)}{\delta\pi^{c,B}+(1-\delta)f^A}$. Otherwise, they prefer to collude in B.

4. The three types of collusion can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$, $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$, $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. These conditions hold if and only if

$$\Pi^{c,A} \geq \pi^{d,A} + \frac{\alpha^A\left(\delta\Pi^{c,B}+f^B\right)}{1-\left(1-\alpha^A\right)\delta} \text{ and } \Pi^{c,B} \geq \frac{\left[1-\left(1-\alpha^A\right)\delta\right]\pi^{d,B}+\alpha^A f^B}{1-\delta}.$$

These inequalities are equivalent to $\alpha^A \leq \bar{\alpha}_L^2 = \min\left\{\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\delta\Pi^{c,B}+f^B}, \frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B}\right\}$. Moreover, note that $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\Pi^{c,A} > \frac{\alpha^A\left(\delta\Pi^{c,B}+f^B\right)}{1-(1-\alpha^A)\delta}$ while $\Pi^{c,AB} > \Pi^{c,A}$ if and only if $\Pi^{c,B} > \frac{\alpha^A f^B}{1-\delta}$. Thus, when the there types of collusion can be sustained as an equilibrium, firms prefer to coordinate in an equilibrium in which they collude in both countries. ■

A.3 Proof of Proposition 3

Suppose that $\delta < \bar{\delta}$. Then, from Lemmas 1 and 2, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A}-\alpha^A f^A}{1-\left(1-\alpha^A\right)\delta}, \; \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta}, \; \Pi^{c,AB} = \frac{\pi^{c,A}+\pi^{c,B}}{1-\delta},$$

respectively.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Note that, since the punishment for deviation is competition in both countries, once a firm decides to violate the collusive agreement, it deviates in both countries. Since $\pi^{c,j} > (1-\delta)\,\pi^{d,j}$ for $j = A, B$, collusion in both countries can always be sustained. Therefore, if $\Pi^{c,A} \geq \pi^{d,A}$, the three types of collusion can be sustained as an equilibrium, while if $\Pi^{c,A} < \pi^{d,A}$, only collusion in country B or collusion in both countries can be sustained as an equilibrium. Finally, note that $\Pi^{c,AB} > \Pi^{c,A}, \Pi^{c,B}$. Thus, firms prefer to coordinate in an equilibrium in which they collude in both countries.

Suppose that $\delta \geq \bar{\delta}$. Then, from Lemmas 1 and 2, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A}-\alpha^A f^A}{1-\left(1-\alpha^A\right)\delta}, \; \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta}, \; \Pi^{c,AB} = \frac{\pi^{c,A}+\pi^{c,B}-\alpha^A\left(f^A+f^B\right)}{1-\left(1-\alpha^A\right)\delta},$$

respectively.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds. For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in both countries, which requires $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Therefore, we must consider four different cases:

1. Only collusion in country B can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\alpha^A > \bar{\alpha}^3 = \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}$ and $\alpha^A > \hat{\alpha}^3 = \frac{\left(\pi^{c,A}+\pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B}$. Thus, if $\alpha^A > \bar{\alpha}^3_H = \max\left\{\bar{\alpha}^3, \hat{\alpha}^3\right\}$, firms only collude in country B and they are never detected.

2. Only collusion in country B or in both countries can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\bar{\alpha}^3 < \alpha^A \leq \hat{\alpha}^3$. If $\hat{\alpha}^3 < \bar{\alpha}^3$, this never holds, while if $\hat{\alpha}^3 > \bar{\alpha}^3$, firms must decide between collusion in B and collusion in both countries. Firms prefer to collude in both countries when $\Pi^{c,AB} > \Pi^{c,B}$ and to collude in B when $\Pi^{c,AB} < \Pi^{c,B}$. Note that $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\alpha^A < \tilde{\alpha}^3 = \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$. Thus, if $\hat{\alpha}^3 < \bar{\alpha}^3$, firms collude in both countries when $\alpha^A < \tilde{\alpha}^3$ and they collude in country B when $\alpha^A > \tilde{\alpha}^3$.

3. Only collusion in country B or collusion in country A can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\hat{\alpha}^3 < \alpha^A \leq \bar{\alpha}^3$. If $\hat{\alpha}^3 > \bar{\alpha}^3$, this never holds, while if $\hat{\alpha}^3 < \bar{\alpha}^3$, firms must decide between collusion in A and collusion in country B. Firms prefer to collude in A when $\Pi^{c,A} > \Pi^{c,B}$ and to collude in B when $\Pi^{c,A} < \Pi^{c,B}$. Note that $\Pi^{c,A} > \Pi^{c,B}$ if and only if $\alpha^A < \breve{\alpha}^3 = \frac{(1-\delta)\left(\pi^{c,A}-\pi^{c,B}\right)}{\delta\pi^{c,B}+(1-\delta)\alpha^A f^A}$. Thus, if $\hat{\alpha}^3 > \bar{\alpha}^3$, firms collude in country A when $\alpha^A < \breve{\alpha}^3$ and they collude in country B when $\alpha^A > \breve{\alpha}^3$.

4. The three types of collusion can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\alpha^A \leq \bar{\alpha}^3_L = \min\left\{\bar{\alpha}^3, \hat{\alpha}^3\right\}$. Firms prefer to collude in A when $\Pi^{c,A} > \max\left\{\Pi^{c,B}, \Pi^{c,AB}\right\}$, i.e., when $\frac{\pi^{c,B}}{f^B} < \alpha^A < \breve{\alpha}^3$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\left\{\Pi^{c,A}, \Pi^{c,B}\right\}$, i.e., when $\alpha^A < \min\left\{\frac{\pi^{c,B}}{f^B}, \tilde{\alpha}^3\right\}$. Finally, firms prefer to collude in B when $\Pi^{c,B} > \max\left\{\Pi^{c,A}, \Pi^{c,AB}\right\}$, i.e., when $\alpha^A > \max\left\{\breve{\alpha}^3, \tilde{\alpha}^3\right\}$. ■

A.4 Proof of Proposition 4

Suppose that firms are only colluding in country A while there is and will always be competition in country B. Assume that $c^A_t = 1$. If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period t is $W^A_t\left(p^A_t = 1 | c^A_t = 1, A\right) = S^{c,A} + 2\left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta} S^{com,A}$, while if firms are not prosecuted, it is $W^A_t\left(p^A_t = 0 | c^A_t = 1, A\right) = \frac{S^{c,A}+2\pi^{c,A}}{1-\delta}$. Since $S^{com,A} > S^{c,A} + 2\pi^{c,A}$, it must be the case that $W^A_t\left(p^A_t = 1 | c^A_t = 1, A\right) > W^A_t\left(p^A_t = 0 | c^A_t = 1, A\right)$.

Suppose that firms are only colluding in country A, there is competition in country B, but as soon as they are prosecuted in country A, firms will start colluding in country B. Assume that $c^A_t = 1$. If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period t is $W^A_t\left(p^A_t = 1 | c^A_t = 1, A\right) = S^{c,A} + 2\pi^{c,A} + \frac{\delta}{1-\delta}\left(S^{com,A} + 2\pi^{c,B}\right)$, while if the firms are not prosecuted, it is $W^A_t\left(p^A_t = 0 | c^A_t = 1, A\right) = \frac{S^{c,A}+2\pi^{c,A}}{1-\delta}$. Since, $S^{com,A} > S^{c,A} + 2\pi^{c,A}$, it must be the case that $W^A_t\left(p^A_t = 1 | c^A_t = 1, A\right) > W^A_t\left(p^A_t = 0 | c^A_t = 1, A\right)$.

Suppose that firms are colluding in both countries and $c_t^A = 1$. If the competition authority of country A decides to prosecute the firms, the expected discounted welfare of country A at period t will be $W_t^A\left(p_t^A = 1|c_t^A = 1, AB\right) = S^{c,A} + 2\left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta}S^{com,A}$. If the competition authority of country A does not prosecute the firms, then the the expected discounted welfare of country A at period t will be $W_t^A\left(p_t^A = 0|c_t^A = 1, AB\right) = \frac{S^{c,A}+2\left(\pi^{c,A}+\pi^{c,B}\right)}{1-\delta}$. $W_t^A\left(p_t^A = 1|c_t^A = 1, AB\right) > W_t^A\left(p_t^A = 0|c_t^A = 1, AB\right)$ if and only if $\delta\left(S^{com,A} - S^{c,A} - 2\pi^{c,A}\right) \geq \delta 2\pi^{c,B} + (1-\delta)\,2f^B$ or, which is equivalent, $\delta \geq \bar{\delta} = \frac{2f^B}{S^{com,A}-S^{c,A}-2\pi^{c,A}+2f^B-2\pi^{c,B}}$. Thus, Lemma 2 holds when Assumption 1 is replaced by Assumption 2.

Suppose that $\delta < \bar{\delta}$. Then, from Lemmas 1 and 2, if firms are colluding in both countries they will never be prosecuted. Therefore, if firms always collude in both countries, the expected discounted profits of a firm are given by:

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1-\delta}$$

Note that there is no other form of collusion that will induce higher discounted expected profits. Moreover, always collude in both countries can be sustained as a subgame perfect Nash equilibrium. In order to prove this, note that for collusion in both countries to be an equilibrium, it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Since $\pi^{c,j} > (1-\delta)\,\pi^{d,j}$ for $j = A, B$, all these conditions hold.

Suppose that $\delta \geq \bar{\delta}$. Then, from Lemmas 1 and 2, if firms are only colluding in country B, they will never be prosecuted, while if they are colluding in country A or in both countries, they will be prosecuted the first time that $c_t^A = 1$. Therefore, there are three types of collusion that firms must consider: 1) collude only in country A until detected, then start colluding in country B; 2) always collude only in country B; and 3) always collude in both countries. The expected discounted profits of a firm associated with each type of collusion are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A + \alpha^A \delta \frac{\pi^{c,B}}{1-\delta}}{1-\left(1-\alpha^A\right)\delta},\ \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta},\ \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1-\left(1-\alpha^A\right)\delta},$$

respectively. In order to deduce $\Pi^{c,A}$ and $\Pi^{c,AB}$ note that $\Pi^{c,A} = \alpha^A\left(\pi^{c,A} - f^A + \delta\Pi^{c,B}\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \delta\Pi^{c,A}\right)$ and $\Pi^{c,AB} = \alpha^A\left(\pi^{c,A} + \pi^{c,B} - f^A - f^B\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \pi^{c,B} + \delta\Pi^{c,AB}\right)$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. Collusion in country B is always an equilibrium because $\Pi^{c,B} \geq \pi^{d,B}$ always holds. For collusion in country A until detected, then collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$. For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$.

1. Only collusion in country B can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and either $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta\pi^{d,A} + f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ and $\alpha^A > \max\left\{\hat{\alpha}^4, \bar{\alpha}^4\right\}$, where $\bar{\alpha}^4 = \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A-\frac{\delta\pi^{c,B}}{1-\delta}}$ and $\hat{\alpha}^4 = \frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B}$.

2. Only collusion in country B and collusion in country A until detected, then collusion in country B can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and $\Pi^{c,AB} < \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta\pi^{d,A} + f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ and $\hat{\alpha}^4 < \alpha^A \leq \bar{\alpha}^4$ or $\delta\pi^{d,A} + f^A \leq \frac{\delta\pi^{c,B}}{1-\delta}$ and $\hat{\alpha}^4 < \alpha^A$. Firms prefer to collude in country A when $\Pi^{c,A} > \Pi^{c,B}$, i.e., when $\alpha^A < \breve{\alpha}^4 = \frac{\pi^{c,A}-\pi^{c,B}}{f^A}$.

3. Only collusion in country B or in both countries can be sustained as an equilibrium when $\Pi^{c,A} < \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta\pi^{d,A} + f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ and $\bar{\alpha}^4 < \alpha^A < \hat{\alpha}^4$. Firms prefer to collude in both countries if $\Pi^{c,AB} > \Pi^{c,B}$, i.e., when $\alpha^A < \tilde{\alpha}^4 = \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$.

4. The three types of collusion can be sustained as an equilibrium when $\Pi^{c,A} \geq \pi^{d,A}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ or, which is equivalent, $\delta\pi^{d,A} + f^A \leq \frac{\delta\pi^{c,B}}{1-\delta}$ and $\alpha^A \leq \hat{\alpha}^4$ or $\delta\pi^{d,A} + f^A > \frac{\delta\pi^{c,B}}{1-\delta}$ and $\alpha^A \leq \min\left\{\bar{\alpha}^4, \hat{\alpha}^4\right\}$. Firms prefer to collude in A when $\Pi^{c,A} > \max\left\{\Pi^{c,B}, \Pi^{c,AB}\right\}$, i.e., when $\frac{\pi^{c,B}}{f^B+\delta\frac{\pi^{c,B}}{1-\delta}} < \alpha^A < \frac{\pi^{c,A}-\pi^{c,B}}{f^A}$. Firms prefer to collude in both countries when $\Pi^{c,AB} > \max\left\{\Pi^{c,A}, \Pi^{c,B}\right\}$, i.e., when $\alpha^A < \min\left\{\frac{\pi^{c,B}}{f^B+\delta\frac{\pi^{c,B}}{1-\delta}}, \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}\right\}$. Finally, firms prefer to collude in B when $\Pi^{c,B} > \max\left\{\Pi^{c,A}, \Pi^{c,AB}\right\}$, i.e., when $\alpha^A > \max\left\{\frac{\pi^{c,A}-\pi^{c,B}}{f^A}, \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}\right\}$. ■

A.5 Proofs of Lemma 3 and Proposition 5

Proof of Lemma 3. As in Lemma 2 we must distinguish two possible cases: when firms are only colluding in country A and when they are colluding in both countries. Suppose that firms are only colluding in country A and $c_t^A = 1$. If competition authority A prosecutes the firms, the expected discounted welfare of country A at period t is $W_t^A(p_t^A = 1|c_t^A = 1, A) = S^{c,A} + 2\left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta}S^{com,A}$, while if firms are not prosecuted, it is $W_t^A(p_t^A = 0|c_t^A = 1, A) = \frac{S^{c,A}+2\pi^{c,A}}{1-\delta}$. Since $S^{com,A} > S^{c,A} + 2\pi^{c,A}$, it must be the case that $W_t^A(p_t^A = 1|c_t^A = 1, A) > W_t^A(p_t^A = 0|c_t^A = 1, A)$. Thus, when firms are only colluding in country A, as soon as $c_t^A = 1$, competition authority A immediately prosecutes the firms.

Suppose there is collusion in both countries and $c_t^A = 1$. If competition authority A decides to prosecute the firms, then, from Lemma 1, competition authority B will detect collusion in country B as well and, hence, firms will also be prosecuted in country B. Therefore, from period $t+1$ there will be competition in both countries. Then, the expected discounted welfare of country A at period t is:

$$W_t^A\left(p_t^A = 1|c_t^A = 1, AB\right) = S^{c,A} + 2\left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta}S^{com,A}.$$

If competition authority A does not prosecute the firms, then with probability α^B competition authority B receives $c_t^B = 1$ and with probability $\left(1-\alpha^B\right)$ it receives $c_t^B = 0$. If $c_t^B = 1$, then competition authority B prosecutes the firms in period t and in period $t+1$ with probability α^A competition authority A will receive $c_{t+1}^A = 1$ and with probability $\left(1-\alpha^A\right)$ it will receive $c_{t+1}^A = 0$. Then, the expected discounted welfare of country A at period t is

$$\begin{aligned} W_t^A\left(p_t^A = 0|c_t^A = 1, AB\right) &= S^{c,A} + 2\pi^{c,A} + 2\pi^{c,B} - \alpha^B 2f^B \\ &+ \alpha^B\delta\left[\alpha^A W_t^A\left(p_t^A = 1|c_t^A = 1, A\right) + \left(1-\alpha^A\right)W_t^A\left(c_t^A = 0, A\right)\right] \\ &+ \left(1-\alpha^B\right)\delta W_t^A\left(p_t^A = 0|c_t^A = 1, AB\right) \end{aligned}$$

where $W_t^A\left(p_t^A = 1|c_t^A = 1, A\right)$ and $W_t^A\left(c_t^A = 0, A\right)$ are given by

$$W_t^A(p_t^A = 1|c_t^A = 1, A) = S^{c,A} + 2\pi^{c,A} + \frac{\delta}{1-\delta}S^{com,A},$$
$$W_t^A\left(c_t^A = 0, A\right) = S^{c,A} + 2\pi^{c,A} + \delta\left[\alpha^A W_t^A(p_t^A = 1|c_t^A = 1, A) + \left(1-\alpha^A\right)W_t^A\left(c_t^A = 0, A\right)\right].$$

Solving for $W_t^A\left(p_t^A=0|c_t^A=1,AB\right)$ we obtain:

$$W_t^A\left(p_t^A=0|c_t^A=1,AB\right)=\frac{\frac{\left[1-\left(1-\alpha^A\right)\delta+\alpha^B\delta\right]\left(S^{c,A}+2\pi^{c,A}\right)}{\left[1-\left(1-\alpha^A\right)\delta\right]}+2\left(\pi^{c,B}-\alpha^Bf^B\right)+\frac{\alpha^A\alpha^B\delta^2S^{com,A}}{\left[1-\left(1-\alpha^A\right)\delta\right](1-\delta)}}{\left[1-\left(1-\alpha^B\right)\delta\right]}.$$

Finally, $W_t^A\left(p_t^A=1|c_t^A=1,AB\right) > W_t^A\left(p_t^A=0|c_t^A=1,AB\right)$ if and only $\Delta S^A - 2\pi^{c,A} \geq \frac{\left[1-\left(1-\alpha^A\right)\delta\right]2\left(1-\alpha^B\right)\left[\delta\pi^{c,B}+(1-\delta)f^B\right]}{\left[1-\left(1-\alpha^B\right)\left(1-\alpha^A\right)\delta\right]\delta}$. ■

Proof of Proposition 5. Suppose that $\Delta S^A-2\pi^{c,A}\geq\frac{\left[1-\left(1-\alpha^A\right)\delta\right]2\left(1-\alpha^B\right)\left[\delta\pi^{c,B}+(1-\delta)f^B\right]}{\left[1-\left(1-\alpha^B\right)\left(1-\alpha^A\right)\delta\right]\delta}$ or, which is equivalent, $\alpha^B>\frac{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]-\delta\left(\Delta S^A-2\pi^{c,A}\right)}{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]+\frac{\delta\left(1-\alpha^A\right)}{1-\left(1-\alpha^A\right)\delta}\delta\left(\Delta S^A-2\pi^{c,A}\right)}$. Then, from Lemmas 1 and 3, if firms collude in both countries or they only collude in country B, they will be prosecuted in country B the first time that $c_t^B=1$, while if they only collude in country A, they will be prosecuted the first time $c_t^A=1$. Therefore, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A}=\frac{\pi^{c,A}-\alpha^Af^A}{1-\left(1-\alpha^A\right)\delta},\ \Pi^{c,B}=\frac{\pi^{c,B}-\alpha^Bf^B}{1-\left(1-\alpha^B\right)\delta},\ \Pi^{c,AB}=\frac{\pi^{c,A}+\pi^{c,B}-\alpha^Bf^B+\frac{\delta\alpha^B\left(\pi^{c,A}-\alpha^Af^A\right)}{1-\left(1-\alpha^A\right)\delta}}{1-\left(1-\alpha^B\right)\delta},$$

respectively. In order to deduce $\Pi^{c,AB}$ note that $\Pi^{c,AB}=\pi^{c,A}+\pi^{c,B}+\alpha^B\left(-f^B+\delta\Pi^{c,A}\right)+\left(1-\alpha^B\right)\left(\pi^{c,A}+\pi^{c,B}+\delta\Pi^{c,AB}\right)$.

Next, we deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium. For collusion in country A to be an equilibrium it must be the case that $\Pi^{c,A}\geq\pi^{d,A}$. For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,B}\geq\pi^{d,B}$. For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,AB}\geq\pi^{d,A}+\pi^{d,B}$. $\Pi^{c,A}\geq\pi^{d,A}$ if and only if $\alpha^A\leq\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$. $\Pi^{c,AB}\geq\pi^{d,A}+\pi^{d,B}$ if and only if $\alpha^B\leq\frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)-\frac{\delta\left(\pi^{c,A}-\alpha^Af^A\right)}{1-\left(1-\alpha^A\right)\delta}+f^B}$. Note that if $\Pi^{c,A}\geq\pi^{d,A}$, then $\Pi^{c,AB}\geq\Pi^{c,B}$. Hence, when collusion in country A and collusion in both countries can be sustained as an equilibrium, firms always prefer to collude in both countries rather than only in country B. Finally, $\Pi^{c,AB}>\Pi^{c,A}$ if and only if $\alpha^B<\frac{(1-\delta)\pi^{c,B}+\alpha^A\left[\delta\left(\pi^{c,A}+\pi^{c,B}\right)+(1-\delta)f^A\right]}{\left[1-\left(1-\alpha^A\right)\delta\right]f^B}$. ■

A.6 Proof of Proposition 6

Part 1. Suppose that the firms are owned by foreign citizens. Fix α^B and consider $P^A=\left\{\alpha^A\in[0,1]:\left(\alpha^B,\alpha^A\right)\in R^B\right\}$, i.e., the set of all α^A for which $\left(\alpha^B,\alpha^A\right)\in R^B$. From Proposition 5 Part 1 (note that the roles of A and B are reversed), we know that $\left(\alpha^B,\alpha^A\right)\in R^B$ implies that, in equilibrium, firms collude in both countries until the first time $c_t^A=1$, when they are prosecuted in country A. Thereafter, they collude in country B until the first time $c_{t+\tau}^B=1$ with $\tau\geq1$, when they are prosecuted in country B. Thereafter, there is competition in both countries. Thus, for $\alpha^A\in P^A$, the expected discounted welfare of country A at period t if $c_t^A=1$ is given by

$$W_t^A\left(c_t^A=1\right)=S^{c,A}+2f^A+\frac{\delta}{1-\delta}S^{com,A},$$

while if $c_t^A = 1$, it is

$$W_t^A\left(c_t^A = 0\right) = S^{c,A} + \alpha^A \delta W_t^A\left(c_t^A = 1\right) + \left(1 - \alpha^A\right)\delta W_t^A\left(c_t^A = 0\right).$$

At $t = 0$, before A selects α^A, the discounted expected welfare of country is given by $W_0^A = \alpha^A W_0^A\left(c_t^A = 1\right) + \left(1 - \alpha^A\right) W_0^A\left(c_t^A = 0\right) - C\left(\alpha^A\right)$. Therefore

$$W_0^A = \frac{S^{c,A} + 2\alpha^A f^A + \frac{\alpha^A \delta}{1-\delta} S^{com,A}}{1 - \left(1 - \alpha^A\right)\delta} - C\left(\alpha^A\right).$$

Note that W_0^A is a C^2 and strictly concave function of α^A, while P^A is an interval because it is just the intersection of linear inequalities. Let α_H be the unique solution to $\left(W_0^A\right)' = 0$, i.e.,

$$\frac{\delta\left(S^{com,A} - S^{c,A}\right) + 2f^A\left(1 - \delta\right)}{\left[1 - \left(1 - \alpha_H\right)\delta\right]^2} = C'\left(\alpha_H\right).$$

Suppose that $\alpha_H \in P^A$. Then, $\alpha_H = \arg\max_{\alpha^A \in P^A} W_0^A$.

Part 2. Suppose that the firms are owned by local citizens and only operate in the domestic market. Assume that $\alpha^A \leq \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}$. Then, firms will collude until the first time $c_t^A = 1$, when they will be prosecuted. Thereafter, there will be competition. Then, the expected discounted welfare of country A at period t if $c_t^A = 1$ is given by

$$W_t^A\left(c_t^A = 1\right) = S^{c,A} + 2\pi^{c,A} + \frac{\delta}{1 - \delta} S^{com,A},$$

while if $c_t^A = 0$ it is

$$W_t^A\left(c_t^A = 0\right) = S^{c,A} + 2\pi^{c,A} + \alpha^A \delta W_t^A\left(c_t^A = 1\right) + \left(1 - \alpha^A\right)\delta W_t^A\left(c_t^A = 0\right)$$

At $t = 0$, before A selects α^A, the discounted expected welfare of country is given by

$$W_0^A = \frac{\left(S^{c,A} + 2\pi^{c,A}\right) + \frac{\alpha^A \delta}{1-\delta} S^{com,A}}{1 - \left(1 - \alpha^A\right)\delta} - C\left(\alpha^A\right).$$

Note that W_0^A is a C^2 and strictly concave function of α^A, while $\alpha^A \leq \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}$ is an interval. Let α_M be the unique solution to $\left(W_0^A\right)' = 0$, i.e.,

$$\frac{\delta\left(S^{com,A} - S^{c,A} - 2\pi^{c,A}\right)}{\left[1 - \left(1 - \alpha_M\right)\delta\right]^2} = C'\left(\alpha_M\right)$$

Suppose that $\alpha_M \leq \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}$. Then, $\alpha_M = \arg\max_{\alpha^A \leq \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}} W_0^A$.

Part 3. Suppose that the firms are owned by local citizens and they also operate in foreign markets. Fix α^B and consider $P^A = \left\{\alpha^A \in [0, 1] : \left(\alpha^A, \alpha^B\right) \in R^A\right\}$, i.e., the set of all α^A for which $\left(\alpha^A, \alpha^B\right) \in R^A$. From Proposition 5 Part 1, we know that $\left(\alpha^A, \alpha^B\right) \in R^B$ implies that, in equilibrium, firms collude in both countries until the first time $c_t^B = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $c^A_{t+\tau} = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries. Thus, for $\alpha^A \in P^A$, the expected discounted welfare of country A is given by:

$$W^A_0 = \left(S^{c,A} + 2\pi^{c,A} + 2\pi^{c,B}\right) + \alpha^B \left(-2f^B + \delta \tilde{W}^A_0\right) + \left(1 - \alpha^B\right) \delta W^A_0,$$

where $\tilde{W}^A_0 = \frac{S^{c,A} + 2\pi^{c,A} + \frac{\alpha^A \delta}{1-\delta} S^{com,A}}{1-(1-\alpha^A)\delta}$. Hence:

$$W^A_0 = \frac{\left[\frac{1-\delta+\alpha^A \delta \alpha^B \delta}{1-(1-\alpha^A)\delta}\right] \left(S^{c,A} + 2\pi^{c,A}\right) + 2\left(\pi^{c,B} - \alpha^B f^B\right) + \frac{\alpha^A \alpha^B \delta^2 S^{com,A}}{[1-(1-\alpha^A)\delta](1-\delta)}}{\left[1 - \left(1 - \alpha^B\right) \delta\right]}.$$

Note that W^A_0 is a C^2 and strictly concave function of α^A, while P^A is an interval because it is just the intersection of linear inequalities. Let α_L be the unique solution to $\left(W^A_0\right)' = 0$, i.e.,

$$\frac{\alpha^B \delta^2 \left(S^{com,A} - S^{c,A} - 2\pi^{c,A}\right)}{\left[1 - (1 - \alpha_L) \delta\right]^2 \left[1 - \left(1 - \alpha^B\right) \delta\right]} = C'(\alpha_L).$$

Suppose that $\alpha_L \in P^A$. Then, $\alpha_L = \arg\max_{\alpha^A \in P^A} W^A_0$.

Part 4. Note that $\frac{\delta\left(S^{com,A}-S^{c,A}\right)+2f^A(1-\delta)}{[1-(1-\alpha)\delta]^2}$, $\frac{\delta\left(S^{com,A}-S^{c,A}-2\pi^{c,A}\right)}{[1-(1-\alpha)\delta]^2}$, and $\frac{\alpha^B\delta^2\left(S^{com,A}-S^{c,A}-2\pi^{c,A}\right)}{[1-(1-\alpha)\delta]^2[1-(1-\alpha^B)\delta]}$ are decreasing in α, while $C'(\alpha)$ is increasing in α. Moreover,

$$\frac{\delta\left(S^{com,A} - S^{c,A}\right) + 2f^A(1-\delta)}{\left[1 - (1 - \alpha) \delta\right]^2} > \frac{\delta\left(S^{com,A} - S^{c,A} - 2\pi^{c,A}\right)}{\left[1 - (1 - \alpha) \delta\right]^2} > \frac{\alpha^B \delta^2 \left(S^{com,A} - S^{c,A} - 2\pi^{c,A}\right)}{\left[1 - (1 - \alpha) \delta\right]^2 \left[1 - \left(1 - \alpha^B\right) \delta\right]}.$$

where the last inequality holds for all α^B. Therefore, $\alpha_H > \alpha_M > \alpha_L$.

Finally, from the implicit function theorem, the derivative of α_L with respect to α^B is given by:

$$\frac{d\alpha_L}{d\alpha^B} = \frac{\delta^2 \left(\Delta S^A - 2\pi^{c,A}\right)(1-\delta)}{\left[1 - (1 - \alpha_L)\delta\right]^2 \left[1 - \left(1 - \alpha^B\right)\delta\right]^2 C''(\alpha_L) + \frac{[1-(1-\alpha^B)\delta] 2\alpha^B \delta^3 (\Delta S^A - 2\pi^{c,A})}{[1-(1-\alpha_L)\delta]}}$$

Since $C''(\alpha_L) \geq 0$, we have $\frac{d\alpha_L}{d\alpha^B} > 0$. ■

A.7 Proof of Proposition 7

Let $\alpha^{B,*}$ be the unique solution to

$$\frac{\delta \Delta S^B + 2f^B(1-\delta)}{\left[1 - \left(1 - \alpha^{B,*}\right)\delta\right]^2} = C'\left(\alpha^{B,*}\right).$$

Assume that $\alpha^{B,*} \in P^B = \left\{\alpha^B \in [0,1] : \left(\alpha^A, \alpha^B\right) \in R^A\right\}$. Then, from Proposition 6 Part 1, $\alpha^{B,*} = \arg\max_{\alpha^B \in P^B} W^B_0$. Thus, $\alpha^B = \alpha^{B,*}$ is a dominant strategy for competition authority B.

Let $\alpha^{A,*}$ be the unique solution to

$$\frac{\alpha^{B,*}\delta^2\left(\Delta S^A - 2\pi^{c,A}\right)}{\left[1-\left(1-\alpha^{A,*}\right)\delta\right]^2\left[1-\left(1-\alpha^{B,*}\right)\delta\right]} = C'\left(\alpha^{A,*}\right).$$

Assume that $\alpha^{A,*} \in P^A = \left\{\alpha^A \in [0,1] : \left(\alpha^A, \alpha^{B,*}\right) \in R^A\right\}$. Then, from Proposition 6 Part 3, $\alpha^{A,*} = \arg\max_{\alpha^A \in P^A} W_0^A$. Thus, $\alpha^A = \alpha^{A,*}$ is the best response of competition authority A to $\alpha^{B,*}$. Therefore, $\left(\alpha^A, \alpha^B\right) = \left(\alpha^{A,*}, \alpha^{B,*}\right)$ is the unique Nash equilibrium.

Since $\frac{\delta\Delta S^B + 2f^B(1-\delta)}{[1-(1-\alpha)\delta]^2}$ and $\frac{\alpha^{B,*}\delta^2\left(\Delta S^A - 2\pi^{c,A}\right)}{[1-(1-\alpha)\delta]^2[1-(1-\alpha^{B,*})\delta]}$ are decreasing in α, while $C'(\alpha)$ is increasing in α, a sufficient condition for $\alpha^{B,*} > \alpha^{A,*}$ is

$$\delta\Delta S^B + 2f^B(1-\delta) > \frac{\alpha^{B,*}\delta^2\left(\Delta S^A - 2\pi^{c,A}\right)}{\left(1-\delta+\alpha^{B,*}\delta\right)}$$

The right hand side of this inequality is increasing in $\alpha^{B,*}$. Hence, if $\delta\Delta S^B + 2f^B(1-\delta) > \delta^2\left(\Delta S^A - 2\pi^{c,A}\right)$, it must be the case that the inequality holds for any α^B. ■

A.8 Proofs of Propositions 8 and 9 and Corollary 2

Equilibrium under no integration. The proof of Proposition 8 is identical to the proof of Proposition 5 Part 1. The only required change in notation is to add the industry superindex $z \in \{x, y\}$.

Welfare under no integration. We compute expected welfare under no integration for each country.

Consider industry x under no integration. Suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A}$. Then, in equilibrium, firms collude in both countries until the first time $c_t^{x,B} = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $c_{t+\tau}^{x,A} = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries. Then, the expected discounted welfare of country A in industry x is:

$$W_0^{x,A} = \left(S^{c,x,A} + 2\pi^{c,x,A} + 2\pi^{c,x,B}\right) + \alpha^{x,B}\left(-2f^{x,B} + \delta\tilde{W}_0^{x,A}\right) + \left(1-\alpha^{x,B}\right)\delta W_0^{x,A},$$

where

$$\tilde{W}_0^{x,A} = \frac{S^{c,x,A} + 2\pi^{c,x,A} + \frac{\alpha^{x,A}\delta}{1-\delta}S^{com,x,A}}{1-\left(1-\alpha^{x,A}\right)\delta}.$$

Hence:

$$W_0^{x,A} = \frac{\left[\frac{1-\delta+\alpha^{x,A}\delta+\alpha^{x,B}\delta}{1-(1-\alpha^{x,A})\delta}\right]\left(S^{c,x,A} + 2\pi^{c,x,A}\right) + 2\left(\pi^{c,x,B} - \alpha^{x,B}f^{x,B}\right) + \frac{\alpha^{x,A}\alpha^{x,B}\delta^2 S^{com,x,A}}{[1-(1-\alpha^{x,A})\delta](1-\delta)}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]}$$

The expected discounted welfare of country B in industry x is

$$W_0^{x,B} = \frac{S^{c,x,B} + 2\alpha^{x,B}f^{x,B} + \frac{\alpha^{x,B}\delta}{1-\delta}S^{com,x,B}}{1-\left(1-\alpha^{x,B}\right)\delta}.$$

Consider industry y under no integration. Suppose that $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B}$. Then, in equilibrium, firms collude in both countries until the first time $c_t^{y,A} = 1$, when they are prosecuted in country A.

Thereafter, they collude in country B until the first time $c_{t+\tau}^{y,B} = 1$ with $\tau \geq 1$, when they are prosecuted in country B. Thereafter, there is competition in both countries. Then, the expected discounted welfare of country B in industry y is:

$$W_0^{y,B} = \frac{\left[\frac{1-\delta+\alpha^{y,B}\delta+\alpha^{y,A}\delta}{1-(1-\alpha^{y,B})\delta}\right]\left(S^{c,y,B} + 2\pi^{c,y,B}\right) + 2\left(\pi^{c,y,A} - \alpha^{y,A} f^{y,A}\right) + \frac{\alpha^{y,B}\alpha^{y,A}\delta^2 S^{com,y,B}}{[1-(1-\alpha^{y,B})\delta](1-\delta)}}{\left[1 - \left(1 - \alpha^{y,A}\right)\delta\right]},$$

while the expected discounted welfare of country A in industry y is

$$W_0^{y,A} = \frac{S^{c,y,A} + 2\alpha^{y,A} f^{y,A} + \frac{\alpha^{y,A}\delta}{1-\delta} S^{com,y,A}}{1 - \left(1 - \alpha^{y,A}\right)\delta}.$$

Equilibrium under integration. Suppose that the competition authorities are integrated and, therefore, they prosecute collusion as soon as it is detected.

Consider industry x. Then, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,x,A} = \frac{\pi^{c,x,A} - \alpha^{x,A} f^{x,A}}{1 - \left(1 - \alpha^{x,A}\right)\delta},\ \Pi^{c,x,B} = \frac{\pi^{c,x,B} - \alpha^{x,B} f^{x,B}}{1 - \left(1 - \alpha^{x,B}\right)\delta},$$
$$\Pi^{c,x,AB} = \Pi^{c,x,A} + \frac{\pi^{c,x,B} - \left[1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\right] f^{x,B}}{1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\delta}.$$

respectively. For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,x,A} \geq \pi^{d,x,A}$. For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,x,B} \geq \pi^{d,x,B}$. For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,x,AB} \geq \pi^{d,x,A} + \pi^{d,x,B}$. $\Pi^{c,x,A} \geq \pi^{d,x,A}$ if and only if $\alpha^{x,A} \leq \frac{\pi^{c,x,A}-(1-\delta)\pi^{d,x,A}}{f^{x,A}+\delta\pi^{d,x,A}}$. $\Pi^{c,x,B} \geq \pi^{d,x,B}$ if and only if $\alpha^{x,B} \leq \frac{\pi^{c,x,B}-(1-\delta)\pi^{d,x,B}}{f^{x,B}+\delta\pi^{d,x,B}}$. $\Pi^{c,x,AB} \geq \pi^{d,x,A} + \pi^{d,x,B}$ if and only if

$$\sum_{j=A,B} \pi^{c,x,j} - \alpha^{x,A} f^{x,j} - \left[1 - \left(1 - \alpha^{x,A}\right)\delta\right]\pi^{d,x,j} \geq \alpha^{x,B}\left(1 - \alpha^{x,A}\right)\left[\delta\pi^{d,x,B} + f^{x,B} - \delta\left(\Pi^{c,x,A} - \pi^{d,x,A}\right)\right].$$

The left hand side of this inequality is positive if and only if $\alpha^{x,A} < \alpha_H = \frac{\pi^{c,x,A}+\pi^{c,x,B}-(1-\delta)\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}{f^{x,A}+f^{x,B}-\delta\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}$, while the right hand side is positive if and only if $\alpha^{x,A} > \alpha_L = \frac{\pi^{c,x,A}-\left(\frac{1-\delta}{\delta}\right)f^{x,B}-(1-\delta)\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}{f^{x,A}+f^{x,B}+\delta\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}$. Moreover, note that $\alpha_L < \alpha_H$. Hence, the inequality holds when $\alpha^{x,A} \leq \alpha_L$ or $\alpha_L < \alpha^{x,A} < \alpha_H$ and $\alpha^{x,B} \leq \frac{\sum_{j=A,B}\pi^{c,x,j}-\alpha^{x,A}f^{x,j}-\left[1-\left(1-\alpha^{x,A}\right)\delta\right]\pi^{d,x,j}}{\left(1-\alpha^{x,A}\right)\left[\delta\pi^{d,x,B}+f^{x,B}-\delta\left(\Pi^{c,x,A}-\pi^{d,x,A}\right)\right]}$. Finally, $\Pi^{c,x,AB} > \Pi^{c,x,A}$ if and only if $\alpha^{x,B} < \frac{\pi^{c,x,B}-\alpha^{x,A}f^{x,B}}{\left(1-\alpha^{x,A}\right)f^{x,B}}$, while $\Pi^{c,x,AB} > \Pi^{c,x,B}$ if and only if $\frac{\pi^{c,x,A}-\alpha^{x,A}f^{x,A}}{1-\left(1-\alpha^{x,A}\right)\delta} > \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B}+(1-\delta)f^{x,B}\right]}{[1-(1-\alpha^{x,B})\delta][1-(1-\alpha^{x,A})(1-\alpha^{x,B})\delta]}$.

Summing up, suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in \bar{R}^{x,A} = \left\{\left(\alpha^{x,A}, \alpha^{x,B}\right) \in [0,1] \times [0,1] : \left(\alpha^{x,A}, \alpha^{x,B}\right)\right\}$ satisfies the following conditions.

$$\begin{gathered}
\alpha^{x,A} \leq \frac{\pi^{c,x,A} - (1-\delta)\, \pi^{d,x,A}}{f^{x,A} + \delta \pi^{d,x,A}}, \\
\left[\alpha^{x,A} \leq \frac{\pi^{c,x,A} - \left(\frac{1-\delta}{\delta}\right) f^{x,B} - (1-\delta)\left(\pi^{d,x,A} + \pi^{d,x,B}\right)}{f^{x,A} + f^{x,B} + \delta\left(\pi^{d,x,A} + \pi^{d,x,B}\right)} \right] \quad or \\
\left[\frac{\pi^{c,x,A} - \left(\frac{1-\delta}{\delta}\right) f^{x,B} - (1-\delta)\left(\pi^{d,x,A} + \pi^{d,x,B}\right)}{f^{x,A} + f^{x,B} + \delta\left(\pi^{d,x,A} + \pi^{d,x,B}\right)} < \alpha^{x,A} < \frac{\pi^{c,x,A} + \pi^{c,x,B} - (1-\delta)\left(\pi^{d,x,A} + \pi^{d,x,B}\right)}{f^{x,A} + f^{x,B} - \delta\left(\pi^{d,x,A} + \pi^{d,x,B}\right)} \quad and \right. \\
\left. \alpha^{x,B} \leq \frac{\sum_{j=A,B} \pi^{c,x,j} - \alpha^{x,A} f^{x,j} - \left[1 - \left(1 - \alpha^{x,A}\right)\delta\right] \pi^{d,x,j}}{\left(1 - \alpha^{x,A}\right)\left[\delta \pi^{d,x,B} + f^{x,B} - \delta\left(\Pi^{c,x,A} - \pi^{d,x,A}\right)\right]} \right], \\
\left[\frac{\pi^{c,x,B} - (1-\delta)\pi^{d,x,B}}{f^{x,B} + \delta \pi^{d,x,B}} < \alpha^{x,B} < \frac{\pi^{c,x,B} - \alpha^{x,A} f^{x,B}}{\left(1 - \alpha^{x,A}\right) f^{x,B}} \right] \quad or \\
\left[\alpha^{x,B} < \min\left\{ \frac{\pi^{c,x,B} - (1-\delta)\pi^{d,x,B}}{f^{x,B} + \delta \pi^{d,x,B}}, \frac{\pi^{c,x,B} - \alpha^{x,A} f^{x,B}}{\left(1 - \alpha^{x,A}\right) f^{x,B}} \right\} \quad and \right. \\
\left. \frac{\pi^{c,x,A} - \alpha^{x,A} f^{x,A}}{1 - \left(1 - \alpha^{x,A}\right)\delta} > \frac{\alpha^{x,A}\left(1 - \alpha^{x,B}\right)\left[\delta \pi^{c,x,B} + (1-\delta) f^{x,B}\right]}{\left[1 - \left(1 - \alpha^{x,B}\right)\delta\right]\left[1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\delta\right]} \right],
\end{gathered}$$

Then, when competition authorities are integrated, firms in industry x collude in both countries until they are detected and prosecuted. Following exactly the same steps, but reversing the roles of A and B and replacing x for y, we obtain that $\bar{R}^{y,B}$. Then, if $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in \bar{R}^{y,B}$ and competition authorities are integrated, firms in industry y collude in both countries until they are detected and prosecuted. This completes the proof of Proposition 9.

Welfare under integration. We compute expected welfare under integration for each country.

Consider industry x under integration. Suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in \bar{R}^{x,A}$. Then, firms collude in both countries until the first time $c_t^{x,A} = 1$, when they are prosecuted in both countries, or until the first time $c_t^{x,A} = 0$ and $c_t^{x,B} = 1$, when they are prosecuted in country B. In the later case, firms keep colluding in country A until the first time $c_{t+\tau}^{x,A} = 1$. Then, the expected discounted welfare of country A in industry x is:

$$\begin{aligned}
\bar{W}_0^{x,A} = {} & \left(S^{c,x,A} + 2\pi^{c,x,A} + 2\pi^{c,x,B}\right) + \alpha^{x,A}\left(-2f^{x,B} + \delta \frac{S^{com,x,A}}{1-\delta}\right) + \\
& + \left(1 - \alpha^{x,A}\right)\alpha^{x,B}\left(-2f^{x,B} + \delta \tilde{W}_0^{x,A}\right) + \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\delta \bar{W}_0^{x,A},
\end{aligned}$$

where

$$\tilde{W}_0^{x,A} = \left(S^{c,x,A} + 2\pi^{c,x,A}\right) + \alpha^{x,A}\delta \frac{S^{com,x,A}}{1-\delta} + \left(1 - \alpha^{x,A}\right)\delta \tilde{W}_0^{x,A}.$$

Hence,

$$\bar{W}_0^{x,A} = \frac{S^{c,x,A} + 2\pi^{c,x,A}}{1 - \left(1 - \alpha^{x,A}\right)\delta} + \frac{2\pi^{c,x,B} - \left[1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\right] 2f^{x,B}}{1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\delta} + \frac{\alpha^{x,A}\delta S^{com,x,A}}{\left[1 - \left(1 - \alpha^{x,A}\right)\delta\right](1-\delta)}.$$

The expected discounted welfare of country B in industry x is:

$$\bar{W}_0^{x,B} = \frac{S^{c,x,B} + \left[1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\right]\left(2f^{x,B} + \frac{\delta S^{com,x,B}}{1-\delta}\right)}{1 - \left(1 - \alpha^{x,A}\right)\left(1 - \alpha^{x,B}\right)\delta}.$$

Consider industry y under integration. $\left(\alpha^{y,B},\alpha^{y,A}\right)\in\bar{R}^{y,B}$. Then, firms collude in both countries until the first time $c_t^{y,B}=1$, when they are prosecuted in both countries, or until the first time $c_t^{y,B}=0$ and $c_t^{y,A}=1$, when they are prosecuted in country A. In the later case, firms keep colluding in country B until the first time $c_{t+\tau}^{y,B}=1$. Then, the expected discounted welfare of country B in industry y is:

$$\bar{W}_0^{y,B}=\frac{S^{c,y,B}+2\pi^{c,y,B}}{1-\left(1-\alpha^{y,B}\right)\delta}+\frac{2\pi^{c,y,A}-\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\right]2f^{y,A}}{1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta}+\frac{\alpha^{y,B}\delta S^{com,y,B}}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right](1-\delta)},$$

while the expected discounted welfare of country A in industry y is:

$$\bar{W}_0^{y,A}=\frac{S^{c,y,A}+\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\right]\left(2f^{y,A}+\frac{\delta S^{com,y,A}}{1-\delta}\right)}{1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta}.$$

Comparison. Suppose that $\left(\alpha^{x,A},\alpha^{x,B}\right)\in R^{x,A}\cap\bar{R}^{x,A}$ and $\left(\alpha^{y,B},\alpha^{y,A}\right)\in R^{y,B}\cap\bar{R}^{y,B}$. Then, the aggregate expected welfare of country $j\in\{A,B\}$ under no integration is $W_0^{x,j}+W_0^{y,j}$, while under integration it is $\bar{W}_0^{x,j}+\bar{W}_0^{y,j}$. Thus, country j is better off under integration than under no integration if and only if $\bar{W}_0^{x,j}-W_0^{x,j}+\bar{W}_0^{y,j}-W_0^{y,j}>0$. Define

$$\Delta W^{x,A}=\bar{W}_0^{x,A}-W_0^{x,A}=\frac{\alpha^{x,A}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]}\left\{\frac{\delta\left(\Delta S^{x,A}-2\pi^{c,x,A}\right)}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]}-\frac{2\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B}+(1-\delta)f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]}\right\},$$

$$\Delta W^{y,A}=\bar{W}_0^{y,A}-W_0^{y,A}=\frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[\delta\Delta S^{y,A}+(1-\delta)2f^{y,A}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}.$$

Then, country A is better off under integration if and only if $\Delta W^{x,A}+\Delta W^{y,A}>0$. Moreover, note that $\left(\alpha^{x,A},\alpha^{x,B}\right)\in R^{x,A}$ implies $\Delta W^{x,A}<0$, while it is clear that $\Delta W^{y,A}>0$.

Define

$$\Delta W_0^{x,B}=\frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta\Delta S^{x,B}+(1-\delta)2f^{x,B}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B}=\frac{\alpha^{y,B}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\frac{\delta\left(\Delta S^{y,B}-2\pi^{c,y,B}\right)}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]}-\frac{2\left(1-\alpha^{y,A}\right)\left[\delta\pi^{c,y,A}+(1-\delta)f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\right\}.$$

Then, country B is better off under integration if and only if $\Delta W^{x,B}+\Delta W^{y,B}>0$. Moreover, note that $\Delta W_0^{x,B}>0$, while $\left(\alpha^{y,B},\alpha^{y,A}\right)\in R^{y,B}$ implies $\Delta W^{y,B}<0$.

Finally, assume that $\pi^{c,z,j}=\pi^c$, $\pi^{d,z,j}=\pi^d$, $\Delta S^{z,j}=\Delta S$, $\alpha^{z,j}=\alpha$, and $f^{z,j}=f$ for all $z\in\{x,y\}$ and $j\in\{A,B\}$. Then

$$\Delta W^{x,A}=\Delta W_0^{y,B}=\frac{\alpha}{\left[1-(1-\alpha)\delta\right]}\left\{\frac{\delta\left(\Delta S-2\pi^c\right)}{\left[1-(1-\alpha)\delta\right]}-\frac{2(1-\alpha)\left[\delta\pi^c+(1-\delta)f\right]}{\left[1-(1-\alpha)^2\delta\right]}\right\},$$

$$\Delta W^{y,A}=\Delta W_0^{x,B}=\frac{\alpha(1-\alpha)\left[\delta\Delta S+(1-\delta)2f\right]}{\left[1-(1-\alpha)^2\delta\right]\left[1-(1-\alpha)\delta\right]}.$$

Therefore:

$$\Delta W^{x,A} + \Delta W^{y,A} = \Delta W^{x,B} + \Delta W^{y,B} = \frac{\left[2 - \alpha - (1-\alpha)^2 \delta\right] \alpha\delta \left(\Delta S - 2\pi^c\right)}{\left[1 - (1-\alpha)\delta\right]^2 \left[1 - (1-\alpha)^2 \delta\right]} > 0.$$

Hence, it is always the case that both countries are better off under autarky. This completes the proof of Corollary 2. ■