

Optimal Income Taxation in Unionized Labor Markets*

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Abstract

We analyze optimal redistributive income taxation in unionized labor markets in the extensive labor-supply model of [Diamond \(1980\)](#) and [Saez \(2002\)](#). We employ a right-to-manage model, where unions bargain with firms over wages in each occupation/sector and firms determine employment. Optimal income taxes and unemployment benefits are shown to be lower in unionized labor markets, because income redistribution from employed to unemployed workers raises wage demands and thereby creates involuntary unemployment. Net participation subsidies (EITC programs) are always desirable for low-skilled workers whose social welfare weight exceeds one. In addition, net participation subsidies could be optimal as well for low-skilled workers whose social welfare weight is below one when labor markets are unionized. Furthermore, increasing the unions' bargaining power may be socially desirable when participation subsidies lead to overemployment. Involuntary unemployment then acts as an implicit tax, which off-sets explicit subsidies on labor participation. However, unions are *never* desirable if labor participation is taxed on a net basis, since the implicit tax of involuntary unemployment only exacerbates distortions of explicit taxes on labor participation. Our simulations demonstrate that optimal taxes and transfers are much less redistributive in unionized labor markets than in competitive labor markets.

Keywords: optimal taxation, unions, wage bargaining, participation

JEL classification: D63, H21, H23, J51, J58

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1 Introduction

Over the past decades, many Western economies have simultaneously experienced sharp increases in inequality and reductions in union membership rates. This pattern, recently documented by [Jaumotte and Buitron \(2015\)](#) and [Kimball and Mishel \(2015\)](#), is illustrated in Figure 1 for the case of the United States. While the negative relationship between unionization and (wage) inequality is well documented empirically, little is known about the consequences of unionization on the effectiveness or desirability of redistributive policies.¹ This is surprising, because unions play a dominant role in labor markets, most notably in continental Europe. In particular, between 49% (Switzerland) and 98% (Austria) of wage earners are covered or affected by union-negotiated labor agreements.² This paper therefore studies optimal income redistribution in unionized labor markets. It asks two main questions: ‘*How should the government optimize income redistribution if labor markets are unionized?*’ And: ‘*Can labor unions be socially desirable if the government wants to redistribute income?*’ Although some papers have analyzed optimal taxation in unionized labor markets, no paper has, to the best of our knowledge, ever studied the desirability of unions for income redistribution.

In this paper, we extend the extensive-margin models of [Diamond \(1980\)](#), [Saez \(2002\)](#) and [Choné and Laroque \(2011\)](#) with unions. Workers are heterogeneous with respect to their costs of participation and their sector- or occupation-specific wage rate. Workers choose whether or not they want to participate, and supply labor on the extensive margin in case they succeed in finding a job.³ In our framework, workers in each sector of the labor market are represented by a sectoral labor union, whose goal it is to maximize the expected utility of its members. Firm-owners own a stock of capital and employ different labor types to produce a final consumption good. Labor market outcomes are determined in a right-to-manage (RtM) setting, due to [Nickell and Andrews \(1983\)](#). This model nests both the monopoly-union (MU) model of [Dunlop \(1950\)](#) and the competitive equilibrium as a special case. These models are considered the canonical union models in the labor literature, jointly with the efficient bargaining (EB) model of [McDonald and Solow \(1981\)](#), which we will analyze in an extension.⁴ In the RtM-model, wages are determined through bargaining between unions and (representatives of) firm-owners. Individual firm-owners, in turn, take wages as given and determine labor demand for each labor type. Unions bid up wages above their market-clearing levels, and, thereby, generate involuntary unemployment. In our baseline model, we assume efficient rationing in the labor market: the burden of involuntary unemployment is borne by the workers with the highest costs of participation. The government acts as the Stackelberg leader and maximizes social welfare by optimally setting a non-linear income tax, unemployment benefits, and profit taxes. Our model

¹The negative relationship between unionization and (wage) inequality is documented in, for instance, [Freeman \(1980, 1991\)](#), [Lemieux \(1993, 1998\)](#), [DiNardo and Lemieux \(1997\)](#), [Machin \(1997\)](#), [Card \(2001\)](#), [Fairris \(2003\)](#), [Card et al. \(2004\)](#), [Checchi and Pagani \(2004\)](#), [Koeniger et al. \(2007\)](#), [Visser and Checchi \(2011\)](#), [Western and Rosenfeld \(2011\)](#), and [Mishel \(2012\)](#).

²These figures are taken from the ICTWSS database on Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts. This database can be freely downloaded from <http://www.uva-aias.net/208>.

³As argued by [Lee and Saez \(2012\)](#), the extensive (participation) margin is often considered the empirically more relevant than the intensive (or hours) margin at the lower part of the income distribution. Empirical evidence in favor of this claim is documented in, among others, [Meyer \(2002\)](#).

⁴See also [Layard et al. \(1991\)](#), [Booth \(1995\)](#), [Boeri and Van Ours \(2008\)](#).

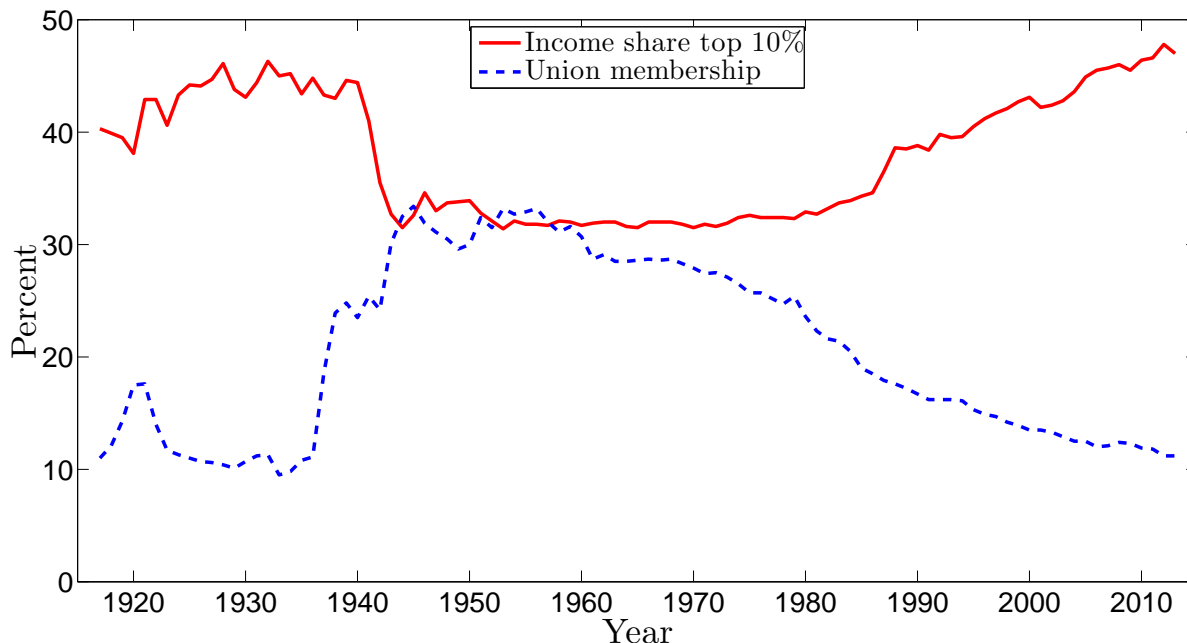


Figure 1: Union membership and inequality (Source: [Kimball and Mishel, 2015](#))

nests the canonical extensive-margin models of [Diamond \(1980\)](#) and [Saez \(2002\)](#) as special cases where unions are absent and wages are exogenously given. [Christiansen \(2015\)](#) extends the models of [Diamond \(1980\)](#) and [Saez \(2002\)](#) with endogenous, competitively determined wages. This model is nested as a special case as well. Our main findings are the following.

First, we answer the question how the optimal tax-benefit system should be adjusted in unionized labor markets. We show that the tax-benefit system is not only geared towards redistributing income, but also to alleviate the distortions from involuntary unemployment. The government redistributes income by taxing workers and providing benefits to the non-employed. However, both taxes on workers and benefits for the unemployed induce unions to bid up wages above market-clearing levels, which results in involuntary unemployment. Therefore, the government optimally lowers income taxes and unemployment benefits to avoid higher involuntary unemployment. By lowering taxes or benefits, unions are motivated to moderate their wage demands, which reduces involuntary unemployment. Optimal participation taxes should therefore be lower the higher is the degree of unionization (i.e., the higher is the bargaining power of unions) – *ceteris paribus*.⁵ Net participation subsidies may even be optimal for workers whose welfare weight is below one, which never occurs if labor markets are competitive, cf. [Diamond \(1980\)](#), [Saez \(2002\)](#) and [Choné and Laroque \(2011\)](#). Involuntary unemployment creates implicit taxes on labor participation which the government wants to offset by providing explicit subsidies on labor participation. EITC programs are thus more likely to be desirable in strongly unionized economies.

Second, we provide an answer to the question whether unions are a useful institution for

⁵Because in an environment with involuntary unemployment participation no longer equals employment, [Jacquet et al. \(2014\)](#) and [Kroft et al. \(2015\)](#) prefer the term *employment tax* over the term *participation tax* to refer to the sum of the income tax and the unemployment benefit. In line with most of the literature, we will continue to use the term ‘participation tax’, keeping the above caveat in mind.

income redistribution. Our main result is that increasing the bargaining power of unions which represent low-skilled workers (i.e., the workers whose social welfare weight exceeds one) raises social welfare, while the opposite holds true for the unions who represent workers whose social welfare weight is below one. Intuitively, in sectors where the workers' welfare weight exceeds one, there is *excessive* labor participation due to positive participation subsidies, see also [Diamond \(1980\)](#), [Saez \(2002\)](#), and [Choné and Laroque \(2011\)](#). Unions reduce these labor-market distortions by demanding higher wages, which lowers employment. Involuntary unemployment acts as an implicit tax on labor participation, which partially off-sets the explicit subsidy on labor participation. By lowering the dead-weight losses resulting from the tax-transfer system, the government can thus redistribute more income, generate higher efficiency, or both.⁶ Consequently, EITC policies and labor unions can be complementary instruments to raise the net incomes of the low-skilled. The reverse is also true: unions are never desirable if labor participation is taxed on a net basis. In that case, implicit taxes from involuntary unemployment only exacerbate explicit taxes on labor participation. Therefore, it is socially optimal to let low-skilled workers organize themselves in a union, whereas labor markets for more productive workers should remain competitive.

Finally, we simulate an empirically reasonable calibration of our model, based on a sufficient statistics approach recently introduced by [Kroft et al. \(2015\)](#). For plausible values of labor-demand and participation elasticities, the optimal tax-benefit system looks quite different if the impact of unions is taken into account. In line with our theoretical predictions, optimal income taxes and unemployment benefits are (significantly) lower if labor markets are unionized. In the U.S., for example, participation tax rates at the bottom of the income distribution may drop from around 30% to -2% and in the Netherlands from 43% to 20%, depending on the degree of unionization. The optimal tax-benefit system may thus feature a strong EITC-component if labor markets are unionized. Our simulations also suggest that the conditions under which unions are desirable are hard to meet empirically. In virtually all our simulations the welfare weight of even the lowest income groups falls short of one. For unions to become desirable under our welfarist criterion, incomes of low-skilled workers would need to fall substantially below levels currently observed in the data. Unions may also be desirable if the government subsidizes participation for non-welfarist reasons, e.g., if the government considers those who work 'more deserving'.

The remainder of this paper is organized as follows. Section 2 discusses the related literature. Section 3 outlines the basic structure of the model and characterizes the general equilibrium for a given set of tax instruments. The question how these instruments should be optimally set is addressed in Section 4. Section 5 subsequently examines how changing the bargaining power unions affects social welfare and characterizes the optimal degree of unionization in each sector. Section 6 investigates the robustness of the results. We present our simulations in Section 7. Section 8 concludes.

⁶This finding echoes the result of [Lee and Saez \(2012\)](#), who show that a binding minimum wage enhances social welfare if the welfare weight of the workers for whom the minimum wage binds exceeds one. For these workers, participation decisions are distorted upwards. Minimum wages can therefore alleviate the distortions induced by taxation, but only in sectors where participation is subsidized on a net basis.

2 Related literature

Our paper relates to five branches in the literature. First, this paper adds to the literature on optimal taxation in unionized labor markets. In a model with multiple labor types and exogenous labor supply, [Palokangas \(1987\)](#) shows that the first-best can be achieved even if wage-setting is influenced by unions. [Fuest and Huber \(1997\)](#), [Koskela and Schöb \(2002\)](#), [Aronsson and Sjögren \(2004a,b\)](#) and [Aronsson and Wikström \(2011\)](#) analyze union models with only one labor type. Provided there are no informational frictions or restrictions on profit taxation, the first-best outcome is achieved as well. [Aronsson and Sjögren \(2003\)](#), [Aronsson et al. \(2005\)](#), [Kessing and Konrad \(2006\)](#), [Aronsson et al. \(2009\)](#) consider models with more than one labor type and allow for endogenous labor-supply responses on the intensive margin. As in [Mirrlees \(1971\)](#), the government can neither observe wage rates nor working hours, which prevents a first-best outcome. Most of these studies find that (un)employment considerations are important to consider when designing tax policies, though their impact on optimal taxes is often ambiguous.⁷ Our paper contributes to this literature by analyzing optimal income taxation in unionized labor markets with extensive rather than intensive labor-supply responses. We also avoid first-best outcomes, since participation costs are unobservable.

Second, there is an extensive literature which analyzes the impact of taxation on wages and employment in union models, see, e.g., [Lockwood and Manning \(1993\)](#), [Bovenberg and van der Ploeg \(1994\)](#), [Koskela and Vilmunen \(1996\)](#), [Fuest and Huber \(1997\)](#), [Sørensen \(1999\)](#), [Fuest and Huber \(2000\)](#), [Lockwood et al. \(2000\)](#), [Bovenberg \(2003\)](#), [Aronsson and Sjögren \(2004b\)](#), [Sinko \(2004\)](#), [van der Ploeg \(2006\)](#), and [Aronsson and Wikström \(2011\)](#). In these models, high unemployment benefits and high income taxes (i.e., high *average* tax rates) improve the position of the unemployed relative to the employed, which leads to higher wage demands and hence, lower employment. High *marginal* tax rates, on the other hand, moderate wage demands, since a larger fraction of marginal wage increases is taxed away by the government. In contrast to models with competitive labor markets, more tax progression is thus associated with higher employment if wage-setting is influenced by unions, provided working hours are fixed. If, however, individuals can also adjust their working hours, the impact of increased tax progressivity on overall employment (i.e., total hours worked) in models with unions becomes ambiguous ([Sørensen, 1999](#), [Fuest and Huber, 2000](#), [Aronsson and Sjögren, 2004b](#), [Koskela and Schöb, 2012](#)). Since we focus on extensive labor-supply responses, our model only features the impact of higher average taxes and benefits on wage demands. By using a discrete tax function, there is no direct wage-moderating effect of tax-rate progressivity. However, we emulate the effects of higher marginal tax rates by introducing proportional employer taxes in one of the extensions.

Third, we extend the analyses of [Diamond \(1980\)](#) and [Saez \(2002\)](#) of optimal taxation with extensive labor-supply responses to settings with finitely elastic labor demand and unions bargaining with firms over wages. [Diamond \(1980\)](#) and [Saez \(2002\)](#) show that the optimal tax-benefit system features participation subsidies for workers whose welfare weight exceeds one,

⁷An exception is [Kessing and Konrad \(2006\)](#), who focus on the impact of unions on (restrictions on) working hours and abstract from involuntary unemployment.

much in the spirit of an Earned Income Tax Credit (EITC).⁸ Our model nests [Diamond \(1980\)](#) and [Saez \(2002\)](#) if labor types are perfectly substitutable. By allowing for varying degrees of unionization, the model analyzed by [Christiansen \(2015\)](#) – who extends the analysis of [Diamond \(1980\)](#) and [Saez \(2002\)](#) with competitively determined wages – is also nested as a special case. Like the aforementioned studies, we also find that low-income workers should optimally be subsidized in an EITC-type program. In addition, we show that with unionized wage-setting, participation subsidies may be optimal as well for workers whose welfare weight is below one.

Fourth, our analysis complements [Christiansen \(2016\)](#), who also studies optimal taxation in a unionized economy in which individuals differ in terms of their (unobservable) participation costs. In his model, however, participation costs refer to the (utility) costs an individual incurs by switching between occupations, whereas in our model participation costs are incurred when moving from unemployment to employment. Furthermore, in [Christiansen \(2016\)](#) there is a single union concerned with wage compression, whereas our baseline model features multiple unions and abstracts from wage inequality within the group of workers represented by the same union. Finally, [Christiansen \(2016\)](#) abstracts from involuntary unemployment, which will prove an important consideration in our paper. Like [Christiansen \(2016\)](#), we show that union responses to taxation are important to consider when designing optimal tax policies. In an extension, we also analyze the case in which a single (utilitarian) union aims to compress the wage distribution. Our main findings remain largely unaffected. In particular, we find that the union’s desire to reduce wage inequality does not provide an *additional* reason why an increase in unionization could be welfare-enhancing.

Fifth, our study relates closely to recent literature on the optimality of minimum-wage policies in conjunction with optimal income taxation. Like unions, a binding minimum wage increases the income of certain groups of workers at the costs of (potentially) generating involuntary unemployment. [Lee and Saez \(2012\)](#) show that introducing a minimum wage for low-skilled workers (whose welfare weight exceeds one) is welfare-enhancing provided unemployment rationing is efficient. If the assumption of efficient rationing is relaxed, a minimum wage need no longer be optimal. [Gerritsen and Jacobs \(2016\)](#) study the optimality of minimum wage policies while allowing for a general rationing schedule and endogenous human capital decisions. In their model, minimum wages could be desirable, since they alleviate the distortions in skill formation. [Hungerbühler and Lehmann \(2009\)](#) also analyze the optimality of a minimum wage, but do so in an economy with matching frictions. In their model, introducing a minimum wage is optimal if the bargaining power of the workers is too low for the Hosios condition to be satisfied. If, however, the government could directly increase the workers’ bargaining power, a minimum wage ceases to be optimal. We also demonstrate that increasing the bargaining power of unions can improve social welfare. However, in contrast to [Hungerbühler and Lehmann \(2009\)](#), we allow the bargaining power of the unions to vary across sectors. We show that only an increase in the bargaining power of unions representing low-skilled workers may improve social welfare.

⁸This finding is confirmed in [Choné and Laroque \(2011\)](#), who consider a very general treatment of optimal taxation with extensive labor-supply responses.

3 Model

We consider an economy which includes workers, unions, firm-owners and a government. The basic structure of the model follows [Diamond \(1980\)](#), with the exception that we consider a finite number of labor types, and, hence, a finite number of unions. Workers supply labor on the extensive margin to different occupations. Within each occupation, workers are represented by a single labor union which negotiates wages with firm-owners. The latter supply capital and produce the final consumption good. The government aims to maximize social welfare by redistributing income between unemployed workers, employed workers, and firm-owners. We assume that the government is the Stackelberg leader relative to all agents in the private sector, including the labor unions. Each union takes tax policy as given and does not internalize the impact of its decisions on the government budget.⁹

3.1 Workers

Workers are heterogeneous in two dimensions: wages and participation costs. There is a discrete number of I occupations or sectors. A worker of type $i \in \mathcal{I} \equiv \{1, \dots, I\}$ can work only in occupation/sector i where she earns wage w_i . We denote by N_i the mass of workers of type i . When working, every worker incurs a monetary participation cost φ , which is private information, and has domain $[\underline{\varphi}, \bar{\varphi}]$, with $\underline{\varphi} < \bar{\varphi} \leq \infty$.¹⁰ The cumulative distribution function of participation costs of workers is denoted by $G(\varphi)$, which is assumed to be identical across sectors.¹¹

Each worker is endowed with one indivisible unit of time and decides whether she wants to work in occupation i or not. All workers derive utility from consumption $c_{i,\varphi}$. They have an identical utility function $u(c_{i,\varphi})$ with $u'(c_{i,\varphi}) > 0$, and $u''(c_{i,\varphi}) < 0$. Consumption of employed workers $c_{i,\varphi}$ equals labor income w_i , minus income taxes T_i and participation costs φ : $c_{i,\varphi} = w_i - T_i - \varphi$. Unemployed workers consume c_u , which equals an unemployment benefit of $-T_u$, hence $c_u = -T_u$. An individual in sector/occupation i with participation costs φ is willing to work whenever

$$u(c_{i,\varphi}) = u(w_i - T_i - \varphi) \geq u(c_u) = u(-T_u). \quad (1)$$

Because we assume that participation is voluntary, this condition is always satisfied if an individual is employed. The reverse, however, need not be true. If for some individuals condition (1) is satisfied, but they are not employed, this simply means that these workers are *involuntarily* unemployed.

For each sector i , equation (1) defines a cut-off $\bar{\varphi}_i$ at which individuals are indifferent between working and not working: $\bar{\varphi}_i \equiv w_i - T_i + T_u$. Higher wages w_i , lower income taxes T_i , and lower unemployment benefits $-T_u$ all raise the cut-off $\bar{\varphi}_i$ and thus, raise labor participation in sector/occupation i .

⁹This assumption is not innocuous. We will come back to this point in Section 8.

¹⁰For analytical convenience, we model participation costs as a pecuniary cost rather than a utility cost, see also [Choné and Laroque \(2011\)](#).

¹¹We could easily allow for a type-specific distribution of participation costs $G_i(\varphi)$, but none of our results would change.

3.2 Firms

There is a group of mass one (representative) firm-owners, who own K units of capital, and employ all types of labor to produce a final consumption good.¹² We distinguish between individual firm-owners who take wages as given when making production decisions, and representatives of firm-owners who bargain with sectoral unions over the sectoral wage. Production is described by a constant-returns-to-scale production function:

$$F(K, L_1, \dots, L_I), \quad F_K(\cdot), F_i(\cdot) > 0, \quad F_{KK}(\cdot), F_{ii}(\cdot), -F_{Ki}(\cdot) \leq 0, \\ \lim_{K \rightarrow 0} F_K(\cdot) = \infty, \quad \lim_{K \rightarrow \infty} F_K(\cdot) = 0, \quad \lim_{L_i \rightarrow 0} F_i(\cdot) = \infty, \quad \lim_{L_i \rightarrow \infty} F_i(\cdot) = 0, \quad (2)$$

where the subscripts refer to the partial derivatives with respect to capital and type i (j) labor. We assume that capital and labor have positive, non-increasing returns in production. Moreover, capital and labor in sector i are co-operant production factors ($F_{Ki} \geq 0$). In deriving our main results, we make the following assumption:

Assumption 1. (Independent labor markets) *Labor productivity in sector i is unaffected by the amount of labor employed in sector $j \neq i$, i.e., $F_{ij}(\cdot) = 0$ for all $i \neq j$.*

Under Assumption 1, a change in employment in one sector does not affect the productivity of workers in other sectors.¹³ This assumption is made for technical convenience, as it ensures that there are no spillover effects between different segments of the labor market. In Section 6.2 we show that all our main results carry over in a modified form to the setting where labor markets are no longer assumed to be independent.

Profits are equal to output minus wage costs:

$$\pi = F(K, L_1, \dots, L_I) - \sum_i w_i L_i. \quad (4)$$

The representative firm maximizes profits taking sectoral wages w_i as given. The first-order conditions for profit maximization are given by:

$$w_i = F_i(K, L_1, \dots, L_I), \quad \forall i. \quad (5)$$

Firms demand labor input until the marginal product of labor is equal to the wage. Under Assumption 1, the demand for labor in sector i is only a function of the wage in sector i :

$$L_i = L_i(w_i), \quad L'_i(\cdot) = 1/F_{ii}(\cdot). \quad (6)$$

¹²Alternatively, we could assume there are sector-specific firms engaged in producing the final consumption good. As long as the government is able to observe (and tax) profits of all firms, none of our results would change.

¹³An example of a production function which satisfies Assumption 1 and the conditions listed in equation (2), is the following:

$$F(K, L_1, \dots, L_I) = K^\alpha \left(\sum_i a_i L_i^{1-\alpha} \right), \quad 0 \leq \alpha < 1, \quad (3)$$

where differences in a_i govern differences in productivity between different types of labor, or, alternatively, the degree to which different types of workers are complementary to capital. The canonical model with exogenous wages is obtained if $\alpha = 0$, in which case labor types are perfect substitutes.

The labor-demand elasticity ε_i in sector i is defined as:

$$\varepsilon_i \equiv -\frac{F_i(\cdot)}{L_i F_{ii}(\cdot)} > 0, \quad (7)$$

which depends only on the amount of type i labor employed, again due to Assumption 1. We can write the indirect profit function as:

$$\begin{aligned} \pi(w_1, \dots, w_I) &= F(K, L_1(w_1), \dots, L_I(w_I)) - \sum_i w_i L_i(w_i), \\ \frac{\partial \pi}{\partial w_i} &= -L_i, \quad \forall i. \end{aligned} \quad (8)$$

where the second line follows from the Envelope theorem. The consumption of firm-owners is denoted by c_f , which equals their profits π minus profit taxes T_f . Their utility is given by:

$$u(c_f) = u(\pi(w_1, \dots, w_I) - T_f). \quad (9)$$

The profit tax is fully non-distortionary, as it affects none of the economic decisions in our model. In Section 5.2, we also analyze the case where the government levies proportional employer taxes on wages. Contrary to a lump-sum profit tax, employer taxes are distortionary, since they affect the firms' labor-demand decisions.

3.3 Unions and labor-market equilibrium

All workers of type i are organized in a sectoral union, which aims to maximize the expected utility of its members.¹⁴ We characterize the labor-market equilibrium in sector i using a version of the Right-to-Manage (RtM) union model, due to [Nickell and Andrews \(1983\)](#). In this model, the wage w_i in sector i is determined through bargaining between the union representing type- i workers and (representatives of) firm-owners. Unions in each sector bargain independently with firm-owners and do not coordinate their actions. Hence, there is no 'leapfrogging' of unions between sectors. Firm-owners in each sector take the agreed-upon wage w_i as given and have the 'right to manage' how much labor they wish to hire. Employment is therefore determined via the labor demand equations (5). A well-known feature of the RtM-model is that it nests both the competitive equilibrium (CE) as well as the monopoly-union (MU) model (due to [Dunlop, 1950](#)) as a special case, each for a specific degree of the union's bargaining power relative to firm-owners. We will discuss these special cases in turn, and then show how our model can be parameterized for any intermediate degree of union bargaining power.

Before doing so, however, we first need to specify how jobs are allocated among workers with different participation costs: which workers become involuntarily unemployed if wages are set above their market-clearing levels? In most of what follows, we will assume labor-market *rationing* is efficient:

¹⁴It is straightforward to allow for different degrees of membership across workers with varying participation costs, or to specify a different union objective. As long as the unions care about the workers who face a positive probability of becoming unemployed and as long as the negotiated wage extends to non-union workers, the qualitative predictions of the model remain intact.

Assumption 2. (Efficient Rationing) *The incidence of involuntarily unemployment is borne by the workers with the highest participation costs φ .*

If labor markets are competitive, all unemployment is voluntary and Assumption 2 is trivially satisfied. If, however, part of unemployment is involuntary, there is no reason to believe that only individuals with the highest participation costs bear the burden of unemployment (see also [Gerritsen and Jacobs \(2016\)](#)). The assumption of efficient rationing, which clearly biases our results in favor of unions, will be relaxed in Section 6.2.

Let $E_i \equiv L_i/N_i$ denote the employment rate among type i workers. Under Assumption 2, individuals with participation costs $\varphi \in [\underline{\varphi}, \hat{\varphi}_i]$ where $\hat{\varphi}_i \equiv G^{-1}(E_i)$, are employed, whereas those with participation costs $\varphi \in (\hat{\varphi}_i, \bar{\varphi}]$ remain unemployed. The expected utility of workers in sector i , and hence the union's payoff, can then be written as

$$\Lambda_i = \int_{\underline{\varphi}}^{\hat{\varphi}_i} u(c_{i,\varphi})dG(\varphi) + \int_{\hat{\varphi}_i}^{\bar{\varphi}} u(c_u)dG(\varphi) = E_i \overline{u(c_i)} + (1 - E_i)u(c_u), \quad (10)$$

where $\overline{u(c_i)} \equiv \int_{\underline{\varphi}}^{\hat{\varphi}_i} u(c_{i,\varphi})dG(\varphi)/E_i$ denotes the average utility of employed workers in sector i . Note that, because participation is voluntary, the fraction of workers willing to participate is always weakly larger than the rate of employment: $E_i = \int_{\underline{\varphi}}^{\hat{\varphi}_i} dG(\varphi) = G(\hat{\varphi}_i) \leq G(\bar{\varphi}_i)$.

If a union in sector i has full bargaining power – a case to which we will refer as the monopoly-union (MU) model – the union chooses the combination of the wage w_i and the rate of employment E_i which maximizes its objective (10) subject to the firm's labor demand curve (5). The equilibrium in the MU-model can thus be found by solving:

$$\begin{aligned} \max_{E_i, w_i, \hat{\varphi}_i} \quad & \Lambda_i = \int_{\underline{\varphi}}^{\hat{\varphi}_i} u(w_i - T_i - \varphi)dG(\varphi) + \int_{\hat{\varphi}_i}^{\bar{\varphi}} u(-T_u)dG(\varphi) \\ \text{s.t.} \quad & w_i = F_i(\cdot), \quad \hat{\varphi}_i = G^{-1}(E_i). \end{aligned} \quad (11)$$

The first-order conditions yield the following optimality condition:

$$1 = \varepsilon_i \frac{u(\hat{c}_i) - u(c_u)}{u'(c_i)w_i}, \quad (12)$$

where $u(\hat{c}_i)$ denotes the utility of the marginally employed worker (i.e., the worker with participation costs $\hat{\varphi}_i$), and $\overline{u'(c_i)}$ is the average marginal utility of employed workers in sector i . If the union has full bargaining power, it sets the wage w_i in sector i such that marginal benefit of raising the wage for the employed with one euro (left-hand side) equals the marginal costs of higher unemployment (right-hand side). The marginal cost of setting the wage above the market-clearing level equals the elasticity of labor demand multiplied with the union wedge $\frac{u(\hat{c}_i) - u(c_u)}{u'(c_i)w_i}$, which denotes the monetized utility difference between the marginally employed and unemployed worker as a fraction of the wage w_i . Importantly, the union wedge is determined solely by the utility loss of the marginally employed worker. Under Assumption 2, the marginal workers are the ones to lose their jobs if the wage is marginally increased. From the union's optimality condition (12), it can readily be verified that a decrease in either the unemployment benefit $-T_u$ or the income tax T_i reduces wage demands. Intuitively, a reduction in either the

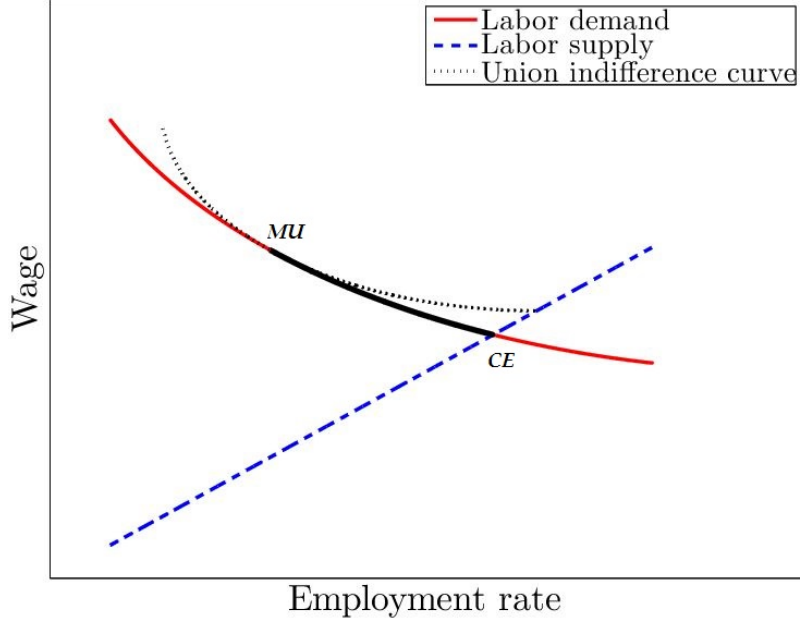


Figure 2: Labor market equilibria in the right-to-manage model

unemployment benefit or the income tax makes employment more attractive than unemployment, which induces the union to increase employment by reducing the wage claim.

In the opposite case where unions have no bargaining power at all, the wage is driven to the point where the marginally employed worker is indifferent between participating and not participating, i.e., $u(\hat{c}_i) = u(c_u)$. In this case, the labor-market outcome corresponds to the competitive outcome. The labor-market equilibrium can be found by combining the firm's labor-demand condition (5) with the labor-supply equation:

$$E_i = G(w_i - T_i + T_u). \quad (13)$$

Since there is no involuntary unemployment, $\hat{\varphi}_i = \bar{\varphi}_i$. Like before, a reduction in either the income tax T_i or the unemployment benefit $-T_u$ leads to higher employment and a lower wage. This time, however, the reduction in employment does not come about through a reduction in the union's wage claim, but rather through an increase in labor participation.

The competitive outcome and the monopoly-union outcome represent the two polar cases in our analysis. We will employ a version of the RtM-model that allows for any intermediate degree of union power. This is graphically illustrated in Figure 2. The competitive equilibrium lies at the intersection of the labor supply curve and the labor demand curve. The MU-outcome, in turn, lies at the point where the union's indifference curve is tangent to the labor demand curve. This point can be found by combining the union's first-order condition (12) with the labor demand schedule (5). In our characterization of the labor-market equilibrium, any point on the bold part of the labor demand curve corresponds to an equilibrium in the RtM-model. The higher (lower) is the union's bargaining power, the closer will the outcome lie to the monopoly-union outcome (competitive outcome).

We characterize the labor-market equilibrium in the RtM-model as follows. Union i 's bargaining power (or, equivalently, the degree of unionization in sector i) is denoted by $\rho_i \in [0, 1]$. This parameter determines which point of the labor demand curve between MU and CE is reached in the wage negotiations. In Appendix A we provide a micro-economic foundation for ρ_i based on the standard RtM-model with Nash-bargaining between unions and firms.¹⁵ Using ρ_i as our measure of the union's bargaining power, we characterize the equilibrium in the RtM-model as follows:

$$\rho_i = \varepsilon_i \frac{u(\hat{c}_i) - u(c_u)}{u'(c_i)w_i}. \quad (14)$$

If $\rho_i = 1$, this outcome corresponds to the equilibrium of in the MU-model and if $\rho_i = 0$, the marginal worker is indifferent between working and not working (i.e., $\hat{c}_i = c_u$) and the competitive outcome applies. Consequently, $0 < \rho_i < 1$ corresponds to any intermediate case of the RtM-model. The higher (lower) is ρ_i , the higher (lower) is the wage. The interpretation of the expression is otherwise the same as in the case of the monopoly-union. The labor market equilibrium can be found by combining equation (14) with the labor-demand schedule (5).

3.4 Government

The government is assumed to maximize a utilitarian social welfare function:¹⁶

$$\mathcal{W} \equiv \sum_i N_i (E_i \overline{u(c_i)} + (1 - E_i)u(c_u)) + u(c_f). \quad (15)$$

The government aims to maximize social welfare by transferring resources between firm-owners, employed workers (in various occupations), and unemployed workers. The government observes the employment status of all workers, as well as wages (assumed to vary across occupations) and profits. Tax policy cannot be conditioned on participation costs φ , which are private information. Consequently, the government cannot redistribute income between workers in the same sector who face different participation costs, and is furthermore unable to distinguish between workers who chose not to participate and those who did not manage to find a job. This results in a second-best problem where the government needs to resort to distortionary taxes and transfers to redistribute income.

In line with the informational assumptions, the government can set income taxes T_i , as well as a profit tax T_f to finance an unemployment benefit $-T_u$ and an exogenous revenue requirement R . Allowing the The government's budget constraint is given by:

$$\sum_i N_i (E_i T_i + (1 - E_i)T_u) + T_f = R. \quad (16)$$

¹⁵In particular, we demonstrate that there exists a monotonic relationship between ρ_i and the weight of the union in the Nash-product. Hence, our short-cut is without loss of generality and allows us to fully analytically trace down the implications of changing union power, while avoiding a number of unimportant technical complications.

¹⁶The utilitarian specification is without loss of generality, since one can allow for stronger redistributive desires by adopting a more concave cardinalization of the utility function, adopt a concave transformation of individual utilities, or introduce Pareto weights for each individual.

3.5 General equilibrium

Wages w_i and employment rates E_i in each sector i follow from simultaneously solving the wage mark-ups (14) and labor demand equations (5). Goods market equilibrium, in turn, requires that all consumption demands and the revenue requirement from the government equal production:

$$F(K, N_1 E_1, \dots, N_I E_I) = \sum_i \int_{\underline{\varphi}}^{\hat{\varphi}_i} N_i (c_{i,\varphi} + \varphi) dG(\varphi) + \sum_i N_i (1 - E_i) c_u + c_f + R. \quad (17)$$

Note that if the budget constraints of workers and firm-owners and the governments budget constraint hold, the economy's resource constraint is satisfied automatically by Walras' law.

3.6 Comparative statics

Before turning to the problem of optimal taxation, we first derive the behavioral responses of the labor-market outcomes (i.e., wages and employment rates) with respect to the tax instruments T_i and T_u . For analytical convenience, in most of what follows we focus on the case where the unions' wage claims respond symmetrically to either an increase in the income tax T_i or an increase in the unemployment benefit $-T_u$, i.e.,

$$\frac{\partial w_i}{\partial T_i} = -\frac{\partial w_i}{\partial T_u}. \quad (18)$$

If the above condition is satisfied, we say there are no *income effects* at the union level. Under this assumption, the equilibrium wage and employment rate in sector i can be written solely as a function of the participation tax rate $t_i \equiv (T_i - T_u)/w_i$, i.e., $w_i = w_i(t_i)$ and $E_i = E_i(t_i)$.¹⁷ The following Lemma describes the behavioral responses.

Lemma 1. *Under Assumptions 1 and 2, and in the absence of income effects at the union level, the wage and employment elasticities with respect to the participation tax rate t_i are given by*

$$\kappa_i \equiv \frac{\partial w_i}{\partial t_i} \frac{1 - t_i}{w_i} = \frac{u'_u w_i (1 - t_i)}{\hat{u}'_i \varepsilon_i E_i / g(\hat{\varphi}_i) + u'_u w_i (1 - t_i) - (\hat{u}_i - u_u) \left(1 + \varepsilon_i \varepsilon_{\varepsilon_i} + \varepsilon_i \frac{(\bar{u}'_i - \hat{u}'_i)}{u'_i} \right)} > 0, \quad (19)$$

$$\eta_i \equiv -\frac{\partial E_i}{\partial t_i} \frac{1 - t_i}{E_i} = \frac{\varepsilon_i u'_u w_i (1 - t_i)}{\hat{u}'_i \varepsilon_i E_i / g(\hat{\varphi}_i) + u'_u w_i (1 - t_i) - (\hat{u}_i - u_u) \left(1 + \varepsilon_i \varepsilon_{\varepsilon_i} + \varepsilon_i \frac{(\bar{u}'_i - \hat{u}'_i)}{u'_i} \right)} > 0, \quad (20)$$

where $\varepsilon_{\varepsilon_i} \equiv \frac{\partial \varepsilon_i}{\partial E_i} \frac{E_i}{\varepsilon_i} = -\left(1 + \frac{1}{\varepsilon_i} + \frac{E_i F_{iii}}{F_{ii}}\right)$ is the elasticity of the labor-demand elasticity with respect to the employment rate. The employment and wage elasticity are related via $\eta_i = \varepsilon_i \kappa_i$.

Proof. See Appendix B. □

According to Lemma 1, an increase in the participation tax (resulting from either an increase in the income tax or the unemployment benefit) raises the union's wage demand, which – through a reduction in labor demand – lowers employment.

¹⁷A sufficient condition for the absence of income effects at the union level is when $u(\cdot)$ is of the CARA-type. In Appendix C.3 we derive the optimal tax structure with income effects, and demonstrate that allowing for income effects brings no substantive economic insights.

4 Optimal taxation

The government optimally chooses unemployment benefits $-T_u$, profit taxes T_f and participation tax rates t_i in order to maximize its objective (15), subject to the budget constraint (16), taking into account the behavioral responses summarized in Lemma 1.

We characterize the optimal tax policy in terms of behavioral responses and social welfare weights for the different groups. The latter are defined as:

$$b_i \equiv \frac{\overline{u'(c_i)}}{\lambda}, \quad b_u \equiv \frac{u'(c_u)}{\lambda}, \quad b_f \equiv \frac{u'(c_f)}{\lambda}, \quad (21)$$

where λ denotes the multiplier on the government's budget constraint. The welfare weight b_j measures the monetized increase in social welfare resulting from a one unit increase in the incomes of individuals belonging to group j . Furthermore, define the labor shares of workers ω_i and the unemployed ω_u as:

$$\omega_i \equiv \frac{N_i E_i}{\sum_j N_j}, \quad \omega_u \equiv \frac{\sum_i N_i (1 - E_i)}{\sum_j N_j}. \quad (22)$$

The following Proposition describes the optimal tax policy:

Proposition 1. *If unemployment benefits $-T_u$, profit taxes T_f and participation tax rates t_i are optimally set, the following conditions must hold:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \quad (23)$$

$$b_f = 1, \quad (24)$$

$$\left(\frac{t_i + \tau_i}{1 - t_i} \right) \eta_i = (1 - b_i) + (b_i - 1) \kappa_i, \quad (25)$$

where $\tau_i \equiv \frac{u(\hat{c}_i) - u(c_u)}{\lambda \omega_i} = \frac{\rho_i b_i}{\varepsilon_i}$ denotes the union wedge.

Proof. See Appendix C.1. □

Equation (23) states that a weighted average of the welfare weights of the employed and unemployed workers should sum to one. Intuitively, the government uniformly raises transfers to all individuals until the marginal utility benefits of doing so (left-hand side) are equal to the unit marginal costs of providing everyone with a marginally higher transfer (right-hand side). This confirms the intuition from Jacobs (2016) that the marginal cost of public funds equals one in the policy optimum even under distortionary taxation. A direct implication is that the social welfare weight of some groups of workers exceeds one, while for others their welfare weight is below one. Because the welfare weight of the non-employed always exceeds the welfare weights of the employed, it must be that $b_u > 1$. For condition (23) to remain valid, there must therefore be at least one group of workers i for whom $b_i < 1$. This, however, does not imply that the welfare weight of *all* employed workers are below one. Depending on the redistributive preferences of the government, there may also be employed workers whose welfare weight is above one. In the remainder, we will refer to workers for whom $b_i > 1$ as low-income, or

low-skilled workers. Equation (24), in turn, states that the government taxes firm-owners until their welfare weight equals one. Since the profit tax is a non-distortionary tax, the government raises profit taxes until it is indifferent between raising firm-owners' consumption with one unit and receiving a unit of public funds.

The optimal participation tax rate t_i is determined by equation (25). The left-hand side of this expression captures the total distortions of participation taxes in sector i , whereas the right-hand side captures the redistributive gains (or losses). The total wedge on labor is $\frac{t_i + \tau_i}{1 - t_i}$ and consists of the explicit tax on participation t_i , and the union wedge, which acts as an *implicit* tax on labor. In words, τ_i denotes the monetized loss in social welfare if the marginal worker in sector i loses employment, as a fraction of gross earnings in sector i . The implicit tax, which measures the distortions from unions bidding up wages above the market-clearing level, is proportional to the union's bargaining power and inversely related to the labor-demand elasticity. Hence, it is zero if either the union has no bargaining power ($\rho_i = 0$), or if labor demand is infinitely elastic ($\varepsilon_i \rightarrow \infty$). In the latter case, unions refrain from demanding a wage that is above the market-clearing level, since doing so would result in a complete breakdown of employment.

Equation (25) shows that the government should set lower participation taxes in sectors where the welfare gains from lowering involuntary unemployment are high, i.e., in sectors where τ_i is large. By inducing unions to moderate their wage claims, low participation taxes alleviate the welfare costs of involuntary unemployment. Hence, if the welfare costs of unemployment are very high, participation tax rates should optimally be lowered *ceteris paribus*. The total wedge is weighed by the employment elasticity with respect to the participation tax. Therefore, if employment responds strongly to taxation (i.e., if η_i is large), the participation tax should optimally be lowered.

The right-hand side of the expression for the optimal participation tax (25) gives the distributional benefits (or costs) of increasing the participation tax rate in sector i . The first term gives the direct distributional effect of raising the participation tax rate, which transfers resources from workers with welfare weight b_i to the government budget. Second, higher participation taxes raise wage demands, which reduce the income of firm-owners. Participation taxes thus indirectly also redistribute resources from the firm-owners towards the workers via higher wage demands. This is socially desirable if the workers in sector i have a higher welfare weight than the firm owners ($b_i > 1$). In that case, participation taxes should be higher the more responsive are wages to increases in the participation tax (i.e., the higher is κ_i).

Like in Diamond (1980), Saez (2002), and Choné and Laroque (2011) we find that it is optimal to subsidize participation on a net basis (i.e., setting $t_i < 0$) for low-income workers whose welfare weight is above one, i.e., if $b_i > 1$. However, and in stark contrast to the result from these models, subsidizing participation can also be optimal for workers whose welfare weight is below unity ($b_i < 1$). This occurs if the welfare costs of involuntary unemployment resulting from unions is very high, so that the implicit tax τ_i is large. When the costs of involuntary unemployment are high, it might be optimal to subsidize participation even for workers whose welfare weight exceeds one. A reduction in the participation tax lowers wage demands, and, as a result, some workers who were involuntarily unemployed are now able to

find a job. Income taxes (or subsidies) are not only used to redistribute income, but also serve to alleviate the labor-market distortions induced by unions.

Our optimal tax formula nests the one derived in [Saez \(2002\)](#) (for the case with no intensive margin) as a special case. If wages are exogenous, optimal participation taxes are determined via:

$$\frac{t_i}{1-t_i} = \frac{1-b_i}{\gamma_i}, \quad \gamma_i \equiv \frac{\partial G(\bar{\varphi}_i)}{\partial \bar{\varphi}_i} \frac{\bar{\varphi}_i}{G(\bar{\varphi}_i)}. \quad (26)$$

Here, γ_i denotes the participation elasticity in sector i . If labor demand is infinitely elastic, equations (25) and (26) coincide. In this case, unions will always refrain from demanding above market-clearing wages. Consequently, wages are essentially exogenous and the result from [Saez \(2002\)](#) applies. The result from [Saez \(2002\)](#) also holds in a perfectly competitive labor market with endogenous wages, which in our model corresponds to setting $\rho_i = 0$ for all i . Hence, if labor markets are competitive, labor-demand considerations are irrelevant for the characterization of optimal participation taxes. This result is derived as well in [Christiansen \(2015\)](#), and is a version of what is labeled by [Saez \(2004\)](#) the ‘Tax-Formula result’ due to [Diamond and Mirrlees \(1971a,b\)](#).

If $\rho_i > 0$ and labor demand is not perfectly elastic, labor-demand considerations are no longer irrelevant in the optimal tax formulae, since they interfere with the union behavior. In particular, the labor-demand elasticity enters the general-equilibrium employment elasticity η_i , the wage elasticity κ_i , and the union wedge τ_i . Hence, whenever the degree of unionization is non-zero, the labor-supply elasticity is no longer a sufficient statistic to characterize optimal tax policies. This finding is consistent with [Jacquet et al. \(2014\)](#), who also identify a crucial role for the labor-demand elasticity in the expression for the optimal participation tax if labor markets are not competitive.

4.1 Restricted profit taxation

How is the design of optimal tax policies affected when there is a restriction on profit taxation? Many studies have analyzed similar questions in the context of union models, see, e.g., [Fuest and Huber \(1997\)](#), [Koskela and Schöb \(2002\)](#), [Aronsson and Sjögren \(2004b\)](#), [Aronsson and Wikström \(2011\)](#), [Christiansen \(2016\)](#). The following Corollary presents the restricted optimum if the government cannot freely choose the profit tax T_f .

Corollary 1. *For an arbitrary level of profit taxation T_f , the following conditions must hold if unemployment benefits $-T_u$ and participation tax rates t_i are optimally set:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \quad (27)$$

$$\left(\frac{t_i + \tau_i}{1-t_i} \right) \eta_i = (1-b_i) + \left(\frac{(1-b_i)t_i + b_i - b_f}{1-t_i} \right) \kappa_i. \quad (28)$$

Proof. See Appendix [C.2](#). □

Compared to Proposition 1, the expression for the optimal unemployment benefit is unaffected and there is no expression for the optimal profit tax. The expression for the optimal participation tax rate is slightly more involved. The first term in the numerator on the right-hand

side appears purely for mechanical reasons. Because we let the government choose participation tax rates $t_i \equiv (T_i - T_u)/w_i$ (rather than income tax levels T_i), a change in the wage also mechanically redistributes resources from workers in sector i to the government. The associated welfare effect is proportional to $1 - b_i$, and this effect is captured by the first term. Concerning the second term, if there are limits to the extent profits can be taxed, the welfare weight of firm-owners falls short of one, i.e., $b_f < 1$. Firm-owners have higher income than before, so their social welfare weight is lower than in the case without a binding restriction on profit taxation. This provides an additional rationale for levying participation taxes. Higher participation taxes motivate unions to increase their wage demands, which indirectly redistributes resources from firm-owners to workers. The welfare effect is proportional to $b_i - b_f$ and is weighed by the elasticity of wages with respect to the participation tax rate. Equation (28) thus states that the more binding is the restriction on profit taxation (i.e., the lower is b_f), the higher should participation taxes be set, *ceteris paribus*.

5 Desirability of unions and the optimal degree of unionization

5.1 Desirability of unions

The previous section analyzed the optimal tax-benefit system if labor markets are unionized. In this section, we ask the question: can it be socially desirable to allow workers to organize themselves in a union? And, if so, under which conditions? The following Proposition addresses both points.

Proposition 2. *If income taxes and transfers are optimized, and profit taxes are either optimized or exogenously given, increasing the bargaining power ρ_i of the union in sector i raises social welfare if and only if the welfare weight of the workers in sector i exceeds one: $b_i > 1$.*

Proof. See Appendix D. □

According to Proposition 2, it is socially desirable to let low-income workers organize themselves in a union, despite the fact that unions distort an efficient functioning of the labor market. To understand why this is the case, consider the introduction of a union for low-skilled workers in a sector where the wage was initially determined competitively. That is, consider a marginal increase in ρ_i above zero in a sector where $b_i > 1$. The introduction of a union leads to an increase in the wage and an accompanying decrease in the rate of employment, as illustrated in Figure 2. If $b_i > 1$ and labor markets are competitive, participation is subsidized on a net basis in the policy optimum (see equation (26)), i.e., $t_i < 0$. As a result, labor participation is distorted *upwards*: too many low-skilled workers decide to participate. Unions alleviate this distortion by putting upward pressure on the wage, which lowers employment and hence, raises government revenue. Moreover, the rise in the equilibrium wage transfers income from firm-owners (whose welfare weight is one) to employed workers in sector i (whose welfare weight is above one), which again raises social welfare. Furthermore, starting from the competitive labor-market outcome without unemployment, a marginal increase in unemployment does not lead to a utility loss of the workers who lose their job, since rationing is assumed to be efficient. Indeed, following the reduction in employment due to the introduction of the union,

the employed workers who enter unemployment first are indifferent between employment and non-employment. Adding up, the introduction of a union unambiguously raises social welfare if the welfare weight of the workers in this sector is larger than one ($b_i > 1$).

Another way to understand the efficiency-enhancing role of unions is the following. Consider a situation where the tax system is optimally chosen and there is an exogenous increase in the union's bargaining power ρ_i . The increase in the union's bargaining power puts upward pressure on the wage, which brings about a reduction in employment. Now, suppose the government wishes to use its tax instrument to off-set the impact from an increase in the union's bargaining power on labor-market outcomes. In order to do so, it needs to lower the participation tax in sector i . To keep the budget balanced, the loss in tax revenue can be financed, for instance, by raising the profit tax.¹⁸ This policy intervention brings about solely a transfer in resources from firm-owners (whose welfare weight is one) to low-skilled workers (whose welfare weight exceeds one), which is welfare-enhancing if and only if $b_i > 1$. It should be noted, though, that while employment and wages remained unaffected, there is another effect: *voluntary* unemployment is substituted for *involuntary* unemployment. Due to the rise in the net income for the low-skilled, more workers want to participate, while the rate of employment is kept constant. Due to the assumption of efficient rationing, however, none of the new participants would be able to find a job. And since the utilities in both states are the same, the distinction between voluntary and involuntary unemployment are immaterial from a utilitarian perspective. Hence, the total welfare effect of the increase in the degree of unionization and the policy reform is proportional to $b_i - 1$.

The reverse is also true: there is no role for a union in sectors where the social welfare weights of the workers in sector i is smaller than one, i.e., $b_i < 1$. If $b_i < 1$ and labor markets are competitive, participation is taxed on a net basis. Compared to the efficient level, there is now too little employment. Raising union power ρ_i will reduce employment even further, which is accompanied by a loss in tax revenue. If $b_i > 1$, unions exacerbate existing labor-market distortions even further by creating involuntary unemployment.

Interestingly, Proposition 2 generalizes to a setting where profits cannot be fully taxed, as formally established in Appendix D. At first sight, this result may seem counter-intuitive because increasing the union's bargaining power indirectly redistributes resources from firm-owners to workers in the form of higher wages, which may seem desirable if profits cannot be directly taxed. However, the government can already achieve this form of indirect redistribution through the wage channel via the tax-benefit system. As was demonstrated in Corollary 1, participation taxes should be raised if profits cannot be fully taxed, *ceteris paribus*. Higher participation taxes puts upward pressure on wages, which indirectly redistributes income from firm-owners to workers. A restriction on profit taxation does not provide an *additional* reason to introduce unions. If the tax-benefit system is optimized, the only role of unions is to reduce labor-market distortions resulting from overemployment, which occurs whenever $b_i > 1$.¹⁹

¹⁸Increasing the profit tax is only one way to finance the decrease in the income tax for workers in sector i . As long as the welfare costs of raising one unit of revenue with other instruments are equal to one, the argument carries over to other instruments as well.

¹⁹A word of caution is required here. Since the welfare weights are endogenous, they are generally affected by a restriction on profit taxation. It is therefore incorrect to conclude that a restriction on profit taxation does not make unions more (or less) desirable. Hence, we only state that a restriction on profit taxation does not provide

5.2 Optimal degree of unionization

We also take a first pass at addressing the question: what is the socially optimal degree of unionization, and how can the resulting allocation be implemented? Starting with the first, suppose the government could impose the bargaining power of unions ρ_i at no costs, how would it choose to set the bargaining power of each union relative to that of the firm-owners?²⁰ The results are summarized in the next Corollary.

Corollary 2. *If taxes and transfers are set according to Proposition 1, then the optimal degree of unionization in sector i equals $\rho_i^* = \min[\hat{\rho}_i, 1]$ if $b_i \geq 1$, and $\rho_i^* = \max[\hat{\rho}_i, 0]$ if $b_i \leq 1$, where $\hat{\rho}_i$ is the bargaining power of union i required to make the social welfare weight of workers equal to one: $\hat{\rho}_i = \{\rho_i : b_i = 1\}$.*

According to Corollary 2, for workers whose social welfare weight exceeds one (i.e., $b_i \geq 1$), the bargaining power of the union representing these workers should optimally be increased until their social welfare weight equals one. However, if this is not feasible (which can happen if workers have low wages w_i), the next best thing to do is to make the labor union a monopoly-union, i.e., to set $\rho_i^* = 1$. For workers whose social welfare weight smaller than one ($b_i < 1$), the government would like to lower the bargaining power of the union representing them. However, the government can never decrease the degree of unionization below the competitive level due to the voluntary participation constraint.

At this point, the question arises if and how the government can implement the allocation with the socially optimal degree of unionization, because the workers' bargaining power ρ_i is generally not considered a policy instrument. Here we highlight two possibilities: sector-specific employer taxes, or sector-specific minimum wages. Starting with the first, suppose the government levies a proportional, sector-specific employer tax on wages denoted by θ_i . Labor demand in sector i then reads as:

$$w_i(1 + \theta_i) = F_i(K, L_1, \dots, L_I). \quad (29)$$

The corresponding wage mark-up equation is modified to:²¹

$$\frac{\rho_i}{1 + \theta_i} = \varepsilon_i \frac{u(\hat{c}_i) - u(c_u)}{u'(c_i)w_i}. \quad (30)$$

Employer taxes raise the costs of unemployment when the union demands higher wages. Hence, higher (lower) employer taxes induce unions to moderate (increase) wage demands. In that respect, they play a very similar role as marginal tax rate progressivity, from which we have abstracted in our model.²² The next Proposition characterizes how the optimal employer taxes should be set.

an *additional* reason why unions could be desirable.

²⁰Obviously, a thorough analysis requires careful examination of whether, and at what costs, the government is able to affect unions' bargaining power. In this context, [Hungerbühler and Lehmann \(2009, p.475\)](#) remark that: "Whether and how the government can affect the bargaining power is still an open question". They suggest that changing the way in which unions are financed and regulated can affect the workers' bargaining power.

²¹This equation can be derived in completely analogous fashion as in Section 3.3, with the labor demand schedule (5) replaced by equation (29).

²²See also the literature discussed in Section 2.

Proposition 3. *Assume there are no sectors for which it would be socially optimal to increase the union’s bargaining power above the monopoly-union level (i.e., $\hat{\rho}_i^* \leq 1$ for all i) and suppose the government sets the participation tax rates, the unemployment benefits and the profit taxes optimally. Then, the allocation implicitly characterized by Proposition 1 and Corollary 2 can be implemented by levying sector-specific, proportional employer taxes θ_i .*

Proof. See Appendix D.2. □

The government imposes employer taxes (subsidies) if it wishes to decrease (increase) union’s wage demands, which is desirable if $b_i < 1$ ($b_i > 1$). If employer taxes are optimized, the effective union power $\rho_i/(1+\theta_i)$ equals the optimal degree of union power ρ_i^* , as characterized in Corollary 2. The government can thus use employer taxes to steer the union’s bargaining power to its socially optimal level. It is important to note that, as mentioned before, proportional employer taxes serve a very similar role as marginal tax progression, from which we have abstracted in our model. Like employer taxes, progressive marginal tax rates moderate wage demands, since the government penalizes unions to bid up wages.

As a second possibility, the government could use sector-specific minimum wages to implement the optimal second-best allocation with the optimal degree of unionization. Like minimum wages, unions fix a wage in each sector. Therefore, it should come as no surprise that our findings in Corollary 2 bear resemblance to findings obtained in Lee and Saez (2008, 2012).²³ In a two-sector model, they show that it is optimal to increase the minimum wage for the low-skilled workers up to the point where their welfare weight is equal to one, provided that rationing is efficient, labor demand is not perfectly elastic and the tax system is optimally chosen. The reasoning is very similar: if rationing is efficient and participation decisions are distorted upwards, both a minimum wage and the introduction of a union allow the government to redistribute resources in a non-distortionary way towards low-income workers. Such income redistribution should always be carried out until the low-income workers’ welfare weight equals one. The next Proposition makes the formal connection between our results to those derived in Lee and Saez (2012).

Proposition 4. *Assume there are no sectors for which it would be socially optimal to increase the union’s bargaining power above the monopoly-union level (i.e., $\rho_i^* \leq 1$ for all i). Then, the optimal second-best allocation characterized by Proposition 1 and Corollary 2 is equivalent to the optimal second-best allocation with competitive labor markets and optimal sector-specific minimum wages. In addition, the tax-benefit systems required to support these allocations are identical in both cases.*

Proof. See Appendix D.3. □

Like unions, minimum wages are useful if individual participation decisions are distorted upwards as a result of participation subsidies (Lee and Saez, 2012). Optimal sector-specific minimum wages and optimally chosen degrees of unionization are therefore equivalent policy instruments which can both support the same second-best allocation. By appropriate choices of

²³See Proposition 3 in Lee and Saez (2008) or Proposition 2 in Lee and Saez (2012).

union power, the same second-best wage level can be obtained as in the case where the government would directly optimize over wages in each sector. Proposition 4 furthermore states that the optimal tax-benefit system is identical in both cases, which is shown formally in Appendix D.3. This seems surprising, given the fact that unions – unlike minimum wages – react to changes in tax policies. These effects, however, can be ignored if the degree of unionization is at its socially optimal level. In that case, changes in union behavior induced by tax changes have no first-order effect on welfare and, as a result, the tax system which implement the optimal second-best allocation are identical.

6 Robustness analysis

In this section, we investigate the robustness of our results by relaxing the assumptions of independent labor markets (Assumption 1) and efficient rationing (Assumption 2). In addition, we analyze two alternative bargaining structures: one in which a single union bargains with firm-owners over the entire *distribution* of wages, and one in which unions bargain with firms over wages *and* employment in the spirit of the efficient bargaining model due to McDonald and Solow (1981).

6.1 Interdependent labor markets

If Assumption 1 is violated and labor markets are interdependent (such that $F_{ij} \neq 0$ for all $i \neq j$), taxes levied in one sector will also affect labor market outcomes in other sectors. These effects have to be taken into account when designing the optimal tax-benefit system. Proposition 5 generalizes Proposition 1 and characterizes the optimal tax policy when labor markets are not necessarily independent. We maintain the assumptions that income effects are absent and rationing is efficient.

Proposition 5. *If labor markets are interdependent, optimal unemployment benefits $-T_u$, profit taxes T_f , and participation tax rates t_i are determined by:*

$$\sum_i \omega_i b_i + \omega_u b_u = 1, \quad (31)$$

$$b_f = 1, \quad (32)$$

$$\sum_j \omega_j \left(\frac{t_j + \tau_j}{1 - t_j} \right) \eta_{ji} = \omega_i (1 - b_i) + \sum_j \omega_j (b_j - 1) \kappa_{ji}, \quad (33)$$

where the cross elasticities of employment and wages in sector j with respect to participation taxes in sector i are defined as:

$$\eta_{ji} \equiv - \frac{\partial E_j}{\partial t_i} \frac{1 - t_i}{E_j} \left(\frac{w_j (1 - t_j)}{w_i (1 - t_i)} \right), \quad (34)$$

$$\kappa_{ji} \equiv \frac{\partial w_j}{\partial t_i} \frac{1 - t_i}{w_j} \left(\frac{w_j (1 - t_j)}{w_i (1 - t_i)} \right). \quad (35)$$

Proof. See Appendix E. □

Equations (31)–(32) are identical to those stated in Proposition 1, and their explanation is not repeated here. Optimal participation tax rates t_i in equation (33) are modified compared to their counterparts from Proposition 1. If labor markets are interdependent, an increase in the participation tax rate in sector i affects labor market outcomes in potentially all sectors of the economy. This explains the summation in equation (33) over all sectors. As in Proposition 1, the left-hand side gives the marginal cost of higher participation taxes in the form of larger labor-market distortions, whereas the right-hand side gives the distributional benefits (or losses) of higher participation taxes. If the participation tax rate in sector i is increased, the union in sector i responds by increasing its wage claim. *Ceteris paribus*, this leads to a decrease in the rate of employment in sector i . If labor types are complementary (i.e., $F_{ij}(\cdot) > 0$ for $i \neq j$), the decrease in the rate of employment in sector i also brings about a decrease in the productivity of workers, and thus in labor demand, in all other sectors $j \neq i$. Consequently, both employment and wages in all sectors are reduced. The reduction of employment is larger if the (weighted) cross elasticity η_{ji} of employment in sector j with respect to the participation tax rate in sector i is larger. Thus, if the sum of the explicit and implicit tax on participation is positive (negative), i.e., $\frac{t_j + \tau_j}{1 - t_j} > 0$ (< 0), a higher participation tax in sector i exacerbates (alleviates) labor market distortions in sectors j .

The right-hand side of (33) captures the distributional benefits of a higher participation tax in sector i . An increase in the participation tax rate t_i redistributes income from workers in sector i to the government, and hence to everyone else in the economy. The associated welfare effect is proportional to $1 - b_i$. Furthermore, the change in the participation tax in sector i brings about redistribution from firm-owners (whose welfare weight is one in the optimum) to workers in sector i (whose welfare weight is b_i) via a change in w_i . In addition, there are indirect redistributive consequences in all other sectors $j \neq i$, because wages in all other sectors are reduced if participation taxes in sector i are raised and labor types are complementary in production. If the social welfare weights of workers in sectors j are larger than one, i.e., $b_j > 1$, the reduction in wages in sector j due to higher participation taxes in sector i is socially costly, because the social welfare weight of the firm-owners is lower. However, if the social welfare weight of workers in sectors j is smaller than one, i.e., $b_j < 1$, the reduction in wages in sector j is socially beneficial, because the social welfare weight of the firm-owners is higher. The strength of these cross-sectoral redistributions of income between firm-owners and workers is determined by the wage elasticity κ_{ji} in sector j with respect to the participation tax rate in sector i . If labor markets are independent, $\eta_{ji} = \kappa_{ji} = 0$ for all $j \neq i$ and the result from Proposition 1 applies.

Turning to the question whether or not unions are desirable if labor markets are interdependent, we find that Proposition 2 generalizes completely (see Appendix E for details). In particular, an increase in union bargaining power ρ_i raises social welfare if and only if the social welfare weight of the workers exceeds one, i.e., $b_i > 1$. As explained before, increasing the bargaining power of a union in a particular sector allows the government to *de facto* redistribute income in a lump-sum way towards those working in this sector (provided that rationing is efficient). This result holds irrespective of the presence of spillover effects in production. While increasing the union's bargaining power in sector i puts upward pressure on the wage in sector

i , this effect can be perfectly offset by lowering the participation tax t_i in sector i , such that no change in wages and employment in sector i results. If neither the wage nor the employment rate in sector i is affected, the same will be true for the labor market outcomes in sector j , even if labor markets are interconnected. Hence, our earlier argument extends to the case with interdependent labor markets.

6.2 Inefficient rationing

We have deliberately biased our findings in favor of unions by assuming that unemployment rationing is efficient: the burden of involuntary unemployment is borne by the workers with the highest participation costs. However, there are neither theoretical nor empirical reasons to expect that labor market rationing is always efficient, see [Gerritsen \(2016\)](#) and [Gerritsen and Jacobs \(2016\)](#). Therefore, in this section we analyze how the optimal tax formulae should be modified, and under which condition unions are still desirable if the assumption of efficient rationing is relaxed.

We follow [Gerritsen \(2016\)](#) and [Gerritsen and Jacobs \(2016\)](#) by modelling the rationing scheme using conditional employment probabilities:

Definition 1. (Rationing schedule) *The rationing schedule is a function*

$$e_i(E_i, \bar{\varphi}_i, \varphi) \tag{36}$$

which specifies the employment rate $e_i \in [0, 1]$ of type- i workers with participation costs $\varphi \in [\underline{\varphi}, \bar{\varphi}_i]$, for a given rate of employment E_i among type- i workers and participation threshold $\bar{\varphi}_i$ in sector i .

For all values of the employment rate E_i and the participation cut-off $\bar{\varphi}_i$, the following relationship must hold:

$$\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_i(E_i, \bar{\varphi}_i, \varphi) dG(\varphi) \equiv E_i. \tag{37}$$

Summing over the employment probabilities of the workers in sector i (who vary in terms of their participation costs) yields the sectoral employment rate. The function $e_i(\cdot)$ is assumed to be differentiable, increasing in its first argument, and decreasing in its second argument, i.e., $e_i E_i(\cdot), -e_i \bar{\varphi}_i(\cdot) > 0$. An example of a rationing schedule which satisfies the above criteria is a uniform rationing scheme. If rationing is uniform, all workers willing to participate face the same probability of (not) finding a job. This case corresponds to setting $e_i(E_i, \bar{\varphi}_i, \varphi) = E_i/G(\bar{\varphi}_i)$ for all values of $\varphi \in [\underline{\varphi}, \bar{\varphi}_i]$. The following Proposition generalizes the optimal tax formulae to account for inefficient rationing. Like before, we ignore income effects at the union level and re-invoke the assumption of independent labor markets.

Proposition 6. *Suppose the employment probability of worker $\varphi \in [\underline{\varphi}, \bar{\varphi}_i]$ in sector i is $e_i(E_i, \bar{\varphi}_i, \varphi)$. If labor markets are independent, the optimal tax formulae are given by*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \tag{38}$$

$$b_f = 1, \quad (39)$$

$$\left(\frac{t_i + \hat{\tau}_i}{1 - t_i}\right) \eta_i - \left(\frac{\psi_i}{1 - t_i}\right) \gamma_i = (1 - b_i) + (b_i - 1)\kappa_i, \quad (40)$$

where the definition of the union wedge is modified to

$$\hat{\tau}_i \equiv \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{iE_i}(E_i, \bar{\varphi}_i, \varphi) \left(\frac{u(w_i(1 - t_i) - T_u - \varphi) - u(-T_u)}{\lambda w_i} \right) dG(\varphi), \quad (41)$$

and ψ_i denotes the ‘rationing wedge’, which is defined as

$$\psi_i \equiv \frac{e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i)}{E_i/G(\bar{\varphi}_i)} \int_{\underline{\varphi}}^{\bar{\varphi}_i} \frac{e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi)}{\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) dG(\varphi)} \left(\frac{u(w_i(1 - t_i) - T_u - \varphi) - u(-T_u)}{\lambda w_i} \right) dG(\varphi). \quad (42)$$

Finally, $\gamma_i \equiv \frac{\partial G(\bar{\varphi}_i)}{\partial \bar{\varphi}_i} \frac{\bar{\varphi}_i}{G(\bar{\varphi}_i)} > 0$ is the participation elasticity of workers in sector i .

Proof. See Appendix F.1. □

The expression for the optimal unemployment benefit and profit tax are again identical to those stated in Proposition 1 and their explanation is not repeated here. The expression for the optimal participation tax rate (40) equates the distortionary costs of a higher participation tax rate (left-hand side) to the distributional gains of a higher participation tax rate (right-hand side). Compared to equation (25), the expression for the optimal participation tax is modified in two ways. First, with a general rationing scheme, the union wedge $\hat{\tau}_i$ no longer measures the monetized utility loss when the *marginal* worker loses his or her job, but instead the expected utility loss of all *rationed* workers: the workers who lose their job when the wage is marginally increased. Second, in addition to the union wedge $\hat{\tau}_i$, there is a distortion associated to the inefficiency of the rationing scheme. The latter is captured by the ‘rationing wedge’ ψ_i .

To understand the ψ_i -term, consider a decrease in the participation tax rate t_i , but suppose the union refrains from lowering its wage claim, so that also the employment rate remains unaffected. Following the reduction in the participation tax rate, more people want to participate. If rationing is efficient, none of the new participants would be able to find a job, since they all have higher participation costs than the workers who were employed before the reduction in the participation tax rate. However, if rationing is inefficient, this is no longer the case. In particular, a fraction $e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i)$ of the workers who are at the participation margin (i.e., those who are indifferent between employment and non-employment) will succeed in finding a job. With a constant employment rate, this has to come at the cost of other workers losing their job. And since these workers were not necessarily at the participation margin, a welfare loss occurs. The latter is captured by the term ψ_i , which measures the welfare costs associated with an inefficient allocation of jobs over those willing to work. Naturally, the welfare loss of inefficient rationing increases in the participation elasticity γ_i . Furthermore, if rationing is efficient, the employment probability for the marginal participant is zero, i.e., $e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i) = 0$, and there is no rationing wedge: $\psi_i = 0$. If rationing is inefficient, the rationing wedge is positive. According to equation (40), the higher is ψ_i (i.e., the more inefficient the rationing scheme), the *higher* should be the optimal participation tax rate. The intuition is similar to [Gerritsen \(2016\)](#): by setting

a higher participation tax, the government replaces involuntary unemployment by voluntary unemployment, which reduces the inefficiency of labor market rationing. A higher participation tax discourages participation, which induces the workers who care least about finding a job to opt out of the labor market. This, in turn, increases the employment prospects of the workers who experience a higher surplus from working.

To answer the question whether it is still optimal to increase the union's bargaining power, we consider the following policy reform (starting from a situation where taxes are optimally set):

1. Starting from an arbitrary degree of union power $\rho_i \in [0, 1)$, the union's power in sector i is marginally increased.
2. There is a simultaneous reduction in the participation tax rate t_i in sector i such that the gross wage w_i , and hence the rate of employment E_i , is kept constant.²⁴
3. The reduction in the participation tax rate is financed by an increase in the profit tax T_f to ensure that the government budget remains balanced.

The next Corollary gives the condition under which an increase in the union's bargaining power improves social welfare if rationing is no longer efficient:

Corollary 3. *Suppose taxes are set according to Proposition 6. Then, an increase in union i 's bargaining power ρ_i raises social welfare if and only if*

$$b_i > 1 + \left(\frac{\psi_i}{1 - t_i} \right) \gamma_i. \quad (43)$$

Proof. See Appendix F.2. □

This result can be understood as follows. The reduction in the participation tax in sector i raises the income of type- i workers. The welfare effect equals $-N_i E_i b_i w_i dt_i$. The increase in the profit tax lowers the welfare of the firm-owners with $-b_f dT_f = b_f N_i E_i w_i dt_i = N_i E_i w_i dt_i$, where the first equality follows from the balanced-budget assumption and the second from the optimality condition for the profit tax (i.e., $b_f = 1$). By construction of the policy reform, there are no welfare effects associated with changes in equilibrium wages and employment rates. Hence, if labor-market rationing is efficient, these are all the relevant effects and the total welfare effect of the combined increase in the union's bargaining power and the policy reform is proportional to $b_i - 1$. This confirms the result from Proposition 2. This is no longer true if rationing is inefficient. The increase in net earnings in sector i increases participation of type- i workers. Now, if some of the (previously voluntarily) non-employed workers find a job, a welfare loss occurs because – with a constant rate of employment – some participants who experience a surplus from working will not be able to find a job.

The welfare loss of inefficient rationing is captured by the second term on the right-hand side of equation (43) and is proportional to the rationing wedge ψ_i , and the participation

²⁴This implies that also the wages and employment rates in other sectors remain unaffected, even if labor markets are interdependent (see Section 6.1). Hence, our analysis does not require labor markets to be independent.

elasticity γ_i . Corollary 3 states that the more inefficient the rationing scheme, or the higher is the participation elasticity (i.e., the higher are ψ_i or γ_i), the less likely it is that an increase in the bargaining power of the union in this sector is socially desirable, *ceteris paribus*. By increasing net incomes, unions raise the participation rate, while rationing the number of jobs. An inefficient allocation of jobs over those willing to participate results in a welfare loss. The welfare costs of inefficient rationing could be so large that they completely off-set the potential welfare gains of unions. Consequently, if rationing is inefficient, it is no longer guaranteed that increasing the union power in a sector where $b_i > 1$ unambiguously raises social welfare.

6.3 Bargaining over multiple wages

In our model, bargaining takes place at the sectoral level and wages only vary across (and not within) sectors. Each sectoral union faces a trade-off between employment and wages for its members, but does not care about the overall *distribution* of wages. There is, however, ample empirical evidence that a higher degree of unionization is associated with lower wage inequality, also at the sectoral, firm and establishment level. See, for instance, Freeman (1980, 1991), Lemieux (1993, 1998), Machin (1997), Card (2001), DiNardo and Lemieux (1997), Card et al. (2004), Visser and Checchi (2011), Western and Rosenfeld (2011). In a paper related to ours, Christiansen (2016) therefore studies the implications of union-induced wage compression for optimal taxation. He finds that wage compression leads to an inefficient allocation of workers over occupations, but lowers the need to use distortionary taxation for income redistribution. Furthermore, and in line with our results, Christiansen (2016) identifies an important role for labor-demand responses for the design of optimal tax policies.

How are the results derived in the current paper affected when unions care about the distribution of wages? To that end, we consider an alternative formulation of the model. Rather than assuming there are multiple unions bargaining over a *single* wage, we now analyze the setting in which a single union bargains with firm-owners over *all* wages. We assume the union has a utilitarian objective. Under efficient rationing, the union's objective therefore reads:

$$\Lambda = \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right). \quad (44)$$

As in the RtM-model, wages are determined through bargaining, while firms (unilaterally) determine employment. Importantly, we do not make the assumption that labor markets are independent. Put differently, we allow the wage for one group of workers to depend on the rate of employment in other sectors. As in Christiansen (2016), these complementarities in production allow the union to compress the wage distribution. For instance, a decrease in the wage (claim) for high-skilled workers increases employment in the high-skilled sector through the firms' labor-demand responses. If labor types are complementary in production, the increase in high-skilled employment increases the productivity (and hence the wage) for lower skilled workers. The union can exploit these complementarities in order to reduce wage inequality.

We assume wages are determined through Nash-bargaining. Hence, the labor-market equi-

librium can be characterized by solving the following maximization program:²⁵

$$\begin{aligned}
\max_{\{w_i, E_i\}_{i \in \mathcal{I}}} \quad \Omega &= \beta \log \left(\sum_i N_i \int_{\varphi_i}^{G^{-1}(E_i)} (u(w_i(1-t_i) - T_u - \varphi) - u(-T_u)) dG(\varphi) \right) \\
&+ (1 - \beta) \log \left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0} - T_f) \right) \quad (45) \\
\text{s.t.} \quad w_i - F_i(\cdot) &= 0, \quad \forall i \\
G(w_i(1-t_i)) - E_i &\geq 0, \quad \forall i.
\end{aligned}$$

The first (second) set of constraints correspond to the labor-demand (voluntary participation) conditions. The union's bargaining power is denoted by $\beta \in [0, 1]$. Because there is one union, the latter does not vary across sectors. This has an important implication: with a single union, there is no longer a direct mapping between the single union's bargaining power β and sector-specific bargaining power ρ_i , which we can use to study the welfare effects of changing the *sectoral* bargaining power. How we address the question whether an increase in the degree of unionization is desirable in this setting will be made clear below.

In Appendix G.1, we characterize the labor-market equilibrium and discuss some of its properties. Here, we highlight the most important features. First, if the union has no bargaining power ($\beta = 0$), the labor-market equilibrium coincides with the competitive outcome. This should come as no surprise. Without any bargaining power for the workers, wages are lowered up to the point where the marginal worker for each type is indifferent between working and not working. Second, if the union's bargaining power is sufficiently high ($\beta \rightarrow 1$), there is at least one group of workers for whom the wage is raised above the market-clearing level. Again, this is intuitive. Starting from the competitive equilibrium, the union has an incentive to increase wages at the very bottom of the income distribution. If labor types are complementary in production, the increase in income for low-skilled workers comes at the costs of income losses for higher-skilled workers. From a utilitarian perspective, however, the former effect dominates. Consequently, the wage for the lowest skill-group will always be raised above the market-clearing level if the union's bargaining power is sufficiently high. Third, it may not be in the union's best interest to raise *all* wages above the market-clearing level. This is because an increase in the wage for high-skilled workers may lower the incomes for low-skilled workers if there are complementarities in production. A utilitarian union may therefore refrain from demanding an above market-clearing wage for high-skilled workers.

How should taxes be optimally set if there is a single union which bargains with firm-owners over the entire distribution of wages? This question can be analyzed in analogous fashion as in the extension with interconnected labor markets (see Section 6.1). In fact, we find that the result from Proposition 5 generalizes completely. Ignoring income effects, the reduced-form wage and employment equations can again be written as $w_i = w_i(t_1, \dots, t_I)$ and $E_i = E_i(t_1, \dots, t_I)$. And since the optimal tax formulae from Proposition 5 are derived for a general specification of the relationship between the tax instruments and labor-market outcomes, they continue to apply.

²⁵In this formulation, both parties' payoff is taken in deviation from the payoff associated with the disagreement outcome (see the Appendix for details).

Under which condition would an increase in the union’s bargaining power enhance social welfare if the union cares about the distribution of wages? At first sight, this question appears hard to answer. As explained above, an increase in the union’s bargaining power has ambiguous effects on the distribution of wages. The union may either increase wages for all workers, or decide to only increase wages at the bottom at the expense of the wages for high-skilled workers. Because of this ambiguity, we tackle the question by analyzing the welfare effects of a joint increase in the union’s bargaining power and a tax reform which leaves all labor-market outcomes unaffected. If taxes are optimally set, the tax reform has no first-order welfare effects and any change in welfare must necessarily come from the increase in the union’s bargaining power.

To find the policy reform which leaves the labor-market outcomes unaffected following an increase in the union’s bargaining power, let us first define by

$$k(\beta) \equiv \{i : G(w_i(1 - t_i)) > E_i\} \quad (46)$$

the worker types for whom the wage is raised above the market-clearing level. Clearly, $k(\cdot)$ depends – among other things – on the union’s bargaining power $\beta \in [0, 1]$. If the union has no bargaining power ($\beta = 0$), no wage is raised above the market clearing level, and consequently $k(\cdot)$ is empty. On the other hand, $k(\beta)$ contains at least one element when $\beta = 1$. A utilitarian union always has an incentive to increase the wages for the lowest-skill group, even if this comes at the costs of lower wages for higher-skilled workers. Now, consider a marginal increase in β which leaves $k(\beta)$ unaffected. The rise in the union’s bargaining power puts further upward pressure on the wages of workers $i \in k(\beta)$ for whom the wage already exceeds the market-clearing level (the ‘direct’ effect). Through spillovers in production, the wages for the other worker types $j \notin k(\beta)$ will be affected as well (the ‘indirect’ effect). Now, consider a tax reform which leaves all labor-market outcomes (i.e., wages and employment rates in all sectors) unaffected. We show in Appendix G.3 that such a tax reform requires *only* an adjustment in the participation tax rates t_i for those workers whose wage exceeds the market-clearing level (i.e., for the worker types $i \in k(\beta)$). Intuitively, if the adjustment in the tax system offsets the ‘direct’ effects, there will also be no ‘indirect’ effects. The reform which leaves labor-market outcomes unaffected can be found by solving the following system of equations for the changes in the participation tax rates:

$$\forall i \in k(\beta) : \sum_{j \in k(\beta)} \frac{\partial w_i(t_1, \dots, t_I, \beta)}{\partial t_j} dt_j^* + \frac{\partial w_i(t_1, \dots, t_I, \beta)}{\partial \beta} d\beta = 0, \quad (47)$$

where the functions $w_i = w_i(t_1, \dots, t_I, \beta)$ are implicitly characterized as the solution to the maximization problem (45). In words, the tax reform off-sets the impact from an increase in the degree of unionization on wages. Clearly, if this is the case, then also the rates of employment are unaffected. A change in the union’s bargaining power and a tax reform which satisfies (47) therefore leaves all labor-market outcomes unaffected.

Now, consider the following reform:

1. Starting from a situation where taxes are optimally set, the union’s bargaining power β

is marginally increased.

2. There is a simultaneous change in the participation tax rates t_i for $i \in k(\beta)$ which satisfies equation (47).
3. The change in the participation tax rate is financed by an increase in the profit tax T_f to ensure that the government budget remains balanced.

The next Proposition addresses the question whether an increase in the union's bargaining power leads to an improvement in welfare:

Proposition 7. *Suppose there is a single utilitarian union which bargains with firm-owners over the entire distribution of wages. Furthermore, assume the tax-benefit system is set according to Proposition 5. Then, an increase in the union's bargaining power leads to an increase in social welfare if and only if*

$$\sum_{i \in k(\beta)} \omega_i (b_i - 1) (-dt_i^*) > 0, \quad (48)$$

where the dt_i^* 's solve equation (47) and $k(\beta)$ is defined by equation (46).

Proof. See Appendix G.3 □

Proposition 7 is an intuitive counterpart of Proposition 2: an increase in the union's bargaining enhances social welfare if and only if doing so allows the government to increase the incomes of workers whose social welfare weight (on average) exceeds one. To see why this is so, consider a sector where the wage is raised above the market clearing level. Both an increase in the union's bargaining and an increase in the participation tax put further upward pressure on the wage, *ceteris paribus*. To prevent a wage increase following a rise in the union's bargaining power, the participation tax rate for workers from this sector has to be lowered: $dt_i^* < 0$. For a constant gross income w_i , the reduction in the participation tax rate implies an increase in the net income for workers in sector i . Hence, what Proposition 7 states is that an increase in the union's bargaining power improves social welfare if and only if the increase in the union's bargaining power increases the net incomes of workers whose social welfare weight 'on average' exceeds one. Put differently: if the union is mostly inclined to increase wages above the market-clearing level for workers whose welfare weight exceeds one, raising the union's bargaining power is desirable from a social welfare point of view.

Another way to understand this result is as follows. Depending on the union's bargaining power, there is a (potentially empty) subset of wages which exceed the market-clearing level. Marginally increasing the union's bargaining power puts upward pressure on these wages (the 'direct' effect). Through complementarities in production, the other wages are affected as well (the 'indirect' effect). The effects on the entire distribution of wages can be off-set by lowering the participation taxes only for workers whose wages are directly affected. Lowering the participation taxes for these workers induces the union to lower the wage claims in the corresponding sectors, thereby off-setting the impact from an increase in the union's bargaining power. Because the tax reform off-sets the direct effect, there will also be no spill-over effects.

By design, the combined increase in the union’s bargaining power and the tax reform leaves all labor-market outcomes unaffected. The *net* incomes for the low-skilled workers, however, have increased. The increase in the union’s bargaining power therefore only redistributes resources from the government’s budget to workers whose wage was raised above the market-clearing level. For this reason, increasing the union’s bargaining power raises social welfare if and only if the welfare weight of these workers exceeds one, in weighted average terms. Naturally, the weight attached to a specific group of workers depends on how many workers belong to this group (as captured by ω_i), and by how much the union is inclined to increase these workers’ wage (which determines by how much the tax needs to be lowered to prevent a wage increase, $-dt_i^*$).

The forces which drive the union’s desirability condition are identical as before: unions are desirable (from a social welfare perspective) if participation subsidies lead to upward distortions in labor supply. Importantly, the union’s desire to compress the wage distribution does not provide an *additional* reason why a welfarist government would prefer to have unions over competitive labor markets. Through the tax-benefit system, the government can already influence both the pre- and after-tax wage distribution, and there is no need to rely on the union’s desire or ability to compress the wage distribution.

6.4 Efficient bargaining

Up to this point, we have assumed that bargaining takes place in a right-to-manage setting. Unions and (representatives of) firm-owners make binding agreements on wages. Firm-owners – taking the negotiated wages as given – optimally decide how many workers to hire. [McDonald and Solow \(1981\)](#) have shown that this bargaining structure generally leads to outcomes which are not Pareto efficient. This is because firm-owners do not take into account the impact of their hiring decisions on the union’s objective. This inefficiency can be overcome if unions and firm-owners bargain over both wages *and* employment, as in the efficient bargaining (EB) model of [McDonald and Solow \(1981\)](#). The natural question thus arises how robust our results are – especially those regarding the desirability of unions – if we analyze a setting with efficient bargaining between unions and firms.

We would like to emphasize from the outset that we consider the EB-model less appealing, both theoretically and empirically. The assumption that firms and unions can write contracts on both wages *and* employment levels is problematic, especially if bargaining takes place at the national or industry level. As noted by [Boeri and Van Ours \(2008\)](#), this would require employers’ associations to commit its members to employment levels that are not consistent with profit-maximizing behavior. Furthermore, it is considered a stylized fact that firms unilaterally set employment, even if bargaining takes place at the firm level ([Oswald, 1993](#)). Moreover, the EB-model predicts higher employment levels compared to competitive labor markets, since some profits of firms are converted into jobs. The model also predicts that reductions in union power (or ‘wage moderation’ policies) result in lower rather than higher employment levels. The last two implications contrast with the predictions from the RtM-model, and are difficult to defend empirically. We therefore keep the RtM-model as our baseline.

Keeping these shortcomings in mind, we now analyze a version of the EB-model of [McDonald](#)

and Solow (1981). The main difference of our framework with McDonald and Solow (1981) is that our model features multiple unions and heterogeneity in participation costs among workers represented by each union. To maintain tractability, we therefore re-invoke the assumptions of independent labor markets and efficient rationing. The key feature of the EB-model is that bargaining leads to an efficient outcome, which implies that any potential equilibrium (w_i, E_i) lies on the *contract curve*. The latter is constructed by equating the firm's and union i 's marginal rate of substitution between wages and employment in sector i :

$$\frac{u(w_i - T_i - \hat{\varphi}_i) - u(-T_u)}{E_i u'(w_i - T_i - \varphi)} = \frac{w_i - F_i(\cdot)}{E_i}. \quad (49)$$

Intuitively, if wages and employment are on the contract curve it is no longer possible to raise either union i 's utility while keeping firm profits constant, or vice versa.

The contract curve defines a set of potential labor-market equilibrium outcomes (w_i, E_i) in sector i . Which contract is negotiated depends on the bargaining power of union i relative to that of the firm. We model union i 's bargaining power as its ability to expropriate part of the firm's profits. Alternatively, we could think of the union's bargaining power as its ability to bargain for a wage which exceeds the marginal product of labor. Let σ_i denote the bargaining power of union i . We select the equilibrium in labor market i using the following rent-sharing rule:

$$w_i = (1 - \sigma_i)F_i(\cdot) + \sigma_i\phi_i(\cdot), \quad (50)$$

where $\phi_i(E_i) \equiv \frac{\Phi_i(N_i E_i)}{N_i E_i}$ is the average production of a worker of type- i and Φ_i is the contribution of type- i workers to total production:

$$\Phi_i(N_i E_i) \equiv F(K, N_1 E_1, \dots, N_i E_i, \dots, N_I E_I) - F(K, N_1 E_1, \dots, 0, \dots, N_I E_I). \quad (51)$$

If unions have zero bargaining power, i.e., $\sigma_i = 0$, the EB-model yields the competitive outcome: $w_i = F_i(\cdot)$ and efficiency requires $\hat{\varphi}_i = w_i - T_i + T_u = w_i(1 - t_i) = \bar{\varphi}_i$. If, on the other hand, union i has full bargaining power, i.e., $\sigma_i = 1$, it can offer a contract which leaves no surplus for firm-owners. In the latter case, the wage equals the average productivity and the firm makes zero profits from hiring type- i workers: $w_i N_i E_i = \Phi_i(\cdot)$. We refer to this outcome as the full expropriation (FE) outcome.

The characterization of the equilibrium is graphically illustrated in Figure 3. The contract curve (49) depicts the locus of points where the union's indifference curve and the firm's iso-profit curve are tangent. Which point is selected, depends on the union's bargaining power σ_i . As in the RtM-model, if the union has zero bargaining power, the equilibrium coincides with the competitive outcome. If the union's bargaining power increases, the equilibrium moves along the contract curve towards the FE-equilibrium where the union has full bargaining power. The higher (lower) is the union's bargaining power, the closer is the outcome to the full expropriation (competitive) outcome.

Figure 3 provides three important insights. First, as in the RtM-model, there is involuntary unemployment whenever the degree of unionization is nonzero. In the absence of involuntary unemployment, unions are marginally indifferent to changes in employment. Under efficient

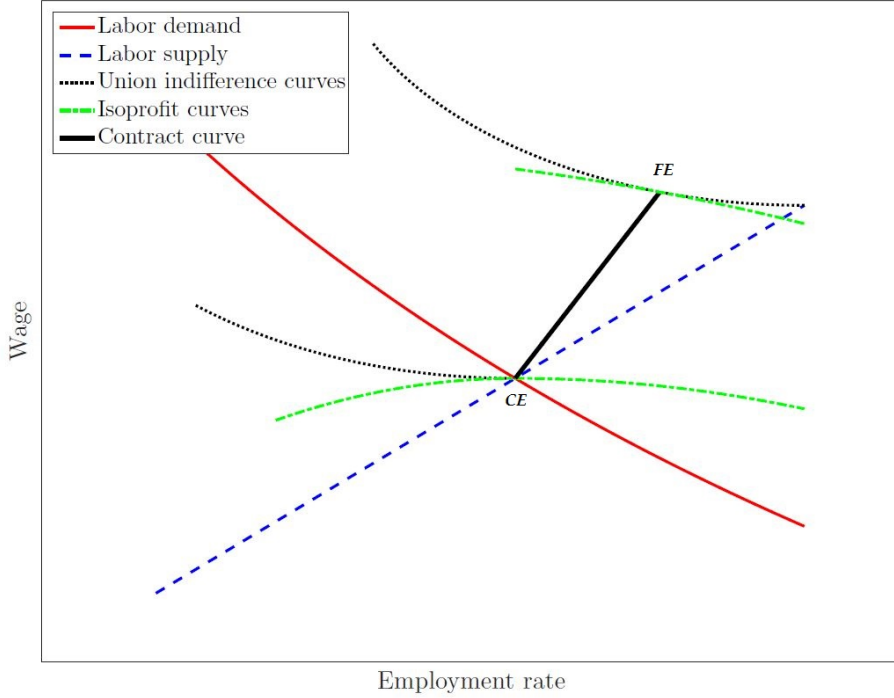


Figure 3: Labor market equilibria in the efficient bargaining model

rationing, unions are always willing to bargain for a slightly higher wage, and accept some unemployment. Second, in contrast to the RtM-model, there is also a labor-demand distortion in the EB-model: the wage exceeds the marginal product of labor whenever $\sigma_i > 0$ (see equation (50)). Consequently, the equilibrium is no longer on the labor-demand curve. The reason is that, when the wage equals the marginal product of labor, firms are indifferent to changes in employment. This is generally not true for unions. Hence, it is possible to find a contract with a lower wage and a higher rate of employment which benefits both parties. In this new contract, the wage exceeds the marginal product of labor and, as a consequence, labor-demand decisions are distorted. Third, and in stark contrast to the RtM-model, an increase in the union's bargaining power in the EB-model will not only result in a higher wage, but also in a *higher* rate of employment. As illustrated in Figure 3, the contract curve is upward sloping. The higher the union's bargaining power, the larger is the share of the bargaining surplus which accrues to union members. Due to the concavity of the utility function $u(\cdot)$, this surplus is translated partly into higher wages, and partly in the form of higher employment.

In the absence of income effects and with independent labor markets, the contract curve (49) and the rent-sharing rule (50) jointly determine the equilibrium wage w_i and employment rate E_i in sector i solely as a function of the participation tax rate t_i .²⁶ If the participation tax rate increases, fewer workers want to participate. In terms of Figure 3, the labor-supply schedule shifts upward. Unions and firm-owners take this into account in the bargaining process and the equilibrium wage (employment rate) will be higher (lower) following the increase in the participation tax. The comparative statics in the EB-model are, therefore, qualitatively the

²⁶The conditions under which income effects are absent in the EB-model are essentially the same as in the RtM-model: income effects are absent at the union level if the individual utility function $u(\cdot)$ is of the CARA-type.

same as in the RtM-model (see Lemma 1). The following Proposition derives optimal taxes and transfers in the EB-model.

Proposition 8. *Let σ_i denote the bargaining power of union i in the EB-model, and let the equilibrium in labor market i be determined by the contract curve (49) and the rent-sharing rule (50). Then, under Assumption 1, Assumption 2, and in the absence of income effects at the union level, optimal unemployment benefits $-T_u$, profit taxes T_f and participation tax rates t_i are determined by:*

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \quad (52)$$

$$b_f = 1, \quad (53)$$

$$\left(\frac{t_i + \tau_i - m_i}{1 - t_i} \right) \eta_i = (1 - b_i) + (b_i - 1) \kappa_i, \quad (54)$$

where all definitions are as before, and $m_i \equiv \frac{w_i - F_i}{w_i} = \sigma_i \left(\frac{\phi_i - F_i}{w_i} \right)$ is the labor-demand wedge. The wage and employment elasticities with respect to the participation tax rate t_i are given by:

$$\kappa_i = \frac{u'_u w_i (1 - t_i) \left(\frac{(1 - m_i)(1 - \sigma_i)}{\varepsilon_i} + m_i \right)}{\frac{\hat{u}'_i E_i}{g(\hat{\varphi}_i)} + u'_u w_i (1 - t_i) \left(\frac{(1 - m_i)(1 - \sigma_i)}{\varepsilon_i} + m_i \right) + (\hat{u}_i - u_u) \left(\frac{(1 - m_i)(1 - \sigma_i)}{m_i \varepsilon_i} - 1 + \frac{(\hat{u}'_i - u'_i)}{u'_i} \right)} > 0, \quad (55)$$

$$\eta_i = \frac{-u'_u w_i (1 - t_i)}{\frac{\hat{u}'_i E_i}{g(\hat{\varphi}_i)} + u'_u w_i (1 - t_i) \left(\frac{(1 - m_i)(1 - \sigma_i)}{\varepsilon_i} + m_i \right) + (\hat{u}_i - u_u) \left(\frac{(1 - m_i)(1 - \sigma_i)}{m_i \varepsilon_i} - 1 + \frac{(\hat{u}'_i - u'_i)}{u'_i} \right)} > 0. \quad (56)$$

Proof. See Appendix H.2. □

The optimality conditions in the EB-model are very similar to their counterparts in the RtM-model, see Proposition 1. Except from differences in the elasticities, the main difference is the presence of the labor-demand distortion m_i in the expression for the optimal participation tax rate t_i . In the EB-model, the equilibrium wage exceeds the marginal product of labor if unions have nonzero bargaining power. Consequently, a decrease in the rate of employment in sector i positively affects the firm's profits. The stronger the impact on profits (i.e., the higher the mark-up m_i), the higher should participation tax rates be optimally set, *ceteris paribus*. Higher participation taxes boost firm profits by lowering employment rates.

The optimal participation tax rate t_i aims to both redistribute income and alleviate the implicit tax from involuntary unemployment and the labor-demand distortion. The implicit tax and the labor demand wedge move the optimal participation tax in opposite directions. On the one hand, employment is too low because unions generate involuntary unemployment, which calls for lower participation taxes. On the other hand, employment is too high because unions create labor-demand distortions leading to excess employment. This calls for a higher participation tax rate. Since both distortions increase in the union's bargaining power, it is no longer unambiguously true that participation taxes should optimally be lower if union power increases. This contrasts the finding from the RtM-model. With only a distortion in labor

supply (in the form of involuntary unemployment) and no distortion in labor demand, a higher bargaining power always lowers the optimal participation tax rate in the RtM-model, *ceteris paribus*.

Are unions still desirable for income redistribution? The next Proposition addresses this question.

Proposition 9. *If taxes and transfer are set according to Proposition 8, increasing the bargaining power σ_i of the union in sector i in the efficient bargaining model raises social welfare if and only if $b_i > 1$.*

Proof. See Appendix H.3. □

Proposition 9 states that the condition under which an increase in union i 's bargaining power is welfare-enhancing in the EB-model is the same as in the RtM-model. Hence, the question whether or not unions are desirable does not depend on the bargaining structure. And this is true despite the fact that both bargaining structures yield opposite conclusions regarding the impact of the union's bargaining power on the rate of employment. The reason is that, in both models, the equilibrium rate of employment with unions is lower than in the absence of unions, *for a given net income from participation*. Put differently, under both bargaining structures, the equilibrium lies left of the labor-supply schedule. As explained before, under efficient rationing unions are always willing to accept some involuntary unemployment. Hence, even though employment increases in the union's bargaining power in the EB-model, *participation* increases even more, resulting in involuntary unemployment. The latter can be desirable if there is too much employment as a result of net subsidies on participation. Therefore, the intuition for the desirability of unions in the RtM-model carries over to the EB-model: unions are only useful if participation subsidies lead to upward distortions in labor supply and hence, result in overemployment.

7 Numerical illustration

This section intends to illustrate numerically how the presence of unions affects the optimal tax-benefit system. To do so, we implement a sufficient-statistics approach developed by Kroft et al. (2015). They show that the optimal tax-benefit system in the presence of wage and (un)employment responses can be calculated using only a limited number of sufficient statistics, and subsequently calibrate the optimal tax-benefit system using estimates of these statistics.²⁷ In this section, we will first identify which statistics are required to calculate the optimal tax-benefit system derived in the current paper. Next, using empirically reasonable values for these statistics, we calculate the optimal tax-benefit system for varying degrees of unionization, both for the U.S. and the Netherlands. These countries provide two interesting benchmarks, since they differ substantially in the extent to which labor markets are unionized. For instance, in 2013 84.8% of all employees in the Netherlands were covered by collective wage bargaining

²⁷In particular, they show these statistics consist of (i) the micro participation elasticity; (ii) the macro participation elasticity, which differs from the micro-elasticity through general-equilibrium effects on wages and employment probabilities; and (iii) the macro employment elasticity to taxation.

agreements, while the corresponding figure for the U.S. is only 11.9% (ICTWSS database, November 2015).

In our calibration we focus on the RtM-model with independent labor markets, efficient rationing and no income effects at the union level. In the optimal tax formulae derived in Proposition 1, two types of behavioral responses are present: the wage elasticity (κ_i) and the employment elasticity (η_i) with respect to the participation tax rate t_i . These elasticities are related via the labor-demand elasticity $\varepsilon_i = \eta_i/\kappa_i$, which also affects the union wedge τ_i . To calculate the optimal tax-benefit system, we therefore only require estimates for two out of the three elasticities. Of these, the labor demand elasticity is the one most frequently estimated in the empirical literature, see [Lichter et al. \(2014\)](#) for a recent overview. To obtain an estimate for either κ_i or η_i , we rely on the large empirical literature which estimates the impact of participation taxes on *participation* (rather than employment) rates. Using a constant elasticity specification, we can write the labor-market equilibrium conditions as:

$$E_i = \zeta_i(w_i(1 - t_i))^{\pi_i}, \quad \zeta_i > 0 \quad (57)$$

$$E_i = \xi_i w_i^{-\varepsilon_i}, \quad \xi_i > 0. \quad (58)$$

Equation (57) represents the combination of wages and employment rates which – for a given labor-demand elasticity, union power and participation tax rate – solves the markup-equation (14). Hence, it is best thought of as a modified labor-supply schedule, and consequently π_i as a modified labor-supply elasticity (which we proxy using estimates of the participation elasticity).²⁸ Equation (58) gives the labor demand schedule, and ε_i the corresponding labor-demand elasticity. These relationships define the equilibrium wage and employment rate in each sector i as a function of the participation tax rate t_i , i.e., $E_i = E_i(t_i)$ and $w_i = w_i(t_i)$. The relevant elasticities are given by:

$$\eta_i = \frac{\pi_i \varepsilon_i}{\pi_i + \varepsilon_i}, \quad \kappa_i = \frac{\pi_i}{\pi_i + \varepsilon_i}. \quad (59)$$

Using estimates of participation elasticities π_i and labor demand elasticities ε_i , the wage elasticity η_i and employment elasticity κ_i can be calculated using equation (59).

In addition to the optimal tax formulae and the labor-market equilibrium conditions, we require a specification of the welfare weights. Following [Saez \(2002\)](#) and [Kroft et al. \(2015\)](#), we parameterize the latter using the conventional CRRA-specification:

$$b_i = \frac{1}{\lambda(w_i(1 - t_i) - T_u)^\nu}, \quad (60)$$

$$b_u = \frac{1}{\lambda(-T_u)^\nu}, \quad (61)$$

where λ refers to the multiplier on the government's budget constraint and ν measures the concavity of the social welfare function (or, equivalently, of the individual utility function). Note that this specification of the welfare weights is defined only over consumption bundles and ignores participation costs of the working individuals.

²⁸These elasticities would coincide if unions are absent, i.e., if $\rho_i = 0$.

7.1 Simulations U.S.

To calibrate the model, we combine estimates of the labor demand and participation elasticities with information on the current tax-benefit system and the current labor market outcomes. For the U.S., the latter information is obtained from [Kroft et al. \(2015\)](#). For a detailed description of the data set, we refer to their paper and the accompanying Online Appendix. Here, we only restate the information that is directly relevant for our purposes. The sample of single women is divided into four categories, based on educational attainment. For each of these categories, information is provided regarding (i) the sample size, (ii) the rate of employment, (iii) the average wage, and (iv) the average tax bill. Also, the (average) transfer received by the unemployed is observed. We use a labor-demand elasticity of 0.6 in our baseline simulations, assumed constant across sectors. This value is well within the range of estimates reported in [Lichter et al. \(2014\)](#). For the participation elasticity, we choose a value of 0.4 in our baseline simulations, again assumed not to vary across sectors. This value lies somewhat in between the estimates obtained for primary earners (which are typically lower) and the estimates that are obtained from exploiting EITC variation (which tend to be somewhat higher). [Table 1](#) provides a summary of all the inputs we use in our simulations.

Table 1: Simulation inputs U.S.

	(1)	(2)	(3)	(4)
	High-school dropout	High-school graduate	Some college	Bachelors degree plus
Average wage (w_i)	10021	16925	21503	36547
Employment rate (E_i)	0.459	0.714	0.802	0.892
Average income tax (T_i)	312	3079	4733	10430
Unemployment benefit ($-T_u$)	2070	2070	2070	2070
Labor force shares ($N_i/\sum_j N_j$)	0.138	0.332	0.298	0.233
Labor-demand elasticity (ε_i)	0.6	0.6	0.6	0.6
Participation elasticity (π_i)	0.4	0.4	0.4	0.4

Descriptive statistics are obtained from [Kroft et al. \(2015\)](#) and an earlier version of their paper. The values for T_i and T_u are calculated for single women without children.²⁹ Values for the wage, income tax and unemployment benefit are in dollars.

For details regarding the simulations, we refer to [Appendix I](#). The main results are presented in [Figures 4–6](#). [Figure 4](#) plots the current and optimal second-best allocations. [Figure 5](#) displays the current and optimal participation tax rates against the gross earnings. In both graphs, the current tax system is compared to the optimal tax-benefit system under (i) competitive labor markets (i.e., $\rho_i = 0$ for all i), (ii) a scenario with an intermediate degree of unionization (i.e., $\rho_i = 1/2$ for all i), (iii) a scenario with monopoly unions (i.e., $\rho_i = 1$ for all i). Finally, [Figure 6](#) plots the social welfare weights of the different income groups.

[Figure 5](#) gives our most important finding: optimal participation taxes are substantially lower in more unionized labor markets. This is in line with the theoretical prediction from [Proposition 1](#). In our simulations, the presence of unions may lower the optimal participation

²⁹The N_i observations (which we use to calculate the population shares) also include women with children. Unfortunately, we could not correct for differences in the number of children between the different groups based on the descriptive statistics.

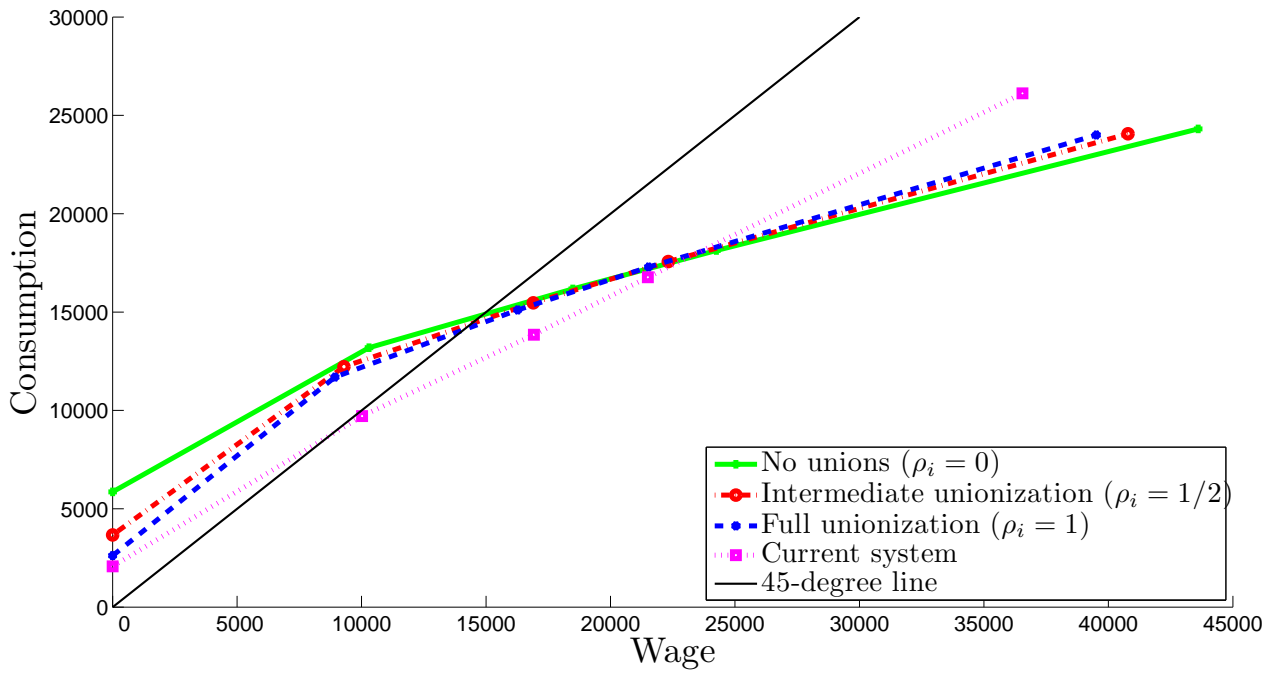


Figure 4: Optimal allocation (U.S.)

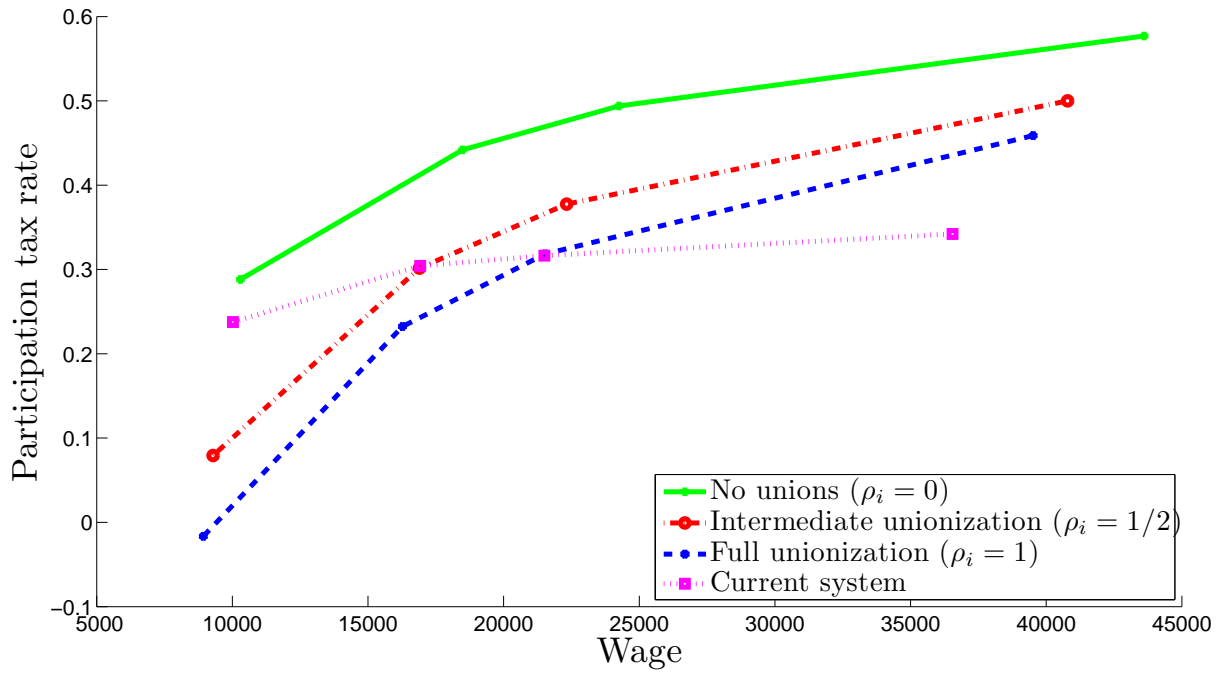


Figure 5: Participation tax rates (U.S.)

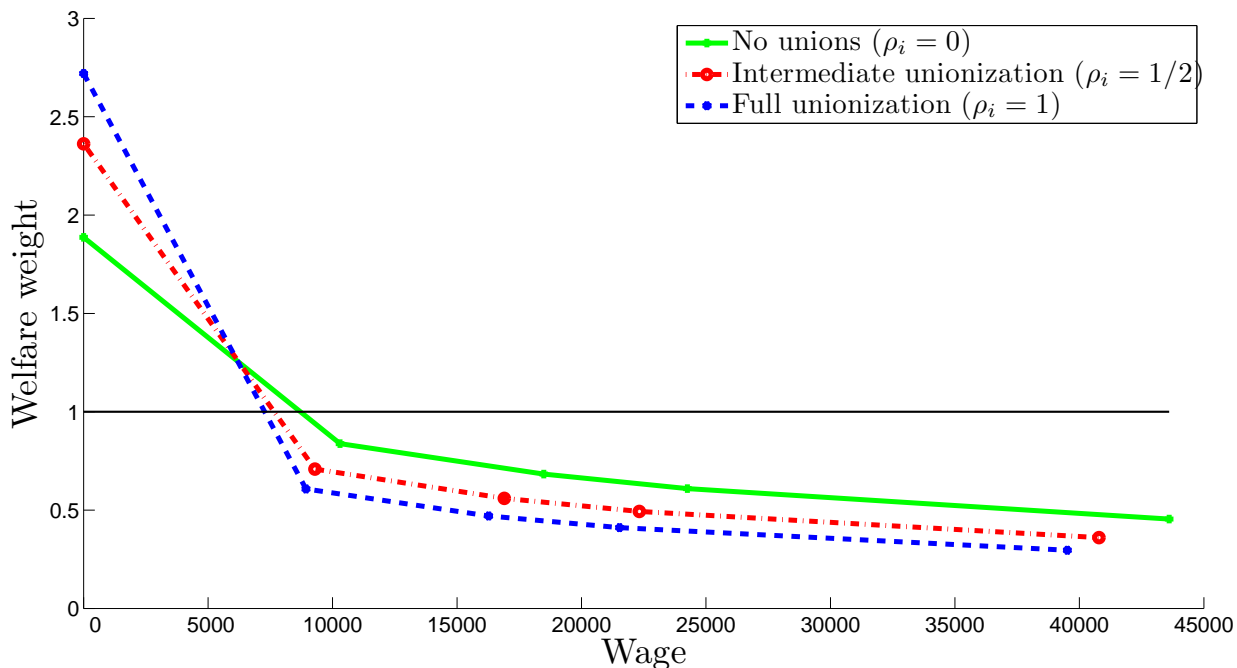


Figure 6: Welfare weights (U.S.)

tax rates from around 30% to approximately -2% at the bottom of the income distribution. The reduction in participation taxes is brought about both by lower income taxes, but most notably by lower unemployment benefits (see Figure 4). Figure 5 furthermore indicates that participation subsidies (i.e. negative participation taxes) for low-income workers are only optimal if the degree of unionization is very close to one. The reason is that, when the degree of unionization is high, so are the welfare costs of involuntary unemployment. High costs of unemployment call for a reduction in participation taxes, so as to induce unions to moderate their wage claims, which leads to higher employment. When the costs of unemployment are sufficiently high, the government may therefore subsidize participation even if the welfare weight of the working poor falls short of one (see also Proposition 1). Indeed, as can be seen from Figure 6, the only group of workers whose social welfare exceeds one are the unemployed. Hence, we never find the ‘classical’ case of an EITC based on a welfare weight of the working poor which exceeds one. The optimal participation subsidy for the working poor in our simulations with a high degree of unionization therefore reflects the government’s desire to alleviate the distortions induced by unions.

The welfare weights of the different groups of workers are plotted in Figure 6. In our baseline simulations, the welfare weight for all groups of workers is below one. According to Proposition 2, an increase in the degree of unionization (irrespective for which type of workers) would therefore always lead to lower social welfare. This finding, however, should be interpreted with caution, since it relies heavily on the specification of the social welfare weights. Furthermore, we ignored participation costs in the definition of the welfare weights, which biases our results against unions, while the assumption of efficient rationing biases the results in favor of unions. Therefore, it remains rather unclear whether in our model unions could be a desirable institution

for redistribution. The more robust finding from our simulations is that unions (strongly) reduce optimal participation tax rates.

7.2 Simulations Netherlands

For the Netherlands, we obtain data for the current tax-benefit system and labor market outcomes from CPB Netherlands Bureau for Economic Policy Analysis. The labor market is divided into five education levels. For each level of education, we observe the employment rate, the average wage and the average tax bill. In addition, we observe the fraction of workers at each level of education. Hence, we observe the empirical counterparts of the labor force shares rather than the group sizes. We assume a non-employment benefit of 8000 euro, based on estimates in [Zoutman et al. \(2016\)](#). In addition, we again use a labor-demand elasticity of 0.6, assumed constant across education levels. The participation elasticity is assumed to be 0.16, as in [Zoutman et al. \(2016\)](#), who base this estimate on an extensive study of labor-supply elasticities in the Netherlands by [Mastrogiacomo et al. \(2013\)](#). Compared to the simulations for the U.S., the Dutch participation elasticity is considerably lower. However, the US sample includes only single women, whereas the Dutch data include all types of households. The inputs for the Dutch simulations are summarized in the next Table.

Table 2: Simulation inputs Netherlands

	(1) Primary education	(2) Lower secondary education	(3) Upper secondary education	(4) Bachelor degree	(5) Master degree
Average wage (w_i)	22912	25430	30661	42344	59886
Employment rate (E_i)	0.646	0.771	0.879	0.927	0.917
Average income tax (T_i)	5471	6771	9120	14587	22423
Unemployment benefit ($-T_u$)	8000	8000	8000	8000	8000
Labor force shares ($N_i / \sum_j N_j$)	0.081	0.230	0.432	0.174	0.083
Labor-demand elasticity (ε_i)	0.6	0.6	0.6	0.6	0.6
Participation elasticity (π_i)	0.16	0.16	0.16	0.16	0.16

Data obtained from CPB Netherlands Bureau of Economic Policy Analysis. Values for the wage, income tax and unemployment benefit are in euros.

The simulation results are presented in Figures 7–9. Compared to the U.S., the current Dutch system more closely resembles that of a negative income tax (NIT) rather than an earned income tax credit (EITC), as can be seen by comparing Figures 4 and 7. The U.S. tax-benefit system combines a low level of a guaranteed income with substantial in-work benefits. The Dutch system provides a more generous guaranteed income, which is subsequently taxed away at higher rates.

Again, we confirm our theoretical prediction that optimal participation tax rates should be lowered when the impact of unions is taken into account, as can be seen in Figure 8. For the lowest skill group, optimal participation tax rates drop from roughly 43% to approximately 21%. Unlike in the simulations for the U.S., we never find that a negative participation tax is optimal in the Netherlands, because, on average, the lowest-skilled group earns a considerably

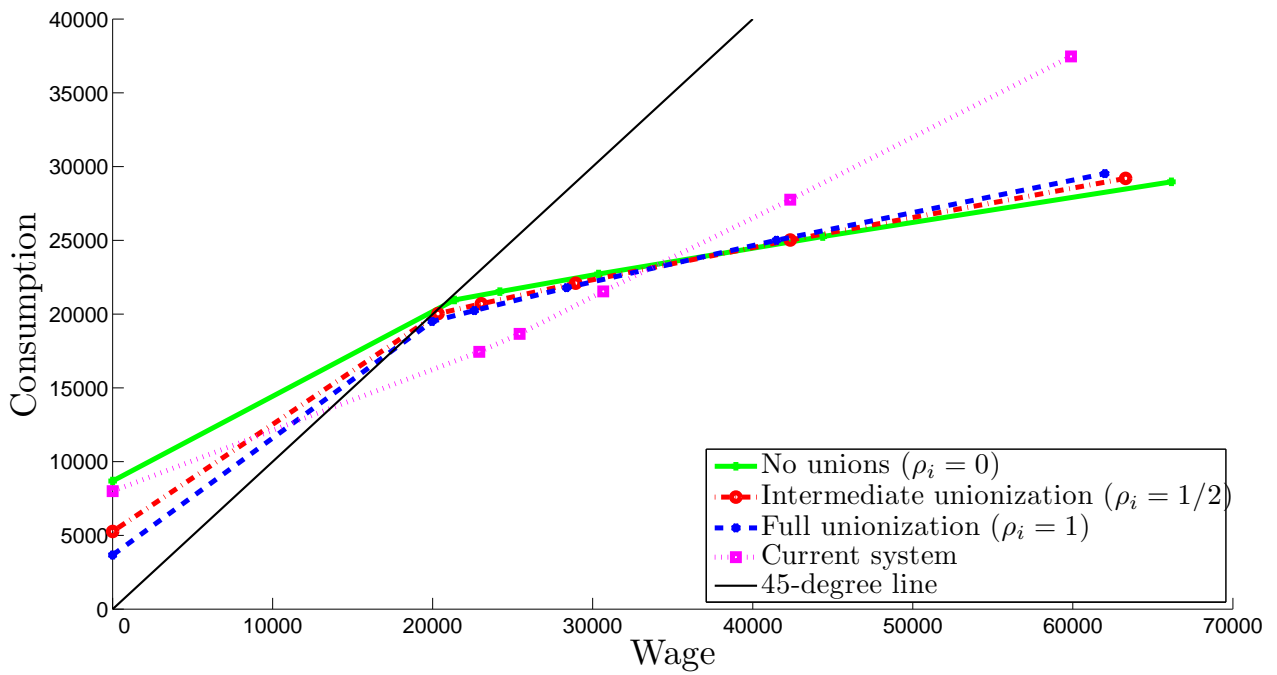


Figure 7: Optimal allocation (Netherlands)

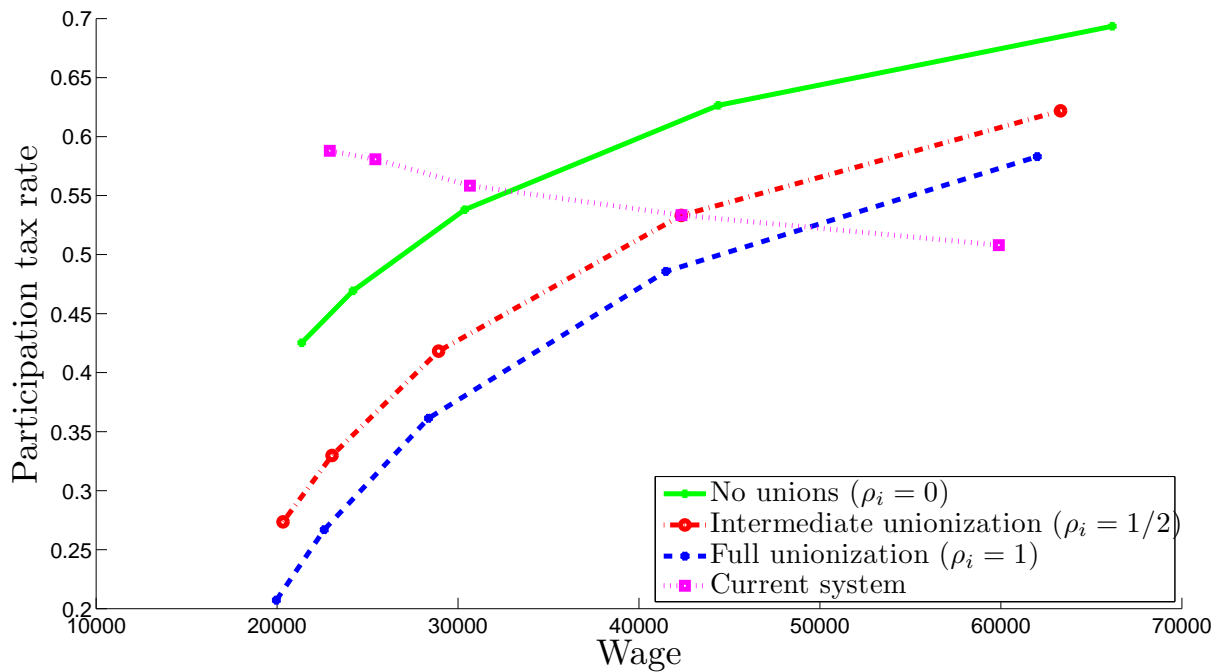


Figure 8: Participation tax rates (Netherlands)

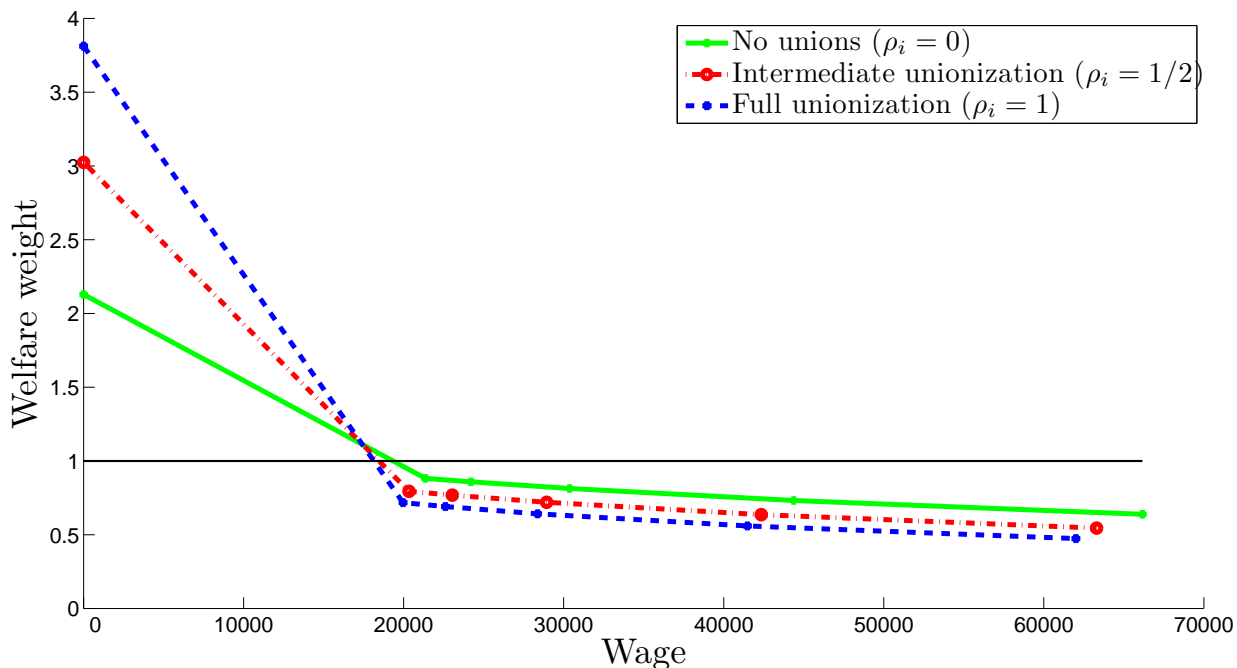


Figure 9: Welfare weights (Netherlands)

higher wage than in the U.S. (compare the first columns of Tables 1 and 2). This significantly lowers the equity gains from providing in-work benefits.

Since a participation subsidy is never found to be optimal, it is immediately implied that unions are not a desirable institution for redistribution. Indeed, from Figure 9 it can be seen that the welfare weights of all working individuals falls below one, irrespective of the degree of unionization. Average incomes for the lowest-skilled workers would have to fall considerably below the levels currently observed in the data to obtain a welfare weight exceeding unity. Again, we emphasize that this finding should be interpreted with caution, as it heavily relies on the specification of the social welfare function.

8 Conclusions

The aim of this paper has been to answer two questions concerning optimal income redistribution in unionized labor markets. With respect to the question, ‘*How should the government optimize income redistribution when labor markets are unionized?*’, our most important finding is that the optimal tax-benefit system is not only used to redistribute income, but also serves to alleviate the distortions induced by unions. In particular, we show that participation taxes should be lower the larger are the welfare gains from lowering involuntary unemployment. Intuitively, low income taxes and low benefits motivate the unions to moderate wage demands, which results in less involuntary unemployment. It may therefore be optimal to subsidize participation on a net basis even for workers whose welfare weight falls short of one, something that can never be optimal if labor markets are competitive (see, e.g., Diamond, 1980, Saez, 2002, Choné and Laroque, 2011 and Christiansen, 2015). Our simulations, based on a sufficient-statistics approach introduced

by [Kroft et al. \(2015\)](#), suggest that optimal participation taxes should be substantially lower if labor markets are strongly unionized. Indeed, in strongly unionized labor markets, the optimal tax-benefit system features a strong EITC-component.

Concerning the question ‘*Can labor unions be socially desirable when the government wants to redistribute income?*’, we show that increasing the bargaining power of the unions representing workers whose welfare weight exceeds one is welfare-enhancing, while the opposite holds true for workers whose welfare weight is below one. This finding bears close resemblance to [Lee and Saez \(2012\)](#), who show that introducing a minimum wage for workers whose social welfare weight exceeds one is welfare-improving. Since [Diamond \(1980\)](#), it is well known that workers whose welfare weight exceeds one should optimally receive a participation subsidy (i.e., receive an income transfer which exceeds the non-employment benefit). Consequently, participation decisions for these workers are distorted upwards, which results in *overemployment*. By bidding up wages, unions can reduce the distortions from taxation. However, in the typical case where participation is taxed on a net basis, employment is already too low and increasing the bargaining power of unions representing these workers lowers social welfare. Our simulations indicate that, at least in our model, the case in favor of unions – even if they would only represent the interest of the lowest-income groups – appears to be rather weak, because participation is hardly ever subsidized on a net basis.

In our baseline model, we assumed a Right-to-Manage model of unions, independent labor markets, efficient rationing of unemployment, and wage bargaining at the sectoral level. Our two main findings – optimal taxes are less redistributive in unionized labor markets and unions are desirable only when participation is subsidized – are robust to changing any of these assumptions, with one exception. Only if we consider an efficient bargaining model, which we deem less plausible theoretically and empirically, do we find that stronger unions do not necessarily call for less redistributive optimal taxes.

We have made some assumptions that warrant further research. First, we assumed throughout that the government is the Stackelberg leader relative to all agents in the private sector, including the unions. This assumption has not gone uncontested in the literature. Since [Calmfors and Driffill \(1988\)](#), it is well understood that unions may internalize some of the macroeconomic and fiscal impacts of their decisions in wage negotiations.³⁰ Since our model features multiple unions, that may all vary in their bargaining power, it appears most natural to assume that the government is the Stackelberg leader. However, it remains to be seen whether our results generalize to a setting where unions and the government interact strategically. Second, we have abstracted from intensive-margin considerations and from the wage-moderating effect of tax progression (see, e.g., [Aronsson and Sjögren, 2004b](#)). It remains interesting to see how our results would be affected if the union’s decisions would be influenced by marginal tax rates, especially if the model would be extended to include an intensive margin. These provide interesting avenues for future research.

³⁰In particular, [Boeters and Schneider \(1999\)](#) and [Aronsson and Wikström \(2011\)](#), among others, consider the case where the union is the Stackelberg leader. In both these studies, and in contrast to our paper, there is only one type of labor and consequently only one union, which is assumed to have full bargaining power.

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A Derivation of ρ_i from the right-to-manage model

In this Appendix, we derive the relationship between our measure of the union's bargaining power ρ_i and the bargaining power in the Nash product that is more commonly used to characterize the equilibrium in the RtM-model (see, for instance, [Boeri and Van Ours, 2008](#)). Under Nash bargaining, we can characterize the labor-market equilibrium by solving the following optimization problem:

$$\begin{aligned} \max_{w_i, E_i} \quad \Omega_i = & \beta_i \log \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u)) dG(\varphi) \right) \\ & + (1 - \beta_i) \log \left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F - \sum_{j \neq i} w_j N_j E_j - T_f) \right) \\ \text{s.t.} \quad & w_i = F_i(\cdot) \\ & G(w_i - T_i + T_u) - E_i \geq 0, \end{aligned} \tag{62}$$

where $\beta_i \in [0, 1]$ is the weight attached to the union's payoff in the Nash product, and $F(\cdot)|_{E_i=0}$ is the output the firm generates if it would not reach an agreement with union i (in which case it is assumed that no type- i workers will become employed). Note that in the above specification, the payoff for each party is taken in deviation from the payoff associated to the disagreement outcome.³¹ It is important to take the final constraint (the voluntary participation constraint) explicitly into account, as it will bind for small values of β_i . If this is the case (i.e., if β_i is close to zero), the labor-market equilibrium is characterized by the final two conditions, which jointly define the competitive equilibrium.

The Lagrangian reads as:

$$\begin{aligned} \mathcal{L} = & \beta_i \log \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u)) dG(\varphi) \right) \\ & + (1 - \beta_i) \log \left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0} - \sum_{j \neq i} w_j N_j E_j - T_f) \right) \\ & + \lambda_i (w_i - F_i(\cdot)) + \mu_i (G(w_i - T_i + T_u) - E_i). \end{aligned} \tag{63}$$

³¹If the firm and union i do not reach an agreement, it is assumed that type- i workers become unemployed (and receive benefits $-T_u$), and that firms continue their operations employing workers of type $j \neq i$. None of our results would change if we ignore the payoffs from the disagreement outcome.

The first-order conditions are given by:

$$w_i : \frac{\beta_i}{(\bar{u}_i - u_u)} \bar{u}'_i - \frac{(1 - \beta_i)}{(u_f - u_f^{-i})} u'_f N_i E_i + \lambda_i + \mu_i G'_i = 0 \quad (64)$$

$$E_i : \frac{\beta_i}{(\bar{u}_i - u_u)} E_i (\hat{u}_i - u_u) - \lambda_i F_{ii} - \mu_i = 0 \quad (65)$$

$$\lambda_i : w_i - F_i = 0 \quad (66)$$

$$\mu_i : \mu_i (G_i - E_i) = 0, \quad (67)$$

where $u_f^{-i} \equiv u(F(\cdot)|_{E_i=0} - \sum_{j \neq i} w_j N_j E_j - T_f)$ is the utility firm-owners attain if they fail to reach an agreement with union i . If $\beta_i = 1$, equations (64)-(65) imply that $\mu_i = 0$ and the equilibrium coincides with the one derived in the MU-model. For small values of β_i , on the other hand, the constraint $G_i = E_i$ becomes binding, and the labor market equilibrium coincides with the competitive outcome.³² This happens for all values of $\beta_i \in [0, \beta_i^*]$, where $\beta_i^* \in (0, 1)$ is implicitly defined by:³³

$$\frac{\beta_i^*}{1 - \beta_i^*} = \frac{E_i(\bar{u}_i - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\bar{u}'_i}. \quad (68)$$

For values of $\beta_i \in [\beta_i^*, 1]$, we thus have $\mu_i = 0$. Combining equations (64)-(65) then leads to

$$\frac{\beta_i}{E_i(\bar{u}_i - u_u)} \left(E_i \bar{u}'_i + \frac{(\hat{u}_i - u_u)}{F_{ii}} \right) - \frac{(1 - \beta_i)}{(u_f - u_f^{-i})} u'_f N_i E_i = 0, \quad (69)$$

or, alternatively,

$$1 - \left(\frac{1 - \beta_i}{\beta_i} \right) \frac{E_i(\bar{u}_i - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\bar{u}'_i} = \varepsilon_i \frac{(\hat{u}_i - u_u)}{\bar{u}'_i w_i}. \quad (70)$$

Defining the left-hand side of this equation as

$$\rho_i \equiv 1 - \left(\frac{1 - \beta_i}{\beta_i} \right) \frac{E_i(\bar{u}_i - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\bar{u}'_i}, \quad (71)$$

we arrive at our equilibrium condition in the RtM-model, as given by equation (14). Clearly, if $\beta_i = 1$, $\rho_i = 1$ as well and the MU-model applies. When $\beta_i = \beta_i^*$, from equation (68) it follows that $\rho_i = 0$ and the equilibrium coincides with the competitive outcome. Hence, there exists a direct relationship between our measure of the union's bargaining power ρ_i and the Nash-bargaining parameter β_i :

$$\rho_i = \begin{cases} 0 & \text{if } \beta_i \in [0, \beta_i^*), \\ 1 - \frac{(1 - \beta_i)}{\beta_i} \frac{E_i(\bar{u}_i - u_u)}{(u_f - u_f^{-i})} \frac{u'_f N_i}{\bar{u}'_i} & \text{if } \beta_i \in [\beta_i^*, 1]. \end{cases} \quad (72)$$

³²This can be verified by setting $\beta_i = 0$. Equations (64)-(65) then imply that $\mu_i > 0$.

³³This equation is obtained by setting $G_i = E_i$ and $\mu_i = 0$ in the system of first-order conditions. The reason is that, at exactly this value of β_i , the constraint $G_i = E_i$ becomes binding.

B Derivation elasticities

This appendix derives the elasticities of wages and employment rates to the tax instruments. If Assumption 1 is satisfied and income effects at the union level are absent (in which case $\partial E_i/\partial T_i = -\partial E_i/\partial T_u$ and $\partial w_i/\partial T_i = -\partial w_i/\partial T_u$), the equilibrium wage and employment rate in sector i can be written solely as a function of the participation tax $T_i - T_u$ or, equivalently, the participation tax rate $t_i = (T_i - T_u)/w_i$. Hence, we can write $E_i = E_i(t_i)$ and $w_i = w_i(t_i)$. The relevant elasticities can be derived using the labor-market equilibrium conditions:

$$w_i = F_i(E_i) \quad (73)$$

$$\overline{\rho_i u'(w_i(1-t_i) - T_u - \varphi) w_i} = \varepsilon_i(E_i)(u(w_i(1-t_i) - T_u - G^{-1}(E_i)) - u(-T_u)), \quad (74)$$

where

$$\overline{u'(w_i(1-t_i) - T_u - \varphi)} \equiv E_i^{-1} \int_{\varphi}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) \quad (75)$$

denotes the average marginal utility of the employed workers. Log-linearizing the labor-market equilibrium conditions around the solution, we obtain:

$$\frac{dE_i}{E_i} = -\varepsilon_i \frac{dw_i}{w_i}, \quad (76)$$

$$\frac{d\overline{u'_i}}{\overline{u'_i}} + \frac{dw_i}{w_i} = \frac{d\varepsilon_i}{\varepsilon_i} + \frac{d(\hat{u}_i - u_u)}{\hat{u}_i - u_u}. \quad (77)$$

Without income effects, T_u affects E_i and w_i only through its impact on t_i . Using this insight, we can linearize the relevant sub-parts of the last equation:

$$\frac{d\overline{u'_i}}{\overline{u'_i}} = \frac{\overline{u''_i} w_i (1-t_i)}{\overline{u'_i}} \left(\frac{dw_i}{w_i} - \frac{dt_i}{1-t_i} \right) + \frac{(\hat{u}'_i - \overline{u'_i})}{\overline{u'_i}} \frac{dE_i}{E_i} \quad (78)$$

$$\frac{d\varepsilon_i}{\varepsilon_i} = \varepsilon_{\varepsilon_i} \frac{dE_i}{E_i} \quad (79)$$

$$\frac{d(\hat{u}_i - u_u)}{\hat{u}_i - u_u} = \frac{\hat{u}'_i w_i (1-t_i)}{\hat{u}_i - u_u} \left(\frac{dw_i}{w_i} - \frac{dt_i}{1-t_i} \right) - \frac{\hat{u}'_i E_i}{g(\hat{\varphi}_i)(\hat{u}_i - u_u)} \frac{dE_i}{E_i}, \quad (80)$$

where $\varepsilon_{\varepsilon_i}$ is the elasticity of the labor demand elasticity with respect to the rate of employment:

$$\varepsilon_{\varepsilon_i} \equiv \frac{\partial \varepsilon_i}{\partial E_i} \frac{E_i}{\varepsilon_i} = - \left(1 + \frac{1}{\varepsilon_i} + \frac{E_i F_{iii}}{F_{ii}} \right). \quad (81)$$

Substituting the subparts, as well as the log-linearized labor-demand equation in the log-linearized mark-up equation allows us to solve for the relative changes in the wage and em-

ployment rate in sector i :

$$\frac{dw_i}{w_i} = \frac{u'_u w_i (1 - t_i)}{\hat{u}'_i \varepsilon_i E_i / g(\hat{\varphi}_i) + u'_u w_i (1 - t_i) - (\hat{u}_i - u_u) \left(1 + \varepsilon_i \varepsilon_{\varepsilon_i} + \varepsilon_i \frac{(\bar{u}'_i - \hat{u}'_i)}{u'_i} \right)} \frac{dt_i}{1 - t_i} \quad (82)$$

$$\frac{dE_i}{E_i} = - \frac{\varepsilon_i u'_u w_i (1 - t_i)}{\hat{u}'_i \varepsilon_i E_i / g(\hat{\varphi}_i) + u'_u w_i (1 - t_i) - (\hat{u}_i - u_u) \left(1 + \varepsilon_i \varepsilon_{\varepsilon_i} + \varepsilon_i \frac{(\bar{u}'_i - \hat{u}'_i)}{u'_i} \right)} \frac{dt_i}{1 - t_i} \quad (83)$$

C Optimal taxation

C.1 Full optimum

The Lagrangian associated with the government's optimization problem can be written as:

$$\begin{aligned} \mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1 - t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\ & + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right). \end{aligned} \quad (84)$$

When differentiating with respect to the policy instruments, we have to take into account the dependency of w_i and E_i on t_i . The first-order conditions (assumed to be necessary and sufficient) are given by:

$$T_u : - \sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \lambda \sum_i N_i = 0 \quad (85)$$

$$T_f : -u'_f + \lambda = 0 \quad (86)$$

$$\begin{aligned} t_i : & -N_i E_i w_i (\bar{u}'_i - \lambda) + \frac{\partial E_i}{\partial t_i} (N_i (\hat{u}_i - u_u) + \lambda N_i t_i w_i) \\ & + \frac{\partial w_i}{\partial t_i} \left(N_i E_i \bar{u}'_i (1 - t_i) - N_i E_i u'_f + \lambda N_i E_i t_i \right) = 0. \end{aligned} \quad (87)$$

To obtain the first result from the proposition, divide equation (85) by $\lambda \sum_i N_i$ and use the definitions of the welfare weights (21) and the labor shares (22). The second result can be found by dividing equation (86) by λ and imposing the definition of b_f . The final result can be found as follows. First, substitute $u'_f = \lambda$ in equation (87), and divide by $\lambda N_i w_i$. Next, use the definitions of the welfare weight b_i from equation (21), the union wedge $\tau_i = \frac{u(\hat{c}_i) - u(c_u)}{\lambda w_i}$, as well as the wage elasticity κ_i and the employment elasticity η_i from equations (19)-(20), and rearrange.

C.2 Restricted profit taxation

To derive the optimal participation tax rate in the presence of a restriction on profit taxation (in which case $b_f < 1$), divide equation (85) by $\lambda N_i w_i$ and use the definitions of the welfare weights b_i and b_f from equation (21), the union wedge $\tau_i = \frac{u(\hat{c}_i) - u(c_u)}{\lambda w_i}$, as well as the wage

elasticity κ_i and the employment elasticity η_i from equations (19)-(20):

$$\left(\frac{t_i + \tau_i}{1 - t_i}\right) \eta_i = (1 - b_i) + \left(\frac{b_i - b_f + (1 - b_i)t_i}{1 - t_i}\right) \kappa_i. \quad (88)$$

If profit taxation is unrestricted, we have $b_f = 1$, and the result from Proposition 1 applies.

C.3 Income effects

If there are income effects, changes in the unemployment benefit $-T_u$ do not only affect E_i and w_i through their impact on participation tax rates t_i . Therefore, we write $E_i = E_i(t_i, T_u)$ and $w_i = w_i(t_i, T_u)$. Now, only the expression for the optimal unemployment benefit has to be modified. The first-order condition – the counterpart of equation (85) – reads:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial T_u} &= - \sum_i N_i E_i \bar{w}'_i - \sum_i N_i (1 - E_i) u'_u + \lambda \sum_i N_i \\ &+ \sum_i \frac{\partial E_i}{\partial T_u} (N_i (\hat{u}_i - u_u) + \lambda N_i t_i w_i) \\ &+ \sum_i \frac{\partial w_i}{\partial T_u} \left(N_i E_i \bar{w}'_i (1 - t_i) - N_i E_i u'_f + \lambda N_i E_i t_i \right) = 0. \end{aligned} \quad (89)$$

To simplify this expression, divide by $\lambda \sum_i N_i$, and impose $b_f = 1$. Furthermore, use the property

$$\frac{\partial E_i}{\partial x_i} = \frac{\partial E_i}{\partial w_i} \frac{\partial w_i}{\partial x_i} \quad (90)$$

for $x_i \in \{T_i, t_i\}$. Here, $\partial E_i / \partial w_i = 1 / F_{ii}(\cdot)$ is the slope of the labor-demand curve. Then, combine equations (87), (89) and (90) to obtain:

$$\sum_i \omega_i b_i + \omega_u b_u = 1 - \sum_i \omega_i (1 - b_i) \iota_i, \quad (91)$$

where $\iota_i \equiv w_i \frac{\partial E_i}{\partial t_i} / \frac{\partial E_i}{\partial T_u}$. This expression generalizes equation (21) to the case with income effects.

To obtain an expression for ι_i , combine the mark-up and the labor-demand equation:

$$\begin{aligned} \rho_i \int_{\varphi}^{G^{-1}(E_i)} u'(F_i(\cdot)(1 - t_i) - T_u - \varphi) dG(\varphi) F_{ii}(\cdot) \\ + u(F_i(\cdot)(1 - t_i) - T_u - G^{-1}(E_i)) - u(-T_u) = 0. \end{aligned} \quad (92)$$

We can then use implicit differentiation of equation (92) to obtain an expression for ι_i :

$$\iota_i = 1 - \frac{u'_u}{\hat{u}'_i - (\hat{u}_i - u_u) \frac{u''_i}{u'_i}}. \quad (93)$$

If income effects are absent (or if there are no unions), we have $\iota_i = 0$, and equation (91) coincides with the first result stated in Proposition 1.³⁴

³⁴It can be shown that a sufficient condition for this to be the case is that the individual utility function $u(\cdot)$ is of the CARA-type.

D Desirability of unions

In this Appendix, rather than deriving our results in terms of sufficient statistics, we explicitly take the labor-market equilibrium conditions into account as constraints in the government's optimization problem. The latter then reads as:

$$\begin{aligned}
\max_{T_u, T_f, \{t_i, w_i, E_i\}_{i=1}^I} \mathcal{W} &= \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\
&\quad + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) \\
\text{s.t.} \quad &\sum_i N_i (T_u + E_i t_i w_i) + T_f = R \\
&w_i = F_i(\cdot), \forall i \\
&\rho_i \int_{\underline{\varphi}}^{G^{-1}(E_i)} u'(w_i(1-t_i) - T_u - \varphi) dG(\varphi) F_{ii}(\cdot) \\
&\quad + u(w_i(1-t_i) - T_u - G^{-1}(E_i)) - u(-T_u) = 0, \forall i.
\end{aligned} \tag{94}$$

The corresponding Lagrangian is given by:

$$\begin{aligned}
\mathcal{L} &= \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\
&\quad + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \kappa \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right) \\
&\quad + \sum_i \lambda_i (w_i - F_i(\cdot)) + \sum_i \mu_i \left(\rho_i \int_{\underline{\varphi}}^{G^{-1}(E_i)} u'(w_i(1-t_i) - T_u - \varphi) dG(\varphi) F_{ii}(\cdot) \right. \\
&\quad \left. + u(w_i(1-t_i) - T_u - G^{-1}(E_i)) - u(-T_u) \right).
\end{aligned} \tag{95}$$

The first-order conditions are:

$$T_u : - \sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \kappa = 0 \tag{96}$$

$$T_f : -u'_f + \kappa = 0 \tag{97}$$

$$t_i : -w_i N_i E_i (\bar{u}'_i - \kappa) - w_i \mu_i \left(\rho_i E_i \bar{u}''_i F_{ii} + \hat{u}'_i \right) = 0 \tag{98}$$

$$w_i : (1-t_i) N_i E_i \bar{u}'_i - N_i E_i u'_f + t_i N_i E_i \kappa + \lambda_i + (1-t_i) \mu_i \left(\rho_i E_i \bar{u}''_i F_{ii} + \hat{u}'_i \right) = 0 \tag{99}$$

$$E_i : N_i (\hat{u}_i - u_u) + \kappa N_i t_i w_i - \lambda_i F_{ii} + \mu_i \left(\rho_i \hat{u}'_i F_{ii} + \rho_i E_i \bar{u}''_i F_{iii} - \hat{u}'_i / G'_i \right) = 0 \tag{100}$$

$$\kappa : \sum_i N_i (T_u + E_i t_i w_i) + T_f - R = 0 \tag{101}$$

$$\lambda_i : w_i - F_i = 0 \tag{102}$$

$$\mu_i : \rho_i E_i \bar{u}'_i F_{ii} + \hat{u}_i - u_u = 0. \tag{103}$$

This system implicitly characterizes the optimal tax-benefit system, as well as the equilibrium values for wages, employment rates, and Lagrange multipliers.

To examine how an increase in the degree of unionization ρ_i in sector i affects social welfare, differentiate the Lagrangian (95) with respect to ρ_i , and apply the Envelope theorem:

$$\frac{\partial \mathcal{W}}{\partial \rho_i} = \frac{\partial \mathcal{L}}{\partial \rho_i} = \mu_i E_i \overline{u}'_i F_{ii}. \quad (104)$$

Since $E_i \overline{u}'_i F_{ii} < 0$ (provided that labor demand is not perfectly elastic), the expression in equation (104) is positive if and only if $\mu_i < 0$. To determine the sign of μ_i , consider equation (98). By the concavity of the individual utility function $u(\cdot)$ and the production function $F(\cdot)$, $\rho_i E_i \overline{u}'_i F_{ii} + \hat{u}'_i > 0$. Denoting by $b_i = \overline{u}'_i / \kappa$, it follows that

$$\mu_i < 0 \quad \Leftrightarrow \quad b_i > 1. \quad (105)$$

Hence, an increase in ρ_i leads to an increase in social welfare if and only if $b_i > 1$. Importantly, nowhere in the proof is it necessary to assume that income effects are absent, that labor markets are independent, or that profit taxation is unrestricted (in which case $b_f = 1$). Proposition 2 thus generalizes to settings with income effects, interdependent labor markets or a binding restriction on profit taxation.

D.1 Optimal degree of unionization

Suppose the government could also optimally determine the bargaining power of each union ρ_i . If we denote by $\underline{\nu}_i \geq 0$ the Kuhn-Tucker multiplier on the restriction that $\rho_i \geq 0$ and by $\bar{\nu}_i \geq 0$ the multiplier on the restriction that $1 - \rho_i \geq 0$, the first-order condition with respect to ρ_i (obtained from differentiating the Lagrangian (95) augmented with the additional restrictions) is given by

$$\rho_i : \quad \mu_i E_i \overline{u}'_i F_{ii} + \underline{\nu}_i - \bar{\nu}_i = 0. \quad (106)$$

The above expression should be considered alongside equations (96)-(103). In an interior optimum (i.e. where the optimal $\rho_i \in (0, 1)$), the Kuhn-Tucker conditions require that $\underline{\nu}_i = \bar{\nu}_i = 0$. Equations (106) and (98) then imply that in these sectors $b_i = 1$. If the solution is at the boundary, then by the Kuhn-Tucker conditions it must be that either $\bar{\nu}_i = 0$ and $\underline{\nu}_i > 0$ or $\underline{\nu}_i = 0$ and $\bar{\nu}_i > 0$. If labor demand is not perfectly elastic, equation (106) implies that $\mu_i > 0$ in the first case (in which case $b_i < 1$) and $\mu_i < 0$ in the second case (in which case $b_i > 1$). The optimal degree of unionization thus equals $\rho_i = \min[\rho_i^*, 1]$ if $b_i \geq 1$, and $\rho_i = \max[\rho_i^*, 0]$ if $b_i \leq 1$, where ρ_i^* is the bargaining power of the union for which $b_i = 1$.

D.2 Employer Taxes

To prove Proposition 3, first rewrite the Lagrangian (95) to take explicitly into account employer taxes:

$$\begin{aligned}
\mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\
& + u(F(\cdot) - \sum_i w_i(1+\theta_i)N_iE_i - T_f) + \kappa \left(\sum_i N_i(T_u + E_i(t_i + \theta_i)w_i) + T_f - R \right) \\
& + \sum_i \lambda_i(w_i(1+\theta_i) - F_i(\cdot)) + \sum_i \mu_i \left(\frac{\rho_i}{1+\theta_i} \int_{\underline{\varphi}}^{G^{-1}(E_i)} u'(w_i(1-t_i) - T_u - \varphi) dG(\varphi) F_{ii}(\cdot) \right. \\
& \left. + u(w_i(1-t_i) - T_u - G^{-1}(E_i)) - u(-T_u) \right), \tag{107}
\end{aligned}$$

where the final constraints reflect the labor-market equilibrium in the presence of employer taxes, as given by equations (29)-(30). The first-order conditions are given by:

$$T_u : - \sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \kappa = 0 \tag{108}$$

$$T_f : -u'_f + \kappa = 0 \tag{109}$$

$$t_i : -w_i N_i E_i (\bar{u}'_i - \kappa) - w_i \mu_i \left(\frac{\rho_i}{(1+\theta_i)} E_i \bar{u}''_i F_{ii} + \hat{u}'_i \right) = 0 \tag{110}$$

$$\begin{aligned}
w_i : & (1-t_i)N_iE_i\bar{u}'_i - N_iE_iu'_f + t_i\kappa N_iE_i \\
& + \lambda_i(1+\theta_i) + (1-t_i)\mu_i \left(\frac{\rho_i}{(1+\theta_i)} E_i \bar{u}''_i F_{ii} + \hat{u}'_i \right) = 0 \tag{111}
\end{aligned}$$

$$\begin{aligned}
E_i : & N_i(\hat{u}_i - u_u) + \kappa N_i(t_i + \theta_i)w_i - \lambda_i F_{ii} \\
& + \mu_i \left(\frac{\rho_i}{(1+\theta_i)} \hat{u}'_i F_{ii} + \frac{\rho_i}{(1+\theta_i)} E_i \bar{u}''_i F_{iii} - \hat{u}'_i / G'_i \right) = 0 \tag{112}
\end{aligned}$$

$$\kappa : \sum_i N_i(T_u + E_i(t_i + \theta_i)w_i) + T_f - R = 0 \tag{113}$$

$$\lambda_i : w_i(1+\theta_i) - F_i = 0 \tag{114}$$

$$\mu_i : \frac{\rho_i}{(1+\theta_i)} E_i \bar{u}'_i F_{ii} + \hat{u}_i - u_u = 0. \tag{115}$$

The welfare effect of introducing employer taxes is:

$$\begin{aligned}
\frac{\partial \mathcal{W}}{\partial \theta_i} &= -w_i N_i E_i (u'_f - \kappa) + \lambda_i w_i - \mu_i \frac{\rho_i}{(1+\theta_i)^2} E_i \bar{u}'_i F_{ii} \\
&= -\mu_i \frac{\rho_i}{(1+\theta_i)^2} E_i \bar{u}'_i F_{ii} > 0 \Leftrightarrow b_i < 1, \tag{116}
\end{aligned}$$

where the second equality follows from equations (109)–(111) (which imply $u'_f = \kappa$ and $\lambda_i = 0$) and the final condition follows from (110) (which implies $b_i < 1$ if $\mu_i > 0$). Hence, the government wishes to introduce employer taxes (subsidies) in sectors where $b_i < 1$ ($b_i > 1$), provided the degree of unionization is nonzero. The government would prefer to do so until the bargaining equation is no longer a binding constraint: $\mu_i = 0$. From equation (108), it is

clear this cannot always be achieved, since the concavity of the individual utility function $u(\cdot)$ implies $b_u > b_i$ for all i . In sectors where $b_i < 1$, the best thing the government can do is to approximate the situation without unions arbitrarily closely by increasing employer taxes. Effective union power $\rho_i/(1 + \theta_i)$ then approaches zero, and equation (115) implies $G_i = E_i$. The optimal allocation is then characterized by:

$$-\sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \kappa = 0 \quad (117)$$

$$-u'_f + \kappa = 0 \quad (118)$$

$$\sum_i N_i (T_u + E_i (t_i + \theta_i) w_i) + T_f - R = 0 \quad (119)$$

$$w_i (1 + \theta_i) - F_i = 0 \quad (120)$$

$$\begin{cases} \bar{u}'_i - \kappa = 0 \\ \kappa (t_i + \theta_i) w_i + \hat{u}_i - u_u = 0 \end{cases} \quad (121)$$

$$\begin{cases} G_i - E_i = 0 \\ \kappa (t_i + \theta_i) w_i + E_i (\bar{u}'_i - \kappa) / G'_i = 0. \end{cases} \quad (122)$$

The system above is very similar to the system which characterizes the optimal allocation with the socially optimal degree of unionization. This allocation is characterized implicitly by equations (96)–(103) and the first-order condition for the degree of unionization (106). To see why these systems implement the same allocation, replace w_i in the system (96)–(103) and (106) by $w_i(1 + \theta_i)$ and $t_i w_i$ by $(t_i + \theta_i) w_i$. Upon making this change of variables, it can readily be verified that the equilibrium consumption bundles for the unemployed, employed and firm-owners, and the rates of employment implied by both systems coincide. Hence, the allocation with the socially optimal degree of unionization can be implemented using employer taxes, provided the degree of unionization is non-zero.

D.3 Minimum wages

If there are no unions, voluntary participation requires that the rate of employment equals the fraction of workers willing to participate in sector i : $G(w_i(1 - t_i)) = E_i$. With a potentially binding (sector-specific) minimum wage, however, this constraint is replaced by:

$$G(w_i(1 - t_i)) \geq E_i, \quad (123)$$

which holds with equality if the (sector-specific) minimum wage does not bind, and with an inequality if it does. Letting the government maximize with respect to minimum wages (in addition to the tax instruments) is thus identical to letting the government maximize subject to $G(w_i(1 - t_i)) \geq E_i$ rather than the voluntary participation constraint $G(w_i(1 - t_i)) = E_i$.

The Lagrangian for this maximization problem is given by:

$$\begin{aligned}
\mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\
& + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \kappa \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right) \\
& + \sum_i \lambda_i (w_i - F_i(\cdot)) + \sum_i \mu_i (G(w_i(1-t_i)) - E_i). \tag{124}
\end{aligned}$$

The Kuhn-Tucker conditions are given by:

$$T_u : - \sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \kappa = 0 \tag{125}$$

$$T_f : -u'_f + \kappa = 0 \tag{126}$$

$$t_i : -w_i N_i E_i (\bar{u}'_i - \kappa) - w_i \mu_i G'_i = 0 \tag{127}$$

$$w_i : (1 - t_i) N_i E_i (\bar{u}'_i - u'_f) + \lambda_i + (1 - t_i) \mu_i G'_i = 0 \tag{128}$$

$$E_i : N_i (\hat{u}_i - u_u) + \kappa N_i t_i w_i - \lambda_i F_{ii} - \mu_i = 0 \tag{129}$$

$$\kappa : \sum_i N_i (T_u + E_i t_i w_i) + T_f - R = 0 \tag{130}$$

$$\lambda_i : w_i - F_i = 0, \tag{131}$$

$$\mu_i : \mu_i (G_i - E_i) = 0 \quad \mu_i \geq 0, \quad G_i \geq E_i. \tag{132}$$

Without minimum wages, the final condition (132) would be replaced by the voluntary participation constraint $G_i = E_i$. The first point of Proposition 4 claims that – under the assumption there is no sector for which the government would want to increase ρ_i above unity (i.e., provided $\bar{\nu}_i = 0$ for all i) – the allocation implied by the system (125)–(132) coincides with the allocation implied by equations (96)–(103) if combined with the first-order condition for the degree of unionization (106).

To see why this is true, first observe that the first-order conditions with respect to T_u , T_f , κ and λ_i are identical. Next, in both optimization problems, the first-order conditions with respect to T_f , t_i and w_i together imply that $\lambda_i = 0$ for all i . Furthermore, with control over the unions' bargaining power ρ_i and assuming the restriction $\rho_i \geq 1$ does not bind, it must be that either $\mu_i = 0$ (in which case $b_i = 1$), or $\nu_i = 0$ (in which case $G_i = E_i$). In the context of minimum wage policies, the Kuhn-Tucker conditions imply either $\mu_i = 0$ (in which case $b_i = 1$) or $\mu_i > 0$ (in which case $G_i = E_i$). In both cases, if $\mu_i > 0$, the first-order condition with respect to t_i and E_i can be combined to substitute out for μ_i . The conditions which implicitly

characterize T_u, T_f, t_i, w_i, E_i and κ in both optimization problems are then given by:

$$-\sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \kappa = 0 \quad (133)$$

$$-u'_f + \kappa = 0 \quad (134)$$

$$\sum_i N_i (T_u + E_i t_i w_i) + T_f - R = 0 \quad (135)$$

$$w_i - F_i = 0 \quad (136)$$

$$\begin{cases} \bar{u}'_i - \kappa = 0 \\ \kappa t_i w_i + \hat{u}_i - u_u = 0 \end{cases} \quad (137)$$

$$\begin{cases} G_i - E_i = 0 \\ \kappa t_i w_i + E_i (\bar{u}'_i - \kappa) / G'_i = 0, \end{cases} \quad (138)$$

where condition (137) applies in sectors with a binding minimum wage (or a positive degree of unionization in the social optimum), and vice versa for condition (138). Given the equivalence of the system of equations which characterize the social optimum, the allocation with the socially optimal degree of unionization can be implemented by introducing sector-specific minimum wages.

E Interdependent labor markets

The Lagrangian is the same as in Proposition 1:

$$\begin{aligned} \mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1 - t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\ & + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right). \end{aligned} \quad (139)$$

If labor markets are interconnected, however, we have to take into account that the wage and employment rate in sector i are also affected by taxes levied in sector $j \neq i$. Ignoring income effects, these relationships can be written as $w_i = w_i(t_1, t_2, \dots, t_I)$ and $E_i = E_i(t_1, t_2, \dots, t_I)$.³⁵

The first-order conditions read

$$T_u : -\sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \lambda \sum_i N_i = 0 \quad (140)$$

$$T_f : -u'_f + \lambda = 0 \quad (141)$$

$$\begin{aligned} t_i : & -N_i E_i w_i (\bar{u}'_i - \lambda) + \sum_j \frac{\partial E_j}{\partial t_i} (N_j (\hat{u}_j - u_u) + \lambda N_j t_j w_j) \\ & + \sum_j \frac{\partial w_j}{\partial t_i} (N_j E_j \bar{u}'_j (1 - t_j) - N_j E_j u'_f + \lambda N_j E_j t_j) = 0. \end{aligned} \quad (142)$$

³⁵The case with income effects can be analyzed in analogous fashion as is done in Appendix C.1.

The first two results from Proposition 5 follow directly from equations (140)–(141). To arrive at the final result, divide the final expression by $\lambda w_i \sum_j N_j$ and impose $b_f = 1$. One then obtains

$$\omega_i(1 - b_i) + \sum_j \omega_j \frac{\partial E_j}{\partial t_i} \frac{1}{E_j} (t_j + \tau_j) \frac{w_j}{w_i} + \sum_j \omega_j \frac{\partial w_j}{\partial t_i} (b_j - 1) \frac{1 - t_j}{w_i}. \quad (143)$$

The latter can be rewritten as

$$\sum_j \omega_j \left(\frac{t_j + \tau_j}{1 - t_j} \right) \eta_{ji} = \omega_i(1 - b_i) + \sum_j \omega_j (b_j - 1) \kappa_{ji}, \quad (144)$$

where the elasticities are given by

$$\eta_{ji} \equiv - \frac{\partial E_j}{\partial t_i} \frac{1 - t_i}{E_j} \frac{w_j(1 - t_j)}{w_i(1 - t_i)}, \quad (145)$$

$$\kappa_{ji} \equiv \frac{\partial w_j}{\partial t_i} \frac{1 - t_i}{w_j} \frac{w_j(1 - t_j)}{w_i(1 - t_i)}. \quad (146)$$

Finally, as stated in Appendix D, the proof regarding the desirability of unions requires no assumption on the cross-derivatives $F_{ij}(\cdot)$ and hence, generalizes completely to a setting with interdependent labor markets.

F Inefficient rationing

F.1 Optimal taxation

To prove the result stated in Proposition 6, we start by characterizing some properties of the rationing scheme. The general rationing scheme is given by

$$\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_i(E_i, \bar{\varphi}_i, \varphi) dG(\varphi) \equiv E_i. \quad (147)$$

Under the assumption that the function $e_i(\cdot)$ is differentiable with respect to its first and second argument, the following relationships must hold:³⁶

$$\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{iE_i}(E_i, \bar{\varphi}_i, \varphi) dG(\varphi) = 1, \quad (148)$$

$$\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) dG(\varphi) + e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i) G'(\bar{\varphi}_i) = 0. \quad (149)$$

Instead of deriving the labor-market equilibrium conditions for a general rationing schedule, we assume that the rationing schedule implicitly defines relationships $E_i = E_i(t_i)$ and $w_i = w_i(t_i)$. Hence, we ignore income effects and assume interdependent labor markets. The Lagrangian

³⁶This follows from differentiating equation (147) with respect to E_i and $\bar{\varphi}_i$.

associated to the government's problem reads:

$$\begin{aligned} \mathcal{L} \equiv & \sum_i N_i \left(u(-T_u) + \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_i(E_i, \bar{\varphi}_i, \varphi) (u(w_i(1-t_i) - T_u - \varphi) - u(-T_u)) dG(\varphi) \right) \\ & + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right). \end{aligned} \quad (150)$$

In the absence of income effects, and with a perfect profit tax, we can derive

$$T_u : \quad - \sum_i N_i E_i \bar{u}_i' - \sum_i N_i (1 - E_i) u_u' + \lambda \sum_i N_i = 0 \quad (151)$$

$$T_f : \quad -u_f' + \lambda = 0, \quad (152)$$

which leads to the first two results from the proposition. To derive the final result, differentiate the Lagrangian with respect to t_i , and set the resulting expression to zero:

$$\begin{aligned} t_i : \quad & - N_i E_i w_i (\bar{u}_i' - \lambda) - w_i N_i \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i} (u_i - u_u) dG(\varphi) \\ & + \frac{\partial w_i}{\partial t_i} \left((1 - t_i) N_i E_i \bar{u}_i' + (1 - t_i) N_i \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i} (u_i - u_u) dG(\varphi) - N_i E_i u_f' + \lambda N_i E_i t_i \right) \\ & + \frac{\partial E_i}{\partial t_i} \left(\lambda N_i t_i w_i + N_i \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{iE_i} (u_i - u_u) dG(\varphi) \right) = 0, \end{aligned} \quad (153)$$

where

$$\bar{u}_i' \equiv \int_{\underline{\varphi}}^{\bar{\varphi}_i} \frac{e_i(E_i, \bar{\varphi}_i, \varphi)}{E_i} u'(w_i(1-t_i) - T_u - \varphi) dG(\varphi) \quad (154)$$

denotes the expected utility of the employed workers, and $u_i(\varphi) \equiv u(w_i(1-t_i) - T_u - \varphi)$ measures the utility of the worker with participation costs $\varphi \in [\underline{\varphi}, \bar{\varphi}_i]$ who is employed in sector i . To proceed, divide equation (153) by $N_i E_i w_i \lambda$ and impose $b_f = 1$. In addition, denote by

$$\hat{\tau}_i \equiv \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{iE_i}(E_i, \bar{\varphi}_i, \varphi) \left(\frac{u(w_i(1-t_i) - T_u - \varphi) - u(-T_u)}{\lambda w_i} \right) dG(\varphi) \quad (155)$$

the expected utility loss of the rationed workers, i.e., the workers who lose their job if the employment rate E_i is marginally reduced.³⁷ After some rearranging, we obtain

$$\left(\frac{t_i + \hat{\tau}_i}{1 - t_i} \right) \eta_i = (1 - b_i) + (b_i - 1) \kappa_i + \frac{\kappa_i - 1}{E_i} \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) \frac{(u_i(\varphi) - u_u)}{\lambda} dG(\varphi). \quad (156)$$

³⁷Note that, by equation (148), the terms $e_{iE_i}(\cdot)$ integrate to one, so that the term defined as $\hat{\tau}_i$ is indeed an expected value.

Next, observe that $\kappa_i - 1 = \frac{\partial \bar{\varphi}_i}{\partial t_i} \frac{(1-t_i)}{\bar{\varphi}_i}$. In addition, using equation (149), we can rewrite the last part of the previous expression as:

$$\begin{aligned} & \frac{\kappa_i - 1}{E_i} \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) \frac{(u_i(\varphi) - u_u)}{\lambda} dG(\varphi) = \\ & - \frac{\partial \bar{\varphi}_i}{\partial t_i} \frac{(1-t_i)}{\bar{\varphi}_i} \frac{e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i) G'(\bar{\varphi}_i)}{E_i} \int_{\underline{\varphi}}^{\bar{\varphi}_i} \frac{e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi)}{\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) dG(\varphi)} \frac{(u_i(\varphi) - u_u)}{\lambda} dG(\varphi). \end{aligned} \quad (157)$$

Then, define

$$\begin{aligned} \psi_i \equiv & \frac{e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i)}{E_i/G(\bar{\varphi}_i)} \int_{\underline{\varphi}}^{\bar{\varphi}_i} \frac{e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi)}{\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) dG(\varphi)} \\ & \left(\frac{u(w_i(1-t_i) - T_u - \varphi) - u(-T_u)}{\lambda w_i} \right) dG(\varphi) \end{aligned} \quad (158)$$

and

$$\gamma_i \equiv - \frac{\partial G(\bar{\varphi}_i)}{\partial t_i} \frac{1-t_i}{G(\bar{\varphi}_i)}. \quad (159)$$

After substituting all these definitions in equation (156), we arrive at

$$\left(\frac{t_i + \hat{\tau}_i}{1-t_i} \right) \eta_i - \left(\frac{\psi_i}{1-t_i} \right) \gamma_i = (1-b_i) + (b_i-1)\kappa_i. \quad (160)$$

F.2 Desirability of unions

To study the welfare effects of the reform described in 6.2, one can differentiate the Lagrangian with respect to t_i and T_f under the assumption that the reform (i) is budget neutral, (ii) leaves the labor-market outcomes (i.e., w_i and E_i) unaffected, while keeping in mind that a change in the participation tax rate affects the participation margin $\hat{\varphi}_i = w_i(1-t_i)$. The welfare effect is then:

$$\begin{aligned} \frac{dW}{\lambda} = & - N_i E_i b_i w_i dt_i - b_f dT_f \\ & - N_i \int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) \frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda} dG(\varphi) w_i dt_i. \end{aligned} \quad (161)$$

The first term reflects the (direct) change in the workers' utility in sector i following the change in the income tax, whereas the second term reflects the change in the utility of firm-owners induced by a change in the profit tax. The third term reflects the utility loss that is due to a change in the participation margin: if t_i is lowered, more people want to participate. If some of these workers find a job (which may happen if rationing is not fully efficient), and the rate of employment is kept constant, it must be that some workers other workers lose their jobs and experience a utility loss.

Under the balanced-budget assumption, $N_i E_i w_i dt_i + dT_f = 0$. In addition, the government can levy a non-distortionary profit tax ($b_f = 1$). Using these results, and equation (149), we

can rewrite the change in social welfare as:

$$\begin{aligned} \frac{dW}{\lambda} &= -N_i E_i \left(b_i - 1 - e_i(E_i, \bar{\varphi}_i, \bar{\varphi}_i) \frac{G'(\bar{\varphi}_i)}{E_i} \right. \\ &\times \left. \int_{\underline{\varphi}}^{\bar{\varphi}_i} \frac{e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi)}{\int_{\underline{\varphi}}^{\bar{\varphi}_i} e_{i\bar{\varphi}_i}(E_i, \bar{\varphi}_i, \varphi) dG(\varphi)} \frac{u(w_i - T_i - \varphi) - u(-T_u)}{\lambda} dG(\varphi) \right) w_i dt_i. \end{aligned} \quad (162)$$

Given that t_i is lowered in the policy experiment (such that $dt_i < 0$), the welfare effect is positive provided that the term in between brackets is positive. Using the definitions for ψ_i and γ_i , this is the case if:

$$b_i > 1 + \left(\frac{\psi_i}{1 - t_i} \right) \gamma_i. \quad (163)$$

G Bargaining over multiple wages

G.1 Labor-market equilibrium

Assume there is one union with a utilitarian objective and denote the union's bargaining power by $\beta \in [0, 1]$. Under Nash-bargaining, the solution is characterized by solving the following maximization problem:

$$\begin{aligned} \max_{\{w_i, E_i\}_{i \in \mathcal{I}}} \Omega &= \beta \log \left(\sum_i N_i \int_{\varphi_i}^{G^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u)) dG(\varphi) \right) \\ &+ (1 - \beta) \log \left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0, \forall i} - T_f) \right) \\ \text{s.t. } w_i - F_i(\cdot) &= 0, \quad \forall i \\ G(w_i - T_i + T_u) - E_i &\geq 0, \quad \forall i. \end{aligned} \quad (164)$$

As was done in Appendix A, the payoffs of both parties are taken in deviation from the payoff associated with the disagreement outcome. The Lagrangian is:

$$\begin{aligned} \mathcal{L} &= \beta \log \left(\sum_i N_i \int_{\varphi_i}^{G^{-1}(E_i)} (u(w_i - T_i - \varphi) - u(-T_u)) dG(\varphi) \right) \\ &+ (1 - \beta) \log \left(u(F(\cdot) - \sum_i w_i N_i E_i - T_f) - u(F(\cdot)|_{E_i=0, \forall i} - T_f) \right) \\ &+ \sum_i \lambda_i (w_i - F_i(\cdot)) + \sum_i \mu_i (G(w_i - T_i + T_u) - E_i). \end{aligned} \quad (165)$$

The first-order conditions:

$$w_i : \frac{\beta}{\sum_j N_j E_j (\bar{u}_j - u_u)} N_i E_i \bar{u}'_i - \frac{1 - \beta}{u_f - \underline{u}_f} N_i E_i \bar{u}'_f + \lambda_i + \mu_i G'_i = 0 \quad (166)$$

$$E_i : \frac{\beta}{\sum_j N_j E_j (\bar{u}_j - u_u)} N_i (\hat{u}_i - u_u) - N_i \sum_j \lambda_j F_{ji} - \mu_i = 0 \quad (167)$$

$$\lambda_i : w_i - F_i = 0 \quad (168)$$

$$\mu_i : \mu_i (G_i - E_i) = 0. \quad (169)$$

These conditions implicitly characterize the labor-market equilibrium, and can be manipulated to obtain the following insights.

First, if the union has zero bargaining power ($\beta = 0$), the equilibrium coincides with the competitive outcome (i.e., $G_i = E_i$ and $w_i = F_i$ for all i). To see why this is true, substitute $\beta = 0$ in the first-order conditions for w_i and E_i . Next, use the first of these to substitute out for λ_i in the second and rearrange:

$$\underbrace{\mu_i (N_i G'_i F_{ii} - 1)}_{<0} + N_i \sum_{j \neq i} \underbrace{\mu_j G'_j F_{ji}}_{\geq 0} = N_i \underbrace{\frac{u'_f}{u_f - \underline{u}_f}}_{>0} \underbrace{\sum_j N_j E_j F_{ji}}_{=-F_{Ki}K < 0}. \quad (170)$$

These inequalities follow from the assumptions of co-operant factors of production and constant returns to scale. Constant returns to scale implies $\sum_j N_j E_j F_{ji} = -F_{Ki}K \leq 0$, whereas non-increasing marginal productivity and co-operant factors of production imply $F_{ii} \leq 0 \leq F_{ji}$. Now, suppose there is a sector in which $G_i > E_i$ (i.e., the wage is above the market-clearing level). Then, from the Kuhn-Tucker conditions, it must be that $\mu_i = 0$. Because of the nonnegativity of all multipliers, however, the above condition cannot be satisfied (unless all labor types would be perfect substitutes: $F_{ii} = F_{ij} = F_{Ki} = 0$ for all i, j). This is a contradiction. Therefore, $G_i = E_i$ for all i . Intuitively, if $\beta = 0$, firm-owners have all the bargaining power, and the equilibrium coincides with the competitive outcome.

Second, if the union has full bargaining power ($\beta = 1$), there is at least one skill-type for whom the wage exceeds the market-clearing level (i.e., there exists an i such that $G_i > E_i$). To see why this is true, suppose $\beta = 1$. In this case, the union has full bargaining power and sets wages in order to maximize the expected utility of all workers. Starting from the competitive equilibrium (where $G_i = E_i$ for all i), lowering the employment rate – or increasing the wage – for the workers with the highest average marginal utility (such that $\bar{u}'_i > \bar{u}'_j$ for all $j \neq i$) increases the union's objective, i.e.

$$-N_i \sum_j N_j E_j F_{ji} \bar{u}'_j = -N_i \left(N_i E_i F_{ii} \bar{u}'_i + \sum_{j \neq i} N_j E_j F_{ji} \bar{u}'_j \right) < 0, \quad (171)$$

provided not all labor types are perfect substitutes (in which case $F_{ii} = F_{ij} = 0$ for all i, j). The sign follows from the assumption on the production function. Now, if $\bar{u}'_i > \bar{u}'_j$ for all $j \neq i$, the first term on the right-hand side dominates the remaining terms. Marginally increasing the wage for the lowest-income workers above the market-clearing level thus increases the union's

objective and therefore, cannot be consistent with maximizing behavior from the union. When $\beta = 1$, there must be at least one sector where the wage exceeds the market-clearing level.

G.2 Optimal taxation

How should taxes be optimally set? In the absence of income effects, the first-order conditions implicitly characterize the equilibrium wages and employment rates as a function the participation tax rates: $w_i = w_i(t_1, \dots, t_I)$ and $E_i = E_i(t_1, \dots, t_I)$.³⁸ These reduced-form equations can be used to derive the optimal tax formulae. This case is identical to the one with multiple unions and interdependent labor markets, which is analyzed in Appendix E. There, the equilibrium wages and employment rates were also written as a function of all the participation taxes. The optimal tax formulae (written in terms of elasticities) therefore remain unaffected.

G.3 Desirability of unions

To study the desirability of unions, we analyze the welfare effects of a joint increase in the degree of unionization β and a tax intervention which leaves all labor-market outcomes unaffected. Since in the policy optimum any tax reform has no welfare effects, any change in welfare must necessarily be the result of the change in the union's bargaining power.

What tax reform offsets any impact of the increase in the union's bargaining power on labor market outcomes? Importantly, the tax reform cannot include a change in the participation tax for workers whose wage is at the market-clearing level. To see why this is true, consider the labor-market equilibrium condition in a sector i where the wage is at the market-clearing level:

$$G_i(F_i(\cdot)(1 - t_i)) = E_i. \quad (172)$$

A change in the t_i in this sector needs to be accompanied by a change in either $F_i(\cdot)$ or E_i . For this to be the case, at least one employment rate needs to be adjusted. However, the intervention is intended keep employment rates unaffected. Hence, it must be that $dt_i = 0$ in sectors where $G_i = E_i$. In sectors where $G_i > E_i$, the tax reform which leaves all labor-market outcomes unaffected can be found by solving

$$\forall i \in k(\beta) : \sum_{j \in k(\beta)} \frac{\partial w_i(t_1, \dots, t_I, \beta)}{\partial t_j} dt_j^* + \frac{\partial w_i(t_1, \dots, t_I, \beta)}{\partial \beta} d\beta = 0, \quad (173)$$

for the dt_j^* 's.

Since the combined increase in the union's bargaining power and the tax reform leaves all labor-market outcomes unaffected, there is only a transfer of resources from the government's budget to workers for whom the wage is raised above the market-clearing level (i.e., for whom $G_i > E_i$). The welfare effect is thus equal to

$$\frac{d\mathcal{W}}{\lambda} = \sum_{i \in k(\beta)} N_i E_i (1 - b_i) dt_i^*,$$

³⁸Like we did in Appendix A, for technical convenience we ignore the dependency of the variables on the profit tax.

where λ is the multiplier on the government's budget constraint. Next, divide the latter by $\sum_i N_i > 0$. The remaining term is positive if and only if

$$\sum_{i \in k\beta} \omega_i (1 - b_i) dt_i^* > 0.$$

H Efficient bargaining

H.1 Derivation elasticities

The partial equilibrium in labor market i is obtained by combining the contract curve (49) and the rent-sharing rule (50):

$$\overline{u'(w_i(1 - t_i) - T_u - \varphi)}(w_i - F_i(E_i)) = u(w_i(1 - t_i) - T_u - G^{-1}(E_i)) - u(-T_u), \quad (174)$$

$$w_i = (1 - \sigma_i)F_i(E_i) + \sigma_i q_i(E_i). \quad (175)$$

In the absence of income effects, these equations define $E_i = E_i(t_i)$ and $w_i = w_i(t_i)$. To derive the relevant elasticities, let us start by linearizing the rent-sharing rule:

$$\frac{dw_i}{w_i} = - \left((1 - m_i) \frac{(1 - \sigma_i)}{\varepsilon_i} + m_i \right) \frac{dE_i}{E_i}, \quad (176)$$

where $m_i \equiv (w_i - F_i)/w_i$ measures the mark-up of the wage over the marginal product of labor, as a fraction of the wage. If the union's bargaining power is zero, $\sigma_i = 0$ and $m_i = 0$ and equation (176) reduces to the linearized labor-demand equation.

Linearizing the contract curve yields:

$$\frac{d\overline{u'_i}}{\overline{u'_i}} + \frac{d(w_i - F_i)}{w_i - F_i} = \frac{d(\hat{u}_i - u_u)}{\hat{u}_i - u_u}. \quad (177)$$

The linearized sub-parts are given by:

$$\frac{d\overline{u'_i}}{\overline{u'_i}} = \frac{\overline{u''_i} w_i (1 - t_i)}{\overline{u'_i}} \left(\frac{dw_i}{w_i} - \frac{dt_i}{1 - t_i} \right) + \frac{(\hat{u}'_i - \overline{u'_i})}{\overline{u'_i}} \frac{dE_i}{E_i} \quad (178)$$

$$\frac{d(w_i - F_i)}{w_i - F_i} = \frac{1}{m_i} \left(\frac{dw_i}{w_i} + \frac{(1 - m_i)}{\varepsilon_i} \frac{dE_i}{E_i} \right) \quad (179)$$

$$\frac{d(\hat{u}_i - u_u)}{\hat{u}_i - u_u} = \frac{\hat{u}'_i w_i (1 - t_i)}{(\hat{u}_i - u_u)} \left(\frac{dw_i}{w_i} - \frac{dt_i}{1 - t_i} \right) - \frac{\hat{u}'_i E_i}{g(\hat{\varphi}_i)(\hat{u}_i - u_u)} \frac{dE_i}{E_i}. \quad (180)$$

Solving for the changes in employment and wages yields:

$$\frac{dE_i}{E_i} = \frac{-u'_u w_i (1 - t_i)}{\frac{\hat{u}'_i E_i}{g(\hat{\varphi}_i)} + u'_u w_i (1 - t_i) \left(\frac{(1-m_i)(1-\sigma_i)}{\varepsilon_i} + m_i \right) + (\hat{u}_i - u_u) \left(\frac{(1-m_i)(1-\sigma_i)}{m_i \varepsilon_i} - 1 + \frac{(\hat{u}'_i - u'_i)}{u'_i} \right)} \frac{dt_i}{1 - t_i} \quad (181)$$

$$\frac{dw_i}{w_i} = \frac{u'_u w_i (1 - t_i) \left(\frac{(1-m_i)(1-\sigma_i)}{\varepsilon_i} + m_i \right)}{\frac{\hat{u}'_i E_i}{g(\hat{\varphi}_i)} + u'_u w_i (1 - t_i) \left(\frac{(1-m_i)(1-\sigma_i)}{\varepsilon_i} + m_i \right) + (\hat{u}_i - u_u) \left(\frac{(1-m_i)(1-\sigma_i)}{m_i \varepsilon_i} - 1 + \frac{(\hat{u}'_i - u'_i)}{u'_i} \right)} \frac{dt_i}{1 - t_i}. \quad (182)$$

The elasticities are now as given in Proposition 8.

H.2 Optimal taxation

The derivation is very similar as in Appendix C.1. Start with the Lagrangian:

$$\begin{aligned} \mathcal{L} \equiv & \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\ & + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right). \end{aligned} \quad (183)$$

Differentiating with respect to T_u , T_f and t_i yields:

$$T_u : \quad - \sum_i N_i E_i \bar{u}'_i - \sum_i N_i (1 - E_i) u'_u + \lambda \sum_i N_i = 0, \quad (184)$$

$$T_f : \quad -u'_f + \lambda = 0, \quad (185)$$

$$\begin{aligned} t_i : \quad & -N_i E_i w_i (\bar{u}'_i - \lambda) + \frac{\partial E_i}{\partial t_i} (N_i (\hat{u}_i - u_u) + u'_f N_i (F_i - w_i) + \lambda N_i t_i w_i) \\ & + \frac{\partial w_i}{\partial t_i} (N_i E_i \bar{u}'_i (1 - t_i) - N_i E_i u'_f + \lambda N_i E_i t_i) = 0. \end{aligned} \quad (186)$$

The first two expressions from Proposition 9 are obtained by dividing the first expression by $\lambda \sum_i N_i$ and the second expression by λ , and imposing the definitions of the welfare weights and the labor force shares from equations (21)-(22). The expression for the optimal participation tax rate t_i is obtained by substituting $u'_f = \lambda$ in equation (186) and divide the expression by $N_i E_i \lambda w_i$. After imposing the definitions of the union wedge $\tau_i = \frac{u(\hat{c}_i) - u(c_u)}{\lambda w_i}$, the mark-up $m_i = \frac{w_i - F_i}{w_i}$ and the elasticities κ_i and η_i as defined in equations (55)-(56), one arrives at the final expression stated in Proposition 9.

H.3 Desirability of unions

As was done in Appendix D, in order to determine how a change in the union's bargaining power affects social welfare, we formulate the Lagrangian taking explicitly the labor-market

equilibrium conditions into account:

$$\begin{aligned}
\mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u(w_i(1-t_i) - T_u - \varphi) dG(\varphi) + \int_{G^{-1}(E_i)}^{\bar{\varphi}} u(-T_u) dG(\varphi) \right) \\
& + u(F(\cdot) - \sum_i w_i N_i E_i - T_f) + \lambda \left(\sum_i N_i (T_u + E_i t_i w_i) + T_f - R \right) \\
& + \sum_i \kappa_i N_i (w_i - (1 - \sigma_i) F_i(\cdot) - \sigma_i q_i(\cdot)) \\
& + \sum_i \mu_i N_i \left(\int_{\underline{\varphi}}^{G^{-1}(E_i)} u'(w_i(1-t_i) - T_u - \varphi) dG(\varphi) (F_i(\cdot) - w_i) \right. \\
& \left. + E_i (u(w_i(1-t_i) - T_u - G^{-1}(E_i)) - u(-T_u)) \right). \tag{187}
\end{aligned}$$

To determine how a change in the union's bargaining power affects social welfare, differentiate the Lagrangian with respect to σ_i , and apply the Envelope theorem:

$$\frac{\partial \mathcal{W}}{\partial \sigma_i} = \kappa_i (F_i - q_i). \tag{188}$$

Because the production function $F(\cdot)$ is concave in E_i , the latter expression is positive if and only if $\kappa_i < 0$. To determine the sign of κ_i , differentiate the Lagrangian with respect to t_i , w_i and T_f :

$$t_i : -w_i N_i E_i (\bar{u}'_i - \lambda) - w_i \mu_i N_i \left(E_i \bar{u}''_i (F_i - w_i) + E_i \hat{u}'_i \right) = 0 \tag{189}$$

$$\begin{aligned}
w_i : & (1-t_i) N_i E_i \bar{u}'_i - N_i E_i u'_f + t_i N_i E_i \lambda + \kappa_i \\
& + (1-t_i) \mu_i N_i \left(E_i \bar{u}''_i (F_i - w_i) + E_i \hat{u}'_i \right) - \mu_i N_i E_i \bar{u}'_i = 0. \tag{190}
\end{aligned}$$

$$T_f : -u'_f + \lambda = 0 \tag{191}$$

Combining these yields:

$$\kappa_i - \mu_i N_i E_i \bar{u}'_i = 0.$$

Substituting for μ_i using equation (189):

$$\kappa_i = -N_i E_i \left(\frac{\bar{u}'_i (\bar{u}'_i - \lambda)}{\bar{u}''_i (F_i - w_i) + \hat{u}'_i} \right). \tag{192}$$

Combining equations (188)–(190), it follows that an increase in σ_i increases social welfare if and only if the term on the right-hand side of expression (192) is positive:

$$b_i > 1. \tag{193}$$

This completes the proof.

I Simulations

This Appendix provides additional information regarding the simulations. The objective is to calculate the optimal tax-benefit system for varying degrees of union power ρ_i . In order to do so, we numerically solve equations which characterize the policy optimum. Since we focus on calculating the optimal tax-benefit system, we ignore firm-owners in our simulations.³⁹ Under Assumption 1 and Assumption 2, the policy optimum is characterized by (see Proposition 1):

$$\omega_u b_u + \sum_i \omega_i b_i = 1, \quad (194)$$

$$\left(\frac{t_i + \frac{\rho_i b_i}{\varepsilon_i}}{1 - t_i} \right) \eta_i = (1 - b_i) + (b_i - 1) \kappa_i, \quad (195)$$

where we substituted out for the union wedge using $\tau_i = \frac{\rho_i b_i}{\varepsilon_i}$. The government's budget constraint reads:

$$\sum_i N_i (T_u + E_i t_i w_i) - R = 0. \quad (196)$$

The labor-market equilibrium conditions and the welfare weights are assumed to take the following form:

$$E_i = \zeta_i (w_i (1 - t_i))^{\pi_i} \quad (197)$$

$$E_i = \xi_i w_i^{-\varepsilon_i} \quad (198)$$

$$b_i = \frac{1}{\lambda (w_i (1 - t_i) - T_u)^\nu} \quad (199)$$

$$b_u = \frac{1}{\lambda (-T_u)^\nu}. \quad (200)$$

We numerically solve the system (194)–(200) for the tax instruments t_i and T_u , the labor-market outcomes w_i and E_i , the welfare weights b_i and b_u , and the multiplier on the government's budget constraint. Values for R , ζ_i , and ξ_i are calibrated using the current values of wages, employment rates and the tax system that is in place (see Table 1). Following Saez (2002) and Kroft et al. (2015), we choose $\nu = 1$ in our baseline simulations. Finally, we set the (modified) participation elasticity $\pi_i = 0.4$ ($\pi_i = 0.16$) for the U.S. (Netherlands) and the labor-demand elasticity $\varepsilon_i = 0.6$. We solve these equations for the following three scenarios:

1. Competitive labor markets without unions: $\rho_i = 0$ for all i ,
2. Intermediate union power: $\rho_i = 1/2$ for all i ,
3. Monopoly unions: $\rho_i = 1$ for all i .

³⁹As long as profits can be taxed, the optimal tax formulae for T_u and t_i are identical, irrespective of whether there are firm-owners. Furthermore, the condition on the desirability of unions is unaffected by any restriction on profit taxation.