Economic Growth with Locked-in Fertility: Under- and Over-Investment in Education*

Masao Nakagawa[a], Asuka Oura[b], and Yoshiaki Sugimoto[c,†]

[a] Graduate School of Social Sciences, Hiroshima University, Japan

[b] Department of Modern Economics, Faculty of Economics, Daito Bunka University, Japan

[c] Faculty of Economics, Kansai University, Japan

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Abstract

This research argues that the rigidity of fertility decisions brings about over- and under-investment in education, thereby delaying human capital accumulation and economic growth. In underdeveloped stages, large family sizes prevent parents from revising their education plans upward even if their children are unexpectedly competent. In advanced stages, by contrast, having fewer children leaves a generous budget for education, which discourages downward revisions for children who are unexpectedly incompetent. The impact of these 'lock-in effects' depends in part on the degree of parental altruism.

Keywords: Rigidity; Over- and Under-Investment; Education; Growth.

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†Corresponding author. 3-3-35 Yamate-cho, Suita, Osaka 564-8680, Japan. Tel.: +81 6 6368 0615; fax: +81 6 6339 7704. E-mail address: sugimoto@kansai-u.ac.jp (Y. Sugimoto).

1 Introduction

It is widely recognized that investment in human capital is one of the steady means to improve the welfare of individuals and the growth performance of an economy. Despite the recognition, under-investment in education prevails in developing countries. A large fraction of households cannot afford the expenses for schooling in the presence of borrowing constraints. On the other hand, the recognition brought the issue of over-investment in higher education on developed countries. Students of these countries do not necessarily reward the financial aid from their parents.[1] Using the data on UK graduates between 1985 to 1990, an empirical analysis by Chevalier (2003) calculated that the wage loss of over-educated workers was amount to 22% to 26%.[2]

A plausible conjecture from these facts is that the developed economies did, to some extent, go through a transition from under- to over-investment in education. It appears that no single theory has shed light on the underlying mechanism of the dynamic phenomenon. Existing theories are only partially satisfactory in this respect. The literature on inequality and growth, which has flourished since the 1990s, asserts the possibility of under-investment in human capital in the presence of capital market imperfections (cf. Galor and Zeira, 1993; Moav, 2002; Mookherjee and Ray, 2003). By contrast, another strand of literature argues that information imperfections, along with market imperfections, may induce precautionary reactions of individuals toward over-investment in human capital (cf. Gould et al., 2001; Aiyagari et al., 2002).[3] None of these studies encompass the aforementioned transition.

Motivated by these observations, this research develops a growth theory to analyze the optimality of private education in comparison with the perfect foresight choice. It focuses on the interaction between childbirth and education investment in the growth process. The rigidity of fertility decisions, along with imperfect foresight into children's abilities, restrain

[1] For the 2009 academic year in Japan, for instance, no less than 39.3 percent of high school dropouts was due to the failure of adaptation to school life and study (the Japanese Ministry of Education, Culture, Sports, Science and Technology. `http://www.mext.go.jp/b_menu/houdou/22/12/1300746.htm`).

[2] In this line of empirical research is Sicherman (1991), who relies on the PSID (Panel Study of Income Dynamics) data for the late 1970s.

[3] Gould et al., (2001) considers the eroding effect of technological progress, which is biased and random across sectors, on human capital. Aiyagari et al. (2002) highlight the lack of insurance markets for ability as well as that of loan markets.

the adjustment between the quantity and the quality of children.[4] The resulting education investment affects human capital accumulation, technological progress, and the return on education investment in the future. The irreversibility of childbirth is therefore a key factor for growth performance both in the short and in the long run.

The growth model presented later features four key elements. First, parental education investment is the only determinant of children's human capital. Hence no conflict arises between parents and children over the household education policy.[5] Second, childbirth is the irreversible investment in the quantity of children, as proposed by Fraser (2001) and Doepke and Zilibotti (2005).[6] Once determined, the number of children to raise is not adjustable in either directions and thus the non-education cost of child rearing is interpreted as a sunk cost. Third, childbirth is accompanied by idiosyncratic, unexpected ability shocks on children. While these shocks may induce parents to revise their initial education plan, their ex post reactions is constrained by the irreversibility of childbirth.

Figure 1 presents two extreme cases that give insight into the lock-in effects of childbirth on education decisions.[7] Panel (a) shows optimization by a household whose children are,

[4] See Becker and Lewis (1973) for the formulation of the quantity-quality trade-off faced by parents. Goldstein et al. (2003, p. 487, Table 2) compare mean personal ideal family size and mean personal expected family size for young women by using the Eurobarometer 2001 survey. They report that the former measure is smaller than the latter by 0.2 to 0.4 points in major European countries (p. 486). A similar pattern applies to the United States (Hagewen and Morgan, 2005, p. 509, Figure 1). These disparities are consistent with this paper's view that the budget constraint is binding in child rearing; i.e., parents can raise funds for desirable education at the cost of the family size.

[5] There is a strand of literature analyzing child-rearing strategies and parent-child conflict in various contexts. Weinberg (2001) develops a static agency model in which an altruistic parent motivates her child to make efforts by a pecuniary means. The author finds that in the presence of the subsistence level of consumption, the effort level increases with parental income only at low income levels. Akabayashi (2006) develops a dynamic theory that explains child maltreatment by a parent who imperfectly observes the accumulation of the child's human capital. The tough love model of Bhatt and Ogaki (2008) shows that parents leave little transfers to impatient children so that poor consumption in childhood makes them more patient.

[6] In relation to schooling, a recent study by de la Croix and Doepke (2009) focused on the lock-in effect of fertility decisions on individuals' voting preferences, in accounting for the differences in public education systems across countries. The assumption of perfect irreversibility would be relaxed by dividing the period of childbirth into two so that unexpected ability shock occurs between them. This approach is taken by Iyigun (2000) for different research objectives from the present paper. The author develops a growth model with no uncertainty (i.e., no lock-in effect of childbirth) and demonstrates that the timing of childbearing is delayed by the accumulation of human capital.

[7] The model presented in Figure 1 is not identical to the one introduced in Section 2, in which the budget constraint is non-linear. Figure 1 nonetheless conveys the essence of the lock-in effects of childbirth that are analyzed later.

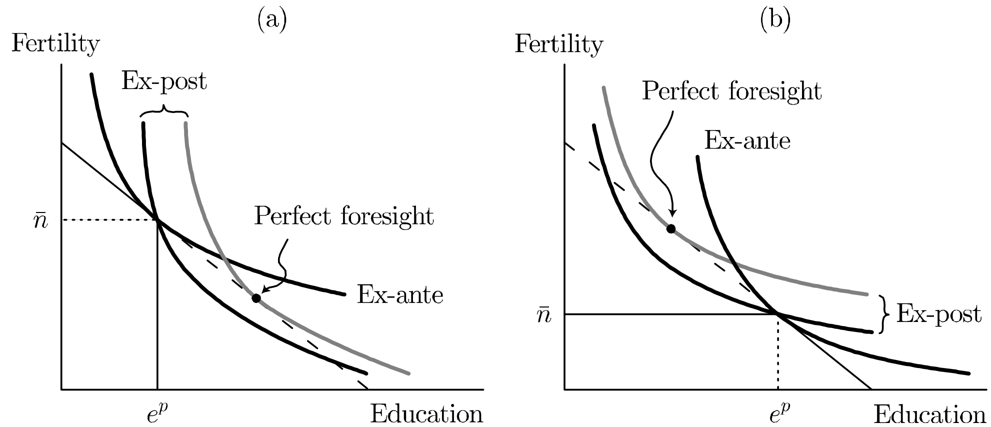

Figure 1. The Lock-in Effects of Childbirth on Education.

in fact, more competent than expected. At the time of childbirth, the household does not observe the true ability of children and its ex ante (before the observation of true ability) optimal choice occurs at $(\bar{n}, e^P)$, where the ex ante indifference curve is tangent to the budget line including the broken line. The quantity of children, $\bar{n}$, is locked into a large level in prospect of small education investment indicated by e^P.

In the diagram, "Perfect foresight" indicates the point at which the ex post indifference curve (i.e., the indifference curve under perfect foresight) is tangent to the original budget line. However, this point is beyond the actual budget constraint, which is kinked at $\bar{n}$. That is, because the household observes the true ability after childbirth, it is not possible to reduce the quantity of children to achieve any higher education level than e^P. The ex post optimal decision is therefore to carry out the initial education plan, e^P, with no change in the family size. The opposite case is explained by panel (b), in which children are in fact less competent than expected.

Taking the two types of lock-in effects into account, the theory developed below demonstrates the following scenario of economic development. In early stage of development, where poor technology is unfavorable for education investment, households place greater importance on the quantity than the quality of children at the time of childbirth. The resulting quantity of children, fixed into a large size, squeezes the budget and thereby constrains education investment for high ability children. In this situation, the ability-based provision of child care subsidies mitigates under-investment in education and accelerates technological progress.

In later stage of development, by contrast, households place more importance on the quality of children at the time of childbirth. The quantity of children, locked into a smaller size, leaves households the budget to spare, which induces education investment for children who are unexpectedly incompetent. Under the circumstance, the ability-based provision of child care subsidies will mitigate over- as well as under-investment in education. The adverse effect of under-investment may remain dominant in the long run, in part depending on the ability distribution.

The rest of the present paper consists of the following sections. Section 2 describes the structure of the baseline model and considers optimal decisions and individuals' attitudes toward education. The last part of the section aggregate individuals' choices. Section Appendix 1 demonstrates the aforementioned dynamic relationship between education investment and technological progress. The proofs of mathematical results are given in the Appendix.

2 The Baseline Model

The economy has an overlapping-generations structure and operates over an infinite discrete time horizon, $t = 0, 1, 2 \cdots$.[8] A single homogeneous goods is produced in one sector by employing human capital, and technological progress occurs through learning by doing.

Individuals give rise to children and then may provide education support for them, depending on their ability observed after birth. Childbirth is irreversible investment in the quantity of children: The family size cannot be either reduced or enlarged at the time of observing true ability. As a feature of the baseline model, individuals have perfect foresight into all aspects, including the ability of children they intend to have.

2.1 Firms

In perfectly competitive environments, firms generate a single homogeneous good by employing human capital (i.e., efficiency unit of labor) with a linear technology. The level of output per worker in period t, denoted as y_t, is determined through the production function

$$y_t = A_t H_t / N_t, \tag{1}$$

[8] This is an extension of the model developed by Galor and Weil (2000), who explore the mechanism underlying the demographic transition in the long-term growth process.

where $A_t > 0$, H_t, and N_t are the levels of technology, employed human capital, and working population, respectively, in period t. For the sake of simplicity, the price of the final good is normalized to unity. As a result of profit maximization by price-taking firms, H_t maximizes the aggregate profit $A_t H_t - w_t H_t$, where w_t is the market wage rate per unit of human capital in period t. In the competitive labor market considered herein, w_t is adjusted so that the resulting profit is neither negative nor infinity large, leading to $w_t = A_t$.

2.2 Households

A new generation is born at the beginning of each period and lives for two periods. Generation t, born in period $t-1$, comprises a continuum of individuals existing on the interval $[0, N_t]$. They raise children at the opportunity cost of working in the production sector.

2.2.1 Environment

Consider the lifetime of an individual $i \in [0, N_t]$ of generation t, born in period $t-1$. In the first period (childhood), the individual engages in skill acquisition possibly with parental assistance. In the second period (adulthood or parenthood), the individual acquires $h_t^i > 0$ efficiency units of labor and gives birth to n_t^i units of identical children. Child rearing incurs $(\delta + e_t^i)$ efficiency units of labor per child, where $\delta > 0$ and e_t^i are the fixed and the education cost, respectively.[9] Wage income from the remaining labor is used up for consumption, c_t^i, so that no bequests are left to the offsprings. It follows that the budget constraint is

$$c_t^i \leq w_t[h_t^i - n_t^i(\delta + e_t^i)]. \tag{2}$$

Utility of individual i of generation t, u_t^i, depends on not only consumption in adulthood but also aggregate income of his/her children. Each of these children, indexed by $j \in [0, N_{t+1}]$, acquires $h_{t+1}^{i,j}$ efficiency units of labor in period $t+1$. Taking these into account, the utility function is formulated as

$$u_t^i = (1-\alpha)\ln c_t^i + \alpha \ln\left(w_{t+1} n_t^i h_{t+1}^{i,j}\right), \tag{3}$$

where $\alpha \in (0, 1)$ measures a degree of altruism.

[9] Parents may either train their children on their own or hire a teacher from the outside by paying $w_t e_t^i$ per child.

Human capital is the composite of basic skills (i.e., raw labor) and non-basic skills, the latter of which is the source of heterogeneity within generations. Let $a_t^i \in (0, 1]$ be the ability level of children born from parent i in period t, and let $l(g_{t+1})$ be the level of raw labor they acquire, where $g_{t+1} \geq 0$ denotes the growth rate of technology between periods t and $t + 1$. Then the production function of human capital is

$$\begin{aligned} h_{t+1}^{i,j} &= \bar{h} \times \min\{a_t^i e_t^i + l(g_{t+1}), 1\} \\ &\equiv h(a_t^i e_t^i, g_{t+1}), \end{aligned} \tag{4}$$

where $\bar{h} > 0$, $0 < l(0) \leq 1$, $l'(g_{t+1}) < 0 \ \forall g_{t+1} \geq 0$, and $\lim_{g_{t+1} \to \infty} l(g_{t+1}) = 0$. In other words, $h(a_t^i e_t^i, g_{t+1})$ is a kinked linear function that is bounded by $\bar{h}$.[10] The multiplicative term $a_t^i e_t^i$ is viewed as the level of educational attainment, which has a one-to-one relationship with non-basic skills. The formulation in Eq. (4) incorporates skill-biased technological progress, in the sense that the acceleration of technology growth raises the wage ratio of educated to uneducated (unskilled) workers.[11]

2.2.2 Optimization

Individuals aim to maximize their own utility as price takers. Substituting Eqs. (2) and (4) into Eq. (3), the maximization problem faced by parent i in period t is

$$\{n_t^i, e_t^i\} = \arg\max \left\{(1 - \alpha) \ln[h_t^i - n_t^i(\delta + e_t^i)] + \alpha \ln[n_t^i h(a_t^i e_t^i, g_{t+1})]\right\}, \tag{5}$$

subject to $(n_t^i, e_t^i) \geq 0$. Childbirth precedes education investment, and the fertility decision is made with education planning. Despite the time lag, actual education spending e_t^i is chosen with n_t^i because perfect foresight into a_t^i makes the initial education plan consistent. In this circumstance, the rigidity constraint on n_t^i (referred to as 'the rigidity constraint' hereafter) is unbinding.

First consider the fertility decision. The objective function exhibits the logarithmic form

[10] As will become clear, these nonstandard properties keep the model tractable without affecting the qualitative results.

[11] Skill-biased technological progress is formulated by Galor and Moav (2002) and others (to be added). See Moav (200?) for a human capital production function that is kinked with respect to education, as in Eq. (4).

and the strict concavity with respect to n_t^i. Hence the first-order optimality condition yields

$$n_t^i(\delta + e_t^i) = \alpha h_t^i, \tag{6}$$

where $n_t^i > 0$ and e_t^i is the planned and actual level of education investment. The parent makes a balance between the quality and the quality of children so as to keep the child-rearing cost at αh_t^i.

After childbirth, parent i in period t observes the ability level a_t^i as expected and then invests in education.[12] Substitution of Eq. (6) into Eq. (5) reveals that a fixed fraction of income, $1 - \alpha$, is spent on consumption and therefore

$$e_t^i = \arg\max \frac{h(a_t^i e_t^i, g_{t+1})}{\delta + e_t^i}, \tag{7}$$

subject to $e_t^i \geq 0$. Namely, e_t^i maximizes aggregate human capital of his/her children under the resource constraint given by Eq. (6). In order to simplify the exposition, it is assumed that when they are indifferent, parents choose to invest in education as much as possible. Then it follows that the optimal education choice is made in a discrete fashion such that

$$\begin{aligned} e_t^i &= \begin{cases} 0 & \text{if } a_t^i < \tilde{a}_t^*; \\ [1 - l(g_{t+1})]/a_t^i & \text{if } a_t^i \geq \tilde{a}_t^*. \end{cases} \\ &\equiv e^*(a_t^i, g_{t+1}), \end{aligned} \tag{8}$$

where $\tilde{a}_t^* = l(g_{t+1})/\delta \equiv \tilde{a}^*(g_{t+1})$.[13] In the last case above, $e^*(a_t^i, g_{t+1})$ is decreasing in a_t^i because higher ability lessens the education cost to acquire the maximum skill level $\bar{h}$. The acceleration of technology growth encourages education investment by making basic skills economically disadvantageous.

It would be plausible to presume that education investment is not advantageous in underdeveloped stages where technology grows sluggishly. That is to say, the fixed cost of child

[12] Siblings may not be born simultaneously. It is assumed thatt when childbirth is sequential, their (identical) atility level is unveiled after the youngest child is born.

[13] In what follows, $f_x(x, y)$ denotes the partial derivative of function f with respect to x. Similarly, $f_{xx}(x, y)$ and $f_{xy}(x, y)$ denote the second and the cross devivative, respectively. As for the notation of e_t^i and A_t^i, note that there is no need to use the superscript j instead of i, because children raised by a same parent are identical.

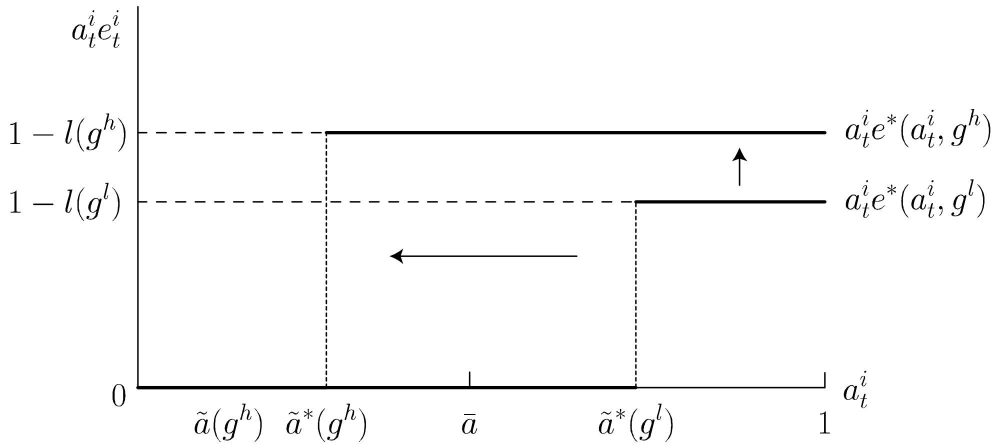

Figure 2. Ability, Education, and Technology: The Perfect Foresight Case. The diagram illustrates how the relationship between ability a_t^i and educational attainment $a_t^i e_t^i$ changes with the growth rate of technology g_{t+1}. The relationships for $g_{t+1} = g^l$ and for $g_{t+1} = g^h$ are shown by $a_t^i e^*(a_t^i, g^l)$ and $a_t^i e^*(a_t^i, g^h)$, respectively, where $g^h > g^l > g^e$. In either case, the education decision is discrete with respect to a_t^i, and a rise in g_{t+1} increases not only the maximum level of education attainment but also the fraction of educating households.

rearing δ is sufficiently small to satisfy[14]

$$l(0) > \delta, \tag{A1}$$

implying that $\delta < 1$. In light of the properties of $l(g_{t+1})$ in Eq. (4), this conditions ensures the existence of a unique value $g^e > 0$ such that $\tilde{a}^*(g^e) = 1$; namely, $g^e \equiv l^{-1}(\delta)$.

To summarize, the function $e^*(a_t^i, g_{t+1})$ exhibits the following properties. First, it is nondecreasing in a_t^i and in g_{t+1}, respectively, because the marginal product of education investment increases with the ability level and because technological progress is biased toward educated workers. Second, the discrete optimal choice is due to the kinked linear production function of human capital, which generates corner solutions. Third and finally, any member of generation $t+1$ does no invest in child education unless g_{t+1} reaches g^e.

Figure 2 illustrates how the relationship between ability a_t^i and educational attainment $a_t^i e_t^i$ changes with the growth rate of technology g_{t+1} in the perfect foresight environment. The relationships for $g_{t+1} = g^l$ and for $g_{t+1} = g^h$ are shown by $a_t^i e^*(a_t^i, g^l)$ and $a_t^i e^*(a_t^i, g^h)$,

[14]Alternatively, if $l(0) < \delta$, then $l(g_{t+1}) < \delta$ for any $g_{t+1} \geq 0$. In light of Eq. (6), this implies that the average fertility rate of uneducated workers never reaches the reproductive level—a result that would not be supported by historical evidence.

respectively, where $g^h > g^l > g^e$. Since $g^l > g^e$, the critical ability level $\tilde{a}^*(g^l)$ is below unity and there is a fraction of households observing $a_t^i > \tilde{a}^*(g^l)$. Their children have educational attainment $a_t^i e_t^i = 1 - l(g^l)$ to acquire the maximum skill level $h_{t+1}^{i,j} = \bar{h}$, whereas the others acquire basic skills $l(g^l)$ with no parental support. As the vertical and the horizontal arrows indicate, a rise in g_{t+1} increases not only the maximum level of education attainment but also the fraction of educating households.

2.3 Macroeconomic Variables

While there is no macroeconomic uncertainty, individuals receive idiosyncratic ability shocks in their childhood. At the beginning of period t, the ability level a_t^i is assigned at random to children born from parent i. Suppose that a_t^i is identically and independently distributed for i and t according to the cumulative distribution function F, where $F(0) = 0$, $F'(a) > 0$ $\forall a \in (0,1)$, and $F(a) = 1$ $\forall a \geq 1$. In other words, a_t^i is household specific and is mutually independent not only within but also across generations. Thus, despite that siblings are identical, there is no genetic inheritance of ability within dynasties.

2.3.1 Population, Human Capital, and Final Output

Substituting Eq. (8) into Eq. (6), the level of working population in period $t+1$ is

$$\begin{aligned} N_{t+1} &= \int_0^{N_t} n_t^i di \\ &= \psi^*(g_{t+1}) \int_0^{N_t} h_t^i di, \end{aligned} \tag{9}$$

where $N_0 > 0$ is historically given and[15]

$$\begin{aligned} \psi^*(g_{t+1}) &\equiv \alpha \int_0^1 \frac{1}{\delta + e^*(a, g_{t+1})} dF(a) \\ &= \alpha \left[\frac{1}{\delta} F(\tilde{a}_t^*) + \int_{\tilde{a}_t^*}^1 \frac{a}{a\delta + 1 - l(g_{t+1})} dF(a) \right], \end{aligned} \tag{10}$$

[15] In Eq. (9), the skill level of of individual i of generation t, born from parent j, is $h_t^i = h_t^{j,i} = h(a_t^j e^*(a_t^j, g_t), g_t)$, which is independent of his/her education spending $e_t^i = e^*(a_t^i, g_{t+1})$. Likewise, in Eq. (11), h_t^i is independent of e_t^i and thus of the skill level of his/her children, $h_{t+1}^{i,j} = h(a_t^i e_t^i, g_{t+1})$.

where $\tilde{a}_t^* = \tilde{a}^*(g_{t+1})$. Since $\tilde{a}^*(g^e) = 1$ and $\tilde{a}^{*\prime}(g_{t+1}) < 0 \ \forall g_{t+1} \geq 0$, $\psi^*(g_{t+1})$ is a continuous function such that $\psi^*(g_{t+1}) > 0$ and $\psi^{*\prime}(g_{t+1}) < 0 \ \forall g_{t+1} \geq 0$ and $\psi^*(g^e) = \alpha/\delta$.[16] The negative derivative is due to the substitution effect: A rise in g_{t+1} induces households to shift their resources from the quantity to the quality of children in period t.

In a similar way to Eq. (9), the evolution of aggregate human capital is

$$\begin{aligned} \int_0^{N_{t+1}} h_{t+1}^j dj &= \int_0^{N_t} n_t^i h_{t+1}^{i,j} di \\ &= \phi^*(g_{t+1}) \int_0^{N_t} h_t^i di, \end{aligned} \tag{11}$$

where the distribution of $h_0^i > 0$ are historically given and the growth factor is defined as

$$\begin{aligned} \phi^*(g_{t+1}) &\equiv \alpha \int_0^1 \frac{h(ae^*(a, g_{t+1}), g_{t+1})}{\delta + e^*(a, g_{t+1})} dF(a) \\ &= \alpha \bar{h} \left[\tilde{a}_t^* F(\tilde{a}_t^*) + \int_{\tilde{a}_t^*}^1 \frac{a}{a\delta + 1 - l(g_{t+1})} dF(a) \right], \end{aligned} \tag{12}$$

where $\tilde{a}_t^* = \tilde{a}^*(g_{t+1})$. $\phi^*(g_{t+1})$ is a continuous function such that $\phi^*(g_{t+1}) > 0$ and $\phi^{*\prime}(g_{t+1}) < 0 \ \forall g_{t+1} > 0$ and $\phi^*(g^e) = \alpha\bar{h}$. The negative derivative is attributed to the erosion effect of technological progress on basic skills.

Let n_t be the ratio of the child to the adult population in period t, which is the average fertility rate in the single-parent economy. Furthermore, let h_t be the average level of human capital in period t. Then Eqs. (9) and (11) yield

$$h_t \equiv \int_0^{N_t} h_t^i di / N_t = \frac{\phi^*(g_t)}{\psi^*(g_t)}; \tag{13}$$

$$n_t \equiv \frac{N_{t+1}}{N_t} = \frac{\psi^*(g_{t+1})\phi^*(g_t)}{\psi^*(g_t)}. \tag{14}$$

A rise in g_t has ambiguous effects on h_t (and thus on n_t through the income effect) because the aforementioned two forces counteract with each other. While the erosion effect depresses the

[16] As follows from Eqs. (8), (10), and (12),

$$\begin{aligned} \psi^{*\prime}(g_{t+1}) &= \alpha \left\{ [1 - l(g_{t+1})] \frac{l'(g_{t+1})}{\delta^2} F'(\tilde{a}_t^*) + \int_{\tilde{a}_t^*}^{\bar{a}} \frac{al'(g_{t+1})}{[a\delta + 1 - l(g_{t+1})]^2} dF(a) \right\}; \\ \phi^{*\prime}(g_{t+1}) &= \alpha\bar{h} \left[\frac{l'(g_{t+1})}{\delta} F(\tilde{a}_t^*) + \int_{\tilde{a}_t^*}^{\bar{a}} \frac{al'(g_{t+1})}{[a\delta + 1 - l(g_{t+1})]^2} dF(a) \right]. \end{aligned}$$

individual skill level h_t^i, the substitution effect in period $t-1$ decreases the working population N_t. A rise in g_{t+1} discourages the average fertility rate n_t through the substitution effect in period t.

As shown by Eq. (6), child rearing incurs αh_t^i efficiency units of labor per worker and the remainder is employed in the production sector. It thus follows from Eqs. (1) and (13) that output per worker in period t is $y_t = (1-\alpha)A_t h_t$, which grows at the rate of g_t in a steady-state equilibrium where g_t is constant.

2.3.2 Technology

In the economy considered herein, the creation of new technology is a by-product of economic activities by adult individuals. Whether working at home or outside, they may come up with new ideas that are utilized for final-good production. Specifically, the variation $A_{t+1} - A_t$ increases proportionally with the aggregate amount of human capital, rather than the volume of employment in the production sector, in period $t+1$.[17] Noting Eq. (11), the evolution of technology is described as

$$\begin{aligned} A_{t+1} &= A_t + B\int_0^{N_{t+1}} h_{t+1}^i di, \\ &= A_t + B\phi^*(g_{t+1})\int_0^{N_t} h_t^i di, \end{aligned} \tag{15}$$

where $B > 0$ measures the degree of learning by doing and $A_0 > 0$ is historically determined.

2.4 The Dynamical System

This section characterizes the evolution of the perfect foresight economy by exploring the dynamic interaction between technology and human capital. As will become apparent, the growth rate of technology converges to a positive level in the long run.

Let x_t be the ratio of technology A_t to aggregate human capital $\int_0^{N_t} h_t^i di$ (hereafter, referred to as the technology-labor ratio). In light of Eq. (15), the growth rate of technology is determined in a self-fulfilling way:

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = \frac{B\phi^*(g_{t+1})}{x_t}, \tag{16}$$

[17] As shown in the Appendix, the alternative formulation would preserve the qualitative properties of the dynamical system at the cost of exposition.

where g_{t+1} in the last term is viewed as the expected value. Eq. (16) implies a one-to-one, adverse relationship between g_{t+1} and x_t:

$$x_t = \frac{B\phi^*(g_{t+1})}{g_{t+1}}, \tag{17}$$

where, as shown earlier, $\phi^*(g^e) = \alpha\bar{h}$, $\phi^*(g_{t+1}) > 0$, and $\phi^{*\prime}(g_{t+1}) < 0$ $\forall g_{t+1} \geq 0$. The growth rate g_{t+1} is associated negatively with the technology-labor ratio x_t through two channels. As Eq. (16) implies, a rise in g_{t+1} results either from a decrease in A_t or from an increase in aggregate human capital in period $t+1$. Owing to the adverse effect on the growth factor $\phi^*(g_{t+1})$, the latter change requires a greater amount of human capital in period t.

It follows from Eqs. (15) and (17) that

$$x_{t+1} = \frac{x_t}{\phi^*(g_{t+1})} + B = B\left(\frac{1}{g_{t+1}} + 1\right), \tag{18}$$

where the second equality is explained by the aforementioned two channels. A rise in g_{t+1} is caused either by a decrease in A_t, which depresses A_{t+1}, or by an increase in aggregate human capital in period $t+1$, which enhances A_{t+1} less than proportionally. Either of them leads to a decline in x_{t+1}, according to the first equality in Eq. (18).

Eqs. (17) and (18) show that x_t is linked with x_{t+1} through g_{t+1}. These two equations therefore constitute a one-dimentional, first-order autonomous system for x_t. Given the initial condition $x_0 > 0$, the system nails down the trajectory of x_t and accordingly those of the other endogenous variables including g_t.

The existence of the steady-state equilibrium $x_{t+1} = x_t$ hinges on the property of the function $\phi^*(g_{t+1})$. Suppose tentatively that the supremum of human capital, $\bar{h}$, is large enough to satisfy

$$\delta \geq l(\alpha\bar{h} - 1), \tag{A2}$$

implying that $\bar{h} > 1$ under Eq. (A1).[18] As Lemma 1 below implies, Eq. (A2) generates a steady-state equilibrium where part of households invests in education.

[18] As will become apparent, Eq. (A2) is a necessary condition for technological progress triggers education investment in the imperfect foresight environment.

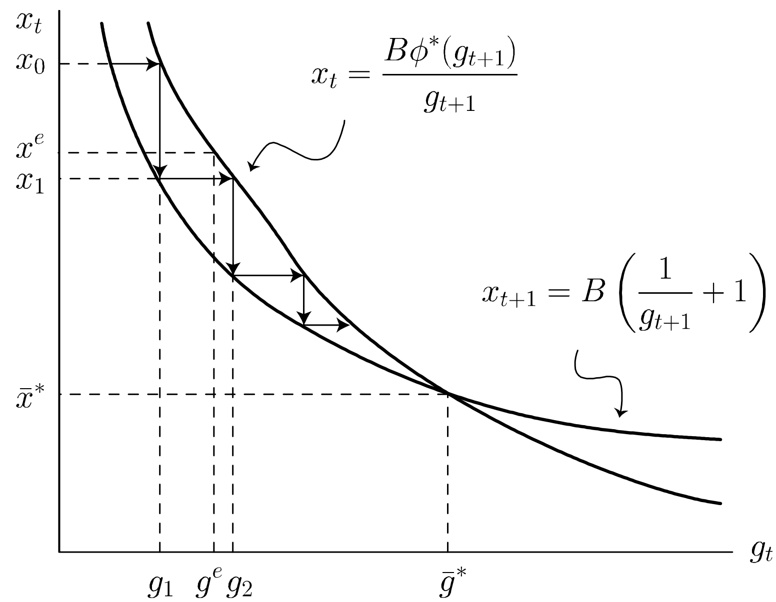

Figure 3. The Dynamical Sytem for the Perfect Foresight Economy. The system exhibits global stability. As long as the initial condition x_0 is positive, the technology-labor ratio, x_t, and the growth rate of technology, g_t, monotonically converge towards their respective steady-state levels, $\bar{x}^*$ and $\bar{g}^*$.

Lemma 1 *Under Eqs. (A1) and (A2), there exists a unique value* $\bar{g}^* \geq g^e$ *such that* $\phi^*(\bar{g}^*) = 1 + \bar{g}^*$ *and*

$$x_{t+1} - x_t \begin{cases} < 0 & \text{if } x_t > \bar{x}^*; \\ = 0 & \text{if } x_t = \bar{x}^*; \\ > 0 & \text{if } x_t < \bar{x}^*, \end{cases}$$

where $\bar{x}^* \equiv B\phi^*(\bar{g}^*)/\bar{g}^* > 0$.

Proof. Eqs. (A2) and (12) reveal that $1 + g^e \leq \alpha\bar{h} = \phi^*(g^e)$, where $g^e \equiv l^{-1}(\delta) > 0$ under Eq. (A1). Since $\phi^*(g_{t+1})$ is decreasing in g_{t+1}, there exists a unique value $\bar{g}^* \geq g^e$ such that

$$1 + g_{t+1} - \phi^*(g_{t+1}) \begin{cases} < 0 & \text{if } g_{t+1} < \bar{g}^*; \\ = 0 & \text{if } g_{t+1} = \bar{g}^*; \\ > 0 & \text{if } g_{t+1} > \bar{g}^*. \end{cases}$$

Hence, the result follows from Eqs. (17) and (18). $\square$

According to these results, Figure 3 graphically represents the dynamical system of the baseline model in the g_t-x_t plane. Observe that the system exhibits global stability. As long as the initial condition x_0 is positive, the technology-labor ratio, x_t, and the growth rate of technology, g_t, monotonically converge towards their respective steady-state levels, $\bar{x}^*$ and $\bar{g}^*$. Provided that $x_0 > \bar{x}^*$, for instance, x_t decreases over time and correspondingly g_t grows in the opposite direction. In this case g_t reaches g^e in the transition process to the steady-state level $\bar{g}^*$—an event that triggers education investment to a limited extent. $\bar{g}^*$ would be lower than g^e if Eq. (A2) was unsatisfied.

3 The Mainline Model

Now it is ready to develop an extended model that plays a central role in the present paper. The economy operates in the same way as before, except that households do not know the ability of children until they are born. While they determine education spending according to unexpected ability shocks, the rigidity constraint binds their ex post decisions somewhat to their initial plans.

3.1 Households

The observation of true ability divides the optimization process of households into two steps.

3.1.1 Ex-Ante Optimization

Suppose that all parents believe that their children will be born with average ability $\bar{a} \in (0, 1)$. Because the fertility decision is controllable at this point of time, ex ante optimization is analogous to that in the baseline model. The only difference is that the common belief on ability brings about homogeneous choices within generations.

Let e_t^p be the planned level of education investment in the beginning of period t. Then in view of Eq. (5), the objective function for parent i in period t is modified to

$$\{n_t^i, e_t^p\} = \arg\max \left\{(1-\alpha)\ln[h_t^i - n_t^i(\delta + e_t^p)] + \alpha \ln[n_t^i h(\bar{a}e_t^p, g_{t+1})]\right\}, \tag{19}$$

subject to $(n_t^i, e_t^p) \geq 0$. The only difference from Eq. (5) is that the expected ability level $\bar{a}$ is now used in place of the true level a_t^i. The counterpart of Eq. (6) is then

$$n_t^i(\delta + e_t^p) = \alpha h_t^i, \tag{20}$$

where $n_t^i > 0$. This shows that as in the baseline model, there is a quantity-quality trade-off at the time of childbirth. The parent makes a balance between the quality and the quality of children so as to keep the child-rearing cost at αh_t^i.

As for the education planning, applying the result of Eq. (8) yields

$$\begin{aligned} e_t^p &= e^*(\bar{a}, g_{t+1}) \\ &= \begin{cases} 0 & \text{if } 0 \leq g_{t+1} < g^p; \\ [1 - l(g_{t+1})]/\bar{a} & \text{if } g_{t+1} \geq g^p, \end{cases} \end{aligned} \tag{21}$$

where g^p is defined as a critical value such that $a^*(g^p) = \bar{a}$ (and therefore $g^p > g^e$).[19] It follows $e^*(\bar{a}, g_{t+1})$ is a nondecreasing, discontinuous function such that $\lim_{g_{t+1} \to g^p - 0} e^*(\bar{a}, g_{t+1}) = 0$ and $e^*(\bar{a}, g^p) = (1 - \bar{a}\delta)/\bar{a}$. The discontinuity at $g_{t+1} = g^p$ is due to the previous assumption that households invest as much resources as possible in education when they are indifferent. Section 3.3 relaxes this assumption and allows the alternative choices of e_t^p between 0 and $(1 - \bar{a}\delta)/\bar{a}$.

3.1.2 Ex-Post Optimization

After childbirth, parent i in period t unexpectedly observes the true ability level, a_t^i, and accordingly reconsider education spending.[20] Now that the quantity of children is taken as given, the actual level of education investment, e_t^i, is

$$e_t^i = \arg\max \left\{ (1 - \alpha) \ln[h_t^i - n_t^i(\delta + e_t^i)] + \alpha \ln h \left(a_t^i e_t^i, g_{t+1} \right) \right\}, \tag{22}$$

subject to $e_t^i \geq 0$. Unlike in the baseline model, n_t^i cannot be reduced to finance the education cost, and thus the quality of children has a trade-off relationship with consumption, not with the quantity of children.[21] While the unexpected ability shock induces the revision of education spending toward the perfect foresight level, it is constrained by the irreversibility of the ex ante decision. Since n_t^i is determined to finance the planned cost, e_t^p, the rigidity

[19] Noting the relationship $l(0) > \delta > \bar{a}\delta$ from Eq. (A1), the properties of $l(g_{t+1})$ in Eq. (4) ensure that $g^p \equiv l^{-1}(\bar{a}\delta) > g^e \equiv l^{-1}(\delta) > 0$. As for the second case in Eq. (21), introducing heterogeneity would not alter the central arguments of the present paper, although it would complicate the exposition of the model.

[20] There is no need to assume that children are born simultaneously. If children are born sequentially, their (identical) atility level is assumed to be unveiled after the birth of the last child.

[21] In the baseline model, parental consumption is constant at $(1 - \alpha) w_t h_t^i$ regardless of the way of child rearing, because the marginal impact of education spending on consumption is neutralized by the associated change in the quantity of children.

of n_t^i binds the ex post education decision somewhat to e_t^p, possibly making it suboptimal. This is the lock-in effect of childbirth on e_t^i.

In view of Eq. (4), the objective function in Eq. (22) is strictly concave as long as e_t^i is sufficiently small. Substitution for n_t^i from Eq. (20) into Eq. (22) reveals that the ex post education decision is summarized as

$$\begin{aligned} e_t^i &= \begin{cases} 0 & \text{if } a_t^i < \tilde{a}_t; \\ e_t^p + \delta(1-\alpha)(1-\tilde{a}_t^*/a_t^i) & \text{if } \tilde{a}_t \leq a_t^i < \hat{a}_t; \\ [1-l(g_{t+1})]/a_t^i & \text{if } a_t^i \geq \hat{a}_t. \end{cases} \\ &\equiv e(a_t^i, g_{t+1}), \end{aligned} \tag{23}$$

where $e_t^p = e^*(\bar{a}, g_{t+1})$, $\tilde{a}_t^* = l(g_{t+1})/\delta$, and[22]

$$\begin{aligned} \tilde{a}_t &= \frac{(1-\alpha)l(g_{t+1})}{e_t^p + (1-\alpha)\delta} \equiv \tilde{a}(g_{t+1}); \\ \hat{a}_t &= \frac{1-\alpha l(g_{t+1})}{e_t^p + (1-\alpha)\delta} \equiv \hat{a}(g_{t+1}). \end{aligned} \tag{24}$$

The second case in Eq. (23) are of particular importance from a comparative perspective with Eq. (8). First, the gap between e_t^i and e_t^p indicates the revision of education spending in response to the unexpected ability shock. Second, the size of the gap, which represents the degree of the lock-in effect, depends in part on the altruism parameter α. Noting that $n_t^i = \alpha h_t^i/(\delta + e_t^p)$ in Eq. (22), a large value of α encourages childbirth for given income level, magnifies the marginal impact of education spending on consumption, and accordingly brings e_t^i near e_t^p. Third, higher ability stimulates education investment because a rise in a_t^i improves the marginal productivity of education investment in skill formation.[23] This implies that weak ability shocks limit the ex-post adjustment of education spending.

By rewriting $e(a_t^i, g_{t+1})$ as $e(a_t^i, g_{t+1}; \alpha)$, Lemma 2 below asserts that in any situation, the lock-in effect of childbirth becomes extreme when individuals are substantially altruistic toward their children.

Lemma 2 *$\lim_{\alpha \to 1} e(a_t^i, g_{t+1}; \alpha) = e^*(\bar{a}, g_{t+1})$ $\forall a_t^i \in (0, 1]$ and $g_{t+1} \geq 0$.*

Proof. See the Appendix. □

[22] Note that $0 < \tilde{a}(g_{t+1}) < \hat{a}(g_{t+1})$ $\forall g_{t+1} \geq 0$ because, in light of Eqs. (4) and (21), $\bar{h} > l(g_{t+1}) > 0$ and $e^*(\bar{a}, g_{t+1}) \geq 0$ $\forall g_{t+1} \geq 0$.

[23] This explains the fact that $\tilde{a}(g_{t+1}) < \hat{a}(g_{t+1})$ for any $g_{t+1} > 0$.

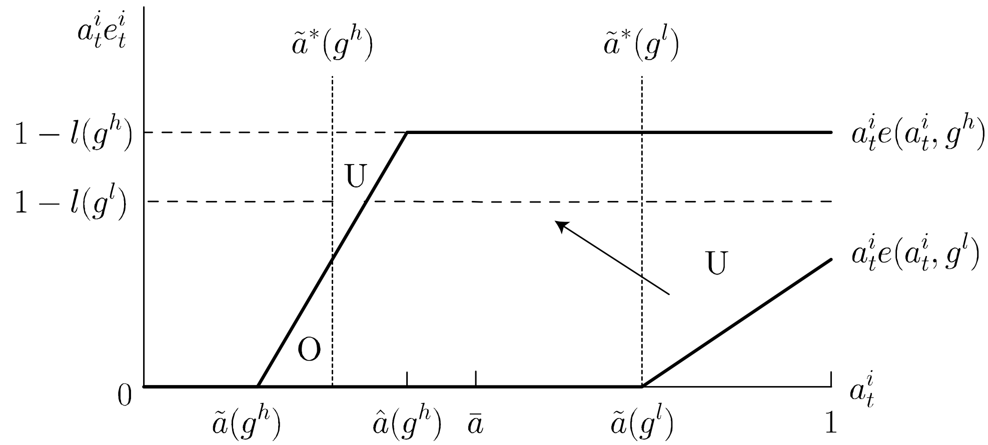

Figure 4. Ability, Education, and Technology: The Imperfect Foresight Case. The diagram illustrates how the relationship between ability a_t^i and educational attainment $a_t^i e_t^i$ changes with the growth rate of technology g_{t+1}. The relationships for $g_{t+1} = g^l$ and for $g_{t+1} = g^h$ are shown by $a_t^i e(a_t^i, g^l)$ and $a_t^i e(a_t^i, g^h)$, respectively, where $g^h > g^p > g^l > g^e$. As the slanting arrow indicates, a rise in g_{t+1} expand the fraction of educating households and their education spending. The two regions labeled U—the one is a trapezoid and the other is a triangle—indicate under-investment in education, whereas the region O indicates over-investment.

Figure 4 depicts how the relationship between ability a_t^i and educational attainment $a_t^i e_t^i$ changes with the growth rate of technology g_{t+1} in the imperfect foresight environment. The relationships for $g_{t+1} = g^l$ and for $g_{t+1} = g^h$ are shown by $a_t^i e(a_t^i, g^l)$ and $a_t^i e(a_t^i, g^h)$, respectively, where $g^h > g^p > g^l > g^e$. The irreversibility of the fertility decision makes some properties distinctive from those in Figure 2. They are addressed in the following section.

3.2 Macroeconomic Variables

Aggregate variables are expressed in the same way as in the baseline model, by modifying $\phi^*(g_{t+1})$ and $\psi^*(g_{t+1})$ in Eqs. (12) and (10) in Section 2.3. In light of Eqs. (21) and (23), the counterparts of them are given by

$$\psi(g_{t+1}) \equiv \frac{\alpha}{\delta + e^*(\bar{a}, g_{t+1})}; \tag{25}$$

$$\phi(g_{t+1}) \equiv \psi(g_{t+1}) \int_0^1 h(ae(a, g_{t+1}), g_{t+1}) dF(a), \tag{26}$$

where, unlike in the baseline model, $\psi(g_{t+1})$ is a nonincreasing, discontinuous function such that $\psi(g^p) = \alpha/\delta$ and $\lim_{g_{t+1} \to g^p + 0} \psi(g_{t+1}) = \alpha\bar{a}$. Applying these results to Eqs. (14) and

(13), the average levels of human capital and fertility are expresses as

$$\begin{aligned} n_t &= \frac{\alpha h_t}{\delta + e^*(\bar{a}, g_{t+1})}; \\ h_t &= \int_0^1 h(ae(a, g_t), g_t) dF(a). \end{aligned}$$

The expressions for h_t and for n_t are simplified by the homogeneity of fertility decisions within generations. It then follows that $\lim_{\alpha \to 1} e_t^i = e_t^p$.

These results demonstrate that the aggregate level of human capital supplied to the production sector in period t, H_t, is

$$H_t = h_t N_t \left[1 - \frac{\alpha}{\delta + e^*(\bar{a}, g_{t+1})} \left(\delta + \int_0^1 e(a_t, g_{t+1}) dF(a) \right) \right],$$

where the multiplicative term in the square brackets is viewed as the time cost of child rearing. Then it follows from Eq. (1) that output per worker in period t, y_t, depends on A_t, g_t, and g_{t+1}. It grows at the rate of g_t in a steady-state equilibrium where g_t is constant.

3.3 The Dynamical System

This subsection scrutinizes the basic properties of the dynamical system by dividing the domain of x_t into three. It demonstrates the existence of a steady-state equilibrium that is unique and globally stable. A poverty trap emerges provided that the upper limit of individual human capital, $\bar{h}$, is not sufficiently large.

Replacing $\phi^*(g_{t+1})$ in Eq. (17) with $\phi(g_{t+1})$ from Eq. (26), the evolution of the technology-labor ratio $x_t \equiv A_t / H_t$ is derived from the following two equations:

$$x_t = \frac{B\phi(g_{t+1})}{g_{t+1}}; \tag{27}$$

$$x_{t+1} = B\left(\frac{1}{g_{t+1}} + 1 \right), \tag{28}$$

where x_0 is historically given and the second equation is identical to Eq. (18). As in the baseline model, g_{t+1} is the channel through which x_t is linked with x_{t+1}. The problem peculiar to the present model is the aforementioned discontinuity of $\phi(g_{t+1})$ at $g_{t+1} = g^p$, for which the ex ante optimal solutions exist on the continuous set $[0, (1 - \bar{a}\delta)/\bar{a}]$. The discontinuity generates an interval of x_t on which x_t is not mapped into g_{t+1}. In order to resolve this

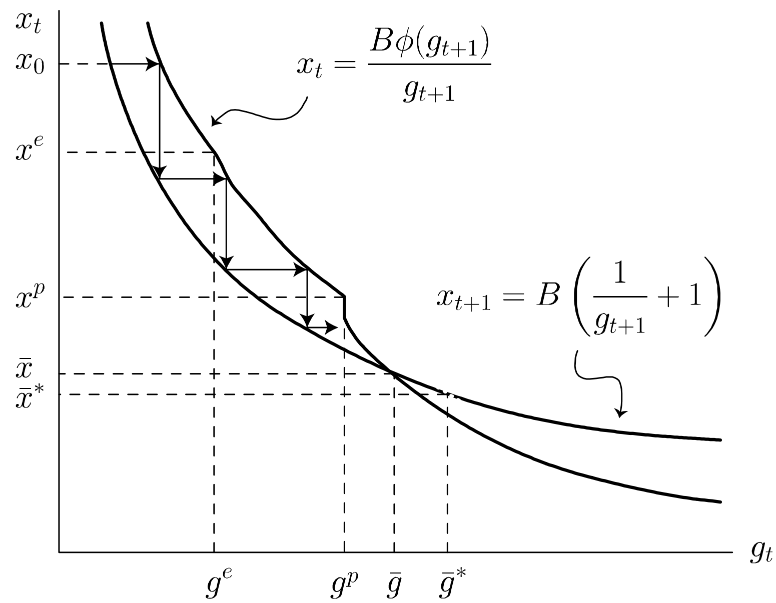

Figure 5. The Dynamical System for the Imperfect Foresight Economy. While the system exhibits global stability as in Figure 3, x_t and g_t respectively converge toward the higher and the lower steady-state levels, $\bar{x}$ and $\bar{g}$. The inferior performance is attributed to the constrained resource allocation between the quantity and the quality of children. This distortion effect would be dominant over the budget constraint if the altruism parameter α is sufficiently large.

technical problem, it is assumed that given $g_{t+1} = g^p$, members of generation t may choose any level of e_t^p between 0 and $(1 - \bar{a}\delta)/\bar{a}$ in the same way.

Figure 5 graphically represents the dynamical system for the imperfect foresight environment. There are two critical levels for education investment, g^e and g^p, which divide the domain of g_{t+1} into three. As will become apparent, $\phi(g_{t+1})$ is a continuous and decreasing function on $\mathbb{R}_{++}/\{g^p\}$. The vertical segment of $\phi(g_{t+1})$ at $g_{t+1} = g^p$ is attributed to the assumption on the education choice introduced above.

3.3.1 Stage I: $x_t > x^e$

Suppose tentatively that $0 \leq g_{t+1} < g^e$ and thus $\tilde{a}^*(g_{t+1}) > 1$. In view of Eq. (21), $e_t^p = 0$ and all members of generation t plan no education investment. Since this yields $\tilde{a}_t = l(g_{t+1})/\delta \equiv \tilde{a}^*(g_{t+1})$ from Eq. (24), Eq. (23) reveals that they pursue the initial plan: $e_t^i = e(a_t^i, g_{t+1}) = 0 \ \forall a_t^i \in (0, 1]$.

With no parental support, children acquire only basic skills and population growth is the single driving force of economic growth. In light of Eq. (4), Eqs. (25) and (26) in Case I are written as $\psi(g_{t+1}) = \alpha/\delta$ and

$$\phi(g_{t+1}) = \alpha\bar{h}\tilde{a}^*(g_{t+1}) = \phi^*(g_{t+1}), \tag{29}$$

where $\phi^*(g^e) = \alpha\bar{h}$ and $\phi^{*\prime}(g_{t+1}) < 0 \; \forall g_{t+1} \geq 0$. Since the dynamical system exhibits the same properties as the ones in the baseline model, Eqs. (A1) and (A2) ensure the nonexistence of steady-state equilibrium (i.e., poverty trap) in Case I.

Corollary 1 *Under Eqs. (A1) and (A2),* $0 < g_{t+1} < g^e$ *and* $x_{t+1} < x_t$ *if* $x_t > x^e$*, where* $x^e \equiv B\phi^*(g^e)/g^e > 0$.

Proof. Eqs. (27) and (29) reveal that $0 < g_{t+1} < g^e$ and $\phi(g_{t+1}) = \phi^*(g_{t+1})$ if $x_t > x^e$, where $g^e \equiv l^{-1}(\delta) > 0$ under Eq. (A1). Then the result follows from the proof of Lemma 1. □

Consistent with Corollary 1, Figure 5 depicts the curve for x_t above the one for x_{t+1} on the interval where $0 < g_{t+1} < g^e$. If Eq. (A2) was unsatisfied, they would intersect with each other on this interval and a globally-stable steady-state equilibrium would occur at the intersection.

3.3.2 Stage II: $x^p < x_t \leq x^e$

Suppose that $g^e \leq g_{t+1} < g^p$ and thus $\bar{a} < \tilde{a}^*(g_{t+1}) \leq 1$, with equality only if $g_{t+1} = g^e$. Then Eq. (21) yields $e_t^p = 0$ as in Stage I, meaning that all households plan not to spend on education. Substitution of this result into Eq. (24) reveals that $\tilde{a}_t = \tilde{a}^*(g_{t+1})$ and $\hat{a}_t = [1 - \alpha l(g_{t+1})]/[(1-\alpha)\delta] > 1$.[24] Thus it follows from Eq. (23) that

$$e_t^i = \begin{cases} 0 & \text{if } a_t^i < \tilde{a}_t^*; \\ (1-\alpha)\delta(1 - \tilde{a}_t^*/a_t^i) & \text{if } \tilde{a}_t^* \leq a_t^i \leq 1, \end{cases} \tag{30}$$

where $\bar{a} < \tilde{a}_t^* \leq 1$. Unlike in Case I, the initial plan is modified by households whose children are unexpectedly and significantly competent. The upward revision is, however, limited by the irreversibility of childbirth, and thus is insufficient for the children to acquire $\bar{h}$ units of human capital. The other households carry out the initial plan, producing unskilled labor. The resulting educational attainment is represented by $a_t^i e(a_t^i, g^l)$ in Figure 4.

[24] The inequality is because $\tilde{a}^*(g_{t+1}) \equiv l(g_{t+1})/\delta \leq 1$ in Case II and $\delta < 1$ under Eq. (A1).

In light of Eq. (4), Eqs. (25) and (26) are now expressed as $\psi(g_{t+1}) = \alpha/\delta$ and

$$\begin{aligned}\phi(g_{t+1}) &= \alpha\bar{h}\left[\tilde{a}_t^* + (1-\alpha)\int_{\tilde{a}_t^*}^{1}(a-\tilde{a}_t^*)dF(a)\right] \\ &\equiv \phi^{II}(g_{t+1}), \end{aligned} \tag{31}$$

where $\phi^{II}(g^e) = \alpha\bar{h}$, $\phi^{II}(g^p) > \alpha\bar{h}\bar{a}$, and $\phi^{II\prime}(g_{t+1}) < 0 \ \forall g_{t+1} \geq 0$.[25] The function $\phi^{II}(g_{t+1})$ is defined on $\mathbb{R}_+$ on the condition that $e_t^p = 0$ regardless of the value of g_{t+1}.

$$\begin{aligned}\phi^*(g_{t+1}) &\equiv \alpha\int_0^1 \frac{h(ae^*(a,g_{t+1}),g_{t+1})}{\delta + e^*(a,g_{t+1})}dF(a) \\ &= \alpha\bar{h}\left[\tilde{a}_t^* F(\tilde{a}_t^*) + \int_{\tilde{a}_t^*}^{1}\frac{a}{a\delta + 1 - l(g_{t+1})}dF(a)\right], \end{aligned} \tag{32}$$

$$\begin{aligned}\psi(g_{t+1}) &= \frac{\alpha}{\bar{a}\delta + 1 - l(g_{t+1})}; \\ \phi(g_{t+1}) &= \bar{h}\bar{a}\psi(g_{t+1})\omega(g_{t+1}) \equiv \phi^{III}(g_{t+1}), \end{aligned} \tag{33}$$

where

$$\begin{aligned}\omega(g_{t+1}) \equiv\ & l(g_{t+1})[(1-\alpha)F(\tilde{a}_t) + \alpha F(\hat{a}_t)] \\ &+[1 - l(g_{t+1}) + (1-\alpha)\bar{a}\delta]\int_{\tilde{a}_t}^{\hat{a}_t}\frac{a}{\bar{a}}dF(a) \\ &+1 - F(\hat{a}_t).\end{aligned}$$

The first property of $\phi^{II}(g_{t+1})$ implies the continuity of $\phi(g_{t+1})$ at $g_{t+1} = g^e$, whereas the latter show that $\phi(g_{t+1})$ remains above $\alpha\bar{h}\bar{a}$ in Case II. In order to analyze the development process toward the advanced stage, suppose tentatively that the supremum of individual human capital, $\bar{h}$, is large enough to satisfy

$$\bar{a}\delta \geq l(\alpha\bar{h}\bar{a} - 1), \tag{A2'}$$

which is a more restrictive assumption than Eq. (A2).

Lemma 3 *Under Eqs. (A1) and (A2′), $g^e \leq g_{t+1} < g^p$ and $x_{t+1} < x_t$ if $x^p < x_t \leq x^e$, where $x^p \equiv B\phi^{II}(g^p)/g^p > 0$.*

[25] The first two properties are obtained from Eq. (31) by noting that $\tilde{a}^*(g^e) = 1 > \tilde{a}^*(g^p) = \bar{a}$ by definition. Regarding the last property, a simple algebra yields $\phi^{II\prime}(g_{t+1}) = \alpha\bar{h}\tilde{a}^{*\prime}(g_{t+1})[(1-\alpha)F(\tilde{a}^*(g_{t+1})) + \alpha]$.

Proof. Eq. (A2') implies that $1 + g^p \leq \alpha\bar{h}\bar{a}$, where $g^p \equiv l^{-1}(\bar{a}\delta) > 0$ under Eq. (A1). On the other hand, Eqs. (27) and (31) imply that $g^e \leq g_{t+1} < g^p$ and $\alpha\bar{h}\bar{a} < \phi(g_{t+1})$ if $x^p < x_t \leq x^e$. Using these results with Eqs. (27) and (28) establishes the lemma. $\square$

Eq. (A2') eliminates the possibility of a stage steady-state equilibrium in Case II, where education investment occurs only among households observing above-average abilities. Consistent with Lemma 3, Figure 5 depicts the curve for x_t above the one for x_{t+1} on the interval where $g^e \leq g_{t+1} < g^p$.

3.3.3 Stage III: $0 < x_t \leq x^p$

For analytical convenience, Stage III is subdivided into two. First, suppose that $g_{t+1} > g^p$ and thus $0 < \tilde{a}^*(g_{t+1}) < \bar{a}$. Then in view of Eq. (21), the planned level of education spending is $e_t^p = [1 - l(g_{t+1})]/\bar{a} > 0$, which is large enough for average ability children to acquire the maximum skill level $\bar{h}$. Substitution of this result into Eq. (24) reveals that $\tilde{a}_t \leq \tilde{a}_t^* < \hat{a}_t < \bar{a}$, where $\tilde{a}_t^* = l(g_{t+1})/\delta$ and[26]

$$\begin{aligned} \tilde{a}_t &= \frac{(1-\alpha)l(g_{t+1})\bar{a}}{1 - l(g_{t+1}) + (1-\alpha)\bar{a}\delta} \equiv \tilde{a}^{III}(g_{t+1}); \\ \hat{a}_t &= \frac{[1 - \alpha l(g_{t+1})]\,\bar{a}}{1 - l(g_{t+1}) + (1-\alpha)\bar{a}\delta} \equiv \hat{a}^{III}(g_{t+1}). \end{aligned} \tag{34}$$

Then it follows from Eq. (23) that the ex post education decision is

$$e_t^i = \begin{cases} 0 & \text{if } a_t^i < \tilde{a}_t; \\ [1 - l(g_{t+1})]/\bar{a} + \delta(1-\alpha)(1 - \tilde{a}_t^*/a_t^i) & \text{if } \tilde{a}_t \leq a_t^i < \hat{a}_t; \\ [1 - l(g_{t+1})]/a_t^i & \text{if } a_t^i \geq \hat{a}_t, \end{cases} \tag{35}$$

where $\tilde{a}_t \leq \tilde{a}_t^* < \hat{a}_t$. Children with $a_t^i \leq \tilde{a}_t$ receive no parental support and become unskilled labor. Their ability levels are so low that, contrary to the initial prospect, education investment is not advantageous at all. Advanced skills are acquired by children with $a_t^i \in (\tilde{a}_t, \hat{a}_t)$. Although they may receive more than e_t^p units of parental support, their skill levels do not reach $\bar{h}$. The most skilled are children with $a_t^i \geq \hat{a}_t$. Unlike in Stage II, they receive sufficient support to acquire $\bar{h}$ units of human capital. The resulting educational attainment is represented by $a_t^i e(a_t^i, g^h)$ in Figure 4.

[26] These quantitative relationships are derived by noting that $l(g_{t+1}) < \bar{a}\delta < \delta$ and $l'(g_{t+1}) < 0 \;\forall g_{t+1} > g^p$.

Using these results with Eq. (4), Eqs. (25) and (26) are now expressed as

$$\begin{aligned}\psi(g_{t+1}) &= \frac{\alpha}{\bar{a}\delta + 1 - l(g_{t+1})};\\ \phi(g_{t+1}) &= \bar{h}\bar{a}\psi(g_{t+1})\omega(g_{t+1}) \equiv \phi^{III}(g_{t+1}),\end{aligned} \tag{36}$$

where

$$\begin{aligned}\omega(g_{t+1}) \equiv\; & l(g_{t+1})[(1-\alpha)F(\tilde{a}_t) + \alpha F(\hat{a}_t)]\\ & +[1 - l(g_{t+1}) + (1-\alpha)\bar{a}\delta] \int_{\tilde{a}_t}^{\hat{a}_t} \frac{a}{\bar{a}} dF(a)\\ & +1 - F(\hat{a}_t).\end{aligned}$$

The function $\phi^{III}(g_{t+1})$ is defined on $\mathbb{R}_+$ on the condition that $e_t^p = [1 - l(g_{t+1})]/\bar{a} > 0$ for any $g_{t+1} \geq 0$.

Lemma 4 *Under Eqs. (A1),* $\phi^{III}(g^p) < \alpha\bar{h}\bar{a}$ *and* $\phi^{III\prime}(g_{t+1}) < 0$ $\forall g_{t+1} \geq g^p$.

Proof. See the Appendix. □

Lemma 4 implies that unlike in Case II, Eq. (A2') does not eliminate the possibility of a steady-state equilibrium where $g_t > g^p$.

Second, suppose that $g_{t+1} = g^p$ and thus $\tilde{a}^*(g_{t+1}) = \bar{a}$. As assumed earlier, members of generation t may choose any level of e_t^p between 0 and $(1 - \bar{a}\delta)/\bar{a}$ in the same way. Since $\phi^{II}(g^p)$ and $\phi^{III}(g^p)$ are based on the condition that $e_t^p = 0$ and $e_t^p = (1-\bar{a}\delta)/\bar{a}$, respectively, it follows that ϕ maps the value g^p into the set $[\phi^{III}(g^p), \phi^{II}(g^p)]$. In other words, by using Eq. (17),

$$g_{t+1} = g^p \quad \text{if } x_t \in [B\phi^{III}(g^p)/g^p, x^p], \tag{37}$$

where $x^p \equiv B\phi^{II}(g^p)/g^p$ and $\phi^{III}(g^p) < \alpha\bar{a}\bar{h} < \phi^{II}(g^p)$ from Eq. (31) and Lemma 4.

The main results for Stage III are summarized as follows.

Lemma 5 *Under Eqs. (A1) and (A2′),* $g_{t+1} \geq g^p$ *if* $0 < x_t \leq x^p$, *and*

$$x_{t+1} - x_t \begin{cases} < 0 & \text{if } \bar{x} < x_t \leq x^p;\\ = 0 & \text{if } x_t = \bar{x};\\ > 0 & \text{if } 0 < x_t < \bar{x}, \end{cases} \tag{38}$$

where $\bar{x} \in (0, x^p)$ *is a unique value.*

Proof. See the Appendix. □

Consistent with Lemma 5, Figure 5 depicts the curve for x_t crossing the one for x_{t+1} from above at $\bar{g} > g^p$, where a unique steady-state equilibrium arises. Such a case requires that $\bar{h}$ is large enough to have $\phi^{III}(g^p) > 1 + g^p$.[27] Alternatively, the intersection may occur at $g_t = g^p$ because Eq. (A2') is not a sufficient condition to eliminate this possibility. In this case the steady-state value of x_t is $\bar{x} = B(1/g^p + 1)$.

4 The Stages of Economic Development

This section describes the stages of economic development in comparison with the baseline model. As will become clear, the rigidity constraint is the key factor for the comparative analysis. It constrains the ex post decisions of households and thereby generates under- or over-investment in education at individual level. The resulting resource allocation between the quantity and the quality of children possibly retards the accumulation of aggregate human capital, technological progress, and growth in output per worker.

Section 3.3 demonstrates that the dynamical system is characterized by global stability. The technology-labor ratio x_t may either increase or decrease monotonically toward the steady-state level $\bar{x}$, depending on the initial condition. In order to make the stages of economic development comprehensive, the analysis below focuses on the economy starting out with a technology-labor ratio such that $x_0 > x^e$. Given the initial condition, x_t monotonically grows over time and the economy goes through the three development stages presented above.

4.1 Stage I: Population-Driven Economic Growth

The economy initially develops over Stage I, where $x_t > x^e$ and thus $0 < g_{t+1} < g^e$. This is the underdeveloped stage in which economic growth is driven by population expansion. As shown in Section 3.3.1, technology grows so sluggishly that education investment is not advantageous for any households at any point of time. As a result, $e_t^i = e_t^p = 0$ for $a_t^i \in (0, 1]$. Since this is also the case in the perfect foresight environment, unexpected ability shocks has

[27] The proof of Lemma 4 implies that $\phi^{III}(g^p)$ becomes infinitely large as $\bar{h}$ approaches inifinity.

no influence on the growth process over Stage I, leading to $\phi(g_{t+1}) = \phi^*(g_{t+1})\ \forall g_{t+1} \in (0, g^e)$. The rigidity constraint is unbinding and no lock-in effect arises in this stage. All the resources reserved for child rearing, αh_t^i, are devoted to the quantity of children, and the resulting population expansion fuels technological progress.

4.2 Stage II: Under-Investment in Education

Monotonic growth in x_t leads the economy to Stage II, where $x^p < x_t \leq x^e$ and thus $g^e \leq g_{t+1} < g^p$. This intermediate stage of development encompasses *under*-investment in education among part of households[28] As in Stage I, households give birth to children in prospect of no education spending. Contrary to the prospect, education investment turns out to be advantageous for those whose children were considerably underestimated (i.e., $a_t^i > \tilde{a}_t \geq \bar{a}$).

While the positive ability shocks induce upward revisions of education spending toward the perfect foresight levels, the rigidity of n_t^i binds the ex post decisions somewhat to the initial plan. Since n_t^i is large compared to their income levels, the upward revisions entails a great sacrifice of consumption. As a consequence, those households cannot afford sufficient education support for their children.[29] The resulting educational attainment in relation to the ability level is represented by $a_t^i e(a_t^i, g^l)$ in Figure 4. Since the diagram depicts $a_t^i e(a_t^i, g^h)$ below the perfect foresight level, $1 - l(g^l)$, on $(\tilde{a}_t, 1]$, the trapezoid area U indicates the degree of under-investment in education. Households observing lower ability have no incentive to revise the education plan, because the initial plan is consistent with the decision that would be made under the perfect foresight environment.

Owing to the rigidity constraint, the accumulation of aggregate human capital is under the influence of unexpected ability shocks through two opposing channels. The first channel concerns the efficiency of resource allocation. Eq. (7) asserts that households with perfect foresight maximizes children's human capital, $n_t^i h_{t+1}^{i,j}$, on the condition that the budget for child rearing is within αh_t^i. By comparing Eqs. (8) and (30), this indicates the possibility that

[28] Through out the present paper, the term 'under-investment' is defined in terms of the individual level of human captial compared to the perfect foresight level.

[29] Since the economy has no capital markets, no one can take out loans for education. The authors would like to thank Prof. Masaru Inaba for pointing this out.

unexpected ability shocks bring about inefficient resource allocation between the quantity and the quality of children. This adverse effect is referred to as the *distortion effect* in what follows.

The second channel concerns the resource constraint. As follows from Eqs. (20) and (30), parental labor devoted to child rearing is $n_t^i(\delta + e_t^i) \geq \alpha h_t^i$, with equality only if $e_t^i = 0$. In other words, unexpected ability shocks expands the budget for child rearing by stimulating education investment. This positive effect is referred to as the *budget effect*. In light of Lemma 2, the distortion effect is dominant when the altruism parameter α is sufficiently large. This is the intuitive explanation for Lemma 6 below, in which $\phi(g_{t+1})$ and $\phi^*(g_{t+1})$ are respectively rewritten as $\phi(g_{t+1}; \alpha)$ and $\phi^*(g_{t+1}; \alpha)$.

Lemma 6 *Under Eq.* (*A*1), $\lim_{\alpha \to 1}[\phi(g_{t+1}; \alpha) - \phi^*(g_{t+1}; \alpha)] < 0 \; \forall g_{t+1} \in (g^e, g^p)$.

Proof. See the Appendix. □

The lemma asserts the possibility that unexpected ability shocks lower the growth rate of human capital and thereby retard technological progress over Stage II.

4.3 Stage III: The Emergence of Over-Investment

The economy eventually enters Stage II, where $0 < x_t \leq x^p$ and thus $g_{t+1} \geq g^p$. This advanced stage is characterized by *over*- as well as *under*-investment in education.[30] Unlike in the previous stages, households give birth to children in prospect of education investment, so that they can acquire $\bar{h}$ units of human capital. Their family sizes, which are small compared to their income levels, leave the households affluent resources when true abilities are observed. As represented by $a_t^i e(a_t^i, g^h)$ in Figure 4, the resulting educational attainment exhibits various patterns depending on the ability level.

If $a_t^i < \tilde{a}_t$, the rigidity constraint is not binding the ex post education decisions. Since $\tilde{a}_t \leq \tilde{a}_t^*$, the ability shocks induce downward revisions of education spending to the perfect foresight level, which is zero. Those negative shocks are so large in magnitude that, contrary to the initial prospect, education investment is not advantageous at all. Accordingly, $0 = e^*(a_t^i, g_{t+1}) = e_t^i < e_t^p$ and thus $n_t^i(\delta + e_t^i) < \alpha h_t^i$.

[30] Through out the present paper, the term 'under-investment' is defined in terms of the individual level of human captial compared to the perfect foresight level.

As a feature of Stage III, over-investment in education occurs if $\tilde{a}_t < a_t^i < \tilde{a}_t^*$. While the ability shocks induce downward revisions of education spending as in the case above, their impacts are not strong enough to completely dissuade education investment against the initial plan. Since small-scale households have affluent resources at hand, spending some of them on education does not entail a great sacrifice of consumption. Accordingly, $0 = e^*(a_t^i, g_{t+1}) < e_t^i < e_t^p$ and thus $n_t^i(\delta + e_t^i) < \alpha h_t^i$. Figure 4 depicts $a_t^i e(a_t^i, g^h)$ above the perfect foresight level on $(\tilde{a}_t, \tilde{a}_t^*)$, and the triangle area O represents the degree of over-investment.

By contrast, under-investment in education occurs if $\tilde{a}_t^* < a_t^i < \hat{a}_t$. Unlike in the previous cases, the ability shocks induce upward revisions of education spending to the perfect foresight level, which is greater than e_t^p. This is because households find that skill acquisition by their children is much more costly than expected.[31] Since the weak ability shocks limit the upward revisions to some extent, children do not receive sufficient parental support to acquire $\bar{h}$ units of human capital. Accordingly, $0 < e_t^p < e_t^i < e^*(a_t^i, g_{t+1})$ and thus $n_t^i(\delta + e_t^i) > \alpha h_t^i$. Figure 4 depicts $a_t^i e(a_t^i, g^h)$ below the perfect foresight level on $(\tilde{a}_t^*, \tilde{a}_t)$, and the triangle area U represents the degree of under-investment.

If $\hat{a}_t \leq a_t^i < \bar{a}$, the rigidity constraint is not binding the ex post education decisions. While the ability shocks induce upward revisions of education spending as in the case above, their impacts are sufficiently strong to allow a greater degree of the revisions. Furthermore, the relatively small gap between a_t^i and $\bar{a}$ implies that the planned budget e_t^p nearly covers the cost to acquire $\bar{h}$ units of human capital. Accordingly, $0 < e_t^p < e_t^i = e^*(a_t^i, g_{t+1})$ and thus $n_t^i(\delta + e_t^i) > \alpha h_t^i$.

Finally, if $a_t^i > \bar{a}_t$, the rigidity constraint is not binding as in the case above. Since skill acquisition is not as much costly as expected, children acquire $\bar{h}$ units of human capital with less than e_t^p units of parental support. Accordingly, $0 < e_t^i = e^*(a_t^i, g_{t+1}) < e_t^p$ and thus $n_t^i(\delta + e_t^i) < \alpha h_t^i$.

Turning to the discussion on growth performance, the results above reveal that $n_t^i(\delta + e_t^i) \gtreqless \alpha h_t^i$, depending on the direction of the ex-post adjustment in education. This indicates the possibility that unexpected ability shocks promote human capital accumulation through the

[31] This means that $[1 - l(g_{t+1})]/a_t^i > e_t^p = [1 - l(g_{t+1})]/\bar{a}$ if $\tilde{a}_t^* < a_t^i < \hat{a}_t$, where $\hat{a}_t < \bar{a}$ in Stage III.

budget effect. Nevertheless, provided that α is sufficiently large, the distortion effect becomes dominant as in Stage II, and thus Lemma 7 below follows similarly.

Lemma 7 *Under Eq.* (A1), $\lim_{\alpha\to 1}[\phi^*(g_{t+1};\alpha) - \phi(g_{t+1};\alpha)] > 0 \ \forall g_{t+1} > g^p$.

Proof. See the Appendix. □

The lemma asserts the possibility that unexpected ability shocks delay technological progress over Stage III. The discussions so far derive the main result of the present paper presented below.

Proposition 1 *Under Eqs.* (A1) *and* (A2′), *suppose that the altruistic parameter* α *is sufficiently large. Then, the economy converges toward a steady-state equilibrium with a lower growth rate of technology compared to the perfect foresight level.*

5 Concluding Remarks

This research has elucidated the role of irreversible childbirth in economic growth. In the stage of underdevelopment, unexpected ability shocks on newborn children induce some households to revise their education plans upward. Their ex post reactions, however, may be constrained by the predetermined family size, which squeezes the household budget as a sunk cost. Fertility decline in the growth process therefore lessens the sunk cost and promotes education investment, easing borrowing constraints on children who aim to raise the money for education. In the more developed stage, the rigidity constraint is binding in the opposite direction. That is, households are unable to have children additionally even if they do not invest in education against the initial plan. For the second best, households tend to invest their resources on the existing children, leading to over-investment in education.

These results derive two notable policy implications. First, a policy reform that discourages childbirth lessens the sunk cost of child rearing and promotes education investment in the underdeveloped stage. Technological progress fueled by the increased share of skilled workers ultimately alters households' (ex ante) stances toward education and brings about fertility decline. Second, raising child care subsidies in the developed stage will mitigate over-investment in education and accelerate growth in the working population. The resulting expansion of the government budget improves the welfare of future generations at the cost of the elderly-related service in the initial reform period.

While the central thesis of the present research is intuitive, the developed theory builds on a number of simplifying assumptions. A more general theory would allow for the following aspects. First, parental human capital or ability would be one of the key factors in the formation of the expectation of children's ability. In this case, ability shocks after childbirth would not be totally unexpected. Second, parents would more or less make a coordination of private education policy with their children, which would moderate under- and over-investment in education. Third, fertility adjustment to prepare for future education expenses should be bounded below (that is, the minimum number of children is 1) because in reality investment in the quantity of children is a discrete choice. Unbounded fertility adjustment would make over-investment in education more likely to occur in the developed stage. Lastly, it is desirable to examine the possibility of Pareto improvement in the presence of capital markets, through which the government can transfer resources across nonadjoining generations. These issues should be addressed in future research.

Appendix: Technical Discussions

Proof of Lemma 2. Suppose that $0 \leq g_{t+1} < g^p$. Then $e^*(\bar{a}, g_{t+1}) = 0$ from Eq. (21), and Sections 3.3.1 and 3.3.2 reveal that for any $a_t^i \in (0, 1]$,

$$0 \leq e(a_t^i, g_{t+1}; \alpha) \leq (1 - \alpha)[\delta - l(g_{t+1})].$$

Hence the result follows. Next, suppose that $g_{t+1} \geq g^p$. Then $e^*(\bar{a}, g_{t+1}) = [1 - l(g_{t+1})]/\bar{a}$ from Eq. (21), and Section 3.3.3 reveals that for any $a_t^i \in [\tilde{a}_t, 1]$,

$$E(a_t^i; \alpha) \leq e(a_t^i, g_{t+1}; \alpha) \leq E(\hat{a}_t; \alpha),$$

where $E(a; \alpha) \equiv e^*(\bar{a}, g_{t+1}) + (1 - \alpha)[\delta - l(g_{t+1})/a]$. As α goes to unity, $\tilde{a}_t$ approaches zero whereas $\hat{a}_t$ approaches the constant value $\bar{a}$. Hence the result follows. $\square$

Proof of Lemma 4. Noting that $l(g^p) = \bar{a}\delta < 1$ under Eq. (A1), Eq. (34) yields $\hat{a}^{III}(g^p) = \bar{a}$ and

$$\tilde{a}^{III}(g^p) = \frac{(1-\alpha)\bar{a}\delta}{1-\alpha\bar{a}\delta}\bar{a} \equiv \theta\bar{a},$$

where $\theta \in (0,1)$. It then follows from Eq. (36) that

$$\phi^{III}(g^p) = \alpha\bar{h}\bar{a}[1 - F(\bar{a}) + \bar{a}\delta F(\theta\bar{a}) + \Delta_1],$$

where $\bar{a}\delta < 1$ and

$$\Delta_1 \equiv \int_{\theta\bar{a}}^{\bar{a}} \left[\frac{a}{\bar{a}}(1-\alpha\bar{a}\delta) + \alpha\bar{a}\delta\right] dF(a) < \int_{\theta\bar{a}}^{\bar{a}} dF(a).$$

Hence $\phi^{III}(g^p) < \alpha\bar{h}\bar{a}$. Now use Eq. (36) again to find that, $\forall g_{t+1} \geq 0$,

$$\phi^{III\prime}(g_{t+1}) = \frac{\alpha\bar{h}\bar{a}l'(g_{t+1})}{\bar{a}\delta + 1 - l(g_{t+1})}\left[\frac{\Delta_2 + \Delta_3}{\bar{a}\delta + 1 - l(g_{t+1})} + \Delta_4\right],$$

where $l'(g_{t+1}) < 0$, $l(g_{t+1}) < 1$, and

$$\begin{aligned}
\Delta_2 &\equiv l(g_{t+1})[(1-\alpha)F(\tilde{a}) + \alpha F(\hat{a})] + 1 - F(\hat{a}); \\
\Delta_3 &\equiv -\alpha\delta\int_{\tilde{a}}^{\hat{a}} a dF(a); \\
\Delta_4 &\equiv \alpha F(\hat{a}) + (1-\alpha)F(\tilde{a}); \\
\tilde{a} &= \tilde{a}^{III}(g_{t+1}); \qquad \hat{a} = \hat{a}^{III}(g_{t+1}).
\end{aligned}$$

It follows that if $g_{t+1} \geq g^p$, then $\tilde{a} < \hat{a} \leq \bar{a}$, $\Delta_3 \geq -\alpha\bar{a}\delta F(\hat{a})$, and thus $\phi^{III\prime}(g_{t+1}) < 0$. $\square$

Proof of Lemma 5. As follows from Eq. (17), Eq. (36), and Lemma 4, $g_{t+1} > g^p$ if $0 < x_t < B\phi^{III}(g^p)/g^p$, where $g^p \equiv l^{-1}(\bar{a}\delta) > 0$ under Eq. (A1). Thus in light of Eq. (37), $g_{t+1} \geq g^p$ if $0 < x_t \leq x^p$. The sign of $x_{t+1} - x_t$ is equal to that of $1 + g_{t+1} - \phi(g_{t+1})$, as follows from Eqs. (27) and (28). Now, there are three points to be noted. First, Eq. (A2') yields $1 + g^p < \alpha\bar{a}\bar{h}$. Second, Eq. (37) implies that $\phi(g^p)$ corresponds to the set $[\phi^{III}(g^p), \phi^{II}(g^p)]$, where $\phi^{III}(g^p) < \alpha\bar{a}\bar{h} < \phi^{II}(g^p)$. Third, $\phi^{III\prime}(g_{t+1}) < 0$ and $\phi(g_{t+1}) = \phi^{III}(g_{t+1})\ \forall g_{t+1} > g^p$. These ensure the existence of a unique value $\bar{g} \geq g^p$ such that

$$1 + g_{t+1} - \phi(g_{t+1}) \begin{cases} < 0 & \text{if } g^p < g_{t+1} < \bar{g}; \\ = 0 & \text{if } g_{t+1} = \bar{g}; \\ > 0 & \text{if } g_{t+1} > \bar{g}, \end{cases}$$

If $1+g^p \geq \phi^{III}(g^p)$, then $\bar{g} = g^p$ and Eq. (38) is satisfied only by $\bar{x} = B(1+g^p)/g^p \in (0, x^p)$, where $x^p \equiv B\phi^{II}(g^p)/g^p$. Alternatively, if $1+g^p < \phi^{III}(g^p)$, then $\bar{g} > g^p$ and Eq. (38) is satisfied only by $\bar{x} = B\phi^{III}(\bar{g})/\bar{g} \in (0, B\phi^{III}(g^p)/g^p)$. □

Proof of Lemmas 6 and 7. Suppose that $g_{t+1} > g^e$, where $g^e \equiv l^{-1}(\delta) > 0$ under Eq. (A1). Since Lemma 2 asserts that $e(a_t^i, g_{t+1}; \alpha)$ approaches $e^*(\bar{a}, g_{t+1})$ as α goes to unity, the spending on child rearing, $n_t^i(\delta + e_t^i)$, approaches αh_t^i as α goes to unity in the imperfect foresight environment, where $e_t^i = e(a_t^i, g_{t+1}; \alpha)$ and $n_t^i = \alpha h_t^i/[\delta + e^*(\bar{a}, g_{t+1})]$ from Eqs. (20) and (21). In addition, Eqs. (7) and (8) reveal that for any $a_t^i \in (0,1]$, the education choice in the perfect foresight environment, $e^*(a_t^i, g_{t+1})$, maximizes $n_t^i h_{t+1}^{i,j}$ under the constraint $n_t^i(\delta + e_t^i) = \alpha h_t^i > 0$. Lastly, in light of Eqs. (8) and (21), there is a continuous set $I \subset (0,1]$ such that $e^*(\bar{a}, g_{t+1}) \neq e^*(a_t^i, g_{t+1})$ $\forall a_t^i \in I$. These results indicate that the lack of perfect foresight strictly decreases aggregate human capital $\int_0^{N_t} n_t^i h_{t+1}^{i,j} di$ for given $h_t^i > 0$ and $g_{t+1} > g^p$. Hence the lemma follows from Eqs. (12) and (26). □

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