Advocacy NGOs

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March 31, 2017

Abstract

We develop a theoretical model in which NGOs financed by foreign donors engage in two types of activities in a developing country: service provision and advocacy. In the model, service provision relieves poverty, but these aid resources risk embezzlement by corrupt authorities. Advocacy can encourage the local population to demand more transparency to the authorities, reducing embezzlement at the cost of investing fewer efforts in direct poverty alleviation. We find that in general advocacy will be under-provided because its benefit, improved governance, has the characteristics of a public good. NGOs can remedy to this under-provision by coordinating their actions, but because this coordination threatens the rents of the local authorities, officials will respond to coordination attempts by cracking down on NGOs. Full coordination is therefore undesirable: crackdown of NGOs will be too strong, which reduces service provision and hurts beneficiaries.

Keywords: NGOs, Autocracy, Advocacy, Campaining, Aid Effectiveness, Coordination.

JEL Codes: L3, F3, F5, O19.

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1 Introduction

In the last few years, the international community has multiplied efforts directed at improving foreign aid effectiveness. At the High Level Fora on Aid effectiveness (Paris, 2005; Accra, 2008; Busan 2011), parts signing the agreements committed to transform the landscape of development assistance. Participant countries pledged to involve local citizens into shaping development strategies, particularly through the explicit recognition of Nongovernmental Organisations as pillars of the implementation of effective aid. From being practically absent at the initial debates at the High Level Fora, NGOs became formally recognised as full and equal participants to the negotiations at the Busan summit in 2011. Accompanying this formal recognition, donor countries have considerably increased their financial support to NGOs: roughly one-fifth of all official OECD aid money was delivered through NGOs in 2013, double of what it was just ten years ago (OECD, 2014). When private donations are added, Worthington and Pipa (2011) estimate that the amount of aid delivered by NGOs is roughly equal to that of aid delivered through official channels.

1.1 The Challenge of Aid Harmonization

The increased reliance on the NGO sector raises the issue of coordination among organizations evolving in a fractioned sector. At the High Level Fora on aid effectiveness, the agreements stressed the importance of harmonizing aid among donors, without clearly stating how this harmonization can be achieved. Relying on a decentralized, atomistic, and heterogeneous multitude of NGOs, each of them driven by their own intrinsic objectives challenges the idea of harmonizing aid. The lack of coordination is one of the main criticisms addressed to the NGO sector, of which Haiti is the most poignant example. Likewise, relying on the NGO sector contradicts another engagement of the donor community: to conceder the ownership of development strategies financed through foreign aid to beneficiary governments. NGOs often have objectives that are divergent, if not opposed, to those of host governments. Examples include human rights associations defying repressive authorities, NGOs that advocate gender equality in patriarchal societies, or environmental NGOs that oppose heavy infrastructure projects.

Recent years have seen a surge of tensions between high-handed rulers and the non-governmental sector. Governments of countries such as Venezuela, Zimbabwe, Russia, Egypt, and several others have stringently audited, fined, intimidated, and ultimately expelled NGOs from their territories (The Economist, 2014). While rent-seeking authorities can take advantage of the presence of service-providing NGOs, directly through extortion or indirectly through the mismanagement of fungible public funds, they also risk being confronted to a sector that encourages beneficiaries into demanding greater accountability to the ruling elite. NGOs can allocate their resources between two activities: the provision of poverty-alleviating services, such as health and education, and the financing of advocacy activities. If effective, advocacy activities can motivate tar-
geted beneficiaries into appropriating aid projects that are intended for them. However, if advocating for improved governance results in a crackdown on the aid sector, which cripples the amounts of aid provided, then one wonders if it is worth for NGOs to engage in advocacy at all.

In this paper, we contribute to the literature by explicitly modelling the trade-off between the provision of welfare-enhancing services and the engagement into governance-enhancing advocacy. Two main assumptions drive our model: first, we assume that the advocacy content of the activities of the NGO is not contractible ex-ante with the host government. Second, we assume that the benefit of advocacy, namely improved governance, has the characteristics of a public good, in that better governance is non-rival and non-excludable. In our model, three types of agents interact: the host government, NGOs, and the local population. In particular, we focus on the description of the numerous choices and constraints that NGOs face: in our set-up, NGOs interact with each other, with the local authorities, and with the local population. NGOs look for an optimal allocation of their efforts between service provision and advocacy, the latter of which brings upon the benefits of improved governance. We assume that NGOs are heterogeneous in their incomes and their preferences towards advocacy, and moreover, they can coordinate with each other.

Our results show that advocacy efforts will typically be under-provided because of the public good characteristics of better governance. While free riding inefficiencies undermine the provision of advocacy efforts, addressing this inefficiency through an increased focus on governance, or through intensified coordination efforts, makes the government strengthen its crackdown on NGOs. As a result, although some degree of advocacy is desirable, increasing it to the point of abolishing free riding inefficiencies is not optimal because corrupt authorities will react by shutting down too many NGOs compared to a second-best optimum. We prove the existence of such a second-best optimal degree of advocacy and we derive its properties, in particular its incompatibility with a state where there is no free riding in advocacy. A direct policy implication is that donors cannot – and should not – expect that all forms of free riding disappear, at least not through a bottom-up approach only.

1.2 Literature Review

Scholars in social sciences highlight the role of NGOs as promoters of good governance as frequently as they emphasise their role as service providers, but rarely do they expose the constraints faced by NGOs when they strive to empower people (Edwards and Hulme, 1996). The economic literature outlines the potential comparative advantages of nonprofits in the provision of public services relative to governments or for-profits (Hansmann, 1980; Hart and Moore, 1998; Besley and Ghatak, 2001, 2005). The literature dealing with the political economy of multilateral or bilateral foreign aid, can be traced back at least half a century (Fei and Paauw, 1965; Mosley, 1986; Boone, 1996; Acemoglu et al., 2003; Mekasha and Tarp, 2011). Studying the political economy of foreign aid when it is provided by private nonprofits is, however, a relatively recent endeavour.
The literature on aid efficiency typically models the problem of aid allocation as a principal-agent relationship between multilateral and bilateral donors (the principals) and recipient governments (the agents). In this context, the literature claims that aid conditionality and aid coordination are required to enhance the governance of recipient countries (Svensson, 2000; Azam and Laffont, 2003; Torsvik, 2005; Bourguignon and Platteau, 2012; Platteau et al., 2014). However, donors often lack credibility regarding their willingness to coordinate and enforce sanctions, which challenges the feasibility of this approach. Instead, many authors see in a bottom-up approach the most effective way towards improved governance (Easterly, 2007; Banerjee and Duflo, 2011). Indeed, if donor countries cannot commit to demand governance improvements from the top, then they should design a way of increasing the bargaining power of the grassroots from the bottom. In order to do this, relying on the NGO sector appears as the most feasible alternative. In practical terms, the Busan Partnership legitimates NGOs as vital service providers and advocates of the poor, assigning them a double mission: to produce a tangible output that relieves poverty, and to advocate institutional change that grants new rights to the poor (Edwards and Hulme, 1996; Bebbington et al., 2008).

Our paper presents the first attempt at modelling NGOs when they carry out both advocacy efforts and service provision. It does not deal with activities targeted at lobbying authorities; such issues usually concern governments of developed countries (Couttenier and Hatte, 2013). Instead, and in line with models of aid coordination, such as those of Platteau et al. (2014) and Torsvik (2005), we consider a setting with multiple donors and one government of a poor country. Their conclusion is that aid coordination is always desirable for beneficiaries of aid, although it can be costly and even detrimental for donors. Focusing on the NGO sector brings about different conclusions. In our setting, too much coordination can be detrimental not only to donors, but also to beneficiaries. The main difference is that, unlike multilateral donors contracting with governments, NGOs operate in the field. This feature means that NGOs interact with the local population, must integrate local politics, and are constrained by the legislation of the host country.

The paper unfolds as follows. Section 3 sets and solves the basic model. Section 4 develops the model towards specifications in which NGOs are able to cooperate or have political preferences. Section 5 discusses the model in light of its policy implications. Section 6 concludes.

2 Basic Model

2.1 Set Up

Three types of players interact in this model. These are: the government of the host country (denoted $G$), the citizens it governs ($C$), and a set of $N$ NGOs willing to operate in the country, indexed by $i = \{1, \ldots, n, \ldots, N\}$. We assume
that due to the atomistic nature of the NGO sector, \( N \) is arbitrarily high.

Actions of the players occur in the following sequence: the government chooses the number \((n)\) of NGOs authorized to operate in the country; once authorized, each NGO has to decide how much of its exogenous budget \( b_i \) it allocates between investing in providing services \((s_i)\) and investing in advocacy \((v_i)\); in the last step, citizens exposed to advocacy and aware of the existence of projects can engage in costly appropriation efforts \((a)\), which allow them to obtain ownership over aid projects.

We define ‘appropriation efforts’ as any costly initiatives engaged by citizens, which allow them to discourage embezzlement by government officials. Appropriation efforts can take the form of protests, petitions, trials, votes for the opposition, or other similar endeavours. Citizens obtain ownership of a share \( 0 < \omega(a) \leq 1 \) of aid projects \( s_i \) when they engage in a level of effort \( a \). To make matters simple, specify ownership gain technology \( \omega(a) \) as an isoelastic function up to a threshold \( \tilde{a} \) such that

\[
\omega(a) = \kappa (1 + a)^{\frac{1}{1 + \rho}} \quad \text{if} \quad 0 < a < \tilde{a} \\
\omega(a) = 1 \quad \text{if} \quad a \geq \tilde{a}
\]

with \( \rho \geq 0, \ 0 < \kappa \leq 1 \) and \( \tilde{a} = \kappa^{-(1+\rho)} - 1 \). \( \rho \) measures the ability of the government to withstand appropriation efforts without having to make concessions. \( \kappa \) marks the baseline ownership level when there are no appropriation efforts. If at the baseline there is full ownership \((\kappa = 1)\), then there is no room for appropriation gains. Ownership gains \( \omega(a) \) are increasing and concave in a. The assumption of concavity guarantees that there is an interior solution for \( a \), and reflects a process of gradual gain of ownership through more efficient appropriation efforts. A convex function would generate corner solutions (either \( a = 0 \) or \( a = 1 \)), more suited to model civil conflict or government overthrows. In this paper, we maintain the focus on non-drastic changes.

The payoff function of the government writes

\[
U^G = [1 - \omega(a)] s. \tag{1}
\]

in which \( s = \sum_i^N s_i \) is the aggregate amount of aid invested in development projects. The payoff of corrupt authorities increases with the amount of resources they can embezzle and reduces with level of appropriation efforts exerted by citizens. Appropriation efforts are costly, which we make explicit in the citizen’s payoff function:

\[
U^P = \omega(a)s - \frac{a}{\nu \lambda} \tag{2}
\]

in which \( v = \sum_i v_i \) are the aggregate advocacy efforts provided by the NGO sector. Citizens benefit from the projects to the extent that they obtain actual ownership over them. They can increase their ownership through appropriation efforts, made cheaper by the NGO’s advocacy input. Advocacy reduces the marginal cost of petitioning the government by providing informational, legal, political, or other types of intangible assistance. The intangible nature of
these services makes them both simultaneously non-contractible and impossible to embezzle. The efficiency of advocacy efforts is variable and depends on how convincing the arguments of the NGO are, and on the degree to which beneficiaries are willing to listen. Parameter $\lambda$ measures the ability of NGOs to mobilize appropriation efforts through advocacy, with low values denoting low responsiveness of citizens to campaigning.

The aforementioned payoff functions of both government and citizens are highly stylized in order to concentrate attention in the modelling of the NGO sector. In the benchmark case, the simplest payoff function for any NGO is

$$U^\text{NGO}_i = \omega(a)s_i$$

which it maximizes subject to the constraint $b_i = s_i + v_i$. In this simple case, an NGO cares about the size of its own project and on the degree of ownership of the project by its beneficiaries. While advocacy does not entail direct benefits, the NGO understands that it drives the appropriation efforts of the beneficiaries, which in turn makes the project more valuable. Through this section, we model NGOs as imperfectly altruistic: they only care about the well-being created by their own project, but not about the well-being created by the overall NGO sector. This assumption is relaxed in the extensions.

![Figure 1: Timing of the model.](image)

The timing of the game is simple; a three-stage, one-shot game is sufficient to derive our main results. In the first stage, the government knows the income distribution of all NGOs willing to enter the country and authorizes operations for $n$ NGOs. In the second stage, all authorized NGOs simultaneously allocate their budget between providing services ($s_i$) and investing in advocacy ($v_i$). At the last stage, citizens decide upon the effort they put into appropriating the
projects \((a)\). Finally, all players receive their payoff. This mechanism is schemed in Figure 1.

The choice of the number of players (NGOs) by the government or the continuous action space for NGOs makes the classic extensive-form representation of the game impractical to display, nonetheless we can easily solve it by backward induction and subgame perfect Nash equilibria.

2.2 Analysis

The model is solved by backward induction. At the last stage, \(n \geq 1\) NGOs active in the country have invested \(\Sigma_i b_i\) monetary units, of which \(\Sigma_i s_i\) financed development projects, and \(\Sigma_i v_i\) financed advocacy. Citizens choose the level of effort put into appropriating aid projects by solving the following program:

\[
\begin{align*}
\text{Max}_a & \quad \omega(a) \Sigma_i s_i - \frac{a}{(\Sigma_i v_i)^\lambda} \\
\text{s.t} & \quad b_i = s_i + v_i \\
\omega & = \left[\frac{\kappa^{1+\rho}}{1 + \rho} \left(1 + \frac{1}{a}\right)^{1+\rho}\right]^{\frac{1}{\rho}} \quad \text{[Non-Distribution Constraint]}, \\
\omega & = \left[\frac{\kappa^{1+\rho}}{1 + \rho} \left(1 + \frac{1}{a}\right)^{1+\rho}\right]^{\frac{1}{\rho}} \quad \text{[3rd stage equilibrium condition, (5)]}, \\
\text{and} & \quad (s^*_j, v^*_j) \in \arg\max_{s_j, v_j} \omega s_j, \forall j \neq i \quad \text{[Best response of } n-1 \text{ NGOs]}
\end{align*}
\]

\(1 \text{Akin to a budget constraint, the term non-distribution constraint is usually employed when referring to nonprofits since Hansmann (1980).}\)
where $a^*$ and the case in which $a^* = 0$ are made implicit to alleviate notation. Solving the programme of the NGOs yields the following solution:

$$v^* = \frac{\lambda}{1 + \lambda + n\rho} b$$  \hspace{1cm} (6)

and

$$s^* = \frac{1 + n\rho}{1 + \lambda + n\rho} b.$$  \hspace{1cm} (7)

in which $b = \Sigma_i b_i$ are the aggregated budgets.

**Proposition 1 (Aggregate investments in service provision and advocacy)**

If $n$ NGOs are allowed to operate in the country, then

1. if advocacy and appropriation efforts have high returns, then NGOs invest a larger share of aid in advocacy;
2. if advocacy increases, then so do appropriation efforts;
3. if aid is fractioned, then NGOs free ride more on advocacy;
4. both advocacy and service provision monotonically increase with the number of NGOs.

**Proof 1** The total derivative maximizing the objective function yields the following equilibrium condition:

$$\frac{d\omega}{ds} s_i + \omega = 0 \Rightarrow \Sigma_{i=1}^n \left[ \frac{d\omega}{ds} s_i + \omega \right] = 0 \Rightarrow \frac{d\omega}{ds} s + \omega n = 0$$

since $ds/ds_i = 1$. The logarithmic derivative of the 3rd stage equilibrium condition (5) with respect to $s$ yields

$$\rho \frac{d\omega}{ds} \frac{s}{\omega} = 1 + \frac{dv}{ds} \frac{s}{v} = -1.$$  \hspace{1cm} (8)

By the non-distribution constraint, $dv/ds = -1$. Combining the two previous equations states the following second stage equilibrium condition:

$$\frac{v}{s} = \frac{\lambda}{1 + n\rho}.$$  \hspace{1cm} (9)

Plugging this condition in the non-distribution constraint yields the desired solution. □

A fragmented aid sector makes advocacy prone to free riding because its benefit, improved governance, has the characteristics of a public good. Corrupt officials have thus an incentive to sow discord among NGOs, inducing them to free ride more on advocacy.

We can disaggregate the levels of advocacy and investment in projects at the individual level to display the second-stage subgame Nash equilibrium:

$$s_i^* = \frac{1 + n\rho}{1 + \lambda + n\rho} b.$$
and
\[ v_i^* = b_i - \frac{1 + n\rho}{1 + \lambda + n\rho} \bar{b} \]
in which \( \bar{b} = b/n \). Richer NGOs are willing to contribute more to advocacy, i.e. to the public good. As the average budget increases, so does the incentive to free ride on other contributors.

This result is standard in the theory of public goods, although not necessarily realistic in our context; in reality, politically motivated NGOs actively exert advocacy efforts, no matter how small they are. On the contrary, large NGOs can be timid militants. We address the question of political motivation in the next section.

Before looking at the optimization problem of the government, it is useful to show that free riding creates inefficiencies. Because we are concerned with the effectiveness of aid, our well-being criterion is the payoff function of the citizens only. We defend this choice with two arguments: (i) interests of NGOs should be aligned with those of their beneficiaries, so that the well-being of NGOs should be perfectly correlated with the well-being of citizens; and (ii) we care about ownership of aid. By adding the well-being of corrupt officials, we would care about absolute levels of aid, disregarding the issue of its distribution.

From the perspective of NGOs, the reaction function of the citizens, \( \omega = \frac{s^{1/\rho}}{\rho v^{1/\rho}} \), displays an inverse U-shaped relationship between advocacy and appropriation efforts, as shown in Figure 2. It reaches its maximum level when \( v_{\text{max}} = \text{argmax} \omega = \frac{\lambda + 1}{\lambda + \rho} b \).

If the choices of NGOs result in corner solutions (all aid is invested in either projects or advocacy), there are no changes in appropriation efforts and ownership remains at a baseline \( \omega(0) \). Intuitively, without advocacy efforts, there are no changes in appropriation efficiency, and without projects, there is no reason to change baseline appropriation efforts.

A low \( \lambda \) indicates that advocacy has little impact on the cost of appropriation efforts. This can happen because the message of the NGO is too weak or because beneficiaries are already highly organized and have little to gain from the NGO’s input. Conversely, high returns of advocacy are most likely to occur when beneficiaries are disorganized and uninformed.

To compute the first-best level of advocacy, plug the FOC of the citizens back in to their payoff function. Evaluated at their optimized level, the utility of citizens is \( U^C = \omega s^{1/\rho} v^{1/\rho} \).

Which yields the following optimal level of advocacy:
\[ v^{\text{op}} = \frac{\lambda + 1}{\lambda + \rho + \rho} b. \]

**Proposition 2 (Free-ridding Inefficiency)** If the amount of aid that a country receives is fixed, then a single NGO should manage all of it.
Figure 2: Levels of advocacy at the equilibrium, optimum, and maximum appropriation.

**Proof 2** The total derivative of the citizens payoff relative to $s$ is $$\frac{d\omega}{ds} + \omega = 0$$ Plugging in (8) yields $$v = \frac{\rho}{1 + \rho}$$ Combining this with the non-distribution constraint yields the desired solution. □

Figure 2 plots this optimum over the reaction function of the citizens. An indifference curve shows the optimal decentralized choice of an NGO when the sector is competitive.

The equilibrium choice of the NGO is the tangency point between its indifference curve and the reaction function of citizens. As fragmentation increases among NGOs, each will choose lower levels of advocacy, drifting away from the optimal monopolistic situation. Advocacy levels that maximize appropriation efforts are not optimal either, unless these efforts are efficient in achieving ownership (if $\rho$ is low). Otherwise, when intents of gaining ownership have no real effect due to the tight grip of the authorities, encouraging appropriation is wasteful. Next, we close the model by solving the problem of rent-seeking officials.

Turn now to the first stage. To alleviate notation, we drop the asterisk from equilibrium values $v^*$ and $s^*$. Likewise, and without loss of generality, we assume continuity of the number of NGOs, imposing the restriction that, if allowed to operate, there should be at least one (i.e. $n \in [1, \infty[\ )$. If the government could contract the advocacy content of aid, it would fix it to zero. However, this contract is unenforceable, because it is too demanding to monitor and to disentangle what part of aid is a service and which is nurturing political leadership. From the point of view of the authorities, the only observable variable is the presence or not of NGOs, given that they deliver the authorizations to operate. The program of the government is
Max \( n\) \((1 - \omega)s\)

\[
\begin{align*}
\text{s.t.} \quad \bar{b}_n &= v + s, \quad \text{[Aggregated Non-Distribution Constraints]}, \\
\omega &= \left[ \frac{\kappa^{1+\rho}sv^\lambda}{1 + \rho} \right]^{\frac{1}{\rho}} \quad \text{[3rd stage equilibrium condition, (5)]}, \\
\text{and} \quad \frac{v}{s} &= \frac{\lambda}{1 + n\rho} \quad \text{[2nd stage equilibrium condition, (9)]}.
\end{align*}
\]

Because \( s(n) \) is a bijective and monotonically increasing function of \( n \), solving this problem is akin to choosing the optimal \( s \) for the government. The first order condition expressed in terms of the total derivative of the objective function with respect to \( s \) is

\[
1 - \omega = \frac{d\omega}{ds} s. \tag{12}
\]

This equilibrium condition states that the marginal benefit of allowing one additional unit of service provision (the captured share \( 1 - \omega \) of that unit) must equal the marginal cost of having the beneficiaries’ share improve. To obtain a more explicit expression of the right-hand side of this equation, take logarithmic derivative of the third stage equilibrium condition with respect to \( s \):

\[
\frac{d\omega}{ds} \frac{s}{\omega} = \frac{1}{\rho} \left[ 1 + \frac{1}{1 + \lambda \frac{dv}{ds} + n\rho} \right] \tag{13}
\]

The second term inside the square brackets is proportional to the elasticity of advocacy to services provision.

**Lemma 1** The elasticity of advocacy to services provision is

\[
\varepsilon = \frac{dv}{ds} v = \frac{1}{1 + \lambda \frac{1}{\rho n + \frac{\lambda}{1 + \lambda + \rho n}}} \tag{14}
\]

In which \( \varepsilon \in [0, 1] \) decreases in \( n \) and \( \rho \) and increases in \( \lambda \), with \( \lim_{n \to 0} \varepsilon(n) = 1 \) and \( \lim_{n \to \infty} \varepsilon(n) = 0 \).

The appendix proves this statement. To each percentage increase of service provision, there is an increase in advocacy, which is marginally declining as the free riding problem aggravates. Combining (12), (13) and (1) obtains the 1st stage equilibrium condition:

\[
1 - \omega = \frac{1 + \lambda \varepsilon}{1 + \rho + \lambda \varepsilon} \tag{15}
\]

Denote the left-hand side \( MB(n) \) for marginal benefit and the right-hand side \( MC(n) \) for marginal cost. \( MB(n) \) and \( MC(n) \) are non-increasing, and as a
result of continuity and their limit conditions, are single-crossing if the following condition holds:

$$\frac{\rho}{1 + \rho + \lambda} > \kappa$$  \hspace{1cm} (16)

This condition states that the government will allow NGOs only if it can withstand appropriation efforts with ease, if the baseline ownership by beneficiaries is low, and if advocacy has sufficiently low returns. Figure 3 plots both functions and the equilibrium condition. Marginal costs are declining due to the free riding properties of our problem: allowing more NGOs increases ownership, but in a marginally declining manner.

Let $MU(n) = MB(n) - MC(n) = 0$ be the marginal payoff of $n$. Table 1 summarizes the sign of the partial derivatives of functions $MB(n)$ and $MC(n)$ for any $n$. The appendix details these computations. In line with proposition 1, the parameters that increase the share of total aid devoted to service provision increase the marginal benefits and reduce the marginal costs for the government.

Figure 3: Marginal Benefits and Costs of allowing one additional unit NGO

<table>
<thead>
<tr>
<th></th>
<th>$\partial MB(n)$</th>
<th>$\partial MC(n)$</th>
<th>$\partial MU(n)$</th>
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<tr>
<td>$\partial \rho$</td>
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<td>$\partial \rho$</td>
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<td>$\partial \kappa$</td>
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Table 1: Partial derivatives for $MB(n)$ and $MC(n)$ function.
Proposition 3 (Number of NGOs allowed by the Government) If condition (16) holds, the authorities cannot observe each individual NGO’s income, and cannot contract advocacy content, then they will deliver a unique number of permits \( n^* \), which maps a unique level of services \( s^* \). The number of NGOs allowed will increase if

1. NGOs’ advocacy has low returns,
2. appropriation efforts have low returns,
3. agencies are poorer on average, and
4. baseline ownership is low.

Proof 3 Notice that at the crossing point \( \partial MC(n^*)/\partial n > \partial MB(n^*)/\partial n \). By implicit derivation and using the partial derivatives stated above:

\[
\frac{dn^*}{d\lambda} = -\frac{\partial MU(n^*)/\partial \lambda}{\partial MU(n^*)/\partial n^*} < 0; \quad \frac{dn^*}{d\rho} = -\frac{\partial MU(n^*)/\partial \rho}{\partial MU(n^*)/\partial n^*} > 0;
\]

and

\[
\frac{dn^*}{db} = -\frac{\partial MU(n^*)/\partial b}{\partial MU(n^*)/\partial n^*} < 0. \quad \square
\]

The results provided in proposition 3 are quite intuitive. The government will be more tolerant of the NGO sector if it does not consider it a threat. The model predicts that areas under tight grip by corrupt authorities, or where NGOs have little influence, will host more NGOs, which will have high incentives to free ride on each other.

Corollary 1 In a regime under which the government has a tight control on the access of NGOs, beneficiaries are better off when the government does not feel its rents threatened by the NGO sector.

Figure 3 shows that parameters increasing the share of aid devoted to services shift the \( MB(s) \) curve upwards, and shift the \( MC(s) \) curve downwards. Because both curves are positively sloped and single crossing, it is easy to see that parameters making aid more service-intensive increase the equilibrium number of NGOs, and increase the level of ownership by beneficiaries. Disturbingly, beneficiaries are better off with a government untroubled by the actions of the NGO sector.

Corollary 2 If the government has more information on NGOs, it will deliver more permits and extract larger rents. This slightly increases governance, and improves the overall well-being of beneficiaries.

The model makes the simplistic assumption that the number of permits \( n \) is the only strategy available to the government. Other observable characteristics of NGOs can allow the government to expand its strategy set: incomes, activities in other countries, reputation, and so on, are signals correlated with political engagement. If the government knows the distribution of these correlated signals,
it can set threshold values such that it will not authorize any NGO above them.
If the threshold monotonically changes with \( n \), then choosing it is equivalent to fixing a certain number of admissions. For instance, we saw that richer NGOs are more likely to be politically active. Then, the government will not admit any NGO richer than a threshold budget \( \tilde{b}(n^*) \) in the income distribution. The comparative statics from \( \lambda \) and \( \rho \) on the threshold are thus known. We have

\[
\frac{d\tilde{b}(n^*)}{d\lambda} = \frac{dB(n^*)}{dn^*} \frac{dn^*}{d\lambda} < 0 \quad \text{and} \quad \frac{d\tilde{b}(n^*)}{d\rho} = \frac{dB(n^*)}{dn^*} \frac{dn^*}{d\rho} < 0.
\]

The government will allow richer NGOs in areas where they are less influential or where appropriation efforts are less effective. Notice that, by fixing such thresholds, allowed NGOs are poorer on average, which shifts the \( MB(n) \) curve upwards, increasing the number of NGOs allowed, total service provision, and slightly improving ownership. If the authorities can perfectly observe the income of NGOs, this improves the situation of both beneficiaries and authorities.

3 Extensions of the Model

In the previous section, we assumed that NGOs take their decisions independently and selfishly. In reality, NGOs often try to coordinate through non-binding contractual arrangements, for example by signing the Istanbul Declaration (Istanbul Principles, 2011). By lacking any kind of enforcement, these documents usually remain vague agreements on common values\(^2\). In this section, we explore how changes in the objective functions of NGOs affect the wellbeing of target populations.

In order to do this, we apply our model to two different objective functions: first, NGOs give different weights to recipient’s ownership relative to the size of their project; second, NGOs form partnerships among each other. We show that these objective functions do not improve the welfare of beneficiaries.

3.1 Focusing on Recipient’s Ownership

First, consider the case where some NGOs decide to focus more citizen’s ownership. In this case, NGOs give more weight to overall beneficiaries appropriation relative to the focus put on the success of their own project. Each individual NGO has the following objective function:

\[
\omega \frac{1}{\phi_i} s_i
\]

in which \( 1/\phi_i \) measures NGO \( i \)'s focus on citizens’ ownership. If \( \phi_i = 1 \), we are in the benchmark model. The NGO, quite pragmatically, cares about how much of the service provision it finances do beneficiaries actually own. When \( \phi_i \) tends

\(^2\) If these agreements where enforceable, then they should be studied through coalitional game theory. In our model, unilateral deviation is always profitable: NGOs find it always individually beneficial to free ride on other’s advocacy. Under these circumstances, coalitional rationality is violated (Maschler et al., 2013).
toward zero, on the contrary, the NGO focuses mainly in improving ownership, at the cost of pulling resources away from service provision. If, on the contrary, $\phi_i$ tends toward infinity, then the NGO is unconcerned with appropriation: what matters to it is to report large expenditures in service provision, whether end-line beneficiaries appropriate them or not.

Using the same resolution method than in the previous section, we obtain:

$$v^* = \frac{\lambda}{1 + \lambda + \rho n \bar{\phi}} b$$  \hspace{1cm} (17)

and

$$s^* = \frac{1 + \rho n \bar{\phi}}{1 + \lambda + \rho n \bar{\phi}} b$$  \hspace{1cm} (18)

where $\bar{\phi} = \frac{1}{n} \sum_{i=1}^{n} \phi_i$ is the average weight put on ownership. At the individual level, we have

$$s_i = \frac{\phi_i}{\bar{\phi}} \frac{\lambda}{1 + \lambda + \rho n \bar{\phi}} b$$

and

$$v_i = b_i - \frac{\phi_i}{\bar{\phi}} \frac{\lambda}{1 + \lambda + \rho n \bar{\phi}} b.$$

Results are as expected. NGOs more focused on ownership dedicate more time to advocacy. The description that we get of the NGO sector is now richer: the emphasis put on ownership by all other NGOs determines the relative effort devoted to advocacy for any single NGO. For example, if an NGO is pragmatic, in the sense that it cares only about the efficiency of its own project ($\phi_i = 1$) and if around it all other NGOs are unconcerned with ownership, then it will have to exert more effort in advocacy despite the fact that this action in not in its initial orientation.

Recall by proposition 2 that the free-riding problem disappears when

$$v^{OP} = \frac{\lambda}{1 + \lambda + \rho} b$$

In our framework, it is easy to find that NGOs can avoid the free riding problem if

$$\bar{\phi} = \frac{1}{n}.$$  \hspace{1cm} (19)

Once $n$ NGOs are allowed, NGOs willing to make aid more efficient should optimally increase the overall focus on ownership when the sector is more fractioned. Donors might frown upon NGOs focusing too much on ownership, taking it as naive idealism. However, this naivet can improve interim efficiency (after admission of $n$ NGOs) to a certain extent because it allows counterbalancing free riding inefficiencies.
3.2 NGOs form partnerships

Assume now that NGOs decide to establish partnerships among each other. To make this simple, suppose that the objective function of any NGO becomes

\[ \omega \left( s_i + \gamma \sum_{j \neq i} s_j \right) \]

in which \( \gamma \) denotes the weight that NGOs give to the projects all other NGOs. If \( \gamma = 1 \), an NGO gives as much importance to its own project as to the sum of all other projects. By proceeding by the same method used in the previous section, at equilibrium we obtain

\[ v^* = \frac{\lambda}{1 + \lambda + \rho \frac{n}{1+\gamma(n-1)}} b \]  
(20)

and

\[ s^* = \frac{1 + \rho \frac{n}{1+\gamma(n-1)}}{1 + \lambda + \rho \frac{n}{1+\gamma(n-1)}} b. \]  
(21)

The greater the weight attached to the projects of others, the more an NGO invests in advocacy. Free riding in advocacy is diminished by giving to service provision the characteristics of a public good as well: NGOs start free riding on service provision too. Because they are subject to a non-distribution constraint, the result is that the proportion spent in each activity evens out. The optimal level of campaigning provided in (13) is reached when

\[ \gamma^* = 1 \]  
(22)

In words, if NGOs fully internalize the performance of all other NGOs, then the free riding problem disappears.

Both principles, focusing more on ownership and forming partnerships among NGOs, make the social optimum attainable at the second stage of the game. However, if the authorities anticipate that the adoption of these principles threatens their rents, then they will update their optimal level of delivered authorizations.

The appendix details how the \( MB(n) \) and \( MC(n) \) curves shift following changes of \( \phi \) and \( \gamma \), summarized in table 2.

<table>
<thead>
<tr>
<th>Partial Derivative</th>
<th>( MB(n) )</th>
<th>( MC(n) )</th>
<th>( MU(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \partial \gamma )</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( \partial \phi )</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2: Partial derivatives for \( MB(n) \) and \( MC(n) \) function.

**Proposition 4 (Anticipation by the Government)** If a rent-seeking authorities anticipate that NGOs will adopt principles committing them to reduce free
riding in advocacy, then they will reduce the number of NGOs authorized to operate in the country.

**Proof 4** Identical to the proof of proposition 3, we obtain by implicit differentiation of \( MU(n^*) \)

\[
\frac{dn^*}{d\phi} = -\frac{\partial MU(n^*)/\partial \phi}{\partial MU(n^*)/\partial n^*} > 0; \quad \text{and} \quad \frac{dn^*}{d\gamma} = -\frac{\partial MU(n^*)/\partial \gamma}{\partial MU(n^*)/\partial n^*} < 0. \quad \Box
\]

Figure 4 displays how the marginal benefit-marginal cost curves change following attempts to reduce free riding inefficiencies\(^3\). Because both curves shift in opposite directions, and are downward-sloping and single-crossing, trying to reduce free riding (by either increasing \( \gamma \) or decreasing \( \phi \)) shifts the equilibrium point to the north-west. The effect is such that even if appropriation efforts become cheaper, the reward of appropriation reduces; the net effect is that appropriation efforts become less valuable.

![Figure 4: Attempts to reduce free-riding reduce the well-being of beneficiaries.](image)

**Corollary 3** If corrupt authorities regulate the access of NGOs and anticipate efforts to reduce free riding inefficiencies, then the welfare of citizens diminishes when NGOs attempt to reduce free riding. The second-best optimum is obtained when the NGO sector is uncoordinated, and unconcerned with governance. This is

\[
\gamma^{**} = 0 \quad \text{and} \quad \phi^{**} = \infty
\]

\(^3\)Recall that \( s \) is a bijective increasing function of \( n \).
By (19) and (22), it is impossible to maximize the welfare of beneficiaries and to wipe out free riding in advocacy simultaneously.

This negative result depends on the fact that the authorities perfectly anticipate that NGOs will coordinate their efforts. Authorities optimize the number of permits depending on the expected value of $\phi$. In the next subsection, we explore how results change when the authorities can perfectly observe the degree at which each NGO focuses on ownership.

### 3.3 Complete information

Suppose there exist two types of NGOs with parameters $\phi_h > \phi_l$ among the $N$ NGOs willing to enter the country. Let $n_h + n_l = N$ define the size of each group, and as before denote $\bar{\phi}$ the average lack of interest in ownership. By definition of an average $\phi_h > \bar{\phi} > \phi_l$. If the authorities can perfectly observe the types, they first allow all high-type NGOs and optimize $n(\phi_h)$; if the optimal $n^*(\phi_h) \leq n_h$, then only high type NGOs are allowed. In this scenario, and by proposition 4, since allowed NGOs are less politicized, more of them are allowed and beneficiaries’ ownership increases slightly.

If $n^*(\phi_h) \leq n_h$ does not hold, the authorities will deliver permits to all high-type NGOs and allow a few low-type NGOs in. The $MB(n)$ and $MC(n)$ curves will display a kink at $n^h$ with the property that

$$\frac{\partial MB(n)}{\partial \phi_h} \geq \frac{\partial MB(n)}{\partial \phi} \geq \frac{\partial MB(n)}{\partial \phi_l} \quad \text{and} \quad \frac{\partial MC(n)}{\partial \phi_h} \leq \frac{\partial MC(n)}{\partial \phi} \leq \frac{\partial MC(n)}{\partial \phi_l}$$

![Figure 5: Complete versus incomplete information on the types of NGOs](image-url)
Figure 5 plots the $MB(n)$–$MC(n)$ functions and the optimal number of permits delivered under complete and incomplete information. Under complete information, initially marginal benefits are higher and marginal costs are lower than in the incomplete information case because only high types are allowed in. If there are rents that can still be appropriated after all high types are allowed, authorities will allow a few low types, which rapidly decrease marginal benefits and increase marginal cost for the authorities. At limit conditions (if all $N$ NGOs willing to enter are allowed or if there are no NGOs) complete and incomplete information yield the same costs and benefits.

**Corollary 4** The number of NGOs allowed under complete information ($n^{ci}$) is larger than the number of permits delivered under incomplete information ($n^{ii}$). Welfare of the beneficiaries improves when authorities have complete information.

This result is in line with corollary 2. Denote $\phi$ the average depoliticization under complete information. By choosing all high types, it follows that $\phi n^{ci} \geq \phi n^{ii}$. Comparing the advocacy to service provision ratio ($v/s$) in both regimes we obtain:

$$\frac{\lambda}{1 + \phi n^{ci}} < \frac{\lambda}{1 + \rho n^{ci}}$$

Asymmetries of information increase the advocacy content of aid, partially attenuating the free riding inefficiency. However, because this makes the authorities shrink the NGO sector too much, beneficiaries are overall better off when authorities have complete information despite the fact that free riding issues are exacerbated.

4 Discussion

The recent surge in the number of development NGOs justify our assumptions on the atomicity of the sector, substantiating our representation of NGOs as agents, and host governments as principals. Competition among NGOs reduces their bargaining power in front of host governments, akin to what occurs in markets. NGOs will deliver aid as long as their participation constraint is satisfied: although, in the model, we normalize the outside option to zero, in reality outside options are such that host governments do not have to worry about attracting NGOs. Additionally, many NGOs perceive providing inefficient aid as an improvement over not providing aid at all: NGOs will enter the country if host countries give them the opportunity. By the late 1990’s, donors saw in the provision of services by NGOs an insufficient mean of promoting development. Poverty-alleviating services were only treating the symptoms, not the causes, of structural poverty; NGOs agreed that institutional change was required to achieve effective long-run development (Mansuri and Rao, 2012). Institutional change meant advocating for new rights for the poor, and curtailing privileges of the ruling elite. This shift toward advocacy was easier said than done: it bore the
risks of confronting the local authorities, of frightening donors concerned with diplomatic affairs, and required substantial coordination efforts among NGOs, in particular to agree upon a shared vision of institutional change. Even when NGOs overcome these institutional barriers, advocacy might not have the hoped influence on the local population. The message might be too weak, or too unconvincing, or outright wrong in the eyes of beneficiaries. For example, Boulding and Gibson (2009) finds that NGOs are conductive of political change in small and poor municipalities in Bolivia, which are politically disorganized, but not in larger, richer municipalities, in which trade unions and political parties out noise NGOs. The sensitivity to the message is captured in the model by parameter $\lambda$. Even if people are willing to listen, fear of repressive governments may deter civilians from demanding new rights. Among several examples, one could quote the assassination of 17 local workers of Action Contre la Faim (a French NGO) by the Sri Lankan military in 2006 (The Independent, 2006). The repressive capacity of the state is captured by parameter $\rho$.

Changing institutional arrangements injures the privileges of the established elite. In the model, the conflict of interests arising between the elite and intended beneficiaries over aid resources is very stylized. In it, the elite steal whatever beneficiaries do not appropriate. This exaggeration impersonates more subtle phenomena of embezzlement: nongovernmental aid reduces demands for accountability addressed to the state, redirecting public funds toward private interest; procedures in situ can impose that third part-contracts, for instance in construction, are procured by fraudulent enterprises; or corrupt officials might outright extort and blackmail NGO workers. A frequent practice is that local bureaucrats organize schemes to purchase alimentary aid at subsidized prices to resell it at market prices (Centre Franais de Recherche sur le Renseignement, 2007). Another way in which authorities can extract rents from nongovernmental aid is by allowing them to operate only where they will be less effective. A collective of NGOs operating in Zambia denounced the adoption of a bill prohibiting NGOs from operating in certain domains and geographical regions (Freedom House, 2014). The model predicts that governments will permit NGOs where they will be unable to promote institutional change, in line with what NGOs operating in Zambia fear. Similar stringent laws were adopted in Russia (The Economist, 2013), Venezuela (Human Rights Watch, 2013) and Egypt (The Guardian, 2010), among others. NGOs understand that coordination efforts can improve their prospects of achieving institutional change, by increasing their bargaining power. The intent of NGOs is to integrate in the debate at the High Level Forum on aid effectiveness. In 2011, an Open Forum for Civil Society Organizations adopted the Istanbul Principles (2011), which is a list of eight commitments addressed at improving the efficiency of nongovernmental aid. In the model, we explore how the adoption of such principles would affect the well-being of target populations. We apply our model to two principles: first, NGOs are encouraged by the principles to ‘focus on citizens’ ownership’ (parameter $\phi$); second, NGOs are called to ‘pursue partnerships with other NGOs’ (parameter $\gamma$). We show that the strict adherence to these principles does not necessarily improve the welfare of beneficiaries, because full coordination will reduce the
incentives of host governments to deliver permits to NGOs. Donor states have
the political grit to ask from recipient countries adequate regulation of the NGO
sector. German Chancellor Angela Merkel appealed Russian president Vladimir
Putin to give NGOs a chance’, following the crackdown of the sector in 2013;
this appeal remains unheard (Al Jazeera, 2013).

5 Conclusion

In this paper, we develop a model of the political economy of nongovernmental
organizations. We focus in the case in which NGOs deliver foreign aid in weakly
institutionalized countries. In a game-theoretical framework, players are a large
number of heterogeneous NGOs, the government of a developing country, and
the people living in this country. NGOs can take two actions: they provide
poverty-alleviating services, and they advocate the rights of the poor. Advocacy
encourages beneficiaries to demand better accountability to their government.
Because the benefits of advocacy have the characteristics of a public good, ad-
vocacy will be typically under-provided. NGOs can remedy the problems of free
riding by increasing efforts in coordination. However, these efforts threaten the
rents of corrupt officials, who will respond by cracking down on the operations
of NGOs. The costs of this crackdown can outweigh the benefits from investing
in advocacy, turning a situation where no one free rides in advocacy into an
undesirable outcome.

We strongly stylize the behaviour of the government and the local population
to focus on the NGO sector. The assumption that all government officials are
pure rent-seekers, or that the local population is a monolithic decision-taking unit
are certainly unrealistic. However, by presenting a reduced-form optimization
program for the local population and for the government, we are able to elaborate
further on the constraints and strategies adopted by NGOs while maintaining
analytical tractability.

Results of our model suggest that proponents of a bottom-up approach to
development should not go too far in dismissing the role of multilateral and
bilateral donors (Easterly, 2007; Banerjee and Duflo, 2011). While merits of
NGOs in the field can be plentiful, they require an enabling environment allows
their initiatives to thrive, while their actions are legitimately and democratically
monitored. Bilateral and multilateral donors have the diplomatic and economic
power capable of demanding that such enabling environment is set in place.

6 Appendix

Proof of Lemma 1 Replace the value of $n$ in (9) by its non-distribution con-
straint value $n = (s + v)/\bar{b}$. This yields

$$\lambda v s = 1 + \rho (s + v)/\bar{b}$$

(23)
Implicit differentiation of \( v \) by \( s \) and multiplication by \( s/v \) obtains:

\[
\varepsilon = \frac{dv}{ds} \frac{s}{v} = \frac{\lambda s - \rho s}{\lambda s + \theta v}
\]

Plugging back in (23) gives

\[
\varepsilon = \frac{1 + v\rho/b}{1 + v\rho/b + b\rho/b}
\]

which easily reduces to the desired equation using (6). □

**Comparative statics of the MB\((n)\) and MC\((n)\) curves holding \( n \) constant.**

We provide the comparative statics for the full model, i.e including parameters \( \overline{\phi} \) and \( \gamma \) included in the extensions. This addition does not affect the comparative statics of the basic model and diminishes redundant computations. In the second stage Nash equilibrium we obtain:

\[
s^* = \frac{1 + \rho \overline{\gamma}_n}{1 + \lambda + \rho \overline{\gamma}_n} \overline{b}n \quad \text{and} \quad v^* = \frac{\lambda}{1 + \lambda + \rho \overline{\gamma}_n} \overline{b}n
\]

6.1 **Comparative statics for MB\((n)\) = 1 − \( \omega \)**

Taking the logarithm of the third stage equilibrium condition (5) we obtain

\[
\ln(\omega) = \frac{(1 + \rho)\ln(\kappa) + \ln(s) + \lambda \ln(v) - \ln(1 + \rho)}{\rho}
\]

Partial derivatives yield

\[
\nabla \omega(\rho, \lambda, \overline{b}, \kappa, \gamma, \overline{\phi}) = \frac{1}{\rho} \begin{bmatrix}
\left( \frac{\partial s}{\partial \rho} \frac{1}{\lambda} + \lambda \frac{\partial v}{\partial \rho} \frac{1}{v} \right) & \left( \frac{\partial \ln(\kappa)}{\partial \rho} - \frac{1}{1 + \rho} - \frac{\ln(\omega)}{\rho} \right)
\end{bmatrix}
\]

By \( \overline{b}n = s + v \):

\[
\nabla \omega(\rho, \lambda, \overline{b}, \kappa, \gamma, \overline{\phi}) = \frac{1}{\rho} \begin{bmatrix}
\left( \frac{1}{s} - \lambda \frac{1}{v} \right) & \left( \frac{\partial \ln(\kappa)}{\partial \rho} - \frac{1}{1 + \rho} - \frac{\ln(\omega)}{\rho} \right)
\end{bmatrix}
\]
By \( \frac{s}{v} = \frac{1 + \rho \bar{\phi}^{n}}{\lambda} \)

\[
\nabla \omega(\rho, \lambda, \bar{b}, \kappa, \gamma, \bar{\phi}) = \frac{1}{\rho} \left[ -\frac{\rho \bar{\phi}^{n}}{1 + \gamma(n-1)} s \right]
\]

By \( \omega(a) = \kappa(1 + a)^{\frac{1}{1 + \rho}} \)

\[
\nabla \omega(\rho, \lambda, \bar{b}, \kappa, \gamma, \bar{\phi}) = \frac{1}{\rho} \left[ -\frac{\rho \bar{\phi}^{n}}{1 + \gamma(n-1)} s \right]
\]

By \( s^{*} = \frac{1 + \rho \bar{\phi}^{\frac{n}{1 + \gamma(n-1)}}}{1 + \lambda + \rho \bar{\phi}^{\frac{n}{1 + \gamma(n-1)}}} \ln \)

\[
\nabla \omega(\rho, \lambda, \bar{b}, \kappa, \gamma, \bar{\phi}) = \begin{bmatrix}
< 0 \\
> 0 \\
> 0 \\
> 0 \\
< 0
\end{bmatrix}
\]

\( \nabla M B(\rho, \lambda, \bar{b}, \kappa, \gamma, \bar{\phi}) = \begin{bmatrix}
> 0 \\
< 0 \\
< 0 \\
< 0 \\
> 0
\end{bmatrix} \)

6.2 Comparative statics for \( MC(n) = \frac{1 + \lambda \varepsilon}{1 + \rho + \lambda \varepsilon} \)

Proceeding like in the proof of lemma 1, we obtain the elasticity for the general case

\[
\varepsilon = \frac{1}{1 + \frac{1}{\rho \bar{\phi}^{\frac{n}{1 + \gamma(n-1)}}} + \frac{1}{1 + \lambda + \rho \bar{\phi}^{\frac{n}{1 + \gamma(n-1)}}}}
\]

23
The denominator in the second term of the denominator decreases in $\rho$ and $\phi$, and increases in $\gamma$. $\epsilon$ moves in the same direction. It is easy to verify that:

$$\nabla MC(\rho, \lambda, \bar{b}, \kappa, \gamma, \phi) = \begin{bmatrix} < 0 \\ > 0 \\ 0 \\ 0 \\ > 0 \\ < 0 \end{bmatrix}$$
References


