

# Global warming as an asymmetric public bad

Louis-Gaëtan Giraudet, Céline Guivarch\*

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## Abstract

We extend the canonical dynamic game of global warming to capture three stylized facts: (i) while most countries are expected to suffer damages, some might enjoy short-term benefits; (ii) countries' exposure to impacts bears little relation to their mitigation capabilities; (iii) some adaptation technologies, such as air conditioning, may exacerbate warming. These various sources of asymmetry add free driving to the classical free riding problem. This opens up possibilities for excessive mitigation in a non-cooperative regime. Moreover, it restricts the possibilities of Pareto improvements without transfers. Finally, it can provide a rationale for differentiating Pigouvian prices across countries.

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\*Centre International de Recherche sur l'Environnement et le Développement (CIRED), Ecole des Ponts ParisTech. Email: giraudet@centre-cired.fr, guivarch@centre-cired.fr

## 1. Introduction

Anthropogenic global warming has brought the problem of public goods to an unprecedented scale. Human activities generate greenhouse-gas (GHG) emissions, which warm the atmosphere. In turn, the atmosphere connects all humans. Therefore, virtually every single individual on the planet is at the same time contributing to and being affected by global warming.

The problem has traditionally been modeled as a dynamic public bad imposing damages on identical countries (van der Ploeg and de Zeeuw, 1992; Dockner and Long, 1993). In this view, global-warming mitigation is subject to free riding: because any country can reap benefits from another's effort without bearing its cost, all mitigate too little in equilibrium and too much global warming ensues. The problem can in theory be solved by implementing a global Pigouvian price on GHG emissions, with the caveat that emergence of the supra-national entity in charge of managing it is rather speculative.

In reality, quite trivially, countries are all but identical. As we shall see in this paper, accounting for asymmetries between countries can change the public-bad nature of global warming and provide new insights into its collective management. We are specifically interested in three sources of asymmetries, which we illustrate with the four largest players among sovereign countries: China, the U.S., India and Russia, altogether covering half of global GHG emissions.

First, the economic impacts of a temperature increase, though net negative at the global scale, can be positive at the regional one (Arent et al., 2014). Recent projections point to strongly negative effects in India, moderately negative effects in China and the U.S. and strongly positive effects in Russia (Samson et al., 2011; Burke et al., 2015; Lemoine and Kapnick, 2015). This provides Russia with weaker incentives to mitigate the problem than the U.S., China and, especially, India.

Second, some adaptation technologies feed back to the climate system. For instance, an increase in global temperature reduces the need for space heating but increases the need for air conditioning (Auffhammer and Mansur, 2014; Davis and Gertler, 2015; Barreca et al., 2016).<sup>1</sup> Such retroactions affect mitigation incentives. Consider a game played by Russia and India. Mitigation in the former prevents emissions due to air conditioning from increasing

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<sup>1</sup>At least conceptually, a similar effect can be envisioned in agriculture: warming may increase agricultural productivity in previously unsuitable areas but intensify the use of fertilizers or irrigation in more vulnerable areas, which in turn might generate GHG emissions.

in the latter. Failure to internalize this positive externality might cause under-mitigation in Russia, which is but another form of free riding. Yet in turn, mitigation in India prevents emissions due to space heating from decreasing in Russia. Failure to internalize this negative externality might lead India to over-mitigate, which we refer to as *free driving*.

Third, besides adaptation technologies, mitigation capabilities too are asymmetric, with little connection between the two. Mitigation costs tend to be low in China and Russia and high in India and the U.S. (Nordhaus, 2015; Aldy et al., 2016). This further affects strategic behaviors. Knowing that mitigation costs are relatively low in Russia, India will anticipate little free riding there and respond with relatively little free driving in equilibrium. This might result in high emission levels. Now consider that India is playing against the U.S.. As both countries face damages, they are both expected to free ride. Knowing that the U.S. has fewer mitigation capabilities, India will anticipate much free riding there and respond with relatively less free riding in equilibrium. This might result in low emission levels.

In this paper, we examine the net effect of those conflicting externalities on global temperature and social welfare. We extend the canonical dynamic-game model of global warming to accommodate asymmetries in exposure to impacts, adaptation technologies and mitigation capabilities. We consider two players, a warm region and a cold one. Adaptation to global warming increases the former player's GHG emissions and decreases the latter's. The players can invest at uneven cost to mitigate emissions. This structure is general enough to encompass both the canonical identical-player framework and the more empirically relevant asymmetric one. Global temperature is assumed to have a linear effect on players' payoffs. This simplification, as compared to the prevailing quadratic assumption, ensures subgame perfectness of the Nash equilibrium and permits welfare analysis. It reflects a short-term perspective, which we think is pertinent to international negotiations.

We start with re-demonstrating the classical free-riding result in the special case of identical damages. We additionally find that cooperation is systematically Pareto-improving in this context. Yet as soon as damages differ across players, Pareto improvements are restricted to a subset of parameter combinations. Otherwise, cooperation is only Hicks-Kaldor improving, necessitating transfers between players. Furthermore, when impact heterogeneity is so strong that the cold region enjoys small benefits, its free-riding behavior is counterbalanced by free driving in the warm region. The net effect on global warming is found to depend on five sets of parameter combinations. In short, if low mitigation costs are paired with severe impacts, free driving can compensate free riding, hence inducing too little warming. In the more empirically relevant case of a positive correlation between mitigation costs and

impact severity, equilibrium temperature is too high. It can even be higher than if players were acting myopically, a regime reminiscent of a Pre-Kyoto era and perhaps still relevant to some countries' behavior. Finally, we find that unless players are myopic, heterogeneity of impacts commands a differentiation of Pigouvian instruments. The warm region should even subsidize GHG-intensive adaptation if the cold region expects benefits from global warming.

Our model provides economic interpretations of bilateral agreements between large emitters, such as the U.S.-China Joint Presidential Statement on Climate Change.<sup>2</sup> More generally, it sheds an economic light on the pledge system forming the basis of the Paris Agreement recently adopted by the Parties to the United Nations Framework Convention on Climate Change (UNFCCC).<sup>3</sup> Instead of relying on the top-down solution of a global price on GHG emissions, the Agreement is based on Nationally-Determined Contributions (NDCs) supported by domestic policies and complemented with international transfers. Can such a bottom-up architecture sustain a Coasian bargaining process or is it simply conducive to a low-ambition Nash equilibrium? Our analysis pinpoints parameter combinations which determine answers to the question. For instance, a naïve interpretation of the model suggests that the first option is plausible in the U.S.-China case. Still, further empirical research is needed to elicit key parameters. In parallel, the robustness of the predictions should be assessed by relaxing the deterministic, two-player and linear-state assumptions.

Ultimately, we aim to contribute to refining the concept of public good. Together with other notions such as impure public bads (Kotchen, 2005) or climate clubs (Nordhaus, 2015), our asymmetric public bad conceptualization suggests that there is more to the global-warming problem than a uniform negative externality. Specifically, we argue that strategic behavior in fact creates multiple externalities. This opens up possibilities for under-supply of a public bad and amends the textbook argument of a uniform tax as the efficient solution.

The paper is organized as follows. Section 2 provides theoretical and empirical background. Section 3 introduces the model. Section 4 characterizes steady states and optimal trajectories. Section 5 examines the trade-offs between steady-state temperature and intertemporal welfare. Section 6 discusses efficient policy. Section 7 concludes.

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<sup>2</sup><https://www.whitehouse.gov/the-press-office/2016/03/31/us-china-joint-presidential-statement-climate-change>

<sup>3</sup><https://unfccc.int/resource/docs/2015/cop21/eng/109r01.pdf>

## 2. Background

### 2.1. Related models of global warming

Fundamentally, global warming is a spatial and intertemporal problem. The two dimensions have been integrated in dynamic games, with most applications to mitigation.<sup>4</sup> In contrast, research on adaptation is still in its infancy and borrows from a broader set of frameworks.

**Dynamic games of mitigation.** Research on dynamic games of global warming is broadly reviewed in Missfeldt (1999), Jørgensen et al. (2010) and Calvo and Rubio (2013). Most studies have been concerned with how players set their emission levels, or, equivalently, their mitigation level. Indeed, if global warming is regarded as a public bad, then mitigation is the mirror image of a public good.

The canonical model has identical players deriving utility from GHG emissions and suffering damages from GHG concentration, that is, cumulative emissions. The payoff function increases in a linear-quadratic manner in emissions and decreases quadratically in cumulative emissions. The game consists of the players maximizing their payoff with respect to emissions (or mitigation thereof), subject to emissions accumulation, net of a natural sink. It is usually set up in continuous time, hence as a differential game.

Two solution concepts can be used to solve non-cooperative differential games: the open-loop Nash equilibrium, which emerges when players simultaneously commit to an intertemporal schedule of actions at the outset of the game, and the feedback Nash equilibrium, which emerges when players simultaneously react to the state of the system at each stage. Both equilibria are time-consistent: if players start on the equilibrium path, it is optimal for them to stick to it. But only the feedback equilibrium has the additional property of subgame perfectness: if one player deviates from his or her equilibrium strategy at some stage, others will still react optimally to the state of the system, so that further playing feedback strategies will still be a Nash equilibrium. With open-loop strategies, the players remain off equilibrium path once one has deviated.

The differential-game approach to global warming was pioneered by van der Ploeg and de Zeeuw (1992) and Hoel (1993). The authors found that linear feedback strategies magnify the incentives to free ride, hence lead to a higher equilibrium concentration of GHGs than open-loop strategies. Subsequent analyses of the canonical model have focused on feedback

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<sup>4</sup>The two dimensions remain mostly studied separately, following the seminal works of Nordhaus (1993) in dynamic optimization and Barrett (1994) and Carraro and Siniscalco (1998) in static game theory.

solutions.<sup>5</sup> Dockner and Long (1993) found that if the players' discount rate is low enough, non-linear strategies can establish the socially optimal level of warming. Rubio and Casino (2002) found that this result required that the initial stock of pollution be above socially optimal. Wirl (2007) placed further restrictions on the result by requiring the elasticity of marginal utility to be increasing.

All the above-mentioned analyses assume identical players. To our knowledge, only Martin et al. (1993) have considered asymmetric players in the sense that some are harmed by global warming while others benefit. The feedback Nash equilibrium is computed numerically, which limits analytic comparison with related studies. More recently, Zagonari (1998) has considered heterogeneity in damages and players' bargaining power. The author finds that linear feedback strategies can lead to more mitigation, thus less warming, than socially optimal.

**Models of adaptation.** Adaptation does not have as clear-cut a definition as mitigation. In the simplest sense, mitigation is a publicly-driven initiative motivated by the existence of a market failure – the free-riding problem – while adaptation is a private, efficient response to the consequences of the market failure – excessive warming. In this view, mitigation provides first-order benefits and adaptation is only second-order.

Some nuances however motivate a closer look at adaptation. First, adaptation can have a public-good dimension. As discussed by Mendelsohn (2000), adaptation in water control may require joint efforts from countries sharing an aquifer. The problem can become first-order for global warming if, as we touched in the introduction, some adaptation measures generate GHG emissions. Second, some authors make a distinction between proactive (or anticipatory, or *ex ante*) and reactive (or *ex post*) adaptation (Lecocq and Shalizi, 2007). While the former is equivalent to efficient adaptation, the latter suggests that some market failures might bias expectations in relation to adaptation.

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<sup>5</sup>Subgame perfectness makes feedback solutions intellectually palatable. Yet in the context of global warming, we find them quite heroic. Feedback would rely on a monitoring platform providing real-time information about countries' emissions and their effect on local climates. As of today, it is possible to attribute GHG emissions to world regions (Ciais et al., 2013), though not without imperfections, as the recent Chinese example illustrates (Liu et al., 2015). Further, the recent dispute over a possible warming hiatus illustrates how challenging it still is to predict the effect of GHG emissions (Marotzke and Forster, 2015; Rajaratnam et al., 2015). In this light, we argue that open loop better mimics the information structure currently shared in international negotiations.

Economic models of adaptation are based on dynamic optimization (Bréchet et al., 2012), static game theory (Ebert and Welsch, 2011; Farnham and Kennedy, 2014), and, less frequently, dynamic games combining the two (Buob and Stephan, 2011; Ingham et al., 2013). The question receiving most attention is whether mitigation and adaptation are complements or substitutes (Kane and Shogren, 2000), with the latter view prevailing so far.

To our knowledge, the question of asymmetries between countries, which we are specifically interested in in this paper, is only addressed by Farnham and Kennedy (2014). Yet the authors confine their attention to the size of the countries. No analysis seems to take into account the fact that some adaptation technologies can generate GHG emissions.

## 2.2. Insights from empirical studies and integrated assessments

**Impacts and adaptation.** Global warming is expected to have a net negative impact, with important asymmetries. Early studies have forecasted moderate impacts, e.g., 2% of gross domestic product (GDP) losses for a 2.5 degree Celcius ( $^{\circ}\text{C}$ ) increase in global temperature, with income gains as high as 4% in Eastern Europe and losses as high as 8% in Africa (Tol, 2009). Recent projections based on empirical works exploiting variations over time and across space suggest substantially larger impacts (Dell et al., 2014). Burke et al. (2015) project a 23% reduction of global income per capita by 2100, with domestic impacts equal to -92% in India, -42% in China, -36% in the U.S. and +419% in Russia. While the distribution of impacts varies across assessments, the relatively high vulnerability of India is corroborated by other studies (Samson et al., 2011; Lemoine and Kapnick, 2015). Figure 1 displays the distributions found in Tol (2009) and Burke et al. (2015).

Impacts are expected to be most asymmetric in economic sectors such as building energy use, agriculture, water supply and tourism (Arent et al., 2014). In particular, warming is expected to increase agricultural productivity in some places while deterring it in other places (Reilly et al., 1994; Auffhammer and Schlenker, 2014). In the building sector, higher temperature and heat waves are empirically found to damage health (Deschenes, 2014) and stimulate adoption of air conditioning systems (Davis and Gertler, 2015). This adaptation strategy effectively reduces mortality due to heat waves (Barreca et al., 2016). It also increases energy use (Cian et al., 2013; Auffhammer and Mansur, 2014). Despite a decrease in space heating needs, this is projected to result in a net increase in global GHG emissions by the end of the century (Isaac and van Vuuren, 2009; Hasegawa et al., 2016). Altogether, these effects can be sizable, as building energy use and agriculture currently account for 6% and 24% of global GHG emissions, respectively (Victor et al., 2014).

Importantly, geographical disparities seem to be the prime determinant of global wealth inequalities. A causal effect has been identified in the past (Hsiang and Meng, 2015) and global warming is expected to amplify it (Mendelsohn et al., 2006).

**Mitigation.** Mitigation costs are highly heterogeneous across world regions (Clarke et al., 2014). Albeit scarce, country-level assessments suggest that mitigation costs are low in Russia and China and high in India and the U.S. (Nordhaus, 2015; Aldy et al., 2016). Little correlation therefore exists between exposure to impacts and mitigation costs, at least for these four countries, as illustrated in Figure 2. In particular, India suffers a double jeopardy with both high exposure and high mitigation costs.

### 3. Model

We build a differential-game model of global warming integrating asymmetries in exposure to impacts, adaptation technologies and mitigation capabilities. Space heating and air conditioning in buildings are used as an example.

#### 3.1. Setup

We consider an economy with two players. A cold region, noted  $c$ , predominantly uses energy for space heating. A warm region, noted  $w$ , predominantly uses energy for air conditioning. The model is summarized in Figure 3.

**Adaptive mitigation.** Energy use  $e^i(\cdot, \cdot)$  to Region  $i \in \{c, w\}$  reads:

$$(1) \quad \begin{cases} e^c(q, T) \equiv \epsilon - q - bT \\ e^w(q, T) \equiv \epsilon - q + dT \end{cases}$$

Regions seek to maintain thermal comfort and perfectly adapt to global warming: Temperature  $T$  increases cooling needs in the warm region and can reduce heating needs in the cold one. As a result, the former's net energy use increases at a marginal rate  $d > 0$  and the latter's decreases at a marginal rate  $b$ . Throughout, the notions of impact of and adaptation to global warming are used interchangeably. Parameter  $d$  or a negative value of  $b$  represent local *damages* raising adaptation costs, while a positive value of  $b$  represents local *benefits* generating adaptation savings.



Under such perfect adaptation, each region  $i$  controls its mitigation investment  $q^i$ . Without loss of generality, mitigation lowers energy use at the same marginal rate of 1 in both regions. Likewise, both regions face the same irreducible energy use  $\epsilon > 0$ , assumed to be high enough for expressions (1) to be non-negative.

Mitigation is purchased in Region  $i$  at a specific quadratic cost  $m^i(\cdot)$ , with  $\delta^i > 0$ :

$$(2) \quad m^i(q) \equiv \delta^i q^2 / 2$$

In the optimization problems considered below, the objective function to Region  $i$  will be the total cost  $\gamma^i(\cdot, \cdot)$  formed by energy expenditures (with the price of energy normalized to 1, assuming away depletion problems) and mitigation costs:

$$(3) \quad \gamma^i(q, T) \equiv e^i(q, T) + m^i(q)$$

**Global warming.** Energy use in both regions generates GHG emissions which accumulate in the atmosphere, with some amount captured by a natural sink (e.g. forests, oceans) at a marginal rate  $s > 0$ . The transition equation describing the evolution of the state variable  $T$  from its initial level  $T^0$  reads (the dot refers to the temporal derivative):

$$(4) \quad \begin{aligned} \dot{T} &= e^c(q^c, T) + e^w(q^w, T) - sT \\ &= 2\epsilon - (q^c + q^w) - (s + b - d)T \end{aligned}$$

The state variable  $T$  identifies (up to two multiplicative constants) cumulative energy use, the atmospheric GHG concentration and the global mean surface temperature. Implicitly, we hold the GHG intensity of energy constant. Therefore, mitigation in our example is better interpreted as improved energy efficiency (e.g., insulation, efficient heating and cooling systems) than substitution of renewable energy sources for fossil fuels. Moreover, we assume a linear response of the atmospheric temperature to the GHG concentration, a reasonable approximation of physical processes – at least for CO<sub>2</sub> (Matthews et al., 2009).

**Stability condition.** Convergence of the system toward a stable steady state requires  $(d\dot{T}/dT)_{T_\infty} < 0$ . This imposes that what we shall refer to as the physico-economic absorption capacity be positive:

$$(5) \quad s + b - d > 0$$

In other words, the natural sink  $s$  needs to absorb more warming than the net economically harmful fraction  $d - b$ .

### 3.2. Public-good regimes: A taxonomy

Our model is general enough to address a variety of public-good regimes. Based on the sense and magnitude of local-impact parameters  $d$  and  $b$ , we use the following taxonomy:

- $d > b$ : Global warming is a public *bad*.
- $d < b$ : Global warming is a public *good*.
- $bd > 0$ : Global warming is a *non-uniform* public good/bad;
  - if  $d = b$ , it is *symmetric*,
  - otherwise, it is *asymmetric*.
- $bd < 0$ : Global warming is a *uniform* public good/bad;
  - if  $d = -b$ , it is *homogeneous*,
  - otherwise, it is *heterogeneous*.

The taxonomy defines seven types of goods, which are illustrated in Figure 4. Since  $d = -b$  is a symmetry axis, we will confine our attention to the upper-left semi-plane, where  $d \geq -b$ . We will further focus on the public-bad semi-plane (weakly defined as  $d \geq b$ ), most relevant to global warming. The resulting domain of interest – the upper corner where  $d \geq |b|$  – covers the vast majority of the projections displayed in Figure 1.

### 3.3. Model specificities

We summarize here how our model relates to the mitigation and adaptation models reviewed in Section 2.1. Its structure is similar to that of the canonical mitigation model (van der Ploeg and de Zeeuw, 1992; Dockner and Long, 1993), with a set of differences. A straightforward yet unessential one is that we minimize costs instead of maximizing utility. More important differences include:

- (i) *Extension to adaptation:* We model adaptation as a perfect adjustment, not a control variable as in related models.<sup>6</sup> With respect to the terminology reviewed in Section 2.1, adaptation in our model is proactive when players are forward-looking and reactive when they are myopic. Moreover, adaptation is conducted at the private level, but, since it affects the state of the system, also has a public dimension. Therefore, it is neither a pure public nor private good (or bad), but rather a blend of the two, which some authors refer to as an impure public good (or bad) (Kotchen, 2005).
- (ii) *Generalization:* Related models have focused on uniform public goods. Our model can address a broader set of goods, in particular asymmetric ones, which, according to Figure 1, are empirically most relevant.
- (iii) *Simplification:* Payoff functions in most dynamic-game models of global warming are quadratic in the state variable. In contrast, following a few others (e.g., Jørgensen and Zaccour, 2001; Beccherle and Tirole, 2011), we adopt a linear assumption. Within such a structure, the open-loop and feedback Nash equilibria are known to coincide (Fershtman, 1987). The model therefore reconciles the flexibility of an open-loop information structure with the property of subgame perfectness. It allows us to derive closed-form solutions for all variables of interest, including intertemporal welfare, which is typically not commented on in quadratic models. From an empirical perspective, the linear assumption can be relevant at the global scale, though less at the local one (Dell et al., 2014; Burke et al., 2015). It seems acceptable to model short-term impacts.

## 4. Optimal paths

Standard analysis of a public bad consists in comparing the social optimum with a strategic equilibrium. We model the former as a cooperative equilibrium and the latter as an open-loop Nash equilibrium. Both solution concepts rely on perfect foresight. In addition, we characterize the myopic equilibrium which emerges when the players ignore the intertemporal effects of their and their competitor's actions on the system.

The exposition starts with the general problem that generates the Nash equilibrium ( $N$ ). Some formulas will also apply to the myopic ( $M$ ) and socially optimal ( $S$ ) equilibria. They are indexed by  $k$ , with  $k \in \{N, S, M\}$ . Recall that we confine our attention to  $d \geq |b|$ .

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<sup>6</sup>Because mitigation and adaptation are separable and the latter is not a control variable, the question of substitutability vs. complementarity, which receives most attention in related papers, does not apply here.

#### 4.1. Nash equilibrium ( $N$ )

Non-cooperative, forward-looking Player  $i$  minimizes its private cost (3) discounted at some common rate  $r > 0$ , subject to the transition equation:

$$(6) \quad \begin{aligned} & \text{Minimize } \int_0^{+\infty} \gamma^i(q^i, T) e^{-rt} dt \\ & \text{subject to (4), } q^i \geq 0, T^0 \text{ given} \end{aligned}$$

Each player's program is solved by minimizing the specific current-value Hamiltonian ( $-i$  denotes the competitor):

$$(7) \quad \mathcal{H}_N^i(T, q^i, q^{-i}, \lambda_N^i) \equiv \gamma^i(q^i, T) - \lambda_N^i [e^i(q^i, T) + e^{-i}(q^{-i}, T) - sT]$$

Player  $i$  faces a specific co-state variable  $\lambda_N^i$  associated with the GHG concentration, which can be interpreted as the economic value the player attributes to warming.

The first-order necessary conditions for optimal mitigation are met when  $\forall i \in \{c, w\}$ ,  $\partial \mathcal{H}_N^i / \partial q^i = 0$ .<sup>7</sup> Then, the marginal cost of mitigation  $\delta^i q^i$  equalizes its marginal benefit in terms of reduced energy expenditures, net of the marginal opportunity cost of warming,  $1 - \lambda^i$ . The resulting mitigation effort  $q^i$  decreases with the mitigation cost  $\delta^i$ :

$$(8) \quad q_k^i = \begin{cases} (1 - \lambda_k^i) / \delta^i & \text{if } \lambda_k^i \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

with  $k = N$ . Player  $i$ 's co-state equation is

$$(9) \quad \dot{\lambda}_N^i - r\lambda_N^i = -\frac{\partial \mathcal{H}_N^i}{\partial T} \Rightarrow \begin{cases} \dot{\lambda}_N^c = (r + b - d + s)\lambda_N^c - b \\ \dot{\lambda}_N^w = (r + b - d + s)\lambda_N^w + d \end{cases}$$

and the transversality condition is

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<sup>7</sup>Throughout, the Hamiltonians are convex with respect to both the control and state variables. The first-order necessary conditions are thus also sufficient for minimization. Existence and uniqueness of the fixed points is guaranteed.

$$(10) \quad \lim_{t \rightarrow \infty} e^{-rt} \lambda_k^i(t) T_k(t) = 0$$

with  $k = N$ . Setting  $\lambda_N^i = 0$  in Equations (9) provides the value of the co-state variables at steady state:

$$(11) \quad \begin{cases} \lambda_N^c(\infty) = \frac{b}{r+b-d+s} \\ \lambda_N^w(\infty) = \frac{-d}{r+b-d+s} \end{cases}$$

Assuming that  $d \geq |b|$  and the stability condition (5) is met,  $\lambda_N^w(\infty) \leq 0$  and  $|\lambda_N^w(\infty)| \geq |\lambda_N^c(\infty)|$ . That is, global warming has a relatively large negative value to the warm region and a relatively small value to the cold region – positive if  $b \geq 0$  and negative otherwise.

In all equilibria  $k \in \{N, S, M\}$ , the steady-state temperature is obtained by setting  $\dot{T} = 0$ :

$$(12) \quad T_k^\infty \equiv T_k(\infty) = \frac{2\epsilon - \sum_i q_k^i(\infty)}{b - d + s}$$

The state and co-state trajectories are given by the differential system formed by Equations (4) and (11). It can be shown that solutions have the following forms (with  $U^i, V$  unknown):

$$(13) \quad \begin{cases} \forall i \in \{c, w\} \quad \lambda_k^i(t) = U^i e^{(r+b-d+s)t} + \lambda_k^i(\infty) \\ T_k(t) = T_k^\infty + \frac{e^{(r+b-d+s)t}}{r+2(b-d+s)} \sum_i \frac{U^i}{\delta^i} + V e^{-(b-d+s)t} \end{cases}$$

with  $k = N$  here. The stability condition (5) and the transversality conditions (10) impose  $U^i = 0$ . The co-state variables are stationary in current value and, thanks to Equation (8), so are mitigation efforts. We therefore drop temporal arguments in further notations:

$$(14) \quad \forall t \quad \begin{cases} \lambda_k^i(t) = \lambda_k^i(\infty) \equiv \lambda_k^i \\ q_k^i(t) = q_k^i(\infty) \equiv q_k^i \end{cases}$$

The value of  $V$  can now easily be found, leading to the following warming path:

$$(15) \quad T_k(t) = T_k^\infty - (T_k^\infty - T^0)e^{-(b-d+s)t}$$

This can be used to derive the present-value cost to Player  $i$ :

$$(16) \quad \begin{aligned} \Gamma_k^i &= \int_0^{+\infty} \gamma_k^i(q_k^i, T_k(t)) e^{-rt} dt \\ &= \frac{\epsilon - q_k^i + \alpha^i T_k^\infty + \delta^i (q_k^i)^2 / 2}{r} - \frac{\alpha^i (T_k^\infty - T^0)}{r + b - d + s} \text{ with } \begin{cases} \alpha^c = -b \\ \alpha^w = d \end{cases} \end{aligned}$$

Finally, the present-value social cost is  $\Gamma_k^{c+w} \equiv \Gamma_k^c + \Gamma_k^w$ .

#### 4.2. Social optimum ( $S$ )

Cooperative, forward-looking players jointly minimize their present discounted cost, subject to the transition equation:

$$(17) \quad \begin{aligned} &\text{Minimize}_{q^c, q^w} \int_0^{+\infty} \sum_{i \in \{c, w\}} \gamma^i(q^i, T) e^{-rt} dt \\ &\text{subject to (4), } q^c \geq 0, q^w \geq 0, T^0 \text{ given} \end{aligned}$$

The current-value Hamiltonian to be minimized is:

$$(18) \quad \mathcal{H}_S(T, q^c, q^w, \lambda) \equiv \sum_{i \in \{c, w\}} \gamma^i(q^i, T) - \lambda [e^c(q^c, T) + e^w(q^w, T) - sT]$$

with  $\lambda$  the co-state variable associated with the GHG concentration.  $\lambda$  is the social value of global temperature, now identically faced by both players:

$$(19) \quad \lambda_S^c = \lambda_S^w \equiv \lambda$$

The first-order necessary condition for optimality ( $\forall i \in \{c, w\} \partial \mathcal{H}_S / \partial q^i = 0$ ) leads to the mitigation effort defined in Equation (8) with  $k = S$ . There is now a single transversality condition, defined in Equation (10) with  $k = S$ , and a single co-state equation:

$$(20) \quad \dot{\lambda} - r\lambda = -\frac{\partial \mathcal{H}_S}{\partial T} \Rightarrow \dot{\lambda} = (r + b - d + s)\lambda - b + d$$

The co-state variable at steady state, defined by  $\dot{\lambda} = 0$ , is:

$$(21) \quad \lambda = \frac{b - d}{r + b - d + s}$$

In our public-bad regime ( $d > |b|$ ), assuming the steady state is stable (Eq. (5)), the social value of warming is negative ( $\lambda < 0$ ).<sup>8</sup> It is the sum of the regions' private values:  $\lambda_N^c + \lambda_N^w = \lambda$ .<sup>9</sup>

The temperature path and the present-value social cost are those defined in Equations (15) and (16), respectively, with  $k = S$ . Note that here, the differential system (13) only comprises two equations, namely (4) and (20).

#### 4.3. Myopic equilibrium ( $M$ )

Non-cooperative, myopic Player  $i$  minimizes its private instantaneous cost  $\gamma^i(q^i, T)$  with respect to its own mitigation effort  $q^i$ . The players do not incorporate the transition equation (4), hence do not consider any co-state variable. Assuming the stability condition (5) is met, the myopic equilibrium can be derived from Equations (8), (15) and (16), with  $k = M$  and using the following notation abuse:

$$(22) \quad \lambda_M^c = \lambda_M^w \equiv 0$$

The differential system (13) here boils down to only one equation, (4).

## 5. Comparative welfare analysis

For each equilibrium  $k \in \{N, S, M\}$ , mitigation efforts are obtained by replacing the co-state variable in Equation (8) by its expression in (11), (20) and (22). Replacing mitigation in

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<sup>8</sup>If global warming were a public good ( $b > d$ ), the stability condition (5) would always be satisfied and  $\lambda$  would be positive.

<sup>9</sup>To put the analysis in the perspective of recent discussions, the  $\lambda_N^i$ s can be seen as domestic social costs of carbon (Fraas et al., 2016) while  $\lambda$  can be seen as the global social cost of carbon (Guivarch et al., 2016).

(12) by the resulting expressions provides the steady-state temperature. Lastly, total costs are obtained by inserting all aforementioned expressions into Equation (16).

We discuss here the welfare effects associated with the resulting formulas, fully listed in Table 1. We refer to Pareto improvements when decentralization makes both players better-off, i.e., lowers their present discounted cost, and to Hicks-Kaldor improvements when one player ends up better-off, the other ends up worse-off and the joint present discounted cost is lower. Unlike the former, the latter necessitates transfers between players.

The following mitigation-cost ratio will prove useful in the discussion:

$$(23) \quad \mu \equiv \delta^w / \delta^c$$

### 5.1. Uniform public bad ( $d \geq -b > 0$ )

When both regions face negative impacts, the classical free-riding problem arises. The players under-mitigate under myopia and, to a lesser extent, under perfect foresight:

$$(24) \quad \begin{cases} \forall i \in \{c, w\} q_S^i > q_N^i > q_M^i > 0 \\ T_S^\infty < T_N^\infty < T_M^\infty \\ \Gamma_S^{c+w} < \Gamma_N^{c+w} < \Gamma_M^{c+w} \end{cases}$$

Forward-looking, non-cooperative players fail to internalize the negative externality they impose on their competitor, but at least they internalize their and their competitor's effect on warming. There is a direct equivalence here between low mitigation, high warming and high social cost (or low social welfare). Unless damages are identical ( $b = -d$ ), the degree of free-riding differs across regions ( $q_N^c < q_N^w$ ), even if they have identical mitigation capabilities ( $\delta^c = \delta^w$ ).

Cooperation is, by construction, welfare-improving. Depending on the relative magnitude of the impact ratio  $-d/b$  and the mitigation ratio  $\mu$ , it can generate Pareto improvements:

$$(25) \quad \begin{cases} -d/b < 1/\sqrt{2\mu} & \Rightarrow \Gamma_S^c < \Gamma_N^c \text{ and } \Gamma_S^w > \Gamma_N^w & \text{(Hicks-Kaldor improvement)} \\ 1/\sqrt{2\mu} \leq -d/b < \sqrt{2/\mu} & \Rightarrow \Gamma_S^c < \Gamma_N^c \text{ and } \Gamma_S^w \leq \Gamma_N^w & \text{(Pareto improvement)} \\ -d/b \geq \sqrt{2/\mu} & \Rightarrow \Gamma_S^c \geq \Gamma_N^c \text{ and } \Gamma_S^w < \Gamma_N^w & \text{(Hicks-Kaldor improvement)} \end{cases}$$



and

$$(26) \quad \begin{cases} -d/b \leq 1 + 2/\mu & \Rightarrow \Gamma_S^c \leq \Gamma_M^c \text{ and } \Gamma_S^w < \Gamma_M^w & \text{(Pareto improvement)} \\ -d/b > 1 + 2/\mu & \Rightarrow \Gamma_S^c > \Gamma_M^c \text{ and } \Gamma_S^w < \Gamma_M^w & \text{(Hicks-Kaldor improvement)} \end{cases}$$

When players do not cooperate, moving from myopia to perfect foresight is also socially desirable:

$$(27) \quad \Gamma_N^c < \Gamma_M^c \text{ and } \Gamma_N^w < \Gamma_M^w \quad \text{(Pareto improvement)}$$

In the special case where the players are fully identical ( $b = -d$  and  $\delta^c = \delta^w$ ), which corresponds to the canonical model, the transitions from equilibrium  $M$  to  $N$  to  $S$  are systematically Pareto-improving.

## 5.2. Non-uniform public bad ( $d \geq b > 0$ )

The equivalence between low equilibrium temperature and low social cost vanishes as soon as one player enjoys benefits, that is, when one considers a non-uniform public bad.<sup>10</sup>

**Mitigation.** When the public bad is asymmetric, the Nash equilibrium is such that the cold region free rides while the warm region over-mitigates, which we refer to as *free driving*:

$$(28) \quad \begin{cases} 0 \leq q_N^c < q_M^c \leq q_S^c \\ 0 < q_M^w \leq q_S^w < q_N^w \end{cases}$$

The cold region fails to internalize the negative externality it imposes on the warm region; in turn, the latter fails to internalize the positive externality it exerts on the former. Note that mitigation by the cold region is now lower under perfect foresight than under myopia and hits the non-negativity constraint if  $d > r + s$ . In the special case where players are symmetric ( $b = d$ ), myopic and socially-optimal mitigation efforts are confounded ( $q_M^i = q_S^i$ ).

<sup>10</sup>In addition, in the least asymmetric case where  $d > b = 0$  then  $0 \leq q_N^c = q_M^c < q_S^c$  and  $0 < q_M^w < q_S^w = q_N^w$ . It follows that  $T_S^\infty < T_N^\infty < T_M^\infty$ . Transitions from equilibrium  $M$  to  $N$  to  $S$  are only Hicks-Kaldor improving:  $\forall i \in \{c, w\} \Gamma_S^i < \Gamma_N^i < \Gamma_M^i$ .

**Global warming.** Under-mitigation by both countries under myopia leads to an inefficiently high level of warming:

$$(29) \quad \begin{cases} d > b \Rightarrow T_M^\infty > T_S^\infty \\ d = b \Rightarrow T_M^\infty = T_S^\infty \end{cases}$$

How the Nash-equilibrium level of warming compares to that of other equilibria is more ambiguous. In the special case where the players face the same mitigation cost ( $\delta^c = \delta^w$ ), they behave identically in both the myopic ( $q_M^c = q_M^w$ ) and socially optimal equilibria ( $q_S^c = q_S^w$ ). Their Nash-equilibrium mitigation efforts only differ according to the different impact they face. Since by assumption, local damages offset local benefits, free riding exceeds free driving. The equilibrium level of warming is then inefficiently high but lower than under myopia, just like in the uniform public bad case.

Outside of this identical-mitigation case, the balance between free riding and free driving is a priori ambiguous and depends on the relative magnitude of  $d/b$  and  $\mu$ . Warming is too high if the impact ratio is lower than the inverse mitigation ratio, and too low otherwise:

$$(30) \quad \begin{cases} d/b > 1/\mu \Rightarrow T_N^\infty > T_S^\infty \\ d/b = 1/\mu \Rightarrow T_N^\infty = T_S^\infty \\ d/b < 1/\mu \Rightarrow T_N^\infty < T_S^\infty \end{cases}$$

The latter implication holds when relatively low mitigation costs are combined with relatively high damages. Then, the warm region is relatively more willing to free drive, which can result in excessive aggregate mitigation.

The possibility of insufficient warming, perhaps counter-intuitive, is not new. Previous models built on a linear-quadratic structure and focused on uniform public bad regimes have found that it could occur under restrictive conditions related to the discount rate and marginal utility (Dockner and Long, 1993; Zagonari, 1998; Rubio and Casino, 2002; Wirl, 2007). Our contribution is to uncover new conditions. As we saw in the previous subsection, insufficient warming cannot occur in our simple, linear-state framework if both impacts are negative or if mitigation is identical across countries. Its occurrence is therefore due to a combination of asymmetric impacts and heterogeneous mitigation.

Warming under perfect foresight is larger than under myopia if the impact ratio is lower than the mitigation ratio, and lower otherwise:

$$(31) \quad \begin{cases} d/b < \mu \Rightarrow T_N^\infty > T_M^\infty \\ d/b = \mu \Rightarrow T_N^\infty = T_M^\infty \\ d/b > \mu \Rightarrow T_N^\infty < T_M^\infty \end{cases}$$

The former implication holds when the warm region suffers a "double jeopardy," that is, relatively high damages combined with relatively high mitigation costs. Then, there is relatively little free riding, which can result in an aggregate mitigation effort lower than under myopia.

**Economic efficiency.** Cooperation has the same welfare effect under perfect foresight as with a heterogeneous public bad:

$$(32) \quad \begin{cases} \Gamma_S^{c+w} < \Gamma_N^{c+w} \\ d/b < 1/\sqrt{2\mu} \\ 1/\sqrt{2\mu} \leq d/b < \sqrt{2/\mu} \\ d/b \geq \sqrt{2/\mu} \end{cases} \Rightarrow \begin{cases} \Gamma_S^c < \Gamma_N^c \text{ and } \Gamma_S^w > \Gamma_N^w & \text{(Hicks-Kaldor improvement)} \\ \Gamma_S^c < \Gamma_N^c \text{ and } \Gamma_S^w \leq \Gamma_N^w & \text{(Pareto improvement)} \\ \Gamma_S^c \geq \Gamma_N^c \text{ and } \Gamma_S^w < \Gamma_N^w & \text{(Hicks-Kaldor improvement)} \end{cases}$$

However, the transition from myopia is no longer systematically Pareto-improving:

$$(33) \quad \Gamma_S^{c+w} < \Gamma_M^{c+w}, \Gamma_S^c > \Gamma_M^c \text{ and } \Gamma_S^w < \Gamma_M^w \text{ (Hicks-Kaldor improvement)}$$

Unlike with a heterogeneous public bad, the desirability of non-cooperative players moving from myopia to perfect foresight is also much restricted. Most transitions deteriorate total welfare, weakly when only one player ends up worse-off and strongly when both players do; Hicks-Kaldor improvements can only occur for high impact ratios:

$$(34) \quad \left\{ \begin{array}{ll} d/b < \mu/2 & \Rightarrow \Gamma_N^c < \Gamma_M^c, \Gamma_N^w > \Gamma_M^w \text{ and } \Gamma_N^{c+w} > \Gamma_M^{c+w} \\ & \text{(weak deterioration)} \\ \mu/2 \leq d/b < 2\mu & \Rightarrow \Gamma_N^c \geq \Gamma_M^c, \Gamma_N^w > \Gamma_M^w \text{ and } \Gamma_N^{c+w} > \Gamma_M^{c+w} \\ & \text{(strong deterioration)} \\ 2\mu \leq d/b < 1 + \mu + \sqrt{1 + \mu + \mu^2} & \Rightarrow \Gamma_N^c > \Gamma_M^c, \Gamma_N^w \leq \Gamma_M^w \text{ and } \Gamma_N^{c+w} > \Gamma_M^{c+w} \\ & \text{(weak deterioration)} \\ d/b \geq 1 + \mu + \sqrt{1 + \mu + \mu^2} & \Rightarrow \Gamma_N^c > \Gamma_M^c, \Gamma_N^w < \Gamma_M^w \text{ and } \Gamma_N^{c+w} \leq \Gamma_M^{c+w} \\ & \text{(Hicks-Kaldor improvement)} \end{array} \right.$$

### 5.3. Summary

Figure 5 summarizes welfare effects. The five combinations of interest between adaptation and mitigation capabilities are displayed in Figure 6 in the  $(\mu, d/b)$  plane, with labels  $i - v$ . Figure 7 maps the corresponding Nash equilibria  $N_{i-v}$  in the welfare-temperature space. The myopic equilibrium  $M$  is systematically warmer and less efficient than the social optimum  $S$ . When damages are relatively high (Domain  $i$ ), the Nash equilibrium  $N_i$  entails intermediate levels of warming and efficiency. This is also the only possible outcome in a uniform public bad regime. With respect to the empirical estimates put together in Figure 2,  $N_i$  could be the Nash equilibrium of a game played by the U.S. and China, two big players facing similar damages. Moreover, since the former's mitigation cost is less than double that of the latter's, according to Figure 5, a transition from the Nash equilibrium to the social optimum could be Pareto-improving.

When mitigation costs are lower in the warm region (Domains  $ii$  and  $iii$ ), free driving is exacerbated, leading to excessive mitigation, hence too little warming ( $N_{ii}$  and  $N_{iii}$ ). When impacts are not too asymmetric (Domain  $iii$ ), the outcome is less efficient than the myopic one. In addition to being counter-intuitive, these outcomes seem implausible, as the empirical studies reviewed in Section 2.2 offer little support for the underlying conditions. Yet very tentatively,  $N_{ii}$  or  $N_{iii}$  could be the Nash equilibrium of a game played by the U.S. and India. When mitigation turns out cheaper in the cold region (Domains  $iv$  and  $v$ ), which seems more plausible, free driving is reduced, leading to insufficient mitigation, hence too much warming ( $N_{iv}$  and  $N_v$ ). Warming can even be higher than under myopia ( $N_v$ ).  $N_{iv}$  and  $N_v$  could be the Nash equilibrium of a game played by India and Russia.

## 6. Policy

The previous section has shown in a two-player framework that rather than one-fold, the externality generated by global warming is multi-fold.<sup>11</sup> The multiplicity is masked in related models by the assumption that countries are myopic, or forward-looking but identical. As we will develop in the present Section, recognizing it can provide a rationale for implementing country-specific Pigouvian instruments.

### 6.1. Pigouvian prices

Let us start with adopting the viewpoint of a supra-national social planner seeking to internalize the externalities associated with the global public bad. Pigouvian prices  $p^i$  are imposed on the GHG emissions of each player  $i$ . Recycling of the tax receipts (if  $p^i > 0$ ) or funding of the subsidy payments (if  $p^i < 0$ ) are operated in a lump-sum manner. Optimal values of the  $p^i$ s are set by the social planner by backward induction in a two-stage game: In the second stage, the players respond optimally to a given  $p$ ; in the first stage, the planner seeks the values of the  $p^i$ s that maximize social welfare (that is, minimize joint cost).

The second stage is equivalent to solving the same private optimization programs as in Section 4 with the objective functions modified as follows:  $e^i(q^i, T)(1 + p^i) + m(q^i)$ . Let  $\rho_k^i$  be the new co-state variable to player  $i$  associated with each solution  $k \in \{M, N\}$ . The new first-order condition for privately optimal mitigation is:

$$(35) \quad (q_k^i)_{\text{after incentive}} = \frac{1 + p_k^i - \rho_k^i}{\delta^i}$$

and the steady-state value of the co-state variables (with the  $\lambda$ s defined in Section 4):

$$(36) \quad \rho_k^i = (1 + p_k^i)\lambda_k^i$$

In the first stage, the planner minimizes the sum of the modified cost functions, subject to temperature increase. This leads to socially optimal mitigation efforts  $q_S^i = (1 - \lambda)/\delta^i$ . Matching the right-hand side with that of Equation (35) provides optimal prices:

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<sup>11</sup>Extending the logic to  $n$  players, the number of externalities would be  $n(n - 1)$ , as every contributor to the public bad affects all others.

$$(37) \quad p_k^i = \frac{\lambda_k^i - \lambda}{1 - \lambda_k^i}$$

**Myopia.** With  $\lambda_M^i \equiv 0$ , the optimal price is the same to both players:  $p_M^c = p_M^w = -\lambda > 0$ . Since the players neither internalize their nor their competitor's actions, it is as if they were facing a global externality equal to the aggregate damage. Such a negative externality should be internalized through a uniform tax on emissions.

**Perfect foresight.** Nash players internalize the consequences of their and their competitor's actions on themselves, but not the consequence of their actions on their competitor. The following prices should be implemented to address the problem:

$$(38) \quad \begin{cases} p_N^c = \frac{d}{r-d+s} > 0 \\ p_N^w = \frac{-b}{r+b+s} \end{cases}$$

In the special case where the players suffer identical damages ( $b = -d$ ), they face the same tax rate  $p_N^c = p_N^w = d/(r-d+s)$ . This rate is smaller than that they would face under myopia ( $-\lambda = 2d/(r-2d+s)$ ). Otherwise, the tax rates differ. If the cold region enjoys benefits ( $b > 0$ ), GHG emissions should even be subsidized in the warm region ( $p_N^w < 0$ ).<sup>12</sup> Whatever its sign, the price to the cold region is systematically smaller than to the warm region ( $|p_N^c| > |p_N^w|$ ). Lastly, note that the prices do not depend on mitigation costs.

These results are at odds with the textbook recommendation of a single price to internalize the global negative externality. The result has already been challenged in the literature, mostly on equity grounds (Chichilnisky and Heal, 1994; Hoel, 1996; d'Autume et al., 2016). Here we put forward a simple efficiency argument: unless players are myopic or identical, strategic behavior creates multiple externalities, hence calling for multiple prices. This is solely due to heterogeneity of impacts and unrelated to heterogeneity of mitigation costs.

Although it is not systematically optimal in our framework, a unique price still has merits in other contexts. In particular, it can avoid so-called emission leakage which may occur if emitting activities can be relocated in low-price countries or sectors. Moreover, if we extend the logic developed in our model to  $n$  countries, each one would have to internalize the net

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<sup>12</sup>In practice, as agreement on the phasing out of fossil fuel subsidies is getting broader, such a subsidy program would rather target air conditioners.

externality it imposes on the  $n - 1$  other players. We conjecture that as  $n$  grows, the domestic prices each player should face converge toward the socially optimal value.

## 6.2. International transfers

Implementing a uniform Pigouvian price is usually thought of as the responsibility of a supra-national entity. Such an intervention is relevant to the myopic and homogeneous context studied above. When it comes to the country-specific instruments needed in other contexts, however, it is perhaps more natural to think of implementation as a domestic matter.

Let us consider the following negotiation architecture: forward-looking countries engage in Coasian bargaining to reach the social optimum; this results in national emission reduction commitments; those commitments are supported by domestic Pigouvian instruments recycled (or funded) in a lump-sum manner. That is, tax receipts (or subsidy payments) remain confined domestically.

In light of our model, such a negotiation can be Pareto-improving, letting both countries better-off without requiring transfers. As illustrated in Figure 5, this occurs within the corridor where  $1/\sqrt{2\mu} \leq |d/b| \leq \sqrt{2/\mu}$ . Otherwise, Region  $i$  ends up better-off while  $-i$  ends up worse-off. The negotiation is only Hicks-Kaldor improving and requires  $i$  to transfer an amount  $\tau$  to  $-i$ . Any positive value of  $\tau$  can be agreed upon as long as the following participation constraints are satisfied:

$$(39) \quad \Gamma_S^{-i} - \Gamma_N^{-i} < \tau < \Gamma_N^i - \Gamma_S^i$$

When the discrepancy of impacts is so large that  $|d/b| > \sqrt{2/\mu}$ , the transfers operate from the warm to the cold region. While this might conform to economic reasoning, according to which better-off agents are typically willing to compensate worse-off agents, it faces legal and ethical difficulties. As the responsibility of warm regions in anthropogenic global warming tends to be low, such a transfer is indeed inconsistent with the UNFCCC's principle of countries acting according to their "common but differentiated responsibilities and respective capabilities" (Paris Agreement, Article 2, paragraph 2).

In the narrower space where  $|d/b| < 1/\sqrt{2\mu}$ , transfers operate from the cold to the warm region, which is more in accord with the UNFCCC's principle. This could occur if low mitigation costs in the cold region allow for substantial free driving in the Nash equilibrium.

According to the empirical estimates put together in Figure 2, this could be relevant to bargaining between the U.S. and India.<sup>13</sup>

## 7. Conclusion

The global-warming problem has traditionally been modeled as a global externality imposing identical damages on all contributors. This view fails to capture strategic behaviors that arise when one considers a small set of big countries responsible for the bulk of global GHG emissions, as is the case with China, the U.S., India and Russia. A growing body of empirical literature points to asymmetries between those countries in terms of projected impacts, adaptation technologies and mitigation capabilities. We show in this paper that such asymmetries create multiple externalities, which leads to new predictions regarding global temperature, social welfare and efficient policy.

We studied the problem within a differential-game model integrating mitigation of and adaptation to global warming. The model is general enough to address the whole spectrum of public-good regimes. Its two-player, linear-state, deterministic structure is parsimonious enough to allow for welfare analysis. In turn, these assumptions confine the model to certain impacts, namely those associated with temperature, to short-term, non-catastrophic views and to certain economic sectors, such as buildings and agriculture, where adaptation to damages is GHG-intensive. Within this scope, three main insights arise. First, possibilities exist for non-cooperative behavior to generate too little global warming. This can occur if benefit prospects and low mitigation capabilities in one country induce the other to free ride substantially. The conditions for this counter-intuitive outcome however lack empirical support. Second, reaching the social optimum requires more transfers than what the canonical model predicts. Third, unless players are myopic or foresee identical damages, the bilateral externalities justify implementation of country-specific Pigouvian prices.

Our theoretical approach can help rationalize some real-world behaviors, in particular the intended NDCs (INDCs) submitted by the Parties to the UNFCCC (Aldy et al., 2016). The free-riding behavior predicted by the model seems to be quite plausible. For instance, Russia, which mainly expects benefits in the short run, submitted only modest reductions of net GHG emissions. Similarly, Canada's exit from the Kyoto protocol in 2011 can be interpreted as a free-riding move – admittedly undone since by a fairly ambitious INDC.

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<sup>13</sup>In this case, tax receipts in the cold region can directly be used to finance subsidy programs in the warm region. Otherwise, international and domestic transfers need not interact.



Bilateral agreements also seem to fit well in our framework. A naïve interpretation of empirical estimates in light of the model suggests that Coasian bargaining between the U.S. and China may be Pareto improving, thereby allowing progress against low-ambition Nash equilibrium without transfers. Furthermore, the myopic behavior considered in our model can be useful to model the few countries which, for a variety of reasons (e.g., civil war), do not take part in international climate negotiations. Against these propositions, evidence of the free-driving behavior predicted by our model is scarce. It can provide a reasonable interpretation for activism by Pacific Islands, a small entity which global warming puts at severe risk. But it is of little help to interpret India’s INDC, a country also exposed to severe damages yet committing to modest GHG emission reductions.<sup>14</sup> This illustrates a limitation of our economic approach, which ignores legal and ethical motivations such as the UNFCCC’s principle of countries acting according to their “common but differentiated responsibilities and respective capabilities.”

Our main contribution is to address the question of the gainers and losers of global warming, an issue early pointed out as crucial (Schelling, 1992). Most of the discussion has focused on how equity weights should be used when one aggregates impacts (Azar, 1999; Anthoff et al., 2009). We show that prior to requiring correction by equity weights, asymmetries matter for overall (unweighted) efficiency. More generally, we contribute to developing new economic concepts to address global warming. Some authors have started to incorporate real-world frictions into the canonical public-good model. Kotchen (2005) uses the concept of *impure* public good to take into account the possibility that mitigation yields private benefits (e.g., alleviation of local pollution in China) in addition to collective ones. That is another type of free driver. More recently, Nordhaus (2015) proposes to form climate *clubs* to overcome free riding, thereby conveying a notion of price differentiation among members and non-members. Along these lines, our *asymmetric* conceptualization concurs to stress that differentiated policies promoted by bottom-up initiatives can be efficient solutions to the global-warming problem.

We see several interesting extensions to our model. On the modeling side, non-linear impacts should be considered (Burke et al., 2015). One difficulty is that it would bring non-linearities into the transition equation, which would seriously impair analytical tractability

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<sup>14</sup>Another interesting example is that of the Organization of the Petroleum Exporting Countries (OPEC). Considering the high exposure to impacts and high (opportunity) cost of mitigation of its members, our model would predict them to free drive. Yet in practice, they show reluctance to engage in global-warming action.

(Xepapadeas, 2010). On the policy side, more research is needed on the international architecture which can generate differentiated Pigouvian prices. A global refunding system appears as a fruitful approach (Gersbach and Winkler, 2012).

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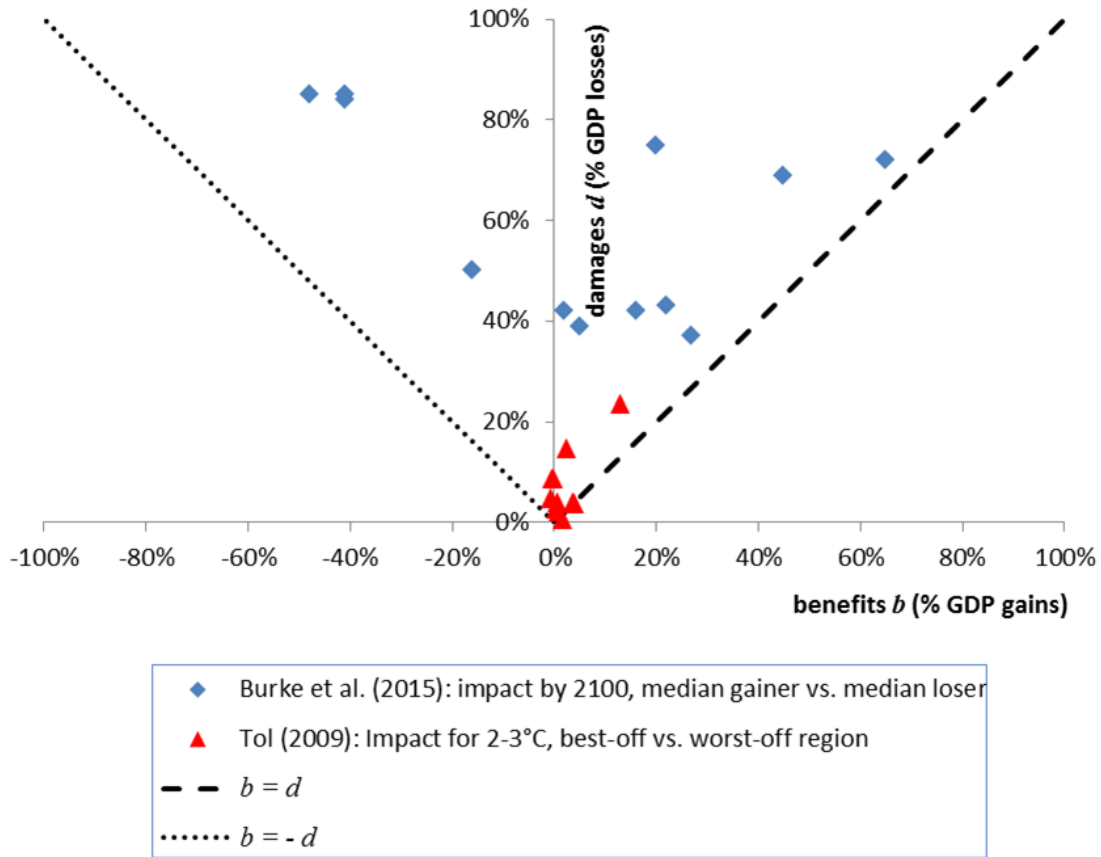
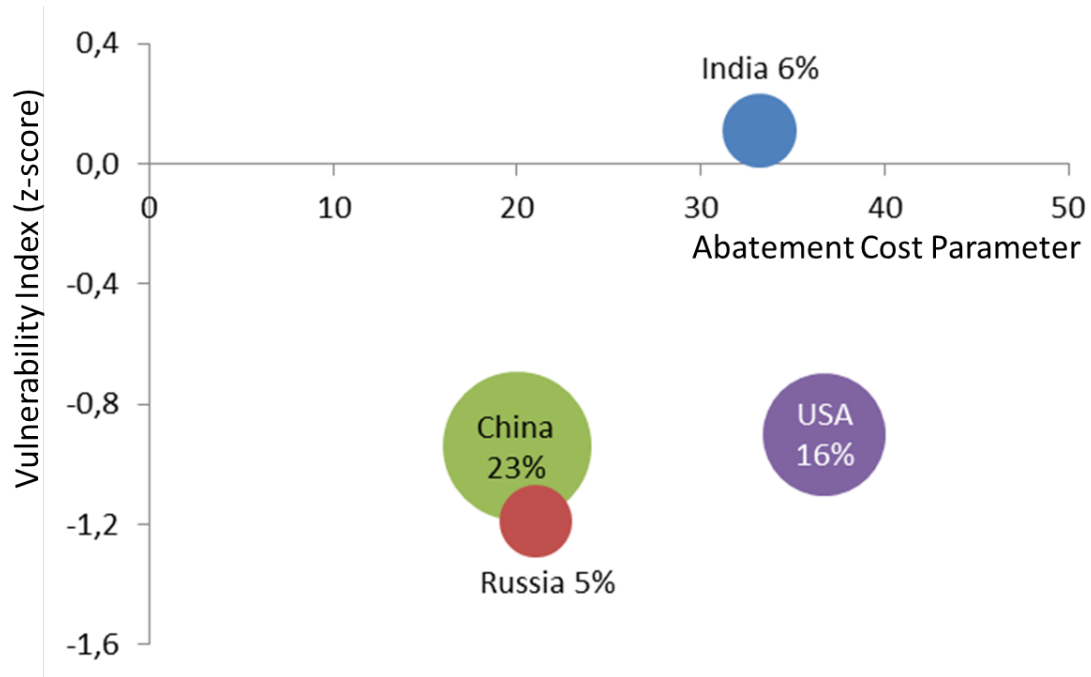


FIGURE 1. **Projections of global-warming impacts.** Each mark is a pair of impact estimate for a representative 'gainer' and a representative 'loser,' from two datasets. The red-triangle data come from Tol (2009, Table 1). Estimates come from 13 integrated assessments performed between 1993 and 2006. Impacts are calculated in percentage of gross domestic product (GDP), for levels of warming ranging from 1.0 to 3.0°C. The gainer and loser considered here are the best-off and worst-off region of each assessment. The blue-diamond data come from Burke et al. (2015, Extended Data Table 3). The 12 estimates result from simulations using four different climate models and three different growth scenarios. Impacts are calculated in % of GDP by 2100. The gainer and loser considered here are respectively the 75th and 25th percentiles of the impact distribution. Note that in the negative part of the horizontal axis, the gainers face damages.





**FIGURE 2. Adaptation and mitigation capabilities of the four largest GHG emitters among sovereign countries.** The horizontal axis depicts dimensionless abatement cost parameters. These are the inverse of the estimates reported in Nordhaus (2015, Table B-5), which, according to the author, are "inversely proportional to the cost of reducing a given unit of emissions." The vertical axis depicts the  $z$ -scores of a vulnerability index computed by Samson et al. (2011). The area of the disks represents the share of countries' GHG emissions reported in Olivier et al. (2014).

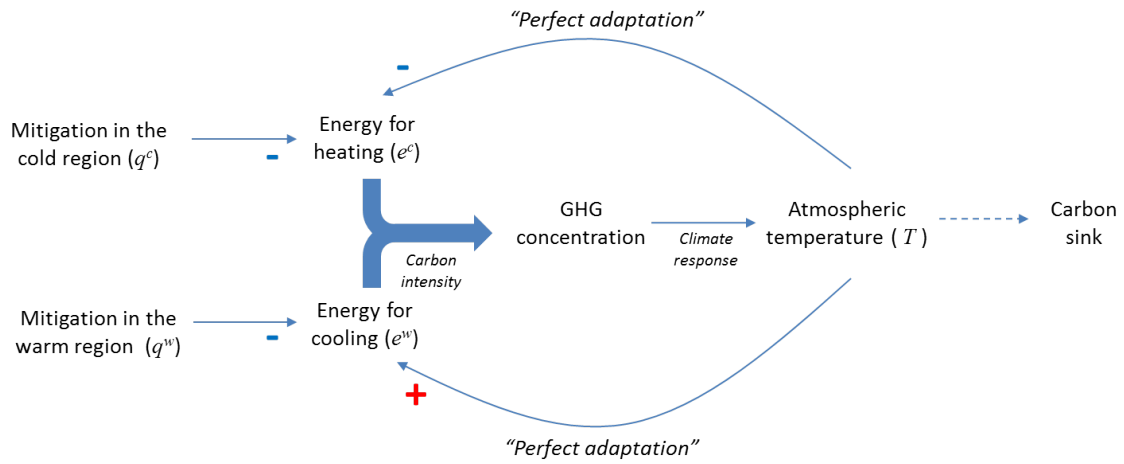


FIGURE 3. **Schematic view of the model.** The (-) and (+) signs represent negative and positive retroactions, respectively. Thick arrows denote stock accumulation. The dashed arrow denotes system outflow.

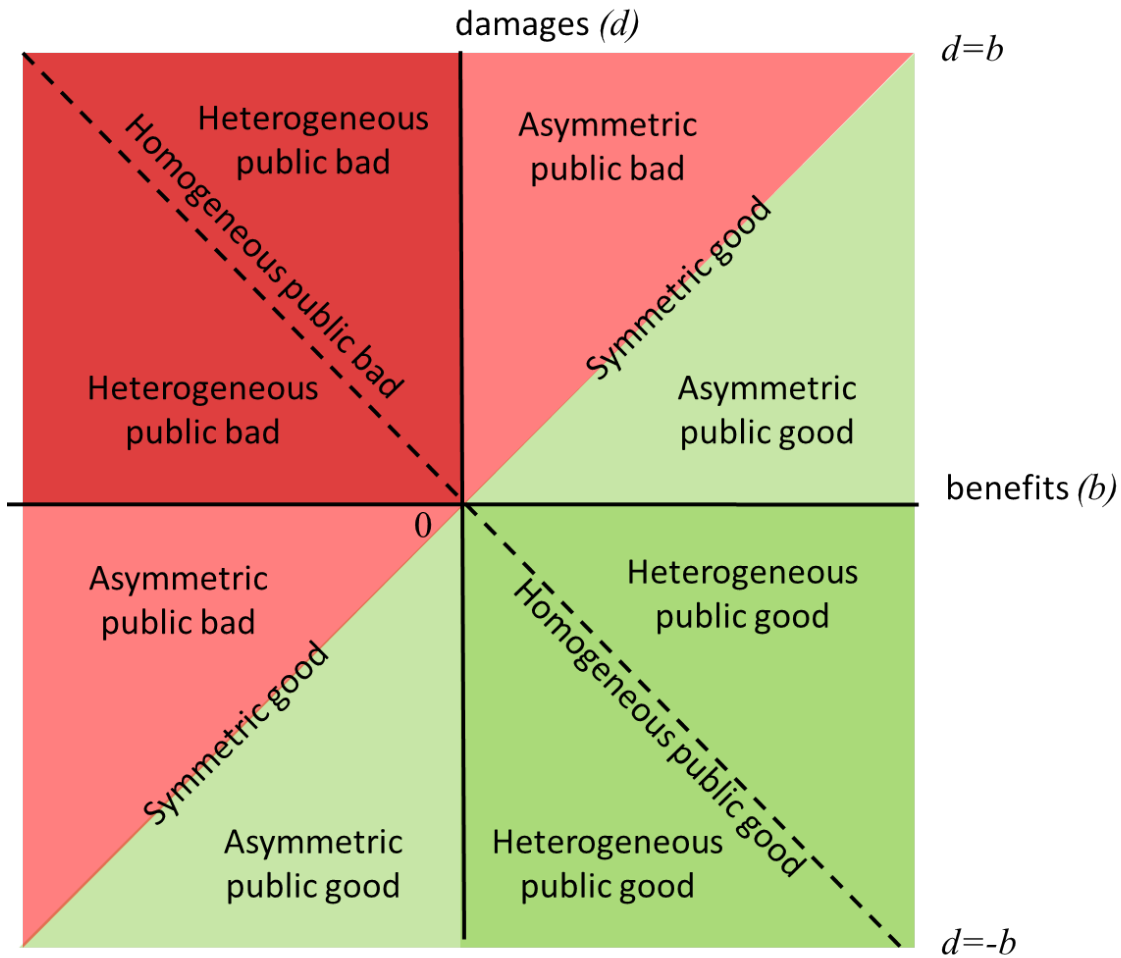


FIGURE 4. **Taxonomy of public-good regimes.** This graph illustrates the regimes defined in Section 3.2. Since  $d = -b$  is a symmetry axis, we focus on the upper semi-plane, where  $d \geq -b$ . Moreover, we focus on public bads, hence the red area where  $d \geq b$ . The resulting domain of interest, where  $d \geq |b|$ , covers most of the projection estimates displayed in Figure 1. Note that existing differential-game models of global warming tend to focus on the dark-red region (uniform, i.e., homogeneous and heterogeneous, public bads).

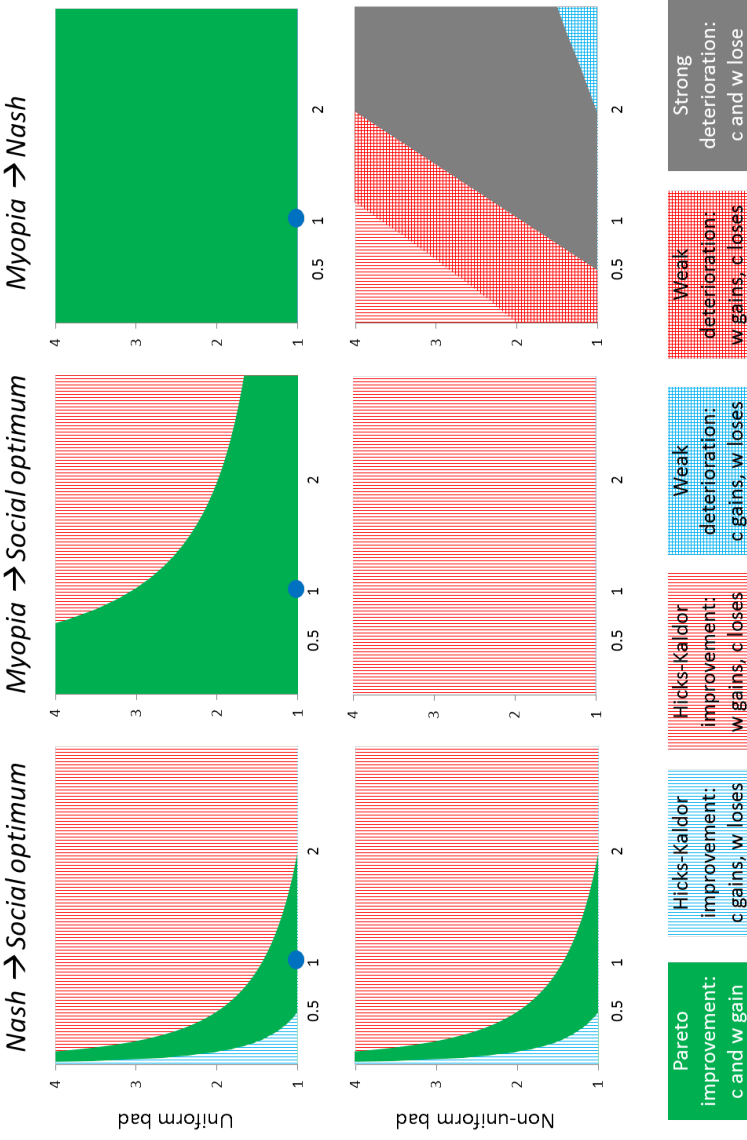


FIGURE 5. **Welfare effects of transitions.** The different graphs illustrate Equations (25), (26), (27), (32), (33) and (34). The horizontal axis depicts the mitigation ratio  $\mu$ . The vertical axis depicts  $-d/b$  in the first row and  $d/b$  in the second row. The  $(\mu = 1, -d/b = 1)$  points in the first row correspond to the outcomes with the canonical identical-player model.

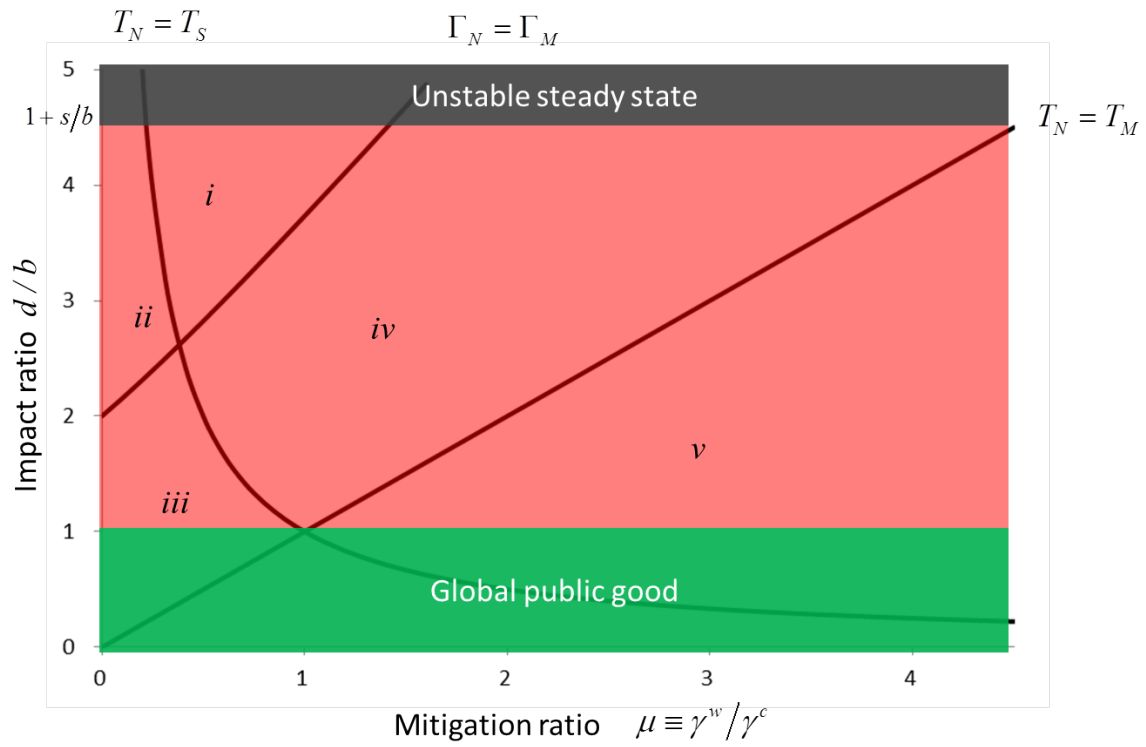


FIGURE 6. Mitigation-adaptation combinations for a non-uniform public bad. Equations (5), (30), (31) and (34) define five domains of  $(\mu, d/b)$ -combination corresponding to different tradeoffs between global warming and social welfare, as depicted in Figure 7.

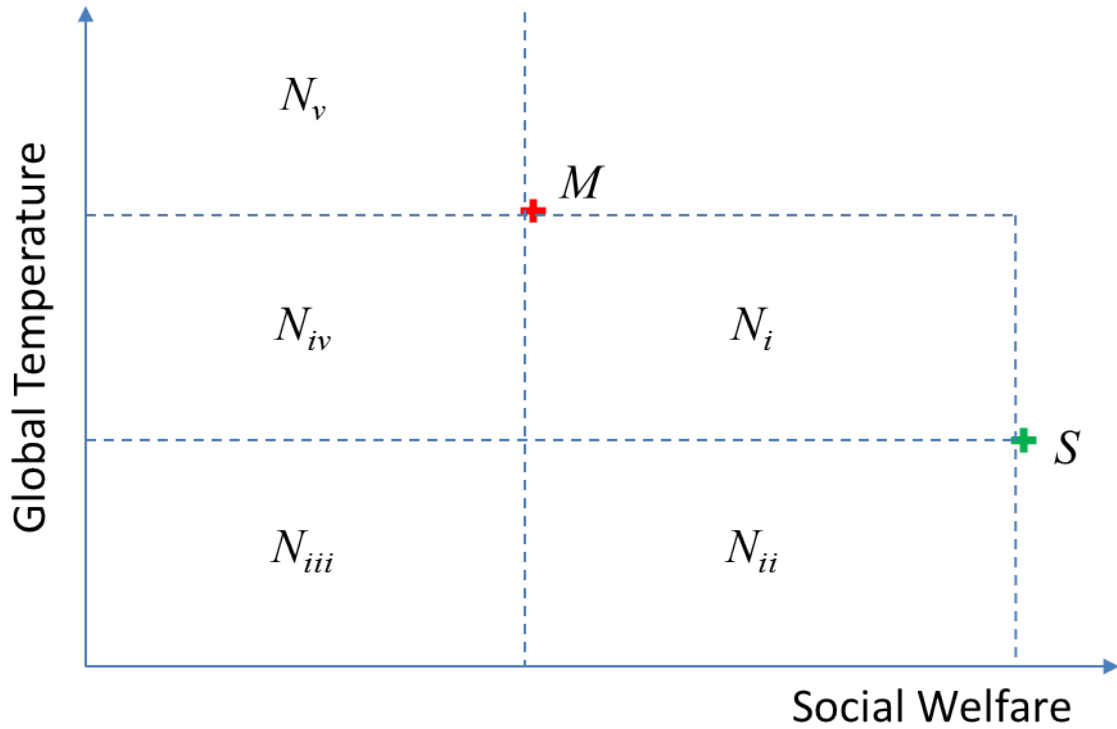


FIGURE 7. **Welfare-temperature tradeoffs for a non-uniform public bad.** For any arbitrary location of the social optimum  $S$ , the myopic equilibrium  $M$  entails both a higher temperature and lower welfare. These two points define five zones where the Nash equilibria  $N_{i-v}$  can be located, depending on the five domains of mitigation-adaptation combinations  $i - v$  depicted in Figure 6. Note that the Nash equilibrium with a uniform public bad would be located in zone  $N_i$ .

TABLE 1. Closed-form solutions of the model. By assumption,  $d \geq |b|$ . Moreover, assuming the stability condition (5) is satisfied, all denominators are positive.

	Nash equilibrium	Social optimum	Myopic equilibrium
	$k \equiv N$	$k \equiv S$	$k \equiv M$
$\lambda_k^c$	$\frac{b}{r+b-d+s}$	$\frac{-(d-b)}{r+b-d+s}$	0
$\lambda_k^w$	$\frac{-d}{r+b-d+s}$		
$q_k^c = \begin{cases} (1 - \lambda_k^c)/\delta^c & \text{if } \lambda_k^c \leq 1 \\ 0 & \text{otherwise} \end{cases}$	$= \begin{cases} \frac{1}{\delta^c} \frac{r-d+s}{r+b-d+s} & \text{if } d \leq r+s \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\delta^c} \frac{r+s}{r+b-d+s}$	$\frac{1}{\delta^c}$
$q_k^w = \begin{cases} (1 - \lambda_k^w)/\delta^w & \text{if } \lambda_k^w \leq 1 \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\delta^w} \frac{r+b+s}{r+b-d+s}$	$\frac{1}{\delta^w} \frac{r+s}{r+b-d+s}$	$\frac{1}{\delta^w}$
	Transition	Transition	Transition
	$j \equiv N \rightarrow S$	$j \equiv M \rightarrow S$	$j \equiv M \rightarrow N$
$\Delta_j \Gamma^\infty = \frac{-\Delta_j q^c - \Delta_j q^w}{b-d+s}$	$\frac{\delta^c b - \delta^w d}{\delta^c \delta^w (r+b-d+s)(b-d+s)}$	$\frac{-(d-b)(\delta^c + \delta^w)}{\delta^c \delta^w (r+b-d+s)(b-d+s)}$	$\frac{\delta^w b - \delta^c d}{\delta^c \delta^w (r+b-d+s)(b-d+s)}$
$\Delta_j \Gamma^c = \frac{-\Delta_j q^c + \Delta_j [(q^c)^2] \delta^c / 2}{r} - \frac{b(b-d+s) \Delta_j \Gamma^\infty}{r+b-d+s}$	$\frac{\delta^w d^2 - 2\delta^c b^2}{2r\delta^c \delta^w (r+b-d+s)^2}$	$\frac{(d-b)(\delta^w(b+d) + 2b\delta^c)}{2r\delta^c \delta^w (r+b-d+s)^2}$	$\frac{-b^2 \delta^w + 2bd\delta^c}{2r\delta^c \delta^w (r+b-d+s)^2}$
$\Delta_j \Gamma^w = \frac{-\Delta_j q^w + \Delta_j [(q^w)^2] \delta^w / 2}{r} + \frac{d(b-d+s) \Delta_j \Gamma^\infty}{r+b-d+s}$	$\frac{\delta^c b^2 - 2\delta^w d^2}{2r\delta^c \delta^w (r+b-d+s)^2}$	$\frac{-(d-b)(\delta^c(b+d) + 2d\delta^w)}{2r\delta^c \delta^w (r+b-d+s)^2}$	$\frac{-d^2 \delta^c + 2bd\delta^w}{2r\delta^c \delta^w (r+b-d+s)^2}$
$\Delta_j \Gamma^{c+w} = \Delta_j \Gamma^c + \Delta_j \Gamma^w$	$\frac{-\delta^w d^2 - \delta^c b^2}{2r\delta^c \delta^w (r+b-d+s)^2}$	$\frac{-(d-b)^2 (\delta^c + \delta^w)}{2r\delta^c \delta^w (r+b-d+s)^2}$	$\frac{-b^2 \delta^w - d^2 \delta^c + 2bd(\delta^c + \delta^w)}{2r\delta^c \delta^w (r+b-d+s)^2}$