Optimal taxation with public good provision for reduction of envy[*]

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Abstract

We examine optimal taxation and public good provision by a government which takes reduction of envy into consideration as one of the constraints. We adopt the notion of extended envy-freeness proposed by Diamantaras and Thomson (1990), called λ-equitability. Different from the existing paper by Nishimura (2003a,b), this paper introduces public good provision by the government into his model, and analyzes both pure public good case and excludable public good case. Under the provision of pure public good, we show that distorting original Samuelson is desirable in order to relax the equitability as well as self-selection constraints. On the other hand, under the provision of excludable public good, the effect of λ-equitability appears not only the provision rule for public good but also the pricing rule for user fees.

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Keywords: Income taxation, Public good provision, Envy, λ-equivalence, Excludable public good

1 Introduction

All over the world, income inequality becomes the biggest problem which leads to chaotic society unless reducing it, and the solution is redistributive policy by a government. There are several ways to redistribute collected incomes from rich ones to poor ones. For instance, the government levies taxes on workers' incomes, and transfers the wealth from rich to poor. Another is to provide public services which are useful for everyone, but to which people with low income would have more limited access if the government did not provide them. With reducing complaints between members in society, the policymaker sets the optimal policy for income redistribution and implementation of such public projects.

In an economy where agents have different skill levels, there are several ethical reasons to consider redistribution, and these are the background for mitigating inequality. One of them is envy. An agent envies the other agent if he prefers the other's commodity bundle to his own. We call envy-free allocation where there is no envy for every agent. In the context of income taxation with endogenous labor supply, agents with high skill cannot envy those with lower skills because of self-selection constraint; on the other hand, lower skilled agents must envy more skilled workers. It is difficult to apply the original envy-free constraint, but we replace the weaker and cardinal criterion proposed by Diamantaras and Thomson (1990), called λ-equitability, and examine the optimal policy schedule under not only self-selection or incentive compatible constraint, but also reduction of envy constraint.

In this paper, we analyze two models of optimal income tax with public good provision: pure public good and excludable public good. The former starts from Boadway and Keen (1993) and the latter does from Hellwig (2005). In providing public good without exclusion and rivality, the government sets the optimal policy constraint on λ-equitablility as well as self-selection and tax revenue. Under these cases, we show that the optimal provision rule includes an effect of λ-equiltability as well as self-selection unless agents' preference has several properties. More rigorously, if individuals have additive and separable preferences, only an effect of λ-equiltability remains and under-provision is optimal. Moreover, we examine the optimal provision rule of public goods with use exclusion and surcharge under the reduction of envy, assuming that individuals have additive and separable preferences and differ in both preferences for public goods and earnings ability. The consideration of λ-equiltability affects not only the amount of public goods but also the level of user fees. The main findings is that the government should refrain from collecting tax revenue from user fees to reduce envy, which means that the optimal level of user fees decrease due to the effect of λ-equiltability.

Related Literatures

We tick off papers related to this project which is categorized into taxation with public good provision and optimal taxation under reduction of envy. In advance, some of readers think that the latter means taxation under cases where a policymaker has objective satisfying fair

distributive taste like maximin or Pigou-Dalton principle.[1] However, this paper allows her to set reduction of envy as constraint, not objective. With regard to optimal taxation for reduction of envy, Nishimura (2003b) studies optimal nonlinear income taxation under constraints about reduction of envy, which shows that the marginal tax rate increases only if leisure is luxury. Also, Nishimura (2003a) examines optimal commodity taxation for reduction of envy. Both papers adopt particular envy-free notion suggested by Diamantaras and Thomson (1990), and we follow this manner, but our paper introduces public good provision by the government which differs from these two papers.

As to optimal nonlinear income taxation with public good provision, Boadway and Keen (1993) investigates optimal income taxation with pure public good provision, and they show that a government provides a public good following modified Samuelson rule which embraces self-selection term. It means that the policymaker reduces the provision level of public good when the valuation of agents mimicking low-skill is greater than that of mimicked low-skilled ones so as to redistribute more taxed wealth. Nava et al. (1996) studies optimal nonlinear income taxation and linear commodity taxation with pure public good provision, and some of our results are related to that paper. Hellwig (2005) allows a policymaker to exclude agents who value a public good less than the surcharge set by her, which shows that a utilitarian government sets zero surcharge at optimum since revenues from increasing the surcharge is dominated by social welfare for reducing the fee, and that the revenue effect becomes stronger as the government is more risk-averse since it is better to utilize more surcharge fees for redistribution.

This remainder of this paper is organized as follows. Section 2 examines the optimal provision rule for pure public goods under the reduction of envy, and section 3 extends to the model with linear commodity taxation. Section 4 analyzes the optimal provision rule for pure public goods and the optimal pricing rule for user fees under the reduction of envy. Section 5 offers concluding remarks.

2 Optimal income taxation with public good provision for reduction of envy

We consider an two-class economy in which each agent ($i = H, L$) possesses an exogenous skill level w_i, where $w_H > w_L > 0$. Without loss of generality, the population of each agent is equal to one. They earn their income by labor supply, and their earnings are the product of unit wage (or skill level) and the amount of labor supply. The government collects taxes on their income, and she can schedule it nonlinearly. In addition, she provides public good by collected taxes.

At first, we assume three kinds of goods, consumption (or after-tax income) $c \in \mathbb{R}_+$, labor

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[1]For instance, Fleurbaey and Maniquet (2006) derives the optimal income tax schedule in settings where social planner maximizes social index satisfying several axioms for fairness and inequality aversion. In this paper, before deriving the optimal policy, they characterize the social index meeting several axioms.

supply l and public good $G \in \mathbb{R}_+$. Also, it is assumed that each worker provides the amount of labor at most $\bar{l}$, so he chooses l between 0 and $\bar{l}$. Every agent shares the identical utility function $U(c_i, G, l_i)$, and U is differentiable, strictly concave, and increasing in c and G and decreasing in l. Let Y be the labor income, and if agent i with skill w_i earns labor income Y_i, we can replace the expression with $U(c_i, G, \frac{Y_i}{w_i})$. In providing public good, the government must incur its production cost $\phi(G)$ with an increasing, strictly convex, and differentiable function. For all goods except for public good, a good with subscript i means the one which agent i enjoys.

Here, we assume that the government wants to achieve constrained Pareto-efficient allocation, so under several constraints with high-skilled agents guaranteeing utility level $\bar{u}$, she wants to maximize low-skilled utility. It is obvious that she faces the resource constraint. Let $T : \mathbb{R} \to \mathbb{R}$ be the income tax function, and agent i's budget constraint is written as $c_i = w_i l_i - T(w_i l_i)$. So, the government's resource constraint is

$$T(w_L l_L) + T(w_H l_H) = w_L l_L - c_L + w_H l_H - c_H \geq \phi(G). \tag{1}$$

Like Stiglitz (1982), we assume that the policymaker cannot observe agents' skill directly but their earned income, so we require that she deals with the information asymmetricity called *self-selection* constraint. We formulate it as follows:

$$U(c_i, G, l_i) \geq U(c_j, G, \frac{w_j}{w_i} l_j) \tag{2}$$

for any $i, j = H, L$ with $i \neq j$. Finally, we impose ethical constraint for reducing envy. We adopt λ-equitability introduced by Diamantaras and Thomson (1990) (as λ envy-free) and used in Nishimura (2003a,b). A taxpayer compares his own bundle with the contractive or expanded bundle starting from the worst $(c, l) = (0, \bar{l})$. Given $\lambda \leq 1$, we set the λ-equitability constraint as follows:

$$U(c_i, G, l_i) \geq U(\lambda c_j, G, \bar{l} - \lambda(\bar{l} - l_j)) \tag{3}$$

for any $i, j = H, L$ with $i \neq j$. If λ is unity, λ-equitability must be equivalent to envy-free, but such an allocation is not implementable.[2] So, we assume $\lambda < 1$. If self-selection constraint for high-skilled agent is satisfied, λ-equitability constraint for $i = H$ is also satisfied.[3] Consequently, we focus only on λ-equitability constraint for low-class.

Summarizing the above, we write down the policymaker's optimization problem as follows:

$$\max_{\{c_i, l_i\}_{i=L,H}, G} U(c_L, G, l_L) \tag{4}$$

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[2]If inequality (2) for $i = H$ holds, the utility of high-skilled should be strictly greater than that of low-skilled. Therefore, equation (3) for low-skilled agents is not satisfied under $\lambda = 1$ since low-skilled envies high-skilled.

[3]By self-selection constraint for high-skilled agent, the following inequality holds:

$$U(c_H, G, l_H) \geq U(c_L, G, \frac{w_L}{w_H} l_L) > U(c_L, G, l_L) > U(\lambda c_L, G, \bar{l} - \lambda(\bar{l} - l_L))$$

Thus, λ-equitability constraint for high skilled agent is satisfied.

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subject to

$$U(c_H, G, l_H) \geq \bar{u}$$

$$w_L l_L - c_L + w_H l_H - c_H \geq \phi(G)$$

$$U(c_i, G, l_i) \geq U(c_j, G, \frac{w_j}{w_i} l_j) \quad \text{where } i, j = H, L \text{ with } i \neq j \tag{5}$$

$$U(c_L, G, l_L) \geq U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$$

The Lagrangian is

$$\mathcal{L}(c_L, c_H, l_L, l_H, G; \gamma, \delta_r, \delta_{sH}, \delta_{sL}, \delta_e) =$$
$$U(c_L, G, l_L) + \gamma\{U(c_H, G, l_H) - \bar{u}\}$$
$$+\delta_r\{w_L l_L - c_L + w_H l_H - c_H - \phi(G)\} \tag{6}$$
$$+\delta_{sH}\{U(c_H, G, l_H) - U(c_L, G, \frac{w_L}{w_H} l_L)\} + \delta_{sL}\{U(c_L, G, l_L) - U(c_H, G, \frac{w_H}{w_L} l_H)\}$$
$$+\delta_e\{U(c_L, G, l_L) - U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))\}$$

where $\gamma, \delta_r, \delta_{sH}, \delta_{sL}$ and δ_e are Lagrangian multipliers corresponding to constraints individually. Note that this problem is almost the same as Boadway and Keen (1993), but the difference is the constraint of λ-equitability. The first-order conditions with respect to the Lagrangian are shown in Appendix A.

2.1 Provision rule of public good

This subsection exhibits optimal public good provision rule as well as marginal income tax rate. For simplicity, we stick to cases in which self-selection constraint for low-skilled agent does not bind, *i.e.*, $\delta_{sL} = 0$. As seen in Boadway and Keen (1993), the optimal provision rule includes self-selection term, which amounts to playing an important role in redistribution. If the mimicker puts more weight on public good based on private consumption than mimicked one with low-skill, then the government should reduce its production and transfer the tax revenue to agents in low-class. In addition, to relax the λ-equitability constraint, the government increases or decreases the amount. For instance, if the evaluation of public good for private good at the λ-scaled bundle $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$ is higher than that at the one high-skilled agent receives, then she must reduce the provision level in order to redistribute more income. Let U_a^i be the partial derivative of $U(c_i, G, l_i)$ with respect to a, $\hat{U}_a^i$ be that of $U(c_j, G, \frac{w_j}{w_i} l_j)$ with respect to a, and $\bar{U}_a$ be that of $U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$ for a, where $a = c, G, l$.

Formally, we can derive the optimal rule with respect to public goods provision in the next proposition.

Proposition 1. *Under nonlinear optimal income tax with constraints listed in (5), the optimal provision rule is characterized by:*

$$\sum_{i=L,H} MRS_{Gc}^i + \frac{\delta_{sH}}{\delta_r} \hat{U}_c^L (MRS_{Gc}^L - \hat{MRS}_{Gc}) + \frac{\delta_e}{\delta_r} \bar{U}_c^H (\lambda \times MRS_{Gc}^H - \bar{MRS}_{Gc}) = \phi'(G). \tag{7}$$

where $MRS^i_{Gc} \equiv \frac{U^i_G}{U^i_{c_i}}$ is the agent with skill w_i's marginal rate of substitution (abbreviated by MRS) for G measured by c_i, $\hat{MRS}_{Gc} \equiv \frac{\hat{U}_G}{\hat{U}_{c_i}}$ is the mimicker's MRS between c and G and $\bar{MRS}_{Gc} \equiv \frac{\bar{U}_G}{\bar{U}_{c_i}}$ is the MRS measured at $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$.

The first term is the sum of agent i's marginal rate of substitution for public good G measured by private consumption c_i, and the second term is the effect by incentive constraint. The third term is the novel one, which reflects the effect on λ-equitability constraint, and the implication is similar to that of incentive constraint. Boadway and Keen (1993) shows that the original Samuelson rule for public good provision is replicated when each agent's preference is represented by $U(H(c_i, G), l_i)$, that is, c_i and G are weakly separable with l_i in the utility function. In this case, while the second bracket in the left hand side is zero, it is ambiguous whether the third bracket is zero. We present two special cases: one is the case in which the original Samuelson rule for public good provision is optimal and the other is the case in which the deviation from the original rule is optimal. In other words, the expression for the preference is not sufficient to hold the first-best rule in the presence of λ-equitability constraint.

First of all, we consider a quasi-linear utility function with respect to leisure, which is represented by:

$$U(c_i, G, l_i) = f(G, c_i) - l_i$$

where, $f(\cdot)$ is strictly concave function with respect to G and c_i, and $f(\cdot)$ is homogeneous degree of $k > 0$ in c_i. Note that the sub-utility $f(G, c)$ is not separable between (G, c) because of the homogeneity, and that the marginal rate of substitution increases proportionally as c increases under the sub-utility. Under the preference, the third term can be rewritten as follows:

$$\frac{\delta_e}{\delta_r} \lambda \bar{u}^H_c \left(\frac{f_G(G, c_H)}{f_{c_H}(G, c_H)} - \frac{f_G(G, \lambda c_H)}{\lambda f_{c_H}(G, \lambda c_H)} \right) = \frac{\delta_e}{\delta_r} \lambda \bar{u}^H_c \left(\frac{f_G(G, c_H)}{f_{c_H}(G, c_H)} - \frac{\lambda^k f_G(G, c_H)}{\lambda \times \lambda^{k-1} f_{c_H}(G, c_H)} \right) = 0$$

Since the second bracket in equation (7) vanishes, the original Samuelson rule is realized under the utility function. Next, we assume that the consumer preference is given by:

$$U(c_i, G, l_i) = u(G) + v(c_i) - l_i$$

where, $u(\cdot)$ and $v(\cdot)$ are strictly concave functions. In addition, $v(\cdot)$ is homogeneous degree of $k > 0$. The difference between the former and the latter is whether G and c_i are separable with each other in the utility function. The utility function v has similar property to f, but the marginal rate of substitution varies different from f as consumption c increases. Under the preference, the third term can be rewritten as follows:

$$\frac{\delta_e}{\delta_r} \lambda \bar{u}^H_c \left(\frac{u'(G)}{v'(c_H)} - \frac{u'(G)}{\lambda v'(\lambda c_H)} \right) = \frac{\delta_e}{\delta_r} \lambda \bar{u}^H_c \left(\frac{u'(G)}{v'(c_H)} - \frac{u'(G)}{\lambda^k v'(c_H)} \right) < 0$$

This means that under-provision is optimal. We summarize the findings as follows.

Corollary 1. *The original Samuelson rule for public good provision is optimal if the utility function is $U(c_i, G, l_i) = f(G, c_i) - l_i$, where $f(\cdot)$ is homogeneous degree of $k > 0$ in c_i. On the other hand, the original Samuelson rule for public good provision are downwardly distorted if the utility function is $U(c_i, G, l_i) = u(G) + v(c_i) - l_i$, where $v(\cdot)$ is homogeneous degree of $k > 0$.*

More generally, this corollary sheds light on that the term for reduction of envy vanishes when the marginal rate of substitution proportionally changes as consumption changes in the same direction, *i.e.*, $\bar{MRS}_{Gc} = \lambda MRS_{Gc}$. Marginal rate of substitution under the sub-utility $f(G, c)$ meets the condition while that under $u(G) + v(c)$ does not because $\bar{MRS}_{Gc} = \lambda^{1-k} MRS_{Gc}$ and the coefficient λ^{-k} still remains in multiplying λ^{-1}.

2.2 Marginal Income Tax Rate

We check the marginal income tax rate. Basically, we derive them in the same way as Nishimura (2003b). We show ones under redistributive cases, $\delta_{sL} = 0$ and $\delta_{sH} > 0$. Next proposition gives the marginal tax rates.

Proposition 2. *Under redistributive cases that $\delta_{sL} = 0$ and $\delta_{sH} > 0$,*

1. Marginal income tax rate at the bottom

$$T'(w_L l_L) = \frac{\delta_{sH} \hat{U}_c^L}{\delta_r} \left[MRS^L(y, c) - \hat{MRS}^H(y, c) \right] > 0$$

where $MRS^L(y, c) = -\frac{1}{w_L} \frac{U_l^L}{U_c^L}$ and $\hat{MRS}^H(y, c) = -\frac{1}{w_H} \frac{\hat{U}_l^H}{\hat{U}_c^H}$

2. Marginal income tax rate at the top

$$T'(w_H l_H) = \frac{\lambda \delta_e \bar{U}_c^H}{\delta_r w_H} \left[MRS_{lc}^H - \bar{MRS}_{lc} \right]$$

where $MRS_{lc}^H \equiv -\frac{U_l^H}{U_{c_H}^H}$ is the marginal rate of substitution for l_H measured by c_H, and $\bar{MRS}_{lc} \equiv -\frac{\bar{U}_l}{\bar{U}_{c_H}}$ the marginal rate of substitution measured at $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$.

This is consistent with Nishimura (2003b).[4] For the marginal income tax rate for the low-skilled agents, it must be positive due to self-selection constraint for agents with high skill, as shown by Stiglitz (1982). On the other hand, the marginal tax rate on the top is different from the standard result presented by Stiglitz (1982) since the term which represents

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[4]Nishimura (2003b) also examines these marginal tax rates under when self-selection constraint for low-skilled workers binds.

the effect to λ-equitability constraint appears.[5] Nishimura (2003b) shows that if the income elasticity of leisure is greater (less) than 1, MRS^H_{lc} is greater (less) than $M\bar{R}S_{lc}$, which means that the marginal income tax rate on the top must be positive (negative).[6] Of course, if $MRS^H_{lc} = M\bar{R}S_{lc}$, it must be zero. Also, if the equitability constraint does not bind, in other words, $\delta_e = 0$, then it must be 0.

3 Extension

In this section, we examine the optimal provision rule for public goods when the government employs not only labor income taxes but also commodity taxes. We assume that the government can only levy linear commodity taxes since it cannot observe individuals' consumption levels.

Again, we define the identical utility function of agent i as the following form: $U(c_i, x_i, G, l_i)$, where c_i is a numeraire commodity and x_i another commodity. Producer price of commodity x are constant and normalized to unity for simplicity. While the government cannot impose any taxes on the numeraire good, it imposes proportional commodity tax t on x_i. The other notations are followed by the above section.

Following Mirrlees (1976) and Jacobs and Boadway (2014), we decompose individual optimization into two stages. At the first stage, each agent chooses the amount of labor supply given nonlinear income taxes, which leads to determine disposable income $R_i \equiv w_i l_i - T(w_i l_i)$. At the second stage, each agent expenses disposable income to consume a numeraire and another commodity. We suppose that individuals anticipate the outcome of the second stage at the first stage. Now, we formally turn to the analysis of individuals' problem. In the second stage, given $\{p, R_i, G, l_i\}$, agent i chooses c_i and x_i to maximize the utility $U(c_i, x_i, G, l_i)$ subject to the budget constraint $c_i + px_i = R_i$, where $p \equiv 1 + t$ is the consumer price with respect to another commodity. The first-order conditions with respect to c_i and x_i yield

$$\frac{U^i_x}{U^i_c} = p \tag{8}$$

The optimal solutions with respect to a numeraire and another commodity are denoted by $c^*_i \equiv c(p, R_i, G, l_i)$ and $x^*_i \equiv x(p, R_i, G, l_i)$ respectively. As a result, substituting these solution into the utility function yields the indirect utility function $V_i \equiv V(p, R_i, G, l_i) \equiv U(c^*_i, x^*_i, G, l_i)$. Let V^i_p, V^i_R, V^i_G, and V^i_l be the partial derivative of V_i with respect to p, R_i, G, and l respectively. From the Roy's identity and the Slutsky decomposition, we can get the following relationship.

$$-\frac{V^i_p}{V^i_R} = x^*_i \tag{9}$$

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[5]Note that the difference in the marginal rate of substitution between consumption and labor, not efficiency-unit labor, between the envying and the envies agent is useful information to the government since λ-equitability considers a proportional shrinkage of the envied agent's bundle.

[6]According to the definition of Nishimura (2003b), if the income elasticity of leisure is greater (less) than 1, leisure is a luxury (necessity).

$$\frac{\partial x_i^*}{\partial p} = \frac{\partial \tilde{x}_i}{\partial p} - \frac{\partial x_i^*}{\partial R_i} \cdot x_i^* \tag{10}$$

$$\frac{\partial x_i^*}{\partial G} = \frac{\partial \tilde{x}_i}{\partial G} + \frac{\partial x_i^*}{\partial R_i} \frac{V_G^i}{V_R^i} \tag{11}$$

$$\frac{\partial c_i^*}{\partial p} = \frac{\partial \tilde{c}_i}{\partial p} - \frac{\partial c_i^*}{\partial R_i} \cdot x_i^* \tag{12}$$

$$\frac{\partial c_i^*}{\partial G} = \frac{\partial \tilde{c}_i}{\partial G} + \frac{\partial c_i^*}{\partial R_i} \frac{V_G^i}{V_R^i} \tag{13}$$

where $\tilde{c}_i$ and $\tilde{x}_i$ indicates the compensated demand function of individual i for numeraire and the taxable good, respectively.

In the first stage, each agent chooses the amount of labor supply to maximize the indirect utility V_i subject to $R_i = w_i l_i - T(w_i l_i)$. The first-order condition is given by:

$$-\frac{V_l^i}{w_i V_R^i} = -\frac{U_l^i}{w_i U_c^i} = 1 - T'(w_i l_i) \tag{14}$$

Proceedings as above, the government faces the budget constraint, self-selection constraint to prevent high-skilled from mimicking low-skilled, and λ-equitability constraint for reducing envy. We formulate them respectively as follows:

$$\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] \geq \phi(G) \tag{15}$$

$$V(p, R_H, G, l_H) \geq V(p, R_L, G, \frac{w_L}{w_H} l_L) \equiv \hat{V} \tag{16}$$

$$V(p, R_L, G, l_L) \geq U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H)) \equiv \bar{V} \tag{17}$$

To sum up, the restricted Pareto optimization problem to the government is given by:

$$\max_{\{R_i, l_i\}_{i=L,H}, p, G} V(p, R_L, G, l_L) \tag{18}$$

subject to

$$\begin{aligned}
& V(p, R_H, G, l_H) \geq \bar{u} \\
& \sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] \geq \phi(G) \\
& V(p, R_H, G, l_H) \geq V(p, R_L, G, \frac{w_L}{w_H} l_L) \\
& V(p, R_L, G, l_L) \geq U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))
\end{aligned} \tag{19}$$

The Lagrangian is

$$
\begin{aligned}
\mathcal{L}(p, R_L, R_H, l_L, l_H, G; \mu, \gamma, \delta, \eta) =& V(p, R_L, G, l_L) + \mu\{V(p, R_H, G, l_H) - \bar{u}\} \\
&+ \gamma\{\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] - \phi(G)\} \\
&+ \delta\{V(p, R_H, G, l_H) - V(p, R_L, G, \frac{w_L}{w_H}l_L)\} \\
&+ \eta\{V(p, R_L, G, l_L) - U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))\}
\end{aligned}
\tag{20}
$$

where μ, γ, δ, and η are Lagrangian multipliers corresponding to constraints respectively. The first-order conditions with respect to the Lagrangian are shown in Appendix B.

Before analyzing the provision rule for public goods, it is useful to explore optimal linear commodity tax rate. Let $\hat{V}_p$, $\hat{V}_R$, and $\hat{V}_G$ be the partial derivative of $\hat{V}$ with respect to p, R, and G. The linear commodity tax rate is characterized by the following proposition.

Proposition 3. *Suppose that the allocations are restricted by the reduction of envy. The optimal commodity tax rate under nonlinear labor income tax and public goods provision is given by:*

$$
t \sum_{i=L,H} \frac{\partial \tilde{x}_i}{\partial p} = \frac{\delta}{\gamma}\hat{V}_R(x_L^* - \hat{x}) + \frac{\lambda\eta}{\gamma}\left[\bar{U}_c \frac{\partial \tilde{c}_H}{\partial p} + \bar{U}_x \frac{\partial \tilde{x}_H}{\partial p}\right]
\tag{21}
$$

where, $\hat{x} \equiv x(p, R_L, G, \frac{w_L}{w_H}l_L)$ is the mimicker's demand for another commodity and $\bar{U}_r$ ($r = c, x$) is the derivative of r at λ-scaled bundle $(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))$.

On the right hand side, the first term is self-selection effect, and if agent's utility is separable between commodity part and labor supply term, then it must vanish. We see this effect frequently in existing literatures of mixed taxation, but the second term is the original part for reducing envy as seen in Nishimura (2003a,b). Each term in the bracket is the inner product of the marginal utility of the low-skilled agent and the substitution effect of the compensated demand, which reflects the reduction of envy through discouragement of consumption by the high-skilled agent due to taxation. Moreover, the second term in the right hand side can be rewritten as follows:[7]

$$
\frac{\lambda\eta}{\gamma}\left[\bar{U}_c \frac{\partial \tilde{c}_H}{\partial p} + \bar{U}_x \frac{\partial \tilde{x}_H}{\partial p}\right] = \frac{\partial \tilde{x}_H}{\partial p}\bar{U}_c\left[\frac{\bar{U}_x}{\bar{U}_c} - \frac{U_x^H}{U_c^H}\right] \equiv \frac{\partial \tilde{x}_H}{\partial p}\bar{U}_c\left[\bar{MRS}_{cx} - MRS_{cx}\right]
\tag{22}
$$

If the envying agent prefers the taxable good to the numeraire more than the envied agent, i.e., $\bar{MRS}_{cx} > MRS_{cx}$, it is taxed more heavily. This term remains even if the utility function

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[7]Using individuals' budget constraint $c_i + px_i = R_i$, the following relationships holds:

$$
\frac{\partial \tilde{c}_H}{\partial p} = -p\frac{\partial \tilde{x}_H}{\partial p}
$$

is weakly separable between labor and consumption and a public good taken together, that is, $U(H(c_i, x_i, G), l_i)$, while the first term in the right hand side in equation (21) vanishes. To replicate the Atkinson and Stiglitz (1976) theorem (hereafter A-S theorem), we assume the following functional form: $H(f(c_i, x_i), G)$, where $f(\cdot)$ is homothetic. In this case, the second term in the right hand side of equation (22) vanishes, which means that commodity taxation is superfluous. The sufficient condition to hold the A-S theorem is slightly different from Nishimura (2003a,b) since we impose the additional restriction which is the weak separability between all private consumptions and a public good.

Now, we turn to the characterization of the optimal provision rule for public goods. The optimal rule with respect to public goods provision can be derived in the next proposition.

Proposition 4. *Under linear commodity tax in addition to nonlinear income tax, the optimal provision rule taking the reduction of envy into account is characterized by:*

$$\sum_{i=L,H} \frac{V_G^i}{V_R^i} + \frac{\delta}{\gamma} \hat{V}_R \left[\frac{V_G^L}{V_R^L} - \frac{\hat{V}_G}{\hat{V}_R} \right] - \frac{\eta}{\gamma} \left[\bar{U}_c \frac{\partial \lambda \tilde{c}_H}{\partial G} + \bar{U}_x \frac{\partial \lambda \tilde{x}_H}{\partial G} + \bar{U}_G \right] = \phi'(G) - t \sum_{i=H,L} \frac{\partial \tilde{x}_i}{\partial G} \quad (23)$$

where $\bar{V}_k$ is the derivative of $\bar{V} = U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))$ with respect to $k = G, R$.

In the left-hand side, the first term amounts to the sum of evaluation for public good G based on marginal utility for disposable income R, and the second term is self-selection effect. The remaining part corresponds to λ-equitability effect which is different from Nava et al. (1996). The part consists of two effects. The first is the indirect effect which is the inner product of the marginal utility of the low-skilled agent and the substitution effect of the compensated demand, which reflects the reduction of envy through discouragement of consumption by the high-skilled agent due to the provision of public good. The second is the direct effect which reduces envy by decreasing the amount of a public good. In the right-hand side, the first term is the marginal cost for public good, and the second term is analogous to Nava et al. (1996), which means the impact on indirect tax revenue in increasing the provision level. This is done through the compensated effects on consumption of the change in the level.

When can we apply the original Samuelson rule in this case? The third term in the left hand side of equation (23) can be manipulated to yield:

$$-\frac{\eta}{\gamma} \left[\bar{U}_c \frac{\partial \lambda \tilde{c}_H}{\partial G} + \bar{U}_x \frac{\partial \lambda \tilde{x}_H}{\partial G} + \bar{U}_G \right] = \frac{\eta}{\gamma} \bar{U}_c \left[\lambda \frac{U_G^H}{U_c^H} - \frac{\bar{U}_G}{\bar{U}_c} \right] + \frac{\lambda \eta}{\gamma} \frac{\partial \tilde{x}_H}{\partial G} \bar{U}_c \left[\frac{U_x^H}{U_c^H} - \frac{\bar{U}_x}{\bar{U}_c} \right]$$

$$\equiv \frac{\eta}{\gamma} \bar{U}_c \left[\lambda MRS_{Gc} - M\bar{R}S_{Gc} \right] + \frac{\lambda \eta}{\gamma} \frac{\partial \tilde{x}_H}{\partial G} \bar{U}_c \left[MRS_{cx} - M\bar{R}S_{cx} \right]$$

$$(24)$$

Following by the analysis above, if the agent's utility is expressed by $U(H(c_i, x_i, G), l_i)$, then the second term in the left hand side of equation (23) which is the self-selection term vanishes. In addition, if the function H meets the following functional form: $H(f(c_i, x_i), G)$, where $f(\cdot)$ is homothetic, the second term in the right hand side of equation (24) must disappear since

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$MRS_{cx} = M\bar{R}S_{cx}$ holds. At the same time, the second term in the right hand side of equation (23) also vanishes since t is zero as shown above. Therefore, as in the analysis without linear commodity tax, whether to deviate from the original Samuelson rule depends on the first term in the right hand side of equation (24).

Now, we present two special cases. First, we put an additional assumption that H is homogeneous degree of j in f on $H = H(f(c_i, x_i), G)$. The first term in the right hand side of equation (24) can be rewritten as:

$$\frac{\lambda\eta}{\gamma}\bar{U}_c\left(\frac{H_G(f(c_H,x_H),G)}{H_c(f(c_H,x_H),G)f_c(c_H,x_H)} - \frac{H_G(f(\lambda c_H,\lambda x_H),G)}{\lambda H_c(f(\lambda c_H,\lambda x_H),G)f_c(\lambda c_H,\lambda x_H)}\right)$$
$$= \frac{\lambda\eta}{\gamma}\bar{U}_c\left(\frac{H_G(f(c_H,x_H),G)}{H_c(f(c_H,x_H),G)f_c(c_H,x_H)} - \frac{\lambda^{kj}H_G(f(c_H,x_H),G)}{\lambda \times \lambda^{kj-1}H_c(f(c_H,x_H),G)f_c(c_H,x_H)}\right) = 0$$

Therefore, λ-equitability term disappears and the original Samuelson rule is replicated.

Second, we assume that H is additive and separable between G and f such as $H = \phi(G) + f(c_i, x_i)$, where $\phi(\cdot)$ is strictly concave function. Using the property of homothetic function f, the first term in the right hand side of equation (24) can be rewritten as:

$$\frac{\lambda\eta}{\gamma}\bar{U}_c\left(\frac{\phi'(G)}{f_c(c_H,x_H)} - \frac{\phi'(G)}{\lambda f_c(\lambda c_H,\lambda x_H)}\right) = \frac{\lambda\eta}{\gamma}\bar{U}_c\left(\frac{\phi'(G)}{f_c(c_H,x_H)} - \frac{\phi'(G)}{\lambda^k f_c(c_H,x_H)}\right) < 0$$

Therefore, λ-equitability term remains and leads to the under-provision. In other words, the original Samuelson rule cannot be applied.

4 The provision of excludable public good under the reduction of envy

In this section, we allow the government to provide public good with use-exclusion. Put it differently, it can impose user fees on those who enjoy public goods. We consider an economy in which high- and low-skilled individuals have heterogeneous taste for the public good, that is, they are characterized by two dimensions. Wage rate is denoted by w_i, $i = H, L$, where $w_H > w_L > 0$, and their preference for a public good, θ, is distributed according to the cumulative distribution function $F(\theta)$ with the strictly positive and continuously differentiable density function $f(\theta)$ over $[\underline{\theta}, \bar{\theta}]$. We assume that $0 = \underline{\theta} < \bar{\theta} < \infty$. The proportion of high-skilled individuals is π_H and the proportion of low-skilled individuals is π_L. Indicator B represents individuals who obtain the benefits from a public good and indicator NB individuals who are excluded from a public good. As used in Hellwig (2005), their utility function is described by

$$U_i^j = \mathbf{1}(j) \cdot \theta G + x_i^j - v(\ell_i^j) \qquad i = H, L \text{ and } j = B, NB \qquad (25)$$

where x_i^j denotes the private consumption of type ij individuals, and ℓ_i^j is the labor supply of type ij individuals. On the other hand, $v(\cdot)$ denotes the disutility of labor supply and is

strictly increasing, strictly convex, and continuously differentiable. The assumption on the utility function is useful to avoid the multidimensional heterogeneity problem.

The government can observe labor income $Y_i^j \equiv w_i \ell_i^j$ but cannot check each individual's skill directly, so it can levy nonlinear income taxes $T(\cdot)$ on Y_i^j. In addition, the government imposes admission fees p on those who access to a public good with wage rate . The budget constraint that type i individuals face is given by $x_i^j = Y_i^j - \mathbf{1}(i) \cdot p - T(Y_i^j)$.

4.1 Intensive Margin

Individuals ij choose the amount of labor supply by solving the following optimization problem.

$$max_{\ell_i^j} \ U_i^j = \mathbf{1}(j) \cdot \theta G + w_i \ell_i^j - \mathbf{1}(j) \cdot p - T(w_i \ell_i^j) - v(\ell_i^j)$$

The first-order condition yields

$$\frac{v'(\ell_i^j)}{w_i} = 1 - T'(w_i \ell_i^j) \tag{26}$$

where $v'(\cdot) \equiv \frac{\partial v}{\partial \ell_i^j}$ denotes the marginal disutility of labor. This suggests that the amount of labor supply does not depend on the benefits from a public good, and therefore, we have $\ell_i \equiv \ell_i^B = \ell_i^{NB}$.

4.2 Extensive Margin

Individuals decide whether or not to get access to a public good. Individuals with type (θ, w_i) obtain utility $\theta G + x_i^B - v(\ell_i)$ if they have access to a public good, and utility $x_i^{NB} - v(\ell_i)$ if they are excluded. Therefore, they choose access to a public good if and only if

$$\theta \geq \frac{x_i^{NB} - x_i^B}{G} = \frac{p}{G} \equiv \hat{\theta} \tag{27}$$

where $\hat{\theta}$ is interpreted as the net gain from being excluded from a public good. We derive the equality in equation (27) using individual's budget constraint. Equation (27) means that, if public goods preferences of individuals are greater (lower) than the threshold $\hat{\theta}$, they (do not) access to a public good. Moreover, it is rewritten as:

$$p = \hat{\theta} G \tag{28}$$

That is, type i individuals prefer to make use of public good paying admission fees p if the benefit θG that they draw from the enjoyment of a public good exceeds p.

4.3 The Government

The budget constraint of the government takes the following form:

$$\int_{\hat{\theta}}^{\bar{\theta}} p f(\theta) d\theta + \sum_i \pi_i T(Y_i) = \phi(G)$$

$$\Rightarrow \quad (1 - F(\hat{\theta}))\hat{\theta}G + \sum_i \pi_i[w_i\ell_i - x_i^{NB}] = \phi(G) \tag{29}$$

The first term is the aggregate revenue from admission fees. The second term represents the aggregate revenue from income taxes. On the other hand, $\phi(\cdot)$ is a strictly increasing, strictly convex, and continuously differentiable cost function of a public good.

Following Hellwig (2005), we consider the maximization of the social welfare function instead of the general Pareto optimization problem. We assume that the social welfare function is Bergson-Samuelson criterion which is represented as follows.

$$
\begin{aligned}
\mathcal{W} \equiv{} & \pi_H\left[\int_{\hat{\theta}}^{\bar{\theta}} W(\theta G + x_H^B - v(\ell_H))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(x_H^{NB} - v(\ell_H))f(\theta)d\theta\right] \\
& + \pi_L\left[\int_{\hat{\theta}}^{\bar{\theta}} W(\theta G + x_L^B - v(\ell_L))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(x_L^{NB} - v(\ell_L))f(\theta)d\theta\right] \\
={} & \pi_H\left[\int_{\hat{\theta}}^{\bar{\theta}} W(\theta G - \hat{\theta}G + x_H^{NB} - v(\ell_H))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(x_H^{NB} - v(\ell_H))f(\theta)d\theta\right] \\
& + \pi_L\left[\int_{\hat{\theta}}^{\bar{\theta}} W(\theta G - \hat{\theta}G + x_L^{NB} - v(\ell_L))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(x_L^{NB} - v(\ell_L))f(\theta)d\theta\right]
\end{aligned}
\tag{30}
$$

where W is an increasing and concave function, that is, $W' \geq 0$ and $W'' \leq 0$.

In the second best environment, the government cannot observe earning abilities which are private information of individuals. By revelation principle, it suffices to induce individuals reveal their true types for earning ability to maximize the objective of the government. Since the government can observe whether or not to access to a public good, they cannot mimic ones in the other group. Therefore, we consider only the incentive constraint within each group as follows:

$$x_H^B - v(\ell_H) \geq x_L^B - v(\frac{w_L}{w_H}\ell_L) \tag{31}$$

$$x_H^{NB} - v(\ell_H) \geq x_L^{NB} - v(\frac{w_L}{w_H}\ell_L) \tag{32}$$

Note that if the incentive constraint on high-skilled agents who are excluded from public goods is satisfied, the incentive constraint on those who enjoy the benefit from public goods is also satisfied.[8] As a result, it is sufficient to focus on equation (32).

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[8]Suppose that equation (32) is satisfied. Subtracting p on both sides,

$$x_H^B - v(\ell_H) = x_H^{NB} - p - v(\ell_H) \geq x_L^{NB} - p - v(\frac{w_L}{w_H}\ell_L) = x_L^B - v(\frac{w_L}{w_H}\ell_L)$$

In addition, the government takes envy with respect to individuals' consumption-leisure bundles into consideration. Introducing the concept of Diamantaras and Thomson (1990), the government must implement an allocation satisfying the following λ-equitability constraint within each group:

$$x_L^B - v(\ell_L) \geq \lambda x_H^B - v(\bar{\ell} - \lambda(\bar{\ell} - \ell_H)) \tag{33}$$

$$x_L^{NB} - v(\ell_L) \geq \lambda x_H^{NB} - v(\bar{\ell} - \lambda(\bar{\ell} - \ell_H)) \tag{34}$$

If $\lambda = 1$, these constraints require an envy-free allocation, in other words, no individual envies another individual. However, as long as the incentive constraint within each group is satisfied, low-skilled individuals always envy high-skilled ones but not vice versa. This can be shown from equation (31) and (32):

$$x_H^B - v(\ell_H) \geq x_L^B - v(\frac{w_L}{w_H}\ell_L) > x_L^B - v(\ell_L) \tag{35}$$

$$x_H^{NB} - v(\ell_H) \geq x_L^{NB} - v(\frac{w_L}{w_H}\ell_L) > x_L^{NB} - v(\ell_L) \tag{36}$$

Therefore, we assume that $\lambda < 1$. Moreover, if the λ-equitability constraint on low-skilled agents who receive the benefit from a public good is satisfied, the λ-equitability constraint on those who are not interested in the benefit from public goods is also satisfied.[9] Consequently, we concentrate only on the equation (33).

To sum up, the government chooses an allocation $\{x_i^{NB}, \ell_i, \hat{\theta}, G\}$ to maximize the social welfare function (equation (30)) subject to the government's budget constraint (equation (29)), the incentive constraint (equation (32)), and the λ-equitability constraint (equation (33)). The corresponding Lagrangian is

$$\mathcal{L}(x_L^{NB}, x_H^{NB}, \ell_L, \ell_H, G, \hat{\theta}; \gamma, \delta, \eta) = \mathcal{W}$$
$$+ \gamma\{(1 - F(\hat{\theta}))\hat{\theta}G + \sum_i \pi_i[w_i\ell_i - x_i^{NB}] - \phi(G)\}$$
$$+ \delta\{x_H^{NB} - v(\ell_H) - x_L^{NB} + v(\frac{w_L}{w_H}\ell_L)\}$$
$$+ \eta\{x_L^{NB} - \hat{\theta}G - v(\ell_L) - \lambda(x_H^{NB} - \hat{\theta}G) + v(\bar{\ell} - \lambda(\bar{\ell} - \ell_H))\} \tag{37}$$

where γ, δ, and η are Lagrangian multipliers corresponding to constraints individually. Using the first-order conditions with respect to the Lagrangian shown in Appendix C, we characterize the optimal provision rule for public goods for the reduction of envy.

Thus, equation (31) is satisfied.

[9]Suppose that equation (33) is satisfied, and then we transform it using individuals' budget constraints:

$$x_L^{NB} - p - v(\ell_L) \geq \lambda(x_H^{NB} - p) - v(\bar{\ell} - \lambda(\bar{\ell} - \ell_H))$$

Since $\lambda < 1$, it is enough to check equation (33) only with respect to the λ-equitability constraint.

Proposition 5. *Under nonlinear income tax, the optimal provision rule taking the reduction of envy into account is characterized by:*

$$\frac{\sum_i \pi_i \int_{\hat{\theta}}^{\bar{\theta}} (\theta - \hat{\theta}) W'(\theta G - \hat{\theta} G + x_i^{NB} - v(\ell_i)) f(\theta) d\theta}{\gamma} + (1 - F(\hat{\theta}))\hat{\theta} + \frac{\eta}{\gamma}\hat{\theta}(\lambda - 1) = \phi'(G) \quad (38)$$

where, $\gamma = \sum_i \pi_i \left[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + x_i^{NB} - v(\ell_i)) f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(x_i^{NB} - v(\ell_i)) f(\theta) d\theta \right] - \eta(\lambda - 1)$

The new terms coming from the use exclusion and the reduction of envy appear. The second term in the left hand side represents the marginal benefit due to the increase in revenue from user fees. The third term expresses the marginal loss caused by the argument of envy owing to the increase of user fees. These terms distort the level of public goods upwardly or downwardly, and the effect depends on the level of user fees. Therefore, we examine the level of user fees. Combining (C.3) with (C.5), the formula is characterized by:

$$\underbrace{\hat{\theta} G f(\hat{\theta}) + \frac{\eta}{\gamma} G (1 - \lambda) F(\hat{\theta})}_{efficiency\ loss} = \underbrace{G(1 - F(\hat{\theta})) \sum_i \pi_i g_{NB}^i - G F(\hat{\theta}) \sum_i \pi_i g_B^i}_{equity\ gain} \quad (39)$$

where, $g_{NB}^i \equiv \frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(x_i^{NB} - v(\ell_i)) f(\theta) d\theta}{\gamma}$ *and* $g_B^i \equiv \frac{\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + x_i^{NB} - v(\ell_i)) f(\theta) d\theta}{\gamma}$. These are the marginal social welfare weight for individuals with skill level w_i in group j, and measure the relative value of the government that gives an additional 1\$ to type ij individuals.

Equation (39) implies an *equity − efficiency tradeoff*. A small reform, such that p increases, distorts the decision making on the extensive margin. That is, individuals with lower preferences for a public good tend to hope the exclusion of public goods. Therefore, revenues from user fees decrease, which is expressed by the first term in the left hand side. On the other hand, the right hand side is the net welfare gains from the redistribution between groups, and the first and second terms describe the government's redistributive tastes for each group. If the government prefers to redistribute from group B to NB, that is, the first term in the right-hand side is greater than the second term, user fess are charged to increase revenues and raise consumption levels.

From equation (39), the equity gain crucially depends on the assumption on the social welfare function. We suggest two polar cases: Utilitarian and Rawlsian. If the social welfare function is utilitarian, the equity gain is zero because the government is not interested in the redistribution. This means that efficiency loss is always larger than equity gain if the government imposes user fees. Therefore, $p = 0$ is optimal. On the other hand, when the objective of the government is Rawlsian, the equity gain which is equal to $G(1 - F(\hat{\theta}))$ remains because the aim is to improve the utility of lowest-skilled agents.[10] That is, the level of p is determined by the comparison between the loss and the gain. However, not

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[10]The objective of the Rawlsian government is to maximize the worst-off individuals' utility, i.e., $x_L^{NB} - v(\ell_L)$.

	Social welfare	G	$\hat{\theta}$	x_L^{NB}	x_H^{NB}
Case I					
No envy and public goods	1.3125	0.25	0	1.5	3.375
Case II					
Envy and public goods	1.21324	0.247152	0	1.13251	3.15974

Table 1: Simulation results under Utilitarian

only interior solution but also a trivial case ($p = 0$ and $G = 0$) satisfies equation (39) and (C.4) simultaneously. In the section, we focus only on the analytical result under the interior solution, and compare the welfare level under the interior solution with that under the trivial case in the numerical section.

On the occasion, we characterize the optimal provision rule for public goods under Utilitarian and Rawlsian as follows:

$$\mathbb{E}(\theta) + \frac{\eta}{\gamma}\lambda\left(\mathbb{E}(\theta) - \frac{\mathbb{E}(\theta)}{\lambda}\right) = \phi'(G) \tag{40}$$

$$(1 - F(\hat{\theta}))\hat{\theta} + \frac{\eta}{\gamma}\hat{\theta}(\lambda - 1) = \phi'(G) \tag{41}$$

where, $\mathbb{E}(\theta) \equiv \int_0^{\bar{\theta}} \theta f(\theta)d\theta$. Equation (40) is the optimal provision rule under Utilitarian and the result suggests that under-provision is optimal, which is consistent with Corollary 1. On the other hand, equation (41) is that under Rawlsian. The envy term distorts downwardly the level of public goods.

4.4 Numerical examples

To assess our theoretical results in terms of the provision of excludable public goods, we present several numerical examples. The objective is to clarify the effect of λ-equitability to the amount of public goods under Utilitarian and Rawlsian social welfare function, and in particular, the effect to the level of the user fee under Rawlsian social welfare function.

In the simulation, we set the following assumptions. First, we assume that the disutility of labor $v(\cdot)$ takes an isoelastic form: $v(\ell_i) = \ell_i^{1+1/e}/(1 + 1/e)$, where $e > 0$. Following by Bisin and Rampini (2006), e is unity. Second, public goods preferences θ are distributed as the uniform function $F(\theta) = \theta/\bar{\theta}$ on the interval $[0, \bar{\theta} = 1]$. Third, Japanese unit labor cost, annual indicators, in 2011 is 0.5, which is obtain from OECD Statistics. Then, we normalize low-type individuals' parameter θ^1 to unity, high-type individuals' one is assumed to be $\theta^2 = 2$. Finally, we assume that $\pi_H = \pi_L = 0.5$ and $\lambda = 0.99$.

	Social welfare	G	$\hat{\theta}$	x_L^{NB}	x_H^{NB}
Case I					
No envy and No public goods	1.14286	0	0	1.30612	3.26531
Case II					
No envy and public goods	1.15848	0.125	0.5	1.32175	3.28093
Case III					
Envy and no public goods	1.12545	0	0	1.19471	3.20215
Case IV					
Envy and public goods	1.14078	0.123409	0.496805	1.20942	3.21733

Table 2: Simulation results under Rawlsian

5 Conclusion

In this paper, we analyze optimal policy for income taxation with public good provision by a government when she is concerned with ethical constraint, reduction of envy. The first part deal with the policymaker providing pure public good, and in the remaining part, we consider that the policymaker provides excludable public good with user fee. As the new constraint, she adopts λ-equitability borrowed from Diamantaras and Thomson (1990). In providing public good, we derive the optimal provision rule as well as marginal income tax rate in optimal policy. Though the income tax part is parallel to the result in Nishimura (2003a,b), the modified provision rule includes the effect of reducing envy, which is different from modified Samuelson rule in Boadway and Keen (1993). In order to relax the ethical constraint, she adjusts the amount of provided public good, comparing the evaluation for low-skilled agents with that at the referred commodity bundle. For instance, if an agent with the envied bundle puts more weight on public good than low-skilled agent, she must decrease the provision level in order to make use of more taxed incomes for redistribution. As the extension, we add taxable consumption good and the linear commodity tax, and study both the optimal tax rate and provision rule of public good.

Next, we allow a policymaker to exclude some of agents from public good by setting user fee. Under constraints on reduction of envy and truth-telling for agents, we derive the optimal provision rule with user fees when the government has Bergson-Samuelson social welfare function. The level of provision is determined by trade-off between efficiency loss and equity gain. Note that this is parallel to Hellwig (2005) except for the nouveau part coming from λ-equitability constraint. As the user fee increases, the efficiency loss also increases or λ-equitability constraint becomes tighter. In addition to these policies in general, we examine the two extreme cases: utilitarian or Rawlsian policymaker. In utilitarian case, the optimal surcharge must be 0 since the equity gain also equals 0 while public good is always provided. On the other hand, in Rawlsian case, there are two possible cases: the provision is implemented with positive user fee or no provision. Analytically, we cannot find which is better, but we simulate the proficiency numerically. According to our simulation result,

Ralwsian government provides public good with user fee excluding agents who evaluate it less.

In the end, there are two policy implications in our model. First of all, in paying attention to reduction of envy, the government must deal with the envied λ-scale bundle relative to the original bundle. Second, in Rawlsian policymaker's mitigating envy, she should decrease the level of user fees. Finally, we think It interesting to remove the implicit assumption that λ-equitability binds. As λ sufficiently small, this constraint will not bind, so studying the problem as well as the comparative statics on λ may be good for future works.

Appendix A

Differentiating Lagrangian (6) with respect to c_L, c_H, l_L, l_H and G,

$$\frac{\partial \mathcal{L}}{\partial c_H} = (\gamma + \delta_{sH})U_c^H - \delta_r - \delta_{sL}\hat{U}_c^L - \delta_e\lambda\bar{U}_c = 0 \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial c_L} = (1 + \delta_{sL} + \delta_e)U_c^L - \delta_r - \delta_{sH}\hat{U}_c^H = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial l_H} = (\gamma + \delta_{sH})U_l^H + \delta_r w_H - \delta_{sL}\frac{w_H}{w_L}\hat{U}_l^L - \delta_e\lambda\bar{U}_l = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial l_L} = (1 + \delta_{sL} + \delta_e)U_l^L + \delta_r w_L - \delta_{sH}\frac{w_L}{w_H}\hat{U}_l^H = 0 \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial G} = (\gamma + \delta_{sH})U_G^H + (1 + \delta_{sL} + \delta_e)U_G^L - \delta_r\phi'(G) - \delta_{sL}\hat{U}_G^L - \delta_e\bar{U}_G - \delta_{sH}\hat{U}_G^H = 0 \tag{A.5}$$

Suppose that $\delta_{sH} > 0$ and $\delta_{sL} = 0$. Rearranging (A.1) and (A.3) yields the optimal marginal income tax rate at the top. On the other hand, we can derive the marginal tax rate on the bottom by combining equation (A.2) with equation (A.4). This result follows Stiglitz (1982) analogously, and it must be positive. The provision rule for public good is obtained by substituting equation (A.1) and (A.2) into (A.5). □

Appendix B

Differentiating Lagrangian (18) with respect to p, R_L, R_H, and G,

$$\frac{\partial \mathcal{L}}{\partial p} = (1 + \eta)V_p^L + (\mu + \delta)V_p^H - \delta\hat{V}_p - \eta\bar{V}_p + \gamma\sum_{i=H,L}[x_i^* + (p-1)\frac{\partial x_i^*}{\partial p}] = 0 \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial R_H} = (\mu + \delta)V_R^H - \eta\bar{V}_R - \gamma + \gamma(p-1)\frac{\partial x_H^*}{\partial R_H} = 0 \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial R_L} = (1 + \eta)V_R^L - \delta \hat{V}_R - \gamma + \gamma(p-1)\frac{\partial x_L^*}{\partial R_L} = 0 \tag{B.3}$$

$$\frac{\partial \mathcal{L}}{\partial G} = (1 + \eta)V_G^L + (\mu + \delta)V_G^H - \delta \hat{V}_G - \eta \bar{V}_G + \gamma \sum_{i=H,L}(p-1)\frac{\partial x_i^*}{\partial G} - \gamma \phi'(G) = 0 \tag{B.4}$$

$\square$

Appendix C

$$\frac{\partial \mathcal{L}}{\partial x_H^{NB}} = \pi_H \left[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta}G + x_H^{NB} - v(\ell_H))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(x_H^{NB} - v(\ell_H))f(\theta)d\theta \right] - \pi_H \gamma + \delta - \eta \lambda = 0 \tag{C.1}$$

$$\frac{\partial \mathcal{L}}{\partial x_L^{NB}} = \pi_L \left[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta}G + x_L^{NB} - v(\ell_L))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(x_L^{NB} - v(\ell_L))f(\theta)d\theta \right] - \pi_L \gamma - \delta + \eta = 0 \tag{C.2}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}} = -G \sum_i \pi_i \int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta}G + x_i^{NB} - v(\ell_i))f(\theta)d\theta + \gamma G[(1-F(\hat{\theta})) - \hat{\theta}f(\hat{\theta})] + G\eta(\lambda-1) = 0 \tag{C.3}$$

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_i \pi_i \int_{\hat{\theta}}^{\bar{\theta}} (\theta - \hat{\theta})W'(\theta G - \hat{\theta}G + x_i^{NB} - v(\ell_i))f(\theta)d\theta + \gamma(1-F(\hat{\theta}))\hat{\theta} - \gamma \phi'(G) + \eta \hat{\theta}(\lambda-1) = 0 \tag{C.4}$$

Combining (C.1) with (C.2) yields:

$$\gamma = \sum_i \pi_i \left[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta}G + x_i^{NB} - v(\ell_i))f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(x_i^{NB} - v(\ell_i))f(\theta)d\theta \right] - \eta(\lambda-1) \tag{C.5}$$

$\square$

References

Atkinson, A. B., Stiglitz, J. E., 1976. The design of tax structure: direct versus indirect taxation. Journal of Public Economics 6, 55–75.

Bisin, A., Rampini, A. A., 2006. Markets as beneficial constraints on the government. Journal of Public Economics 90, 601–629.

Boadway, R., Keen, M., 1993. Public goods, self-selection and optimal income taxation. International Economic Review 34, 463–478.

Diamantaras, D., Thomson, W., 1990. A refinement and extension of the no-envy concept. Economics Letters 33, 217–222.

Fleurbaey, M., Maniquet, F., 2006. Fair income tax. The Review of Economic Studies, 55–83.

Hellwig, M. F., 2005. A utilitarian approach to the provision and pricing of excludable public goods. Journal of Public Economics 89, 1981–2003.

Jacobs, B., Boadway, R., 2014. Optimal linear commodity taxation under optimal non-linear income taxation. Journal of Public Economics 117, 201–210.

Mirrlees, J., 1976. Optimal tax theory: a synthesis. Journal of Public Economics 6, 327–358.

Nava, M., Schroyen, F., Marchand, M., 1996. Optimal fiscal and public expenditure policy in a two-class economy. Journal of Public Economics 61, 119–137.

Nishimura, Y., 2003a. Optimal commodity taxation for reduction of envy. Social Choice and Welfare 21, 501–527.

Nishimura, Y., 2003b. Optimal non-linear income taxation for reduction of envy. Journal of Public Economics 87, 363–386.

Stiglitz, J. E., 1982. Self-selection and pareto efficient taxation. Journal of Public Economics 17, 213–240.