Climate Policy: How to deal with ambiguity?

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Abstract

In this paper we study the impact of ambiguity and ambiguity attitudes on optimal adaptation and mitigation decisions when the future environmental quality is ambiguous and the decision maker's (DM) preferences are represented by the MMEU model. We show that a more risk averse DM should soften the environmental policy whereas an increase in ambiguity aversion may display opposite results. We also focus on the induced effects of changes in ambiguity, captured by the arrival of additional information. We show that when a bad news arrives, it can be good for the environment as the DM is more likely to increase the effort of both adaptation and mitigation. When more than one new scenarios arrive, our conclusions might be altered depending on which news dominates one or the other. Finally, when all economic instruments are endogeneized, we state that the DM must always invest in adaptation, in addition of a lump-sum refund towards agents. Then, we perform a numerical exercise to evaluate the impact of ambiguity aversion on the optimal levels of mitigation as well as the sharing between redistribution and adaptation.

Keywords: Environmental risks; Ambiguity; Mitigation; Adaptation; Non-expected utility.
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1 Introduction

Anthropogenic emissions of greenhouse gases (GHG) are commonly admitted to be responsible for the observed climatic changes over the last century. And maybe more importantly, it is clear that

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if the stock of emissions continues to rise in the next decades, more and more changes should break out. Then, many questions like "how, how much or how fast will our environment respond to such a phenomenon?" arise. Answers to such questioning can be found in integrated assessment models, which have often been used to quantify the impact of human activity on the environment and to provide welfare evaluation of environmental policies. However, most of the time those big models abstract from uncertainties that naturally surround environmental issues but rather employ deterministic setups.[1] We argue that despite the remarkable accomplished work made by climate scientists and experts, the information we have about future climate risks is still not of sufficient quality so that it is required to adopt alternative approaches to evaluate welfare consequences of climate change. In this paper, we intend to mobilize frameworks and tools from decision theory, which account for uncertainty, in order to discuss optimal strategies to be adopted to fight against climate change.

Indeed, scientists or climate experts use to apply well-established and broad biological principles in order to empirically predict the behavior of our environment, our climate, face to an increasing concentration of greenhouse gases. But the very sophisticated and complex models involved, as well as our imperfect understanding of all concerned physical and ecological processes, make hard to precisely design an appropriate policy. In addition and beyond the long-time horizon considered, technological and socio-economic uncertainties arise since we can not accurately forecast neither future emissions nor the policy that will be chosen to thwart the harmful effects of climate change. Eventually, all the contributions, all the predictions found in the literature can be disputable on the ground of physical and economic parameters choices, statistical methods, data and the like (Tebaldi and Knutti, 2007). As a consequence of the intrinsic grey areas of such models, estimates can even become contradictory or at least exhibit substantial discrepancies. Meinshausen *et al.* (2009) or Millner *et al.* (2013), in their article, have reported a set of estimated probability density functions of climate sensitivity[2] provided by the literature to illustrate the imprecise feature of such predictions. An other striking example would be the AR5 (Fifth Assessment Report) offered by the IPCC[3] (2014) which presents estimations based on four distinct scenarios, each of them revealing different climate sensitivities, but also different implicit hypothesis on the economic and

[1]Some examples of recent contributions that deal with uncertainty, ambiguity or learning about future risks in a dynamic framework could be Keller *et al.* (2004), Karp and Zhang (2006), Millner *et al.* (2013) or Berger *et al.* (2017).

[2]According to Millner *et al.* (2013), climate sensitivity is a crucial summary statistic that captures how the climate responds to increase in greenhouse gases concentration.

[3]IPCC: Inter-governmental Panel of experts on Climate Change.

demographic patterns of the world economy. Hence it comes that, with respect to an increase in global temperature over the 21^{th} century, the estimations substantially vary between +0.3 to +4.8°C. Finally, the famous "*Nordhaus-Stern controversy*" can also be used as a relevant illustration of wide disparate beliefs about the policy to be implemented (Nordhaus, 2007; Stern, 2007).

Face to such imperfect knowledge, to potential disagreements between experts, how is it possible to interpret the whole information without referring to subjective feelings? How to combine or to aggregate the (too many) estimates? (see Knutti, 2010 and Knutti *et al.*, 2010 for a presentation of the challenges involved by the multiple climate models available.) How to properly design an effective environmental policy? In this context where the variability of predictions prevails, it becomes vital to find the keys to help or guide the decision making process and this is the main goal of our contribution.

Firs of all, in the present paper we consider that public authorities may undertake two types of actions to fight against climate change: *mitigation* and *adaptation* strategies.[4] To that extent, we follow, among others, Kane and Shogren (2000) or Buob and Stephan (2011) who underline that the mix policy (mitigation/adaptation) has been so far too much neglected in current policy making, although it appears to be more and more relevant. Basically, mitigation consist in curtailing GHG emissions to lower the likelihood that damaged states of nature occur - it could be for instance a permanent rise in air temperature or sea level - while adaptation should reduce the severity of damage, when it occurs. Moreover, we consider one of the main currently used economic instruments for controlling emissions of GHGs which is a carbon tax.[5] In order to complete the big picture, we also focus on the crucial issue of tax recycling since it participates to make a policy acceptable or not. Hence, we consider that the collected revenues from the carbon tax are partially redistributed as lump sum payments to consumers while the remaining share of fiscal resources is devoted to public investment in adaptation.[6]

Finally, we dwell at length on the role played by an imprecise information on optimal public decisions. In fact, we argue that the overall picture is better featured by ambiguity (or what some authors like Heal and Millner (2014) call "model uncertainty") rather than risk. Hence, we

[4]Let us notice that, in this set-up, we do not consider private efforts of prevention against harmful effects of climate change, as costs to bear are often too large to be afforded by individuals.

[5]The counter-part of such a tool would be the implementation of a permits trading system, like the European Union Emissions Trading Scheme. In our set-up nonetheless, the nature of the instrument does not really matter, so that we formally choose to model a carbon tax.

[6]We thus depart from the literature on the double dividend (for a survey, see Bovenberg and Goulder, 2002) as we concentrate on the decision making process rather than the overall efficiency of a fiscal system.

account for a lack of precise probabilistic information about the occurrence of different possible events associated with climate change. In this context of ambiguity, we depart from the seminal Subjective Expected Utility (SEU) model provided by Savage (1954) and thus, our contribution lies in the determination of the optimal environmental policy design according to a model of preferences representation that embeds ambiguity aversion. Indeed, the former SEU model has often been criticized as a descriptive model since it fails to take into account the quantity of available information and the difference between risk and ambiguity attitudes, the Ellsberg paradox being a striking illustration (Ellsberg, 1961). Hopefully, there have been a series of theoretical advances in decision theory which provide us with more appropriate representations of preferences (for a survey see Etner *et al.*, 2012 or Gilboa and Marinacci, 2013). Among more general models that have been brought out to represent preferences depending on the information structure, we use the MMEU model proposed by Ghirardato and Marinacci (2002). Alternative setup that permits to disentangle risk and ambiguity attitudes could be investigated like the one proposed by Klibanoff *et al.* (2005), so-called the KMM model. Indeed, it has recently been used to tackle the issue of climate change, namely by Lange and Treich (2008), Millner *et al.* (2013), Lemoine and Traeger (2016) and Berger *et al.* (2017). However, we depart from these contributions in many dimensions. Eventually, the closest work to ours is the paper by Berger *et al.* (2017). The authors first study the impact of uncertainty aversion on optimal mitigation level in a theoretical model. Then, they use expert data on climate change catastrophes to calibrate a Dynamic Integrated Climate Economy (DICE) model introducing unknown catastrophe probabilities. It allows them to quantify the impact of uncertainty and uncertainty aversion on the optimal level of emissions abatement. The main differences between their model and ours is that they consider a single state of nature with ambiguous probability (corresponding to a catastrophic event) and a unique policy instrument (emissions abatement). Moreover we grant a special attention to changes in the information structure and its impacts on the optimal policy design. In particular, we are interested in the consequences of changes in ambiguity, that is the arrival (disappearance) of new (obsolete) scenarios, in addition of changes in ambiguity aversion.

By introducing both mitigation and adaptation as available instruments to improve the ambiguous future environmental quality, and assuming that preferences under ambiguity are represented by the MMEU model, we aim at designing an optimal policy mix but also at providing some policy recommendation in the way to recycle green fiscal resources. We show that ambiguity attitudes may significantly influence the optimal mitigation/adaptation policy mix: More precisely, an increase in risk aversion incites the DM to reduce the effort of both adaptation and mitigation while a more ambiguity adverse DM tends to implement a more stringent environmental policy. Then, we concentrate on potential changes in the levels of ambiguity. In particular, we consider that scientific research may influence the informational structure in that the set of information may reduce or grow. Say otherwise, we wonder about the induced effects of a new information, be it good, neutral or bad. After defining the nature of the information, we show that when a bad news arrives, it can be good for the environment as the DM is more likely to increase the effort of both adaptation and mitigation. Conversely, a more optimistic new scenario tends to soften the environmental policy. When many new scenarios arrive, our conclusions might be altered, depending on which news (the good or the bad one) dominates one or the other. Finally, when all policy instruments are endogeneized, we state that the DM must always invest in adaptation in addition of a lump-sum refund towards agents. Then, we perform a numerical exercise to provide some policy recommendations. We assess the impact of a change in ambiguity aversion on the effort of mitigation but also on the optimal design of the policy, that is the sharing between adaptation and redistribution.

The paper is organized as follows. In section 2 we introduce the general framework and present in more details our two policy instruments. In section 3 we investigate the model of ambiguity when redistribution is exogenous. In section 4, we explore the trade-off between mitigation and adaptation when wealth redistribution is endogenous and we propose a numerical example. Finally, Section 5 concludes.

2 The Framework

2.1 Risk mitigation and risk adaptation

According to recent recommendations provided by the IPPC and as it has been observed during last international climate negotiations, an effective climate policy should involve a range of various adaptation and mitigation actions (Parry, 2007). In climate change issues, the effort of mitigation may then encompass all actions that cut down the flow of GHGs in the atmosphere and thus change the probability distribution over future climate states (Heal and Kriström, 2002). According to the AR5 and the experts of the IPCC (2014), there exists a substantial potential for mitigation of global GHG emissions that could be exploited thanks to future energy infrastructure investments, the spread of low-carbon technologies, the improvement in crop and grazing land management (to increase soil carbon storage), the reduction of deforestation and so on. In our framework, a public decision maker can implement a mitigation policy *via* a carbon tax (T) in order to curb down emissions of pollution. Formally, it reduces the potential level of production (y_0) and thus, agents' disposal income. The tax can be interpreted as sacrificed productive resources in order to pollute less or to produce more efficiently. Only the remaining share of production can indeed be consumed by agents. Hence, it turns out that income *per capita*[7] y negatively depends on taxes T meanwhile, the later enable to improve the probability distributions of future climate states.

As for the revenue of the taxes, it can be recycled through a public investment in adaptation or a lump-sum refund towards agents. We denote by $\rho \in [0,1]$ the share of those public receipts redistributed to consumers, as a form of green vouchers. Then, mechanically, $(1-\rho)T$ represents the amount invested in adaptation. Notice that

This public policy aims at reducing the damages that come along with a given climate state and thus at limiting the vulnerability to environmental hazards[8]. For instance, in agriculture, it could take the form of changes in crop varieties, in water management strategies or in planting schedule. Similarly, adaptation strategies in coastal zones could aim at implementing well-planned retreats, changes in land-use, protections with hard and soft structures likes dikes and beach fills. Formally, mitigation and adaptation strategies display different outcome on the future environment that we describe now.

First, the environmental quality denoted Q is considered to be a random variable which takes its values in the interval $[\underline{Q}, \overline{Q}]$. We denote by F and f, the distribution and the density function of Q respectively, and we assume that they depend on the amount of collected tax, T. More precisely, the larger the tax revenues, the better the distribution of Q in the sense of the first order stochastic dominance (hereafter FSD):

$$T > T' \Longrightarrow \Pr\left(Q \geq q \mid T\right) \geq \Pr\left(Q \geq q \mid T^{'}\right) \Leftrightarrow F(q,T) \leq F(q,T') \ \forall q \in \mathbb{R}$$

which implies that $\frac{\partial F(q,T)}{\partial T} \leq 0$. Moreover, we assume that the distribution function, F, is convex in its second argument, $\frac{\partial^2 F(q,T)}{\partial T^2} \geq 0$. Therefore, the greater T the less its effect in term of a change in distribution of Q is.

Second, if a share $(1-\rho)T$ of the fiscal revenues is invested in adaptation in order to reduce the

[7] We could alternatively consider that y is the individual wage and thus, the marginal productivity of labor.

[8] Adaptation may be disputable as a mean to preserve our future environment, since it may allow to pollute more because it reduces damages (See an illuminating discussion on this specific point in Schumacher, 2016.). Nonetheless, we do account for such perverse or counter-productive effects, rather than its useful feature and we emphasize its crucial role in international negotiations as it may favor the success of a binding agreement (Ayong le Kama and Pommeret, 2016).

severity of the damage then, the future random environmental quality become $Q + \varphi((1-\rho)T)$, where the function $\varphi(x)$ characterizes the adaptation technology. We merely assume that $\varphi'(x) > 0$, $\varphi''(x) \leq 0$ and $\lim\limits_{x \to 0} \varphi'(x) = +\infty$.

2.2 Preferences representation

In this paper, we investigate the choices made by the public DM when she faces ambiguity. She maximizes a welfare function, which is the sum of individuals utilities. For simplicity, we suppose that all agents are identical and derive utility from consumption, c, and environmental quality, q, so that the utility function is additively separable: $U(c, q) = u(c) + v(q)$, where u and v are assumed to be increasing and concave in each argument.

The representative agent consumes the whole revenue that is composed of the production revenue, $y(T)$, and public transfers, ρT, $c(T) = y(T) + \rho T$. Nonetheless, notice that an increase in the amount of collected tax can not raise the total income to be consumed, as stated in the following assumption.

Assumption 1 *$\forall T$, $\forall \rho \in [0, 1]$, $c'(T) = y'(T) + \rho < 0$ and $c''(T) = y''(T) < 0$.*

Of course, whenever Assumption 1 were not to be satisfied, then the DM should levy the highest possible tax in order to maximize agents' utility. There would not be any trade-off and the decision problem would become trivial.

Let us now focus on the random feature of environmental quality. According to the illustrations provided in Introduction, we consider that different scenarios are proposed to evaluate the future possible evolution of the environment. Thus, there does not exist a unique probability distribution but rather many ones, each of them being associated to a scenario called θ. Following the introduction of this ambiguity, the distribution function $F(q, T)$ writes now $F_\theta(q, T)$. Without loss of generality, we assume an infinity of possible scenarios that can be ranked from the worst to the best one: $\theta \in \Theta = [\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} < \overline{\theta}$. Hence, a lower value of θ deteriorates the distribution of Q in the sense of the first order stochastic dominance:

$$\forall T,\ \theta > \theta' \Rightarrow F_\theta(q, T) \leq F_{\theta'}(q, T),\ \forall q \in \mathbb{R}.$$

Moreover, and in line with the usual hypothesis made in the literature, the cost of abatement is assumed to be increasing and convex with the level of pollution. Hence, it implies that mitigation is more efficient in the worst scenario than in the best one:

Assumption 2 $\forall q$, $\left|\dfrac{\partial F_{\underline{\theta}}(q,T)}{\partial T}\right| \geq \left|\dfrac{\partial F_{\overline{\theta}}(q,T)}{\partial T}\right|$.

To evaluate decisions in the presence of incomplete and ambiguous knowledge, the theoretical literature proposes a set of models, the prominent one being the Subjective Expected Utility set-up proposed by Savage in 1954. In this model, according to her subjective beliefs, the DM should opt for a scenario $\tilde{\theta}$ that she believes being the right one among all possible scenarios: $\tilde{\theta} \in [\underline{\theta}, \overline{\theta}]$. Then she evaluates decisions according to it, ignoring all the others. Everything should happen as if the DM arbitrarily chooses a prior distribution of probabilities and thus is neutral to ambiguity. Several criticisms have been addressed to the SEU model, mainly related to the fact that it does not allow to explicitly disentangle risk from ambiguity but also, ambiguity attitudes from risk attitudes.

Alternative models representing preferences under ambiguity exist but, while allowing to separate risk and ambiguity attitudes, they differ mainly in the type of available information and beliefs. In our framework, the available information takes the form of a set of scenarios and no belief can be associated to any of them. One well suited model of preferences representation in this context is the MMEU setup proposed by Ghirardato and Marinacci (2002). When ambiguity is characterized by a set of priors (here the scenarios), this model evaluates decisions as a weighted sum of the highest and the lowest expected utility compatible with the set of priors. The weight granted to the lowest expected utility measures the DM's ambiguity aversion. The evaluation of a representative agent's welfare, for a given T and ρ is:

$$V(T,\rho) = u(c(T)) + \int_{\underline{Q}}^{\overline{Q}} v(q + \varphi((1-\rho)T))[\alpha f_{\underline{\theta}}(q,T) + (1-\alpha) f_{\overline{\theta}}(q,T)]dq \qquad (1)$$

where $\alpha \in [0,1]$ captures ambiguity aversion.

3 Optimal Mitigation

Since equity is one of the major challenges in global climate change negotiations, it is vital to grant a specific care to the distribution of economic burdens. Hence, finding publicly acceptable solutions for de-carbonizing is a clear priority (Metcalf, 2009). To comply with such goals we aim at providing policy recommendations with regards to both the optimal level of the carbon tax but also the way it can be recycled.

In order to figure out all the mechanisms at stake, we first consider that the proportion of the tax revenues paid back to households (ρ) is given.[9] In any case, the optimal amount of fiscal resources levied by the DM is the solution of the welfare maximization. We denote by T^* the optimal interior solution, which satisfies the usual First-Order Condition, $\frac{\partial V(T,\rho)}{\partial T} = 0$, written below:

$$
\begin{aligned}
-\left(y'(T)+\rho\right)u'(c(T)) \quad &= \quad (1-\rho)\varphi'(\cdot)\left[\int_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(\alpha f_{\underline{\theta}}(q,T)+(1-\alpha)f_{\overline{\theta}}(q,T)\right)dq\right] \\
&\quad + \int_{\underline{Q}}^{\overline{Q}} v(q+\varphi(\cdot))\left(\alpha\frac{\partial f_{\underline{\theta}}(q,T)}{\partial T}+(1-\alpha)\frac{\partial f_{\overline{\theta}}(q,T)}{\partial T}\right)dq \qquad (2)
\end{aligned}
$$

The optimal level of tax implemented by the DM equalizes its marginal cost in terms of consumption with its marginal benefit in terms of an improved environmental quality. Notice that the marginal benefit is twofold: It consists, on the one hand, in a marginal benefit derived from adaptation, so that when the tax rises, the investment in adaptation becomes mechanically larger. On the other hand, there exists a marginal benefit derived from mitigation, so that when the levied fiscal resources are larger, the distribution improves. In addition, we observe that the optimal level T^* is a function of relevant parameters for our purpose like ρ and α. Then, we can easily see that an increase in the share devoted to redistribution (ρ) leads to a higher optimal green tax. Indeed, when the effort of adaptation reduces, the DM is incited to increase the fiscal pressure in order to compensate for. Let us turn now to the main goal of the paper that is the impact of ambiguity and risk on decisions in climate change issues.

3.1 Risk and Ambiguity Aversion

Risk Aversion. The relation between risk attitudes and prevention decisions, such as mitigation[10], have been broadly discussed in the literature (see, for instance, Ehrlich and Becker, 1972; Eeckhoudt and Gollier, 2005). From these studies, it appears that risk attitudes display an ambiguous impact on mitigation. Indeed, in a one period setting, neither usual risk aversion nor prudence are able to fully explain decisions: When individuals are expected utility maximizers, an increase in risk aversion does not always trigger more prevention while prudence can reduce it. In our model,

[9] In section 4, all choice variables will be endogeneized.

[10] Mitigation would correspond to self-protection and adaptation to self-insurance in general risk analysis (see Ehrlich and Becker (1972) for more details on these risk reduction tools).

the intuition is similar and goes as follows: First, mitigation by modifying the probabilities of the different outcomes corresponds to a FSD improvement. Second, a larger effort of mitigation, that is in our paper a raise in taxation, drives up adaptation, which improves the environmental quality in each state of nature. This second effect corresponds to an other FSD improvement. These two first effects encourage prevention while by lowering the net income, that is the current consumption, mitigation entails a utility deterioration. Consequently, choices cannot be compared on the basis of risk aversion alone. However, we obtain a clear-cut result when the DM exhibits constant absolute or relative risk aversion (CARA or CRRA utility functions). These classes of utility functions are currently used in the decision making under risk and ambiguity (see for instance Gollier, 2001) and in particular in climate change issues (Pyndick, 2012; Berger *et al.*, 2017). Accordingly, we can state the following proposition:

Proposition 1 *Under Assumptions 1-2 and if the environmental utility function v is CARA or CRRA then, an increase in risk aversion lowers mitigation.*

Proof. See Appendix A. ■

For constant absolute or relative risk aversion (and constant prudence), when risk aversion increases, agents favor the risk-less source of welfare that is consumption in our model. In other words, a risk averse and a prudent DM selects a level of taxation lower than the one chosen by a neutral DM. This leads to a reduced effort of mitigation but also of adaptation. Then, the environmental policy to be implemented is more lenient.

Ambiguity Aversion. From equation (2), we can see that the marginal cost of taxation is not influenced by the DM's ambiguity aversion while the latter only impacts the marginal benefit. Indeed, a more ambiguity averse DM over-evaluates the likelihood of the worst possible scenario and thus, over-evaluates the marginal benefit of adaptation. Hence, ambiguity aversion raises the marginal benefit of taxation. In addition, the DM might over-evaluate the marginal benefit of mitigation but only if mitigation is more efficient for the worst scenario than for the best one (see Assumption 2). Then, in this case, ambiguity aversion also leads to a larger benefit from green taxation. This is formally established in the following proposition.

Proposition 2 *Under Assumptions 1-2, an increase in ambiguity aversion increases the effort of mitigation.*

Proof. See Appendix B. ■

Proposition 2 allows us to underscore the role played by ambiguity aversion in the decision process. Contrary to risk aversion, ambiguity aversion encourages mitigation (*via* heavier taxes). This positive relationship has been indeed already highlighted in some recent contributions. (see Alary *et al.*, 2013; Berger *et al.*, 2017; Millner *et al.*, 2013). In these papers, ambiguity aversion can increase the optimal level of prevention (in the sense of mitigation) under some conditions including risk or prudence attitudes but in KMM setups. In addition, in these studies, prevention only affects two states of nature and thus no condition on distribution function is necessary. In our framework with a distribution function on environmental quality depending on prevention, we need some assumptions on these distribution functions, but no condition on utility function is required. The fiscal policy implies that when taxes rise, so does the investment in adaptation. Yet, ambiguity aversion leads to more mitigation but also more adaptation under assumption 2. Otherwise said, ambiguity aversion triggers more stringent environmental policies.

3.2 Changes in Ambiguity

As mentioned in Introduction, the many contributions on climate change issue participate to create ambiguity and such informational structure clearly affects the design of the policy to be implemented. What is interesting also to take into account is the potential changes in ambiguity degree, due for instance to scientific research advances. In this section, we focus on the consequences involved by the arrival of a new information (an additional scenario) that makes the set of considered scenarios evolves. Such a configuration would be defined by an increase in the degree of ambiguity. Although we do no explicitly consider so far a dynamic framework, we aim at assessing the impact of additional information, depending on their nature. As an illustration, we may refer to the evolution of the set of scenarios provided by the IPCC over time. This variability in the set of considered scenarios can alternatively be understood as a proximate for disagreements among climate experts.

In our model, ambiguity is characterized by a set of initially given scenarios Θ. More scenarios modify ambiguity by increasing the set of possible probability distributions. In a first step, we assess the impact on decisions of a new scenario's arrival and then, in a second step, we consider the situation where more than one new scenarios are introduced.

Built on our preferences representation model, a new scenario potentially influences the optimal decision depending on how it modifies the bounds of the interval. Hence, only two scenarios matter: those giving the worst and the best welfare evaluation. Consequently, a new scenario potentially alters the optimal decision only if it is worse than the worst existing scenario or if it is better than the best one. Accordingly, we can distinguish three relevant configurations defined as below:

Definition 1 *We say that a new scenario, $\widehat{\theta}$, is a*

(i) ***bad news*** *if* $\forall T$, $\forall q$, $F_{\underline{\theta}}(q,T) \leq F_{\widehat{\theta}}(q,T)$ *and* $\left|\partial F_{\widehat{\theta}}(q,T)/\partial T\right| > \left|\partial F_{\underline{\theta}}(q,T)/\partial T\right|$.

(ii) ***good news*** *if* $\forall T$, $\forall q$, $F_{\widehat{\theta}}(q,T) \leq F_{\overline{\theta}}(q,T)$ *and* $\left|\partial F_{\widehat{\theta}}(q,T)/\partial T\right| < \left|\partial F_{\overline{\theta}}(q,T)/\partial T\right|$.

(iii) ***neutral news*** *if* $\forall T$, $\forall q$, $F_{\underline{\theta}}(q,T) > F_{\widehat{\theta}}(q,T) > F_{\overline{\theta}}(q,T)$.

A bad news is defined as a new scenario, $\widehat{\theta}$, which becomes the new worst scenario with the same properties as $\underline{\theta}$ (so that it satisfies the FSD and Assumption 2). Similarly, a good news replaces the initial best scenario in the set of of all considered scenarios. As for the neutral news, it is a scenario that does not change the bounds of the interval and thus is not taken into account by the DM. Then, we obtain the following results:

Proposition 3

- *A bad news increases taxes and thus the level invested in mitigation if and only if the DM is not fully optimistic ($\alpha \neq 0$).*
- *A good news decreases taxes and thus the level invested in mitigation if and only if the DM is not fully pessimistic ($\alpha \neq 1$).*
- *A neutral news has no effect neither on taxes nor on mitigation.*

Proof. See Appendix C. ■

When experts estimate that a new worst scenario can appear, which is a *bad news*, the marginal benefit of mitigation is over-evaluated and the DM is incited to levy more taxes. This leads to increase mitigation and mechanically adaptation. To that extent and borrowing the words of Freeman *et al.* (2015), bad news could be good for environment:[11] Bad news induces the DM to implement a more active environmental policy. On the contrary, in case of a *good news*, the marginal benefit of mitigation is under-evaluated and the DM is incited to cut down the effort of mitigation. Notice

[11] In fact, the authors discuss whether good news about climate sensitivity might be in fact bad news in the sense that it lowers societal well-being.

that for extreme ambiguity attitudes, $\alpha = 0$ or $\alpha = 1$, the DM considers only one scenario and ignores the bad news or the good news, respectively.
In the AR5, it appears that the more optimistic scenario according to which emissions concentration will maintain under 430 ppm over the 21^{st} century is no more considered (with respect to previous reports) or is said to be *very unlikely*. The same holds for concentration levels under 550 ppm over the same period. Hence, everything goes as if the best scenarios disappear, meaning that the new best scenario $\hat{\theta}$ is less good than the previous ones. This could be understood as a *bad news*. However, consistently with our definition, this configuration must be interpreted as the reverse of a *good news*. Ultimately, this configuration should drive the DM to intensify her efforts of mitigation which is still a good news for the future environment.

Let us now take a step further by allowing the arrival of more than one new scenario. Interestingly, in that configuration, conclusions might differ. In particular, it depends on the type of combination between good and bad news. Suppose that among all the potential new scenarios, there exists at least one good news and one bad one.

Whatever the total number of new scenarios, the DM is only concerned with the ones that effectively modify the bounds of the interval. Consequently, what matters are the potential impacts of at least two news scenarios that would simultaneously affect the lower and the upper bound of the considered interval. More precisely, the global effect on T depends on the distance between new and initial scenarios and how it is sensitive to the effort of mitigation. To that extent, the relative weight of the good and bad news is determined by the DM's degree of pessimism, α_{DM}. If $\alpha_{DM} > 1/2$, the DM is called pessimistic otherwise she is optimistic. Our results are the following:

Proposition 4 *Let us define a threshold value* $0 \leq \alpha_0(T^*) \leq 1$ *such that*

$$\alpha_0(T^*) \equiv \left[1 + \frac{\Gamma(\tilde{\theta}, \underline{\theta}, T^*)}{\Gamma(\overline{\theta}, \hat{\theta}, T^*)} \right]^{-1}$$

with

$$\Gamma(\theta_1, \theta_2, T) = \int_{\underline{Q}}^{\overline{Q}} \left[-(1-\rho)\varphi'(\cdot)v''(q+\varphi(\cdot)) \left(F_{\theta_1}(q,T) - F_{\theta_2}(q,T)\right) + v'(q+\varphi(\cdot)) \left(\frac{\partial F_{\theta_2}(q,T)}{\partial T} - \frac{\partial F_{\theta_1}(q,T)}{\partial T} \right) \right] dq$$

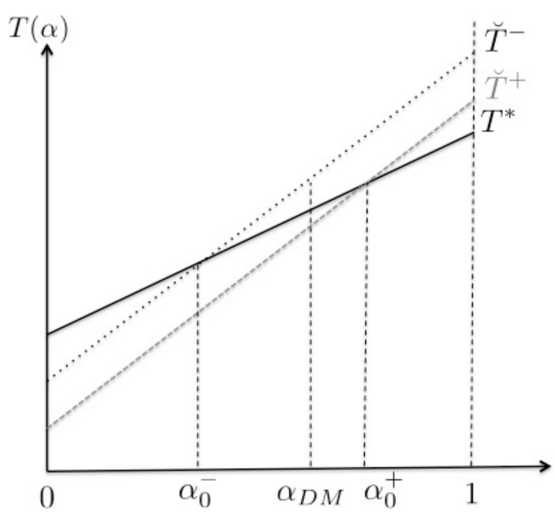
A change in ambiguity induces the DM to increase the effort of mitigation iff she is sufficiently *pessimistic,* $\alpha_{DM} > \alpha_0(T^*)$.

Proof. See Appendix D. ■

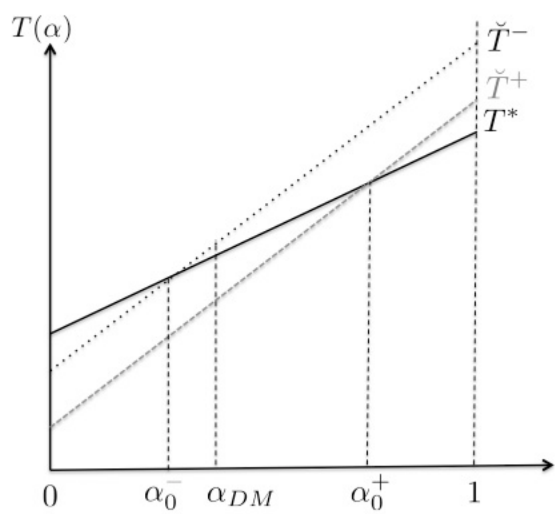
The role played by the degree of ambiguity aversion, α_{DM}, is crucial to explain the changes induced by a modification of ambiguity. When the DM is sufficiently pessimistic, $\alpha_{DM} > \alpha_0(T^*)$, she over-weights the bad news and thus increases the mitigation effort. Similarly, when she is sufficiently optimistic, $\alpha_{DM} < \alpha_0(T^*)$, she over-weights the good news and thus lowers the mitigation effort.

The threshold value, $\alpha_0(T^*)$, is the degree of pessimism such that the new scenarios let unchanged the optimal effort of mitigation. It notably depends on the relative strength of the bad vs the good news, so-called the *informational effect*. More precisely, it is negative (positive) when the deterioration involved by the bad news, $\tilde{\theta}$, is larger (lower) than the improvement induced by the good one, $\widehat{\theta}$. For instance, we argue that if the bad news dominates, $\alpha_0(T^*)$ is small.

If $\alpha_{DM} = \alpha_0(T^*)$, the DM does not change her decision since the *informational effect* vanishes. Else, the *informational effect* becomes relevant as the effort of mitigation changes. The sense of variation depends on a second effect called the *ambiguity aversion effect*. We identify a pessimism (optimism) effect when $\alpha_{DM} > (<)\alpha_0(T^*)$, such that the level of mitigation increases (decreases). Consequently, these two effects should be considered simultaneously. According to their magnitude, the DM can augment or reduce the effort of mitigation (see Figure 1).



(a) A weak pessimistic Decision Maker



(b) A weak optimistic Decision Maker

Fig. 1. *Change in ambiguity*

First, we highlight two configurations with regards to the *informational effect* depending on whether it is negative or positive. In the former case, we denote $\alpha_0(T^*)$ by $\alpha_0^-(T^*)$ and in the latter one by $\alpha_0^+(T^*)$ respectively. Second, we distinguish two types of DM: A pessimistic and an optimistic one.

Let us begin with a pessimistic DM who initially implements T^*. On the one hand, pessimism incites the DM to increase mitigation; On the other hand, a negative (positive) *informational effect* tends to increase (decrease) mitigation. Following Figure 1(a), when the *informational effect* is negative, $\alpha_0^-(T^*)$ is relatively small, $\alpha_{DM} > \alpha_0^-(T^*)$, The both effects are in the same direction: the effort of mitigation augments, $\breve{T}^- > T^*$. When the informational effect is positive, the two effects are opposite. The *pessimism effect* overcomes the *positive informational effect* if the DM is very pessimistic, $\alpha_{DM} > \alpha_0^+(T^*)$. Else, if she is weakly pessimistic, $\alpha_{DM} < \alpha_0^+(T^*)$, the *informational effect* outweighs and the effort of mitigation diminishes, $\breve{T}^+(T^*) < T^*$.

Let us now consider an optimistic DM. On the one hand, optimism incites the DM to reduce mitigation; On the other hand, a negative (positive) informational effect tends to increase (decrease) mitigation. Let us see on Figure 1(b) how choices are modified when the DM is optimistic. If the *informational effect* is positive $\alpha_0^+(T^*)$ is relatively high and as the DM is optimistic, $\alpha_{DM} < \alpha_0^+(T^*)$, she reduces mitigation, $\breve{T}^+ < T^*$. Now, if the *informational effect* is negative, it could be possible that an optimistic DM increases the effort of mitigation. In that case, $\alpha_0^-(T^*)$ is relatively small. If the DM is weakly optimistic, $\alpha_{DM} > \alpha_0^-(T^*)$, and she augments the effort of mitigation, $\breve{T}^+ > T^*$.

4 Trade-off between Mitigation and Adaptation

Once we have identified the consequences of an ambiguous knowledge with regards to the climate issue on the optimal environmental policy, we aim at providing some policy recommendations, which involve both the nature of the effort to be undertaken (adaptation vs mitigation) and its distributional feature. Suppose then that the authority can choose both the level of taxes and the distribution of its revenue, ρ. The target of the DM is twofold: a better (or at least not so much deteriorated) environmental quality and a high enough utility associated to consumption.

4.1 Optimal choice

The optimal interior solution, that is the couple (T^*, ρ^*), satisfies simultaneously $\frac{\partial V(T^*,\rho^*)}{\partial T^*} = 0$ and $\frac{\partial V(T^*,\rho^*)}{\partial \rho^*} = 0$.

As for the optimal level of the tax (T^*), we directly refer to equation (2). With regards to the optimal level of redistribution (ρ^*), we obtain:

$$Tu'(c(T)) - T\varphi'(\cdot)\left[\int_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(\alpha f_{\underline{\theta}}(q,T) + (1-\alpha)f_{\overline{\theta}}(q,T)\right)dq\right] = 0 \tag{3}$$

We show that for any DM, it is optimal to choose a strictly positive environmental tax and this, independently of her ambiguity attitude. The redistribution rate of the tax is always strictly lower than 1, which means that the investment in adaptation is always strictly positive. This conclusion might depart from the one of Schumacher (2016) who shows that adaptation is not well fitted to fight climate change.

Proposition 5 *Under assumptions 1-2, the optimal policy is characterized by a strictly positive tax rate ($T^* > 0$), and a strictly positive investment in adaptation ($\rho^* < 1$).*

Proof. See Appendix E. ■

Mitigation and adaptation are two inseparable instruments of the environmental policy in the sense that a positive environmental tax always implies a positive investment in adaptation.

Let us consider a DM who can now choose the level of redistribution. It comes easily that T^* is increasing with ρ: At the optimum, $\frac{\partial^2 V}{\partial T \partial \rho} > 0$. The greater the capital invested in mitigation the greater is the redistribution. Then, the question is how the decision of mitigation is affected when the DM chooses the optimal level of redistribution. Let us suppose that the initial given level of redistribution (ρ^{initial}) is larger than the optimal one ρ^*, meaning that the effort of adaptation is initially small. Giving the opportunity for the DM to optimally choose redistribution implies that mitigation reduces $(T^* < T(\rho^{\text{initial}}))$ while adaptation $((1-\rho^*)T^*)$ could increase.[12]

It is often claimed that mitigation and adaptation to climate change are complementary strategies for dealing with climate change (AR5). However, only a few studies explicitly consider adaptation and mitigation as policy responses to climate change (see among others Kane and Shogren, 2000; Buob and Stephan, 2011).

[12] We can derive clear cut results when we perform our numerical example, see Section 4.2.

The question now is to understand how these two instruments vary with a change in ambiguity and risk aversion. Let us begin with risk aversion.
As previously, we study the impact of the "environmental" risk aversion by considering CARA and CRRA utility functions.

Proposition 6 *If the environmental utility function is CARA or CRRA, then an increase in risk aversion decreases environmental taxes T (for a given ρ) and increases the transfers, ρ (for a given T). The total effects are unclear.*

Proof. See Appendix F. ■

We can distinguish two opposite effects on mitigation and adaptation.
Firstly, a *positive direct effect*: An increase in risk aversion leads to a decrease in mitigation and adaptation. The effect on taxes is the same as in the previous section. Concerning the part invested in adaptation, a riskier averse agent prefers the risk-less source of welfare and increases wealth redistribution. Secondly, these effects can be reversed by a *negative return effect*. Indeed, as mitigation and redistribution are complements ($\frac{\partial^2 V}{\partial T \partial \rho} > 0$), the decrease in mitigation (taxes) incites the DM to decrease redistribution (ρ), and conversely. This second effect is opposite to the first one and consequently the total effect is ambiguous.

Proposition 7 *Under assumptions 1-2, an increase in ambiguity aversion increases environmental taxes T (for a given ρ), and decreases the transfers, ρ (for a given T). The total effects are unclear.*

Proof. See Appendix G. ■

Like for risk aversion, we can distinguish two opposite effects on mitigation and adaptation.
A *positive direct effect*: An increase in ambiguity aversion leads to an increase in taxes and a decrease in the part intended to wealth redistribution. The effect on taxes is the same as in the previous section. Concerning the part invested in adaptation, a more ambiguity averse agent over-evaluates the likelihood of the worst possible scenario, and thus over-evaluates the marginal benefit of adaptation. Thus, it incites her to increase the level of adaptation (or decrease the wealth redistribution). A *negative return effect*: The increase in mitigation (taxes) incites the DM to increase the redistribution (ρ), and conversely. This second effect is opposite to the first one and consequently the total effect is once again ambiguous.

The impact of ambiguity aversion and risk aversion are less clear than in the previous section due to the presence of some return effects. That is why we propose a numerical example in the following section.

4.2 Numerical example

Consistently with the previous sections, let us now present a simulation exercise to explore with more details some of the implications of our theoretical model.[13] To isolate the different effects induced either by risk aversion or ambiguity aversion on optimal decisions and, in order to emphasize the role played by environmental preferences, we assume that the utility of wealth is linear such that $u(c) = c$ and that the utility for environmental quality is of CARA type: $v(x) = -\frac{1}{\beta}\exp^{-\beta x}$, where β is the constant coefficient of absolute risk aversion. In addition, the relation between income and taxes is given by $y(T) = \frac{y_0}{1+0.01T}$ with $y_0 > 0$ and the adaptation technology is represented by $\varphi(x) = x^\gamma$ with $\gamma \in [0, 1]$. Finally, the random variable follows a mixed exponential distribution where θ_0 captures the existence of an ideal scenario, that might be out of reach. The density function, for a given θ, is given by $f_\theta(q, T) = \frac{1}{(1+T)\theta}\exp^{-\frac{q}{\theta}} + (1 - \frac{1}{(1+T)})\frac{1}{\theta_0}\exp^{-\frac{q}{\theta_0}}$. And the distribution function is $F_\theta(q, T) = 1 - \frac{1}{1+T}\exp^{\frac{-q}{\theta}} - (1 - \frac{1}{1+T})\exp^{\frac{-q}{\theta_0}}$. We can easily check that such a probability distribution satisfies all the assumptions made in the theoretical framework proposed in the paper.

In order to perform the maximization and numerically derive the optimal couples (T, ρ) for given values of ambiguity aversion, we arbitrarily choose the following baseline parameters (See Table 1). Concerning ambiguity attitude, studies find strong support for ambiguity aversion. This is true as well in experiments with students (see for instance Cho and Sarin 2001), as with a representative sample of a general population (Dimmock *et al.* 2016) and also for risk professionals (see Cabantous 2007 for insurers). Moreover, experimental, and also neuro-economic studies conclude to the independence of ambiguity and risk attitudes. Borghans et al. (2009) show that the determinants of risk and ambiguity attitudes are different, and Hsu *et al.*(2005) conclude that the brain areas activated in the two decision situations are not the same. In our illustration, we evaluate the welfare function for values of α going from $[0, 1]$ and varying following an incremental increase of 0.05. The resulting list of optimal couples (T, ρ) was sufficiently exhaustive to obtain clear trends.

Our objective is at first to study the impact of risk and ambiguity aversion on the optimal couple (T, ρ). Figure 2 plots the change in taxation as a function of the coefficient of ambiguity

[13] The numerical results are obtained with Mathematica 9.

	Parameter	Value
Elasticity of adaptation	γ	0.5
Worst scenario	$\underline{\theta}$	$\frac{2}{3}$
Best scenario	$\overline{\theta}$	2
Ideal scenario	θ_0	10
Potential production	y_0	100
Initial risk aversion	β	0.5
Good news	$\hat{\theta}$	3
Bad news	$\tilde{\theta}$	$\frac{1}{3}$

Table 1: Parameters' value

aversion. Similarly, Figure 3 plots the change in redistribution as a function of the same coefficient. We obtain unequivocal results so that mitigation efforts increase with ambiguity aversion whereas the share of fiscal resources refunded towards agents reduces. We can thus observe that mitigation and adaptation are complements in the sense that an increase in the carbon tax comes along with a simultaneous development of adaptation strategies (Figure 4). In our example, the direct effects are greater than the return effect.

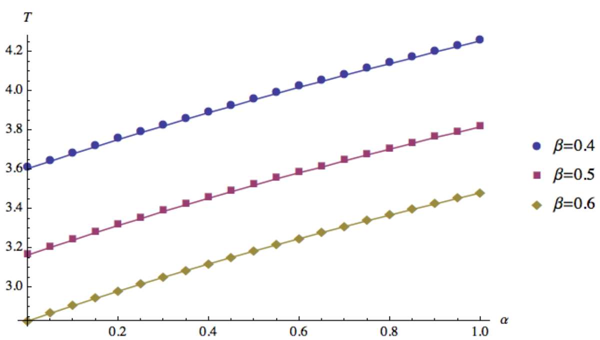


Fig. 2: Mitigation and ambiguity aversion

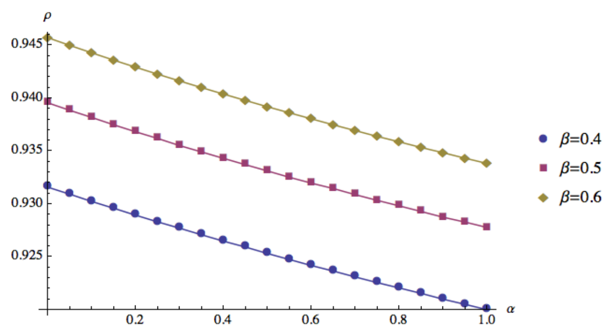


Fig. 3: Redistribution and ambiguity aversion

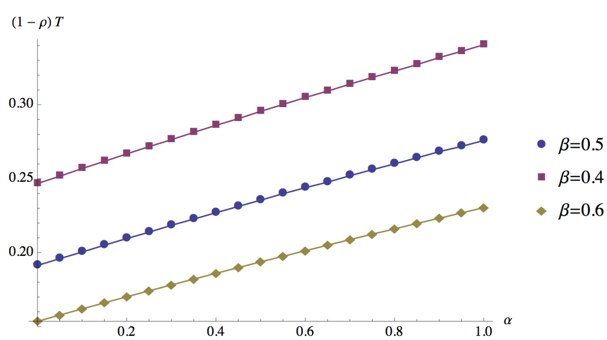


Fig. 4: Adaptation and ambiguity aversion

In addition, these three graphs show how the level of tax and redistribution is modified when risk aversion changes. In particular, we can observe that the more risk adverse is the DM, β higher, the lower the efforts of both mitigation and adaptation, for a given attitudes towards ambiguity. As a consequence, an increase in ambiguity aversion entails a more severe environmental policy whereas an increase in risk aversion softens it.

In this particular example it appears that an increase in ambiguity aversion has lower impact on taxes than an increase in risk aversion. An increase in the set of scenarios would probably modify this result. To conclude, it appears that both risk aversion and ambiguity aversion have to be estimated in order to characterize the optimal mitigation and adaptation policies.

The other important conclusion is the optimality of a policy mix (mitigation and adaptation) for any level of risk and ambiguity aversion.

Let us now introduce additional scenarios implying a change in ambiguity. On figure 5 and 6 are depicted the changes in mitigation and redistribution following a change in ambiguity, respectively.

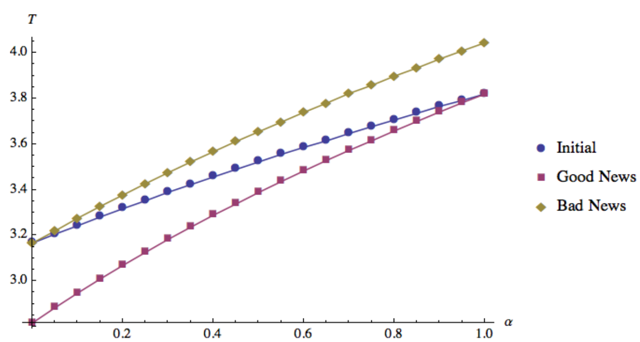


Fig. 5: Mitigation and changes in ambiguity

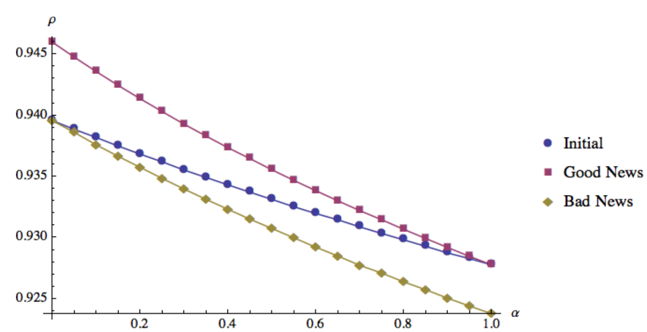


Fig. 6: Redistribution and changes in ambiguity

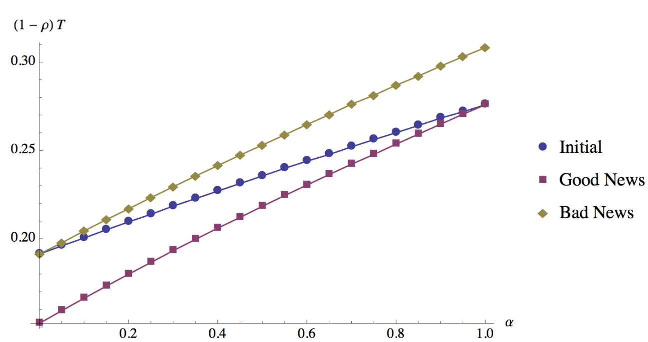


Fig. 7: Adaptation and changes in ambiguity

We can observe that, as the weight given to the worst and the better distribution evolves, the carbon tax moves closer (further) to the initial solution when a good (bad) news arrives. We obtain similar result with regards to the redistribution. This can be explained focusing on the arrival of a good news. Indeed, as α grows the weight given to the better scenario increases. For an extreme value, $\alpha = 0$, the only scenario considered is the best one. When a good news arrives, the better scenario is even better and thus the effort of mitigation is lower and the redistribution is higher. When, $\alpha = 1$, we found the benchmark solution since the only considered scenario is the worst, which is identical in both configuration (with and without the additional scenario). These results extend results in Proposition 3 when the DM makes trade-off between mitigation and adaptation.

Finally, let us observe the results when two new scenarios are introduced. Similarly to the case of only one decision (Proposition 4), there exist some thresholds values, α_0 on the level of ambiguity aversion (see Figures 8-10). If the DM is sufficiently pessimistic, $\alpha_{DM} > \alpha_0$, she increases the effort of mitigation, she decreases the redistribution rate and finally, increases the effort of adaptation. Else, she increases the effort of both mitigation and adaptation.

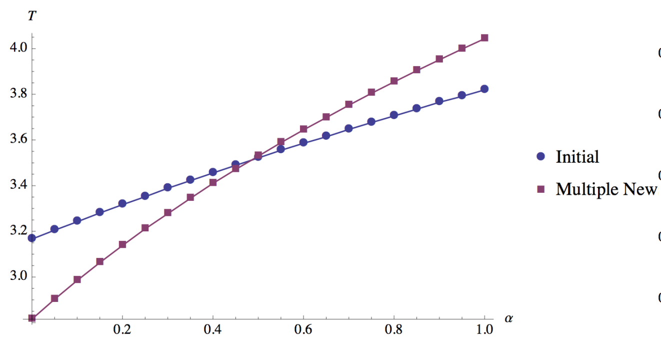


Fig. 8: Mitigation when two scenarios arrive

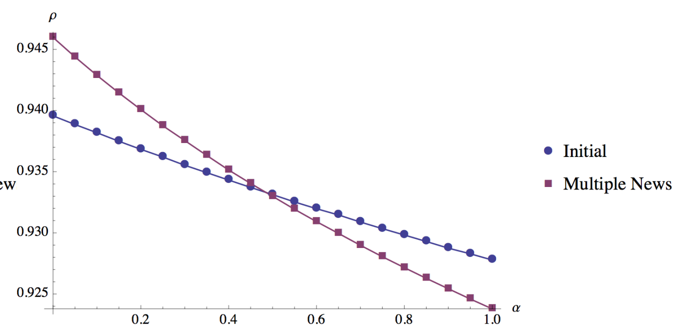


Fig. 9: Redistribution when two scenarios arrive

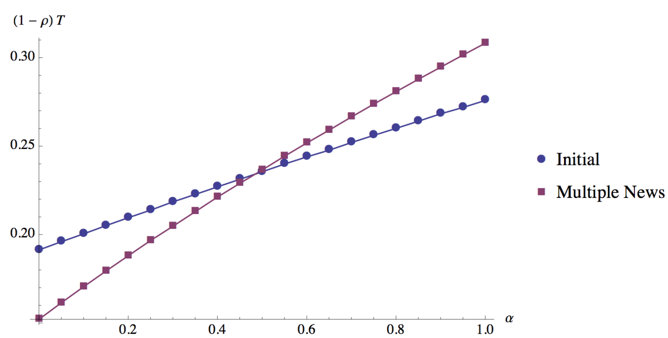


Fig. 10: Adaptation when two scenarios arrive

5 Conclusion

In this paper, we have assessed the impact of ambiguity and ambiguity attitudes on the optimal design of an environmental policy that aims at fighting against climate change. to that aim, we use a MMEU model of preferences representation that allows us to capture the ambiguous feature of the future environmental quality through the existence of many different scenarios. We show that ambiguity aversion does play a significant role in designing optimal environmental policy, and differently from risk aversion. We also state that a change in the information structure may have a positive impact by triggering more efforts of both mitigation and adaptation, when bad scenarios appear in the new set of information. We also argue that an optimal policy should favor a mixed strategy, that is developing mitigation and adaptation while implementing a system of lump-sum refund towards agents. This result of complementary can be reinforced when many new different scenarios are to be considered.

While we allow for the arrival of new information, our framework does not account for a dynamic setting that clearly should be investigated soon in the future.

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6 Appendices

A Proof of Proposition 1

Using integration by parts, equation (2) can be written as follows:

$$-(y'(T)+\rho)u'(c) = \int_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))h(q)dq, \tag{A.1}$$

with $h(q) \equiv (1-\rho)\varphi'(\cdot)\left(\alpha f_{\underline{\theta}}(q,T) + (1-\alpha)f_{\overline{\theta}}(q,T)\right) - \left(\alpha\frac{\partial F_{\underline{\theta}}(q,T)}{\partial T} + (1-\alpha)\frac{\partial F_{\overline{\theta}}(q,T)}{\partial T}\right)$, $h(q) > 0$ and $h'(q) > 0$.

Then, we consider two specific functions to describe the utility derived from environmental quality:

- If v is a CARA utility function given by $v(x) = -\frac{1}{\beta}\exp(-\beta x)$, where β is the coefficient of absolute risk aversion, then the FOC (A.1) writes:

$$-(y'(T)+\rho)u'(y(T)+\rho T) = \int_{\underline{Q}}^{\overline{Q}} \exp\left(-\beta(q+\varphi(\cdot)\right) h(q)dq \tag{A.2}$$

Using the implicit functions theorem, we get that $sign\dfrac{dT^*}{d\beta} = sign - \dfrac{\frac{\partial^2 V(T,\rho)}{\partial T\partial\beta}}{\frac{\partial^2 V(T,\rho)}{\partial T^2}}$. Since the maximization problem is concave, it comes that $\frac{\partial^2 V(T,\rho)}{\partial T^2} < 0$. Thus, the nature of the impact of risk aversion on optimal mitigation is given by the sign of $\frac{\partial^2 V(T,\rho)}{\partial T\partial\beta}$ such that :

$$\frac{\partial^2 V(T,\rho)}{\partial T\partial\beta} = -\int_{\underline{Q}}^{\overline{Q}} (q+\varphi(\cdot))\exp\left(-\beta(q+\varphi(\cdot))\right) h(q)dq < 0 \tag{A.3}$$

Thus, T^* is a decreasing function of β.

- If v is a CRRA utility function given by $v(x) = \frac{x^{1-\gamma}}{1-\gamma}$, where γ the coefficient of relative risk aversion, then the FOC (A.1) becomes:

$$-(y'(T)+\rho)u'(y(T)+\rho T) = \int_{\underline{Q}}^{\overline{Q}} [q+\varphi(\cdot)]^{-\gamma} h(q)dq \tag{A.4}$$

Using the implicit functions theorem, the sign of $\dfrac{dT^*}{d\gamma}$ is also given by the sign of the following expression:

$$\frac{\partial^2 V(T,\rho)}{\partial T\partial\gamma} = -\int_{\underline{Q}}^{\overline{Q}} \ln(q+\varphi(\cdot)) \times [q+\varphi(\cdot)]^{-\gamma} h(q)dq < 0 \tag{A.5}$$

Thus, T^* is a decreasing function of γ.

B Proof of Proposition 2

To determine the sign of $\frac{dT^*}{d\alpha}$, it is required to differentiate (2) with respect to the degree of ambiguity aversion, α. Then, integrating by parts yields:

$$\begin{aligned}\frac{\partial^2 V(T,\rho)}{\partial T \partial \alpha} \quad = \quad & -(1-\rho)\varphi'(\cdot)\int\limits_{\underline{Q}}^{\overline{Q}} v''(q+\varphi(\cdot))\left[F_{\underline{\theta}}(q,T)-F_{\overline{\theta}}(q,T)\right]dq \\ & -\int\limits_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left[\frac{\partial F_{\underline{\theta}}(q,T)}{\partial T}-\frac{\partial F_{\overline{\theta}}(q,T)}{\partial T}\right]dq \end{aligned} \tag{B.6}$$

Through Assumption 2 and since $v'' < 0$, it comes that $\frac{\partial^2 V(T,\rho)}{\partial T \partial \alpha}$ is positive so as $\frac{dT^*}{d\alpha}$.

C Proof of Proposition 3

Let us consider a new scenario $\hat{\theta}$, associated with a new welfare function denoted $\hat{V}(T,\rho)$ and a subsequent "new" optimal tax $\hat{T} = \arg\max \hat{V}(T,\rho)$. Three configurations might occur:

- When $\hat{\theta}$ is worst than all the initial scenarios then $F_{\underline{\theta}}(q,T) \leq F_{\hat{\theta}}(q,T)$. We can state that $\hat{T} > T^* \Leftrightarrow \frac{\partial V(\hat{T},\rho)}{\partial T} < \frac{\partial \hat{V}(\hat{T},\rho)}{\partial T}$. This inequality is satisfied if

$$\begin{aligned} & \alpha(1-\rho)\varphi'(\cdot)\left[\int\limits_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(f_{\underline{\theta}}(q,T)-f_{\hat{\theta}}(q,T)\right)dq\right] \\ & +\alpha\int\limits_{\underline{Q}}^{\overline{Q}} v(q+\varphi(\cdot))\left(\frac{\partial f_{\underline{\theta}}(q,T)}{\partial T}-\frac{\partial f_{\hat{\theta}}(q,T)}{\partial T}\right)dq < 0 \end{aligned} \tag{C.7}$$

Integrating by parts, the inequality above rewrites:

$$\begin{aligned} & -\alpha(1-\rho)\varphi'(\cdot)\left[\int\limits_{\underline{Q}}^{\overline{Q}} v''(q+\varphi(\cdot))\left(F_{\underline{\theta}}(q,T)-F_{\hat{\theta}}(q,T)\right)dq\right] \\ & -\alpha\int\limits_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(\frac{\partial F_{\underline{\theta}}(q,T)}{\partial T}-\frac{\partial F_{\hat{\theta}}(q,T)}{\partial T}\right)dq < 0 \end{aligned} \tag{C.8}$$

Through Assumption 2, this inequality is always verified for $\alpha > 0$, meaning that $\hat{T} > T^*$.

- When $\hat{\theta}$ is better that the existing ones, then $F_{\hat{\theta}}(q,T) \leq F_{\overline{\theta}}(q,T)$. As previously, $\hat{T} > T^* \Leftrightarrow \frac{\partial V(\hat{T},\rho)}{\partial T} < \frac{\partial \hat{V}(\hat{T},\rho)}{\partial T}$. We deduce that this inequality is satisfied if

$$(1-\alpha)(1-\rho)\varphi'(\cdot)\left[\int_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(f_{\overline{\theta}}(q,T) - f_{\hat{\theta}}(q,T)\right) dq\right]$$
$$+(1-\alpha)\int_{\underline{Q}}^{\overline{Q}} v(q+\varphi(\cdot))\left(\frac{\partial f_{\overline{\theta}}(q,T)}{\partial T} - \frac{\partial f_{\hat{\theta}}(q,T)}{\partial T}\right) dq < 0 \tag{C.9}$$

Integrating by parts, the inequality above rewrites:

$$-(1-\alpha)(1-\rho)\varphi'(\cdot)\left[\int_{\underline{Q}}^{\overline{Q}} v''(q+\varphi(\cdot))\left(F_{\overline{\theta}}(q,T) - F_{\hat{\theta}}(q,T)\right) dq\right]$$
$$-(1-\alpha)\int_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(\frac{\partial F_{\overline{\theta}}(q,T)}{\partial T} - \frac{\partial F_{\hat{\theta}}(q,T)}{\partial T}\right) dq < 0 \tag{C.10}$$

Through Assumption 2, this inequality is never verified for $\alpha < 1$, meaning that $\hat{T} < T^*$.

- When $\hat{\theta}$ is a neutral additional scenario and such that $F_{\underline{\theta}}(q,T) > F_{\hat{\theta}}(q,T) > F_{\overline{\theta}}(q,T)$. We easily show that $V(T,\rho) = \hat{V}(T,\rho)$. Hence, there is no effect on the optimal tax : $T = \hat{T}$.

D Proof of Proposition 4

Let us consider two additional scenarios $\tilde{\theta}$, being worst than all the other ones and $\widehat{\theta}$, better that the existing ones. Recall that, by definition, we have

1. $F_{\tilde{\theta}}(q,T) \geq F_{\underline{\theta}}(q,T)$ and $F_{\overline{\theta}}(q,T) \geq F_{\widehat{\theta}}(q,T)$,
2. $\frac{\partial F_{\underline{\theta}}(q,T)}{\partial T} \geq \frac{\partial F_{\tilde{\theta}}(q,T)}{\partial T}$ and $\frac{\partial F_{\widehat{\theta}}(q,T)}{\partial T} \geq \frac{\partial F_{\overline{\theta}}(q,T)}{\partial T}$.

Let us now denote by $\breve{T}$, the optimal level in this case, that is the one which maximizes the following welfare function:

$$\begin{aligned}\breve{V}(T;\rho) &= u(y(T)+\rho T)+\alpha\int\limits_{\underline{Q}}^{\overline{Q}} v(q+\varphi\left((1-\rho)T\right) f_{\widehat{\theta}}(q,T)dq \\ &\quad +(1-\alpha)\int\limits_{\underline{Q}}^{\overline{Q}} v(q+\varphi\left((1-\rho)T\right) f_{\tilde{\theta}}(q,T)dq,\end{aligned}$$

Then, $\breve{T} > T^* \Leftrightarrow \frac{\partial V(\breve{T},\rho)}{\partial T} < \frac{\partial \breve{V}(\hat{T},\rho)}{\partial T}$. This inequality is satisfied if

$$\begin{aligned}&\alpha(1-\rho)\varphi'(\cdot)\left[\int\limits_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(f_{\underline{\theta}}(q,T)-f_{\tilde{\theta}}(q,T)\right)dq\right] \\ &+(1-\alpha)(1-\rho)\varphi'(\cdot)\left[\int\limits_{\underline{Q}}^{\overline{Q}} v'(q+\varphi(\cdot))\left(f_{\overline{\theta}}(q,T)-f_{\hat{\theta}}(q,T)\right)dq\right] \\ &+\alpha\int\limits_{\underline{Q}}^{\overline{Q}} v(q+\varphi(\cdot))\left(\frac{\partial f_{\underline{\theta}}(q,T)}{\partial T}-\frac{\partial f_{\tilde{\theta}}(q,T)}{\partial T}\right) \\ &+(1-\alpha)\int\limits_{\underline{Q}}^{\overline{Q}} v(q+\varphi(\cdot))\left(\frac{\partial f_{\overline{\theta}}(q,T)}{\partial T}-\frac{\partial f_{\hat{\theta}}(q,T)}{\partial T}\right)dq<0 \end{aligned} \tag{D.11}$$

By integrating by parts, the inequality above rewrites:

$$\begin{aligned}&-(1-\rho)\varphi'(\cdot)\left[\int\limits_{\underline{Q}}^{\overline{Q}} v''(.)\left[\alpha\left(F_{\underline{\theta}}(q,T)-F_{\tilde{\theta}}(q,T)\right)+(1-\alpha)\left(F_{\overline{\theta}}(q,T)-F_{\widehat{\theta}}(q,T)\right)\right]dq\right] \\ &-\int\limits_{\underline{Q}}^{\overline{Q}} v'(.)\left[\alpha\left(\frac{\partial F_{\underline{\theta}}(q,T)}{\partial T}-\frac{\partial F_{\tilde{\theta}}(q,T)}{\partial T}\right)+(1-\alpha)\left(\frac{\partial F_{\overline{\theta}}(q,T)}{\partial T}-\frac{\partial F_{\widehat{\theta}}(q,T)}{\partial T}\right)\right]dq<0\end{aligned}$$

which is equivalent to

$$\alpha > \alpha_0 \equiv \frac{-(1-\rho)\varphi'(\cdot)\int v''(.)\left(F_{\overline{\theta}}-F_{\widehat{\theta}}\right)dq+\int v'(.)\left(\frac{\partial F_{\overline{\theta}}}{\partial T}-\frac{\partial F_{\widehat{\theta}}}{\partial T}\right)dq}{-(1-\rho)\varphi'(\cdot)\int v''(.)\left(F_{\overline{\theta}}-F_{\widehat{\theta}}+F_{\tilde{\theta}}-F_{\underline{\theta}}\right)dq+\int v'(.)\left(\frac{\partial F_{\widehat{\theta}}}{\partial T}-\frac{\partial F_{\overline{\theta}}}{\partial T}+\frac{\partial F_{\underline{\theta}}}{\partial T}-\frac{\partial F_{\tilde{\theta}}}{\partial T}\right)dq}$$

Consequently, $\breve{T} > T^*$ iff $\alpha > \alpha_0$.

E Proof of Proposition 5

Using the two First Order Conditions, equations (2) and (3) respectively evaluated at $T = 0$ and $\rho = 1$, we have:

$$\begin{aligned}\frac{\partial V(0,\rho)}{\partial T} &= (y'(0)+\rho)u'(y(0)+\rho T) \\ &+(1-\rho)\varphi'(0)\left[\int_{\underline{q}}^{\overline{q}} v'\left(q+\varphi\left(0\right)\right)\left(\alpha f_{\overline{\theta}}(q,T)+(1-\alpha)f_{\underline{\theta}}(q,T)\right)dq\right] \\ &+\int_{\underline{Q}}^{\overline{Q}} v\left(q+\varphi\left(0\right)\right)\left(\alpha\frac{\partial f_{\overline{\theta}}(q,0)}{\partial T}+(1-\alpha)\frac{\partial f_{\underline{\theta}}(q,T)}{\partial T}\right)dq\end{aligned}$$

$$\begin{aligned}\frac{\partial V(T,1)}{\partial \rho} &= Tu'(y(T)+T) \\ &-z\varphi'(0)\left[\int_{\underline{Q}}^{\overline{Q}} v'\left(q+\varphi\left(0\right)\right)\left(\alpha f_{\overline{\theta}}(q,T)+(1-\alpha)f_{\underline{\theta}}(q,T)\right)dq\right]\end{aligned}$$

Since $\varphi'(0) \to +\infty$ by assumption, $\frac{\partial V(0,\rho)}{\partial T} > 0$ and $\frac{\partial V(T,1)}{\partial \rho} < 0$.

F Proof of Proposition 6

First of all, let us recall that we only consider interior solutions. By Assumptions 1-**??**, the second derivatives $\frac{\partial^2 V(T,\rho)}{\partial T^2}$ and $\frac{\partial^2 V(T,\rho)}{\partial \rho^2}$ are negative and $\frac{\partial^2 V(T,\rho)}{\partial T \partial \rho} > 0$ at the optimum. Moreover, the determinant of the Hessian matrix, $\Delta = \frac{\partial^2 V(T,\rho)}{\partial T^2}\frac{\partial^2 V(T,\rho)}{\partial \rho^2} - \left(\frac{\partial^2 V(T,\rho)}{\partial T \partial \rho}\right)^2$, is supposed positive.

The proof is similar to proof of Proposition 1. We consider two specific functions to describe the utility derived from environmental quality.

- If v is CARA, $v(x) = -\frac{1}{\beta}\exp\left(-\beta x\right)$, the FOCs (2) and (3) write:

$$\frac{\partial V(T,\rho)}{\partial T} = (y'(T)+\rho)u'(y(T)+\rho T) + \int_{\underline{Q}}^{\overline{Q}} \exp\left(-\beta(q+(1-\rho)\varphi(.))\right)h(q)dq \tag{F.12}$$

with $h(q) = \left[(1-\rho)\varphi'(.) \times \left(\alpha f_{\overline{\theta}} + (1-\alpha) f_{\underline{\theta}}\right) - \left(\alpha \frac{\partial F_{\overline{\theta}}}{\partial T} + (1-\alpha)\frac{\partial F_{\underline{\theta}}}{\partial T}\right)\right]$ a positive function of q, and

$$\frac{\partial V(T,\rho)}{\partial T} = Tu'(y(T)+\rho T) - \int_{\underline{Q}}^{\overline{Q}} \exp\left(-\beta(q+(1-\rho)\varphi(.))\right) \ell(q) dq \qquad \text{(F.13)}$$

with $\ell(q) = \alpha f_{\overline{\theta}} + (1-\alpha) f_{\underline{\theta}}$.

The effect of risk aversion is given by a change in β, the Absolute Risk Aversion coefficient. Thus, we have to determine the expressions of $\frac{\partial^2 V(T,\rho)}{\partial T \partial \beta}$ and $\frac{\partial^2 V(T,\rho)}{\partial \rho \partial \beta}$:

$$\frac{\partial^2 V(T,\rho)}{\partial T \partial \beta} = -\int_{\underline{Q}}^{\overline{Q}} (q+(1-\rho)\varphi(.)) \exp\left(-\beta(q+(1-\rho)\varphi(.))\right) h(q) dq < 0$$

$$\frac{\partial^2 V(T,\rho)}{\partial \rho \partial \beta} = +\int_{\underline{Q}}^{\overline{Q}} (q+(1-\rho)\varphi(.)) \exp\left(-\beta(q+(1-\rho)\varphi(.))\right) \ell(q) dq > 0$$

The total effects of a change in risk aversion are given by:

$$\frac{dT}{d\beta} = \frac{V_{\rho T} V_{\rho\beta} - V_{T\beta} V_{\rho\rho}}{\Delta} \text{ and } \frac{d\rho}{d\beta} = \frac{V_{\rho T} V_{T\beta} - V_{TT} V_{\rho\beta}}{\Delta}.$$

with $V_{ij} = \frac{\partial^2 V}{\partial i \partial j}$.

We can distinguish two effects:

1. For a given ρ, an increase in β incites the DM to decrease T, this is described by $-V_{T\beta}V_{\rho\rho}$ which is negative. Simultaneously, for a given T, an increase in β incites the DM to increase ρ, this is described by $-V_{TT}V_{\rho\beta}$ which is positive.
2. As T and ρ are substitutes in the sense where $V_{T\rho} > 0$, the decrease in T incites the DM to decrease ρ, $V_{\rho T}V_{T\beta} < 0$, and the increase in ρ incites the DM to increase T, $V_{\rho T}V_{\rho\beta} > 0$.

The global effects are not well determined.

- Now consider that v is CRRA, $v(x) = \frac{x^{1-\gamma}}{1-\gamma}$, the FOCs (2) and (3) write:

$$\frac{\partial V(T,\rho)}{\partial T} = (y'(T)+\rho)u'(y(T)+\rho T) + \int_{\underline{Q}}^{\overline{Q}} [q+(1-\rho)\varphi(.)]^{-\gamma} h(q)dq \tag{F.14}$$

$$\frac{\partial V(T,\rho)}{\partial \rho} = Tu'(y(T)+\rho T) - \int_{\underline{Q}}^{\overline{Q}} [q+(1-\rho)\varphi(.)]^{-\gamma} \ell(q)dq \tag{F.15}$$

The effect of risk aversion is given by a change in γ, the Relative Risk Aversion coefficient. Thus, we have to determine the expressions of $\frac{\partial^2 V(T,\rho)}{\partial T \partial \gamma}$ and $\frac{\partial^2 V(T,\rho)}{\partial \rho \partial \gamma}$:

$$\frac{\partial^2 V(T,\rho)}{\partial T \partial \gamma} = -\int_{\underline{Q}}^{\overline{Q}} \ln(q+(1-\rho)\varphi(.))[q+(1-\rho)\varphi(.)]^{-\gamma} h(q)dq < 0$$

$$\frac{\partial^2 V(T,\rho)}{\partial \rho \partial \gamma} = +\int_{\underline{Q}}^{\overline{Q}} \ln(q+(1-\rho)\varphi(.))[q+(1-\rho)\varphi(.)]^{-\gamma} \ell(q)dq > 0$$

The total effects of a change in risk aversion are given by:

$$\frac{dT}{d\gamma} = \frac{V_{\rho T}V_{\rho\gamma} - V_{T\beta}V_{\rho\rho}}{\Delta} \text{ and } \frac{d\rho}{d\gamma} = \frac{V_{\rho T}V_{T\gamma} - V_{TT}V_{\rho\gamma}}{\Delta}.$$

We can distinguish two same effects as in the first case:

1. For a given ρ, an increase in γ incites the DM to decrease T, $-V_{T\gamma}V_{\rho\rho} < 0$. Simultaneously, for a given T, an increase in γ incites the DM to increase ρ, $-V_{TT}V_{\rho\gamma} > 0$.
2. As T and ρ are substitutes, the decrease in T incites the DM to decrease ρ, $V_{\rho T}V_{T\gamma} < 0$, and the increase in ρ incites the DM to increase T, $V_{\rho T}V_{\rho\gamma} > 0$.

The global effects are not well determined.

G Proof of Proposition 7

The proof is similar to the proof of Proposition 6.

To obtain the effects of ambiguity aversion, α, we need the expressions of $\frac{\partial^2 V(T,\rho)}{\partial T \partial \alpha}$ and $\frac{\partial^2 V(T,\rho)}{\partial \rho \partial \alpha}$:

$$\begin{aligned}
\frac{\partial^2 V(T,\rho)}{\partial T \partial \alpha} &= -(1-\rho) z \varphi'(.) \left[\int_{\underline{Q}}^{\overline{Q}} v''\left(q + \varphi\left((1-\rho)T\right)\right) \left(F_{\overline{\theta}}(q,T) - F_{\underline{\theta}}(q,T)\right) dq \right] \\
& \quad - \int_{\underline{Q}}^{\overline{Q}} v'\left(q + \varphi\left((1-\rho)T\right)\right) \left(\frac{\partial F_{\overline{\theta}}(q,T)}{\partial T} - \frac{\partial F_{\underline{\theta}}(q,T)}{\partial T} \right) dq > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 V(T,\rho)}{\partial \rho \partial \alpha} &= -T \varphi'(.) \left[\int_{\underline{Q}}^{\overline{Q}} v'\left(q + \varphi\left((1-\rho)T\right)\right) \left(f_{\overline{\theta}}(q,T) - f_{\underline{\theta}}(q,T)\right) dq \right] \\
&= T \varphi'(.) \left[\int_{\underline{Q}}^{\overline{Q}} v''\left(q + \varphi\left((1-\rho)T\right)\right) \left(F_{\overline{\theta}}(q,T) - F_{\underline{\theta}}(q,T)\right) dq \right] < 0
\end{aligned}$$

The total effects of a change in ambiguity aversion are thus given by:

$$\frac{dT}{d\alpha} = \frac{V_{\rho T} V_{\rho\alpha} - V_{T\alpha} V_{\rho\rho}}{\Delta} \text{ and } \frac{d\rho}{d\alpha} = \frac{V_{\rho T} V_{T\alpha} - V_{TT} V_{\rho\alpha}}{\Delta}$$

As for risk aversion, we can distinguish two effects:

1. For a given ρ, an increase in α incites the DM to increase T, this is described by $-V_{T\alpha} V_{\rho\rho}$ which is positive. Simultaneously, for a given T, an increase in α incites the DM to decrease ρ, this is described by $-V_{TT} V_{\rho\alpha}$ which is negative.

2. As T and ρ are substitutes in the sense where $V_{T\rho} > 0$, the increase in T incites the DM to increase ρ, $V_{\rho T} V_{T\alpha} > 0$, and the decrease in ρ incites the DM to decrease T, $V_{\rho T} V_{\rho\alpha} < 0$.

The global effects are unclear.