

# Who controls the controller? A dynamical model of corruption

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## Abstract

The aim of this paper is to give a partial answer to the question made in the title. Several works analyse the evolution of the corruption in different societies. Most of such papers show the necessity of several controls displayed by a central authority to deterrence the expansion of the corruption. However there is not much literature that addresses the issue of who controls the controller. This article aims to approach an answer to this question. Indeed, as it is well known. in democratic societies an important role should be played by citizens. We show that politically active citizens can prevent the spread of corruption.

## 1 Introduction

In February 2014, the European Union published its first ever anti-corruption report. Over 41 pages, it concluded that bribery, tax evasion, cronyism, embezzlement, political fraud, and the like, cost the European economy 120 billion euros at year, just short of the EU annual budget. Corruption is costly, but it deprives citizens of more than money. There is a lot of empirical and theoretical evidence showing that high and rising corruption increases income inequality and poverty.

In this paper we conclude that citizens are key in the fight against corruption, because in a democratic country they have the possibility to exert pressure demanding the government to combat this scourge.

There is a profuse economic literature related with the topic of administrative and political corruption. Pioneering works in the area are [Rose-Ackerman, S. (a) ] and [Rose Ackerman, S. (b) ]. A basic insight that emerges from many studies is the self-enforcing nature of corruption: in an environment where corruption is the norm, corruption tends to persist and to be imitated, see for instance [Lui, Francis T. (a)] [Lui, Francis T. (b) ]; [Sah, R.]; [Mishra, Ajit] In recent works the evolution of the corruption in a given society is modeled using the evolutionary game theory. Even when initially individuals choice their strategies independently, after some time, they compare the obtained payoffs and copy the apparently more profitable strategy. Under this evolutionary approach and under given social conditions, corruption, can become a dominant strategy. See for instance [Accinelli, E.; Carrera, E. (a)], [Accinelli, E.; Carrera, E. (b)]. In much of this literature the conditions under which the public officials are willing to be corrupted are analyzed. These officials must ensure compliance with the law, payment of taxes by citizens, compliance with rules aimed at preventing pollution and annoying sounds, etc.. But often, officials themselves are willing to accept bribes from citizens who do not want to be punished for breaching the rules of coexistence. See for instance [Accinelli, E.; Carrera, E.; Policardo, L.]. The increasing of official corruption, in turn, creates incentives for the development of the corrupt behavior and in this way the society as a whole becomes corrupt. The question about how to avoid the evolution of corruption is not easy to be answered however of the great importance.

On the other hand, in many specialized papers it is considered that the central authority, the government and or central agencies, should play an important role to deterrence and to control the evolution of corruption. The government is considered as a benevolent planner trying to maximize the social welfare. But many times, individuals who are members of these central agencies (political elites) can benefit by the evolution of corruption among officials. In such cases these agents act maximizing their own self-interests rather than being compliant agents maximizing the social welfare. Models of this behavior are considered for instance in [Becker, G.], and [Grossman, G. M. and Helpman, E], but the question that remains unanswered is: who and how controls the controller?

An interesting discussion on this point is introduced in [Hurwicz, L.]. In the cited work, the author retakes a question posed by the Latin author Juvenal: *Quis custodiet ipsos custodes?* This is a Latin locution variously translated as “Who will guard the watchmen?”, “Who will guard the guards?”, or “Who will guard the guards themselves?”. Originally the problem was posed by Plato in the *Republic* ([Plato]) in his work on government and morality, to control the ones who exercise positions of power . The perfect

society, as described by Socrates, is based on workers, slaves and merchants. The guard class is to protect the city. The question before Socrates is "who will keep the guardians?" Or "who will protect us from the protectors?" Plato's answer to this question is that they will take care of themselves. According to Plato it would suffice for the guard to perform their function with honesty, to make them believe that they are better than those to whom they render their service and that therefore, it is his responsibility to watch and to protect the inferiors. The latin quotation is attributed to Juvenal written in his work Satires (I/II century AD, written five centuries after Plato) (see [Juvenal]). In that work he referred to the inability to control the marital fidelity (Satire VI, lines 346-348). The author concluded that to keep spouses under control is not possible because guardians can be bribed. For similar considerations a finite succession of guardians of guards does not seem to be a solution. So, Juvenal suggests in this satiric way, that the problem to guard the guards has no solution, i.e., there is no way to control the guardians. In conclusion, until now, we have two different absolute answers to the same question.

The aim of this paper is to give a partial answer to this question without recourse to an endless succession of guard of guards. We argue that an infinite cycle of guardians is not necessary to control the government. When citizens are voters in a democratic country two levels of guards are enough, under the assumption that it will be possible for the citizens to vote for a new government with the expectation that this government will be cost efficient in controlling corruption and will practice appropriate fines against corrupts. In our work we have officials, the individuals that must be controlled, the government is a first level of guards, and the second level is made up of citizens. We conclude that we can not be as pessimistic as Juvenal nor as optimistic as Plato.

We do not give an exact definition of corruption<sup>1</sup>, moreover we consider only one of its forms of expression, the abuse of the officials, more interested in their own profits, rather than fulfilling their duties. We show that when there is not a perspective to elect a government that it is cost efficient against corruption and practices appropriate fines to corrupts, corruption is a self-reinforcing mechanism having cycles of corruption with corrupt government or corrupt officials or both, depending upon the cost efficiency in fighting against corruption and of the penalization of the officials.

However, when the citizens have the perspective of voting for a government that it is cost efficient against corruption and practices appropriate fines to corrupts, the persistence of corrupt behavior in a democratic country depends on the degree of intolerance of citizens with respect to this behavior. To measure this degree of intolerance we introduce an index. This index is a function of the percentage of corrupt officials existing in each time, and is a measure of the probabilities that a government will be re-elected. We focus on the dynamics of corruption, and we analyze how certain patterns of behavior may evolve, and give the conditions under which the stationary points become stable. In particular, we want to show how — if the parameters of the model are exogenous — a sudden decrease in corruption might occur as consequence of changes in the intolerance index.

The rest of the work is organized as follows: In the next section we introduce a formal model of a process that involves, official citizens and government. To analyze the evolution of the corruption we consider a normal form game with three players. In section (2.4) we consider the corruption as a self reinforcement mechanism. In section (3) we consider a dynamical system to explain the evolution of the corruption in a society. We analyze the relationships between dynamical and Nash equilibria. The analysis of the stability of the equilibria are given in section (3.2). In section 4 we analyze with some detail the role of the index of intolerance of corruption. In the last section we present some conclusions.

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<sup>1</sup>Different ways of defining corruption and its limitations are considered in [Jain, A. K.].

## 2 The model

Consider an economy or society, where the central authority is elected by universal suffrage of citizens. By central authority or national government, we understand the president and his political sector. They make up the ruling elite. The members of the government can be reelected or not after each period of government through universal suffrage. The president and members of his political sector, in turn, appoint public officials who may or may not be renewed by the new government. These officials are in charge of carrying out the legal and administrative management of the government and serve directly to the citizens when they require to carry out this type of formalities before the central authority. At the end of each election period, these officials must choose between two different behaviors namely, properly fulfilling its role or, when her participation is required by a citizen, he fulfils his duty as long as the citizen pay for it a certain amount of money.

We call an honest or non-corrupt official the one that chooses to unconditionally fulfil its functions, otherwise we call the official a dishonest or corrupt official. Sometimes, a dishonest official is colluded with a member of the central authority and both take advantage for this behavior. Several examples of this kind of collusion are considered in [Thompson D.] and [Lessig, L.].

In general, corruption can be defined as the misuse of public power for private benefit. For instance, government official collect bribes for providing permits, licenses, passage through costumers, or avoiding the entrance to competitors in a given market. Such behavior may give room to an increase of the dishonest behavior in the whole society.

Following [Shleifer, A. and Vishny, R.], we define the governmental corruption as the complicity of the government (the ruling elite) with officials that sell government property for personal gain.

But even when some members of the government can be attracted to acting in collusion with dishonest officials, it is necessary to consider that the government is interested in being re-elected for the next period, and they know that this happens only if citizens are satisfied with the performance of the government. Citizens will judge the performance of the central authority through the work of officials who deal directly with them. We assume as mandatory, that at the end of every period each citizen must vote to re-elect or not the government. Citizens prefer a non corrupt government, but they do not have complete information about the behavior of the government. They know this information only in an indirect way, and only if they have taking contact with some official.

If the current government is not re-elected then a new government takes the place of the former.

We summarize the activity of the government saying that it must choose between to follow a corrupt behavior or a non-corrupt behavior, meaning to act in complicity with corrupt officials, or alternatively, punishing them.

The model can be formalized as a normal form game with three different populations. The sets of pure strategies are as follows:

1. Officials must choose between two pure strategies: to be corrupt or not, respectively symbolized by  $O_c$  and  $O_{nc}$ , so that we have  $\Gamma_O = \{O_c, O_{nc}\}$ .
2. The central authority or government must choose in the set of pure strategies  $\Gamma_G = \{G_c, G_{nc}\}$ . A corrupt policy (meaning to collude with corrupt officials) is symbolized by  $G_c$  while an honest or non-corrupt policy is denoted by  $G_{nc}$ . This represents to the behaviour of the political elites.
3. At the end of every electoral period, citizens must choose between to re-elect the government or not. We assume that every citizen prefers a non-corrupt government to a corrupt one, however they do not have perfect information about the governmental corruption. They perceive the corruption through the behavior of the official, and through other factors that may be, for instance, be due to the actions of the media, or due to other political, social and economical factor.

The payoffs for officials and government are represented in the following table.

	$G/O$	$G_c$	$G_{nc}$	
$C_R \rightarrow$	$O_c$	$W + M_c - M_g, M_g - W + V_{G_c}$	$W + M_c - M, M - W - e + V_{G_{nc}}$	
	$O_{nc}$	$W - M'_g, M'_g - W + V_{G_c}$	$W, -W + V_{G_{nc}}$	(1)

Where:

- By  $W$  we symbolize the wage of the officials which is paid by the government.
- $M$  is the fine imposed by an honest government to a dishonest official.
- $M_c$  corresponds to the bribe that a dishonest official takes from a citizen when his participation is required.
- $M_g$  is the amount that the dishonest official must pay to his partner in the government.
- $M'_g$  is the amount that an honest official must pay to a dishonest government to keep his position or because they do not want to be punished for breaching the rules of coexistence. We note that  $M'_g$  can be negative, i.e., a reward given by the corrupt government to honest officials. When  $M'_g$  is positive, it can also be considered as legal appropriation of the officials welfare due to solidarity ideological reasons.
- $e$  is the cost associated with the capture of a corrupt official. We assume that this cost is a measure of the governmental efficiency in fight against corruption.
- $V_{G_c}$  and  $V_{G_{nc}}$  correspond respectively to the value that a corrupt government and a non-corrupt government assign to be re-elected for the next period.

Government must choose between two possible behaviours or pure strategies: to be corrupt, or to be honest (or non-corrupt). Analogously, officials must choose between being corrupt and being honest. Citizens have the power to vote to reelect the government and not reelect the government, which can be regarded as their pure strategies. So, in our setting, there are three populations. The government and officials populations (that in our model we assume to be disjoint populations) are the players. Furthermore, there is a third population of citizens, that are not players of the game in classical sense, so we do not consider explicit payoff for citizens, but their actions (for simplicity, we can see them as pure strategies), voting or not for reelection, have a definite influence on the corruption game, by means of the index of intolerance to corruption. We assume that citizens prefer an honest government to a non-corrupt one, but their choice is randomized, because the information they have is not complete. We consider that the probability of re-electing one government or another will depend on their relationship with the officers. Typically, this probability decreases as the number of corrupt officers increases, although in this work we consider this probability as exogenous. Furthermore, this probability also reflects political and mediatic beliefs, as well as social and economic beliefs citizens have about the government. This will play a role through the index of intolerance to corruption, so that citizens will have an influence in the strategic decisions of the government. In other words, since governments seek to be maintained in functions, their payoffs (and their expected payoff, as we will see) have a component that reflects the political pressure and influence from the citizens and their intolerance to corruption, ultimately being very relevant in the government's strategy that will be selected by evolution in the dynamics that we will introduce. We do not go into considerations about the fact that some citizens may be involved in acts of corruption, this possibility may be considered in a future work.

## 2.1 Government and officials utilities

The von Neumann-Morgenstern utility theorem shows that, under certain axioms of rational behaviour, a decision-maker faced with risky outcomes of different choices will behave as if he is maximizing the expected values of some function (the von Neumann-Morgenstern utility function) defined over the potential outcomes at some specified point in the future. We will follow this point of view to describe the behaviour of the agents involved in our model. We assume that the values of the utility function associated with each choice, are the potential profits in each state of the world.

The total payoff of a dishonest government corresponds to

$$E(G_c)(t) = Nn_c(t)M_g + Nn_{nc}(t)M'_g - NW + R_{G_c} , \quad (2)$$

and the total payoff of a honest government corresponds to

$$E(G_{nc})(t) = -NW + (M - e)Nn_c(t) + R_{G_{nc}} , \quad (3)$$

where

- $Nn_c(t)$  is the quantity of corrupt officials in time  $t$ ,  $Nn_{nc}(t)$  the quantity of honest officials in time  $t$ , and so  $n_c + n_{nc} = 1$ .
- $R_{G_c}$  and  $R_{G_{nc}}$  are the expected profits by governments in case of re-election, i.e.,  $R_{G_c} = V_{G_c}q_{G_c}$  and analogously for a non-corrupt government  $R_{G_{nc}} = V_{G_{nc}}q_{G_{nc}}$ , where  $q_{G_c}$  and  $q_{G_{nc}}$  are respectively, the probabilities that a corrupt and a non-corrupt government get re-elected.

We denote by  $g_c$  the probability that the government follows a corrupt policy. We shall see later that this probability is determined endogenously. Note that  $g_{nc} = 1 - g_c$  is the probability that the government follows a non-corrupt policy. The expected profit of a dishonest official is given by

$$E(O_c)(t) = (W + M_c - M_g)g_c(t) + (W + M_c - M)g_{nc}(t) . \quad (4)$$

The expected profit of an honest official is given by

$$E(O_{nc})(t) = (W - M'_g)g_c(t) + Wg_{nc}(t) . \quad (5)$$

## 2.2 Government and officials characteristics

We will use the following characterizations of corrupt and non-corrupt governments.

To simplify the writing we can consider

$$A = -M_g + M + M'_g, \quad B = M_c - M \quad (6)$$

$$A' = N(M_g - M'_g - M + e), \quad B' = NM'_g + R_{G_c} - R_{G_{nc}}$$

- (i) The non-corrupt government practices appropriate fines if  $M > M_c$ , ( $B < 0$ ), and it practices inadequate fines if  $M < M_c$ , ( $B > 0$ ). Let the non-corrupt government re-election threshold be

$$T_1 = NM'_g + R_{G_c} .$$

- (ii) The non-corrupt government has high re-election power if  $V_{G_{nc}}q_{G_{nc}} > T_1$ , ( $B' < 0$ ), and it has low re-election power if  $V_{G_{nc}}q_{G_{nc}} < T_1$ , ( $B' > 0$ ). Let the non-corrupt government efficiency threshold be

$$T_2 = R_{G_{nc}} - R_{G_c} + N(M - M_g) .$$

- (iii) The non-corrupt government is cost efficient in fighting against corruption if  $e < T_2/N$ , ( $A' + B' < 0$ ) and it is cost inefficient in fighting against corruption if  $e > T_2/N$ , ( $A' + B' > 0$ ).
- (iv) The corrupt government penalizes more honest officials than dishonest officials if  $M'_g > M_g - M_c$ , ( $A + B > 0$ ), and penalizes more dishonest officials than honest officials if  $M'_g < M_g - M_c$ , ( $A + B < 0$ ).

It is possible to consider the case  $M'_g < M_g - M_c$ . This could happen if a corrupt government is charging corrupt officials ( $M_g$ ) more than the bribe that the corrupt officials receive ( $M_c$ ) plus the amount ( $M'_g$ ) that an honest official must pay to a dishonest government to keep his position. This kind of corruption of the ruling elites can be considered like a legal corruption associate to self-imposed laws charging severe fines to officials that are corrupt by breaking these laws and so the (ideological) solidarity with the government.

### 2.3 The Index of Intolerance to Corruption

The intolerance of citizens towards corrupt acts is important regarding the evolution of corruption in society. We define the index of intolerance to corruption as follows:

**Definition 1 (The Index of Intolerance to Corruption)** *Let  $q_{G_{nc}}$  be the probability that a corrupt government is re-elected given that the percentage of corrupt officials is  $n_c$  and let  $q_{G_c}$  be the probability that a non-corrupt government is re-elected. We define the index of intolerance to corruption by the difference:*

$$D_{it} = q_{G_{nc}} - q_{G_c}. \quad (7)$$

This index captures the social sensibility to the corruption. Note that:

$$\begin{aligned} R_{G_{nc}} - R_{G_c} &= V_{G_{nc}} q_{G_{nc}} - V_{G_c} q_{G_c} = \\ &= (V_{G_{nc}} - V_{G_c}) q_{G_{nc}} - V_{G_c} (q_{G_c} - q_{G_{nc}}) = \\ &= (V_{G_{nc}} - V_{G_c}) q_{G_{nc}} + V_{G_c} D_{it}. \end{aligned} \quad (8)$$

Because corruption is willfully hidden, it is not easy to measure it directly [Seligson, M.]. There have been many attempts to solve this problem but they have all came up with limitations, see for instance [Campbell, S. V.] and [Mauro, P.]. However, we consider that citizens perceive the degree of corruption of government through the services that officials provide. Consequently, the indignation that corrupt services cause among citizens can help to stop corruption.

### 2.4 Corruption as a self-reinforcing mechanism

Given that we assume a rational behavior of the different agents involved, it follows that, the quantity of dishonest official increases if and only if  $E(O_c) > E(O_{nc})$ , i.e.

$$(W + M_c - M_g)g_c + (W + M_c - M)(1 - g_c) > (W - M'_g)g_c + Wg_{nc}. \quad (9)$$

After some algebra we obtain the following statements:  $E(O_c) > E(O_{nc})$  if and only if

$$g_c > \frac{M - M_c}{M - M_g + M'_g} = -\frac{B}{A} \quad (10)$$

and  $E(G_c) > E(G_{nc})$  if and only if

$$n_c > \frac{(R_{G_{nc}} - R_{G_c}) - NM'_g}{N(M_g - M'_g - M + e)} = -\frac{B'}{A'}. \quad (11)$$

The next proposition summarizes these facts

**Proposition 1** *Officials prefer to choose a dishonest behaviour if and only if the government corruption is large enough, and reciprocally a high number of corrupt officials encourage governmental corruption.*

**Remark 1** *Note that if the fines are relatively low relative to what a corrupt officer can obtain as an illegal payment for his services, i.e., if  $M_c \geq M$  then, when the government is non-corrupt, it is more profitable for the officials to follow a corrupt conduct. This situation would raise the number of corrupt officials and consequently, the government would prefer over time to be corrupt. More precisely, this will happen as soon as the inequality (11) is verified.*

A general conclusion can be obtained from proposition (1) and summarized in the following way: corruption corrupts. More explicitly, this proposition says that corruption is a self-reinforcing mechanism. The question now is how to break down this process.

The answer is in the degree of intolerance of citizens.

Note that substituting (8) in the inequality (11) it follows that government prefers the corrupt strategy if and only if

$$n_c > \frac{[(V_{G_{nc}} - V_{G_c})q_{G_{nc}} + V_{G_c}D_{it}] - NM'_g}{A'}. \quad (12)$$

If we consider the additional hypothesis that the profits that a political group in power can obtain in case of being re-elected are the same whether it is corrupt or not, then equation (12) simplifies and the role of the index of intolerance is very clear:

$$n_c > \frac{V_{G_c}D_{it} - NM'_g}{A'}. \quad (13)$$

The next corollary holds.

**Corollary 1** *If citizens are sufficiently intolerant with the bad services provided by corrupt officials, then, according to (12) or (13), it becomes more unlikely that there are enough corrupt officials so that governments prefer to be corrupt, so that the government loses incentives to tolerate or to allow corruption. Insofar as that the degree of tolerance of citizens for the services of corrupt officials decreases or, equivalently, insofar the degree of intolerance for corrupt services increases, the government prefers to punish corrupt officials.*

However, note that the strategy “to be corrupt” can be a dominant strategy for the government if its efficiency to capture corrupt officials is low, or equivalently the cost to catch the corrupt officials is high, i.e., if  $e > M - M_g$ <sup>2</sup>. The cost to catch the corrupt officials is higher in those countries where the effectiveness of the legal system is low, and in this case, and also when the intolerance index is low, we may be in presence of a negative cycle where an inefficient legal system becomes a cause and a consequence of corruption. We will analyse how exogenous changes in these and other quantities can change the processes of evolution of corruption and revert the spreading of corruption.

Let us analyse the social evolution of corruption by means of the replicator dynamics.

### 3 The evolution of corruption

To explain the social evolution of corruption, we shall follow an evolutionary approach. This approach is based on the fact that strategies that make a person do better than others will be retained, while strategies

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<sup>2</sup>We assume that costs associated with the capture of a corrupt official are funded by sanctions that a non-corrupt government obtains from fines to corrupt officials. Certainly if this cost exceeds the total amount of fines collected, the government will have to appeal to other sources to perform this task. This point is not considered in this work.



that lead to failure will be abandoned. The success of a strategy is measured by its relative frequency in the population at any given time. Strategies change over time as a function of their relative success in an environment that is made up of other players that keep changing their own strategies adaptively.

Initially people decide their strategies independently. We assume that individuals in every time try to improve his welfare and that they follows a myopic behaviour, because officials and government can not forecast the consequences of the changes in the relative frequency of their strategies can provoke. In addition we consider that officials do not know with absolute accuracy the likelihood that the government act corruptly, neither the government knows exactly the percentage of corrupt officials.

Periodically, they compare the obtained returns and after some time, some of them update their strategic choices, switching for the, apparently, most profitable strategies. So, in each period, the percentage of individuals that follows a given strategy increases if the expected payoff of such strategy is greater than the average payoff obtained by the population. Otherwise, if the expected payoff is performing worse than the average, that strategy becomes less frequent in the population. The dynamical system summarizing these facts is the replicator dynamics (see [Weibull, W. J.]). In other words, the replicator dynamics considers that the difference between the expected payoff of a strategy and the average payoff of all strategies is the per-capita change in the frequency of the strategy in the population. In our case, since we have only two pure strategies for each players, the replicator dynamics is even simpler: the player (or population) change their strategy by simply evaluating which is the strategy that is yielding more expected profit. Along time, more profitable strategies become the most widely used. Other dynamics can be considered, such as, for instance, some kind of imitation, where players imitate the behaviour of the most successful agent, or best-response dynamics, where the strategy that is best response to the opponent's strategy is chosen.

The amount of officials following one or another strategy may change. We have that  $n_i(t)$  is the percentage of corrupt officials following the strategy  $i \in \{O_c, O_{nc}\}$ . By  $n(t) = (n_c(t), n_{nc}(t))$ . By  $\dot{n}_i$  we represent the derivative with respect to the time of the percentage of official following the strategy  $i$ .

We symbolize the distribution of the officials over the set of pure strategies, in each time  $t$ , by  $g(t) = (g_c(t), g_{nc}(t))$  the mixed strategy of the government in time  $t$ . By  $\dot{g}_i$  we represent the derivative with respect to time of the probability  $g_i(t)$  that the government follows strategy  $i$ .

All these variables are time depending, but to simplify we do not write the variable  $t$ .

According with the replicator dynamics discussed above, the growth rate of corrupt officials is given by the following differential equation:

$$\dot{n}_c = n_c[E(O_c) - \bar{E}] = n_c(1 - n_c)(E(O_c) - E(O_{nc})) , \quad (14)$$

where  $\bar{E} = n_c E(O_c) + n_{nc} E(O_{nc})$  is the expected payoff of the officials when they follow a corrupt strategy, and  $E(O_c)$  and  $E(O_{nc})$  denote, respectively, the expected value of a corrupt behavior and a non-corrupt behavior by an official, given a distribution  $g$  over the government behavior.

To measure the evolution of the governmental corruption we use  $g_c$  as an index measuring the percentage of corrupt acts committed in public offices regarding the total of acts performed in these government agencies<sup>3</sup>. We endogenize the probability of a government being corrupt by considering the index  $g_c$  that represents the percentage of corrupt acts made by a government on the total acts of government performed as the probability that government follows a corrupt strategy. In other words, this will be the mixed strategy of the government over his set of pure strategies. Then, in a similar way, we obtain that the evolution of the government policy can be represented by the following dynamical system.

$$\dot{g}_c = g_c(1 - g_c)(E(G_c) - E(G_{nc})) , \quad (15)$$

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<sup>3</sup>Most indexes measuring corruption actually measure proxies for corruption because corruption is a difficult phenomenon to measure. An example of such an empirical index of the perceived governmental corruption is Transparent International's (TI) Corruption Perceptions Index (CPI). This index captures information about administrative and political aspects of corruption. However, its use has not come without criticism (see [Campbell, S. V.]).

where  $E(G_c)$  and  $E(G_{nc})$  represent, respectively, the expected value of a corrupt behavior and a non-corrupt behavior by the government, given a distribution  $n$  of the officials over their available strategies and the degree of intolerance to corruption  $D_{it}$ .

This dynamical system with four equations can be summarized in the following system with only two differential equations:

$$\begin{aligned}\dot{n}_c &= n_c(1 - n_c)(E(O_c) - E(O_{nc})) \\ \dot{g}_c &= g_c(1 - g_c)(E(G_c) - E(G_{nc}))\end{aligned}\tag{16}$$

After some algebra we obtain:

$$\begin{aligned}\dot{n}_c &= n_c(1 - n_c)(Ag_c + B) \\ \dot{g}_c &= g_c(1 - g_c)(A'n_c + B')\end{aligned}\tag{17}$$

### 3.1 Nash equilibria and steady-states

The dynamical system (17) has the following four dynamic equilibria corresponding to pure strategies of the game. Depending on the value of parameters, these points may or may not correspond to Nash equilibria for the sub-game played by officials and government. In this framework, there are four possible equilibria in pure strategies and one in mixed strategies.

1. The corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$  is a Nash Equilibrium if and only if

$$E(O_c) \geq E(O_{nc}) \text{ and } E(G_c) \geq E(G_{nc}) .$$

2. The corrupt officials equilibrium  $(n_c^2, g_c^2) = (1, 0)$  is a Nash Equilibrium if and only if

$$E(O_c) \geq E(O_{nc}) \text{ and } E(G_c) \leq E(G_{nc}) .$$

3. The corrupt government equilibrium  $(n_c^3, g_c^3) = (0, 1)$  that it is a Nash Equilibrium if and only if

$$E(O_c) \leq E(O_{nc}) \text{ and } E(G_c) \geq E(G_{nc}) .$$

4. The non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  is a Nash Equilibrium if and only if

$$E(O_c) \leq E(O_{nc}) \text{ and } E(G_c) \leq E(G_{nc}) .$$

On the other hand, the dynamical system (16) has an interior dynamic equilibrium that it is a mixed Nash equilibrium. If  $A$  and  $A'$  are not equal to zero, then the point  $(\bar{n}_c^T, \bar{g}_c^T)$  is a steady state, where

$$\bar{n}_c^T = -\frac{B'}{A'} = \frac{(R_{G_{nc}} - R_{G_c}) - NM'_g}{N(M_g - M'_g - M + e)} \quad \text{and} \quad \bar{g}_c^T = -\frac{B}{A} = \frac{M - M_c}{M - M_g + M'_g} .$$

Note that, in our framework, this equilibrium makes sense if

$$0 < \bar{n}_c^T = \frac{(R_{G_{nc}} - R_{G_c}) - NM'_g}{N(M_g - M'_g - M + e)} \leq 1 \quad \text{and} \quad 0 \leq \bar{g}_c^T = \frac{M - M_c}{M - M_g + M'_g} < 1$$

are satisfied. The steady state  $(\bar{n}_c^T, \bar{g}_c^T)$  is a mixed Nash equilibria for the game, i.e.  $E(O_c) = E(O_{nc})$  and  $E(G_c) = E(G_{nc})$ . We note that if  $\bar{n}_c^T$  is equal to 0 or 1, then  $(\bar{n}_c^T, g_c)$  is a steady equilibrium for any  $g_c$ ; and if  $\bar{g}_c^T$  is equal to 0 or 1, then  $(n_c, \bar{g}_c^T)$  is a steady equilibrium for any  $n_c$ .

### 3.2 Stability of equilibria

The Hartman-Grobman theorem states that the orbit structure of a dynamical system in a neighbourhood of a hyperbolic equilibrium point is topologically equivalent to the orbit structure of the linearised dynamical system.

Assuming that  $A$  and  $A'$  are non-zero, then the point  $(n_c^T, g_c^T) = (B'/A', -B/A)$  is a steady state for the dynamical system. The linearisation at this point is given by the matrix:

$$J\left(-\frac{B'}{A'}, -\frac{B}{A}\right) = \begin{bmatrix} 0 & -\frac{A'B}{A^2}(B+A) \\ -\frac{AB'}{A'^2}(B'+A') & 0 \end{bmatrix}$$

The eigenvalues of this matrix are:

$$\lambda = \pm \sqrt{\frac{B'B}{AA'}(B'+A')(B+A)}.$$

Thus, if  $\frac{B'B}{AA'}(B'+A')(B+A) > 0$  then this point is a saddle point for the dynamics. In other cases the Hartman-Grobman theorem is not conclusive, because the matrix  $J$  has eigenvalues with zero real parts, meaning that the point is not hyperbolic.

The corruption equilibrium  $(n_c^B, g_c^B) = (1, 1)$  corresponding to a fully corrupt society where all officials are corrupt and government always acts in a corrupt way. The matrix corresponding to the linearisation is

$$J(1, 1) = \begin{bmatrix} -(A+B) & 0 \\ 0 & -(A'+B') \end{bmatrix}$$

The eigenvalues are  $\lambda_1 = -(A+B)$  and  $\lambda_2 = -(A'+B')$ . Hence, the corruption equilibrium  $(n_c^B, g_c^B) = (1, 1)$  is stable if the non-corrupt government is cost inefficient in fighting against corruption and the corrupt government penalizes more honest officials than dishonest officials.

The officials corrupt equilibrium  $(n_c^2, g_c^2) = (1, 0)$  corresponds to a situation where all officials are corrupt but the government always acts in an honest way. The linearisation is

$$J(1, 0) = \begin{bmatrix} -B & A' \\ 0 & A'+B' \end{bmatrix}$$

The eigenvalues are  $\lambda_1 = -B$  and  $\lambda_2 = A'+B'$ . Hence, the officials corrupt equilibrium  $(n_c^2, g_c^2) = (1, 0)$  is stable if the non-corrupt government is cost efficient in fighting against corruption but practices inadequate fines.

The government corrupt equilibrium  $(n_c^3, g_c^3) = (0, 1)$  corresponds to a situation where the government acts in a corrupt way, but officials are forced to be honest. The linearisation is

$$J(0, 1) = \begin{bmatrix} A+B & A' \\ 0 & -B' \end{bmatrix}$$

The eigenvalues are  $\lambda_1 = A+B$  and  $\lambda_2 = -B'$ . Hence, the government corrupt equilibrium  $(n_c^3, g_c^3) = (0, 1)$  is stable if the corrupt government penalizes more dishonest officials than honest officials and the non-corrupt government has low re-election power.

The non-corruption equilibria  $(n_c^4, g_c^4) = (0, 0)$  corresponds to a situation where there is a strictly ruled government country. The linearisation is

$$J(0, 0) = \begin{bmatrix} B & 0 \\ 0 & B' \end{bmatrix}$$

This matrix has two real eigenvalues  $\lambda_1 = B$  and  $\lambda_2 = B'$ . Hence, the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  is stable if the non-corrupt government practices appropriate fines and it has high re-election power.

Finally, we observe that the corruption and non-corruption equilibria can be simultaneously stable. When both are stable the other two boundary equilibria are unstable and the interior equilibrium is a saddle.

### 3.3 The transition paths: Possible cases

For each time  $t$  we say that the pair  $(n_c(t), g_c(t))$  defines the state of corruption of the society in time  $t$ . Thus, given the dynamical system (17) and an initial condition in time  $t = t_0$  (i.e., an initial state of corruption),  $(n_c(t_0), g_c(t_0)) = (n_{c_0}, g_{c_0})$ , we say that  $\xi(\cdot, (n_{c_0}, g_{c_0})) \rightarrow \mathfrak{R}^2$  is a solution of the dynamical system with such initial condition if and only if  $\xi(t, (n_{c_0}, g_{c_0}))$  verifies the system (17), and  $\xi(t_0, (n_{c_0}, g_{c_0})) = (n_{c_0}, g_{c_0})$ . Classic theorems in the theory of differential equations show that once an initial condition is fixed, there is a unique solution for the differential equation and that the function  $\xi(t, \cdot) : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$  is smooth, i.e., the solution of the dynamical system (17) is smooth with respect to initial conditions (see, for instance [Hirsh, M.; Smale, S.; Devanay, R.]).

**Definition 2 (The trajectory of corruption)** *Given the dynamical system (17) and an initial condition in time  $t = t_0$ , we define the trajectory of the corruption, as the set  $\Gamma \subset \mathfrak{R}^2$  given by:*

$$\Gamma = \{(n_c(t), g_c(t)) = \xi(t, (n_c(t_0), g_c(t_0))), \forall t \geq t_0\}.$$

Note that each trajectory defines a set of possible future states of corruption, i.e., for each initial condition, there is only one set of possible future states (since we do not consider shocks and stochastic effects in this work). So, the corruption in a given society, once the initial condition is fixed, evolves along a trajectory.

Given the dynamical system (17) and an initial condition the set of possible states for all  $t > t_0$  will be called the *transition path*.

This transition path is given by the set of possible states of corruption, from a fixed initial time  $t = t_0$  until the system rests in a dynamical equilibrium.

To analyze the possible evolution of the corruption, i.e., the possible transition paths in a given society, we will separate our analysis in several sections depending of the values of the quantities that determine the payoffs of the game.

### 3.4 Non-corrupt government with high re-election power and appropriate fines

In this subsection, we assume that  $B < 0$  and  $B' < 0$ . Hence the non-corrupt government has high re-election power and uses appropriate fines. In this case the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  is always asymptotically stable.

- (1) When  $A > -B$  and  $A' > -B'$  then  $A + B > 0$  and  $A' + B' > 0$ . From these conditions the following inequalities are verified:  $A > 0, A' > 0, 0 < -\frac{B}{A} < 1$ , and  $0 < -\frac{B'}{A'} < 1$ , implying the existence of a mixed equilibrium in the interior of the unit square. In this case we also have that  $\frac{B'B}{AA'}(B' + A')(B + A) > 0$ , so the Hartman-Grobman theorem can be applied to the mixed equilibrium, yielding a saddle point. In this case the corruption equilibrium and the non-corruption equilibrium are asymptotically stable. See Figure (1) for the general picture of the dynamics in this case.

This is a good example of ongoing spontaneous coordination. Note that this case corresponds to a social situation where:

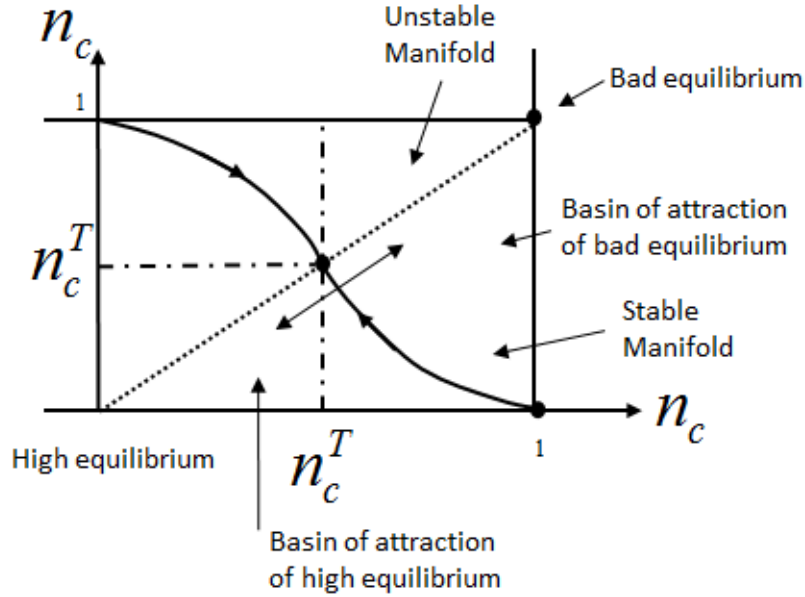


Figure1

Figure 1: The dynamics of the system, with the basin of attraction of the equilibria and the mixed Nash equilibrium.

1. The amount  $M$  of the fine imposed by a non-corrupt government to a corrupt official is relatively high, meaning that it is greater than the bribe  $M_c$  the official takes from citizens, i.e.,  $M > M_c$ .
2. The inequality  $M'_g > M_g - M_c$  is verified. Recall that  $M'_g$  is the amount that an honest official must pay to a dishonest government to keep his place. This means that a corrupt government punishes honest behaviour more than dishonest behavior.
3. The government is inefficient to catch corrupts officials, or equivalently,  $e$  is relatively high (relatively high costs to combat corruption).
4. The non-corrupt government has high re-election power. This may be written as  $V_{G_c} D_{it} > NM'_g + (V_{G_{nc}} - V_{G_c})q_{G_{nc}}$ . This occurs if the index of intolerance is high enough, and the government is interested in being re-elected.

In this case corruption can be regarded as a social trap. If the initial distributions of officers and government actions correspond with a point in the basin of attraction of corruption equilibrium, then officials and government have incentives to act in a corrupt way. Thus, the general levels of corruption will increase, and corruption becomes a self-enforcing mechanism over time.

However, the basin of attraction of the corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$  decreases when the interior equilibrium gets close to the corruption equilibrium  $(n_c^1, g_c^1)$ , i.e., when  $A + B$  and  $A' + B'$  tend to zero. Hence, the basin of attraction of the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  is large when the non-corrupt government's costs in fighting corruption are close to the non-corrupt government efficiency threshold, i.e.  $e$  is close to  $T_2$ , and the corrupt government penalizes honest officials similarly to dishonest officials, i.e.  $M'_g$  close to  $M_g - M_c$ .

The cost efficiency  $e$  can get closer to the threshold  $T_2$  because of different reasons: (a) non-corrupt government is able to decrease the value of its efficiency cost to capture the corrupt officials; (b) the efficiency threshold  $T_2$  rises due for instance to an increase in the index of tolerance, an increase in the valuation of re-election by a corrupt government, an increase in the probability of a non-corrupt government being re-elected or to the increase of the fine imposed by a non-corrupt government to a dishonest official.

Hence, the levels of corruption that were increasing can suddenly change if the degree of intolerance of citizens increases. If the government believes that this change in intolerance can take place then (depending also on the value that the government assigns to be re-elected), it may result in a change in the basin of attractions of the corruption and non-corruption equilibria, making some paths that would initially evolve towards the corruption equilibrium now evolve towards the non-corruption equilibrium. This possibility is supported in the following fact:

**Remark 2** *The basin of attraction of the corruption equilibrium  $(n_c^B, g_c^B) = (1, 1)$  decreases when the index of intolerance increases.*

Thus, the index of intolerance of citizens with respect to corruption, if high enough, and if the government is interested in being re-elected can play an important role at the time to control the controller acting as a servomechanism correcting the evolution of corruption. It acts as a barrier stopping corruption, since, under several circumstances, it can reverse a process of growing corruption. The higher it is, the more difficult it gets that corruptions grows and develops within the government. In Figure (2) we plot some trajectories of the system that exemplify the previous remark. For the same initial conditions with different model parameters, corresponding to an increase in the degree of intolerance, we see that initial conditions originally in the basin of attraction of the corruption equilibrium are instead converging to the non-corruption equilibrium. This illustrates the shrinking of the basin of attraction of the corruption equilibrium as the degree of intolerance grows.

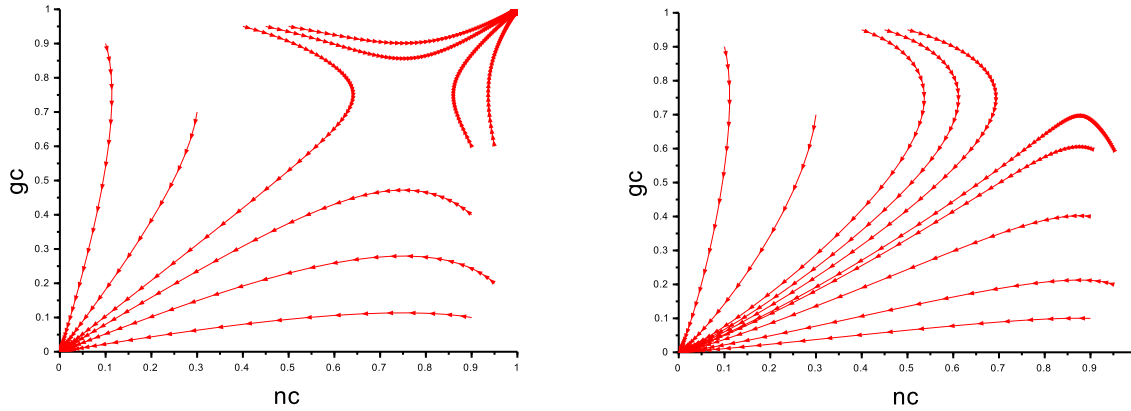


Figure 2: Some trajectories of the system for the same initial conditions with different parameters. Left-hand side: lower degree of intolerance. Right-hand side: higher degree of intolerance.

- (2) Assuming that  $A > -B$  and  $A' < -B'$  it follows that  $(A + B) > 0$ ,  $(A' + B') < 0$  then there is not a mixed Nash equilibrium because either  $-\frac{B'}{A'} > 1$  or  $-\frac{B'}{A'} < 0$ . The corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$  is a saddle point, as well as the officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$ , and the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$  is a repulsor. In this case there is a unique

asymptotically stable dynamic equilibrium and this is the Nash equilibrium without corruption, i.e.,  $(n_c^4, g_c^4) = (0, 0)$ , with all the interior initial conditions being attracted to this point. See Figure (3).

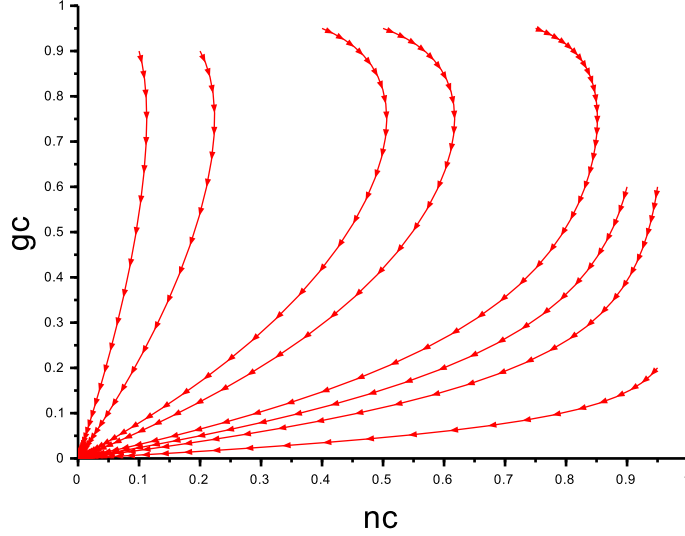


Figure 3: Some trajectories of the system for case (2).

- (3) Assuming that  $A < -B$  and  $A' < -B'$  it follows that  $(A + B) < 0$ ,  $(A' + B') < 0$  then the corruption equilibrium is a repulsor, there is no mixed equilibrium, and there is a unique equilibrium that is asymptotically stable, that is the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$ , with all interior initial conditions being attracted to this point. The officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  and the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$  are saddle points.
- (4) Assuming that  $A < -B$  and  $A' > -B'$  it follows that  $(A + B) < 0$ ,  $(A' + B') > 0$ . Then, there is no mixed equilibrium and the corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$  is a saddle point as well as the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$ , and the officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  is a repulsor. The only asymptotically stable equilibrium is the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$ , with all interior initial conditions being attracted to this point.

In case (2), we have that  $A + B > 0$  and there is no mixed Nash equilibrium. The assumptions are describing a political situation corresponding to: (a) a corrupt government penalizes honest officials more than dishonest officials; (b) an index of intolerance relatively high; and/or (c) a governmental elite with a high interest in being re-elected; and/or (d) the government is highly efficient in fighting corruption, i.e. low values of  $e$ .

### 3.5 Non-corrupt government with low re-election power and inappropriate fines

In this subsection, we assume that  $B > 0$  and  $B' > 0$ . Hence the non-corrupt government has low re-election power and uses inappropriate fines.

- (5) Assuming that  $A < -B, A' < B'$  then the inequalities  $(A+B) < 0, (A'+B) < 0$  hold. Both the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  and the corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$  are repulsors and the mixed Nash equilibrium is a saddle point. The officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  and the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$  are attractors, i.e., government prefers to be honest but officials prefer to be corrupt, or reciprocally, government prefers to be corrupt but officials prefer to be honest. Which one of these two situations occurs is initial condition dependent. In figure (4) we plot some transition paths of the system. Depending on the initial condition, the transition path approach either the officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  or the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$ . The exception is one initial condition that approaches the mixed equilibrium, since that initial condition lies on the stable manifold of the mixed equilibrium. We observe that the stable manifold of the mixed equilibrium is a curve passing through the mixed equilibrium that connects the corruption and the non-corruption equilibrium.

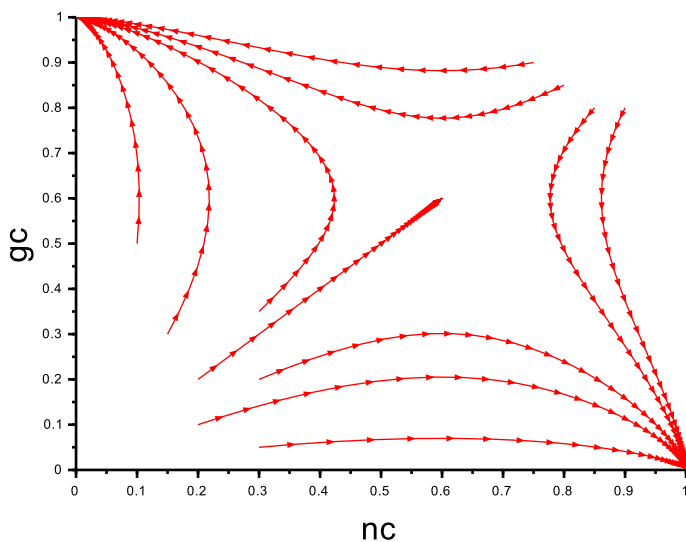


Figure 4: Different trajectories of the system for case (5).

- (6) Assuming that  $A > -B, A' < -B'$  then  $A+B > 0$  and  $A'+B' < 0$ . Hence, the corrupt government penalizes more honest officials than dishonest officials, i.e.  $M_g < M_g - M_c$  ( $A+B < 0$ ), and the non-corrupt government is cost inefficient in fighting against corruption, i.e.  $e > T_2$  ( $A'+B' > 0$ ). There is no mixed Nash equilibrium and the officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  is the only equilibrium point that is asymptotically stable, and all initial conditions in the interior of the unit square are attracted to this equilibrium.
- (7) Assuming that  $A < -B, A' > -B'$  then  $A+B < 0$  and  $A'+B' > 0$ . There is no mixed Nash equilibrium and the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$  is the only equilibrium point that is asymptotically stable, and all initial conditions in the interior of the unit square are attracted to this equilibrium. This case is similar to the previous one when the initial conditions were in the basin of attraction of the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$ .



In case (6) society is evolving to an equilibrium where officials prefer to be corrupt, even with an honest government. Our assumptions imply that governmental fines to punish corrupt behavior are relatively low, and that a non-corrupt government has low re-election power, because citizens perceive this government as a corrupt one. This case is mathematically analogous to the previous one, but very different in its social and political implications. This society evolves to an equilibrium where government is corrupt but officials prefer to be honest. We plot some trajectories of the system for case (6) in figure 5. Summarizing, we can say that this case corresponds to a socio-political situation where government has a relatively high interest in being re-elected and so might try to prevent the spreading corruption. However, the government is unable to diminish corruption because of the government: (a) being focused on re-election; (b) being inefficient; or (c) practicing low fines imposed on corrupt officials.

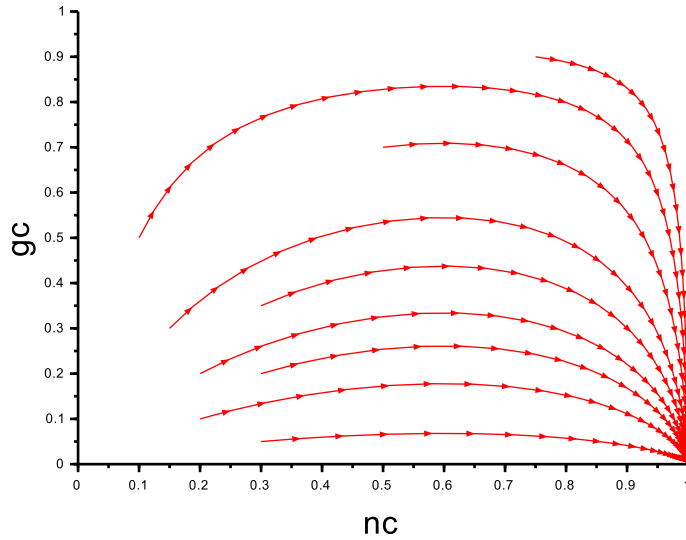


Figure 5: Some trajectories of the system for case (6).

- (8) Assuming that  $A > -B$ ,  $A' > -B'$  then  $A + B > 0$  and  $A' + B' > 0$ . In this case the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  is a repulsor and the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$  and the officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  are saddle points. The only equilibrium point that is asymptotically stable is the non-corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$ , with all interior initial conditions being attracted to this point. We plot some trajectories of the system for this case in figure (6).

In case (8) society is evolving towards full corruption both on the governmental level and on the officials' level, due to general inefficacy of government, low fines to punish corrupt officials, high costs to capture corrupt officials and because of low intolerance index. A corrupt government would have developed ways to protect the corrupts. This extreme situation is characteristic of dictatorships where the dictator confuses his own interests with national interests. It becomes a cause of several social and economic ills.

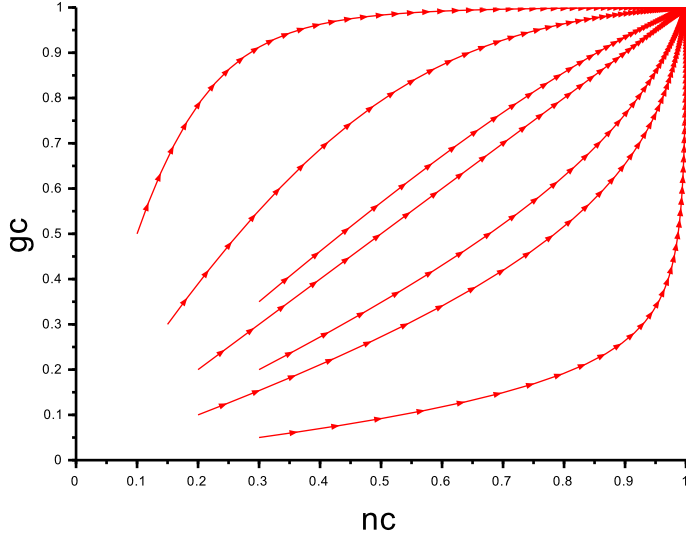


Figure 6: Some trajectories of the system for case (8).

### 3.6 Corruption cycles

In this section we show that the index of intolerance to corruption plays a central role to stop or reverse a process of growing corruption.

Let us consider now the case where  $\frac{B'B}{AA'}(B' + A')(B + A) < 0$ . Note that in this case the Hartman-Grobman's theorem is not applicable, because the eigenvalues of the mixed equilibrium are purely imaginary numbers.

Let us consider the case where  $B < 0$ ,  $B' > 0$  and  $A > -B$ ,  $A' < -B'$ . These inequalities imply that the corruption equilibrium  $(n_c^1, g_c^1) = (1, 1)$  and the non-corruption equilibrium  $(n_c^4, g_c^4) = (0, 0)$  are saddle points. The officials corruption equilibrium  $(n_c^2, g_c^2) = (1, 0)$  and the government corruption equilibrium  $(n_c^3, g_c^3) = (0, 1)$  are also saddle points. These inequalities imply the existence of a mixed Nash equilibrium in the interior of the unit square, and by the previous formula, the eigenvalues of its linearization are purely imaginary numbers. It corresponds to cycles of growth and decline of corruption. Recall that in this case there are low costs to capture corrupt officials, resulting in high efficiency, and there are high fines to punish corrupt officials, but the intolerance index is low. This interplay between these quantities results in the appearance of periodic orbits, as shown in Figure (7). The mixed equilibrium is a focus. The rationale behind this situation is the following. The low index of intolerance causes an increase in government corruption, which in turn causes more officials to prefer to be corrupt. Facing an increasingly bigger number of corrupt officials, government decides to be less corrupt, taking advantage of low costs to capture corrupt officials and high fines, which in turn cause a disincentive for officials to become corrupt, thus increasing the number of honest officials. Hence, the overall levels of corruption in society have declined to the original levels so that the cycle restarts again. A similar situation with the appearance of periodic orbits occurs if  $B > 0$ ,  $B' < 0$  and  $A < -B$ ,  $A' > -B'$ .

Periodic orbits appear naturally if the intolerance index is a function of the percentage of corrupt agents. This index increases when the number of corrupt officials grows, and decreases as does the percentage of corrupt officials. The corrupt political elite feels the pressure of a high index of intolerance, possibly

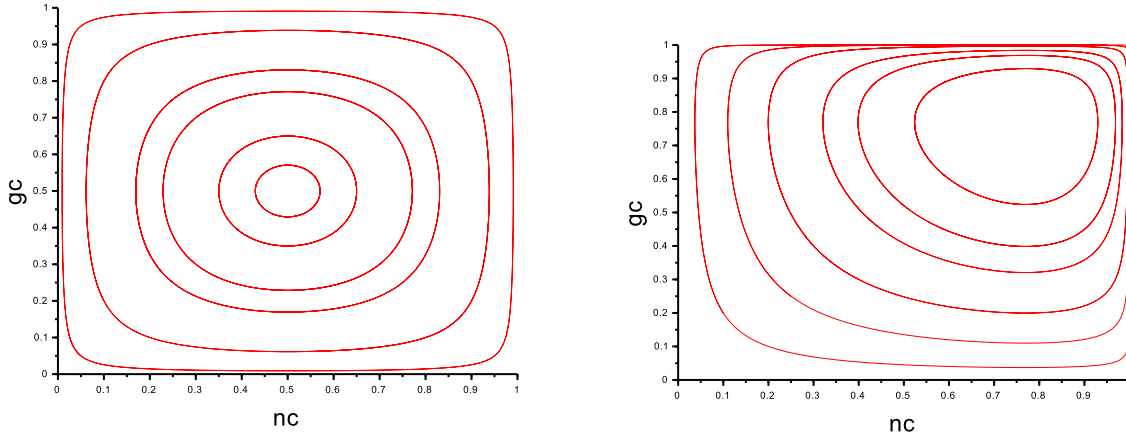


Figure 7: Some trajectories of the system (17) in the case of complex eigenvalues for different model parameters.

reducing its expected value in this case of re-election, because the probability of being re-elected is reduced. As a result, the government corruption is reduced, and government will seek to punish corrupt officials more severely. But by reducing the amount of corrupt officials, the index of intolerance decreases, and therefore the pressure on the government declines, again permitting an increase in governmental corruption and allowing for an increase in the number of corrupt officials, thus restarting the cycle of corruption. We discuss this possibility in more detail in the next section, in which we consider a variable index of intolerance, namely, depending on the relative number of corrupt officials.

## 4 The role of the Index of intolerance

In some cases, corruption can be considered as a social trap [Rothstein, B.]. Under several circumstances, the corruption equilibrium is asymptotically stable. In this case, if the initial distribution of corrupt officials and government's corrupt acts are in the basin of attraction of this equilibrium, neither official nor the government have incentives to act in a non-corrupt way. It is in this sense that we consider the corruption as a self reinforcing mechanism. Corrupt actions by a party encourage corrupt actions by the other. If everybody is corrupt, nobody wants to be honest. To be corrupt is the rational way, because under these initial conditions, the expected value of this behavior is higher than the expected value of the non-corrupt behavior. Under this prospect, corruption looks like a sticky problem that can not be changed for internal agents. This grim prospect is analyzed in several works. See for instance [Rothstein, B.] and [Kornai, I.]. However, the degree of intolerance of citizens to corruption plays an important role to deter corruption. There are examples of success in deterrence of corruption, for instance the cases of Singapore and Honk Kong, see [Root, H.].

If the government has some interest in being re-elected, and the degree of intolerance of citizens is high enough, the situation considered above can be reverted. The basin of attraction of the corruption equilibrium shrinks and the evolution of corruption can be reversed. This possibility shows the leading role that intolerance index can play in the fight against corruption. For instance, if the intolerance index rises enough with the number of corrupt officials, then the corruption equilibrium is not achieved, since in this situation, there is  $\alpha < 1$  such that, for all  $\alpha \leq n_c(t) \leq 1$ , we obtain a high enough degree of intolerance,

yielding, from equation (13)

$$V_{G_c} D_{it}(n_c(t)) > n_c(t) (M_g - M'_g - M + e) + N M'_g ,$$

hence  $E(G_{nc}) > E(G_c)$ .

In case previously considered where  $V_{G_{nc}} = V_{G_c} = V_G$  we have:

$$D_{it} > \frac{1}{V_G} \left[ \frac{n_c(t)}{N} (M_g - M'_g - M + e) + M'_g \right] .$$

Note that under the hypothesis of our model, it is natural to assume that the Intolerance Index grows with the amount of corrupt officials, because the citizens perceive the corruption through the actions of the officials and when the number of corrupt officials, the perception of corruption increases, thus increasing the intolerance of the population. However in future works it will be necessary to complete this index, considering other sources of information for citizens, such as rumours, the press, *etc.*.

## 5 Conclusions

As it is well known, many politicians and ruling elites of many countries and across the world and the whole of the political spectrum are currently involved processes of corruption. Is it possible to deter this process? To give an answer to this question is the main concern of this paper. To do this, we considered an evolutionary model, where the political agents (considered as players of a game in normal form) compare their respective payoffs, and they choose their strategies according with their expectations and the most profitable behaviors end by prevail. Hence the replicator's dynamical appear as a natural mathematical tool to describe the evolution of the corruption inside the society.

Our first conclusion is that corruption corrupts, so that corruption is a self-reinforcing mechanism (see Proposition 1). When the degree of intolerance is relatively low and the political elite in the government has good prospects of being re-elected and a large interest in gain immediate benefits, the country can be in a corruption trap, i.e., a self-reinforcing mechanism where corruption generates more corruption. An external event is a necessary condition for the country to leave this corruption social trap. This is the most worrying situation, because it escapes any self-monitoring mechanism, and then there is no way to control the controller. When there is not a perspective to elect a government that it is cost efficient against corruption and practices appropriate fines to corrupts, there can be corruption cycles or stable equilibria having corrupt governments or corrupt officials or both depending upon the cost efficiency in fighting against corruption and of the value of the penalization of the officials.

However, if the ruling elite has some interest in the re-election this self-reinforcing mechanism can be weakened or broken by a high enough degree of intolerance to corruption by the citizens under the perspective of voting for a government that it is cost efficient against corruption and practices appropriate fines to corrupts. The degree of intolerance of corruption plays an important role to make the government fulfill the role that society has assigned it, even when some of its members are attracted by the individual benefits that corruption offers. Even in situations where corruption tends to expand, if the intolerance index becomes large enough, it can help to stop or even to reverse this regressive process. However, we observe that the intolerance of citizens also depends on other relevant exogenous social, political and economical quantities that are not reflected in the index of intolerance we introduced, and hence not addressed in this paper.

We can summarize the main results of this paper, saying that, if the Index of Intolerance to Corruption is high enough, then it might be possible to stop the growth of corruption, even when the country is

immersed in an increasing process of corruption<sup>4</sup> . Finally we can say that according to what was stated at the beginning of this paper, we have given at least a partial answer to the question that motivated it and that the answer is mainly positive, since it is possible to control the controller, and the main agent in this process is citizens and the Index of Intolerance to Corruption

Certainly, like physics, and astronomy, economics and sociology need practical confirmation of the hypothesis and models considered. In this paper we give a first step to recognize the possibility of fight with success against the corruption. In future works, will be necessary to use statistical data and numerical analysis to confirm, or deny, the model presented here and also to obtain quantitative accurate conclusions. The model can be improved by further studying the quantities introduced, for instance, the degree of intolerance of citizens, a quantity that may be endogenized, possibly yielding different results, and that may be modeled as depending on other political, social and economic quantities. It will be necessary to consider also cross terms and the corresponding non linear effects, and to study other types of dynamics, for instance, the role of the imitative behavior (see [Accinelli, E.; Carrera,E. (b)]). Other accesses to information, like the press and modern media, that can influence the performance of the index of intolerance should also to be incorporated in the model.

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<sup>4</sup>Recent events in South Korea, where citizens unanimously reacted to the corrupt practices of Prime Minister Park Geun-hye suggest that, if the Index of Intolerance of citizens to Corruption is high enough, it is possible to exert political pressure that can maybe result in stopping the growth of corruption. See <http://www.abc.net.au/news/2016-11-15/south-korea-park-geun-hye-hopes-political-crisis-be-contained/8024978> .

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