Child Allowance, Public Investment in Education, and Economic Growth

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ABSTRACT

This study employs an overlapping-generations model featuring public and private education to analyze whether providing child allowances and free high school education influence economic growth. Earlier studies that analyze public and private education (Glomm and Ravikumar, 1992; Cardak, 2004) do not consider endogenous fertility and whether individuals simultaneously choose public and private education. Earlier studies that consider endogenous fertility and child allowances (Groezen, Leers, and Mejidam, 2003) disregard human capital accumulation. This study assumes people can choose both public and private education simultaneously and considers endogenous fertility and human capital accumulation. It introduces both child allowances and investment in public education financed by income taxes. If further considers how raising child allowances or investing in public education affects endogenous fertility, human capital accumulation, and economic growth. This study is motivated by evidence that the burden of meeting children's educational expenses is partly responsible for Japan's declining birthrate. It analyzes whether child allowances and free high school education can improve them. We find it unlikely such policies promote economic growth if they are financed by income taxes that cannot increased indefinitely.

Key words: *Human Capital, Overlapping-Generations, Child Allowance, Free High School Education Bill, Economic Growth.*

JEL Classification: I25, O15, H52

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1. Introduction

Evidence suggests that the burden of educational expenses contributes to Japan's declining birthrate,[1] and its ongoing erosion in economic growth. Publically funded child allowances and educational investment in the form of providing free high school education are thought to relieve these problems, but taxes finance these policies, and excessive taxation might thwart the intended outcomes. If that is the case, government policy must choose between raising child allowances and investment in public education. This study employs an overlapping-generations model featuring public and private education to analyze whether raising child allowances or providing free high school education lifts Japan's fertility rate and economic growth.

Earlier studies typically compare disparate models (Glomm and Ravikumar, 1992; Gradstein and Justman, 1997; Saint and Verdier, 1993) and treat public and private education as complements in developing human capital (Benabou, 1996; Eckstein and Zilcha, 1994; Kaganovich and Zilcha, 1999). Cardak (2004a, b) departs from the literature by analyzing public and private education as mutually exclusive rather than separately comparable alternatives and by assuming individuals choose between them by evaluating their utilities.

Representative studies that analyze child allowances and fertility rates (Exstein and Wolpin, 1985; Bental, 1989; Groezen, Leers, and Mejidam, 2003) outline why individuals' fertility choices may not be socially optimum and how publically funded child allowances could adjust it to the social optimum.

Although Glomm and Ravikumar (1992) and Cardak (2004a, b) consider public and private education, they ignore that individuals can choose both simultaneously. They consider human capital accumulation and assume it is determined by governmental or parental expenditure on education and by parent's human capital endowments, but they ignore child allowances and assume population is constant in each period. Therefore endogenous fertility does not influence private educational expenditures in their models. Although Groezen, Leers, and Mejidam (2003) consider endogenous fertility and child allowances, they assume individuals' labor income equals the wage rate and disregard human capital accumulation.

We assume individuals can simultaneously choose both public and private education to accumulate human capital, and endogenous fertility influences private educational expenditures. We introduce child allowances and investment in public education that are financed by income taxes. We assume that government cannot provide continuously higher child allowances or investment in education because doing so would generate prohibitive tax burdens. Therefore, it must choose one policy or the other. We then consider how both policies influence endogenous fertility, human capital accumulation, and economic growth.

In discussing policies, the term "public investment in education" refers to free high school education, and "private educational expenditures" include parental payments to private schools, cramming schools, and tutors. We find that when child allowances and investment in public education are financed by income taxes that cannot be raised indefinitely, government can improve either the birthrate or human capital accumulation but not both, and neither policy assures economic growth.

Section 2 presents our model, which extends Cardak (2004a) and Groezen, Leers, and Mejidam (2003), our human capital production function. Section 3 analyzes how raising child

[1] Declining Birthrate White Paper (Cabinet Office, 2003) establishes the ideal number of children per households is 2.42, whereas the actual number is 2.07, a historical low. The primary reason households do not bear the ideal number of children is "the cost burden of child care and education" and the ratio amounts to 60.4%.

allowances (reducing investment in public education) or investment in public education (reducing child allowances) influences fertility rates, human capital accumulation, and economic growth.

2. The Model

Consider an overlapping-generations economy extending over an infinite discrete period. Individuals in each generation live two periods and bear children in the second period.

2.1. Human Capital Accumulation

Individual i of generation t is born to a parent endowed with $h_{i,t}$ units of human capital. His parent provides him $e_{i,t}$ units of educational expenditure at time t. He acquires $h_{i,t+1}$ units of human capital at time $t+1$. Therefore,

$$h_{i,t+1} = (E)^{\gamma}\left(\frac{e_{i,t}}{n_{i,t}}\right)^{\delta}(h_{i,t})^{1-\gamma-\delta};\gamma,\delta \in (0,1), \gamma+\delta<1, \tag{1}$$

where E denotes public educational expenditure by government in every period, $e_{i,t}$ is private educational expenditures by individual i of generation $t-1$ at time t, and $n_{i,t}$ is the number of children born to individual i of generation $t-1$ at time t. The efficiency labor employed in the production at time t is defined as equation (2).

$$H_{t+1} = n_{i,t}h_{i,t+1} + \sum\nolimits_{j\neq i} n_{j,t}h_{j,t+1}. \tag{2}$$

In equation (2), $n_{j,t}$ is the number of children born to individual j of generation $t-1$ at time t, $h_{j,t+1}$ is the human capital level of individual j of generation t at time $t+1$, and H_{t+1} is the efficiency of labor employed in production at time $t+1$. j denotes all individuals other than individual i.

2.2. Utility Maximization

Labor income earned by individual i of generation $t-1$ at time t, $y_{i,t}$ is equal to acquired human capital $h_{i,t}$ at time t, as in Glomm and Ravikumar(1992) and Cardak(2004a, b).

$$y_{i,t} = h_{i,t} \tag{3}$$

We exclude the possibility of inheritances. Equation (4) determines the budget constraint of individual i of generation $t-1$.

$$(1-\tau_t)y_{i,t} + n_{i,t}\rho = c_{i,t} + e_{i,t}. \tag{4}$$

In equation (4), τ_t is the income tax rate at time t, ρ is child allowances in every period, and $c_{i,t}$ is consumption of individual i of generation $t-1$ at time t. We assume child allowances and investment in public education are financed by income taxes, revenues from which cannot be increased without imposing undue burdens. Therefore, we assume government sets the tax rate to hold $\rho + E$ constant. Equation (5) determines government's budget constraint.

$$\rho + E = \frac{\tau_t H_t}{n_{i,t} + \sum\nolimits_{j\neq i} n_{j,t}} = \frac{\tau_t\left(n_{i,t-1}h_{i,t} + \sum\nolimits_{j\neq i} n_{j,t-1}h_{j,t}\right)}{n_{i,t} + \sum\nolimits_{j\neq i} n_{j,t}}. \tag{5}$$

In equation (5), $n_{i,t-1}$ and $n_{j,t-1}$ are the number of children born to individual i and j of generation $t-2$ at time $t-1$, and $h_{j,t}$ is the human capital endowment of individual j of generation $t-1$ at time t. From equation (5), τ_t is derived as equation (6).

$$\tau_t = \frac{\left(n_{i,t} + \sum_{j\neq i} n_{j,t}\right)(\rho + E)}{H_t} \tag{6}$$

Individual i of generation $t-1$ chooses $c_{i,t}$, $e_{i,t}$, and $n_{i,t}$ to maximize utility across the two periods.

$$\begin{aligned}
&\underset{c_{i,t},e_{i,t},n_{i,t}}{Maximize}\ u^{t-1} = \alpha \log c_{i,t} + (1-\alpha)\log n_{i,t} + \beta \log h_{i,t+1};\ \alpha \in (0,1), \beta > 0 \\
&subject\ to\quad y_{i,t} = h_{i,t},\ (1-\tau_t)y_{i,t} + n_{i,t}\rho = c_{i,t} + e_{i,t} \\
&\rho + E = \frac{\tau_t H_t}{n_{i,t} + \sum_{j\neq i} n_{j,t}} = \frac{\tau_t\left(n_{i,t-1}h_{i,t} + \sum_{j\neq i} n_{j,t-1}h_{j,t}\right)}{n_{i,t} + \sum_{j\neq i} n_{j,t}} \\
&h_{i,t+1} = (E)^{\gamma}\left(\frac{e_{i,t}}{n_{i,t}}\right)^{\delta}\left(h_{i,t}\right)^{1-\gamma-\delta};\gamma,\delta \in (0,1), \gamma+\delta<1
\end{aligned}$$

From the first-order conditions, the optimal number of children, consumption, and private educational expenditures of individual i of generation $t-1$ at time t are derived as equation (7), (8), and (9)[2].

$$n_t = \frac{(\alpha-\beta\delta)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho+E)\sum_{j\neq i} n_{j,t}\right\}}{E} \tag{7}$$

$$c_t = (1-\alpha)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho+E)\sum_{j\neq i} n_{j,t}\right\} \tag{8}$$

$$e_t = \beta\delta\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho+E)\sum_{j\neq i} n_{j,t}\right\} \tag{9}$$

We assume $\alpha > \beta\delta$. Incorporating equation (7) and (9) into (1), the production function for human capital $h(\rho, E, h_{i,t})$ is derived as equation (10).

$$h_{i,t+1} = h(\rho, E, h_{i,t}) = (E)^{\gamma+\delta}\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\delta}\left(h_{i,t}\right)^{1-\gamma-\delta}. \tag{10}$$

From equation (10), we derive steady-state equilibrium for human capital level as equation (11).

$$h_s^* = \left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}} E \tag{11}$$

In equation (10), $0<1-\gamma-\delta<1$, and the production function for human capital becomes concave. Therefore, all individuals' human capital endowments converge to h_s^*.

[2] Proofs for Equations (7), (8), and (9) appear in the APPENDIX.

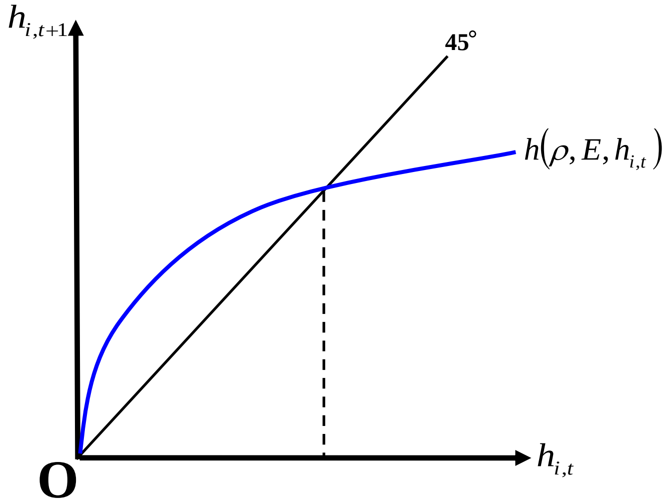

Figure 1. Human Capital Production Function

Incorporating Equation (6) and (11) into (7), we rewrite the optimal number of children born to individual i of generation $t-1$ at time t, n_t, as equation (12).

$$n_t = \frac{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left\{(\rho+E)\sum_{j\neq i} n_{j,t} - H_t\right\}E}{(\alpha-\beta\delta)(\rho+E)H_t - \left\{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}(\rho+E)+H_t\right\}E} \tag{12}$$

We assume the conditions expressed by Equation (13) and (14).

$$(\rho+E)\sum_{j\neq i} n_{j,t} - H_t > 0 \;\Rightarrow\; \rho+E > \frac{H_t}{\sum_{j\neq i} n_{j,t}} \tag{13}$$

$$(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left\{(\rho+E)\sum_{j\neq i} n_{j,t} - H_t\right\} < (\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}(\rho+E)+H_t$$

$$\Rightarrow\; \rho+E < \frac{\left\{1+(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\right\}H_t}{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left(\sum_{j\neq i} n_{j,t} - 1\right)} \tag{14}$$

From equation (13) and (14), $\rho+E$ satisfies the condition expressed by equation (15).

$$\frac{H_t}{\sum_{j\neq i} n_{j,t}} < \rho+E < \frac{\left\{1+(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\right\}H_t}{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left(\sum_{j\neq i} n_{j,t} - 1\right)} \tag{15}$$

3. Education Policies and Economic Growth

We now consider that government's budget constraint presents it with two choices—raise child allowances or investment more in public education—and evaluate consequences of both choices for economic growth. Education Policy 1 calls for raising child allowances, thereby investing less in public education because of its budget constraint. Education Policy 2 would increase investment in public education, thereby reducing child allowances. We assume ρ increases to $\rho'(>\rho)$, E decreases to $E'(<E)$, and $h(\rho, E, h_{i,t})$ shifts to $h(\rho', E', h_{i,t})$ under Education Policy 1. Whereas, ρ decreases to $\rho''(<\rho)$, E increases to $E''(>E)$, and $h(\rho, E, h_{i,t})$ shifts to $h(\rho'', E'', h_{i,t})$ under Education Policy 2.

Following Glomm and Ravikumar (1992) and Cardak (2004a, b), we disregard physical capital accumulation, and economic growth is determined by fertility rates and human capital accumulation.

3.1. Education Policy 1

Education Policy 1 satisfies the condition expressed in equation (16).

$$\rho + E = \rho' + E' \tag{16}$$

From equation (12), (13), and (14), Education Policy 1 results in more children as indicated by equation (17).

$$\begin{aligned} n_t &= \frac{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left\{(\rho+E)\sum_{j\neq i} n_{j,t} - H_t\right\}E}{(\alpha-\beta\delta)(\rho+E)H_t - \left\{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}(\rho+E) + H_t\right\}E} \\ &< \frac{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left\{(\rho+E)\sum_{j\neq i} n_{j,t} - H_t\right\}E'}{(\alpha-\beta\delta)(\rho+E)H_t - \left\{(\alpha-\beta\delta)\left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}(\rho+E) + H_t\right\}E'} \end{aligned} \tag{17}$$

From equation (11), Education Policy 1 constricts human capital accumulation as indicated by equation (18).

$$h_s^* = \left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}} E > \left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}} E' = h_s^{*'} \tag{18}$$

Figure 2 illustrates the shift of the human capital production function from $h(\rho, E, h_{i,t})$ to $h(\rho', E', h_{i,t})$.

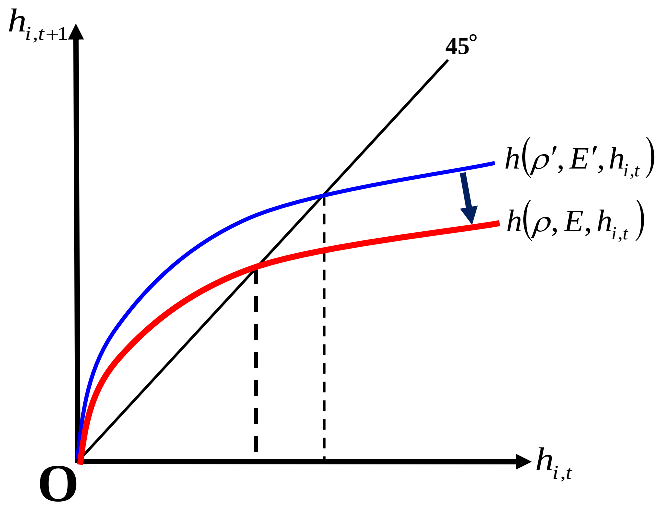

Figure 2. Shift of Human Capital Production Function Under Education Policy 1

Although Education Policy 1 results in more children and improves fertility rate, Equations (17) and (18) reveal it constricts human capital accumulation. From Equation (2), although the result is more children (i.e., higher fertility rate), constricting human capital accumulation impairs economic growth. Therefore, there is no assurance that raising child allowances (reducing investment in public education) promote economic growth.

3.2. Education Policy 2

Education Policy 2 satisfies the condition expressed in Equation (19).

$$\rho + E = \rho'' + E'' \tag{19}$$

From Equation (12), (13), and (14), Education Policy 2 results in more children as indicated by Equation (20).

$$\begin{aligned} n_t &= \frac{(\alpha-\beta\delta)\left(\dfrac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left\{(\rho+E)\sum_{j\neq i} n_{j,t} - H_t\right\}E}{(\alpha-\beta\delta)(\rho+E)H_t - \left\{(\alpha-\beta\delta)\left(\dfrac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}(\rho+E)+H_t\right\}E} \\ &> \frac{(\alpha-\beta\delta)\left(\dfrac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}\left\{(\rho+E)\sum_{j\neq i} n_{j,t} - H_t\right\}E''}{(\alpha-\beta\delta)(\rho+E)H_t - \left\{(\alpha-\beta\delta)\left(\dfrac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}}(\rho+E)+H_t\right\}E''} \end{aligned} \tag{20}$$

From equation (11), Education Policy 2 promotes human capital accumulation as indicated by Equation (21).

$$h_s^* = \left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}} E < \left(\frac{\beta\delta}{\alpha-\beta\delta}\right)^{\frac{\delta}{\gamma+\delta}} E'' = h_s^{*''} \tag{21}$$

Figure 3 illustrates the shift of human capital production function from $h(\rho, E, h_{i,t})$ to $h(\rho'', E'', h_{i,t})$.

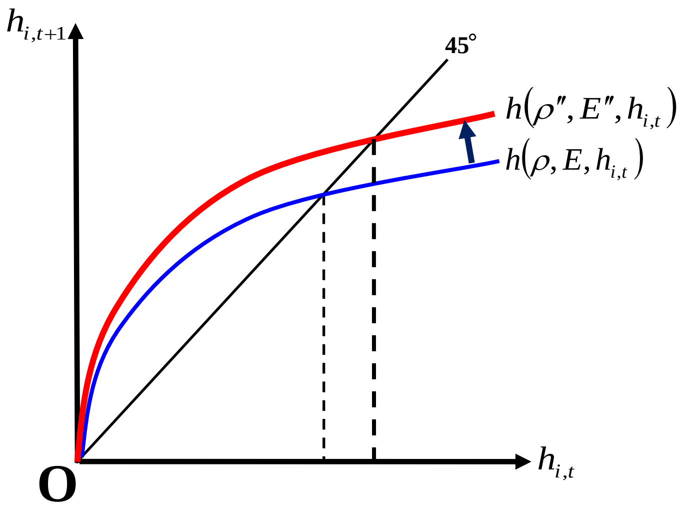

Figure 3. Shift of Human Capital Production Function Under Education Policy 2

Equations (20) and (21) reveal that Education Policy 2 promotes human capital accumulation but results in fewer children and a lower fertility rate. From Equation (2), although decreasing the number of children, (i.e., reduced fertility rate) impairs economic growth, promoting human capital accumulation enhances it. There is again no assurance that increased investment in public education (reducing child allowances) promotes economic growth.

4. Conclusion

Extending Glomm and Ravikumar (1992) and Cardak (2004a,b), we have proposed a model whereby individuals can simultaneously choose both public and private education, and we introduced child allowances and endogenous fertility into the model following Groezen, Leers, and Mejidam (2003). We assumed child allowances and investment in public education (i.e., providing free high school education) are financed by income taxes, revenues form which are fixed to avoid onerous taxation. Moreover, we considered how raising child allowances or investment in public education influences endogenous fertility, human capital accumulation, and economic growth. Three conclusions emerged.

(a) Raising child allowances and reducing investment in public education results in more children and higher fertility rates. However, that policy constricts human capital accumulation.
(b) Increasing investment in public education and reducing child allowances results in fewer children and lower fertility rates. However, that policy promotes human capital accumulation.
(c) Neither policy assures economic growth.

Our overall finding is that government can improve either fertility rates or human capital accumulation but not both, and neither policy is assured of promoting economic growth. Government must decide between emphasizing fertility rates or human capital accumulation.

If resources for child care and investment in public education are financed by national debt rather than taxes, government might be able to boost fertility rates and human capital accumulation simultaneously. Doing so, however, displaces the financial burden to future generations. In short, the Japanese government faces a difficult situation about education.

REFERENCES

Benabou, R. 1996. Heterogeneity, Stratification, and Growth: Macroeconomics Implications of Community Structure and School Finance. *American Economic Review* 86: 584-609.

Bental, B. 1989. The Old Age Security Hypothesis and Optimal Population Growth. *Journal of Population Economics* 1:285-301.

Cabinet Office. 2013. Declining Birthrate White Paper in Japanese. http://www8.cao.go.jp/shoushi/shoushika/whitepaper/measures/w2013/25pdfhonpen/pdf/s1-2.pdf

Cardak, B.A. 2004a. Education Choice, Endogenous Growth and Income Distribution. *Economica* 71: 57-81.

Cardak, B.A. 2004b. Education Choice, Neoclassical Growth, and Class Structure. Oxford Economic Papers, 56: 643-666.

Eckstein, Z. and I. Zilcha. 1999. The Effect of Compulsory Schools on Growth, Income Distribution and Welfare. *Journal of Public Economics*, 54: 339-359.

Eckstein, Z. and K. Wolpin. 1985. Endogenous Fertility and Optimal Population Size. *Journal of Public Economics*, 27: 93-1.

Glomm, G. and B. Ravikumar. 1992. Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality. *Journal of Political Economy*, 100: 818-834.

Gradstein, M. and M. Justman. 1997. Democratic Choice of an Education System: Implications for Growth and Income Distribution. *Journal of Economic Growth*, 2: 169-183.

Kaganovich, M. and I. Zilcha. 1999. Education, Social Security, and Growth. *Journal of Public Economics*, 69: 289-309.

Saint, P.G. and T. Verdier. 1993. Education, Democracy and Growth. *Journal of Development Economics*, 82: 393-408.

van Groezen, B., T. Leers, and L. Mejidam. 2003. Social Security and Endogenous Fertility: Pension and Child Allowance as Siamese Twins, *Journal of Public Economics*, 87: 233-251.

APPENDIX

From equation (3), (4), and (5), u^{t-1} is rewritten as follow:

$$\begin{aligned} u^{t-1} &= (1-\alpha)\log\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho+E)\sum_{j\neq i} n_{j,t} - n_{i,t}E - e_{i,t}\right\} + \alpha\log n_{i,t} + \beta\log h_{i,t+1} \\ &= (1-\alpha)\log\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho+E)\sum_{j\neq i} n_{j,t} - n_{i,t}E - e_{i,t}\right\} + \alpha\log n_{i,t} \\ &\quad + \beta\gamma\log E + \beta\delta\log e_{i,t} - \beta\delta\log n_{i,t} + \beta(1-\gamma-\delta)\log h_{i,t} \end{aligned}$$

The optimal private educational expenditure of individual i of generation $t-1$ at time t is derived as follows.

$$\frac{\partial u^{t-1}}{\partial e_{i,t}} = \frac{-(1-\alpha)}{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E - e_{i,t}} + \frac{\beta\delta}{e_{i,t}} = 0$$

$$e_t = \frac{\beta\delta\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E\right\}}{1-\alpha+\beta\delta} \tag{A-1}$$

The optimal consumption of individual i of generation $t-1$ at time t is derived as follows.

$$c_{i,t} = (1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E - \frac{\beta\delta\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E\right\}}{1-\alpha+\beta\delta}$$

$$c_t = \frac{(1-\alpha)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E\right\}}{1-\alpha+\beta\delta} \tag{A-2}$$

Incorporating Equations (A-1) and (A-2), u^{t-1} is rewritten as follow.

$$\begin{aligned} u^{t-1} = {} & (1-\alpha)\log \frac{(1-\alpha)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E\right\}}{1-\alpha+\beta\delta} + \alpha \log n_{i,t} \\ & + \beta\gamma \log E + \beta\delta \log \frac{\beta\delta\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} E\right\}}{1-\alpha+\beta\delta} \\ & - \beta\delta \log n_{i,t} + \beta(1-\gamma-\delta)\log h_{i,t} \end{aligned}$$

The optimal number of children of individual i of generation $t-1$ at time t is derived as follows.

$$\begin{aligned} \frac{\partial u^{t-1}}{\partial n_{i,t}} = {} & \frac{-(1-\alpha)E}{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E} + \frac{\alpha - \beta\delta}{n_{i,t}} \\ & + \frac{-\beta\delta E}{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t} - n_{i,t}E} = 0 \end{aligned}$$

$$n_t = \frac{(\alpha - \beta\delta)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t}\right\}}{E} \tag{A-3}$$

Incorporating Equation (A-3) into (A-1), the optimal private educational expenditure of individual i of generation $t-1$ at time t is derived as follows.

$$\begin{aligned} e_t = {} & \frac{\beta\delta\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t}\right\}}{1-\alpha+\beta\delta} \\ & - \frac{\beta\delta E}{1-\alpha+\beta\delta} \times \frac{(\alpha - \beta\delta)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t}\right\}}{E} \\ = {} & \beta\delta\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j \neq i} n_{j,t}\right\} \end{aligned}$$

Incorporating Equation (A-3) into (A-2), the optimal consumption of individual i of generation $t-1$ at time t is derived as follows.

$$
\begin{aligned}
c_t &= \frac{(1-\alpha)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j\neq i} n_{j,t}\right\}}{1-\alpha+\beta\delta} \\
&\quad - \frac{(1-\alpha)E}{1-\alpha+\beta\delta} \times \frac{(\alpha-\beta\delta)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j\neq i} n_{j,t}\right\}}{E} \\
&= (1-\alpha)\left\{(1-\tau_t)h_{i,t} + \tau_t H_t - (\rho + E)\sum_{j\neq i} n_{j,t}\right\}
\end{aligned}
$$