Optimal emissions taxation and expected overall underinternalization in a durable goods oligopoly*

Amagoia Sagasta and José M. Usategui†
University of the Basque Country UPV/EHU

March 30, 2017

Abstract

In a durable-goods setting where emissions may occur during use or at the disposal of the good the comparison between the optimal emission tax in a period and marginal environmental damage in that period may not be relevant for environmental policy, as producers' decisions depend on the expected total emission tax per unit produced in each period and the units of the good produced in one period may cause environmental damage in future periods. Using a two-period model with imperfect competition, it is shown that there is overall underinternalization as the total expected optimal emission tax per unit produced in the first period is smaller than the expected overall marginal environmental damage from the last unit produced in that period. This occurs even if there is overinternalization of environmental damage in the first period.

JEL classification codes: Q58, H23, L13.

Keywords: optimal emission taxes, durable good, internalization, environmental damage, imperfect competition.

*The authors thank an anonymous referee and Ana I. Saracho for helpful comments and suggestions on earlier drafts of the paper. The usual disclaimer applies. Financial support from Spain's Ministerio de Economía y Competitividad (ECO2012-35820 and ECO2015-64467-R (MINECO/FEDER)) and from the Departamento de Educación, Universidades e Investigación del Gobierno Vasco (IT-783-13) is gratefully acknowledged.

†Departamento de Fundamentos del Análisis Económico II and BRiDGE. Emails: josemaria.usategui@ehu.eus, amagoia.sagasta@ehu.eus. Corresponding author: José M. Usategui; Departamento de Fundamentos del Análisis Económico II; Facultad de CC. Económicas y Empresariales; University of the Basque Country UPV/EHU; Avda. Lehendakari Agirre, 83; 48015 Bilbao; Spain.

1 Introduction

Economic literature has studied the relationship between environmental policy and market structure. Under perfect competition, external damage is fully internalized when the per unit emission tax equals the marginal environmental damage caused by pollution. However, as first noted by Buchanan (1969), under a monopoly the welfare maximizing emission tax is less than the marginal external damage (see also Barnett (1980)). Production under imperfect competition is below the efficient level, so due to firms' market power, optimal emission taxes under imperfect competition are, in general, below marginal environmental damage. This implies underinternalization of environmental damage. However, the possibility of overinternalization when there is imperfect competition has also been demonstrated in the literature. For instance, Katsoulacos and Xepapadeas (1995) show that under a fixed-number oligopoly the optimal emission tax falls short of the marginal external damage but that with free entry, so that the market structure is determined endogenously, the optimal tax may exceed the marginal environmental damage. Simpson (1995) also shows the possibility of overinternalization under a Cournot duopoly with asymmetric costs of production, in order to redistribute output from the less efficient producer to his more efficient rival.

In a durable goods setting it is known that firms face the time consistency problem noted by Coase (1972) when they produce and sell a durable good and do not have commitment ability. When deciding its future production, a firm has no incentive to take into account the units previously sold that are in the hands of consumers. As consumers anticipate that the firm will increase its future production, thus reducing its future selling price, consumers' willingness to pay for the durable good in the present is reduced. The firm would therefore like to commit to reducing production in the future as this would raise consumers' willingness to pay for the good in the present, increasing firms' profits. An environmental tax in the future increases the firms' future marginal costs, giving them a higher commitment power and increasing their market power. Thus, both imperfect competition and the durability of the good could lead to an optimal tax rate in the future that is lower than the marginal environmental damage in that period. However, time is an essential factor when goods are durable, so it is also necessary to study the effects of an emission tax in the present. Unlike an emission tax in the future, an emission tax in the present is found to increase firms' present marginal costs, reducing their commitment power and therefore also reducing their market power. Moreover, as shown in Section 3 below, production levels in the present and in the future are affected by both the emission tax in the present and the emission tax in the future. As a result, the repercussions of optimal emission taxes on internalization of environmental damage in the present are far from evident.

Analysis of the internalization of environmental damage in the literature on durable goods has centered on the relationship between the optimal emission tax in a period and marginal environmental damage in that period. Several situations where there could be overinternalization of environmental damage in the present have been described. For instance, Boyce and Goering (1997) derive that the optimal emission tax in the present may exceed marginal environmental damage under a monopoly that sells its product, when durability is exogenous and there are increasing returns to scale in production. Runkel (2002) shows that underinternalization results when there is an oligopoly of firms that rent their product and durability is exogenous. However, he finds that overinternalization may result with endogenous product durability. In a context with constant returns to scale in production and exogenous product durability, Runkel (2004) proves that the optimal waste taxes lie below the marginal environmental damage in the present and in the future, under a monopoly that sells its product, but that there may be overinternalization in the present when there is an oligopoly of firms that sell their product. He also extends the analysis of Goering and Boyce (1997) to show that overinternalization may occur under a monopoly that sells its product, with endogenous durability and constant returns to scale in production.

The comparison between the optimal emission tax *in a period* and marginal environmental damage *in the same period* may, however, give an incomplete account of the relationship between emission taxes and environmental damage for a unit of the good produced in that period when emissions are proportional to the stock of the durable good in use or when they occur at the end of the product's lifetime. For emissions of these types the environmental damage from a unit produced in a given period, or part of that damage, occurs in future periods. All emissions in different periods from a unit produced in a given period are taxed, so the analysis of internalization for that period must compare the *total* expected optimal emission tax per unit produced in the period and the expected *overall* marginal environmental damage from the last unit produced in that period. This paper contributes to the literature on internalization when goods are durable by carrying out that comparison.

The overall environmental damage caused in all periods by units produced in each period is considered for the total surplus. Furthermore, firms' decisions in a given period are affected by the total expected optimal emission tax per unit produced in that period, as firms have a long-run perspective.[1] Hence, it is appropiate to study the relationship between the *total* expected optimal emission tax per unit produced in a period and the expected *overall* marginal environmental damage from the last unit produced in that period when the relationship between emission taxes and environmental damage is at issue.

The comparison between the optimal emission tax in a period and marginal environmental damage in that period makes sense only when emissions occur at the time of production. In that case, a unit produced in a given period only causes emissions and environmental damage in that period and thus only pays emission taxes in that period.

In this paper a context with durable goods firms that sell their product under imperfect competition in a two-period model is considered, and the total expected optimal emission tax to be paid per unit produced in one period and the expected overall marginal environmental damage caused by that unit are compared when emissions occur during the use of the durable good and when they occur at the end of the lifetime of the product. It is shown that there is expected overall underinternalization of environmental damage of a

[1] Many firms' decisions are based on the long run. The adoption of new technology, investment decisions and entry and exit decisions are usually envisaged in the long run. Nevertheless, the total expected emission tax is also relevant for established firms that decide on present and future production.

unit produced in the first period, although the optimal emission tax in the first period may imply overinternalization in that period. Hence, we prove that if there is overinternalization in the first period then that overinternalization is offset by underinternalization in the second period. These results may provide insight into some previous results on overinternalization in the literature, for durable goods that may cause pollution in periods other than the production period. Moreover, given that there are major polluting industries which produce durable goods and are highly concentrated (the car and aircraft industries, for instance), our findings on the characteristics of optimal emission taxes may be relevant to environmental policy design.

Considering an oligopoly of producers of the durable good and situations where a unit produced in one period may cause environmental damage in future periods has other implications for the analysis besides enhancing the applicability of our results. If there is an oligopoly of producers then firms have incentives to steal market shares from their rivals in the present and in the future by increasing their sales in the present. In this case there is an additional effect of a variation in the emission tax in the present: a decrease in that emission tax decreases marginal cost in the present and increases the incentives to steal market share and, hence, competition among producers. Considering situations where there may be emissions in future periods from a unit produced in a given period reinforces the influence of an emission tax in one period on production levels in all periods and the result that optimal emission taxes maximize total surplus through the combination of their effects on production levels in all periods. Section 4 shows that this strategic behavior and those intertemporal considerations, together with imperfect competition and durability, interact to cause overall underinternalization of environmental damage.

The paper is organized as follows: Section 2 introduces the model. Firms' decisions on production when there are emission taxes are investigated in Section 3, which also presents the optimal emission taxes that maximize total surplus. In Section 4 it is proved that optimal emission taxes imply overall underinternalization of environmental damage, even though those taxes may imply underinternalization for a unit produced in the present. That section closes with some extensions of the result on overall underinternalization to other contexts. Section 5 contains some concluding remarks.

2 The model

We consider an oligopolistic industry with n ($n > 1$) firms that produce and sell a homogeneous durable good in two discrete periods of time, $t = 1, 2$, in which production occurs at the beginning of each period. Entry into the industry is assumed to be unprofitable or unfeasible. In the second period there is a perfect second-hand market for the durable good.

All agents participating in the market (consumers, firms and the regulator) have perfect and complete information. Those agents use the same discount factor, which is represented by $\beta \epsilon [0, 1]$. Moreover, we assume that potential buyers of the good have perfect foresight.

Denote by q_{1i} and q_{2i}, respectively, the quantity produced and sold by firm i, with $i = 1, ..., n$, in $t = 1, 2$ (for the corresponding quantities at industry level write Q_1 and Q_2). All firms face the same production cost functions. For firm i these cost functions are $C_1(q_{1i})$ in $t = 1$ and $C_2(q_{2i})$ in $t = 2$. It is assumed that $C_t' > 0$ and $C_t'' \geq 0$, where C_t' and C_t'' denote the first and second derivatives of C_t with respect to the quantity produced in period t, with $t = 1, 2$.

The durable good depreciates over time: Only a proportion δ of the units produced in the first period remains in the second period. The durability of the good is assumed to be exogenous. We consider situations where all units produced in $t = 1$ that do not depreciate are also used in $t = 2$. Therefore, $\delta Q_1 + Q_2$ is the quantity of the durable good that is used in $t = 2$.

The inverse demand function for the services of the durable good is the same in each period and is denoted by $p(Q)$, where Q is the total stock available for use in the period. It is assumed that $p'(Q) < 0$ and that marginal revenue from demand for services of the durable good is decreasing for each firm.

The durable good creates negative externalities, which are modeled considering two types of emission: emissions proportional to the stock of product in use in the market and emissions that occur at the end of the lifetime of the product. Hereafter, these two types are referred to as emissions during use and emissions at disposal, respectively.

The emission functions in each period are:

$$E_1 = (1 - \delta + \alpha\delta)Q_1$$

$$E_2 = \delta Q_1 + Q_2$$

where $\alpha = 1$ when emissions occur during use and $\alpha = 0$ when emissions occur at disposal. There are $(1 - \delta)Q_1$ units produced in the first period that cannot be used in period 2 due to depreciation. Those units only cause emissions in the first period under both types of emission. By contrast, the δQ_1 units produced in the first period that can be used in period 2 cause emissions only in period 2 when emissions occur at disposal and cause emissions in both periods when emissions are during use. Hence, the expected emissions per unit produced in period 1 are $1 - \delta + \alpha\delta$ in the first period and δ in the second period. A unit produced in the second period entails emissions of 1 in that period.

Environmental damage in each period is a function of that period's emissions. We denote by $\gamma(E_t)$ the environmental damage in period t, with $t = 1, 2$. It is assumed that $\gamma' > 0$ and $\gamma'' > 0$ for all $E_t \geq 0$.

Let p_t denote the price paid by the buyer of a unit of the durable good in period t, with $t = 1, 2$. Given that the buyers in the first period correctly anticipate the selling price of the good in the second period, they will be willing to pay:

$$p_1 = p(Q_1) + \beta\delta p_2$$

to acquire the good in the first period ($p_1 - \beta\delta p_2$ is the additional price to be paid for being able to use the good in the first period).

The second period selling price will be

$$p_2 = p(\delta Q_1 + Q_2),$$

Our presentation considers the situation where emission taxes are paid by producers. In that case the market prices are p_1 and p_2. However, sometimes emission taxes are charged to buyers, as often occurs when emissions occur during use. When emission taxes are charged to buyers the total price that they pay for the durable good in a given period is the sum of the corresponding total emission tax paid per unit produced in that period plus the market price or price received by sellers in the period. We show in Section 3 that the analysis and results in this paper remain unchanged when emission taxes are paid by consumers rather than producers.

The analysis is modeled as a non-cooperative game in two stages. In the second stage there are two periods $t = 1$ and $t = 2$. Firms engage in quantity competition in each period of the second stage. Their choices are simultaneous in each of those periods. In the first stage the regulator sets emission taxes per unit of emission for each period of the second stage. Those taxes are τ_1 and τ_2, respectively, for $t = 1$ and $t = 2$. We assume that the regulator can commit to emission taxes at the first stage.[2]

3 Firms' decisions with emission taxes and optimal emission taxation

First we analyze the decision problem of each firm in the presence of emission taxes τ_1 and τ_2. Coase (1972) notes that in the absence of explicit contracts of guarantees a durable goods producer that sells its output and that has no commitment ability has a credibility problem. When deciding its output that firm will not take into account the decline in the value of units previously sold that are in the hands of consumers. As consumers have perfect foresight, they realize that each firm will behave in that way in $t = 2$. This causes buyers to substitute current consumption by future consumption and decreases current demand. Thus, the firms are constrained by consumer expectations. In a given period each firm maximizes the present discounted value of profits starting from that period, given that constraint. In order to calculate the intertemporal schedule of production that maximizes the present value of profits for firm i, the maximization problem must be resolved recursively by backward induction: First the optimal production for period $t = 2$ must be determined given any production level in period $t = 1$, and then the optimal production corresponding

[2] This model can be considered as an extension of the monopolistic case considered by Bulow (1982) to situations where there is an oligopoly, environmental damage from use or disposal of the durable good and emissions taxation. Our analysis requires the inclusion of a previous stage where the regulator sets the emission taxes.

to period 1 must be calculated. Hence the solution, a subgame perfect Nash equilibrium in pure strategies, is obtained through backward induction from the last period of the second stage.[3]

At the beginning of period 2 each firm i chooses its second period production to maximize its second period profits, π_2^i, given the quantity sold in the first period. Given that a unit produced in period 1 expects to pay $\delta\tau_2$ in the second period it follows that in $t = 2$ each firm i solves the following problem:

$$\max_{q_{2i}} \ \pi_2^i = \left[p(\delta Q_1 + Q_2))q_{2i} - C_2(q_{2i}) - \tau_2 q_{2i} - \delta\tau_2 q_{1i}\right].$$

subject to $q_{2i} \geq 0$. The first order condition of this maximization problem is:

$$p(\delta Q_1 + Q_2) + p'(\delta Q_1 + Q_2)q_{2i} = C_2'(q_{2i}) + \tau_2 \tag{1}$$

Condition (1) points out that in $t = 2$ marginal revenue for oligopolist i equals total marginal cost. This equation gives the second period production as a function of the remaining first period output.

Consumers know that each firm will choose its second period production to satisfy (1), as they have perfect foresight. Hence, that dependence of the second period production of each firm on first period output is expected by consumers.

In the first period, each firm i, with $i = 1, ..., n$, chooses the level of sales that maximizes the present value of its total profits subject to (1). Hence, given that a unit produced in period 1 expects to pay $(1 - \delta + \alpha\delta)\tau_1$ in emission taxes in that period, it follows that firm i, with $i = 1, ..., n$, solves the following problem in $t = 1$ to maximize the present value of its total profits, $\pi^i = \pi_1^i + \beta\pi_2^i$:

$$\max_{q_{1i}} \ \pi_1^i + \beta\pi_2^i = [(p(Q_1) + \beta\delta p(\delta Q_1 + Q_2) - (1 - \delta + \alpha\delta)\tau_1)q_{1i} - C_1(q_{1i})$$
$$+\beta(p(\delta Q_1 + Q_2)q_{2i} - C_2(q_{2i}) - \tau_2 q_{2i} - \delta\tau_2 q_{1i})]$$

subject to (1). From the maximization of profits of each firm in $t = 2$ it follows

[3]We assume throughout the paper that parameters and functions are such that interior solutions are obtained in each optimization problem.

that $\frac{\partial \pi_2^i}{\partial q_{2i}} = 0$. Hence, the first order condition of this problem is:

$$\begin{gathered} p(Q_1) - (1 - \delta + \alpha\delta)\tau_1 + p'(Q_1)q_{1i} - C_1'(q_{1i}) \\ +\beta\delta p(\delta Q_1 + Q_2) + \beta\delta(\delta + \frac{\partial Q_2}{\partial q_{1i}})p'(\delta Q_1 + Q_2)q_{1i} \\ +\beta(\delta + \frac{\partial Q_{2-i}}{\partial q_{1i}})p'(\delta Q_1 + Q_2)q_{2i} - \beta\delta\tau_2 = 0. \end{gathered} \tag{2}$$

where $Q_{2-i} = Q_2 - q_{2i}$. Condition (2) points out that in $t = 1$ marginal revenue for oligopolist i equals the total marginal cost, given by $C_1'(q_{1i}) + (1-\delta+\alpha\delta)\tau_1 + \beta\delta\tau_2$.

Given τ_1 and τ_2 the quantities sold in each period at the firm and market levels can be obtained from equations (1) and (2) (in those equations the derivatives $\frac{\partial Q_2}{\partial q_{1i}}$ and $\frac{\partial Q_{2-i}}{\partial q_{1i}}$ are evaluated at τ_1 and τ_2). When buyers of the durable good pay for emission taxes they adjust their willingness to pay for the good taking into account τ_1 and τ_2. In that case the term $-\delta\tau_2 q_{1i}$ is not included in the maximization problem for period 2. However, condition (1) remains unchanged, so the analysis and results obtained below also hold when buyers rather than producers pay emission taxes.

We now analyze the emission taxes that maximize total surplus taking into account the effect of emission taxes on the decisions of firms. In this model total surplus is given by:

$$\begin{gathered} \text{Total surplus } (TS) = \text{Consumer surplus} + \text{Profits of firms} \\ + \text{Taxes paid - Emissions damage.} \end{gathered}$$

The optimal emission taxes solve:

$$\max_{\tau_1, \tau_2} TS(\alpha)$$

subject to (1) and (2). The optimal emission taxes in periods 1 and 2 that maximize TS can be written as follows (a detailed analysis can be found in Sagasta and Usategui (2017)):

$$\begin{gathered} \tau_1^* = \gamma'(E_1) + \frac{1}{1-\delta+\alpha\delta}[q_{1i}p'(Q_1) \\ +\beta p'(\delta Q_1 + Q_2)\left(\delta(\delta + \frac{\partial Q_2}{\partial q_{1i}})q_{1i} + \frac{\partial Q_{2-i}}{\partial q_{1i}}q_{2i}\right)] \\ \tau_2^* = \gamma'(E_2) + p'(\delta Q_1 + Q_2)q_{2i} \end{gathered} \tag{3}$$

where the quantities of the good are the optimal production levels that maximize TS.

Conditions (1) and (2) imply that each emission tax affects production levels in both periods, so it follows that optimal emission taxes maximize total surplus through the combination of the effects of each emission tax on production levels.

As a result of durability and imperfect competition, each firm behaves strategically to steal sales from its rivals in the present and in the future. Hence, τ_1^* and τ_2^* simultaneously correct for the distortion in production due to the strategic behavior of oligopolists and for environmental damage, taking into account the implications of the oligopolistic market structure, the durability of the good and the type of emission.

4 Overall underinternalization of environmental damage

Following the literature on optimal emissions taxation of durable goods under imperfect competition, there is said to be underinternalization of the environmental damage in a period if the emission tax in that period is lower than marginal environmental damage in the period. By contrast, if the emission tax in a period is greater than marginal environmental damage in that period then there is overinternalization in the period.

From (3) it follows that there is underinternalization of environmental damage in the second period as $\tau_2^* - \gamma'(E_2) = p'(\delta Q_1 + Q_2)q_{2i} < 0$. Underinternalization in $t = 2$ is caused by the negative imperfect competition effect in that period: Firms have market power, so total production is lower than total production when the market is competitive and the optimal emission tax in $t = 2$ is lower than marginal environmental damage in that period. Nevertheless, Runkel (2004) proves that when emissions occur at disposal there may be overinternalization in the first period ($\tau_1^* > \gamma'(E_1)$), and his analysis can be extended to the case where emissions occur during use. The possibility of overinternalization in the first period can be obtained from our analysis in Section 3. (1) and (2) imply that there is a symmetric market solution for firms' decisions, so equation (1) can be written as:

$$p(\delta Q_1 + Q_2) + p'(\delta Q_1 + Q_2)\frac{Q_2}{n} = C_2'(\frac{Q_2}{n}) + \tau_2$$

From the assumptions on $p(Q)$, this latter equation gives:[4]

$$\frac{\partial Q_2}{\partial q_{1i}} = \frac{\partial Q_2}{\partial Q_1} = -\frac{\delta p'(\delta Q_1 + Q_2) + \delta p''(\delta Q_1 + Q_2)\frac{Q_2}{n}}{p'(\delta Q_1 + Q_2) + p''(\delta Q_1 + Q_2)\frac{Q_2}{n} + \frac{1}{n}(p'(\delta Q_1 + Q_2) - C_2''(\frac{Q_2}{n}))} < 0 \tag{4}$$

and:

$$\delta + \frac{\partial Q_2}{\partial Q_1} = \frac{\delta \frac{1}{n}(p'(\delta Q_1 + Q_2) - C_2''(\frac{Q_2}{n}))}{p'(\delta Q_1 + Q_2) + p''(\delta Q_1 + Q_2)\frac{Q_2}{n} + \frac{1}{n}(p'(\delta Q_1 + Q_2) - C_2''(\frac{Q_2}{n}))} > 0$$

Hence, the sign of $\tau_1^* - \gamma'(E_1)$ in the first equation in (3) is indeterminate.

The literature on optimal emissions taxation of durable goods under imperfect competition has studied the relationship between the optimal emission tax in a period and marginal environmental damage in that period. Nevertheless, as pointed out in the Introduction, comparing the optimal emission tax in a period and marginal environmental damage in that period makes sense only when emissions occur at production, because in that case a unit produced in period t only causes emissions and environmental damage in that period and thus only pays taxes in period t. By contrast, when emissions occur during use or at disposal a unit produced in one period may cause environmental damage in another, future period. In such cases it follows from (1) that firms' decisions in the present are affected by the total expected emission tax per unit produced in that period, as firms have a long-run perspective. In the first period the total expected emission tax per unit produced relevant for firms' decisions is $(1 - \delta + \alpha\delta)\tau_1 + \beta\delta\tau_2$. Moreover, the expected overall marginal environmental damage from the last unit produced in $t = 1$ is $(1-\delta+\alpha\delta)\gamma'((1-\delta+\alpha\delta)Q_1)+\beta\delta\gamma'(Q_2+\delta Q_1)$, where Q_1 and Q_2 are obtained from (1) and (2).

From (3) it emerges that the total expected optimal emission tax paid by

[4] Note that the assumptions on $p(Q)$ imply that $p'(\delta Q_1+Q_2)+\delta p''(\delta Q_1+Q_2)(\delta\frac{Q_1}{n}+\frac{Q_2}{n}) < 0$ and, hence, $p'(\delta Q_1+Q_2)+\delta p''(\delta Q_1+Q_2)\frac{Q_2}{n} < 0$.

any unit produced in the present is:

$$(1-\delta+\alpha\delta)\tau_1^* + \beta\delta\tau_2^* = (1-\delta+\alpha\delta)\gamma'(E_1) + \delta\beta\gamma'(E_2) + q_{1i}p'(Q_1)$$

$$+\beta\delta\left(\delta q_{1i} + q_{2i}\right)p'(\delta Q_1 + Q_2) + \beta\delta q_{1i}\tfrac{dQ_2}{dq_{1i}}p'(\delta Q_1 + Q_2) + \beta\tfrac{dQ_{2-i}}{dq_{1i}}q_{2i}p'(\delta Q_1 + Q_2) \tag{5}$$

We now introduce the following definition:

Definition A. *There is expected overall underinternalization of environmental damage in* $t = 1$ *if:*

$$(1-\delta+\alpha\delta)\tau_1 + \beta\delta\tau_2 < (1-\delta+\alpha\delta)\gamma'(E_1) + \beta\delta\gamma'(E_2),$$

with Q_1 *and* Q_2 *obtained from (1) and (2). There is expected overall overinternalization of environmental damage if the inequality is reversed.*

From this definition it clearly follows that if there is underinternalization in each of the two periods then overall underinternalization is also expected in the first period.[5]

The following can be proved:

Proposition 1. *If firms sell their output and durability is exogenous then even though optimal emissions taxation may imply overinternalization for a unit produced in the first period, there is expected overall underinternalization in* $t = 1$ *for each of the two types of emissions considered.*

Proof: Symmetry between firms, $\frac{\partial Q_{2-i}}{\partial q_{1i}} = \frac{n-1}{n}\frac{\partial Q_2}{\partial Q_1}$ and (4) imply that:

$$\delta + \frac{\partial Q_{2-i}}{\partial Q_1} = \delta\frac{p'(\delta Q_1 + Q_2) + p''(\delta Q_1 + Q_2)\frac{Q_2}{n} + (p'(\delta Q_1 + Q_2) - C_2''(\frac{Q_2}{n}))}{n(p'(\delta Q_1 + Q_2) + p''(\delta Q_1 + Q_2)\frac{Q_2}{n}) + (p'(\delta Q_1 + Q_2) - C_2''(\frac{Q_2}{n}))} > 0$$

Overall underinternalization in $t = 1$ is expected because from (5), $\delta + \frac{\partial Q_2}{\partial Q_1} > 0$ and $\delta + \frac{\partial Q_{2-i}}{\partial Q_1} > 0$ it follows that:

$$(1-\delta+\alpha\delta)(\tau_1^* - \gamma'(E_1)) + \delta\beta(\tau_2^* - \gamma'(E_2))$$

$$= q_{1i}p'(Q_1) + \beta p'(\delta Q_1 + Q_2)\left[\delta(\delta + \tfrac{\partial Q_2}{\partial Q_1})\tfrac{Q_1}{n} + (\delta + \tfrac{\partial Q_{2-i}}{\partial Q_1})\tfrac{Q_2}{n}\right] < 0.$$

[5] If emissions occur at production then Definition A is equivalent to underinternalization in the first period. It can be shown that the optimal emission tax in the first period does not imply overinternalization in that period when emissions occur at production.

■

Therefore, it emerges that though the optimal emission tax in $t = 1$ may imply overinternalization in that period when emissions occur during use or at disposal, there is expected overall underinternalization in $t = 1$ for both types of emissions. In the context considered in this analysis, underinternalization in $t = 2$ offsets the possible overinternalization in $t = 1$ for such emissions.

To provide an intuition for this result, from (5) the difference between the total expected optimal emission tax per unit produced in $t = 1$ and the expected overall marginal environmental damage from the last unit produced in that period can be broken down into the following effects:

$$\underbrace{p'(Q_1)q_{1i}}_{\text{imperfect competition effect}} + \underbrace{\beta p'(\delta Q_1 + Q_2)\delta\left(\delta q_{1i} + q_{2i}\right)}_{\text{durability effect}}$$
$$+ \underbrace{\beta p'(\delta Q_1 + Q_2)\delta \frac{dQ_2}{dq_{1i}} q_{1i}}_{\text{intertemporal inconsistency effect}} + \underbrace{\beta p'(\delta Q_1 + Q_2)\frac{dQ_{2-i}}{dq_{1i}} q_{2i}}_{\text{strategic effect}}$$

The imperfect competition and durability effects are negative but the intertemporal inconsistency and strategic effects are positive, as $\frac{dQ_{2i}}{dq_{1i}}$ and $\frac{dQ_{2-i}}{dq_{1i}}$ are negative.[6] The imperfect competition effect means that total surplus can be maximized with a lower total expected emission tax per unit produced in $t = 1$, given that imperfect competition implies a total production lower than total production when the market is competitive. The durability effect also means that the tax levels necessary to maximize total surplus can be lower, because when firms produce durable goods, they benefit from further reducing production in $t = 1$, since this increases sales in the second period.

The intertemporal inconsistency effect captures the impact of first period sales on second period sales in a context in which producers in period 2 do not take into account the capital loss borne by buyers of the good in the first period. A reduction in production in the first period induces an increase in production in the second period and, hence, the intertemporal inconsistency effect means that a higher total expected emission tax is required to maximize

[6] The imperfect competition, durability and intertemporal inconsistency effects are analogous to those obtained in Boyce and Goering (1997) for a monopolistic seller. Our analysis includes an additional term, the strategic effect, as we consider that there are n firms, with $n \geq 2$.

total surplus.[7] Finally, the strategic effect covers the implications of $n > 1$ (it would be 0 in a monopolistic market). As Carlton and Gertner (1989) state, competition is more intense in industries that produce durable goods than in nondurable goods industries. In a non-durable-good setting, firms respond only to present competition from each other: The future does not matter. In a durable-good setting, the decision about how much to sell today is also affected by rivals' future behavior. When a firm sells a durable good today, it is not only stealing a sale today from its rivals but also stealing their future sales. Thus, the strategic effect increases competition in the market, thus raising the total expected emission tax necessary to maximize total surplus.

Proposition 1 implies that for either type of emissions the positive intertemporal inconsistency and strategic effects are not great enough to offset the negative imperfect competition and durability effects. These latter effects make the total expected emission tax per unit produced in the first period lower than the expected overall marginal environmental damage.

When emissions occur at disposal a unit produced in period 1 may cause environmental damage and pay emission taxes in the first period or in the second period. It is known that if environmental damage is caused in period 1 there may be overinternalization of the emission tax in that period while there is underinternalization of the emission tax in period 2 if environmental damage is caused in this latter period. Proposition 1 implies that this second possibility offsets the first as overall underinternalization is expected for a unit produced in the first period.

When emissions occur during use a unit produced in the first period may cause emissions in both periods. In that case Proposition 1 implies that for that unit of the good the present value of the difference between the expected marginal environmental damage in the second period and the expected emission tax in $t = 2$ offsets for the possible excess of the emission tax in the first period over the marginal environmental damage in that period.[8]

[7] In particular a higher emission tax in the second period is required to limit the increase in production in that period.

[8] Consider that $p(Q) = a - bQ$, environmental damage per unit of emission in each period is constant at γ and marginal costs of production in the first and second periods are constant and given, respectively, by c_1 and c_2. The following are examples of overinternalization

Proceeding as in the proof of Proposition 1, it can also be shown that there is expected overall underinternalization in some situations where environmental damage is non-stationary. This occurs if the environmental damage functions are $\gamma(E_1)$ in the first period and $\kappa\gamma(E_2)$ in $t=2$, where κ is a constant. There may be $\kappa > 1$ due to a reduction in the capacity of the environment to assimilate pollution in $t=2$ when pollution from production of all goods and services in the economy accumulates, or $\kappa < 1$ if environmental damage from emissions from the durable good industry in that period diminishes as a result of decreases in emissions from other industries.[9]

If pollution in the second period is equal to emissions from use or disposal of the durable good in that period plus emissions in the first period multiplied by υ, with $\upsilon > 0$, (durable pollution) then the extension of the previous analysis is immediate.[10] In that case total emissions in period 2 would be:

$$E_2 = (\delta + (1-\delta+\alpha\delta)\upsilon)Q_1 + Q_2$$

and each of those emission units would pay a tax τ_2 in $t=2$. It is possible to proceed as above to show that there is expected overall underinternalization of environmental damage, noting that the maximization problems of firm i should be written as $(\delta + (1-\delta+\alpha\delta)\upsilon)\tau_2 q_{1i}$, rather than $\delta\tau_2 q_{1i}$.[11]

Furthermore, the result in Proposition 1 can also be extended to those situations where emissions occur during use and the units produced in period 1 that are in use in period 2 pollute more in the second period than the units

in the first period and overall underinternalization in this context (we denote the optimal emission taxes by τ_1^* and τ_2^*): i) Example 1: If $\beta = 0.5$, $\delta = 1$, $a = 3.5$, $n = 3$, $c_1 = 2.4$, $c_2 = 0.1$, $b = 1$, $\gamma = 1$ and $\alpha = 1$, then production levels are positive, $\tau_1^* = 1.1313 > \gamma$ and $(1-\delta+\alpha\delta)\tau_1^* + \beta\delta\tau_2^* = 1.256 < 1.5 = \gamma(1-\delta+\alpha\delta+\beta\delta)$, ii) Example 2: If $\beta = 1$, $\delta = 0.5$, $a = n = 3$, $c_1 = 2.4$, $c_2 = 0.1$, $b = 1$, $\gamma = 1$ and $\alpha = 0$ then production levels are positive, $\tau_1^* = 1.19792 > \gamma$ and $(1-\delta+\alpha\delta)\tau_1^* + \beta\delta\tau_2^* = 0.794 < 1 = \gamma(1-\delta+\alpha\delta+\beta\delta)$.

[9] The value of κ affects the possibilities of obtaining overinternalization in the first period. For instance, it is easy to show that if $p(Q)$ is linear and if marginal environmental damage and unit production cost in each period are constant then the set of values of the parameters where there is overinternalization in the first period gets bigger as κ diminishes (and it gets smaller as κ increases).

[10] Benchekroun and Long (1998) analyzes the optimal tax rule when firms produce durable goods and emit pollutants that cause present as well as future damage as the stock of pollution accumulates over time.

[11] If $p(Q)$ is linear and if marginal environmental damage and unit production cost are constant in each period then the set of values of the parameters where there is overinternalization in the first period gets bigger as υ diminishes (and it gets smaller as υ increases).

produced in that period to (for instance, cars often pollute more as they age). If emissions in period 2 of a unit produced in period 1 are ρ times the emissions in a period of a unit produced in that period, with $\rho > 1$, then it can be proved that there is expected overall underinternalization of environmental damage, so that $\rho\delta\tau_2 q_{1i}$ is written instead of $\delta\tau_2 q_{1i}$ in the maximization problems of firm i.

5 Concluding remarks

When emissions occur during use or at disposal optimal emissions taxation may entail an emission tax in $t = 1$ that is higher than the marginal environmental damage in that period. This may occur when the incentives of firms to increase first period production due to both the intertemporal inconsistency effect and the strategic effect are greater than the incentives to produce fewer units in the first period due to the imperfect competition and durability effects. Nevertheless, it is shown that there is overall underinternalization of environmental damage in the first period as the total expected optimal emission tax per unit produced in $t = 1$ is lower than the expected overall marginal environmental damage from the last unit produced in that period. In the context considered, underinternalization in $t = 2$ offsets the possible overinternalization in $t = 1$.

It is relevant to consider overall underinternalization when the units of the durable good produced in a period may cause environmental damage, and therefore be subject to emission taxes, in future periods. First, the overall environmental damage caused in all periods by units produced in each period is considered for total surplus. Moreover, in that situation a firm's first period production decision depends not only on the first period emission taxes paid for units produced in that period but also on the total expected optimal emission taxes to be paid. Hence, a more suitable approach for analyzing overinternalization in the present is to study the relationship between the total expected optimal emission tax per unit produced in the first period and the expected overall marginal environmental damage from the last unit produced in that period. This contribution of the paper could give a new insight to the relationship between optimal emissions taxation and marginal external damage analyzed in the literature on durable goods.

If $C_1'' < 0$ and $C_2'' < 0$ and there is a symmetric solution (assuming that cooperation between firms to concentrate all production in one firm and share the profits obtained is not feasible or not allowed) then proceeding as in Section 4 it follows that the following inequalities are feasible: $\frac{dQ_2}{dq_{1i}} = \frac{dQ_2}{dQ_1} > 0$, $\delta + \frac{dQ_2}{dq_{1i}} < 0$ and $\delta + \frac{dQ_{2-i}}{dq_{1i}} < 0$. Hence, from the proof of Proposition 1 it cannot be assured that there will be overall underinternalization under increasing returns to scale.[12]

If producers can select the durability of the good the cost function for firm i in $t = 1$ is $C_1(q_{1i}, \delta_i)$ and the optimal quantities and durability that maximize $TS(\alpha, \delta)$ cannot be attained with just two emission taxes τ_1 and τ_2. The emission taxes that maximize $TS(\alpha)$ induce quantities and durability levels other than those that maximize $TS(\alpha, \delta)$. Hence, another instrument is required to maximize $TS(\alpha, \delta)$ and it is not possible to proceed as in the analysis of Section 4 to check for overall underinternalization.

If the number of periods is finite but greater than two then there must be a final period when the durable good can be produced and sold. In that last period there is underinternalization because the difference between the optimal emission tax and marginal environmental damage in that period is equal to the imperfect competition effect in the period, as in (3). With a finite number of periods greater than two there are also intermediate periods when there are units of the good that cause environmental damage and have been produced in previous periods. In that case the consideration of overall underinternalization is relevant and overall underinternalization can be expected even if there is overinternalization in the first period or in some intermediate periods. Nevertheless, the resolution of the optimal taxation problem becomes complicated when emissions occur during use or at disposal and it is left for future research. In a model with infinite periods or in a continuous time frame, which may be more adequate in some instances, there is no final period.

[12] Boyce and Goering (1997) analyze the effect of increasing returns to scale on overinternalization in the first period under monopoly.

The approach in such cases considers stationary demand and cost functions.[13] However, our two-period analysis does not assume stationary cost functions. In fact it is not difficult to show that if the inverse demand function of services of the durable good is linear and if marginal environmental damage and unit production cost are constant in each period then overinternalization in the first period requires a greater unit cost in that period than in the second period. In Runkel (2004) it is also explained that overinternalization in the first period is more likely with high unit costs in that period and low unit costs in the second period. Hence, the extension to infinite periods or the consideration of a continuous framework would at least require the appropriate assumptions to be made on the cost functions in different periods to allow for overinternalization in some periods.

6 References

Barnett, A.H. (1980): "The Pigouvian Tax Rule Under Monopoly", *American Economic Review* 70: 1037-1041.

Benchekroun, H. and Long, N.V. (1998): "Efficiency inducing taxation for polluting oligopolists", *Journal of Public Economics* 70: 325-342.

Boyce, J. and Goering, G. (1997): "Optimal taxation of a polluting durable goods monopolist", *Public Finance Review* 25: 522-541.

Buchanan, J.M. (1969): "External Diseconomies, Corrective Taxes, and Market Structure", *American Economic Review* 59: 174-177.

Bulow, J., (1982): "Durable-goods monopolists", *Journal of Political Economics* 101: 729-749.

Carlton, D. and Gertner, R. (1989): "Market Power and Mergers in Durable-Goods Industries." *Journal of Law and Economics* 32: S203-S226.

Coase, R. (1972): "Durability and Monopoly", *Journal of Law and Economics* 15: 143-149.

[13] See Kahn (1986).

Kahn, Ch. (1986): "The durable goods monopolist and consistency with increasing costs", *Econometrica* 54: 275-294.

Katsoulacos, Y. and Xepapadeas, A. (1995): "Environmental policy under oligopoly with endogenous market structure", *Scandinavian Journal of Economics* 97: 411-420.

Runkel, M. (2002): "A note on emissions taxation in durable goods oligopoly", *Journal of Industrial Economics* 50: 235-236.

Runkel, M. (2004): "Optimal emissions taxation under imperfect competition in a durable good industry", *Bulletin of Economic Research* 56: 115-132.

Sagasta, A. and J. M. Usategui (2017): "Time structure of emissions and comparison between optimal emission taxes under selling and under renting in durable goods oligopolies", The Manchester School, doi:10.111/manc.12181.

Simpson, R.D. (1995): "Optimal pollution taxation in a Cournot duopoly", *Environmental and Resource Economics* 6: 359-369.