A unified framework for optimal taxation with undiversifiable risk*

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March 30, 2017

Abstract

This paper considers a model of linear capital taxation for an economy where capital and labor income are subject to idiosyncratic uninsurable risk. To keep the model tractable, we assume that investment decisions are made before uncertainty is realized, so that the realization of the capital and labor income shocks only affects current consumption. In this setting, we are able to jointly analize capital and labor income risk and derive analitical results regarding the optimal taxation of capital. We find that the optimal capital tax is positive in the long run if there is only capital income risk. The reason for this is that the capital tax provides insurance against capital income risk. Furthermore, for high levels of risk, increasing the capital tax may actually induce capital accumulation. On the other hand, if there is only labor income risk the optimal capital tax is zero. The sign of the optimal tax can only be negative if the two types of risk are negatively correlated and labor income risk is large enough.

Keywords: Fiscal Policy, Optimal Taxation, Incidence
JEL Classification: E62, H21, H22

*A previous version of this paper with only capital income risk was circulated with the title "Optimal Taxation with Idiosyncratic Capital Income Risk". We are gratefull to Isabel Correia, Mikhail Golosov, Leonor Modesto, Pedro Teles, and Ivan Werning for helpful comments. Financial support from the ADEMU (H2020, No 649396) project and Fundação para a Ciência e Tecnologia are gratefully acknowledged.

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1 Introduction

This model aims to study individual idiosyncratic risk in a simple framework with both labor income risk and capital income risk. The two types of risk have been studied separately in different models. However, there is no obvious way to bring them together in a tractable framework. This paper will be able to do so by making the simplifying assumption that investment decisions are made before uncertainty is realized. This means that in every period all the ex-ante identical agents will make the same investment decision, and their individual shocks will only affect the value of consumption in the current period. We are thus able to keep the representative agent nature of the model, although within each period the agents will have different consumption levels.

Optimal taxes with idiosyncratic labor income risk have been studied by Aiyagari (1994, 1995), who uses a Bewley model to find that it is optimal to tax capital income in the long run because idiosyncratic risk tends to increase capital accumulation due to precautionary savings. On the other hand, Panousi and Reis (2016) use a model based on Angeletos (2007) to derive the optimal capital income tax when there is capital income risk and find that it may be optimal to tax or subsidize capital, depending on whether aggregate capital lies above or below its first best level. However, the two models are not quite comparable since Aiyagari (1994, 1995) needs exogenous borrowing constraints which would make the model in Angeletos (2007) intractable. Furthermore, dynamic models with idiosyncratic uncertainty are in general very hard to solve analytically because agents are repeatedly hit by different shocks.

We propose an alternative modelling strategy for decisions which makes these kinds of models surprisingly simple, and allows us to incorporate both kinds of idiosyncratic shocks in one single model. The assumption is that agents make their investment decisions before the realization of uncertainty for the current period. Although this is a simplifying assumption, the opposite assumption that is usually made, that agents optimize after they have all the information relative to the current shock, is also extreme and probably reality lies somewhere in between. The advantage of our assumption is that the decision for the state variable is made before uncertainty is realized, and as a consequence the state variable will be the same for all agents, even though they get hit by different shocks. We thus need to keep track of a single state variable, even though agents will get hit by a different shock in each period, which makes individual consumption random. Investment will still be affected by the existence of uncertainty, although it does not depend on its realization. Notice also that as the variance of the idiosyncratic shock goes to zero this model converges exactly to the traditional Ramsey model, where zero taxes on capital are optimal.

One concern with our simplified framework would be that the dynamics in capital might not be able to replicate what happens under incomplete markets, since the investment decision is made before the realization of uncertainty. However, we find that this is not the case. In particular, in our model capital investment decreases when its riskiness increases, as happens in Panousi and Reis (2016). More interestingly, we are able to generate non-monotonicity of capital with respect to the tax rate, as Angeletos and Panousi (2007). In addition to that, the new framework allows us to explore the effect of the covariance between labor and capital income risk. For example, if labor income risk is negatively correlated with capital income risk, then increasing capital riskiness may increase the level of capital.

In our model if there are only labor income shocks the optimal capital income tax is zero. This is different from the Aiyagari (1995) paper, where the optimal capital income tax is positive if there is labor income risk. This is not surprising, however, since the Aiyagari model has borrowing constraints that generate precautionary savings which are absent in our model. Since there is no over-saving for precautionary reasons, there is no need to use capital taxation.

On the other hand, if there is only capital income risk the optimal tax is always positive in our model. Furthermore the optimal tax seems to be increasing in the level of risk. These results are similar to Panousi and Reis (2016) for high levels of capital income risk, where the tax was positive and increasing with risk, but in Panousi and Reis it was also possible to get optimal capital subsidies for low risk levels in order to induce higher risk taking. In our model, the only way we are able to generate optimal capital subsidies is if the labor and capital shocks are negatively correlated and the labor shock is sufficiently strong. The main reason for taxation in this model is insurance, since taxation reduces the randomness in income. In addition to that, for high levels of risk, increasing the capital tax may actually increase the level of capital since it reduces its riskiness.

The paper proceeds as follows. Section 2 sets up the model and derives the competitive equilibrium for the economy. Section 3 derives the optimal policy plan for the benevolent government. Section 4 concludes. The appendix shows the specification of a numerical calibration that is used to highlight the results of the paper.

2 Model Setup

This section sets up a model of linear capital income taxation. Agents are subject to an idiosyncratic capital income shock. They supply labor inelastically and lump sum taxes are allowed. Notice that in this model there is no need to use capital taxation in order to collect revenue, since lump sum taxes are allowed, so any positive (or negative) taxes that arise are purely so for efficiency reasons.

2.1 Households

There is a continuum of measure one of ex-ante identical households who discount the future at rate β. Households derive utility from consumption c_t, and supply one unit of labor inelastically. Each household's lifetime utility is thus given by

$$\sum_{t=0}^{\infty} \beta^t E\left[u(c_t)\right],$$

where u is increasing and concave.

Each period, households receive income w_t from their labor plus a labor income shock ϵ_t, and the after tax return on their capital investment, where r_t denotes the deterministic return on capital, η_t denotes each household's individual shock to this return, and δ is the depreciation rate. Households can spend their income in consumption or invest in capital, and they have to pay lump sum taxes. The shocks η_t and ϵ_t may be correlated but they follow a distribution that is i.i.d. across time and has mean zero. The households' budget constraint for each period is given by

$$c_t + k_{t+1} + T_t = w_t + \epsilon_t + (1-\tau_t)(r_t + \eta_t)k_t + (1-\delta)k_t. \tag{1}$$

The crucial assumption in this paper is that households choose k_{t+1} before the uncertainty in η_t and ϵ_t is realized. This implies that all households (that by assumption have the same initial stock of capital) will make the same investment decisions over time, which in fact allows us to maintain the representative agent of the model by limiting the effect of the idiosyncratic shock realization to current consumption. Households maximize expected utility subject to their budget constraints. Associating the constraint for period t and realization (η_t, ϵ_t) with multiplier $\beta^t \lambda_t(\eta_t, \epsilon_t) f(\eta_t, \epsilon_t)$, we have the following Lagrangian

$$L = \sum_{t=0}^{\infty} \beta^t E\left\{ u\left[c_t(\eta_t, \epsilon_t)\right] - \lambda_t(\eta_t, \epsilon_t) \left[\begin{array}{c} c_t(\eta_t, \epsilon_t) + k_{t+1} + T_t - w_t + \epsilon_t \\ -(1-\tau_t)(r_t + \eta_t)k_t - (1-\delta)k_t \end{array} \right] \right\}.$$

The first order conditions for this problem are

$$\begin{aligned} u'\left(c_t(\eta_t, \epsilon_t)\right) &= \lambda_t(\eta_t, \epsilon_t) \\ \beta^t E\left[\lambda_t(\eta_t, \epsilon_t)\right] &= \beta^{t+1} E\left[\lambda_{t+1}(\eta_{t+1}, \epsilon_{t+1})\left(1 + (1-\tau_{t+1})(r_{t+1} + \eta_{t+1}) - \delta\right)\right]. \end{aligned}$$

Putting the two together, we reach the following Euler Equation

$$E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] = \beta E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)\left(1 + (1-\tau_{t+1})(r_{t+1} + \eta_{t+1}) - \delta\right)\right]. \tag{2}$$

Together with the budget constraints, this equation determines the optimal solution to the households' problem.

2.2 Firms

Each period firms maximize profits given the before taxes prices for capital r_t and labor w_t. They have access to the production function $F(k_t, l_t)$, which has constant returns to scale and is concave. Furthermore, assume that the usual Inada conditions are met.

Firm optimality and market clearing imply that factor prices must equal the marginal productivity of each factor

$$r_t = F_k(k_t, l_t) \quad w_t = F_l(k_t, l_t) \ . \tag{3}$$

Notice that although we have assumed that there is idiosyncratic capital risk, there is still a perfect capital market. This way, each household receives the same return from the market, plus their individual productivity shock. The same is true for the labor income shock.

2.3 Government and Market Clearing

Each period, the government uses lump sum taxes (or transfers) to balance its budget, which is given by

$$T_t = E\left[-\tau_t(r_t + \eta_t)k_t\right] = -\tau_t r_t k_t. \tag{4}$$

The resource constraint for the economy in period t is given by

$$E\left[c_t(\eta_t, \epsilon_t)\right] = F(k_t, l_t) + (1-\delta)k_t - k_{t+1}. \tag{5}$$

Furthermore, total labor demand must equal supply, which is inelastic and equal to one.

2.4 Competitive Equilibrium

Lemma 1 *If a sequence of consumption, capital, and capital taxes meets the conditions below, then it can be implemented as a competitive equilibrium.*

$$\begin{aligned}
c_t(\eta_t, \epsilon_t) + k_{t+1} &= F(k_t, 1) + \epsilon_t + (1-\delta)k_t + (1-\tau_t)\eta_t k_t \\
E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] &= \beta E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)\left(1 + (1-\tau_{t+1})(F_k(k_{t+1}, 1) + \eta_{t+1}) - \delta\right)\right].
\end{aligned}$$

Proof. A competitive equilibrium for our economy is a sequence of consumption, capital, prices, and taxes such that (i) the government's budget constraints are met, (ii) households maximize their utility subject to their budget constraints, (iii) firms maximize their profits given prices, and (iv) markets clear. Conditions (1)-(5) with $l_t = 1$ are thus necessary and sufficient for a competitive equilibrium. The resource constraint does not need to be imposed explicitly since it follows from the households' and the government's budget constraints. The problem can be further simplified by replacing market prices and lump sum taxes from equations (3) and (4) into the remaining equations, which leads to the conditions above. ■

The first equation that characterizes our competitive equilibrium is an 'Individual Resource Constraint' that highlights the fact that households will have different levels of consumption because they are subject to different individual shocks. If we aggregate this condition we will reach our usual aggregate resource constraint. The second equation is one single Euler Equation for all households because the investment decision is made before the shocks are realized.

The capital tax plays two roles: it reduces the impact of capital income risk on consumption, which provides insurance but it also decreases the return on capital for households.

2.5 The Economy Without Taxes

Before we study the effect of capital taxes, let us analyze the economy without taxes ($\tau = 0$) in order to understand the consequences of modelling uncertainty in this way for the behavior of agents and the whole economy. We will focus mostly on the behavior of the capital level.

First notice that if there is no capital income shock ($\eta = 0$), then in steady state households will choose capital such that $\beta (1 + (F_k(k,1) - \delta) = 1$, so the level of capital is equal to the one in a deterministic economy. This is true even when there is individual uncertainty in the labor income, so this model does not exhibit precautionary savings motif, which means that all the effects that traditionally arise from that channel will not be present. The existence of shocks to individual labor income may however delay the convergence to the steady state since it will exacerbate the curvature of the utility function.

If η is stochastic and there is no labor income shock ($\epsilon = 0$) then the Individual Resource Constraint implies that consumption is positively correlated with the capital income shock, which means that the covariance in the Euler Equation is negative. This leads to higher capital productivity and lower level of capital in steady state relative to the deterministic economy.

However, if both η and ϵ are stochastic and negatively correlated, then if the labor income shock is strong enough, consumption may become negatively correlated with the capital income shock, which means that capital will be higher than under complete markets (we will see an example of this in the simulation in section 3.3).

2.6 The Effect of Capital Taxes

In this section, we analyze the effect of capital taxes on the steady state level of capital. In steady state, the level of capital is determined by the following equations

$$\begin{aligned} c(\eta, \epsilon) &= F(k, 1) - \delta k + \epsilon + (1-\tau)\eta k \\ 1 &= \beta\left(1 + (1-\tau)F_k(k, 1) - \delta\right) + \beta(1-\tau)\frac{E\left[u'(c(\eta, \epsilon))\eta\right]}{E\left[u'(c(\eta, \epsilon))\right]}. \end{aligned}$$

If there is no capital income shock ($\eta = 0$), an increase in capital always discourages capital investment since it reduces the return on capital. However, if there is capital income risk, a higher capital tax also reduces the covariance between consumption and the capital income shock, which increases steady state capital. Whether increasing the capital tax reduces or increases the level of capital depends on the trade off between these two effects.

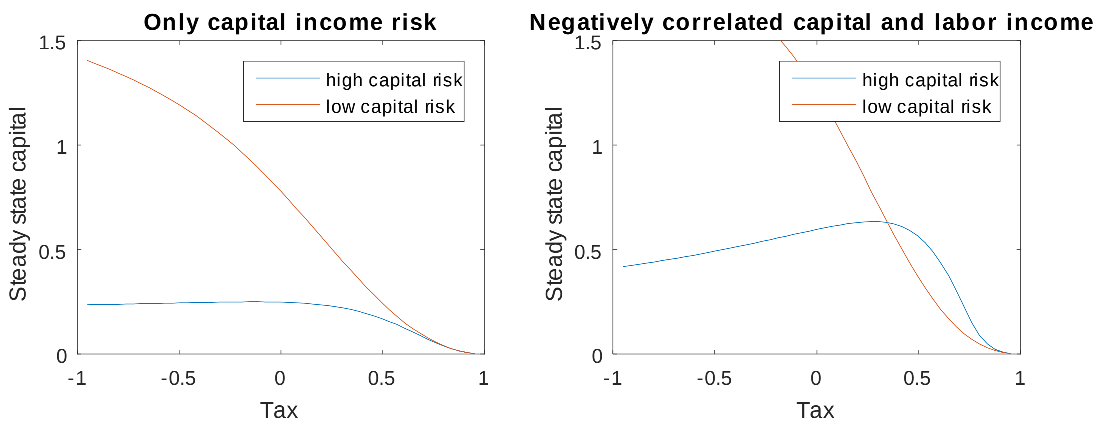

Figure 1 - Monotonicity of capital with respect to the capital tax for different levels of risk

Figure 1 illustrates the dynamics for capital by comparing economies with different levels of capital income risk. The economy on the left has no labor income risk, whereas the economy on the right has a labor income shock which is negatively correlated with the capital income shock. For this calibration of the model, steady state capital would be one if there were complete markets (no idiosyncratic risk) and no taxes. Calibration details are provided in the appendix. With low capital income risk, the economy essentially behaves as under complete markets, with capital decreasing monotonically with the tax. However, for high capital income risk, we can observe a non-monotonicity of the capital level with respect to the tax. This effect can be even stronger if we add a labor income shock, as we can see in the right pane.

3 Optimal Capital Taxes

In this section, we determine the optimal tax profile chosen by a benevolent government who chooses the competitive equilibrium that maximizes average utility in the economy. The government thus solves the following problem

$$
\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \beta^t E\left[u\left(c_t(\eta_t, \epsilon_t)\right)\right] \\
st \quad & c_t(\eta_t, \epsilon_t) + k_{t+1} = F(k_t, 1) + \epsilon_t + (1-\delta)k_t + (1-\tau_t)\eta_t k_t && (6) \\
& E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] = \beta E\left[\begin{array}{c} u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right) \\ \left(1 + (1-\tau_{t+1})(F_k(k_{t+1}, 1) + \eta_{t+1}) - \delta\right) \end{array}\right] && (7)
\end{aligned}
$$

Let us set up the Lagrangian for this problem associating the condition (6) with multiplier $\beta^t \rho_t(\eta_t, \epsilon_t) f(\eta_t, \epsilon_t)$, and condition (7) with multiplier $\beta^t \mu_t$

$$
\begin{aligned}
L \;=\; & \sum_{t=0}^{\infty} \beta^t E\left[u\left(c_t(\eta_t, \epsilon_t)\right)\right] - \sum_{t=0}^{\infty} \beta^t \mu_t E\left[u'\left(c_t(\eta_t, \epsilon_t)\right)\right] \\
& - \sum_{t=0}^{\infty} \beta^t E\left\{\rho_t(\eta_t, \epsilon_t)\left[c_t(\eta_t, \epsilon_t) + k_{t+1} - F(k_t, 1) - \epsilon_t - (1-\delta)k_t - (1-\tau_t)\eta_t k_t\right]\right\} \\
& + \sum_{t=0}^{\infty} \beta^{t+1} \mu_t E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)\left(1 + (1-\tau_{t+1})(F_k(k_{t+1}, 1) + \eta_{t+1}) - \delta\right)\right]
\end{aligned}
$$

The first order condition for consumption in period t with shocks η_t and ϵ_t is given by

$$
u'\left(c_t(\eta_t, \epsilon_t)\right) = \rho_t(\eta_t, \epsilon_t) + \mu_t u''\left(c_t(\eta_t, \epsilon_t)\right) - \mu_{t-1} u''\left(c_t(\eta_t, \epsilon_t)\right)\left(1 + (1-\tau_t)(F_k(k_t, 1) + \eta_t) - \delta\right).
$$

The first order condition for capital in period $t+1$ is given by

$$
\begin{aligned}
E\left[\rho_t(\eta_t, \epsilon_t)\right] \;=\; & \beta E\left[\rho_{t+1}(\eta_{t+1}, \epsilon_{t+1})\left(1 + F_k(k_{t+1}, 1) + (1-\tau_{t+1})\eta_{t+1} - \delta\right)\right] \\
& + \beta \mu_t E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)(1-\tau_{t+1}) F_{kk}(k_{t+1}, 1)\right].
\end{aligned}
$$

Finally, the first order condition for the proportional capital tax in period $t+1$ is given by

$$
-E\left[\rho_t(\eta_t, \epsilon_t)\eta_t k_t\right] = \mu_t E\left[u'\left(c_{t+1}(\eta_{t+1}, \epsilon_{t+1})\right)\left(F_k(k_{t+1}, 1) + \eta_{t+1}\right)\right].
$$

The optimal tax for the initial period is given by $E\left[\rho_0(\eta_0, \epsilon_0)\eta_0\right] k_0 = 0$. Since η has zero expected value, this implies that the covariance between the shock and the multiplier on condition (6) must also be zero. When there is only capital income risk this is achieved with $\tau_0 = 1$, which makes consumption constant in all states of nature.

This is similar to many taxation models, since taxing initial capital does not distort the incentives to save since the initial level of capital is predetermined. However, here the reason to tax capital is not to collect revenue, but to provide insurance.

Let us now turn to analyzing the steady state features of the optimal tax. To do this we will first consider each type of risk separately, and then we turn to analyzing the joint effects of the two.

3.1 Only capital income risk

In this section, we will assume that there is only capital income risk, so $\epsilon = 0$ always.

Proposition 1 *In steady state, the optimal capital tax is strictly positive if there is only capital income risk.*

Proof. We will show that it is not optimal to have zero capital taxes. Furthermore, when the capital tax is equal to zero, utility is increasing in the capital tax, which implies that utility achieves its maximum for a positive tax rate.

If capital taxes are zero, condition (7) does not bind in steady state, which implies that $\mu = 0$. With $\mu = 0$, the derivative of the Lagrangian with respect to the capital tax is given by

$$\left.\frac{\delta L}{\delta \tau}\right|_{\tau=0} = -E\left[\rho(\eta, \epsilon)\eta\right] k.$$

Furthermore, consumption varies positively with the shock η and from the first order condition for consumption we know that $u'\left(c_t(\eta, \epsilon)\right) = \rho_t(\eta, \epsilon)$. Since $\rho_t(\eta, \epsilon)$ is negatively correlated with η, this derivative is strictly positive, and utility can be increased by choosing a positive capital tax. ■

Thus, the existence of capital income risk is a reason that may justify positive capital income taxes. To see how the level of taxes varies with the degree of riskiness we now turn to a numerical simulation of the model.

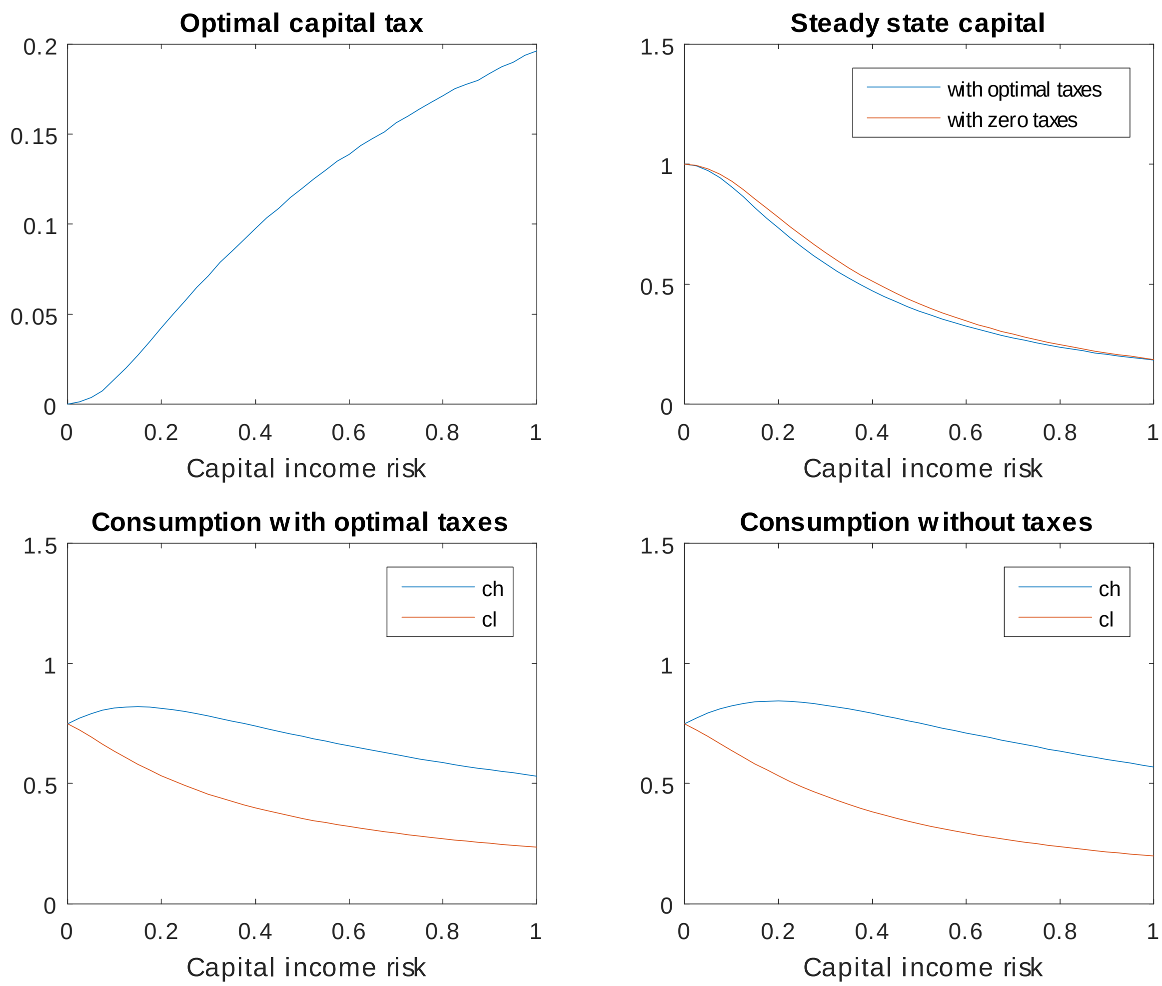

Figure 2: Optimal taxes with only capital income risk

Figure 2 illustrates the optimal taxes in the presence of capital income risk. Calibration details are provided in the appendix. The optimal tax increases monotonically with the magnitude of the shock. Although steady state capital is almost always reduced by the optimal taxes, it does not significantly differ from the level of capital with no taxes. This is because for higher levels of risk, where the optimal tax is higher, the relationship between tax and capital becomes non-monotone, as we saw in figure 1. In this model, what significantly lowers steady state capital is the existence of idiosyncratic risk, and not the optimal tax per se. In the bottom panes we can see that consumption becomes significantly less volatile when the optimal taxes are used.

3.2 Only labor income risk

In this section, we will assume that there is only labor income risk, so $\eta = 0$ always.

Proposition 2 *In steady state, the optimal tax is zero if there is only labor income risk.*

Proof. If $\eta = 0$, the first order condition for the optimal tax becomes $0 = \mu F_k(k,1) E\left[u'\left(c(\eta,\epsilon)\right)\right]$, which implies that $\mu = 0$. If this is the case, the first order condition for capital becomes $1 = \beta\left(1 + F_k(k_{t+1},1) - \delta\right)$, which implies that the capital income tax is zero. ■

With only labor income risk the optimal capital tax is zero in this model. This result differs from Aiyagari, where the optimal tax on capital is positive, but it is not a surprising result since in Aiyagari there was a strong precautionary savings motif due to the borrowing constraints that were imposed.

3.3 Both types of risk

We now assume both types of risk are present, and are possibly correlated.

Proposition 3 *The capital income tax can be either positive or negative when both types of risk are present.*

Proof. We will start by showing that at zero capital taxes, the first order condition for the optimal tax can be either positive or negative, depending on the correlation between the two shocks.

If capital taxes are zero, condition (7) does not bind in steady state, which implies that $\mu = 0$. With $\mu = 0$, the derivative of the Lagrangian with respect to the capital tax is given by

$$\left.\frac{\delta L}{\delta \tau}\right|_{\tau=0} = -E\left[\rho(\eta,\epsilon)\eta\right]k.$$

From the first order condition for consumption we know that $u'\left(c(\eta,\epsilon)\right) = \rho_t(\eta,\epsilon)$. The level of consumption depends on the two shocks in the following way

$$c(\eta,\epsilon) = F(k,1) - \delta k + \epsilon + \eta k.$$

If ϵ and η are positively correlated, then $E\left[\rho(\eta,\epsilon)\eta\right] < 0$, and the first order condition is positive at zero taxes, which means that the optimal tax is positive.

If ϵ and η are are negatively related, then the optimal tax can be either positive or negative. If $\epsilon + \eta k$ and η are positively correlated, then $E\left[\rho(\eta,\epsilon)\eta\right] < 0$, and the optimal tax is still positive. However, if $\epsilon + \eta k$ and η are negatively correlated, then $E\left[\rho(\eta,\epsilon)\eta\right] > 0$, and the optimal tax becomes a subsidy. ■

If there is capital and labor income risk, the optimal capital tax can be either positive or negative. If the two shocks are positively correlated, the optimal capital tax is positive. A more interesting case occurs when the two shocks are negatively correlated, since in this case the optimal capital tax can be positive or negative. To gain a better understanding of the steady state effects, we fix the variance of the labor income shock, but we allow the magnitude of the capital income shock to vary.

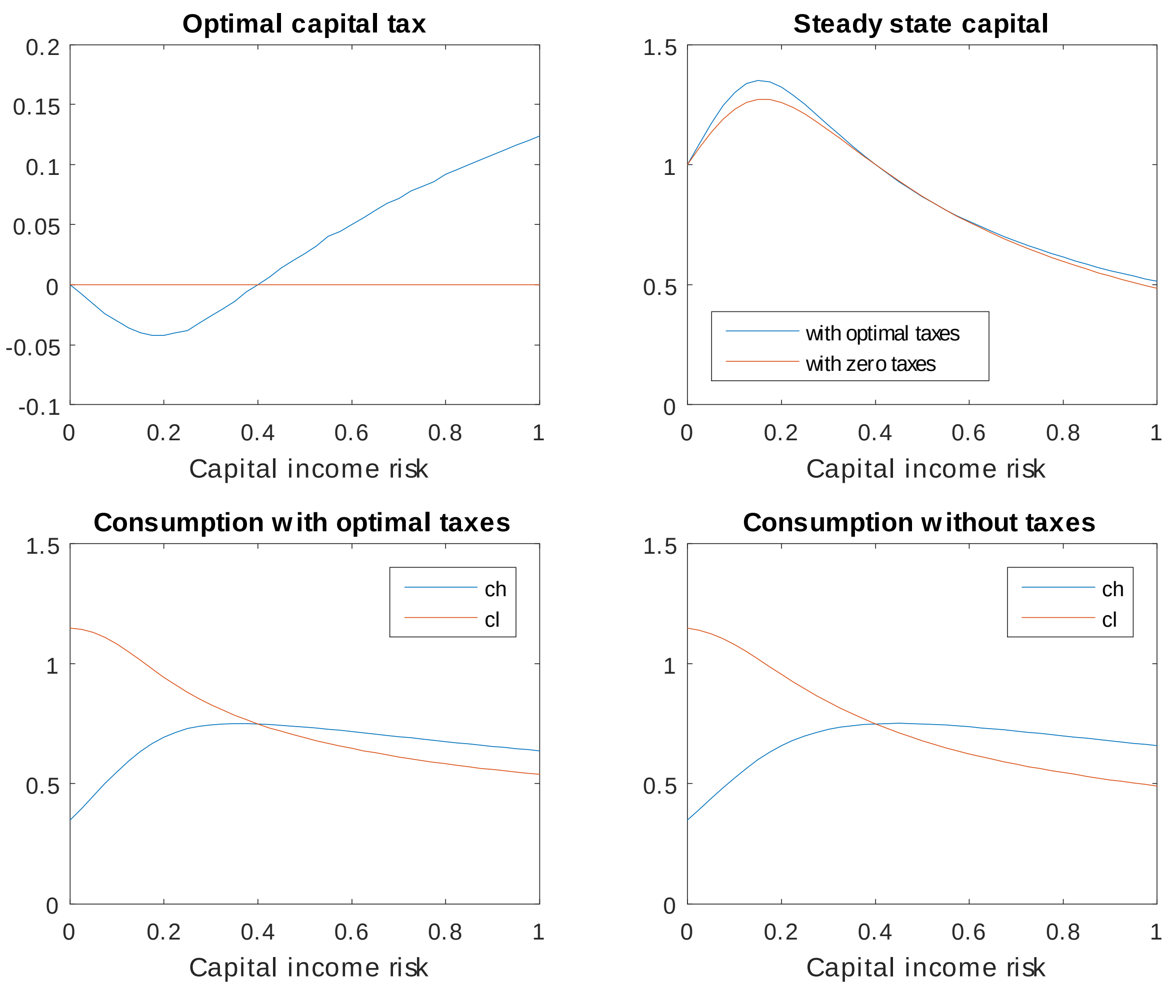

Figure 3: Optimal taxes with negatively correlated capital and labor income risk

Figure 3 illustrates the optimal taxes in the presence of negatively correlated labor and capital income risk. Calibration details are provided in the appendix. In this case, the optimal tax is negative for low capital income risk, and positive for high risk levels. This is reminiscent of Panousi and Reis (2016), where the optimal tax was positive for high levels of risk but negative for low levels of risk (although for different reasons). Now steady state capital tends to be higher when optimal taxes are used. This happens for two reasons. First, for low levels of risk the optimal tax is a subsidy. Second, for high levels of risk the presence of the labor income shock exacerbates the non-monotonicity of capital with respect to the tax rate. Notice that for low levels of risk, capital is now higher than under complete markets, since it provides insurance against the labor income shock. Just like in figure 2, we can see that consumption becomes significantly less volatile when the optimal taxes are used. This is true even when the optimal tax is a subsidy because in this case the capital income shock (which is relatively small) is actually insuring against the labour income shock.

4 Concluding Remarks

If households are subject to idiosyncratic capital and labor income risk it is optimal for taxes to provide partial insurance against the income shocks. In general, this is achieved through a positive capital income shock. However, there is large labor income that is negatively correlated with the capital income shock, the optimal policy may be a capital subsidy. Thus, the conventional wisdom that capital taxes should be zero in the long run no longer applies in this setting.

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5 Numerical Calibration

We assume individual uncertainty follows a discrete uniform distribution where the capital income shock η can take values θ and $-\theta$ with 50% probability each. By varying the value of θ we can change the degree of uncertainty in the model. We will restrict our attention to values of θ such that $\theta \leq 1$. We consider two possible calibrations for the labor income shock ϵ. Either there is no labor income shock and $\epsilon = 0$, or there is a labor income shock which is perfectly negatively correlated with the capital income shock such that $\epsilon = 0.4$ when $\eta = -\theta$ and $\epsilon = -0.4$ when $\eta = \theta$. By varying θ we vary the magnitude of the capital income shock keeping the labor income shock constant.

Utility is isoelastic and the production function is Cobb-Douglas, with the following parameters

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \qquad \text{with } \sigma = 1.5$$
$$F(k,1) = Ak^{\alpha} \qquad \text{with } A = 1 \text{ and } \alpha = 0.5.$$

Furthermore, the discount factor is $\beta = 0.8$ and depreciation is $\delta = 0.25$.

For the calibration in figure 1, we consider a high capital risk of $\theta = 0.8$ and a low capital risk of $\theta = 0.2$. This represents the percentage of the capital investment that is lost if the shock is negative.