Infinite-horizon critical-level leximin principles: Axiomatizations and some general results*

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Abstract

This paper examines egalitarian evaluations of streams of (possibly different dimensional) utility vectors of generations. We propose three infinite-horizon variants of the critical-level leximin principle: the critical-level leximin social welfare relation (SWR), the critical-level leximin overtaking SWR, and the critical-level leximin catching-up SWR. It is shown that the critical-level leximin SWR is characterized by five axioms: finite anonymity, weak existence of critical levels, existence independence, strong Pareto, and Hammond equity. Further, the critical-level leximin overtaking and the critical-level leximin catching-up SWRs are characterized by adding consistency axioms. We prove these results by showing the general result that three of the five axioms: finite anonymity, weak existence of critical levels, and existence independence imply that an evaluation within a generation must be extended between generations. We also evaluate the three SWRs by using population ethics axioms.

Keywords: Intergenerational equity, Variable population social choice, Critical-level leximin principle, Population ethics

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1 Introduction

The standard analysis of intergenerational equity employs the framework of ranking infinite utility streams, where the well-being of each generation is represented

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by a single utility value, and as reviewed by Asheim (2010) and Lauwers (2014), many evaluation relations have been proposed and axiomatically characterized. This framework, however, cannot take into account (i) the diverse levels of well-being of individuals within a generation and (ii) demographic change across generations. Consequently, the evaluation relations established in this framework cannot be applied to economic growth models with endogenous population growth where an evaluation relation for infinite streams of utility vectors of generations is needed (see, e.g., Boucekkine and Fabbri (2003)).

Recent work by Kamaga (2016) presents the framework of ranking streams of utility vectors. This framework is an infinite-horizon extension of variable population social choice initiated by Blackorby and Donaldson (1984), and an intratemporally anonymous and finitely complete quasi-ordering for streams of utility vectors, which we call social welfare relation (SWR), is analyzed. Intratemporal anonymity means that the evaluation does not depend on the identities of individuals in each generation. Analyzing a finitely complete quasi-ordering instead of an ordering is due to that Paretian and intergenerationally anonymous orderings for infinite utility streams cannot be explicitly described (Dubey 2011; Lauwers 2010; Zame 2007) and this impossibility carries over to the current framework.

In Kamaga (2016), three (classes of) SWRs are introduced: the critical-level generalized utilitarian (CLGU) SWR, the critical-level generalized overtaking (CLGO) SWR, the critical-level generalized catching-up (CLGC) SWR. They are infinite-horizon variants of the CLGU ordering introduced by Blackorby and Donaldson (1984) in the finite-horizon framework of variable population social choice.[1] All of them apply the CLGU ordering to the heads of streams of utility vectors but they are different in the treatment of the tails of streams. The CLGU SWR applies the Suppes-Sen grading principle to each generation in the tails, while the CLGO and CLGC SWRs extend the use of the CLGU ordering to succeeding generations in the same way as the overtaking criterion due to von Weizsäcker (1965) and the catching-up criterion in Atsumi (1965) and von Weizsäcker (1965).

The purpose of this paper is to explore and axiomatically characterize egalitarian SWRs for streams of utility vectors. We propose three (classes of) SWRs called *critical-level leximin* (CLL), *critical-level leximin overtaking* (CLLO), *critical-level leximin catching-up* (CLLC) SWRs. They are infinite-horizon variants of the CLL ordering proposed by Blackorby, Bossert, and Donaldson (1996) in the finite-

[1]For the ciritical-level generalized utilitarian ordering, see also Blackorby, Bossert, and Donaldson (1995).

horizon framework of variable population social choice. The CLL SWR applies the CLL ordering to the heads of streams of utility vectors and uses the Suppes-Sen grading principle in each generation in the tails. The CLLO and CLLC SWRs consecutively applies the CLL ordering to the heads of streams.

We show that the CLL SWR is characterized by five axioms: strong Pareto (**SP**), finite anonymity (**FA**), weak existence of critical levels (**WECL**), existence independence (**EI**), and Hammond equity (**HE**). The first four axioms are used in Kamaga (2016) to characterize the CLGU SWR. The difference between the characterization of the CLL SWR and that of the CLGU SWR in Kamaga (2016) is that we use **HE** instead of the axiom called restricted continuity. We characterize the CLLO SWR by additionally using the three consistency axioms that Kamaga (2016) uses to characterize the CLGO and CLGC SWRs, namely, weak preference consistency (**WPC**), strong preference consistency (**SPC**), and indifference consistency (**IC**). Again, the difference between the characterizations of the CLLO and CLLC SWRs and those of the CLGO and CLGC SWRs in Kamaga (2016) is the use of **HE** in place of restricted continuity. Therefore, the difference between our three egalitarian SWRs and the three utilitarian SWRs in Kamaga (2016) can be ascribed to the two properties, **HE** and restricted continuity.

We prove these characterization results through general results identifying the classes of SWRs satisfying some of the axioms used in the characterizations. In particular, we show that (i) the class of all SWRs satisfying **FA**, **WECL**, and **EI** coincides with that of all SWRs that use an ordering on the set of variable dimensional vectors satisfying the corresponding properties of **FA**, **WECL**, and **EI** to evaluate the heads of streams of utility vectors and that (ii) the class of all SWRs that additionally satisfy **WPC** (resp. **SPC**) and **IC** coincides with that of all SWRs that include, as a subrelation, the overtaking (resp. catching-up) criterion associated with an ordering on the set of variable dimensional vectors satisfying the corresponding properties of **FA**, **WECL**, and **EI**. These general characterization results are useful in exploring possible extensions of the well-established orderings in the finite-horizon framework of variable population social choice in the current framework.[2]

We also evaluate the three SWRs by using population ethics axioms, especially focusing on whether the SWRs associated with a positive critical level are com-

[2]General analyses similar to ours are done by Asheim, d'Aspremont, and Banerjee (2010), Asheim and Banerjee (2010), d'Aspremont (2007), Kamaga and Kojima (2009), and Sakai (2010) to examine possible extensions of fixed-population social welfare orderings to the framework of ranking infinite utility streams.

patible with those axioms. We show that additionally imposing an infinite-horizon extension of the axiom requiring the avoidance of the repugnant conclusion due to Parfit (1976, 1982, 1984), the CLLO and CLLC SWRs associated with a positive critical level are characterized. Further, it is shown that using an infinite-horizon extension of the axiom of priority for lives worth living in Blackorby, Bossert, and Donaldson (2005) instead of the axiom of avoidance of the repugnant conclusion, the CLLO and CLLC SWRs associated with a non-negative critical level are characterized. This result means that the CLLO and CLLC SWRs associated with a positive critical level never imply the infinite-horizon version of the very sadistic conclusion in Arrhenius (2000, forthcoming). On the other hand, we will see that the CLLO and CLLC SWRs associated with a positive critical level imply an infinite-horizon version of the weak repugnant conclusion and violate a version of Parfit's (1984) mere addition principle.

The rest of the paper is organized as follows. Sect. 2 presents notation and basic definitions. Sect. 3.1 introduces the CLL SWR and provide its axiomatic characterization. The proof of the characterization result is presented in Sect. 3.2, where we provide general results identifying the class of SWRs satisfying some of the axioms used in the characterization. In Sect. 4.1, we introduce the CLLO and the CLLC SWRs and state their axiomatic characterizations. The proof of them are done by establishing general characterization results in Sect. 4.2. In Sect. 5, we evaluate the three SWRs by using population ethics axioms. Sect. 6 concludes the study.

2 Notation and definitions

Let $\mathbb{R}$ (resp. $\mathbb{R}_{++}$ and $\mathbb{R}_{--}$) be the set of all (resp. all positive and all negative) real numbers and $\mathbb{N}$ be the set of all positive integers. For all $n \in \mathbb{N}$, $\mathbf{1}_n$ is the vector consisting of n ones. The notation for the vector inequality is as follows: for all $n \in \mathbb{N}$ and all $(u_1, \ldots, u_n), (v_1, \ldots, v_n) \in \mathbb{R}^n$, $(u_1, \ldots, u_n) \geq (v_1, \ldots, v_n)$ if and only if $u_i \geq v_i$ for all $i = 1, \ldots, n$; and $(u_1, \ldots, u_n) > (v_1, \ldots, v_n)$ if and only if $(u_1, \ldots, u_n) \geq (v_1, \ldots, v_n)$ and $(u_1, \ldots, u_n) \neq (v_1, \ldots, v_n)$. Further, for all $(u_1, u_2, \ldots), (v_1, v_2, \ldots) \in \mathbb{R}^{\mathbb{N}}$, $(u_1, u_2, \ldots) \gg (v_1, v_2, \ldots)$ if and only if $u_i > v_i$ for all $i \in \mathbb{N}$. For any sets A and B, we write $A \subseteq B$ to mean that A is a subset of B and $A \subset B$ to mean $A \subseteq B$ and $A \neq B$. The empty set is denoted by $\varnothing$. Negation of a statement is indicated by the symbol $\neg$.

We consider the welfarist framework of infinite-horizon variable population social choice presented in Kamaga (2016). Let $\Omega = \cup_{n\in\mathbb{N}}\mathbb{R}^n$, and let $\Omega^{\mathbb{N}}$ be the set of all streams of utility vectors $\boldsymbol{u} = (\boldsymbol{u}^1, \boldsymbol{u}^2, \ldots)$. For all $\boldsymbol{u} \in \Omega$ and all $t \in \mathbb{N}$, $\mathbf{n}(\boldsymbol{u}^t)$ is the number of components in $\boldsymbol{u}^t$, and thus, $\boldsymbol{u}^t = \left(u^t_1, \ldots, u^t_{\mathbf{n}(\boldsymbol{u}^t)}\right)$. For all $\boldsymbol{u} \in \Omega$ and all $t \in \mathbb{N}$, we interpret $\boldsymbol{u}^t$ as the utility distribution among $\mathbf{n}(\boldsymbol{u}^t)$ individuals in t-th generation and we ignore the identities of individuals in each generation. This simplification does not affect the analysis since the binary relations we consider do not depend on the identities of individuals. We employ the convention in population ethics that a utility level of zero represents neutrality and a utility level above zero represents her/his life is worth living.[3]

For any $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ and any $t \in \mathbb{N}$, let $\boldsymbol{u}^{-t}$ denote $(\boldsymbol{u}^1, \ldots, \boldsymbol{u}^t) \in \Omega^t$ and $\boldsymbol{u}^{+t}$ denote $(\boldsymbol{u}^{t+1}, \boldsymbol{u}^{t+2}, \ldots) \in \Omega^{\mathbb{N}}$. Thus, $\boldsymbol{u} = (\boldsymbol{u}^{-t}, \boldsymbol{u}^{+t}) = (\boldsymbol{u}^{-(t-1)}, \boldsymbol{u}^t, \boldsymbol{u}^{+t})$. We refer to $\boldsymbol{u}^{-t}$ as the head of a stream of utility vectors and $\boldsymbol{u}^{+t}$ as the tail of a stream of utility vectors. For any $\boldsymbol{u}, \boldsymbol{v} \in \Omega$ and any $t \in \mathbb{N}$, we write $[\boldsymbol{u}^t, \boldsymbol{v}^t]$ as $[\boldsymbol{u}^t, \boldsymbol{v}^t] = \left(u^t_1, \ldots, u^t_{\mathbf{n}(\boldsymbol{u}^t)}, v^t_1, \ldots, v^t_{\mathbf{n}(\boldsymbol{v}^t)}\right) \in \Omega$. Extending this notation, for any $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ and any $t \in \mathbb{N}$, let $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^t]$ denote the vector in Ω defined by $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^t] = \left(u^1_1, \ldots, u^1_{\mathbf{n}(\boldsymbol{u}^1)}, \ldots, u^t_1, \ldots, u^t_{\mathbf{n}(\boldsymbol{u}^t)}\right)$.

A binary relation on $\Omega^{\mathbb{N}}$ is generically denoted by R. The asymmetric and symmetric parts of R is denoted by P and I, respectively. A binary relation on $\Omega^{\mathbb{N}}$ is quasi-ordering if it is reflexive and transitive. A binary relation R on $\Omega^{\mathbb{N}}$ is *intratemporally anonymous* if and only if, for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$, $\boldsymbol{u}I\boldsymbol{v}$ if, for all $t \in \mathbb{N}$, there exists a bijection $\pi^t : \{1, \ldots, \mathbf{n}(\boldsymbol{u}^t)\} \to \{1, \ldots, \mathbf{n}(\boldsymbol{v}^t)\}$ such that $\boldsymbol{u}^t = \left(v^t_{\pi^t(1)}, \ldots, v^t_{\pi^t(\mathbf{n}(\boldsymbol{u}^t))}\right)$. A binary relation R on $\Omega^{\mathbb{N}}$ is *finitely complete* if and only if $\boldsymbol{u}R\boldsymbol{v}$ or $\boldsymbol{v}R\boldsymbol{u}$ for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\boldsymbol{u}^{+t} = \boldsymbol{v}^{+t}$ for some $t \in \mathbb{N}$. An SWR on $\Omega^{\mathbb{N}}$ is an intratemporally anonymous and finitely complete quasi-ordering. Given binary relations R_A and R_B on $\Omega^{\mathbb{N}}$, we say that R_A is a *subrelation* of R_B if $I_A \subseteq I_B$ and $P_A \subseteq P_B$.

We also consider a binary relation on Ω, which we generically denote by $\succsim$. The asymmetric and symmetric parts of $\succsim$ are denoted by $\succ$ and $\sim$, respectively. A binary relation on Ω is a quasi-ordering if it is reflexive and transitive. A binary relation on Ω is an ordering if it is a complete quasi-ordering.

[3]For a discussion of neutrality and its normalization to zero, see Broome (1993).

3 Critical-level leximin SWR

3.1 Definition and axiomatic characterization

In this section, we introduce an infinite-horizon variant of the critical-level leximin ordering in Blackorby, Bossert, and Donaldson (1996, 2005), which we call *cirical-level leximin* SWR. Then, we state an axiomatic characterization of it. We prove the characterization result in Sect. 3.2 through the analysis of a general infinite-horizon-extension of an ordering on Ω.

Let us begin with the definition of the critical-level leximin ordering on Ω. For all $n \in \mathbb{N}$ and all $\mathbb{R}^n$, the leximin ordering $\succsim_L^n$ on $\mathbb{R}^n$ is defined as follows: for all $(u_1, \ldots, u_n), (v_1, \ldots, v_n) \in \mathbb{R}^n$, (i) $(u_1, \ldots, u_n) \succ_L^n (v_1, \ldots, v_n)$ if and only if there exists $m \leq n$ such that $u_{(m)} > v_{(m)}$ and $u_{(i)} = v_{(i)}$ for all $i < m$; and (ii) $(u_1, \ldots, u_n) \sim_L^n (v_1, \ldots, v_n)$ if and only if $u_{(i)} = v_{(i)}$ for all $i = 1, \ldots, n$, where $u_{(j)}$ and $v_{(j)}$ denote j-th smallest element in $(u_1, \ldots, u_n)$ and $(v_1, \ldots, v_n)$, ties being broken arbitrarily. Given $\alpha \in \mathbb{R}$, the critical-level leximin ordering $\succsim_{L,\alpha}$ associated with α is the ordering on defined as follows: for all $\boldsymbol{u}^t, \boldsymbol{v}^t \in \Omega$ with $n = \mathbf{n}(\boldsymbol{u}^t)$ and $\mathbf{n}(\boldsymbol{v}^t) = m$,

$$\boldsymbol{u}^t \succ_{L,\alpha} \boldsymbol{v}^t \Leftrightarrow \begin{cases} \boldsymbol{u}^t \succ_L^n [\boldsymbol{v}^t, \alpha \mathbf{1}_{n-m}] & \text{if } n \geq m \\ [\boldsymbol{u}^t, \alpha \mathbf{1}_{m-n}] \succ_L^m \boldsymbol{v}^t & \text{if } n < m, \end{cases} \tag{1a}$$

$$\boldsymbol{u}^t \sim_{L,\alpha} \boldsymbol{v}^t \Leftrightarrow \begin{cases} \boldsymbol{u}^t \sim_L^n [\boldsymbol{v}^t, \alpha \mathbf{1}_{n-m}] & \text{if } n \geq m \\ [\boldsymbol{u}^t, \alpha \mathbf{1}_{m-n}] \sim_L^m \boldsymbol{v}^t & \text{if } n < m. \end{cases} \tag{1b}$$

To present the definition of the critical-level leximin SWR on $\Omega^{\mathbb{N}}$, we define the quasi-ordering on Ω called the Suppes-Sen grading principle due to Sen (1970) and Suppes (1966). The Suppes-Sen grading principle $\succsim_S$ is the quasi-ordering on Ω defined as follows: for all $(u_1, \ldots, u_m), (v_1, \ldots, v_n) \in \Omega$, $(u_1, \ldots, u_m) \succsim_S (v_1, \ldots, v_n)$ if and only if $m = n$ and there exists a permutation μ on $\{1, \ldots, n\}$ such that $(u_1, \ldots, u_n) \geq (v_{\mu(1)}, \ldots, v_{\mu(n)})$. It is easy to check that, for all $(u_1, \ldots, u_n), (v_1, \ldots, v_n) \in \Omega$, (i) $(u_1, \ldots, u_n) \succ_S (v_1, \ldots, v_n)$ if and only if there exists a permutation μ on $\{1, \ldots, n\}$ such that $(u_1, \ldots, u_n) > (v_{\mu(1)}, \ldots, v_{\mu(n)})$, and (ii) $(u_1, \ldots, u_n) \sim_S (v_1, \ldots, v_n)$ if and only if there exists a permutation μ on $\{1, \ldots, n\}$ such that $(u_1, \ldots, u_n) = (v_{\mu(1)}, \ldots, v_{\mu(n)})$.

We now present the definition of the critical-level leximin SWR. It applies the critical-level leximin ordering $\succsim_{L,\alpha}$ associated with a given α to heads of streams of utility vectors by reconstructing the heads as vectors in Ω and uses the Suppes-

Sen grading principle $\succsim_S$ to check utility dominance in each generation in tails of streams. Given $\alpha \in \mathbb{R}$, the critical-level leximin SWR R_L associated with α is defined as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\begin{aligned} \boldsymbol{u} R_L \boldsymbol{v} \Leftrightarrow &\text{ there exists } T \in \mathbb{N} \text{ such that } \boldsymbol{u}^t \succsim_S \boldsymbol{v}^t \text{ for all } t > T \text{ and} \\ &[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{2}$$

It is easy to check that, for any $\alpha \in \mathbb{R}$, the associated $\succsim_L$ is an SWR. For any $\alpha \in \mathbb{R}$, the restriction of the associated CLL SWR to $\mathbb{R}^{\mathbb{N}}$ coincides with the leximin quasi-ordering for infinite utility streams introduced by Bossert, Sprumont, and Suzumura (2007).

As we will prove in the next section, the asymmetric and symmetric parts of R_L are characterized as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\begin{aligned} \boldsymbol{u} P_L \boldsymbol{v} \Leftrightarrow &\text{ there exists } T \in \mathbb{N} \text{ such that } \boldsymbol{u}^t \succsim_S \boldsymbol{v}^t \text{ for all } t > T \text{ and} \\ &[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T], \end{aligned} \tag{3a}$$

$$\begin{aligned} \boldsymbol{u} I_L \boldsymbol{v} \Leftrightarrow &\text{ there exists } T \in \mathbb{N} \text{ such that } \boldsymbol{u}^t \sim_S \boldsymbol{v}^t \text{ for all } t > T \text{ and} \\ &[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{3b}$$

By (3b), $\alpha \in \mathbb{R}$ is the unique critical level of utility for all $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ and for all $t \in \mathbb{N}$.

To present an axiomatic characterization of R_L, we consider five axioms for an SWR. We begin with the four axioms used in Kamaga (2016). The axiom of strong Pareto requires the evaluation must to be positively sensitive to individuals' utilities.

Strong Pareto (SP): For all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ such that $\mathbf{n}(\boldsymbol{u}^t) = \mathbf{n}(\boldsymbol{v}^t)$ for all $t \in \mathbb{N}$, if $\boldsymbol{u}^t \geq \boldsymbol{v}^t$ for all $t \in \mathbb{N}$ and there exists $t' \in \mathbb{N}$ such that $\boldsymbol{u}^{t'} > \boldsymbol{v}^{t'}$, then $\boldsymbol{u} P \boldsymbol{v}$.

The finite anonymity asserts the relative ranking of any two streams of utility vectors should be invariant with respect to reordering two generations.

Finite Anonymity (FA): For all $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}, \boldsymbol{z} \in \Omega^{\mathbb{N}}$, if there exist $t_1, t_2 \in \mathbb{N}$ such that $\boldsymbol{u}^{t_1} = \boldsymbol{w}^{t_2}$, $\boldsymbol{u}^{t_2} = \boldsymbol{w}^{t_1}$, $\boldsymbol{v}^{t_1} = \boldsymbol{z}^{t_2}$, $\boldsymbol{v}^{t_2} = \boldsymbol{z}^{t_1}$, and, for all $t \neq t_1, t_2$, $\boldsymbol{u}^t = \boldsymbol{w}^t$ and $\boldsymbol{v}^t = \boldsymbol{z}^t$, then $\boldsymbol{u} R \boldsymbol{v} \Leftrightarrow \boldsymbol{w} R \boldsymbol{z}$.

Since an SWR is finitely complete and transitive, imposing **FA** to an SWR is equivalent to requiring the following stronger property: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbf{N}}$, if there exist $t_1, t_2 \in \mathbb{N}$ such that $\boldsymbol{u}^{t_1} = \boldsymbol{v}^{t_2}$, $\boldsymbol{u}^{t_2} = \boldsymbol{v}^{t_1}$, and $\boldsymbol{u}^t = \boldsymbol{v}^t$ for all $t \neq t_1, t_2$, then $\boldsymbol{u} I \boldsymbol{v}$.

To present the next axiom, we define the notion of a critical level of utility. For any $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ and any $t \in \mathbb{N}$, $\alpha \in \mathbb{R}$ is said to be a critical level for $\boldsymbol{u}$ at the t-th generation if $\boldsymbol{u} I(\boldsymbol{u}^{-(t-1)}, [\boldsymbol{u}^t, \alpha], \boldsymbol{u}^{+t})$. That is, a critical level of utility is the utility level such that the addition of an individual with that utility level does not change the goodness of a stream of utility vectors. The following axiom asserts that a critical level of utility exists for at least one stream of utility vectors at at least one generation.

Weak Existence of Critical Levels (WECL): There exist $t \in \mathbb{N}$, $\alpha \in \mathbb{R}$, and $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ such that $\boldsymbol{u} I(\boldsymbol{u}^{-(t-1)}, [\boldsymbol{u}^t, \alpha], \boldsymbol{u}^{+t})$.

The existence independence axiom formalizes the independence property of an evaluation. It requires that the evaluation for streams with a common tail be independent of any addition of individuals at all generations.

Existence Independence (EI): For all $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \Omega^{\mathbb{N}}$, if there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t = \boldsymbol{v}^t$ for all $t > T$, then $\boldsymbol{u} R \boldsymbol{v} \Leftrightarrow ([\boldsymbol{u}^t, \boldsymbol{w}^t])_{t \in \mathbb{N}} \, R \, ([\boldsymbol{v}^t, \boldsymbol{w}^t])_{t \in \mathbb{N}}$.

Now, we introduce the equity axiom called Hammond equity, which is a reformulation of the axiom introduced by Hammond (1976) in the fixed population social choice.[4] It formalizes the equity property that an order-preserving change that diminishes the inequality of utilities between two conflicting individuals in some generation is socially preferable. Our version of the axiom requires this property hold for at least one generation.

Hammond Equity (HE): There exists $t \in \mathbb{N}$ such that for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\boldsymbol{u}^{t'} = \boldsymbol{v}^{t'}$ for all $t' \neq t$, if $\mathbf{n}(\boldsymbol{u}^t) = \mathbf{n}(\boldsymbol{v}^t)$ and there exist $i, j \in \{1, \dots, \mathbf{n}(\boldsymbol{u}^t)\}$ such that $v_i^t < u_i^t \leq u_j^t < v_j^t$ and $u_k^t = v_k^t$ for all $k \neq i, j$, then $\boldsymbol{u} R \boldsymbol{v}$.

The following theorem presents an axiomatic characterization of R_L in terms of subrelation. It shows that the class of all SWRs satisfying the above five axioms coincides with the class of all SWRs that include R_L associated with a given α as a subrelation.

Theorem 1. *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **SP**, **FA**, **WECL**, **EI**, and **HE** if and only if there exists $\alpha \in \mathbb{R}$ such that R_L associated with α is a subrelation of R.*

By Theorem 1, given $\alpha \in \mathbb{R}$, the associated R_L is the least element in the class of all SWRs satisfying the five axioms. Further, since R_L is finitely complete, this class of SWRs consists only of finitely complete SWRs. By Arrow's (1963) variant of Szpilrajn's (1930) lemma, there exists an ordering extension of R_L in

[4] See Blackorby, Bossert, and Donaldson (1996, 2002, 2005) for a version of the axiom formalized in the finite-horizon framework of variable population social choice.

this class. However, it is non-constructible object since the impossibility of explicit construction of a Paretian and finitely anonymous ordering for infinite utility streams proved by Dubey (2011), Lauwers (2010), and Zame (2007) carries over to the current framework.

3.2 Proof of Theorem 1: General characterizations

We prove Theorem 1 by using characterizations of classes of SWRs satisfying some of the axioms in the Theorem. We begin with the characterization of the class of all SWRs satisfying **FA**, **WECL**, and **EI**. To this end, we consider the three properties of an ordering $\succsim$ on Ω corresponding to **FA**, **WECL**, and **EI**:

Anonymity* (A*): For all $n \in \mathbb{N}$ and all $\boldsymbol{u}^1, \boldsymbol{v}^1 \in \mathbb{R}^n$, if there exists a permutation μ on $\{1, \ldots, n\}$ such that $\boldsymbol{u}^1 = \left(v^1_{\mu(1)}, \ldots, v^1_{\mu(n)}\right)$, then $\boldsymbol{u}^1 \sim \boldsymbol{v}^1$.

Weak Existence of Critical Levels* (WECL*): There exist $\boldsymbol{u}^1 \in \Omega$ and $\alpha \in \mathbb{R}$ such that $\boldsymbol{u}^1 \sim [\boldsymbol{u}^1, \alpha]$.

Existence Independence* (EI*): For all $\boldsymbol{u}^1, \boldsymbol{v}^1, \boldsymbol{w}^1 \in \Omega$, $\boldsymbol{u}^1 \succsim \boldsymbol{v}^1 \Leftrightarrow [\boldsymbol{u}^1, \boldsymbol{w}^1] \succsim [\boldsymbol{v}^1, \boldsymbol{w}^1]$.

To state the characterization result, we define a property of an SWR. Given an ordering $\succsim$ on Ω, we say that an SWR R on $\Omega^{\mathbb{N}}$ is the *extension* of $\succsim$ if for all $T \in \mathbb{N}$ and for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\boldsymbol{u}^{+T} = \boldsymbol{v}^{+T}$,

$$[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \Leftrightarrow \boldsymbol{u} R \boldsymbol{v}. \tag{4}$$

Note that if R is the extension of $\succsim$, R applies $\succsim$ to the heads of streams with a common tail.

The following proposition shows that the class of all SWRs satisfying **FA**, **WECL**, and **EI** coincides with that of all SWRs that apply an ordering $\succsim$ satisfying the three corresponding properties to the heads of streams with a common tail.

Proposition 1. *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **FA**, **WECL**, and **EI** if and only if there exists an ordering $\succsim$ on Ω satisfying **A***, **WECL***, and **EI*** such that R is the extension of $\succsim$.*

To prove Proposition 1, we use two lemmas. The first shows that **FA** (and intratemporal anonymity of an SWR R) together with **EI** imply that any transposition of individuals across generations does not change goodness of streams of utility vectors. The second shows that **WECL** together with **FA** and **EI** imply that the existence of a utility level which is a critical level for all streams and for all generations. The second result is the replication of Theorem 6.9 (i) in Blackorby, Bossert, and Donaldson (2005) obtained in the finite-horizon framework of variable-population social choice. We relegate the proof of them to Appendix.

Lemma 1. *Suppose that an SWR R on $\Omega^{\mathbb{N}}$ satisfies **FA** and **EI**. For all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\mathbf{n}(\boldsymbol{u}^t) = \mathbf{n}(\boldsymbol{v}^t)$ for all $t \in \mathbb{N}$, if there exist $t_1, t_2 \in \mathbb{N}$, $i \in \{1, \ldots, \mathbf{n}(\boldsymbol{u}^{t_1})\}$, and $j \in \{1, \ldots, \mathbf{n}(\boldsymbol{u}^{t_2})\}$ such that $u_i^{t_1} = v_j^{t_2}$, $u_j^{t_2} = v_i^{t_1}$, and $u_k^t = v_k^t$ for all $(k, t) \neq (i, t_1), (j, t_2)$, then $\boldsymbol{u} I \boldsymbol{v}$.*

Lemma 2. *If an SWR R on $\Omega^{\mathbb{N}}$ satisfies **FA**, **WECL**, and **EI**, then there exists $\alpha \in \mathbb{R}$ such that*

$$\boldsymbol{u} I^* (\boldsymbol{u}^{-(t-1)}, [\boldsymbol{u}^t, \alpha], \boldsymbol{u}^{+t}) \text{ for all } t \in \mathbb{N} \text{ and all } \boldsymbol{u} \in \Omega^{\mathbb{N}}. \tag{5}$$

Proof of Propisition 1. (If-part) To show that R satisfies **FA**, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that there exist $t_1, t_2 \in \mathbb{N}$ such that $\boldsymbol{u}^{t_1} = \boldsymbol{v}^{t_2}$, $\boldsymbol{u}^{t_2} = \boldsymbol{v}^{t_1}$, and $\boldsymbol{u}^t = \boldsymbol{v}^t$ for all $t \in \mathbb{N} \setminus \{t_1, t_2\}$. Let $T = \max\{t_1, t_2\}$. Since R is the extension of $\succsim$, we obtain $\boldsymbol{u} R \boldsymbol{v} \Leftrightarrow [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$. Since $\succsim$ satisfies **A***, we have $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$. Thus, $\boldsymbol{u} I \boldsymbol{v}$.

Next, we show that R satisfies **WECL**. Since $\succsim$ satisfies **WECL***, there exists $\boldsymbol{u}^1 \in \Omega$ and $\alpha \in \mathbb{R}$ such that $\boldsymbol{u}^1 \sim [\boldsymbol{u}^1, \alpha]$. Let $\boldsymbol{v} \in \Omega^{\mathbb{N}}$. Since R is the extension of $\succsim$, we obtain $(\boldsymbol{u}^1, \boldsymbol{v}^{+1}) I ([\boldsymbol{u}^1, \alpha], \boldsymbol{v}^{+1})$. Thus, R satisfies **WECL**.

Finally, to show that R satisfies **EI**, let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \Omega^{\mathbb{N}}$ and $T \in \mathbb{N}$, and suppose that $\boldsymbol{u}^t = \boldsymbol{v}^t$ for all $t > T$. Since R is the extension of $\succsim$, we obtain

$$\boldsymbol{u} R \boldsymbol{v} \Leftrightarrow [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$$

and

$$([\boldsymbol{u}^t, \boldsymbol{w}^t])_{t \in \mathbb{N}} R ([\boldsymbol{v}^t, \boldsymbol{w}^t])_{t \in \mathbb{N}} \Leftrightarrow ([\boldsymbol{u}^t, \boldsymbol{w}^t])_{t=1,\ldots,T} \succsim ([\boldsymbol{v}^t, \boldsymbol{w}^t])_{t=1,\ldots,T}.$$

Since $\succsim$ is transitive and it satisfies **A*** and **EI***, we obtain

$$[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \Leftrightarrow ([\boldsymbol{u}^t, \boldsymbol{w}^t])_{t=1,\ldots,T} \succsim ([\boldsymbol{v}^t, \boldsymbol{w}^t])_{t=1,\ldots,T}.$$

Thus, combining the above equivalence assertions, we obtain

$$\boldsymbol{u} R \boldsymbol{v} \Leftrightarrow ([\boldsymbol{u}^t, \boldsymbol{w}^t])_{t \in \mathbb{N}} R ([\boldsymbol{v}^t, \boldsymbol{w}^t])_{t \in \mathbb{N}}.$$

(Only-if-part) We first show the existence of an ordering $\succsim$ on Ω such that R is the extension of it. Given $\boldsymbol{w} \in \Omega^{\mathbb{N}}$, define $\Omega^{\mathbb{N}}_{\boldsymbol{w}}$ by $\Omega^{\mathbb{N}}_{\boldsymbol{w}} = \{\boldsymbol{u} \in \Omega^{\mathbb{N}} : \boldsymbol{u}^{+1} = \boldsymbol{w}^{+1}\}$. Since there exists a bijection from $\Omega^{\mathbb{N}}_{\boldsymbol{w}}$ to Ω, we can define the binary relation $\succsim$ on Ω as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}_{\boldsymbol{w}}$,

$$\boldsymbol{u}^1 \succsim \boldsymbol{v}^1 \Leftrightarrow \boldsymbol{u}R\boldsymbol{v}. \tag{6}$$

Since R is an SWR, $\succsim$ is an ordering. We show that $\succsim$ satisfies (4) if $T = 1$. To show this, let $\boldsymbol{z} \in \Omega^{\mathbb{N}}$ and $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}_{\boldsymbol{w}}$. Then, we obtain, by (6) and **EI**, that

$$\boldsymbol{u}^1 \succsim \boldsymbol{v}^1 \Leftrightarrow \boldsymbol{u}R\boldsymbol{v} \Leftrightarrow ([\boldsymbol{u}^t, \boldsymbol{z}^t])_{t\in\mathbb{N}} R ([\boldsymbol{v}^t, \boldsymbol{z}^t])_{t\in\mathbb{N}} \Leftrightarrow (\boldsymbol{u}^{-1}, \boldsymbol{z}^{+1}) R (\boldsymbol{v}^{-1}, \boldsymbol{z}^{+1}).$$

Thus, $\succsim$ satisfies (4) if $T = 1$.

Next, we show that $\succsim$ satisfies (4) for $T > 1$. Let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and $T \in \mathbb{N}\backslash\{1\}$ and suppose $\boldsymbol{u}^{+T} = \boldsymbol{v}^{+T}$. Let $\ell(\boldsymbol{u})$ denote $\ell(\boldsymbol{u}) = \sum_{t=1}^{T} \mathbf{n}(\boldsymbol{u}^t)$ for all $\boldsymbol{u} \in \Omega^{\mathbb{N}}$. By Lemma 2, there exists $\alpha \in \mathbb{R}$ that satisfies (5). Define $\bar{\boldsymbol{u}}, \bar{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ by $\bar{\boldsymbol{u}}^{+T} = \bar{\boldsymbol{v}}^{+T} = \boldsymbol{u}^{+T}$,

$$\bar{\boldsymbol{u}}^t = [\boldsymbol{u}^t, \alpha \mathbf{1}_{\ell(\boldsymbol{u})-\mathbf{n}(\boldsymbol{u}^t)}] \text{ for all } t \leq T,$$

and

$$\bar{\boldsymbol{v}}^1 = [\boldsymbol{v}^t, \alpha \mathbf{1}_{\ell(\boldsymbol{v})-\mathbf{n}(\boldsymbol{v}^1)}] \text{ and } \bar{\boldsymbol{v}}^t = [\boldsymbol{v}^t, \alpha \mathbf{1}_{\ell(\boldsymbol{u})-\mathbf{n}(\boldsymbol{v}^t)}] \text{ for all } t = 2, \ldots, T.$$

By (5) and the transitivity of R, $\boldsymbol{u}I\bar{\boldsymbol{u}}$ and $\boldsymbol{v}I\bar{\boldsymbol{v}}$. Thus, by transitivity,

$$\boldsymbol{u}R\boldsymbol{v} \Leftrightarrow \bar{\boldsymbol{u}}R\bar{\boldsymbol{v}}. \tag{7}$$

Next, define $\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ as follows: $\tilde{\boldsymbol{u}}^{+T} = \tilde{\boldsymbol{v}}^{+T} = \boldsymbol{u}^{+T}$,

$$\tilde{\boldsymbol{u}}^1 = [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \text{ and } \tilde{\boldsymbol{v}}^1 = [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T],$$

and

$$\tilde{\boldsymbol{u}}^t = \tilde{\boldsymbol{v}}^t = \alpha \mathbf{1}_{\ell(\boldsymbol{u})} \text{ for all } t = 2, \ldots, T.$$

By Lemma 1 and the transitivity of R, we obtain $\bar{\boldsymbol{u}}I\tilde{\boldsymbol{u}}$ and $\bar{\boldsymbol{v}}I\tilde{\boldsymbol{v}}$. Thus, by transitivity,

$$\bar{\boldsymbol{u}}R\bar{\boldsymbol{v}} \Leftrightarrow \tilde{\boldsymbol{u}}R\tilde{\boldsymbol{v}}. \tag{8}$$

Further, by the definitions of $\tilde{\boldsymbol{u}}$ and $\tilde{\boldsymbol{v}}$, we obtain

$$\tilde{\boldsymbol{u}}R\tilde{\boldsymbol{v}} \Leftrightarrow [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \tag{9}$$

Combining (7), (8), and (9), we complete the proof that $\succsim$ satisfies (4). Thus, R is the extension of $\succsim$.

Finally, since R is the extension of $\succsim$ and intratemporally anonymous and it satisfies **EI**, $\succsim$ satisfies **A*** and **EI***. Further, $\succsim$ satisfies **WECL*** since R is transitive and it satisfies **WECL**. □

Next, we present the characterization of all SWRs satisfying **SP** in addition to **FA**, **WECL**, and **EI**. To this end, we define a general infinite-horizon extension of an ordering $\succsim$ on Ω in the way as considered for R_L. Given an ordering $\succsim$ on Ω, we define the binary relation $R_{\succsim}$ on $\Omega^{\mathbb{N}}$ associated with $\succsim$ as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\begin{aligned} \boldsymbol{u} R_{\succsim} \boldsymbol{v} \Leftrightarrow & \text{ there exists } T \in \mathbb{N} \text{ such that } \boldsymbol{u}^t \succsim_S \boldsymbol{v}^t \text{ for all } t > T \text{ and} \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{10}$$

Note that R_L associated with a given α is $R_{\succsim}$ associated with $\succsim_{L,\alpha}$. While R_L is an SWR, there is no guarantee that $R_{\succsim}$ associated with an arbitrary ordering $\succsim$ is an SWR. To address this problem, we consider the property of an ordering $\succsim$ on Ω that corresponds to **SP**.

Strong Pareto* (SP*): For all $n \in \mathbb{N}$ and all $\boldsymbol{u}^1, \boldsymbol{v}^1 \in \mathbb{R}^n$, if $\boldsymbol{u}^1 \geq \boldsymbol{v}^1$ and $\boldsymbol{u}^1 \neq \boldsymbol{v}^1$, then $\boldsymbol{u}^1 \succ \boldsymbol{v}^1$.

The following lemma shows that $R_{\succsim}$ is well defined as an SWR if $\succsim$ satisfies **SP***, **A***, and **EI***. It also provides the characterization of the asymmetric and symmetric parts of $R_{\succsim}$. The proof is relegated to Appendix.

Lemma 3. *Let $\succsim$ be an ordering on Ω satisfying **SP***, **A***, and **EI***. Then, $R_{\succsim}$ is an SWR on $\Omega^{\mathbb{N}}$ and for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,*

$$\begin{aligned} \boldsymbol{u} P_{\succsim} \boldsymbol{v} \Leftrightarrow & \textit{ there exists } T \in \mathbb{N} \textit{ such that } \boldsymbol{u}^t \succsim_S \boldsymbol{v}^t \textit{ for all } t > T \textit{ and} \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T], \end{aligned} \tag{11a}$$

$$\begin{aligned} \boldsymbol{u} I_{\succsim} \boldsymbol{v} \Leftrightarrow & \textit{ there exists } T \in \mathbb{N} \textit{ such that } \boldsymbol{u}^t \sim_S \boldsymbol{v}^t \textit{ for all } t > T \textit{ and} \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{11b}$$

In the following proposition, we present a characterization of the class of all SWRs that satisfy **SP**, **FA**, **WECL**, and **EI**. It shows that this class coincides with the class of all SWRs that include $R_{\succsim}$ associated with an ordering $\succsim$ satisfying the corresponding properties as a subrelation.

Proposition 2. *An SWR R on $\Omega^{\mathbb{N}}$ satisfies* ***SP****,* ***FA****,* ***WECL****, and* ***EI*** *if and only if there exists an ordering $\succsim$ on Ω satisfying* ***SP*****, **A***, **WECL***, and* ***EI***** such that $R_{\succsim}$ is a subrelation of R.*

To prove the proposition, we use the following lemma. The proof of the lemma is relegated to Appendix.

Lemma 4. *Let $\succsim$ be an ordering on Ω satisfying* ***SP*****,* ***A*****, and* ***EI*****. If $R_{\succsim}$ is a subrelation of an SWR R on $\Omega^{\mathbb{N}}$, then R is the extension of $\succsim$.*

Proof of Proposition 2. (If-part) By Lemma 4, R is the extension of $\succsim$. By Proposition 1, R satisfies **FA**, **WECL**, and **EI**. Since $\succsim$ satisfies **SP*** and $R_{\succsim}$ is a subrelation of R, it follows from (11a) that R satisfies **SP**.

(Only-if-part) By Proposition 1, there exists an ordering $\succsim$ on Ω satisfying **A***, **WECL***, and **EI*** such that R is the extension of $\succsim$. Since R satisfies **SP**, it follows from (4) that $\succsim$ satisfies **SP***. We show that $R_{\succsim}$ associated with $\succsim$ is a subrelation of R. To show that $P_{\succsim} \subseteq P$, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose $\boldsymbol{u} P_{\succsim} \boldsymbol{v}$. By (11a), there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t \succsim_S \boldsymbol{v}^t$ for all $t > T$ and $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$. Let $\boldsymbol{w} = (\boldsymbol{u}^{-T}, \boldsymbol{v}^{+T})$. Since R is an SWR and it satisfies **SP**, $\boldsymbol{u} R \boldsymbol{w}$. Since R is the extension of $\succsim$, we obtain $\boldsymbol{w} P \boldsymbol{v}$. By transitivity, $\boldsymbol{u} P \boldsymbol{v}$. Next, to show that $I_{\succsim} \subseteq I$, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose $\boldsymbol{u} I_{\succsim} \boldsymbol{v}$. By (11b), there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t \sim_S \boldsymbol{v}^t$ for all $t > T$ and $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$. Let $\boldsymbol{w} = (\boldsymbol{u}^{-T}, \boldsymbol{v}^{+T})$. Since R is intratemporally anonymous, $\boldsymbol{u} I \boldsymbol{w}$. Since R is the extension of $\succsim$, we obtain $\boldsymbol{w} I \boldsymbol{v}$. By transitivity, $\boldsymbol{u} I \boldsymbol{v}$. □

Proof of Theorem 1. (If-part) Suppose that R_L associated with $\alpha \in \mathbb{R}$ is a subrelation of R. Since $\succsim_{L,\alpha}$ satisfies **SP***, **A***, **WECL***, and **EI***, it follows from Proposition 2 that R satisfies **SP**, **FA**, **WECL**, and **EI**. It is easy to show that R satisfies **HE**, and we omit the proof of it.

(Only-if-part) By Proposition 2, there exists an ordering $\succsim$ on Ω satisfying **SP***, **A***, **WECL***, and **EI*** such that $R_{\succsim}$ is a subrelation of R. Further, since R satisfies **HE** and **FA** and it is transitive, $\succsim$ satisfies the following property corresponding to **HE**.

Hammond Equity* **(HE*****)**: For all $n \in \mathbb{N}$ and all $\boldsymbol{u}^1, \boldsymbol{v}^1 \in \mathbb{R}^n$, if there exists $i, j \in \{1, \ldots, n\}$ such that $v_i^1 < u_i^1 < u_j^1 < v_j^1$ and $u_k^1 = v_k^1$ for all $k \neq i, j$, then $\boldsymbol{u}^1 \succsim \boldsymbol{v}^1$.

By Theorem 6. 13 in Blackorby, Bossert, and Donaldson (2005), if an ordering $\succsim$ on Ω satisfies **SP***, **A***, **WECL***, **EI***, and **HE***, then there exists $\alpha \in \mathbb{R}$ such that $\succsim = \succsim_{L,\alpha}$. □

4 Extensions by overtaking criteria

4.1 Definitions and axiomatic characterizations

In this section, we present the extensions of R_L by using the overtaking criterion due to von Weizsäcker (1965) and the catching-up criterion in Atsumi (1965) and von Weizsäcker (1965), which we call *critical-level leximin overtaking* SWR and *critical-level leximin catshing-up* SWR. These SWR can compare (not all but some) streams of utility vectors even if they have different population sizes in the tails. We provide axiomatic characterizations of them by adding consistency axioms to the set of axioms in Theorem 1. We prove the characterization results in Sect. 4.2 where we characterize generalized overtaking and catching-up SWRs associated with an ordering on Ω.

The critical-level leximin overtaking SWR consecutively applies the critical-level leximin ordering to the heads of streams of utility vectors in the manner of the overtaking criterion. Given $\alpha \in \mathbb{R}$, the critical-level leximin overtaking SWR R_L^O associated with α is defined as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\boldsymbol{u} P_L^O \boldsymbol{v} \Leftrightarrow \text{there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \quad [\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succ_{L,\alpha} [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T], \tag{12a}$$

$$\boldsymbol{u} I_L^O \boldsymbol{v} \Leftrightarrow \text{there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \quad [\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \sim_{L,\alpha} [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]. \tag{12b}$$

We will verify R_L^O is well-defined as an SWR in the next section. For any $\alpha \in \mathbb{R}$, the restriction of the associated CLLO SWR to $\mathbb{R}^{\mathbb{N}}$ coincides with the leximin version of the overtaking criterion, called W-leximin quasi-ordering, for infinite utility streams introduced by Asheim and Tungodden (2004).

The critical-level leximin catching-up SWR consecutively applies the critical-level leximin ordering to the heads of streams of utility vectors in the manner of the catching-up criterion. Given $\alpha \in \mathbb{R}$, the critical-level leximin catching-up SWR R_L^C associated with α is defined as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\boldsymbol{u} R_L^C \boldsymbol{v} \Leftrightarrow \text{there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \quad [\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succsim_{L,\alpha} [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]. \tag{13}$$

For any $\alpha \in \mathbb{R}$, the restriction of the associated CLLC SWR to $\mathbb{R}^{\mathbb{N}}$ coincides with the leximin version of the catching-up criterion, called S-leximin quasi-ordering, for infinite utility streams in Asheim and Tungodden (2004).

In the next section, we will show the asymmetric and symmetric parts of R^C_L are characterized as follows: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\left.\begin{aligned} &\boldsymbol{u} P^C_L \boldsymbol{v} \Leftrightarrow \text{there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \\ &\qquad [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \\ &\qquad \text{and for all } T' \in \mathbb{N}, \text{ there exists } T > T' \text{ such that} \\ &\qquad [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]; \end{aligned}\right\} \tag{14a}$$

$$\begin{aligned} &\boldsymbol{u} I^C_L \boldsymbol{v} \Leftrightarrow \text{there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \\ &\qquad [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{14b}$$

To provide axiomatic characterizations of R^O_L and R^C_L, we consider the three consistency axioms presented in Kamaga (2016). The first two axioms assert, in weak and strong forms, that the strict preference relation of the evaluation must be consistent with the evaluations obtained for streams with a common tail.

Weak Preference Consistency (WPC): For all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$, if $(\boldsymbol{u}^{-t}, \boldsymbol{w}^{+t}) P (\boldsymbol{v}^{-t}, \boldsymbol{w}^{+t})$ for all $t \in \mathbb{N}$ and all $\boldsymbol{w} \in \Omega^{\mathbb{N}}$, then $\boldsymbol{u} P \boldsymbol{v}$.

Strong Preference Consistency (SPC): For all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$, if, for all $\boldsymbol{w} \in \Omega^{\mathbb{N}}$, $(\boldsymbol{u}^{-t}, \boldsymbol{w}^{+t}) R (\boldsymbol{v}^{-t}, \boldsymbol{w}^{+t})$ for all $t \in \mathbb{N}$ and, for all $t' \in \mathbb{N}$, there exists $t > t'$ such that $(\boldsymbol{u}^{-t}, \boldsymbol{w}^{+t}) P (\boldsymbol{v}^{-t}, \boldsymbol{w}^{+t})$, then $\boldsymbol{u} P \boldsymbol{v}$.

Note that **SPC** is stronger than **WPC** since the former allows weak preference relations in its premise.

The next axiom formalizes the consistency for the indifference relation of the evaluation in the same way as **WPC**.

Indifference Consistency (IC): For all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$, if $(\boldsymbol{u}^{-t}, \boldsymbol{w}^{+t}) I (\boldsymbol{v}^{-t}, \boldsymbol{w}^{+t})$ for all $t \in \mathbb{N}$ and all $\boldsymbol{w} \in \Omega^{\mathbb{N}}$, then $\boldsymbol{u} I \boldsymbol{v}$.

The following theorem shows that, adding **WPC** and **IC** to the axioms in Theorem 1, the CLLO SWR is characterized and that, if we strengthen **WPC** to **SPC**, the CLLC SWR is characterized.

Theorem 2. (i) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **SP**, **FA**, **WECL**, **EI**, **HE**, **WPC**, and **IC** if and only if there exists $\alpha \in \mathbb{R}$ such that R^O_L associated with α is a subrelation of R.*

(ii) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **SP**, **FA**, **WECL**, **EI**, **HE**, **SPC**, and **IC** if and only if there exists $\alpha \in \mathbb{R}$ such that R_L^C associated with α is a subrelation of R.*

4.2 Proof of Theorem 2: General characterizations

Given an ordering $\succsim$ on Ω, we define the overtaking criterion associated with $\succsim$ as the following binary relation $R^O_{\succsim}$: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\begin{aligned} \boldsymbol{u} P^O_{\succsim} \boldsymbol{v} \Leftrightarrow & \text{ there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T], \end{aligned} \tag{15a}$$

$$\begin{aligned} \boldsymbol{u} I^O_{\succsim} \boldsymbol{v} \Leftrightarrow & \text{ there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{15b}$$

As shown in the following lemma, $R^O_{\succsim}$ is well defined as an SWR on $\Omega^{\mathbb{N}}$ if $\succsim$ is an ordering satisfying $\mathbf{A}^*$ and $\mathbf{EI}^*$. We relegate the proof to Appendix.

Lemma 5. *Let $\succsim$ be an ordering on Ω satisfying $\boldsymbol{A}^*$ and $\boldsymbol{EI}^*$. Then, $R^O_{\succsim}$ is an SWR.*

Next, given an ordering $\succsim$ on Ω, we define the catching-up criterion associated with $\succsim$ as the following binary relation $R^C_{\succsim}$ on $\Omega^{\mathbb{N}}$: for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,

$$\begin{aligned} \boldsymbol{u} R^C_{\succsim} \boldsymbol{v} \Leftrightarrow & \text{ there exists } T^* \in \mathbb{N} \text{ such that, for all } T \geq T^* \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{16}$$

The following lemma shows that $R^C_{\succsim}$ is an SWR if $\succsim$ is an ordering satisfying $\mathbf{A}^*$ and $\mathbf{EI}^*$. Further, it presents the characterizations of the asymmetric and symmetric parts of $R^C_{\succsim}$. The proof is relegated to Appendix.

Lemma 6. *Let $\succsim$ be an ordering on Ω satisfying $\boldsymbol{A}^*$ and $\boldsymbol{EI}^*$. Then, $R^C_{\succsim}$ is an SWR and for all $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$,*

$$\left.\begin{aligned} \boldsymbol{u} P^C_{\succsim} \boldsymbol{v} \Leftrightarrow & \textit{ there exists } T^* \in \mathbb{N} \textit{ such that, for all } T \geq T^* \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \\ & \textit{and for all } T' \in \mathbb{N}, \textit{ there exists } T > T' \textit{ such that} \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]; \end{aligned}\right\} \tag{17a}$$

$$\begin{aligned} \boldsymbol{u} I^C_{\succsim} \boldsymbol{v} \Leftrightarrow & \textit{ there exists } T^* \in \mathbb{N} \textit{ such that, for all } T \geq T^* \\ & [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]. \end{aligned} \tag{17b}$$

The following proposition presents characterizations of the classes of all SWRs that satisfy (i) **WPC** and **IC** in addition to the axioms in Proposition 1 and (ii) **SPC** instead of **WPC**. It shows that these classes coincide with the classes of all SWRs that include, respectively, $R^O_{\succsim}$ and $R^C_{\succsim}$ associated with $\succsim$ satisfying $\mathbf{A}^*$, $\mathbf{WECL}^*$, and $\mathbf{EI}^*$ as a subrelation.

Proposition 3. (i) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **FA**, **WECL**, **EI**, **WPC**, and **IC** if and only if there exists an ordering $\succsim$ on Ω satisfying $\mathbf{A}^*$, $\mathbf{WECL}^*$, and $\mathbf{EI}^*$ such that $R^O_{\succsim}$ is a subrelation of R.*

(ii) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **FA**, **WECL**, **EI**, **SPC**, and **IC** if and only if there exists an ordering $\succsim$ on Ω satisfying $\mathbf{A}^*$, $\mathbf{WECL}^*$, and $\mathbf{EI}^*$ such that $R^C_{\succsim}$ is a subrelation of R.*

To prove Proposition 3, we use the following lemma. We relegate the proof to Appendix.

Lemma 7. *Let $\succsim$ be an ordering on Ω satisfying $\mathbf{EI}^*$. If $R^O_{\succsim}$ is a subrelation of an SWR R on $\Omega^{\mathbb{N}}$, then R is the extension of $\succsim$.*

Proof of Proposition 3. (If-part of (i)) By Lemma 7, R is the extension of $\succsim$. By Proposition 1, R satisfies **FA**, **WECL**, and **EI**. To show that R satisfies **WPC**, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that $(\boldsymbol{u}^{-t}, \boldsymbol{w}^{+t})P(\boldsymbol{v}^{-t}, \boldsymbol{w}^{+t})$ for all $t \in \mathbb{N}$ and all $\boldsymbol{w} \in \Omega^{\mathbb{N}}$. Since R is the extension of $\succsim$, $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succ [\boldsymbol{u}^1, \dots, \boldsymbol{u}^T]$ for all $T \in \mathbb{N}$. By (15a), $\boldsymbol{u}P^O_{\succsim}\boldsymbol{v}$. Since $R^O_{\succsim}$ is a subrelation of R, we obtain $\boldsymbol{u}P\boldsymbol{v}$. By using (15b) instead of (15a), we can show that R satisfies **IC**, and we omit the proof of it.

(If-part of (ii)) Since $R^O_{\succsim}$ is a subrelation of $R^C_{\succsim}$, R satisfies **FA**, **WECL**, **EI**, and **IC**. By the same argument as the proof of the if-part of (i), we can show that R satisfies **SPC** by using (17a) instead of (15a). Thus, we omit the proof of it.

(Only-if-part of (i)) By Proposition 1, there exists an ordering $\succsim$ on Ω satisfying $\mathbf{A}^*$, $\mathbf{WECL}^*$, and $\mathbf{EI}^*$ such that R is the extension of $\succsim$. We show that $R^O_{\succsim}$ associated with $\succsim$ is a subrelation of R. To show that $P^O_{\succsim} \subseteq P$, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose $\boldsymbol{u}P^O_{\succsim}\boldsymbol{v}$. By (15a), there exists $T^* \in \mathbb{N}$ such that, for all $T \geq T^*$, $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]$. By Lemma 2, there exists $\alpha \in \mathbb{R}$ satisfying (5). Define $\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ by

$$\begin{cases} \tilde{\boldsymbol{u}}^1 = \boldsymbol{u}^1,\ \tilde{\boldsymbol{u}}^t = [\boldsymbol{u}^t, \alpha] \text{ for all } t \in \{2, \dots, T^*\}, \text{ and } \tilde{\boldsymbol{u}}^{+T^*} = \boldsymbol{u}^{+T^*}; \\ \tilde{\boldsymbol{v}}^1 = \boldsymbol{v}^1,\ \tilde{\boldsymbol{v}}^t = [\boldsymbol{v}^t, \alpha] \text{ for all } t \in \{2, \dots, T^*\}, \text{ and } \tilde{\boldsymbol{v}}^{+T^*} = \boldsymbol{v}^{+T^*}. \end{cases}$$

Since R is transitive, we obtain by (5) that $\tilde{\boldsymbol{u}}I\boldsymbol{u}$ and $\tilde{\boldsymbol{v}}I\boldsymbol{v}$. We next define $\bar{\boldsymbol{u}}, \bar{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$

by

$$\begin{cases} \bar{\boldsymbol{u}}^1 = [\boldsymbol{u}^1, \dots, \boldsymbol{u}^{T^*}],\ \bar{\boldsymbol{u}}^t = \alpha \text{ for all } t = 2, \dots, T^*, \text{ and } \bar{\boldsymbol{u}}^{+T^*} = \boldsymbol{u}^{+T^*}; \\ \bar{\boldsymbol{v}}^1 = [\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T^*}],\ \bar{\boldsymbol{v}}^t = \alpha \text{ for all } t = 2, \dots, T^*, \text{ and } \bar{\boldsymbol{v}}^{+T^*} = \boldsymbol{v}^{+T^*}. \end{cases}$$

Since $\succsim$ satisfies **A***, $[\tilde{\boldsymbol{u}}^1, \dots, \tilde{\boldsymbol{u}}^{T^*}] \sim [\bar{\boldsymbol{u}}^1, \dots, \bar{\boldsymbol{u}}^{T^*}]$ and $[\tilde{\boldsymbol{v}}^1, \dots, \tilde{\boldsymbol{v}}^{T^*}] \sim [\bar{\boldsymbol{v}}^1, \dots, \bar{\boldsymbol{v}}^{T^*}]$. Since R is the extension of $\succsim$, we obtain $\tilde{\boldsymbol{u}} I \bar{\boldsymbol{u}}$ and $\tilde{\boldsymbol{v}} I \bar{\boldsymbol{v}}$. Since R is transitive, we obtain $\boldsymbol{u} R \boldsymbol{v} \Leftrightarrow \bar{\boldsymbol{u}} R \bar{\boldsymbol{v}}$. We show $\bar{\boldsymbol{u}} P \bar{\boldsymbol{v}}$ to prove $P^O_{\succsim} \subseteq P$. Recall that $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]$ for all $T \geq T^*$. Thus, $\bar{\boldsymbol{u}}^1 \succ \bar{\boldsymbol{v}}^1$. Since $\succsim$ satisfies **EI***, we obtain $[\bar{\boldsymbol{u}}^1, \dots, \bar{\boldsymbol{u}}^T] \succ [\bar{\boldsymbol{v}}^1, \dots, \bar{\boldsymbol{v}}^T]$ for all $T = 2, \dots, T^*$. Further, since $\succsim$ satisfies **A*** and **EI*** and it is transitive, we obtain $[\bar{\boldsymbol{u}}^1, \dots, \bar{\boldsymbol{u}}^T] \succ [\bar{\boldsymbol{v}}^1, \dots, \bar{\boldsymbol{v}}^T]$ for all $T > T^*$. Since R is the extension of $\succsim$, we obtain $(\bar{\boldsymbol{u}}^{-T}, \boldsymbol{w}^{+T}) P (\bar{\boldsymbol{v}}^{-T}, \boldsymbol{w}^{+T})$ for all $T \in \mathbb{N}$ and all $\boldsymbol{w} \in \Omega^{\mathbb{N}}$. By **WPC**, $\bar{\boldsymbol{u}} P \bar{\boldsymbol{v}}$. We can show $I^O_{\succsim} \subseteq I$ by using (15b) and **IC** instead of (15a) and **WPC**. Thus, we omit the proof of it.

(Only-if-part of (ii)) By the same argument as the proof of the only-if-part of (i), we can show that $R^C_{\succsim}$ associated with $\succsim$ is a subrelation of R. Since $I^O_{\succsim} = I^C_{\succsim}$, it suffices to show $P^C_{\succsim} \subseteq P$. This can be shown by using (17a) and **SPC** instead of (15a) and **WPC**. Thus, we omit the proof of it. □

We state the consequence of adding **SP** to the set of axioms in Proposition 3 as the following corollary.

Corollary 1. (i) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **SP**, **FA**, **WECL**, **EI**, **WPC**, and **IC** if and only if there exists an ordering $\succsim$ on Ω satisfying **SP***, **A***, **WECL***, and **EI*** such that $R^O_{\succsim}$ is a subrelation of R.*

(ii) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **SP**, **FA**, **WECL**, **EI**, **SPC**, and **IC** if and only if there exists an ordering $\succsim$ on Ω satisfying **SP***, **A***, **WECL***, and **EI*** such that $R^C_{\succsim}$ is a subrelation of R.*

We now prove Theorem 2.

Proof of Theorem 2. First, we prove the if-parts of (i) and (ii). Since R_L is a subrelation of R^O_L and R^C_L, it follows from Theorem 1, R satisfies **SP**, **FA**, **WECL**, **EI**, and **HE**. Thus, by Proposition 3, R satisfies **WPC** and **IC** if R^O_L is a subrelation of R, and R satisfies **SPC** and **IC** if R^C_L is a subrelation of R.

Next, we prove the only-if-part of (i). By Theorem 1, there exists $\alpha \in \mathbb{R}$ such that R_L associated with α is a subrelation of R. Thus, R is the extension of $\succsim_{L,\alpha}$. On the other hand, by Proposition 3 (i), there exists an $\succsim$ ordering on Ω satisfying

EI* such that $R^O_{\succsim}$ is a subrelation of R. Since R is the extension of $\succsim_{L,\alpha}$, we obtain by Lemma 7 that $\succsim = \succsim_{L,\alpha}$.

By the same argument using Proposition 3 (ii) instead of Proposition 3 (i), we can prove the only-if-part of (ii). Thus, we omit the proof of it. □

5 Population ethics and positive critical levels

In this section, we evaluate R^O_L and R^C_L associated with a positive critical level by using some population ethics properties for an SWR. We show that R^O_L and R^C_L associated with a positive critical level are characterized by the axiom of avoidance of the repugnant conclusion. Further, we will see that they satisfy the axiom of priority for lives worth living, and thus, they never implies the very sadistic conclusion. We also discuss drawbacks of R^O_L and R^C_L associated with a positive critical level.

We begin with an infinite-horizon reformulation of the repugnant conclusion due to Parfit (1976, 1982, 1984). We say that an SWR R on $\Omega^{\mathbb{N}}$ implies the *repugnant conclusion* if and only if, for any stream of population sizes $(n_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ and for any stream of positive utility levels of generations $(\xi_t)_{t\in\mathbb{N}}, (\epsilon_t)_{t\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}_{++}$ satisfying $(\xi_t)_{t\in\mathbb{N}} \gg (\epsilon_t)_{t\in\mathbb{N}}$, there exists a stream of population sizes $(m_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ with $(m_t)_{t\in\mathbb{N}} \gg (n_t)_{t\in\mathbb{N}}$ such that $(\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}} P^* (\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}}$. The following axiom is presented in Kamaga (2016), which equires the repugnant conclusion to be avoided.

Avoidance of the Repugnant Conclusion (ARC): There exist $(n_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ and $(\xi_t)_{t\in\mathbb{N}}, (\epsilon_t)_{t\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}_{++}$ with $(\xi_t)_{t\in\mathbb{N}} \gg (\epsilon_t)_{t\in\mathbb{N}}$ such that for all $(m_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ with $(m_t)_{t\in\mathbb{N}} \gg (n_t)_{t\in\mathbb{N}}$, $(\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}} R (\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}}$.

Note that **ARC** implies the negation of the repugnant conclusion.

The following theorem shows that if we add **ARC** to the axioms in Theorem 2, R^O_L and R^C_L associated with a positive critical level are characterized.

Theorem 3. (i) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **ARC** and the axioms in Theorem 2 (i) if and only if there exists $\alpha \in \mathbb{R}_{++}$ such that R^O_L associated with α is a subrelation of R.*

(ii) *An SWR R on $\Omega^{\mathbb{N}}$ satisfies **ARC** and the axioms in Theorem 2 (ii) if and only if there exists $\alpha \in \mathbb{R}_{++}$ such that R^C_L associated with α is a subrelation of R.*

Proof. (i) To prove the if-part, suppose that R^O_L associated with $\alpha > 0$ is a subrelation of R. By Theorem 2 (i), we only need to show that R satisfies **ARC**. Let

$\xi_t = \alpha$ and $\epsilon \in (0, \alpha)$ for all $t \in \mathbb{N}$. Then, for any $(m_t)_{t\in\mathbb{N}}, (n_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ with $(m_t)_{t\in\mathbb{N}} \gg (n_t)_{t\in\mathbb{N}}$, we obtain $[\xi_1 \mathbf{1}_{n_1}, \ldots, \xi_t \mathbf{1}_{n_t}] \succ_{L,\alpha} [\epsilon_1 \mathbf{1}_{m_1}, \ldots, \epsilon_t \mathbf{1}_{m_t}]$ for all $t \in \mathbb{N}$. By (12a), we obtain $(\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}} P_L^O (\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}}$. Since R_L^O is a subrelation of R, we have $(\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}} P (\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}}$. Thus, R satisfies **ARC**. Next, we prove the only-if-part. By Theorem 2 (i), R_L^O associated with $\alpha \in \mathbb{R}$ is a subrelation of R. We show $\alpha > 0$ by contradiction. Suppose $\alpha \leq 0$. Let $(n_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ and $(\xi_t)_{t\in\mathbb{N}}, (\epsilon_t)_{t\in\mathbb{N}} \in \mathbb{R}_{++}^{\mathbb{N}}$ with $(\xi_t)_{t\in\mathbb{N}} \gg (\epsilon_t)_{t\in\mathbb{N}}$. Define $(m_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ by $m_t = n_t + 1$ for all $t \in \mathbb{N}$. Then, for all $t \in \mathbb{N}$, $[\epsilon_1 \mathbf{1}_{m_1}, \ldots, \epsilon_t \mathbf{1}_{m_t}] \succ_{L,\alpha} [\xi_1 \mathbf{1}_{n_1}, \ldots, \xi_t \mathbf{1}_{n_t}]$. By (12a), $(\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}} P_L^O (\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}}$. Since R_L^O is a subrelation of R, we have $(\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}} P (\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}}$. Thus, R implies the repugnant conclusion, and this is a contradiction to that R satisfies **ARC**.

(ii) By Theorems 2 (ii) and 3 (i), the if-part is straightforward since R_L^O associated with α is a subrelation of R_L^C associated with α. Further, the only-if-part is also straightforward since, if R_L^C associated with $\alpha \leq 0$ is a subrelation of R, then it contradicts to that R_L^O associated with $\alpha > 0$ is a subrelation of R. □

Next, we consider an infinite-horizon variant of the very sadistic conclusion that is introduced by Arrhenius (2000, forthcoming) in the finite-horizon context of population ethics. Let $\Omega_{++} = \cup_{n\in\mathbb{N}} \mathbb{R}_{++}^n$ and $\Omega_{--} = \cup_{n\in\mathbb{N}} \mathbb{R}_{--}^n$. Following Kamaga (2016), we say that an SWR R on $\Omega^{\mathbb{N}}$ implies the *very sadistic conclusion* if and only if, for any stream of negative utility vectors $\boldsymbol{u} \in \Omega_{--}^{\mathbb{N}}$, there exists a stream of positive utility vectors $\boldsymbol{v} \in \Omega_{++}^{\mathbb{N}}$ such that $\boldsymbol{u} P \boldsymbol{v}$.

To examine whether R_L^O associated with a positive critical level avoids the very sadistic conclusion, we define the infinite-horizon extension of the axiom of priority for lives worth living in Blackorby, Bossert, and Donaldson (2005) as follows.

Priority for Lives Worth Living (PLWL): For all $\boldsymbol{u} \in \Omega_{--}^{\mathbb{N}}$ and all $\boldsymbol{v} \in \Omega_{++}^{\mathbb{N}}$, $\boldsymbol{v} P \boldsymbol{u}$.

Note that **PLWL** implies the negation of the very sadistic conclusion.

The following proposition shows that any SWR that includes R_L^O associated with $\alpha > 0$ avoids the very sadistic conclusion.

Proposition 4. *Suppose that an SWR R on $\Omega^{\mathbb{N}}$ includes R_L^O associated with $\alpha \in \mathbb{R}$ as a subrelation. R satisfies* ***PLWL*** *if and only if $\alpha \geq 0$.*

Proof. To prove the if-part, let $\boldsymbol{u} \in \Omega_{--}^{\mathbb{N}}$, $\boldsymbol{v} \in \Omega_{++}^{\mathbb{N}}$, and $\alpha \geq 0$. Then, for all $T \in \mathbb{N}$, $[\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \succ_{L,\alpha} [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T]$. Since R_L^O associated with α, we obtain by (15a), $\boldsymbol{v} P \boldsymbol{u}$. Next, we prove the only-if-part by contradiction. Suppose R_L^O associated with $\alpha < 0$ is a subrelation of R. Consider $\boldsymbol{u} \in \Omega_{--}^{\mathbb{N}}$ and $\boldsymbol{v} \in \Omega_{++}^{\mathbb{N}}$ such that, for all $t \in \mathbb{N}$, $\boldsymbol{u}^t = (\epsilon, \epsilon)$ with $\epsilon \in (\alpha, 0)$, and $\boldsymbol{v}^t = -\alpha$. Then, for all $T \in \mathbb{N}$,

$[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ_{L,\alpha} [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$. Since R_L^O associated with α, we obtain by (15a), $\boldsymbol{u}P\boldsymbol{v}$. Thus, R violates **PLWL**. □

By Proposition 4, R_L^O and R_L^C associated with a non-negative critical level are characterized by replacing **ARC** with **PLWL** in Theorem 3.

Now, we consider two more issues in population ethics: the weak repugnant conclusion due to Broome (1992) and the mere addition principle in Parfit (1984). We present the infinite-horizon extensions of the weak repugnant conclusion and the mere addition principle in Kamaga (2016). We say that an SWR R on $\Omega^{\mathbb{N}}$ implies the *weak repugnant conclusion* if and only if, for any $(n_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ and any $(\xi_t)_{t\in\mathbb{N}}, (\epsilon_t)_{t\in\mathbb{N}} \in \mathbb{R}_{++}^{\mathbb{N}}$ with $(\xi_t)_{t\in\mathbb{N}} \gg (\epsilon_t)_{t\in\mathbb{N}} \gg (\alpha, \alpha, \ldots)$, there exists $(m_t)_{t\in\mathbb{N}} \in \mathbb{N}^{\mathbb{N}}$ with $(m_t)_{t\in\mathbb{N}} \gg (n_t)_{t\in\mathbb{N}}$ such that $(\epsilon_t \mathbf{1}_{m_t})_{t\in\mathbb{N}} P (\xi_t \mathbf{1}_{n_t})_{t\in\mathbb{N}}$, where $\alpha \in \mathbb{R}_{++}$ is a critical level for all $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ at any $t \in \mathbb{N}$. An SWR R on $\Omega^{\mathbb{N}}$ satisfies the *mere addition principle* if and only if, for all $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ and all $(\xi_t)_{t\in\mathbb{N}} \in \mathbb{R}_{++}^{\mathbb{N}}$, $([\boldsymbol{u}^t, \xi_t])_{t\in\mathbb{N}} R \boldsymbol{u}$.

It is easy to check that R_L^O associated with a positive critical level implies the weak repugnant conclusion, Further, it can be checked that R_L^O associated with a positive critical level α violates the mere addition principle since, for any $\boldsymbol{u} \in \Omega^{\mathbb{N}}$ and any $\xi \in (0, \alpha)$, we obtain $\boldsymbol{u}P([\boldsymbol{u}^t, \xi])_{t\in\mathbb{N}}$. This observation applies to any SWR that includes R_L^O associated with $\alpha > 0$. Thus, we state the following remark.

Remark 1. Every SWR in the classes characterized in Theorem 3 implies the weak repugnant conclusion and violates the mere addition principle.

6 Conclusion

In this paper, we introduced the three infinite-horizon extensions of the critical-level leximin orderings for evaluating streams of utility vectors, namely, the CLL SWR, the CLLO SWR, and the CLLC SWR. Further, we presented an axiomatic characterization of each of them. The proofs of the characterization results are done by the analysis of generalized SWRs associated with a given ordering $\succsim$ on the set |Omega of variable dimensional utility vectors. The general results we obtained are useful for exploring other possible SWRs by using the existing results in variable population social choice.

Our general results, however, suggest a limitation in extending the well-established orderings in variable population social choice to the current framework in the forms of the generalized SWRs we considered. In particular, as we showed in Lemmas 3, 5, and 6, the existence independence property of an ordering $\succsim$ is a sufficient condition for the ordering $\succsim$ to be extended as an SWR in the forms of the generalized SWRs we considered. In the literature of variable population social choice, there have been proposed many orderings that violate the existence independence property, e.g., the average utilitarian ordering (Blackorby, Bossert, and Donaldson, 1999), the number dampened utilitarian ordering (Blackorby, Bossert, and Donaldson, 2005), the rank-discounted ciritical-level generalized utilitarian ordering (Asheim and Zuber, 2014), and a version of the critical-level leximin ordering (Arrhenium, forthcoming). Our general results suggests that we may need other forms of generalized SWRs to extend these ordering to the current framework. We should address this issue in future research.

Appendix: Proofs of Lemmas

Proof of Lemma 1. Let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\mathbf{n}(\boldsymbol{u}^t) = \mathbf{n}(\boldsymbol{v}^t)$ for all $t \in \mathbb{N}$, and suppose that there exist $t_1, t_2 \in \mathbb{N}$, $i \in \{1, \ldots, \mathbf{n}(\boldsymbol{u}^{t_1})\}$, and $j \in \{1, \ldots, \mathbf{n}(\boldsymbol{u}^{t_2})\}$ such that $u_i^{t_1} = v_j^{t_2}$, $u_j^{t_2} = v_i^{t_1}$, and $u_k^t = v_k^t$ for all $(k,t) \neq (i, t_1), (j, t_2)$. To show that $\boldsymbol{u} I \boldsymbol{v}$, we first consider $\bar{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ defined by

$$\bar{\boldsymbol{v}}^{t_1} = \boldsymbol{u}^{t_2},\ \bar{\boldsymbol{v}}^{t_2} = \boldsymbol{u}^{t_1},\ \text{and } \bar{\boldsymbol{v}}^t = \boldsymbol{u}^t \text{ for all } t \neq t_1, t_2.$$

By **FA**, $\boldsymbol{u} I \bar{\boldsymbol{v}}$. Next, we define $\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ by

$$\begin{cases} \tilde{\boldsymbol{u}}^{t_1} = [\boldsymbol{u}^{t_1}, u_i^{t_1}],\ \tilde{\boldsymbol{u}}^{t_2} = [\boldsymbol{u}^{t_2}, u_j^{t_2}],\ \text{and } \tilde{\boldsymbol{u}}^t = [\boldsymbol{u}^t, \boldsymbol{u}^t] \text{ for all } t \neq t_1, t_2; \\ \tilde{\boldsymbol{v}}^{t_1} = [\boldsymbol{u}^{t_2}, u_i^{t_1}],\ \tilde{\boldsymbol{v}}^{t_2} = [\boldsymbol{u}^{t_1}, u_j^{t_2}],\ \text{and } \tilde{\boldsymbol{v}}^t = [\boldsymbol{u}^t, \boldsymbol{u}^t] \text{ for all } t \neq t_1, t_2. \end{cases}$$

By **EI**, $\boldsymbol{u} I \bar{\boldsymbol{v}}$ implies $\tilde{\boldsymbol{u}} I \tilde{\boldsymbol{v}}$. Now, define $\check{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ by

$$\check{\boldsymbol{v}}^{t_1} = [\boldsymbol{u}^{t_1}, u_j^{t_2}],\ \check{\boldsymbol{v}}^{t_2} = [\boldsymbol{u}^{t_2}, u_i^{t_1}],\ \text{and } \check{\boldsymbol{v}}^t = [\boldsymbol{u}^t, \boldsymbol{u}^t] \text{ for all } t \neq t_1, t_2.$$

By **FA**, $\tilde{\boldsymbol{v}} I \check{\boldsymbol{v}}$. Since $\tilde{\boldsymbol{u}} I \tilde{\boldsymbol{v}}$ and R is transitive, it follows $\tilde{\boldsymbol{u}} I \check{\boldsymbol{v}}$. Next, define $\hat{\boldsymbol{v}} \in \Omega^{\mathbb{N}}$ by

$$\hat{\boldsymbol{v}}^{t_1} = [\boldsymbol{v}^{t_1}, u_i^{t_1}],\ \hat{\boldsymbol{v}}^{t_2} = [\boldsymbol{v}^{t_2}, u_j^{t_2}],\ \text{and } \hat{\boldsymbol{v}}^t = [\boldsymbol{u}^t, \boldsymbol{u}^t] \text{ for all } t \neq t_1, t_2.$$

Since R is intratemporally anonymous, we obtain $\check{\boldsymbol{v}} I \hat{\boldsymbol{v}}$. Further, since $\tilde{\boldsymbol{u}} I \check{\boldsymbol{v}}$ and R is transitive, we obtain $\tilde{\boldsymbol{u}} I \hat{\boldsymbol{v}}$. Thus, by **EI**, $\boldsymbol{u} I \boldsymbol{v}$ follows. □

Proof of Lemma 2. By **WECL**, there exist $t^* \in \mathbb{N}$, $\alpha \in \mathbb{R}$, and $\boldsymbol{v} \in \Omega$ such that $\boldsymbol{v} I (\boldsymbol{v}^{-(t^*-1)}, [\boldsymbol{v}^{t^*}, \alpha], \boldsymbol{v}^{+t^*})$. Let $\boldsymbol{u} \in \Omega^{\mathbb{N}}$. We show that $\boldsymbol{u} I (\boldsymbol{u}^{-(t^*-1)}, [\boldsymbol{u}^{t^*}, \alpha], \boldsymbol{u}^{+t^*})$. Let

$\tilde{\boldsymbol{v}} = (\boldsymbol{v}^{-(t^*-1)}, [\boldsymbol{v}^{t^*}, \alpha], \boldsymbol{v}^{+t^*})$ and $\tilde{\boldsymbol{u}} = (\boldsymbol{u}^{-(t^*-1)}, [\boldsymbol{u}^{t^*}, \alpha], \boldsymbol{u}^{+t^*})$. By **EI**, $\boldsymbol{v}I\tilde{\boldsymbol{v}}$ implies $([\boldsymbol{v}^t, \boldsymbol{u}^t])_{t\in\mathbb{N}} I([\tilde{\boldsymbol{v}}^t, \boldsymbol{u}^t])_{t\in\mathbb{N}}$. Note that $[\tilde{\boldsymbol{v}}^{t^*}, \boldsymbol{u}^{t^*}]$ is a rearrangement of $[\boldsymbol{v}^{t^*}, \tilde{\boldsymbol{u}}^{t^*}]$ since $[\tilde{\boldsymbol{v}}^{t^*}, \boldsymbol{u}^{t^*}] = [\boldsymbol{v}^{t^*}, \alpha, \boldsymbol{u}^{t^*}]$. Further, for all $t \in \mathbb{N}\backslash\{t^*\}$, $[\boldsymbol{v}^t, \boldsymbol{u}^t] = [\tilde{\boldsymbol{v}}^t, \boldsymbol{u}^t] = [\boldsymbol{v}^t, \tilde{\boldsymbol{u}}^t]$. Since R is transitive and intratemporally anonymous, $([\boldsymbol{v}^t, \boldsymbol{u}^t])_{t\in\mathbb{N}} I([\tilde{\boldsymbol{v}}^t, \boldsymbol{u}^t])_{t\in\mathbb{N}}$ implies $([\boldsymbol{u}^t, \boldsymbol{v}^t])_{t\in\mathbb{N}} I([\tilde{\boldsymbol{u}}^t, \boldsymbol{v}^t])_{t\in\mathbb{N}}$. By **EI**, we obtain $\boldsymbol{u}I\tilde{\boldsymbol{u}} = (\boldsymbol{u}^{-(t^*-1)}, [\boldsymbol{u}^{t^*}, \alpha], \boldsymbol{u}^{+t^*})$. Since R is transitive and it satisfies **FA**, we can extend this result (established for t^*) to any $t \in \mathbb{N}$. We omit the easy proof of it for the sake of brevity. □

Proof of Lemma 3. To prove that $R_{\succsim}$ is an SWR, we first show that $R_{\succsim}$ is reflexive. Let $\boldsymbol{u} \in \Omega^{\mathbb{N}}$. Since $\succsim$ and $\succsim_S$ are reflexive, we obtain $\boldsymbol{u}^1 \succsim \boldsymbol{u}^1$ and $\boldsymbol{u}^t \succsim_S \boldsymbol{u}^t$ for all $t > 1$. By (10), $\boldsymbol{u}R_{\succsim}\boldsymbol{u}$. Next, to show that $R_{\succsim}$ is transitive, let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \Omega^{\mathbb{N}}$ and suppose that $\boldsymbol{u}R_{\succsim}\boldsymbol{v}$ and $\boldsymbol{v}R_{\succsim}\boldsymbol{w}$. By (10), there exists $T \in \mathbb{N}$ such that $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]$ and $\boldsymbol{u}^t \succsim_S \boldsymbol{v}^t$ for all $t > T$, and there exists $T' \in \mathbb{N}$ such that $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}] \succsim [\boldsymbol{w}^1, \dots, \boldsymbol{w}^{T'}]$ and $\boldsymbol{v}^t \succsim_S \boldsymbol{w}^t$ for all $t > T'$. If $T = T'$, since $\succsim$ and $\succsim_S$ are transitive, we obtain $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succsim [\boldsymbol{w}^1, \dots, \boldsymbol{w}^T]$ and $\boldsymbol{u}^t \succsim_S \boldsymbol{w}^t$ for all $t > T$. Thus, by (10), $\boldsymbol{u}R_{\succsim}\boldsymbol{w}$. Now, consider the case that $T \neq T'$. Without loss of generality, we assume $T > T'$. Since $\succsim$ satisfies **EI***, $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}] \succsim [\boldsymbol{w}^1, \dots, \boldsymbol{w}^{T'}]$ implies $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}, \boldsymbol{w}^{T'+1}, \dots, \boldsymbol{w}^T] \succsim [\boldsymbol{w}^1, \dots, \boldsymbol{w}^T]$. Since $\succsim$ satisfies **SP*** and **A*** and it is transitive, we obtain $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^T] \succsim [\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}, \boldsymbol{w}^{T'+1}, \dots, \boldsymbol{w}^T]$. By transitivity of $\succsim$, $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^T] \succsim [\boldsymbol{w}^1, \dots, \boldsymbol{w}^T]$. Further, since $\succsim_S$ is transitive, $\boldsymbol{u}^t \succsim_S \boldsymbol{w}^t$ for all $t > T$. Thus, by (10), $\boldsymbol{u}R_{\succsim}\boldsymbol{w}$. Next, to show that $R_{\succsim}$ is finitely complete, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^{+T} = \boldsymbol{v}^{+T}$. Since $\succsim$ is complete, we obtain $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]$ or $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^T] \succsim [\boldsymbol{u}^1, \dots, \boldsymbol{u}^T]$. Since $\boldsymbol{u}^t \sim_S \boldsymbol{v}^t$ for all $t > T$, we obtain, by (10), $\boldsymbol{u}R_{\succsim}\boldsymbol{v}$ or $\boldsymbol{v}R_{\succsim}\boldsymbol{u}$. Finally, to show that $R_{\succsim}$ is intratemporally anonymous, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that, for all $t \in \mathbb{N}$, there exists a bijection $\pi^t : \{1, \dots, \mathbf{n}(\boldsymbol{u}^t)\} \to \{1, \dots, \mathbf{n}(\boldsymbol{v}^t)\}$ such that $\boldsymbol{u}^t = \left(v^t_{\pi^t(1)}, \dots, v^t_{\pi^t(\mathbf{n}(\boldsymbol{u}^t))}\right)$. Since $\succsim$ satisfies **A*** and it is transitive, we obtain $\boldsymbol{u}^1 \sim \boldsymbol{v}^1$. Further, we obtain $\boldsymbol{u}^t \sim_S \boldsymbol{v}^t$ for all $t > 1$. Thus, by (10), $\boldsymbol{u}I_{\succsim}\boldsymbol{v}$.

We next prove (11a) and (11b). First, we prove the if-part of (11a). Let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t \succsim_S \boldsymbol{v}^t$ for all $t > T$ and $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]$. By (10), $\boldsymbol{u}R_{\succsim}\boldsymbol{v}$. We show $\neg\boldsymbol{v}R_{\succsim}\boldsymbol{u}$ by contradiction. Suppose $\boldsymbol{v}R_{\succsim}\boldsymbol{u}$. By (10), there exists $T' \in \mathbb{N}$ such that $\boldsymbol{v}^t \succsim_S \boldsymbol{u}^t$ for all $t > T'$ and $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}] \succsim [\boldsymbol{u}^1, \dots, \boldsymbol{u}^{T'}]$. Since we obtain a contradiction to $[\boldsymbol{u}^1, \dots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \dots, \boldsymbol{v}^T]$ if $T = T'$, we consider the case that $T \neq T'$. Without loss of generality, we assume $T > T'$. Since $\succsim$ satisfies **EI***, $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}] \succsim [\boldsymbol{u}^1, \dots, \boldsymbol{u}^{T'}]$ implies $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}, \boldsymbol{u}^{T'+1}, \dots, \boldsymbol{u}^T] \succsim [\boldsymbol{u}^1, \dots, \boldsymbol{u}^T]$. Since $\succsim$ satisfies **SP*** and **A*** and it is transitive, we obtain $[\boldsymbol{v}^1, \dots, \boldsymbol{v}^T] \succsim [\boldsymbol{v}^1, \dots, \boldsymbol{v}^{T'}, \boldsymbol{u}^{T'+1}, \dots, \boldsymbol{u}^T]$. Thus, by transitivity, $[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \succsim [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$. This is a contradiction to $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succ [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. Thus, $\neg \boldsymbol{v} R_{\succsim} \boldsymbol{u}$.

Next, we prove the only-if-part of (11a). Let $\boldsymbol{u},\boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose $\boldsymbol{u} P_{\succsim} \boldsymbol{v}$. By (10), there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t \succsim_S \boldsymbol{v}^t$ for all $t > T$ and $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. We distinguish two cases: (a) $\boldsymbol{u}^t \sim_S \boldsymbol{v}^t$ for all $t > T$ and (b) $\boldsymbol{u}^{T^*} \succ_S \boldsymbol{v}^{T^*}$ for some $T^* > T$. First, consider case (a). We show $\neg[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \succsim [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$ by contradiction. Suppose $[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \succsim [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$. By (10), $\boldsymbol{v} R_{\succsim} \boldsymbol{u}$. This is a contradiction to $\boldsymbol{u} P_{\succsim} \boldsymbol{v}$. Thus, $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succ [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. Next, consider case (b). Since $\succsim$ satisfies $\mathbf{EI}^*$, $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$ implies $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T,\boldsymbol{v}^{T+1},\dots,\boldsymbol{v}^{T^*}] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^{T^*}]$. Since $\succsim$ satisfies $\mathbf{SP}^*$ and $\mathbf{A}^*$ and it is transitive, we obtain $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^{T^*}] \succ [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T,\boldsymbol{v}^{T+1},\dots,\boldsymbol{v}^{T^*}]$. By transitivity, $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^{T^*}] \succ [\boldsymbol{v}^1,\dots,\boldsymbol{v}^{T^*}]$.

To prove the if-part of (11b), let $\boldsymbol{u},\boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t \sim_S \boldsymbol{v}^t$ for all $t > T$ and $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \sim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. By (10), $\boldsymbol{u} R_{\succsim} \boldsymbol{v}$ and $\boldsymbol{v} R_{\succsim} \boldsymbol{u}$, or equivalently, $\boldsymbol{u} I_{\succsim} \boldsymbol{v}$. Next, to prove the only-if-part of (11b), let $\boldsymbol{u},\boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose $\boldsymbol{u} I_{\succsim} \boldsymbol{v}$. By (10), there exists $T \in \mathbb{N}$ such that $\boldsymbol{u}^t \succsim_S \boldsymbol{v}^t$ for all $t > T$ and $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$, and there exists $T' \in \mathbb{N}$ such that $\boldsymbol{v}^t \succsim_S \boldsymbol{u}^t$ for all $t > T'$ and $[\boldsymbol{v}^1,\dots,\boldsymbol{v}^{T'}] \succsim [\boldsymbol{u}^1,\dots,\boldsymbol{u}^{T'}]$. Without loss of generality, we assume $T \geq T'$. Then, we obtain $\boldsymbol{u}^t \sim_S \boldsymbol{v}^t$ for all $t > T$. If $\neg[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \succsim [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$, then by (11a), we obtain $\boldsymbol{u} P_{\succsim} \boldsymbol{v}$, a contradiction to $\boldsymbol{u} I_{\succsim} \boldsymbol{v}$. Thus, $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \sim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. □

Proof of Lemma 4. Let $T \in \mathbb{N}$ and $\boldsymbol{u},\boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\boldsymbol{u}^{+T} = \boldsymbol{v}^{+T}$. By (10), $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$ implies $\boldsymbol{u} R_{\succsim} \boldsymbol{v}$, which in turn implies $\boldsymbol{u} R \boldsymbol{v}$ since $R_{\succsim} \subseteq R$. Next, assume $\boldsymbol{u} R \boldsymbol{v}$. We show $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$ by contradiction. Suppose $\neg[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. Since $\succsim$ is complete, we obtain $[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \succ [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$. By (11a), $\boldsymbol{v} P_{\succsim} \boldsymbol{u}$. Since $P_{\succsim} \subseteq P$, we obtain $\boldsymbol{v} P_{\succsim} \boldsymbol{u}$. This is a contradiction to $\boldsymbol{u} R \boldsymbol{v}$. □

Proof of Lemma 5. First, we prove that $R^O_{\succsim}$ is well defined as a binary relation on $\Omega^{\mathbb{N}}$. To this end, we show that $P^O_{\succsim} \cap I^O_{\succsim} \neq \varnothing$ and that $P^O_{\succsim}$ and $I^O_{\succsim}$ are, respectively, asymmetric and symmetric. We show $P^O_{\succsim} \cap I^O_{\succsim} \neq \varnothing$ by contradiction. Suppose $\boldsymbol{u} P^O_{\succsim} \boldsymbol{v}$ and $\boldsymbol{u} I^O_{\succsim} \boldsymbol{v}$. By (15a) and (15b), there exists $T \in \mathbb{N}$ such that $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \succ [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$ and $[\boldsymbol{u}^1,\dots,\boldsymbol{u}^T] \sim [\boldsymbol{v}^1,\dots,\boldsymbol{v}^T]$. This is a contradiction to that $\succsim$ is a binary relation on Ω. Next, to show that $P^O_{\succsim}$ is asymmetric, suppose $\boldsymbol{u} P^O_{\succsim} \boldsymbol{v}$. By (15a), there is no $T^* \in \mathbb{N}$ such that, for all $T \geq T^*$, $[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \succ [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$. Thus, $\neg \boldsymbol{v} P^O_{\succsim} \boldsymbol{u}$. Now, to show that $I^O_{\succsim}$ is symmetric, suppose $\boldsymbol{u} I^O_{\succsim} \boldsymbol{v}$. By (15b), there exists $T^* \in \mathbb{N}$ such that, for all $T \geq T^*$, $[\boldsymbol{v}^1,\dots,\boldsymbol{v}^T] \sim [\boldsymbol{u}^1,\dots,\boldsymbol{u}^T]$. Thus, $\boldsymbol{v} I^O_{\succsim} \boldsymbol{u}$.

Next, we prove that $R^O_{\succsim}$ is an SWR. First, to show that $R^O_{\succsim}$ is reflexive, let

$\boldsymbol{u} \in \Omega^{\mathbb{N}}$. Then, for all $T \in \mathbb{N}$, $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T]$. By (15a), $\boldsymbol{u} R^O_{\succsim} \boldsymbol{u}$. Next, to show that R^O is transitive, let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \Omega^{\mathbb{N}}$ and suppose that $\boldsymbol{u} R^O_{\succsim} \boldsymbol{v}$ and $\boldsymbol{v} R^O_{\succsim} \boldsymbol{w}$. By (15a) and (15b), there exist $T^* \in \mathbb{N}$ such that (a) $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$ for all $T \geq T^*$ or $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$ for all $T \geq T^*$ and (b) $[\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \succ [\boldsymbol{w}^1, \ldots, \boldsymbol{w}^T]$ for all $T \geq T^*$ or $[\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \sim [\boldsymbol{w}^1, \ldots, \boldsymbol{w}^T]$ for all $T \geq T^*$. Since $\succsim$ is transitive, we obtain that $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succ [\boldsymbol{w}^1, \ldots, \boldsymbol{w}^T]$ for all $T \geq T^*$ or $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{w}^1, \ldots, \boldsymbol{w}^T]$ for all $T \geq T^*$. By (15a) and (15b), $\boldsymbol{u} R^O_{\succsim} \boldsymbol{v}$. Now, to show that $R^O_{\succsim}$ is finitely complete, let $T \in \mathbb{N}$ and $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\boldsymbol{u}^{+T} = \boldsymbol{v}^{+T}$. Since $\succsim$ is complete, we obtain $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$ or $[\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \succsim [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T]$ Since $\succsim$ satisfies **EI***, we obtain that, for all $T' > T$

$$[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \Leftrightarrow [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^{T'}] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^{T'}]$$

and

$$[\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \succsim [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \Leftrightarrow [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^{T'}] \succsim [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^{T'}].$$

Thus, by (15a) and (15b), $\boldsymbol{u} R^O_{\succsim} \boldsymbol{v}$ or $\boldsymbol{v} R^O_{\succsim} \boldsymbol{u}$. Finally, to show that $R^O_{\succsim}$ is intratemporally anonymous, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that, for all $t \in \mathbb{N}$, there exists a bijection $\pi^t : \{1, \ldots, \mathbf{n}(\boldsymbol{u}^t)\} \to \{1, \ldots, \mathbf{n}(\boldsymbol{v}^t)\}$ such that $\boldsymbol{u}^t = \left(v^t_{\pi^t(1)}, \ldots, v^t_{\pi^t(\mathbf{n}(\boldsymbol{u}^t))}\right)$. Since $\succsim$ satisfies $\mathbf{A}^*$ and it is transitive, we obtain $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$ for all $T \in \mathbb{N}$. By (15b), $\boldsymbol{u} I^O_{\succsim} \boldsymbol{v}$. □

Proof of Lemma 6. First, we prove that $R^C_{\succsim}$ is an SWR. By (15a), (15b), and (16), $R^O_{\succsim} \subseteq R^C_{\succsim}$. Thus, by Lemma 5, $R^C_{\succsim}$ is finitely complete. To show that $R^C_{\succsim}$ is reflexive, let $\boldsymbol{u} \in \Omega^{\mathbb{N}}$. Since $\succsim$ is reflexive, we obtain that $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T]$ for all $T \in \mathbb{N}$. By (16), $\boldsymbol{u} R^C_{\succsim} \boldsymbol{v}$. Next, to show that $R^C_{\succsim}$ is transitive, let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \Omega^{\mathbb{N}}$ and suppose that $\boldsymbol{u} R^C_{\succsim} \boldsymbol{v}$ and $\boldsymbol{v} R^C_{\succsim} \boldsymbol{w}$. By (16), there exists T^* such that $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$ and $[\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T] \succsim [\boldsymbol{w}^1, \ldots, \boldsymbol{w}^T]$ for all $T \geq T^*$. Since $\succsim$ is transitive, we obtain $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \succsim [\boldsymbol{w}^1, \ldots, \boldsymbol{w}^T]$ for all $T \geq T^*$. By (16), $\boldsymbol{u} R^C_{\succsim} \boldsymbol{v}$. Finally, to show that $R^C_{\succsim}$ is intratemporally anonymous, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose that, for all $t \in \mathbb{N}$, there exists a bijection $\pi^t : \{1, \ldots, \mathbf{n}(\boldsymbol{u}^t)\} \to \{1, \ldots, \mathbf{n}(\boldsymbol{v}^t)\}$ such that $\boldsymbol{u}^t = \left(v^t_{\pi^t(1)}, \ldots, v^t_{\pi^t(\mathbf{n}(\boldsymbol{u}^t))}\right)$. Since $\succsim$ satisfies $\mathbf{A}^*$ and it is transitive, we obtain $[\boldsymbol{u}^1, \ldots, \boldsymbol{u}^T] \sim [\boldsymbol{v}^1, \ldots, \boldsymbol{v}^T]$ for all $T \in \mathbb{N}$. By (16), $\boldsymbol{u} I^C_{\succsim} \boldsymbol{v}$.

Next, we prove (17a) and (17b). Let R_A and R_B be the binary relations on $\Omega^{\mathbb{N}}$ defined by (17a) and (17b), respectively. We show that $R_A \cup R_B = R^C_{\succsim}$ and R_A and R_B are asymmetric and symmetric. By (17a) and (17b), it is straightforward that R_A is asymmetric and R_B is symmetric. To show that $R_A \cup R_B \subseteq R^C_{\succsim}$, let $\boldsymbol{u}, \boldsymbol{v} \in \Omega^{\mathbb{N}}$ and suppose $(\boldsymbol{u}, \boldsymbol{v}) \in R_A \cup R_B$. By (17a) and (17b), there exists $T^* \in \mathbb{N}$ such that, for all $T \geq T^*$, $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$. By (16), $\boldsymbol{u}R^C_{\succsim}\boldsymbol{v}$. Next, to show that $R^C_{\succsim} \subseteq R_A \cup R_B$, suppose $\boldsymbol{u}R^C_{\succsim}\boldsymbol{v}$. By (16), there exists $T^* \in \mathbb{N}$ such that, for all $T \geq T^*$, $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$. If there exists $T' \geq T^*$ such that, for all $T \geq T'$, $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \sim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$, then we obtain $\boldsymbol{u}R_B\boldsymbol{v}$ by (17b). If there is no $T' \geq T^*$ such that, for all $T \geq T'$, $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \sim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$, then, for all $T' \geq T^*$, there exists $T \geq T'$ such that $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succ [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$, and we obtain $\boldsymbol{u}R_A\boldsymbol{v}$ by (17a). Thus, $(\boldsymbol{u},\boldsymbol{v}) \in R_A \cup R_B$. □

Proof of Lemma 7. Let $T \in \mathbb{N}$ and $\boldsymbol{u},\boldsymbol{v} \in \Omega^{\mathbb{N}}$ with $\boldsymbol{u}^{+T} = \boldsymbol{v}^{+T}$. First, suppose $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$, and we show that $\boldsymbol{u}R\boldsymbol{v}$. Since $\succsim$ satisfies **EI***, we obtain that, for all $T' \geq T$,

$$[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T] \Leftrightarrow [\boldsymbol{u}^1,\ldots,\boldsymbol{u}^{T'}] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^{T'}].$$

Thus, by (15a) and (15b), we have $\boldsymbol{u}R^O_{\succsim}\boldsymbol{v}$. Since $R^O_{\succsim}$ is a subrelation of R, $\boldsymbol{u}R\boldsymbol{v}$. Next, assume $\boldsymbol{u}R\boldsymbol{v}$, and we show $[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$ by contradiction. Suppose $\neg[\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T] \succsim [\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T]$. Since $\succsim$ is complete, we have $[\boldsymbol{v}^1,\ldots,\boldsymbol{v}^T] \succ [\boldsymbol{u}^1,\ldots,\boldsymbol{u}^T]$. Since $\succsim$ satisfies **EI***, we obtain that, for all $T' \geq T$, $[\boldsymbol{v}^1,\ldots,\boldsymbol{v}^{T'}] \succ [\boldsymbol{u}^1,\ldots,\boldsymbol{u}^{T'}]$. By (15a), $\boldsymbol{v}P^O_{\succsim}\boldsymbol{u}$. Since $R^O_{\succsim}$ is a subrelation of R, we obtain $\boldsymbol{v}P\boldsymbol{u}$. This is a contradiction to $\boldsymbol{u}R\boldsymbol{v}$. □

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