Optimal education policy when parental time investments matter

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Abstract

Parents typically invest both time and money in the education of their children. We study the implications for the optimal education policy of taking parental time investments in children's education into account. The children's human capital production function depends on both parental time investments and formal education, either privately purchased or publicly provided. We assume that parents differ in their unobservable ability and that the productivity of the time spent with their children depends on their ability. In the second-best, we assume that the domestic time investment is always unobservable but explore the implications of alternative assumptions on the observability of private purchases of education. We show that the sign of the cross derivative between informal and formal education plays a critical role in the second best optimal education policy.

Keywords: optimal nonlinear taxes, parental time investment, human capital production

JEL Classification: H21, J22

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1 Introduction

The link between parental time with children and children's development has been extensively explored in the psychology and sociology literature, and also more recently in the economics literature (Del Boca et al., 2014; Bernal, 2008).

In this paper we study the implications for the optimal education policy of taking parental time investments in children's education into account. We assume that parents differ in their unobservable ability and that the productivity of the time spent with children depends on their ability. Both informal and formal education enter the children's human capital production function. We show that the sign of the cross-derivative plays a critical role in the second best optimal education policy. We also show that the desirability of public education provision critically depends on whether private purchases of education are observable or not. The latter is consistent with results in previous contributions such as Cremer and Pestieau (2006) and Carta (2013). Those contributions however do not account for alternative assumptions on the complementarity or substitutability between time investments and private purchases of education, which we find play a key role.

Casarico and Sommacal (2012) examine the effects of labor income taxation on growth in an overlapping generations model in which schooling and childcare play a role in the production of human capital. Casarico et al. (2015) characterize the optimal tax policy and quality of day care services in an overlapping generations model in which child care arrangements chosen by parents of different skill types affect the probability that children become high-skilled adults in a type-specific way.

The rest of the paper is organised as follows. In section 2 we present the model and the laissez-faire outcome. In section 3 we derive the first-best benchmark solution. In section 4 we analyze the second-best asymmetric information problem: we assume that the domestic time investment is always unobservable in the second best but explore the effects of alternative assumptions on the observability of private purchases of education, in particular on the desirability of public education provision. We conclude in section 5.

2 The model

We consider a society in which individuals differ in ability w_i. Parents invest both time (e) and money (h) in the education of their children. Parents utility represented by

$$u(c_i) + H(h_i, w_i e_i)$$

where $H(h_i, w_i e_i)$ is the utility derived from the human capital of their kids. The productivity of time investment e_i is the parents own ability w_i. We assume that $H_1 > 0$ and $H_{11} < 0$ (i.e. positive but diminishing marginal returns to private purchases of education): $H_2 > 0$ and $H_{22} < 0$ (i.e. positive but diminishing marginal returns to time investments in education). As for the cross derivative H_{12} we don't assume a particular form and allow for $H_{12}\,(>,=,<)\,0$. The sign of the cross derivative indeed plays a crucial role in the second best results.

In the second best we assume that the domestic time investment is always unobservable but we explore the effects of alternative informational assumptions on the private purchases of education.

We assume throughout that $y = w(1-e)$. Hence, $ew = w - y$. There is no labour-leisure dimension: time is devoted to either work or education of children. In the following subsection we study the shape of the indifference curves focusing alternatively on e and y.

2.1 Indifference curves

In the (e, c) space:

$$\frac{dc_i}{de_i} = -\frac{H_2(h_i, w_i e_i)\, w_i}{u'(c_i)}$$

At a given point (e, c), for given h, it is unclear whether the indifference curve of type-1 or type-2 is steeper: on the one hand $w_2 > w_1$ (and then the indifference curve of type-1 tends to be steeper) but $H_{22} < 0$ so $H_2(h, w_2 e) < H_2(h, w_1 e)$. In the relevant (y, c)-space, however, the single-crossing property holds:

$$\frac{dc_i}{dy_i} = \frac{H_2(h_i, w_i - y_i)}{u'(c_i)}$$

At a given point (y, c), for given h, $H_{22} < 0$ implies $H_2(h, w_2 - y) < H_2(h, w_1 - y)$ and indifference curve of type 2 is steeper.

In the (y, h) space:

$$\frac{dh_i}{dy_i} = -\frac{H_2(h_i, w_i - y_i)}{H_1(h_i, w_i - y_i)}$$

At a given point (y, h), for given c, $H_{22} < 0$ implies $H_2(h, w_2 - y) < H_2(h, w_1 - y)$ and numerator is smaller for type-2, however the denominator depends on the cross derivative: $H_{12} > 0$ means denominator is higher for type-2, so further reducing the slope, but $H_{12} < 0$ would run in the opposite direction making the overall effect on the slope ambiguous.

In the (h, c) space:

$$\frac{dc_i}{dh_i} = -\frac{H_1(h_i, w_i e_i)}{u'(c_i)}$$

At a given point (h, c), for given e: $H_{12} > 0$ means numerator is higher, and $H_{12} < 0$ means numerator is smaller, for type-2.

2.2 Laissez-faire

Each individual i chooses bundle (e_i, c_i, h_i) to maximize $u(c_i) + H(h_i, w_i e_i)$ subject to individual budget constraint $(1 - e_i) w_i = c_i + h_i$. The first order conditions (hereafter FOCs) imply:

$$u'(c_i) = H_1(h_i, w_i e_i) = H_2(h_i, w_i e_i).$$

3 The government problem

The government maximizes a utilitarian social welfare function

$$\sum n_i [u(c_i) + H(h_i, w_i - y_i)]$$

subject to the budget constraint

$$\sum n_i (y_i - c_i - h_i) = 0$$

and relevant self-selection constraints in those cases in which some of the individual choices are non-observable.

Note that we express the problem in terms of (y, c, h) for to be able to compare the first best and the second best solutions as these are the variables that are observable (at least in the benchmark second best problem with observable purchases of education h).

3.1 The first best

In the first best the social planner is able to observe all the individual characteristics. The corresponding Lagrangian is:

$$L = \sum n_i [u(c_i) + H(h_i, w_i - y_i) + \mu(y_i - c_i - h_i)].$$

The FOCs are:

$$\begin{aligned}(c_1) &:\ u'(c_1)-\mu=0\\(c_2) &:\ u'(c_2)-\mu=0\\(y_1) &:\ -H_2(h_1,w_1-y_1)+\mu=0\\(y_2) &:\ -H_2(h_2,w_2-y_2)+\mu=0\\(h_1) &:\ H_1(h_1,w_1-y_1)-\mu=0\\(h_2) &:\ H_1(h_2,w_2-y_2)-\mu=0\end{aligned}$$

Combining the FOCs above we obtain:

$$u'(c_i)=H_1(h_1,w_1-y_1)=H_2(h_i,w_i-y_i)=\mu.$$

Hence:

$$u'(c_1)=u'(c_2)\rightarrow c_1=c_2,$$

and

$$\begin{aligned}H_2(h_1,w_1-y_1) &= H_2(h_2,w_2-y_2),\\H_1(h_1,w_1-y_1) &= H_1(h_2,w_2-y_2).\end{aligned}$$

Decreasing returns to scale is sufficient, although not necessary, to obtain $h_1=h_2$ and $w_1-y_1=w_1e_1{=}w_2e_2=w_2-y_2$.

3.2 The second best

In the second best the social planner is no longer able to observe all individual characteristics. We assume that the domestic time investment is always unobservabe. We explore however the implications of alternative assumptions on the observability of private purchases of education, first considering that private purchases of education are observable, and relaxing subsequently this assumption.

3.2.1 Second best with observable h_i

The Lagrangian in the second best where parental time invested in education e_i is not observable but money invested in education h_i is observable is:

$$\begin{aligned} L &= \sum n_i \left[u\left(c_i\right) + H\left(h_i, w_i - y_i\right) + \mu\left(y_i - c_i - h_i\right)\right] \\ &\quad + \lambda\left[u\left(c_2\right) + H\left(h_2, w_2 - y_2\right) - u\left(c_1\right) - H\left(h_1, w_2 - y_1\right)\right] \end{aligned}$$

The FOCs are:

$$\begin{aligned} (c_1) &: u'\left(c_1\right) - \mu - \frac{\lambda}{n_1} u'\left(c_1\right) = 0 \\ (c_2) &: u'\left(c_2\right) - \mu + \frac{\lambda}{n_2} u'\left(c_2\right) = 0 \\ (y_1) &: -H_2\left(h_1, w_1 - y_1\right) + \mu + \frac{\lambda}{n_1} H_2\left(h_1, w_2 - y_1\right) = 0 \\ (y_2) &: -H_2\left(h_2, w_2 - y_2\right) + \mu - \frac{\lambda}{n_2} H_2\left(h_2, w_2 - y_2\right) = 0 \\ (h_1) &: H_1\left(h_1, w_1 - y_1\right) - \mu - \frac{\lambda}{n_1} H_1\left(h_1, w_2 - y_1\right) = 0 \\ (h_2) &: H_1\left(h_2, w_2 - y_2\right) - \mu + \frac{\lambda}{n_2} H_1\left(h_2, w_2 - y_2\right) = 0 \end{aligned}$$

The solution is characterized by non-distortion at the top:

$$u'\left(c_2\right) = H_2\left(h_2, w_2 - y_2\right) = H_1\left(h_2, w_2 - y_2\right).$$

Alternatively,

$$\frac{H_2\left(h_2, w_2 - y_2\right)}{u'\left(c_2\right)} = 1 \text{ and } \frac{H_1\left(h_2, w_2 - y_2\right)}{u'\left(c_2\right)} = 1.$$

There is distortion at the bottom:

$$(1 - \frac{\lambda}{n_1}) u'\left(c_1\right) = \mu$$

$$\frac{H_2\left(h_1, w_1 - y_1\right)}{u'\left(c_1\right)} = 1 + \frac{\lambda}{\mu n_1}\left[H_2\left(h_1, w_2 - y_1\right) - H_2\left(h_1, w_1 - y_1\right)\right] < 1$$

since $H_{22} < 0$.

$$\frac{H_1\left(h_1, w_1 - y_1\right)}{u'\left(c_1\right)} = 1 + \frac{\lambda}{\mu n_1}\left[H_1\left(h_1, w_2 - y_1\right) - H_1\left(h_1, w_1 - y_1\right)\right]$$

The sign of the distortion depends on sign of the cross-derivative H_{12} :

$$\begin{aligned} H_{12} &> 0 \rightarrow \frac{H_1\left(h_1, w_1 - y_1\right)}{u'\left(c_1\right)} > 1, \\ H_{12} &< 0 \rightarrow \frac{H_1\left(h_1, w_1 - y_1\right)}{u'\left(c_1\right)} < 1. \end{aligned}$$

What does this mean in terms of the distortion on h? Since $H_{11} < 0$, $H_{12} > 0$ is associated with a decrease in h_1 and $H_{12} < 0$ is associated with an increase in h_1. Note that when $H_{12} = 0$ there is no role for a distortion on h_1.[1]

To sum up, there is non-distortion on type-2 individuals. With respect to the first best, since type-2 individuals are able to retain a portion of the information rent we can expect higher c_2, lower y_2, higher $w_2 - y_2 = w_2 e_2$ (hence higher e_2). And then higher h_2 if $H_{12} > 0$ or lower h_2 if $H_{12} < 0$. Type-1 individuals are distorted on both margins: time investment on education e and private purchases of education h. With respect to first best, we can expect lower c_1, higher y_1, lower $w_1 - y_1 = w_1 e_1$ (hence lower e_1). And then lower h_1 if $H_{12} > 0$ or higher h_1 if $H_{12} < 0$, with added distortion because we don't recover the equality of the marginal utilities.

Intuitively, the planner should want to distort h_1 downwards, to make mimicking less attractive, if purchases of education and parental time are positively correlated. With observable h_i we are also able to show that there is no role for public education g, if public and private purchases of education are perfect substitutes in the human capital production function. This is consistent with previous results in the literature such as Cremer and Pestieau (2006) and Carta (2013).

3.2.2 Public education g is redundant if h and g perfect substitutes

We show that public education g is redundant by taking the option of public education g into consideration in the Lagrangian:

$$\begin{aligned} L &= \sum n_i \left[u(c_i) + H(h_i + g, w_i - y_i) + \mu(y_i - c_i - h_i - g)\right] \\ &\quad + \lambda\left[u(c_2) + H(h_2 + g, w_2 - y_2) - u(c_1) - H(h_1 + g, w_2 - y_1)\right] \end{aligned}$$

and calculating $\partial L / \partial g$:

$$\frac{\partial L}{\partial g} = \sum n_i H_1(h_i + g, w_i - y_i) - \mu + \lambda\left[H_1(h_2 + g, w_2 - y_2) - H_1(h_1 + g, w_2 - y_1)\right]$$

We have that:

$$\begin{aligned} H_1(h_1 + g, w_1 - y_1) - \mu - \frac{\lambda}{n_1} H_1(h_1 + g, w_2 - y_1) &= 0 \\ H_1(h_2 + g, w_2 - y_2) - \mu + \frac{\lambda}{n_2} H_1(h_2 + g, w_2 - y_2) &= 0 \end{aligned}$$

[1] In the Appendix we show that this would be the case for an isoelastic function with $\gamma = \rho$, where $\rho \leq 1$ is a measure of the curvature of the isoquant - the elasticity of subtitution is given by $\sigma = 1/(1-\rho)$ - and γ represents the returns to scale.

Multiplying by n_1 and n_2, respectively:

$$\begin{aligned} n_1 H_1 (h_1 + g, w_1 - y_1) - n_1 \mu - \lambda H_1 (h_1 + g, w_2 - y_1) &= 0 \\ n_2 H_1 (h_2 + g, w_2 - y_2) - n_2 \mu + \lambda H_1 (h_2 + g, w_2 - y_2) &= 0 \end{aligned}$$

Summing up:

$$\sum n_i H_1 (h_i + g, w_i - y_i) - \mu + \lambda [H_1 (h_2 + g, w_2 - y_2) - H_1 (h_1 + g, w_2 - y_1)] = 0$$

Public education g just crowds out private investment in education without any further welfare effect.

3.2.3 Second best with unobservable h_i

The government only controls income and disposable income, but not its decomposition between consumption and private education. We call disposable income x as opposed to consumption c. Disposable income is used to purchase both own consumption and private education: $x = c + h$, hence $c = x - h$. The Lagrangian can be rewritten:

$$\begin{aligned} L &= n_i [u (x_i - h_i) + H (h_i, w_i - y_i) - \mu (x_i - y_i)] \\ &\quad + \lambda [u (x_2 - h_2) + H (h_2, w_2 - y_2) - u (x_1 - h_{21}) - H (h_{21}, w_2 - y_1)] \end{aligned}$$

Note that the mimicker chooses the level of private education optimally according to his individual maximization problem given contract (y_i, x_i) :

$$\max_{h_i} u (x_i - h_i) + H (h_i, w_i - y_i)$$

The FOC is given by:

$$-u' (x_i - h_i) + H_1 (h_i, w_i - y_i) = 0$$

This equation defines $h_i (y_i, x_i) = h (w_i; y, x)$ with derivative:

$$\frac{\partial h}{\partial w} = -\frac{H_{12}}{u'' (x_i - h_i) + H_{11}}.$$

The sign positively depends on H_{12}. It follows that:

$$\begin{aligned} h_1 &< h_{21} \text{ if } H_{12} > 0, \\ h_1 &> h_{21} \text{ if } H_{12} < 0. \end{aligned}$$

A type-2 mimicking a type-1 and a type-1 invest now different amounts of (unobservable) private purchases of education h. The sense of the difference depends on the sign of H_{12}.

The FOCs are:

$$\begin{aligned}
(x_1) &\quad : \quad u'\left(x_1 - h_1^*\right) - \mu - \frac{\lambda}{n_1} u'\left(x_1 - h_{21}\right) = 0 \\
(x_2) &\quad : \quad u'\left(x_2 - h_2^*\right) - \mu + \frac{\lambda}{n_2} u'\left(x_2 - h_2^*\right) = 0 \\
(y_1) &\quad : \quad -H_2\left(h_1^*, w_1 - y_1\right) + \mu + \frac{\lambda}{n_1} H_2\left(h_{21}, w_2 - y_1\right) = 0 \\
(y_2) &\quad : \quad -H_2\left(h_2^*, w_2 - y_2\right) + \mu - \frac{\lambda}{n_2} H_2\left(h_2^*, w_2 - y_2\right) = 0
\end{aligned}$$

For type-2 individuals:

$$\left(1 + \frac{\lambda}{n_2}\right) u'\left(x_2 - h_2^*\right) = \mu$$

$$\left(1 + \frac{\lambda}{n_2}\right) H_2\left(h_2^*, w_2 - y_2\right) = \mu$$

$$\frac{H_2\left(h_2^*, w_2 - y_2\right)}{u'\left(x_2 - h_2^*\right)} = 1$$

For type-1 individuals:

$$\left(1 - \frac{\lambda}{n_1}\right) u'\left(x_1 - h_1^*\right) = \mu + \frac{\lambda}{n_1}\left[u'\left(x_1 - h_{21}\right) - u'\left(x_1 - h_1^*\right)\right] > \mu \text{ if } H_{12} > 0$$

$$\frac{H_2\left(h_1^*, w_1 - y_1\right)}{u'\left(x_1 - h_1^*\right)} = \frac{1 + \frac{\lambda}{\mu n_1}\left[H_2\left(h_{21}, w_2 - y_1\right) - H_2\left(h_1^*, w_1 - y_1\right)\right]}{1 + \frac{\lambda}{\mu n_1}\left[u'\left(x_1 - h_{21}\right) - u'\left(x_1 - h_1^*\right)\right]}$$

It is possible to show that now public provision of education g can play a role.

4 Conclusions

In this paper we have explored the implications for the optimal education policy of taking parental time investments in children's education into account. We assumed that parents differ in their unobservable ability and that the productivity of the time spent with children depends on their ability. Both time and money enter the children's human capital production function. We showed that the sign of the cross-derivative plays a critical role in the second best optimal education policy. Previous contributions in the most related literature have generally not considered alternative assumptions on the complementarity or substitutability between time investments and private purchases of education, which we have found play a key role.

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A Isoelastic human capital production function

We have to make sure the properties of the production function $H(h, we)$ are reasonable. In this appendix we study the properties of the family of isoelastic production functions:

$$H(h, we) = \left[\alpha h^{\rho} + \beta (we)^{\rho}\right]^{\frac{\gamma}{\rho}}$$

where $\rho \leq 1$ is a measure of the curvature of the isoquant: the elasticity of subtitution is given by $\sigma = 1/(1-\rho)$. Note that $\rho = 0$ represents the unit elastic Cobb-Douglas production function. α and β represent the relative importance of each factor of production. γ represents the returns to scale, with increasing returns to scale when $\gamma > 1$ and decreasing returns to scale if $\gamma < 1$.

The first, second and cross derivatives are given by:

$$H_h == \gamma\alpha \left[\alpha h^{\rho} + \beta (we)^{\rho}\right]^{\frac{\gamma}{\rho}-1} h^{\rho-1},$$

$$H_{hh} = \alpha\gamma\left[\alpha h^{\rho} + \beta\left(we\right)^{\rho}\right]^{\frac{\gamma}{\rho}-2} h^{\rho-2}\left[\left(\gamma-1\right)\alpha h^{\rho} + \left(\rho-1\right)\beta\left(we\right)^{\rho}\right],$$

$$H_{we} = \gamma\beta\left[\alpha h^{\rho} + \beta\left(we\right)^{\rho}\right]^{\frac{\gamma}{\rho}-1}\left(we\right)^{\rho-1},$$

$$H_{we,we} = \beta\gamma\left[\alpha h^{\rho} + \beta\left(we\right)^{\rho}\right]^{\frac{\gamma}{\rho}-2}\left(we\right)^{\rho-2}\left[\left(\rho-1\right)\alpha h^{\rho} + \left(\gamma-1\right)\beta\left(we\right)^{\rho}\right],$$

$$H_{h,we} = H_{we,h} = \gamma\left(\gamma-\rho\right)\left[\alpha h^{\rho} + \beta\left(we\right)^{\rho}\right]^{\frac{\gamma}{\rho}-2}\alpha h^{\rho-1}\beta\left(we\right)^{\rho-1}.$$

It is relevant to note that the sign of the cross-derivative depends on the the sign of $\gamma - \rho$:

- In the case of constant returns to scale (i.e. $\gamma = 1$), $\rho \leq 1$ ensures that $H_{we,h} \geq 0$ (and it is equal to 0 in the perfect substitutes case $\rho = 1$).
- In the Cobb-Douglas unit elasticity case (i.e. $\rho = 0$), $H_{we,h} > 0$ as long as $\gamma > 0$.
- In the case of perfect substitutes (i.e. $\rho = 1$) decreasing returns to scale ($\gamma < 1$) yields $H_{we,h} < 0$. But more generally in the case of decreasing returns to scale, the sign of $H_{we,h}$ depends on $\gamma - \rho$.