

# Intergenerational transfers in an aging economy trapped in secular stagnation

Gilles Le Garrec\*      Vincent Touzé†

PRELIMINARY AND INCOMPLETE

## Abstract

We characterize the dynamics of secular stagnation as a permanent regime switching from a full employment equilibrium to an underemployment equilibrium. In the latter, the natural interest rate is negative, and the economy is in deflation. Due to the non negativity condition imposed on policy rate, the zero lower bound (ZLB) applies which prevents targeting inflation. The secular stagnation equilibrium is achieved in a standard overlapping generations model with capital accumulation where two market imperfections are introduced: credit rationing and downward nominal wage rigidity. We then show that an aging population can bring the economy back into secular stagnation. To figure out how to escape the secular stagnation trap, we study the impact of transfers from workers to retirees. By lowering savings incentives, they can help the economy get out of the secular stagnation trap. In addition, they can be Pareto improving even if their return, the population growth rate, is lower than the savings return, the interest rate.

**Keywords:** Secular Stagnation, Capital Accumulation, Intergenerational transfers, Zero Lower Bound. **JEL codes:** D91, E31, E52, E62.

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\*corresponding author. Sciences Po - OFCE, 69 quai d'Orsay, 75007 Paris, FRANCE. Email: gilles.legarrec@sciencespo.fr

†Sciences Po - OFCE. Email: vincent.touze@sciencespo.fr

# 1 Introduction

In view of the recent economic situation in the United States and Europe, the state of slow growth and underemployment, coupled with low inflation or even deflation, has been widely discussed in the wake of the famous speech by Larry Summers at the International Monetary Fund (IMF) in 2013, under the label of "secular stagnation" (see Summers, 2013, 2014, 2016; Krugman, 2013; Bernanke, 2015, among many others). The concept of secular stagnation was coined in 1938 in a speech by A. Hansen, which was published in 1939. In Hansen's view, a state of secular stagnation results when an abundance of savings relative to demand for credit pushes up the "natural" interest rate (defined following Wicksell, 1898, as the real rate compatible with full employment) below zero. As a consequence, if the real interest rate remains permanently above the natural rate, the result is a chronic shortage of aggregate demand and investment, with a weakened growth potential. Contrarily to the Keynesian approach, Hansen was worried that the economy would not recover spontaneously from a lack of demand.

According to most promoters of the concept revival, secular stagnation was initiated by the 2008 economic and financial crisis. This crisis was linked to high household debt, which ultimately led to credit rationing. In this context, credit rationing leads to a fall in demand and excess savings. Consequently, the real interest rate falls, the natural rate becoming negative<sup>1</sup>. To counter the crisis, the monetary authorities first reduced their policy rates. However, nominal interest rates cannot, in practice, be forced by central banks to be "too negative". Consequently, the nominal interest rate has reached, or approached, its zero lower bound (ZLB) as illustrated in Figure 1, leaving conventional monetary policy toothless. In this case, the economy plunges into a lasting state of underemployment of labour, characterised by output that is below potential and eventually by deflation.

Somewhat differently, in Hansen's view the central point was that declining population growth in the United States would lead to a large fall in investment that would result in secular stagnation. This population channel finds echo nowadays in the Japanese economic

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<sup>1</sup>Recent empirical studies have evidencing the persistent negativity of natural interest rate since, or not long after, the onset of the recession (Barsky et al., 2014; Laubach and Williams, 2003 and 2015; Cúrdia, 2015; Pescatori and Turunen, 2015).

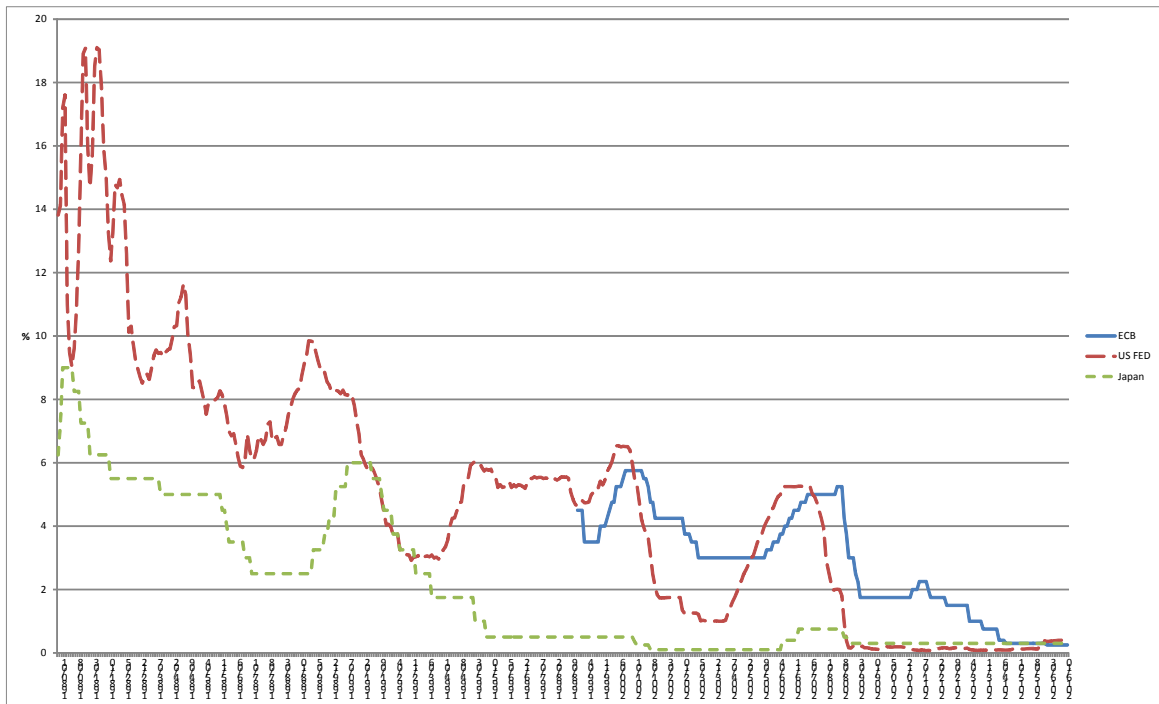


Figure 1: Policy rates (source: ECB, US FED and Bank of Japan)

situation. For example, one can observe in Figure 1 that the nominal interest rate has reached, or approached, its zero lower bound (ZLB) since mid-1990ies. In the same time, one can observe also observe in Figure 2 that Japan exhibits the lowest population growth, with a negative rate since 2010 that is forecasted to persist all the century (United Nations, 2015). Without denying the important role of the 2008 crisis, the low population growth in the Euro zone might also help explaining the difficulties of Europe escaping the stagnation. And if this is the case, Europe following Japan might enter a new very long-term stagnation period that can be qualified of secular. New policies must then be considered in this new period.

In this paper, we propose a tractable theoretical model based on Eggertsson and Mehrotra (2014) and Le Garrec and Touzé (2016) to study the aging population channel of secular stagnation. We develop an overlapping generations model by incorporating imperfections affecting the credit market (borrowing constraint) and the labor market (downward wage rigidity). The central bank conducts monetary policy according to a Taylor rule subject to a ZLB on the nominal interest rate. In line with Le Garrec and Touzé (2016), individuals when young must first borrow on the financial market to buy capital which is productive the following period.

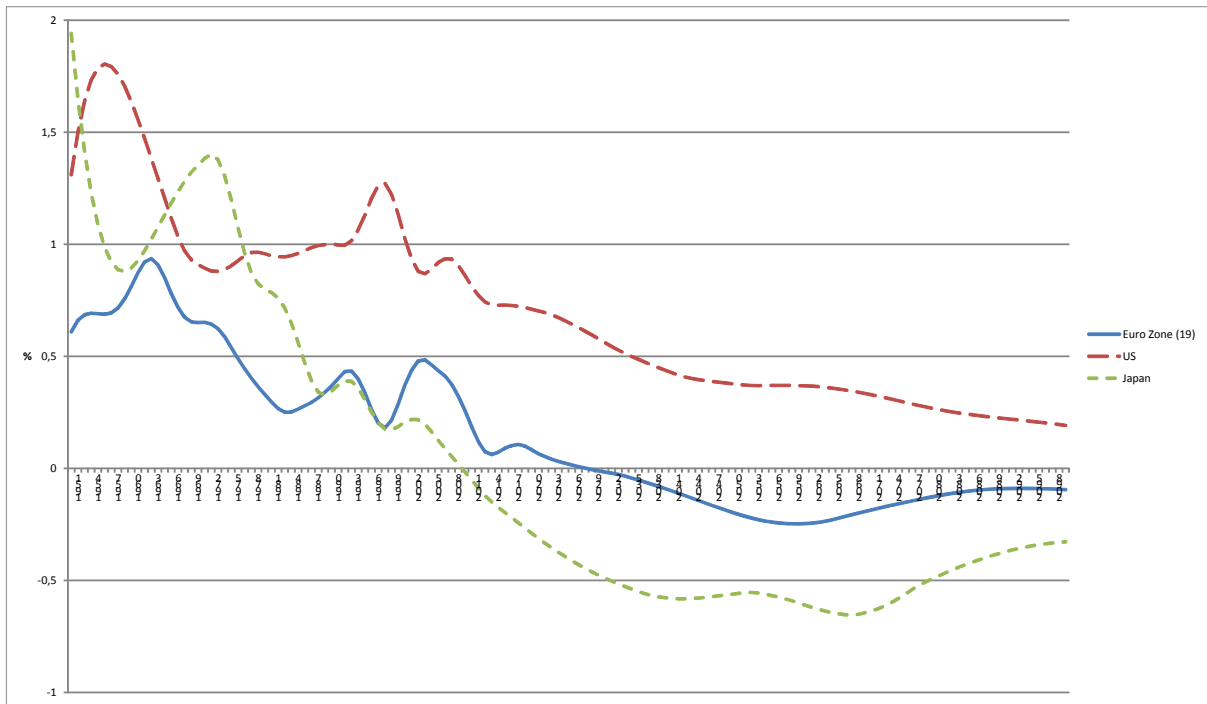


Figure 2: Population growth (source: United Nations, 2015)

In that framework, aging can induce a fall in aggregate demand and an excess in savings over investment opportunities. Consequently, the real interest rate falls. If full employment requires a negative natural interest rate, the economy sinks into a state of persistent deflation, characterised by underemployment and a low output. To figure out how to escape the secular stagnation trap, we study the impact of transfers from workers to retirees. By lowering savings incentives, they can help the economy get out of the secular stagnation trap. In addition, they can be Pareto improving even if their return, the population growth rate, is lower than the savings return, the interest rate.

In addition to Eggertsson and Mehrotra (2014) and Le Garrec and Touz  (2016), our model is also related to the few other models that aim to explain the persistence of the crisis as a potential permanent ZLB situation. In Michau (2015) and Ono (2015), The dynamics in a Ramsey model can be characterized by an overaccumulation of capital and a negative natural interest rate by incorporating a preference for wealth in the Ramsey model. In Kocherlakota (2013), with overlapping generations or a credit constraint, a fall in the price of land can generate a secular stagnation type of equilibrium. Nevertheless, note that for the result to hold, some kind of upward nominal wage rigidity is required. In Caballero and Farhi (2014),

with a perpetual youth OLG model with no capital accumulation, it is the shortage of safe assets that can bring the economy to the ZLB and the associated recession. Accordingly, one privileged way to stimulate aggregate demand and exit recession is for the government to issue safe public debt, and eventually to buy risky private assets with the proceeds. In this model, unlike in ours, an increase in the inflation target can also be efficient to get out of secular stagnation. Finally, in Benigno and Fornaro (2015), who rely on an endogenous growth model with innovation activities, pessimistic expectations are key to explain the fall into recession. Afterward, the economy may be persistently or permanently trapped, because weak growth depresses aggregate demand, pushing the nominal interest rate against the ZLB, while depressed demand reduces profits, hence investment in innovation. In this context, contrarily to ours, any policy that enhances productivity growth can be efficient for exiting the stagnation trap.

This article consists in four parts. In the second section, we present our overlapping generations model with capital accumulation, market imperfections and a Taylor rule. In section three, we characterize the economic dynamics (dynamic time paths and steady states) and the secular stagnation equilibrium. There are three configurations. If the long-run equilibrium is unique, two cases must be considered: either full employment with an inflation target, or secular stagnation with underemployment and deflation. In both cases, the equilibria are globally determined with the dynamics characterized by a unique saddle path. Finally, in a third configuration, the two preceding equilibria coexist with a third equilibrium of full employment and missed inflation target. This equilibrium is indeterminate and the other two are determinate. These determinate steady states are not unique. Therefore, they are determinate only locally, not globally. Section four discusses the intergenerational transfer issues bound up with exiting the secular stagnation trap and its distributional properties. The last section concludes.

## 2 The model

The economy is composed of four types of agents: individuals, firms, a government and a central bank.

We assume that individuals live for three periods: they are successively young, middle-aged-

workers then retired. The number of young individuals,  $N_t$  at date  $t$ , is growing at the constant rate  $n$  so that:

$$N_t = (1 + n) N_{t-1} \quad (1)$$

Competitive firms produce one good, which is both a consumption good and an investment good, by using two factors: labor and capital. The government finances public expenditure by taxing workers (with a balanced budget), and the central bank determines the nominal rate of interest to control inflation.

Accordingly, there are four markets in the economy: good, labor, capital and credit.

## 2.1 Individuals

During the first period of their lives, individuals borrow to invest  $I_{t-1}$  in capital. One period later, the investment is sold to firms with a return equal to  $R_t^k$ . When people are active, they offer inelastically an amount of work  $\bar{l}$  normalized to unity,  $\bar{l} = 1$ , and work for a real wage rate  $w_t$ . They consume  $c_t$  and save such that  $a_{t-1}^m$  is their real net asset. They also pay back their loans plus interest and a lump-sum tax  $T$ . In the final period of life, people consume  $d_{t+1}$ . Assuming that each individual effectively works an identical duration  $l_t \leq \bar{l}$ , the budgetary constraints are as follows:

$$\begin{cases} a_{t-1}^y = -I_{t-1} \\ c_t + a_t^m = w_t l_t + R_t^k I_{t-1} + R_t a_{t-1}^y \\ d_{t+1} = R_{t+1} a_t^m \end{cases} \quad (2)$$

where  $a_t^y$  denotes the net real asset at date  $t$  of a young individual and  $R_t$  the real interest factor.

Note that purely for analytical simplicity, we assume as usual in this type of literature that individuals do not consume in the first period of life (see for example Boldrin and Montes, 2005, and Docquier et al., 2007). Furthermore, it is assumed that there is no altruism, and so individuals start in life with zero assets. The preferences of an individual born in period  $t - 1$  are therefore characterized by the following utility function:

$$U_{t-1} = \log c_t + \beta \log d_{t+1} \quad (3)$$

where  $\beta$  denotes the psychological discount factor. It is easily shown that the optimal behavior of the consumer, obtained by utility maximization of eq. (3) under budgetary constraints (2), yields the following optimal asset:

$$a_t^m = s (w_t l_t + R_t^k I_{t-1} + R_t a_{t-1}^y) \quad (4)$$

where  $s = \frac{\beta}{1+\beta}$  denotes the saving rate.

To characterize the imperfection of financial markets, we assume following Aiyagari (1994), Eggertsson and Krugman (2012), Eggertsson and Mehrotra (2014) or Coeurdacier et al. (2015) that the credit market is rationed as<sup>2</sup>:

$$-a_{t-1}^y \leq \frac{D}{R_t} \quad (5)$$

Such a constraint does not focus on the loanable proportion, but on households' ability in the following period to repay their loans, i.e. to repay the capital borrowed plus interest. If this constraint bites (of course this is assumed when  $R_t^k > R_t$ ), we then have  $a_{t-1}^y = -\frac{D}{R_t}$ .

## 2.2 Firms

From the production side, we assume that the good is produced in a competitive sector characterized by a Cobb-Douglas technology with constant return to scale, such that  $F(K_t, N_{t-1}l_t) = AK_t^\alpha (N_{t-1}l_t)^{1-\alpha}$ , where  $\alpha < 1$  and  $A$  denotes the total factor productivity (TFP). The profit maximization then yields:

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<sup>2</sup>Microeconomic theories of credit rationing are based mostly on the non-observability either of the individual effort (moral hazard) or of skills (adverse selection) (see Stiglitz and Weiss, 1981; Aghion and Bolton, 1997; Piketty, 1997). They all share the common explanation that greater collateral allows one to borrow more.

$$w_t = A(1 - \alpha) k_t^\alpha l_t^{1-\alpha}, l_t \leq 1 \quad (6)$$

and

$$R_t^k = A\alpha k_t^{\alpha-1} l_t^{1-\alpha} + (1 - \delta) \quad (7)$$

where  $k_t = \frac{K_t}{N_{t-1}}$  is the level of capital per worker and  $\delta$  the depreciation rate of capital,  $\delta \in (0, 1]$ .

### 2.3 Wage bargaining and nominal rigidity

Each generation of workers negotiates a contract. We assume that at the beginning of each period a wage negotiation defines the profile of nominal wages throughout the period of activity. For simplicity, we define  $W_t(0)$  and  $W_t(1)$  as the levels of nominal wages at the beginning and end of period  $t$ . Assuming an aversion to a decline in nominal wages during the period, the wage at the end of the period is determined according to:

$$W_t(1) = \max\left(\tilde{W}_t, W_t^*\right) \quad (8)$$

where  $W_t^* = A(1 - \alpha) P_t k_t^\alpha$  denotes the full employment wage rate and  $\tilde{W}_t = \gamma W_t(0) + (1 - \gamma) W_t^*$ ,  $\gamma \in (0, 1)$  characterizes the aversion to the decline in nominal wages, or the degree of downward rigidity of wages. Assuming that wage bargaining leads to setting a constant level of the real wage over the period,  $w_t = \frac{W_t(0)}{P_{t-1}} = \frac{W_t(1)}{P_t}$ , we then have:

$$w_t = \max\left(\frac{(1 - \gamma) A(1 - \alpha) k_t^\alpha}{1 - \frac{\gamma}{\Pi_t}}, w_t^*\right) \quad (9)$$

where  $w_t^*$  denotes the full employment real wage rate and  $\Pi_t$  the inflation factor at date  $t$ . We observe straightforwardly that in this configuration, if the economy is in deflation, then the negotiated real wage level is above its full employment level:  $w_t = \frac{(1-\gamma)(1-\alpha)k_t^\alpha}{1-\frac{\gamma}{\Pi_t}} \geq (1 - \alpha) k_t^\alpha$  if  $\Pi_t \leq 1$ . Indeed, in case of deflation, maintaining both purchasing power and full employment means a lower nominal wage. If the required drop is reduced, and especially if the aversion to a nominal wage decline is strong, then the real wage becomes stronger than the one that would allow full employment.



## 2.4 Central bank: Taylor rule and inflation target

We assume that the monetary authorities want to control inflation. Accordingly, we express the Taylor rule as:

$$1 + i_t = \max \left( 1, (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right) \quad (10)$$

where  $i_t$  denotes the nominal interest rate at date  $t$ ,  $\Pi^* \geq 1$  the official inflation target and  $\phi_\pi > 1$  an inflation gap aversion parameter. When  $1 + i_t = (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi}$ , the Taylor rule will operate and, in this sense, we can say that monetary policy is active. By contrast, when  $i_t = 0$ , the central bank is constrained by the nominal zero lower bound. In this case, we say that monetary policy is inactive. According to equation (10), we can highlight a level of inflation

$$\Pi_{kink} = (1 + i^*)^{\frac{-1}{\phi_\pi}} \Pi^* \quad (11)$$

such that  $i_t \geq 0 \Leftrightarrow \Pi_t \geq \Pi_{kink}$ . In addition, for the inflation target to be reached in the unconstrained regime we set  $1 + i^* = R_{eq} \Pi^*$ , where  $R_{eq}$  denotes the natural interest rate at steady state.

## 2.5 Equilibrium with market imperfections

By assuming that the credit constraint is binding, i.e.  $R_t^k > R_t$ , an equilibrium is defined by clearings in the capital market

$$N_{t-1}k_t = N_{t-1}I_{t-1} \quad (12)$$

and in the credit market

$$N_t a_t^y + N_{t-1} a_t^m = 0 \quad (13)$$

By contrast, knowing that wages are downwardly rigid, the labor market does not necessarily clear. Equations (6) and (9) then yield:

$$l_t = \min(1, \mathcal{L}(\Pi_t)) \quad (14)$$

where  $\mathcal{L}(\Pi_t) = \left(\frac{1-\frac{\gamma}{\Pi_t}}{1-\gamma}\right)^{\frac{1}{\alpha}}$  with  $\mathcal{L}' = \frac{\gamma}{\alpha\Pi^2} \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{1}{\alpha}-1} > 0$ . We then observe that the underemployment of labor is particularly important whenever deflation is strong.

### 3 Characteristics of the secular stagnation equilibrium

#### 3.1 The supply-demand equilibrium of good and the dynamics of capital

The supply of good per worker is obviously determined by:

$$y_t^s = Ak_t^\alpha \min(1, \mathcal{L}(\Pi_t)^{1-\alpha}) + (1-\delta)k_t \quad (15)$$

From the constancy of scale returns, we immediately check that the demand per worker is equal to  $y_t^d = \frac{Y_t}{N_{t-1}} = w_t l_t + R_t^k k_t$ . Knowing that at equilibrium  $k_t = I_{t-1}$  (eq. 12), we deduce from equation (4) that  $w_t l_t + R_t^k k_t = \frac{1}{s} a_t^m - R_t a_{t-1}^y$ . Using equilibrium relations (13) and (14), the dynamics of the population (1) and the credit rationing (5), the aggregate demand per worker can be expressed as:

$$y_t^d = \frac{1+n}{s} I_t + D \quad (16)$$

Typically, this increases with its two components, private investment and public demand. It also decreases with the saving rate  $s$  and increases with the population growth rate  $n$ . The aggregate demand is also impacted through the credit constraint (5). In particular, if  $D$  is lowered, consumption of the elderly is reduced so that  $d_t = R_t a_{t-1}^m = -R_t(1+n)a_{t-1}^y = (1+n)D$ .

In the good market, the supply-demand equilibrium  $y_t^s = y_t^d$  (with eq. 12) then determines the following backward dynamics of the capital stock:

$$k_{t+1} = \frac{1}{1+n} \mathcal{S}(k_t, \Pi_t) \quad (17)$$

where  $\mathcal{S}(k_t, \Pi_t) = s [Ak_t^\alpha \min(1, \mathcal{L}(\Pi_t)^{1-\alpha}) - D + (1-\delta)k_t]$  with  $\mathcal{S}'_k = sR^k > 0$ ,  $\mathcal{S}'_\Pi = \frac{sw}{(1+n)}\mathcal{L}' > 0$  if  $\Pi < 1$  and  $\mathcal{S}'_\Pi = 0$  if  $\Pi \geq 1$ .

If  $\Pi_t \geq 1$ , then the dynamics is  $k_{t+1} = \frac{1}{1+n}\mathcal{S}(k_t, 1)$ . In this case, it is easy to show that if  $D$  is small enough, there exists a unique stable steady state with full employment  $k_{FE}$  for any  $k_0 > k_{unst} > 0$ , where the second steady state  $k_{unst}$  is unstable. The dynamic properties of the capital accumulation process can be characterized by studying  $\Delta k_{t+1} = k_{t+1} - k_t$ . By definition, at the steady state  $k_{FE}$ ,  $\Delta k_{t+1} = 0$ . Therefore, with decreasing productivity, if  $k_t > k_{FE}$  then  $\Delta k_{t+1} < 0$  and if  $k_{FE} > k_t > k_{unst}$  then  $\Delta k_{t+1} < 0$ . Note that this process is independent of inflation, and thus the curve  $\Delta k = 0$  is vertical in the plane  $(k, \Pi)$ . If  $\Pi_t < 1$ , then the dynamics is expressed as  $k_{t+1} = \frac{1}{1+n}\mathcal{S}(k_t, \Pi_t)$ . When the economy falls into deflation, the nominal rigidity of wages eliminates the full employment equilibrium. Thus, a decline in the amount of work will reduce the marginal product of capital. In this case, the equilibrium level of capital will be lowered. The curve describing the locus  $\Delta k_{t+1} = 0$  is growing in the plane  $(k, \Pi)$  when  $\Pi < 1$  and  $sR_t^k < 1$ .

### 3.2 Taylor rule and stabilization of inflation

According to the Fisher equation, we have:

$$R_{t+1} = (1 + i_t) \frac{P_t}{P_{t+1}} = \frac{1 + i_t}{\Pi_{t+1}} \quad (18)$$

Combining equations (10) and (18), we obtain:

$$\Pi_{t+1} = \frac{\max\left(1, (1 + i^*) \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right)}{R_{t+1}} \quad (19)$$

Knowing that  $k_{t+1} = I_t = \frac{D}{R_{t+1}}$ , it yields from eq. (17) that:

$$\frac{1}{R_{t+1}} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1 + n)D}. \quad (20)$$

Including this result in equation (19), the dynamics of inflation becomes:

$$\Pi_{t+1} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1 + n)D} \max\left(1, (1 + i^*) \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \quad (21)$$

For clarity, to examine this forward dynamics of inflation, suppose that  $\Pi_{kin} \geq 1$ . In this case, if  $\Pi_t \geq \Pi_{kin}$ , then this equation becomes  $\Pi_{t+1} = \frac{s}{(1+n)D} (1 + i^*) \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \mathcal{S}(k_t, 1)$  and we obtain:

$$\Delta\Pi_{t+1} = 0 \iff \Pi_t = \Pi^* \left( \frac{(1+n)k_{FE}}{\mathcal{S}(k_t, 1)} \right)^{\frac{1}{\phi_\pi - 1}} \text{ if } \Pi_t \geq \Pi_{\text{sink}} \quad (22)$$

We observe that when monetary policy is active ( $\Pi_t \geq \Pi_{\text{sink}}$ ), then the stabilization of inflation requires that the level of inflation decreases when the level of capital is high (or the interest rate is low). To understand this configuration, rewrite the Fisher equation as  $\Pi_{t+1} = \frac{1+i_t}{R_{t+1}}$ . Stabilizing inflation  $\Delta\Pi_{t+1} = 0$  is then equivalent to  $\Pi_t = \frac{1+i_t}{R_{t+1}}$ , or after log-linearization to  $d\tilde{\Pi}_t = d\widetilde{1+i_t} - d\tilde{R}_{t+1}$ . According to equation (17), increasing the level of capital in  $t$  yields more capital in  $t+1$  and then a lower interest rate in  $t+1$ . Everything else being equal, i.e. with an unchanged nominal interest rate  $d\widetilde{1+i_t} = 0$ , the stabilization of inflation then requires that the level of inflation increases in  $t$  such that  $d\tilde{\Pi}_t = d\tilde{\Pi}_{t+1} = -d\tilde{R}_{t+1} > 0$  when  $d\tilde{k}_t > 0$ . However, an active monetary policy yields an overreaction in the nominal interest rate such that  $d\widetilde{1+i_t} = \phi_\pi d\tilde{\Pi}_t$  where  $\phi_\pi > 1$ . Hence, the initial effect of an increase in the level of capital through a decrease in the interest rate is reversed such that  $d\tilde{\Pi}_t = d\tilde{\Pi}_{t+1} = \frac{d\tilde{R}_{t+1}}{\phi_\pi - 1} < 0$  when  $d\tilde{k}_t > 0$ . Accordingly, we can verify that when monetary policy is inactive ( $\Pi_t < \Pi_{\text{sink}}$ ) and inflation is positive ( $\Pi_t \geq 1$ ), this relationship is in the opposite direction. Indeed, in this case,  $\Pi_{t+1} = \frac{\mathcal{S}(k_t, 1)}{(1+n)D}$  which allows us to define:

$$\Delta\Pi_{t+1} = 0 \iff \Pi_t = \frac{\mathcal{S}(k_t, 1)}{(1+n)D} \text{ if } 1 \leq \Pi_t \leq \Pi_{\text{sink}} \quad (23)$$

Finally, when monetary policy is active in the deflationary area ( $\Pi_t < 1 \leq \Pi_{\text{sink}}$ ), we have  $\Pi_{t+1} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D}$  and we obtain:

$$\Delta\Pi_{t+1} = 0 \iff \Pi_t = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D} \text{ if } \Pi_t < 1 \leq \Pi_{\text{sink}} \quad (24)$$

Differentiating this equation in the neighborhood of  $\Pi = 1$  yields  $\left(1 - \frac{S'_\Pi}{(1+n)D}\right) d\Pi_t = \frac{S'_k}{(1+n)D} dk_t$ . As is obvious,  $\gamma = 0$  yields  $\frac{d\Pi_t}{dk_t} > 0$ . Indeed, in this case there is no wage rigidity, and the curve is strictly similar to the previous one. By contrast, if the wage rigidity is strong enough such that  $\gamma > \frac{\alpha(1+n)D}{\alpha(1+n)D + (1-\alpha)sAk_{\Pi=1}^\alpha}$ , the relation that links the level of capital to inflation in order to guarantee a stable level of inflation again decreases. In that case, the increase in the capital in  $t$  yields a sufficiently strong decrease in employment (due to the gap between the effective

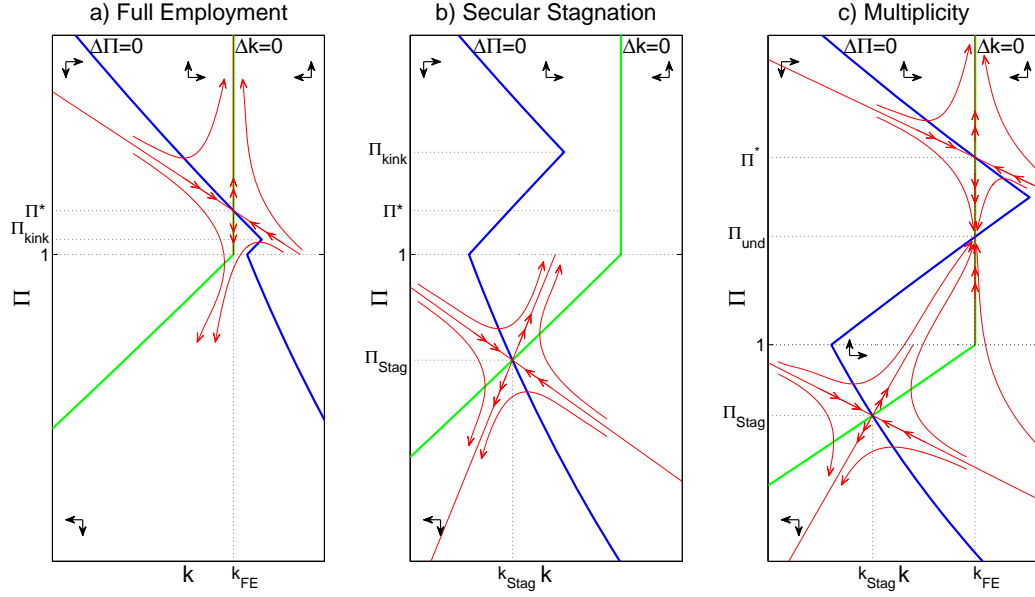


Figure 3: Features of the steady states

and the full employment real wage) so that the product in  $t$  decreases. Therefore, savings in  $t$  decreases as well as the capital in  $t + 1$  such that  $R_{t+1}$  increases. In all of these configurations, it is easy to show that for any given level of  $\Pi_t$ ,  $\frac{\partial \Delta \Pi_{t+1}}{\partial k_t} > 0$ .

Three configurations can be illustrated by representing the curves  $\Delta k$  and  $\Delta \Pi$  in the same phase plane  $(k, \Pi)$ . If the equilibrium is unique, there are only two cases: full employment with inflation target (Fig. 1a) with  $(k_{FE}, \Pi^*)$  the stable steady state, or secular stagnation with underemployment and deflation (Fig. 1b) with  $(k_{Stag}, \Pi_{Stag})$  the stable steady state. In both cases, the equilibria are globally determinate with dynamics characterized by a unique saddle path. Finally, in a third configuration, the two preceding equilibria coexist with a third equilibrium of full employment and missed inflation target:  $(k_{FE}, \Pi_{und})$ . This equilibrium is indeterminate and the other two are determinate. These determinate steady states are not unique. Therefore, they are only locally determinate, not globally.

The question is that of the change from one dynamic time path to another, and in particular, what may explain the fall into secular stagnation starting from a full-employment equilibrium. So we are naturally interested in the declining population growth as an eventual cause of the stagnation.

### 3.3 Aging and secular stagnation

What happens if population is aging such that  $dn < 0$ ? From equation (17), it follows that in the neighborhood of the saddle point steady state defined as  $k_{FE} = \frac{s}{1+n-s(1-\delta)} (Ak_{FE}^\alpha - D)$ , the dynamics of capital is as follows:  $\frac{dk_{t+1}}{k_{FE}} = \varepsilon \frac{dk_t}{k_{FE}} - \frac{dn}{1+n}$ , where  $\varepsilon = \frac{A\alpha k_{FE}^\alpha}{Ak_{FE}^\alpha - D} < 1$ . This yields  $\frac{dk_{FE}}{dn} = -\frac{k_{FE}}{1+n} \frac{1}{1-\varepsilon} < 0$ . As  $k_{FE} = \frac{D}{R_{eq}}$ , we deduce that at the steady state,  $\frac{dR_{eq}}{dn} > 0$ . A decrease in  $n$  automatically implies a decrease in the demand for credit which can be adjusted only by lower interest rates. Consequently, households are less indebted, increasing their future saving capacity and therefore the accumulation of savings.

Finally, it can be shown that if  $D < \left(\frac{A}{\delta + \frac{1+n}{s}}\right)^{\frac{1}{1-\alpha}}$ , then at the steady state we have  $R_{eq} < 1$ , i.e. the natural interest rate ( $r_{eq} = R_{eq} - 1$ ) becomes negative, which can be problematic for monetary policy, as outlined in the introduction. Following a savings glut in the economy, monetary policy can be conducted so as to move toward the zero bound and thereby induce a type of long-term stagnation. Nominal wage rigidity combined with the positivity constraint of the nominal policy interest rate is a key element for displaying a situation of secular stagnation. The following lemma gives a necessary and sufficient condition for the existence of such an equilibrium (Walrasian disequilibrium).

**Lemma 1** *A secular stagnation equilibrium exists and is locally determinate iff  $R_{eq} < 1 \iff n < \tilde{n}$ , where  $\tilde{n} = s \left(\frac{A}{D^{1-\alpha}} - \delta\right) - 1$ . If  $R_{eq} < \frac{1}{\Pi^*} \iff \Pi_{kink} > \Pi^*$ , then the secular stagnation equilibrium is the unique equilibrium.*

**Proposition 2** *All things being equal, any sufficiently low population growth rate  $n$  can make the economy fall into secular stagnation (Fig. 2).*

It is worth noting that the existence of a secular stagnation equilibrium is not due solely to the effects of population aging. In particular, without going further into the details of the model, a credit tightening ( $dD < 0$ ) can also play a role in explaining secular stagnation (Eggertsson and Mehrotra, 2014; Le Garrec and Touz e, 2016).

## 4 Upward transfers

As secular stagnation can exist only as wages are downwardly rigid, we could naturally think that promoting growth and employment proceeds through increasing the flexibility of the labor market. However, in secular stagnation, this would have a paradoxical impact as a decrease in wage rigidity  $\gamma$  tends to reduce production and employment. This result might seem surprising. Indeed, when there is no rigidity, i.e.  $\gamma = 0$ , actual production is always equal to its potential. However, this result can be easily explained. In secular stagnation, a stronger nominal wage flexibility results in recessionary effects, because it generates deflationary pressures, and therefore, as monetary policy is constrained by the ZLB, an increase in the real interest rate ( $R = \frac{1}{\Pi}$ ). Demand, and then effective production, are reduced at equilibrium. Paradoxically, a higher nominal wage flexibility yields an increase in the real wage. Increasing labor market flexibility, unless this becomes total, then has counterproductive effects for the economy (Eggertsson and Krugman, 2012). Another Keynesian paradox, named "Paradox of toil" (Eggertsson, 2010) arises in our setting: as higher productivity generates deflationary pressures, it also leads to lower production and employment.

As secular stagnation arises because of an excess of savings that pushes up the natural interest rate below zero, it appears natural to consider a transfer from the worker to the retirees. Indeed, such a transfer first reduce the disposable income in the middle-aged period. Second, by providing benefits when retired, it reduces the need for savings. However, even if it may allow escaping secular stagnation, such transfer to be accepted must be Pareto-improving.

### 4.1 Escaping secular stagnation

Suppose that the economy is characterized by a unique deflationary secular stagnation equilibrium as shown in Figure 1b ( $\Pi_{kink} > \Pi^*$ ). To get out of such an equilibrium,

$$\begin{cases} a_{t-1}^y = -I_{t-1} \\ c_t + a_t^m = w_t l_t - T + R_t^k I_{t-1} + R_t a_{t-1}^y \\ d_{t+1} = R_{t+1} a_t^m + P_{t+1} \end{cases} \quad (25)$$

such that  $N_{t-1}P_{t+1} = N_tT$  yields according to eq. (1)  $P_{t+1} = (1+n)T$ . In that case, the optimal asset becomes:

$$a_t^m = s(w_t l_t - T + R_t^k I_{t-1} + R_t a_{t-1}^y) - (1-s) \frac{1+n}{R_{t+1}} T \quad (26)$$

Savings is reduce for two reasons. First, contributions reduce the disposable income in the middle-aged period. Second, by providing benefits when retired, it reduces the need for savings.

We now deduce from equations (12) and (4) that  $w_t l_t + R_t^k k_t = \frac{1}{s} a_t^m - R_t a_{t-1}^y + \left(1 + \frac{(1-s)(1+n)}{sR_{t+1}}\right) T$ . Knowing that  $y_t^d = w_t l_t + R_t^k k_t$  and using equilibrium relations (13) and (14), the dynamics of the population (1) and the credit rationing (5), the aggregate demand per worker can be expressed as:

$$y_t^d = \frac{1+n}{s} I_t + D + \left(1 + \frac{(1-s)(1+n)}{sR_{t+1}}\right) T \quad (27)$$

In the good market, the supply-demand equilibrium  $y_t^s = y_t^d$  (with eq. 12) determines the following dynamics of the capital stock  $k_{t+1} = \frac{s}{1+n} \left[ A k_t^\alpha \min(1, l(\Pi_t)^{1-\alpha}) + (1-\delta) k_t - D - \left(1 + \frac{(1-s)(1+n)}{sR_{t+1}}\right) T \right]$  or more relevantly as  $k_{t+1} = I_t = \frac{D}{R_{t+1}}$ :

$$k_{t+1} = \frac{1}{1+n} \mathcal{S}(k_t, \Pi_t, T) \quad (28)$$

where  $\mathcal{S}(k_t, \Pi_t, T) = \frac{s}{1+(1-s)\frac{T}{D}} \left[ A k_t^\alpha \min(1, l(\Pi_t)^{1-\alpha}) + (1-\delta) k_t - D - T \right]$  with  $\mathcal{S}'_T < 0$ .

**Proposition 3** *Assume the economy is trapped in secular stagnation whose necessary condition is  $n < \tilde{n}$ . In that case, there exists a strictly positive intergenerational transfert from the workers to the retirees  $\tilde{T} = \frac{s}{1+(1-s)n} \left( \frac{1+\tilde{n}}{1+n} - 1 \right) D$  such that  $T > \tilde{T}$  allows the economy escaping secular stagnation.*

**Proposition 4** *The minimal level of intergenerational transfert from the workers to the retirees that allows the economy escaping secular stagnation is growing with population aging,  $\frac{\partial \tilde{T}}{\partial n} < 0$ .*



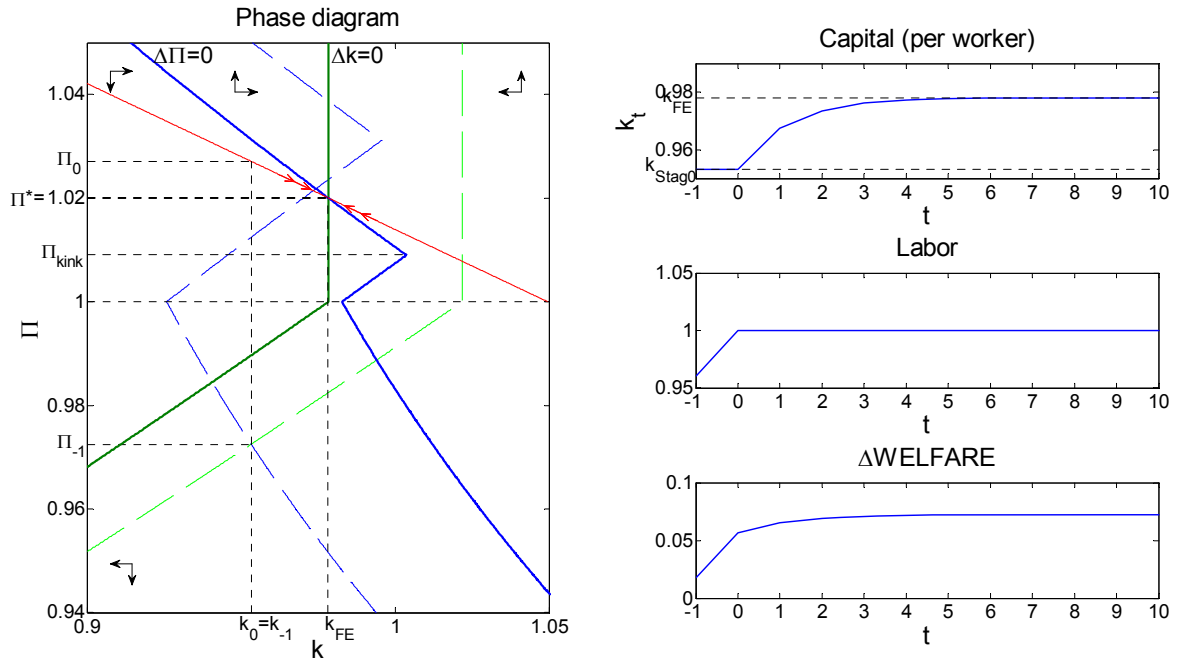


Figure 4: Escaping secular stagnation: a Pareto improving transfer

## 4.2 Welfare analysis

**Proposition 5** *Assume an economy trapped in secular stagnation. Even if  $R (= \frac{1}{\Pi}) > 1 + n$ , it may exist upward transfers that allow the economy escaping secular stagnation and are Pareto improving.*

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# Appendix: Study of the trajectories in the neighborhood of steady states

**Case 1.** Full employment and satisfied inflation target ( $i \geq 0, l = 1$ ) :

The equation (19) can be written  $\frac{D}{k_{t+1}} = \frac{1+i^*}{\Pi^* \phi_\pi} \frac{\Pi_t^{\phi_\pi}}{\Pi_{t+1}}$ . After log-linearization of this equation and the equation (17), we obtain:

$$\tilde{\pi}_{t+1} = \phi_\pi \tilde{\pi}_t + \eta \tilde{k}_t \quad (29)$$

and

$$\tilde{k}_{t+1} = \eta \tilde{k}_t \quad (30)$$

where  $\eta = \frac{\alpha A k^{\alpha-1} + 1 - \delta}{A k^{\alpha-1} - \frac{D+G}{k} + 1 - \delta}$ . In matrix form, the local dynamics can be expressed as follows:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} \phi_\pi & \eta \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

where  $\lambda_1 = \eta$  and  $\lambda_2 = \phi_\pi$  are the two eigenvalues ( $\det(M) = \eta \phi_\pi = \lambda_1 \lambda_2$  and  $\text{tr}(M) = \phi_\pi + \eta = \lambda_1 + \lambda_2$ ). In the neighborhood of the steady state, we have  $\eta < 1$  and  $\phi_\pi > 1$ , the full employment steady state with satisfied inflation target is a saddle point.

**Case 2.** Full employment with unsatisfied inflation target ( $i = 0, l = 1$ ) :

In this case, the equation (17) and its linearized form (30) are still valid. By contrast, we have  $\Pi_{t+1} = \frac{k_{t+1}}{D}$  and then:

$$\tilde{\pi}_{t+1} = \tilde{k}_{t+1} = \eta \tilde{k}_t \quad (31)$$

In this case, the local dynamics is characterized by the following system:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} 0 & \eta \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

where  $\lambda_1 = \eta < 1$  and  $\lambda_2 = 0$  are the two eigenvalues. In this case the dynamics are indeterminate.

**Case 3.** Secular stagnation equilibrium ( $i = 0, l < 1$ ) :

We deduce from (16):

$$\tilde{y}_t = \nu \tilde{k}_{t+1} - \frac{(1-\delta)s}{1+n} \nu \tilde{k}_t \quad (32)$$

where  $\nu = \frac{1}{1 + \frac{s}{1+n} \frac{D+G}{k} - \frac{(1-\delta)s}{1+n}}$ . Furthermore as  $y_t^s = Ak_t^\alpha l_t^{1-\alpha}$ , we deduce  $\tilde{y}_t = \alpha \tilde{k}_t + (1-\alpha) \tilde{l}_t$ .

Profit maximization gives  $(1-\alpha) k_t^\alpha l_t^{-\alpha} = w_t$  and  $l_t = \left(\frac{w_t}{1-\alpha}\right)^{-\frac{1}{\alpha}} k_t$ . We find  $\tilde{l}_t = \frac{-1}{\alpha} \tilde{w}_t + \tilde{k}_t$ , and:

$$\begin{aligned} \tilde{y}_t &= \alpha \tilde{k}_t + (1-\alpha) \left( \frac{-1}{\alpha} \tilde{w}_t + \tilde{k}_t \right) \\ &= \tilde{k}_t - \frac{1-\alpha}{\alpha} \tilde{w}_t \end{aligned} \quad (33)$$

Moreover, given the rigidity of nominal wages expressed by  $w_t = \gamma w_t \frac{1}{\Pi_t} + (1-\gamma)(1-\alpha) k_t^\alpha$ , the log-linear form can be written as follows  $\tilde{w}_t = \frac{\gamma}{\Pi} (\tilde{w}_t - \tilde{\pi}_t) + (1-\gamma)(1-\alpha) \alpha \frac{k^\alpha}{w} \tilde{k}_t$ , where  $\frac{k^\alpha}{w} = \frac{1-\frac{\gamma}{\Pi}}{(1-\gamma)(1-\alpha)}$ . We find:

$$\tilde{w}_t = \alpha \tilde{k}_t - \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} \tilde{\pi}_t \quad (34)$$

By introducing equation (34) in (33), we obtain:

$$\tilde{y}_t = \alpha \tilde{k}_t + \frac{1-\alpha}{\alpha} \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} \tilde{\pi}_t$$

From (32), we find:

$$\tilde{k}_{t+1} = \left( \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \right) \tilde{k}_t + \frac{1-\alpha}{\alpha \nu} \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} \tilde{\pi}_t$$

Moreover, as  $\Pi_{t+1} = \frac{k_{t+1}}{D}$ , we have:

$$\tilde{\pi}_{t+1} = \tilde{k}_{t+1},$$

and we identify the dynamics in its matrix form:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1-\alpha}{\alpha \nu} \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} & \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \\ \frac{1-\alpha}{\alpha \nu} \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} & \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

$\det(M) = 0$  et  $tr(M) = \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} + \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n}$ . If  $\delta = 1$ , then we deduce immediately that the two associated eigenvalues are  $\lambda_1 = \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} + \frac{\alpha}{\nu}$  and  $\lambda_2 = 0$ . A necessary and sufficient condition for a saddle-point equilibrium is  $\lambda_1 > 1$ . This condition is satisfied iff:

$$\nu < \alpha + \frac{1-\alpha}{\alpha} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}}$$

This condition is equivalent to the observation in the space  $(Y, \Pi)$  of a slope of demand  $(\frac{1}{\nu})$  which is greater than that of supply. This condition is always met when the secular stagnation equilibrium exists (see Lemma 1).