Environmental Regulation and Intra-Industry Trade

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Abstract

The paper analyzes the complex interactions between intra-industry trade (IIT) and the environment by extending the Krugman's model of monopolistic competition and trade. Both autarky and free trade equilibria are derived, where output, prices, number of varieties and pollution are determined endogenously. It is found that a unilateral increase in the exogenous environmental tax by a country leads to a fall in output, increase in number of varieties and fall in aggregate pollution in that country. With IIT, if Home is a net exporter, an increase in environmental stringency by Home leads to a fall in its export-competitiveness and output of each variety (scale effect) on the one hand, thus reducing its export demand and raising its import demand. On the other hand, higher prices and lower cost of production induce entry of firms in the Home industry (selection effect), reducing its import demand. Consequently, the aggregate effect on imports remains ambiguous, and the first-order scale effect on exports dominates the second-order effect on imports implying a rise in Home's share of IIT in its overall trade with Foreign. The opposite holds true in case Home is a net importer, where the share of its IIT falls to give way to more inter-industry trade with Foreign. Furthermore, the impact of a rise in environmental tax on aggregate welfare comprises the following effects: a fall in aggregate output (scale effect), rise in number of varieties (selection effect), lowering of aggregate pollution and rise in environmental tax revenue in autarky. With free trade, two additional effects, namely, changes in the level of exports and imports, will also arise. The overall change in aggregate welfare, in both autarky and free trade, depends on the relative strength of these effects, and is in general ambiguous.

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1 Introduction

The trade-environment relationship has two significant aspects that require a deeper economic analysis. The first aspect relates to the environmental implications of trade liberalization, while the second one pertains to the plausible influence of environmental policy stringency on the global distribution of production, and consequent implications for the pattern of international trade, environmental outcomes and aggregate welfare of the trading countries. International trade is driven not only by comparative advantage based on differences in factor endowments across countries, but may also be determined by consumer's preference for variety in goods and economies of scale in the production of each variety. The first approach, which is also the older one, examines the environmental impact of trade based on the traditional Heckscher-Ohlin-Samuelson (HOS) framework, that is, one based on inter-industry trade. The factor-endowment framework is the most commonly studied approach and has a large body of literature dedicated to it (see for instance, Copeland and Taylor (1994, 1995) and Mehra and Das (2008) for a theoretical treatment of this issue, and Antweiler et al. (2001), Shen (2008), Cole and Elliot (2003), Cole (2006) for an empirical analysis). Most analytical papers, such as those by Copeland and Taylor (1994, 1995) and Mehra and Das (2008) discuss the links between environment and trade working through the scale, technique and composition effects. Scale effect refers to the increase in the size of an economy due to trade-liberalization induced increases in market access while the technique effect refers to the impact on pollution due to a change in the production processes by inducing transfer of cleaner technologies or changes in regulations as demand for environmental quality rises with income. Finally, the composition effect refers to a change in pollution due to a change in the relative shares of different goods (polluting versus non-polluting) in production. This means that the actual impact of the composition effect on the environment depends on the determinants of a country's comparative advantage.

The alternative approach examines the environmental impact of trade based on the “new” trade-theoretic framework, that is, one of intra-industry trade (or IIT). IIT refers to trade in similar products, largely determined by the demand for differentiated goods or varieties. IN this case, in addition to the scale and technique effects, the selection effect comes into play, which refers to how a change in the number of product varieties changes the pollution levels. This replaces the composition effect. It is now more relevant to focus on modeling IIT because of its increasing importance in the total world trade.[1] A study by Brulhart (2008) notes that in 2006, 27 percent of global trade, when measured at the 5-digit (finest) level of statistical aggregation, and 44 percent when measured at the 3 digit (coarser) level of statistical aggregation, was IIT. The author also asserts that the share of IIT has been on the rise over the last five decades, suggesting a gradual convergence of the sectoral composition of national economies world-wide.

An interesting extension of the IIT paradigm is the analysis of its interface with environmental issues. Rauscher (1997) uses an IIT framework to analyze the outcome of a change in the environmental policy on the production and number of firms and finds that, due to stricter environmental regulation, output of each variety is reduced while the impact on the number of varieties is ambiguous. Gurtzgen and Rauscher (2000) discuss the impacts of domestic environmental regulation on foreign country's emissions and transboundary pollution through the channel of the pollution leakage effect. This effect works in the following manner: a change in domestic environmental policy changes the number of firms that can operate profitably in Home thus changing the number of available varieties. This, in turn, impacts the consumer behavior and the mark-ups that producers charge in excess of the marginal costs. Therefore, the number of firms and emissions in Foreign are also impacted. Benarroch and Weder (2006) explore the link between environmental taxes and IIT in intermediate goods with an increasing returns pollution function. They find that, under autarky, an exogenous increase in the environmental tax rate of a country raises that country's relative cost of using the dirty intermediate products and increases the relative demand for clean intermediates, which leads to the decrease in pollution and output of the final good. Moreover, international trade in intermediate products leads to an increase in the environmental tax, which depending on the strengths of the scale, composition and technique effects, may or may not lower total pollution relative to autarky. Aralas and Hoehn (2010) have tried to examine the impact of IIT on environment by setting up an analytical model and then testing it empirically. Moving along the same line as Antweiler et al. (2001), they show that the growth in total emissions is dependent on the growth of the scale of the economy (the scale effect), the growth in the emissions intensity of production (the technique effect) and the change in the number of firms or identically, in the number of product varieties (the selection effect). Cole and Elliot (2003) examine empirically whether environmental regulations have any impact on the composition of trade within the new trade-theoretic framework by using cross-sectional trade data for developed and developing countries for the year 1995.

[1] According to the Standard Industrial Trade Classification (SITC), about 25 per cent of the current world trade is IIT.

In terms of the model structure and some initial analysis, our research is the closest to the general equilibrium trade-theoretic framework of Rauscher (1997). However, the cost function in our paper has a more specific functional form as compared to Rauscher (1997), which helps characterize increasing returns to scale more clearly, and derive more definitive results in terms of the impact of change in environmental regulation on the output (via the scale effect), number of varieties produced (via the selection effect) and the aggregate pollution (through a combination of the above two effects). The technique effect is absent in our model because of the assumption of a functional specification under which the aggregate pollution varies in some constant positive proportion with the output and number varieties. Moreover, our research is extended beyond Rauscher (1997) in that we also analyze how the stringency of environmental regulation (environmental taxes) affects the IIT flows (captured by the Grubel-Lloyd (G-L) index) and the aggregate welfare of the trading countries, again working through the scale, selection and environmental (own and spillover) effects.

Some findings of our research are as follows. Under both autarky and free trade, an exogenous unilateral rise in environmental stringency by Home (or Foreign) leads to a fall in the production of each variety (scale effect), increase in its number of varieties (selection effect) and a fall in its aggregate pollution (a combination of the above two effects). These impacts work through changes in the labor employment, wages and price of varieties, through a general equilibrium system, explained later in the text. Moreover, if Home is a net exporter, an increase in the environmental stringency by Home leads to a rise in its share of IIT in its aggregate trade with Foreign, and if Home is a net importer, an increase in environmental stringency by Home leads to a fall in its share of IIT in overall with Foreign. In the first case, this occurs because the negative scale effect dominates the positive selection effect, thus resulting in a fall in the level of exports and a rise in the level of imports, entailing a higher share of IIT in aggregate between the two countries. In the second case, when Home is a net importer, identical effects are at work, but the fall in exports is such in comparison to the rise in imports that it actually lowers the IIT share in aggregate trade between Home and Foreign. Furthermore, the impact of a rise in environmental tax on the welfare of Home and Foreign is found to be ambiguous on account of counteracting effects: a rise in environmental tax of a country reduces its welfare due to lowering of aggregate output (scale effect), but raises welfare due to an increase in the number of varieties available for consumption (selection effect), reduction in its pollution and increase in tax revenue that could potentially be used for eliminating the distortionary effects of taxes elsewhere in the economy. These effects are common for both autarky and free trade regimes. Besides these, in the case of free trade, the changes in level of exports and imports due to a change in the environmental tax rate are taken into account, which provide an additional source of ambiguity to the overall welfare effects.

The rest of the paper is organized as follows. Section 2 lays out the theoretical model and describes the production and consumption sides of this stylized economy. Section 3 solves the autarky and free trade equilibria. It also characterizes the implications of an exogenous change in environmental regulation on trade and pollution levels in the trading countries. This is followed by an analysis of the impact of environmental regulation on the pattern of IIT in Section 4. Section 5 analyzes the impact of environmental regulation on the aggregate welfare levels of the trading countries. Finally, Section 6 concludes.

2 Theoretical Model

We consider an extension of the model of monopolistic competition and trade of Krugman (1980), wherein each firm wields market power, but free entry of firms in the long-run reduces monopoly profits to zero. Each firm can costlessly differentiate its product variety from another, and there exist economies of scale in production. There are two countries: Home and Foreign. Both comprise of consumers, producers and a government that regulates. The two countries have identical technology and preferences and there is no transportation cost between them. Labor is the only factor of production, which is inelastically supplied in a competitive labor market in each country. The cost function is linear in the labor input. The extension of the model takes the following form. The production of each product variety generates environmental pollution. Pollution is proportional to production and its adverse effects are not necessarily confined to the country that emits it; that is, there exist pollution spillovers to the other country as well. Further, each government imposes an exogenous environmental tax rate on the producers of their respective countries. The environmental taxes are the only policy tool available to the respective country governments to regulate pollution.

In what follows immediately, the consumption and production sides of the economy are described, which are later solved to characterize the autarky equilibrium. Next, the case of free trade equilibrium between the two countries is characterized that examines the impact of trade on the output, number of varieties (or firms) and the pollution levels in the two countries in comparison with autarky. Finally, a comparative static analysis is attempted to understand the impact of an exogenous change in the environmental tax in each country on the equilibrium level of the output, number of varieties, and pollution in both Home and Foreign. The analysis is extended to ascertain the effect of an increase in the environmental policy stringency on the share of intra-industry trade, as captured by G-L index, as well as the aggregate welfare in both the countries.

We begin with the consumption side of this stylized economy.

2.1 Consumers

There is a differentiated commodity with potentially infinite varieties, all of which enter symmetrically into the demand function of the consumers. There are l number of individuals (also consumers) in the Home economy and L individuals in the Foreign economy. Each individual is assumed to be endowed with one unit of labor. All the individuals within the Home (Foreign) country are identical, that is, they have the same utility function. Moreover, all the commodities are desirable from the point of the consumers in a likewise manner. Without loss of generality, we focus on the Home country. Consumers wish to consume as many varieties as possible. This "love-for-diversity" or "love-for-variety" is introduced via the following specification of the representative consumer's utility function:

$$u = \sum_{i=1}^{n} c_i^{\theta} + v\left[z + (1-\delta)Z\right]; \quad 0 < \theta < 1, \quad 0 < \delta < 1, \tag{1}$$

where c_i is the consumption of the ith variety, z and Z denote the emissions from the production of the good in the Home and Foreign countries respectively, δ is the pollution spillover parameter and θ is the elasticity of substitution in consumption between different varieties of goods. θ is taken to be a constant in the sense that it does not rise when the number of available varieties in an economy increase.[2] There are n varieties of commodities produced in the Home country. This implies that consumers derive positive utility from the consumption of the good but reap a negative utility or disutility from the pollution in their own country (Home) as well as from pollution spillover from the Foreign country. That is,

$$\frac{\partial u}{\partial c_i} > 0, \quad i = 1, 2, ..., n, \quad \frac{\partial u}{\partial z} < 0 \quad and \quad \frac{\partial u}{\partial Z} < 0.$$

[2]The case of θ being of function of number of varieties is assumed in Gurtzgen and Rauscher (2000). Rauscher (1997) also considers θ to be an increasing function of the number of varieties for one of his results. This is ignored in the current analysis for mathematical tractability.

The consumers maximize their utility with respect to c_i subject to their budget constraint, taking Home's and the Foreign's pollution levels – z and Z – as given. That is, they solve

$$Max_{c_i} \quad \sum_{i=1}^{n} c_i^{\theta} + v(z + (1-\delta)Z) \quad subject \quad to \quad w = \sum_{i=1}^{n} p_i c_i, \tag{2}$$

where w is the wage per unit labor and p_i is the price of ith variety in the Home economy. From the respective first-order condition we get the following inverse demand function for each variety:

$$p_i = \theta\lambda^{-1} c_i^{\theta-1}, \quad i = 1, 2, ...n, \tag{3}$$

where λ is the marginal utility of income.

Further, equilibrium in the product market implies that total demand equals total production, such that

$$lc_i = q_i, \quad i = 1, 2...n, \tag{4}$$

where q_i is the output of the ith variety. Using this, we can now write the inverse demand function as

$$p_i = \theta\lambda^{-1} \left(\frac{q_i}{l}\right)^{\theta-1}. \tag{5}$$

The price elasticity of demand, ϵ_i, can be calculated from the inverse demand function and is given by:

$$\epsilon_i = \frac{1}{1-\theta} = \epsilon. \tag{6}$$

Thus, with θ constant, the price elasticity of demand is also constant, and is positively related to the elasticity of substitution between varieties. As we shall see later on, the size of θ will play an important role in determining some of our results.

2.2 Producers

On the production side, there are n varieties being produced in Home. Since each firm produces only one variety on account of economies of scale, there exist n firms in Home. Firms use identical technology of production wherein the demand for labor is a linear function of output and takes a specific functional form given by:[3]

$$l_i = \alpha + \beta q_i, \quad i = 1, 2, ...n, \tag{7}$$

[3]Rauscher (1997) takes the variable cost of production to be dependent on the quantity produced and on the factor prices for the environmental resource and capital, that is, the emissions tax rate t^e and the rate of interest r. The unit costs is of a generic form given by $c(t^{e,i}, r)$. Further, the fixed cost of production is given as rk^0, where k^0 is some initial fixed capital required for the production of each variety.

where α is the fixed cost of production, β is the marginal cost of production and $\alpha,\ \beta > 0$. The cost function for each firm could be written as:

$$wl_i = w\alpha + w\beta q_i, \quad i = 1, 2, ...n,$$

and the average cost function could be derived by dividing the above by q_i. That is,

$$\frac{wl_i}{q_i} = \frac{w\alpha}{q_i} + w\beta, \quad i = 1, 2, ...n,$$

where, $w\alpha/q_i$ is the fixed cost that declines with an increase in q_i and $w\beta$ is the constant marginal cost. Hence, with positive initial costs, constant marginal costs and thus declining average costs, there exist increasing returns to scale that are internal to the firm.

Pollution is generated as a by-product of production activity, in some constant proportion, γ, of the output level:

$$z^i = \gamma q_i, \quad i = 1, 2, ...n, \tag{8}$$

and aggregate pollution in Home, $z = \sum_{i=1}^{n} z^i = n\gamma q_i$, or,

$$z = n\gamma q, \quad where \quad q = q_i \quad \forall \quad i = 1, 2, ..., n. \tag{9}$$

The Home and the Foreign governments impose exogenous environmental taxes – t and T – per unit output on their respective producers.

Finally, the full employment condition is given by:

$$l = \sum_{i=1}^{n} l_i = \sum_{i=1}^{n} (\alpha + \beta q_i). \tag{10}$$

The producers maximize profits. The profits of the ith producer can be expressed as:

$$\pi_i = p_i q_i - (\alpha + \beta q_i)w - tq_i, \tag{11}$$

where w is the wage rate for labor and is taken as a given by firm i. Using equation (5), the profits of the ith producer can be expressed as

$$\pi_i = \theta\lambda^{-1}\left(\frac{q_i}{l}\right)^{\theta-1} q_i - (\alpha + \beta q_i)w - tq_i. \tag{12}$$

We now characterize the autarky and free trade equilibria.

3 Characterizing the equilibria

3.1 Autarky equilibrium

Let us assume that both Home and Foreign do not trade. Also, without loss of generality, we first focus on the Home country characterization. Under monopolistic competition, the following two conditions will characterize the equilibrium:
1) Marginal Revenue (MR) = Marginal Cost (MC) in the short-run, and,
2) Profits = 0 in the long-run.
The first condition yields the following equation:

$$p_i = \theta^{-1}(\beta w + t), \tag{13}$$

and the second condition yields the condition:

$$p_i = \frac{(\alpha + \beta q_i)w}{(q_i + t)}. \tag{14}$$

Solving equations (13) and (14) simultaneously derives the output for the ith firm to be

$$q_i = \frac{w\alpha\theta}{(1-\theta)(\beta w + t)} = q \quad \forall \quad i = 1, 2, ..., n. \tag{15}$$

This will be true for all i.

We now use the full employment condition in equation (10) to find the number of varieties or the number of firms in each country. That is,

$$l = n(\alpha + \beta q_i),$$

or,

$$n = \frac{l}{\alpha + \beta q_i} \tag{16}$$

Using equation (15), the equilibrium number of firms (or varieties) is derived to be:

$$n = \frac{l(1-\theta)(\beta w + t)}{\alpha[\beta w + (1-\theta)t]}. \tag{17}$$

We get the long-run equilibrium price prevailing under autarky in the Home country by substituting the value of q_i from equation (15) into equation (14) to be:

$$p_i = \frac{\beta w + t}{\theta} = p \quad \forall \quad i = 1, 2, ...n. \tag{18}$$

Notably, given that each firm i in the industry is a monopoly for the ith variety, and uses mark-up pricing over marginal costs $\beta w + t$, it must be that $p_i > \beta w + t$ implying the restriction that

$$0 < \theta < 1. \tag{19}$$

The aggregate pollution in Home country is given by equation (9). Thus, using (15) and (17), the aggregate pollution in Home can be expressed as:

$$z = \frac{\gamma l w \theta}{[\beta w + (1-\theta)t]}. \tag{20}$$

Correspondingly, for the Foreign country we use the same notations but with respective uppercase letters. So the total labor force in Foreign is denoted by L. We derive the symmetric autarky equilibrium equations for output for the Ith commodity (where $I = 1, 2, ..., N$), number of varieties and aggregate pollution for Foreign as:

$$Q_I = \frac{W\alpha\theta}{(1-\theta)(\beta W + T)} = Q \quad \forall \quad I = 1, 2..., N, \tag{21}$$

$$N = \frac{L(1-\theta)(\beta W + T)}{\alpha[\beta W + (1-\theta)T]}, \tag{22}$$

$$P_I = \frac{\beta W + T}{\theta} = P \quad \forall \quad I = 1, 2...N, \tag{23}$$

and

$$Z = \frac{\gamma L W \theta}{[\beta W + (1-\theta)T]}, \tag{24}$$

where T is the unit exogenous environmental tax rate imposed by the Foreign country on its producers. From (19), similar mark-up pricing exists in this case as well.

Based on the expressions in (15)-(24), ignoring subscript i and I, it is easy to show that

Proposition 1: *In the autarky equilibrium, an exogenous unilateral rise in the domestic environmental tax rate in the Home country results in a fall in the production of each firm but raises the number of varieties (or firms) in the Home country. Similar results hold for the Foreign country as well.*

Mathematically, from equations (15) and (17) and equations (21) and (22), we get the following signs of the partials for the Home and Foreign countries respectively:

$$\frac{\partial q}{\partial t} = -\frac{w\alpha\theta}{(1-\theta)(\beta w + t)^2} < 0; \tag{25}$$

$$\frac{\partial n}{\partial t} = \frac{l\theta\beta w(1-\theta)}{\alpha[\beta w + (1-\theta)t]^2} > 0; \tag{26}$$

and

$$\frac{\partial Q}{\partial T} = -\frac{W\alpha\theta}{(1-\theta)(\beta W+T)^2} < 0; \tag{27}$$

$$\frac{\partial N}{\partial T} = \frac{L\theta\beta W(1-\theta)}{\alpha[\beta W+(1-\theta)T]^2} > 0. \tag{28}$$

Intuitively, an increase in the environmental tax rate lowers profits and thus reduces the production of the individual firms (this is the negative scale effect). Given that firms are monopolists in their individual markets, they are able to shift a part of the burden of this tax to the consumers by a fraction of $1/\theta$ (see the price expression in equation (18) for this), thus raising profits for each firm in the short-run. Since the firms can charge a price which is higher than the marginal cost, the amount of price mark-up charged can be calculated as follows:

$$\frac{P-MC}{P} = (1-\theta) > 0 \quad with \quad 0 < \theta < 1.$$

This induces new firms to enter the markets till all firms break even or are just able to cover their fixed cost of production, α, which raises the number of firms in the industry (this is the positive selection effect).[4]

Further,

Proposition 2: *An exogenous and unilateral increase in the stringency of environmental regulation by Home (Foreign) results in a fall in its aggregate pollution under autarky.*

This is proved as follows. Differentiating equation (20) w.r.t. t and equation (24) w.r.t. T, we have,

$$\frac{\partial z}{\partial t} = -\frac{\gamma lw\theta(1-\theta)}{[\beta w+(1-\theta)t]^2} < 0; \tag{29}$$

$$\frac{\partial Z}{\partial T} = -\frac{\gamma LW\theta(1-\theta)}{[\beta W+(1-\theta)T]^2} < 0. \tag{30}$$

Thus, a rise in the environmental tax in each country – Home and Foreign – results in lowering of environmental pollution.

Intuitively, this works through two opposing effects, which can be seen by differentiating equation (9) for Home w.r.t. t that yields

$$\frac{\partial z}{\partial t} = \gamma\left[n\frac{\partial q}{\partial t} + q\frac{\partial n}{\partial t}\right].$$

[4]Droge and Schroder (2005) also derive similar results in the context of doing welfare comparisons of ad valorem and unit taxes under monopolistic competition.

The first term in square brackets in the r.h.s. works through a reduction in the production by each firm (a first-order scale effect), thus leading to a fall in the level of pollution externality; the second term in square brackets in the r.h.s. entails an increase in the number of varieties being produced (or the number of firms in the country), thus leading to a rise in the level of pollution externality (a second-order effect selection effect). Since the first-order scale effect dominates the second-order selection effect, there will be a fall in aggregate pollution in Home, as seen from the direction of change in environmental pollution in (29). Thus, aggregate pollution at the autarky equilibrium falls in Home due to the tightening of environmental regulation.

Notably, given our assumption that environmental pollution is a constant fraction, γ, of output for each firm i, and labor is the only factor of production, there is absence of the technique effect (found by Copeland and Taylor [1994, 1995] and Mehra and Das [2008], both in the context of inter-industry trade). This can be seen by looking at the per unit pollution level, that is,

$$\frac{z}{nq} = \gamma,$$

which implies that,

$$\frac{\partial\left(\frac{z}{nq}\right)}{\partial t} = 0.$$

Similar effects are at work for Foreign as well, thus yielding the direction of change in environmental pollution in (30).

From the above proposition, we can also deduce the following:

Proposition 3: *In the autarky equilibrium, the total output produced in Home country falls as a result of an exogenous rise in its environmental tax rate. Similar results hold for the Foreign country.*

This is proved as follows. The total output produced in the Home country can be written as nq. Differentiating this with respect to t, we get the change in the aggregate output due to change in environmental tax as the sum of the two effects in the r.h.s. of the expression below:

$$\frac{d(nq)}{dt} = \frac{\partial q}{\partial t}n + \frac{\partial n}{\partial t}q.$$

The first term in the r.h.s. in the above expression is the negative scale effect while the second term is the positive selection effect. As the scale effect is a first-order effect and selection effect is a second-order effect, the former dominates the latter that leads to a fall in the total production after an increase in the environmental tax by Home (or Foreign). By using equations (15), (17), (25) and (26), one can ascertain the magnitude of this change in Home on account of an exogenous change in its environmental tax rate to be:

$$-\frac{lw\theta(1-\theta)}{[\beta w+(1-\theta)t]^2}<0.$$

By using (21), (22, (27) and (28), a symmetric expression for the aggregate change in output can be worked out for Foreign as well.

3.2 Free Trade Equilibrium

We now allow for trade to be opened up between the Home and Foreign countries at zero transportation costs. Given the "love-for-variety" preference structure, the consumers in each country will now want to consume both – Home and Foreign – varieties. We denote the free trade equilibrium solutions for variables with a prime, ($\prime$). Further, θ, t and T remain the same as in the case of autarky; these are assumed to be given parametrically. The consumers in Home now maximize their utility with respect to the consumption of the domestically produced varieties (c_i's) as well as the imports of Foreign's varieties (m_Is) at prices p' and P' respectively. That is,

$$u'=\sum_{i=1}^{n}(c_i')^{\theta}+\sum_{I=1}^{N}m_I^{\theta}+v[z'+(1-\delta)Z'];\quad 0<\theta<1,\quad 0<\delta<1, \tag{31}$$

subject to the budget constraint

$$w'=\sum_{i=1}^{n}p'c_i'+\sum_{I=1}^{N}P'm_I,$$

where m_I represents the level of imports of Foreign's varieties by Home consumers. Similarly, the consumers in Foreign maximize their utility with respect to their consumption of domestically produced goods as well as imports from Home, such that

$$U'=\sum_{I=1}^{N}(C_I')^{\theta}+\sum_{i=1}^{n}M_i^{\theta}+v[Z'+(1-\delta)z'];\quad 0<\theta<1,\quad 0<\delta<1, \tag{32}$$

subject to the budget constraint

$$W'=\sum_{I=1}^{N}P'C_I'+\sum_{i=1}^{n}p'M_i,$$

where M_i is level of the imports of Home varieties by the Foreign's consumers. Maximization of the utility of the representative Home and Foreign consumer with respect to their respective consumption of Home and Foreign varieties yields the following first-order conditions:

$$p_i' = (\lambda^{-1}\theta)(c_i')^{\theta-1} = p' \quad \forall \quad i = 1, 2, ..., n', \tag{33}$$

$$P_I' = (\lambda^{-1}\theta)(m_I)^{\theta-1} = P' \quad \forall \quad I = 1, 2, ..., N', \tag{34}$$

$$P_I' = (\lambda^*)^{-1}\theta(C_I')^{\theta-1} = P' \quad \forall \quad I = 1, 2, ..., N', \tag{35}$$

$$p_i' = (\lambda^*)^{-1}\theta(M_i)^{\theta-1} = p' \quad \forall \quad i = 1, 2, ..., n', \tag{36}$$

where λ and λ^* are the marginal utilities of income for Home and Foreign consumers. Further, one would derive symmetric solutions for prices, p' and P', at the free trade equilibrium, similar to the case in autarky equilibrium.

From utility maximization, we get the demand functions for each variety to be:

$$c_i' = \frac{w'(p')^{\frac{1}{\theta-1}}}{\left[n'(p')^{\frac{\theta}{\theta-1}} + N'(P')^{\frac{\theta}{\theta-1}}\right]}, \tag{37}$$

$$m_I = \frac{w'(P')^{\frac{1}{\theta-1}}}{\left[n'(p')^{\frac{\theta}{\theta-1}} + N'(P')^{\frac{\theta}{\theta-1}}\right]}, \tag{38}$$

$$C_I' = \frac{W'(P')^{\frac{1}{\theta-1}}}{\left[n'(p')^{\frac{\theta}{\theta-1}} + N'(P')^{\frac{\theta}{\theta-1}}\right]}, \tag{39}$$

$$M_i = \frac{W'(p')^{\frac{1}{\theta-1}}}{\left[n'(p')^{\frac{\theta}{\theta-1}} + N'(P')^{\frac{\theta}{\theta-1}}\right]}. \tag{40}$$

See Appendix I for the full derivation of these.

With the opening up of trade, the product market clearing equation for Home will be:

$$q_i' = c_i' + M_i = q' \quad \forall \quad i = 1, 2, ..., n', \tag{41}$$

that is, the total production of the ith variety now equals the total domestic demand plus total import demand, which is also symmetric across all the n' varieties in Home. Similarly, for Foreign, the product market clearing equation is given by:

$$Q_I' = C_I' + m_I = Q' \quad \forall \quad I = 1, 2, ..., N'. \tag{42}$$

Using equations (33), (36) and (41), we get the demand for domestic variety as:

$$q' = (p')^{1/(\theta-1)}[(\lambda\theta^{-1})^{1/(\theta-1)} + (\lambda^*\theta^{-1})^{1/(\theta-1)}]. \tag{43}$$

Note that λ and λ^* denote the constant marginal utilities of income for Home and Foreign individuals respectively, and θ is also assumed to be constant. Then, under the assumption that there exist a large number of firms operating in the economy, we can infer that the term inside the square brackets in the r.h.s. of (43) will not be influenced by the decisions of any one firm, since a single firm holds a very small fraction of the total market share. This implies that the demand for domestic variety, q', is dependent only on the respective Home country price, p'. Log differentiating this expression with respect to the own price yields the price elasticity of demand for the ith variety to be $1/(1-\theta)$, which is the same as under autarky, and is a constant (see Appendix II for the detailed derivation).[5] Thus, we get an important result:

Result R1: Home's (Foreign's) variables are a function only of Home's (Foreign's) own respective prices, which in turn, are a function of Home's (Foreign's) environmental tax alone.

This result is significant and will be used later in analyzing the effect of change in environment tax on the pattern of IIT and aggregate welfare of the trading countries.

Next, on the production side, the profits of the ith firm in Home can be written as:

$$\pi' = p'q' - (\alpha + \beta q')w' - tq',$$

Again, the two conditions of monopolistic competition equilibrium have to be satisfied, that is,
(1) MR=MC in the short-run and,
(2) Profits = 0 in the long-run.

These conditions imply that

$$p' = \theta^{-1}(\beta w' + t), \tag{44}$$

and

$$p'q' = (\alpha + \beta q')w' + tq'. \tag{45}$$

Further, the full employment condition for Home is given as:

$$l = n'(\alpha + \beta q'). \tag{46}$$

[5] A similar result can be found in Lancaster (1984) and Gros (1987) who have modeled trade policy in the context of IIT and Rauscher(1997) who has modeled the interaction between environmental policy and intra-industry trade.

Equations (44), (45) and (46) can be solved to get the free trade equilibrium values of prices, outputs and number of varieties. Using $(R1)$ and equation (44) it is evident that the Home country price is independent of Foreign's tax rates. Thus, Home's output, price and number of varieties are independent of Foreign's environmental tax, that is, Home's variables are independent of Foreign policy variable. Using a similar argument for Foreign, the Foreign country variables will also be independent of Home's environmental policies/ taxes.

At the free trade equilibrium, the balance of payments equation is also assumed to hold, which is given by:

$$N'P'm_I = n'p'M_i. \tag{47}$$

Further, the total income of an individual in Home is given by the following budget constraint:

$$w' = \sum_{i=1}^{n} p'c_i' + \sum_{I=1}^{N} P'm_I, \tag{48}$$

which can be also be written as (summing over the number of varieties i and I):

$$w' = n'p'c_i' + N'P'm_I. \tag{49}$$

Using equation (47), we can write the total income in Home as

$$w' = n'p'q'. \tag{50}$$

Similarly, for the Foreign country, the total income is given by:

$$W' = N'P'Q'. \tag{51}$$

By using equations (40), (50), (38) and (51), the demand functions for imports by Home and Foreign can now be written respectively as:

$$m_I = \frac{n'(q')^{\theta}Q'}{[n'(q')^{\theta} + N'(Q)'^{\theta}]}, \tag{52}$$

and

$$M_i = \frac{N'(Q')^{\theta}q'}{[n'(q')^{\theta} + N'(Q)'^{\theta}]}. \tag{53}$$

To analyze the impact of environmental policy changes on different variables, we divide equations (44) and (45) by p' to get that,

$$\theta = \beta(w'/p') + (t/p'), \tag{54}$$

and

$$q' = (\alpha + \beta q')(w'/p') + (t/p')q', \tag{55}$$

where (w'/p') and (t/p') are the real wage rate and real environmental tax rate respectively in Home.

By totally differentiating equations (54) and (55), we get

$$-\beta d(w'/p') = d(t/p'), \tag{56}$$

$$[1 - \beta(w'/p') - (t/p')]dq' - (\alpha + \beta q')d(w'/p') = q'd(t/p'). \tag{57}$$

Moreover, totally differentiating equation (46) and substituting the value of dq' from equation (57), we get,

$$[(\alpha + \beta q')(1 - \beta(w'/p') - (t/p')]dn' + [n'\beta(\alpha + \beta q')]d(w'/p') = -n'\beta q'd(t/p'). \tag{58}$$

The last three equations can be written in matrix form as:

$$\begin{pmatrix} [1 - \beta(w'/p') - (t/p')] & -(\alpha + \beta q') & 0 \\ 0 & -\beta & 0 \\ 0 & [n'\beta(\alpha + \beta q')] & [(\alpha + \beta q')(1 - \beta(w'/p') - (t/p')] \end{pmatrix} \begin{pmatrix} dq' \\ d(w'/p') \\ dn' \end{pmatrix} = \begin{pmatrix} q'd(t/p') \\ d(t/p') \\ -n\beta dq'd(t/p') \end{pmatrix}$$

Solving the matrix system, the impacts of a change in real environmental tax rate by the Home country on output, real wages and the number of varieties (or firms) are derived to be:

$$\frac{dq'}{d(t/p')} = -\frac{\alpha}{\beta[1 - \beta(w'/p') - (t/p')]} < 0, \tag{59}$$

$$\frac{d(w'/p')}{d(t/p')} = -\frac{1}{\beta} < 0, \tag{60}$$

and

$$\frac{dn'}{d(t/p')} = \frac{\alpha n'}{(\alpha + \beta q')[1 - \beta(w'/p') - (t/p')]} > 0. \tag{61}$$

Similarly, for Foreign, the effect of a change in the real environmental tax will be derived to be:[6]

$$\frac{dQ'}{dT} = -\frac{\alpha}{\beta[1 - \beta W' - T]} < 0, \tag{62}$$

$$\frac{dW'}{dT} = -\frac{1}{\beta} < 0, \tag{63}$$

[6] This is based on the fact that the price of Foreign varieties, which is the same across all the varieties in equilibrium, is assumed to be the unity.

and

$$\frac{dN'}{dT} = \frac{\alpha N'}{[\alpha + \beta Q'][1 - \beta W' - T]} > 0. \tag{64}$$

Thus, from the expressions in (59) to (64),

Proposition 4: *: At the free trade equilibrium, if the Home country unilaterally raises its real environmental tax rate and given that $0 < \theta < 1$, there is a fall in its production and real wages while there is an increase in the number of varieties produced in that country. The same results holds for the Foreign country as well.*

The intuition for this result will be as follows. In our model, output and pollution are generated jointly; pollution is proportional to the level of output. Thus, an increase in the real environmental tax is perceived as a rise in the cost of production by each firm. This tends to reduce its output (a negative scale effect) as well as the demand for labor, which is the only factor of production. Moreover, higher costs lead to a fall in the number of firms in the industry (a negative selection effect). This fall in the demand for labor releases labor, thus lowering the wage rate. Furthermore, being monopolists in the market for its variety, each firm passes on a part of this increase in the environmental tax rate to the consumers in terms of higher prices, by a factor of $1/\theta$ (see the discussion on this under autarky). Lower wages and higher prices in the short-run generate positive profits, thus attracting new firms to enter the Home market. This tends to increase the number of varieties produced in Home (a positive selection effect). However, in our case, the positive selection effect dominates the negative selection one, leading to an overall increase in the number of firms/ varieties in the market.

Using a similar IIT framework, Rauscher (1997) also derives the effects associated with a change in environmental policy on the production and number of firms. He too finds that a stricter environmental policy affects production negatively, but it's effect is ambiguous on the number of varieties (or firms). The difference arises because Rauscher (1997) uses a generic cost function, while this study employs a more definitive cost function for analysis, which helps us derive more specific results on the number of firms in the industry.

Moreover,

Proposition 5: *An exogenous and unilateral increase in the stringency of environmental regulation by Home (Foreign) results in a fall in its aggregate pollution at the free trade equilibrium.*

At the free trade equilibrium, the aggregate pollution in (say) Home is:

$$z' = \gamma n' q'. \tag{65}$$

The result in Proposition 5 can be derived by differentiating (65) with respect to real environmental tax and using equations (59) and (61) to get,

$$\frac{\partial z'}{\partial (t/p')} = -\frac{\gamma n' \alpha^2}{\beta(\alpha + \beta q')[1 - \beta(w'/p') - (t/p')]} < 0. \tag{66}$$

And, similarly, for Foreign,

$$\frac{\partial Z'}{\partial T} = -\frac{\gamma N' \alpha^2}{\beta[\alpha + \beta Q'][1 - \beta W' - T]} < 0. \tag{67}$$

The intuition for this result is similar to that for the autarky case (refer to Proposition 2 for this). That is, more implicitly, the differentiation of the expression in (65) with respect to t entails

$$\frac{\partial z'}{\partial t} = \gamma \left[n' \frac{\partial q'}{\partial t} + q' \frac{\partial n'}{\partial t} \right]$$

From the above, it is evident that as the environmental tax rate is raised, it has two opposing effects: first, it reduces the production by each firm (a first-order scale effect) represented by the first term in square brackets in the r.h.s., thus leading to a fall in the level of pollution externality, and, second, it raises the number of varieties being produced (or the number of firms in the country), as shown by the second term in square brackets in the r.h.s., thus leading to a rise in the level of pollution externality (a second-order selection effect). Since the scale effect will dominate the selection effect, it can be inferred that there will be a fall in aggregate pollution as shown in (66).

Similarly, following a similar line of reasoning, the result in (67) can also be explained.

Moreover, we have

Proposition 6: *At the free trade equilibrium, the aggregate output produced in Home falls as a result of an exogenous increase in its environmental tax. Similar results hold for Foreign as well.*

The total output produced in Home can be written as $n'q'$. Differentiating this with respect to t, we get the change in total output due to change in environmental tax as

$$\frac{d(n'q')}{dt} = \frac{\partial q'}{\partial t} n' + \frac{\partial n'}{\partial t} q'.$$

Again using equations (59) and (61), we can write the total change in production or the r.h.s. of the above due to an environmental tax change as:

$$-\frac{n' \alpha^2}{\beta(\alpha + \beta q')[1 - \beta(w'/p') - (t/p')]} < 0.$$

Similar to the reasoning used earlier, the negative scale effect outweighs the positive selection effect and, thus, the aggregate output at the free trade equilibrium declines due to a rise in the real environmental tax. A similar result has been derived by Rauscher (1997) as well. Analogous results can be derived for the Foreign country also.

In the section that follows, the effect of a unilateral environmental tax increase on the pattern of IIT is analyzed.

4 Impact of an Exogenous Change in Environmental Regulation on Share of Intra-Industry Trade

In this section, we seek to find an answer to the question: is the share of intra-industry trade (relative to inter-industry trade) between the Home and Foreign countries affected by a unilateral exogenous increase in the stringency of environmental regulation in any one country. To this end, we first work out an expression for the Grubel-Lloyd (G-L) index, which is the most commonly used indicator of share of IIT (in overall trade) between the two countries, c and d (here Home and Foreign respectively). This can be expressed as:

$$GL_{cd,i} = 1 - \frac{|X_{cd,i} - M_{cd,i}|}{(X_{cd,i} + M_{cd,i})},$$

where $i = 1, 2, ..., n$ is an indicator for the product variety/ firm in the Home industry and X(or M) denotes the exports (or imports) from (to) country c to (from) d respectively. On the one hand, if there exists only inter-industry trade between two countries, $GL_{cd,i}$ takes on a value zero, entailed by the country in consideration either only exporting or only importing good i. On the other hand, if there exists only intra-industry trade then $GL_{cd,i}$ takes a value one, implied by the country in question exporting the same quantity of good i as much at it imports it. Thus, values of G-L index closer to zero (one) indicate more of inter-industry (intra-industry) trade between Home and Foreign.

Next, since Home imports m_I from Foreign and exports M_i to Foreign, substituting these into the G-L index derives the following alternative cases:

Case 1 : Home is a net exporter of ith variety. This means that $M_i > m_I$ and, therefore, the G-L index can be expressed as:

$$GL_{cd,i} = \frac{2m_I}{(M_i + m_I)}.$$

Differentiating the above expression with respect to t, we get,

$$\frac{dGL_{cd,i}}{dt} = \frac{2M_i \frac{dm_I}{dt} - 2m_I \frac{dM_i}{dt}}{(M_i + m_I)^2}.$$

Moreover, we have

$$\frac{dm_I}{dt} = \frac{N'(Q')^{\theta+1}(q')^{\theta-1}[q'\frac{dn'}{dt} + n'\theta\frac{dq'}{dt}]}{[n'(q')^{\theta} + N'(Q')^{\theta}]^2} \lesseqgtr 0,$$

and,

$$\frac{dM_i}{dt} = \frac{[N'n'(Q')^{\theta}(q')^{\theta}(1-\theta)\frac{dq'}{dt} + (N')^2(Q')^{2\theta}\frac{dq'}{dt} - N'(Q')^{\theta}(q')^{\theta+1}\frac{dn'}{dt}]}{[n'(q')^{\theta} + N'(Q')^{\theta}]^2} < 0.$$

Using the the signs of the last two, we get that, in general, $dGL_{cd,i}/dt \gtrless 0$, and the change in the share of IIT in aggregate trade due to a change in the environmental tax is ambiguous. To get a clear sign for this, we use (52) and (53) to write the G-L index as

$$GL_{cd,i} = \frac{2n'(q')^{\theta}Q'}{[n'(q')^{\theta}Q' + N'(Q')^{\theta}q']}.$$

Further, from $R1$, we know that Home's variables are independent of Foreign's policy variables and vice versa for the Foreign's variables. Mathematically, $dq/dT = dn/dT = 0$, and $dQ/dt = dN/dt = 0$. Utilizing this result and differentiating the G-L index with respect to t and T, we get that,

$$\frac{dGL_{cd,i}}{dt} = \frac{2N'(q')^{\theta}(Q')^{\theta+1}[q'\frac{dn'}{dt} + n'(\theta-1)\frac{dq'}{dt}]}{[n'(q')^{\theta}Q' + N'(Q')^{\theta}q']^2},$$

and

$$\frac{dGL_{cd,i}}{dT} = \frac{2n'(q')^{\theta+1}(Q')^{\theta}[-Q'\frac{dN'}{dT} + N'(1-\theta)\frac{dQ'}{dT}]}{[n'(q')^{\theta}Q' + N'(Q')^{\theta}q']^2}$$

Using the results stated in Proposition 4 and with $0 < \theta < 1$, it is easy to see that, $dGL_{cd,i}/dt > 0$ and $dGL_{cd,i}/dT < 0$.

Case 2 : Home is a net importer of the ith variety. Here, $M_i < m_I$ and, therefore, the G-L index is:

$$GL_{cd,i} = \frac{2M_i}{(M_i + m_I)}.$$

Differentiating the above with respect to t, we get that,

$$\frac{dGL_{cd,i}}{dt} = \frac{2m_I \frac{dM_i}{dt} - 2M_i \frac{dm_I}{dt}}{(M_i + m_I)^2}.$$

Again, in view of $\frac{dM_i}{dt} < 0$ and $\frac{dm_I}{dt} \lesseqgtr 0$ we are unable to find a clear direction of change in the share of IIT in overall between the countries. Again, using (52)and (53), we get that,

$$GL_{cd,i} = \frac{2N'(Q')^{\theta}q'}{[n'(q')^{\theta}Q' + N'(Q')^{\theta}q']}.$$

Differentiating the above expression with respect to t and T and using $R1$, one gets that,

$$\frac{dGL_{cd,i}}{dt} = \frac{2N'(q')^{\theta}(Q')^{\theta+1}[-q'\frac{dn'}{dt} + n'(1-\theta)\frac{dq'}{dt}]}{[n'(q')^{\theta}Q' + N'(Q')^{\theta}q']^2}$$

$$\frac{dGL_{cd,i}}{dT} = \frac{2n'(q')^{\theta+1}(Q')^{\theta}[Q'\frac{dN'}{dT} + N'(\theta-1)\frac{dQ'}{dT}]}{[n'(q')^{\theta}Q' + N'(Q')^{\theta}q']^2}$$

Again, using Proposition 4, it is easy to observe that $dGL_{cd,i}/dt < 0$ and $dGL_{cd,i}/dT > 0$.

Proposition 7: *: Assuming that Home is a net exporter and given $0 < \theta < 1$, a unilateral and exogenous increase in the environmental stringency by Home results in a rise in the share of IIT in overall trade between Home and Foreign, while a similar increase in the Foreign's environmental tax rate decreases the share of IIT in aggregate trade between the two trading countries, both measured by the G-L index.*

Intuitively, an exogenous increase in the environmental tax rate by Home leads to a fall in the production of its export variety (this is a first-order negative scale effect). This effect is channelized through the following sub-effects. First, as Home is a net exporter of the ith variety, a higher tax results in a reduction in the competitiveness of Home firms in the world market as its varieties now become costlier than Foreign's, resulting in a fall in its exports (that is, $\frac{dM_i}{dt} < 0$), Second, the higher price of Home varieties induces consumers in Home to change their consumption in favor of the Foreign varieties, thereby having a positive impact on its imports from Foreign (that is, $\frac{dm_I}{dt} > 0$). A lower production in Home reduces the demand for its labor thus lowering the wages and hence the cost of production. This decrease in the cost of production, in addition to the higher taxes and the resultant higher prices in home, attracts new firms to the Home industry (a second-order positive selection effect). Due to the introduction of new varieties in Home, the domestic consumers partly switch back their demand from imported varieties toward Home, leading to a fall in the level of imports ($\frac{dm_I}{dt} < 0$). Consequently, the aggregate effect on imports is second-order and remains ambiguous. However, as expected, the first-order scale effect on exports dominates the second-order effect on imports resulting in a fall in net exports. This raises G-L index, implying a move in favor IIT between Home and Foreign. Additionally, if Foreign raises its environmental tax, the opposite effects will be at work, thus reducing the share of IIT in aggregate trade between Home and Foreign.[7]

On the other hand,

[7] Mathematically, when Home is a net exporter, this can be found by analyzing the derivative of the G-L index

Proposition 8: *: Assuming that Home is a net importer and given that $0 < \theta < 1$, a unilateral exogenous increase in environmental stringency by Home results in a fall in the share of IIT in overall trade, while an increase in the Foreign's environmental tax raises the share of IIT in aggregate trade between the two trading partners, again as captured by G-L index.*

Effects similar to those mentioned in the above proposition are at work here as well. The only difference is that here, Home is a net importer and thus, even though the impact on imports is second-order and ambiguous after the rise in the environmental tax, the effect of the fall in its exports (a first-order effect) dominates, now resulting in a lower share of IIT in aggregate trade between Home and Foreign relative to the case when the environmental tax rate remains unchanged. Similar results hold for Foreign as well.[8]

Propositions 7 and 8 provide an insight into how the pattern of IIT in overall trade, indicated by the G-L index, is affected due to a rise in the environmental policy stringency (in this case an increase in exogenous environmental tax).[9]

We now analyze the changes in aggregate welfare due to an exogenous environmental tax in any country at the two distinct equilibria – autarky and free trade.

5 Aggregate Welfare Impacts of an Exogenous Change in Environmental Regulation

The aggregate welfare impact of an increase in the environmental tax in any one country can be worked out by analyzing the following changes in the individual components of welfare – utility from

with restect to the tax derived to be: $\frac{d}{dt}\frac{2M_{cd,i}}{(X_{cd,i}+M_{cd,i})}$. Dropping the subscripts this will be $\frac{2[XdM/dt-MdX/dt]}{(X+M)^2}$. Since we know that the first-order effect of the fall in exports (dX/dt) dominates the second-order effect on a change in imports (dM/dt), the resultant term in the square bracket is positive implying that the share of IIT in aggregate trade rises with a rise in the environmental tax in Home.

[8]Mathematically, this can be seen more clearly from the following: when Home is a net importer, change in the G-L index is given is $\frac{d}{dt}\frac{2X_{cd,i}}{(X_{cd,i}+M_{cd,i})}$. This is equivalent to $\frac{2[MdX/dt-XdM/dt]}{(X+M)^2}$. Again, we know that the first-order effect of the fall in exports (dX) dominates the second-order effect on a change in imports (dM), hence the resultant term in the square bracket is negative implying that the share of IIT in overall trade falls with a rise in the environmental tax in Home.

[9]Lancaster (1980) uses a model of monopolistic competition to analyze IIT and finds that the protection of the agricultural sector by the country with comparative disadvantage in agriculture might even result in an increase in the IIT in manufactures.

the consumption of final good, disutility from environmental pollution, and finally, environmental tax revenue generated by the government. Note that since the focus of analysis is on the long-run, producers will earn zero profits as the number of firms in the industry would adjust.

First, we consider the case when the two economies are closed. From (1), the utility function of a consumer in autarky is given as follows:

$$u = \sum_{i=1}^{n} c_i^{\theta} + v\left[z + (1-\delta)Z\right], \tag{68}$$

or,

$$u = nc_i^{\theta} + v\left[z + (1-\delta)Z\right]. \tag{69}$$

Since $lc_i = q_i$, the above can be re-expressed as,

$$u = n\Big(\frac{q_i}{l}\Big)^{\theta} + v\left[z + (1-\delta)Z\right]. \tag{70}$$

Further, the total revenue generated through environmental taxation in Home can be expressed as:

$$r = tnq. \tag{71}$$

Adding the above two, the aggregate welfare of Home (denoted by g) will be given by

$$g = n\Big(\frac{q_i}{l}\Big)^{\theta} + v\left[z + (1-\delta)Z\right] + tnq. \tag{72}$$

Differentiating the welfare function w.r.t. t, we get,

$$\frac{dg}{dt} = \left(\frac{q_i}{l}\right)^{\theta}\frac{dn}{dt} + n\theta\left(\frac{q_i}{l}\right)^{\theta-1}\frac{dq}{dt} + \frac{\partial v}{\partial z}\frac{dz}{dt} + nq + tq\frac{dn}{dt} + tn\frac{dq}{dt}. \tag{73}$$

Since Home's environmental tax, t, does not affect Foreign's variables (from $R1$), we have $\frac{dZ}{dt} = 0$. Moreover, the first and the second terms in the r.h.s. denote the selection effect and the scale output effect respectively. Since the scale effect is first-order in nature, it outweighs the second-order selection effect (this is shown later in this section). Also, from our earlier analysis, we know that $\frac{\partial v}{\partial z} < 0$ (since rise in pollution causes disutility to the consumers) and $\frac{dz}{dt} < 0$ (see Proposition 2), thus making this term positive. Finally, the last three terms in the r.h.s. taken together represent the change in revenue due to a change in environmental tax, which also comprises of the scale and selection effects. Substituting from equations (15) and (17), we get the following expression for the change in Home's aggregate welfare under autarky due to a change in its environmental tax rate:

$$\frac{dg}{dt} = \frac{-tw^{\theta}l^{1-\theta}(1-\theta)^{2-\theta}\alpha^{\theta-1}\theta^{\theta+1}(\beta w + t)^{(-\theta)}}{[\beta w + (1-\theta)t]^2} + \frac{\partial v}{\partial z}\frac{dz}{dt} + \frac{\beta w^2 l\theta}{[\beta w + (1-\theta)t]^2}. \tag{74}$$

From the r.h.s. of above, it is easy to observe that the scale effect dominates the selection effect (now the first term) leading to a fall in the total output in Home (see also Proposition 3). The second term is the gain in welfare due to a fall in the disutility from pollution resulting from a rise in environmental tax.[10] The last term is the gain in revenue due to an increase in environmental tax rate. Thus, aggregate welfare change depends on which of these effects dominates: loss from a fall in the total output, gain from lower aggregate pollution and gain in revenue generation, which can be potentially used to offset the effects of distortionary taxes elsewhere in the economy. Hence, the net effect of an increase in environmental tax on aggregate welfare of a country remains ambiguous.

Notably, aggregate welfare in Home (or Foreign) could increase due to a rise in its environmental tax if the marginal disutility due to pollution is very high, that is, if the economy is very sensitive to an increase in pollution. This is because an increase in environmental tax rate in this scenario would provide a large marginal gain to the Home consumers. This effect coupled with the positive revenue generation would overpower the negative scale effect thus increasing the aggregate welfare level of Home. Also, in the special case when $\theta = 1$, the first term in equation (74) vanishes and a definite positive sign of a change in aggregate welfare can be seen. This is because if the varieties are perfect substitutes of each other, then the reduction in output (scale effect) is fully offset by an increase in the number of varieties (selection effect) as now the consumers are indifferent between the old and new varieties. Hence the increase in tax leads to a fall in pollution and a positive revenue generation in the economy implying a rise in aggregate welfare for the Home country.

We next consider the case of free trade. We can write the utility function of the Home consumer as:

$$u' = n'(c_i')^\theta + N'm_I^\theta + v[z' + (1-\delta)Z']. \tag{75}$$

Since the product market clearing condition is now given by $q' = c_i' + M_i$, which can be substituted in the above utility function to derive,

$$u' = n'[q' - M_i]^\theta + N'm_I^\theta + v[z' + (1-\delta)Z']. \tag{76}$$

Similar to the autarky case, we can write the aggregate welfare function of Home (denoted by g') as:

$$g' = n'[q' - M_i]^\theta + N'm_I^\theta + v[z' + (1-\delta)Z'] + tn'q', \tag{77}$$

[10]The actual amount of welfare gain from reduction in aggregate pollution would depend on the specific form of disutility from pollution function, which has not been specified in our analysis.

where the last terms denote the total revenue generated through environmental tax at the free trade equilibrium.

Differentiating the above w.r.t. t, we get,

$$\frac{dg'}{dt} = (c_i')^{\theta}\frac{dn'}{dt}+n'\theta[q'-M_i]^{\theta-1}\left[\frac{dq'}{dt}-\frac{dM_i}{dt}\right]+N'\theta m_I^{\theta-1}\frac{dm_I}{dt}+\frac{\partial v}{\partial z'}\frac{dz'}{dt}+n'q'+tq'\frac{dn'}{dt}+tn'\frac{dq'}{dt}. \quad (78)$$

As the Home's environmental tax does not impact Foreign's pollution (from $R1$), $\frac{dZ'}{dt} = 0$. Similar to autarky, the first term in the r.h.s. of the above expression is again the selection effect which is positive, from $\frac{dn'}{dt} > 0$ (see equation (61)). The second term refers to the change in the demand for Home's varieties w.r.t. a change in the environmental tax. This term is different from the autarky case in that it additionally incorporates the impact of the change in environmental tax on Home's exports. The third term in the r.h.s. denotes the impact on Home's imports. Furthermore, since $\frac{\partial v}{\partial z'} < 0$ and $\frac{dz'}{dt} < 0$ (see equation (**??**)), it makes the fourth term positive implying a gain in welfare due to a fall in pollution. Finally, the last three terms comprise the impact of a change in environmental tax on revenue at the free trade equilibrium. Each of these terms are analyzed in detail below.

To ascertain the signs of $\frac{dm_I}{dt}$ and $\frac{dM_i}{dt}$, we utilize equations (52) and (53), which are differentiated w.r.t. t to get (79) and (80) below.

First,

$$\frac{dm_I}{dt} = \frac{N'(Q')^{\theta+1}(q')^{\theta-1}[q'\frac{dn'}{dt}+n'\theta\frac{dq'}{dt}]}{[n'(q')^{\theta}+N'(Q')^{\theta}]^2} \lesseqgtr 0. \quad (79)$$

In the above, the change in a country's imports due to a change in its tax rate also works through the scale and the selection effects (as discussed in the previous section) and its sign will depend on which of these two effects dominate. Further,

$$\frac{dM_i}{dt} = \frac{[N'n'(Q')^{\theta}(q')^{\theta}(1-\theta)\frac{dq'}{dt}+(N')^2(Q')^{2\theta}\frac{dq'}{dt}-N'(Q')^{\theta}(q')^{\theta+1}\frac{dn'}{dt}]}{[n'(q')^{\theta}+N'(Q')^{\theta}]^2} < 0. \quad (80)$$

As discussed in the previous section, we are able to derive a definitive negative sign for the change in Home's exports due to an increase in its environmental tax. Further, combining equations (59) and (80), we can get the net impact of a change in environmental tax on the demand for Home's varieties to be negative, that is,

$$\frac{dq'}{dt}-\frac{dM_i}{dt} = -\frac{\alpha n'(q')^{\theta}[n'(q')^{\theta}(\alpha+\beta q')+\alpha(1+\theta)N'(Q')^{\theta}+\theta\beta N'(Q')^{\theta}q']}{[n'(q')^{\theta}+N'(Q')^{\theta}]^2} < 0. \quad (81)$$

From the above, it follows that even though $\frac{dM_i}{dt} < 0$, the negative scale effect outweighs the fall in exports, resulting in a reduction in the consumption of Home's variety by its own consumer. Intuitively, a rise in the environmental tax in Home reduces production of Home varieties resulting in a fall in its exports as well. Also, since the tax rate rise results in higher prices of Home varieties, they lose competitiveness in the world market, leading to a reduction in their demand in Foreign as well as domestic markets. Thus, consumption of Home variety by Home consumers falls.

Finally, the last three terms in equation (78) entail a positive change in aggregate revenue generation due to a rise in environmental tax by Home, thus providing a gain in its aggregate welfare.

In sum, the overall sign of the change in (78) has the following effects at work: a gain in welfare from an increase in the number of varieties, from higher tax revenue generation and from an increase in environmental tax rate on one hand, while a loss in welfare due to decreased production and exports on the other. In addition, the effect of an increase in environmental tax does not have a clear impact on imports, thus exacerbating the ambiguity already present in the change in welfare expression.

We next differentiate the aggregate welfare of Home w.r.t. the Foreign's environmental tax rate, T, to get,

$$\frac{dg'}{dT} = n'\theta(q' - M_i)^{\theta-1}\left(-\frac{dM_i}{dT}\right) + N'\theta m_I^{\theta-1}\frac{dm_I}{dT} + m_I^{\theta}\frac{\partial N'}{\partial T} + (1-\delta)\frac{\partial v}{\partial Z'}.\frac{dZ'}{dT}. \tag{82}$$

In the above expression, the first two terms in the r.h.s. refer to a change in Home's exports and imports respectively due to a change in Foreign's environmental tax. The third term is the impact of an increase in the number of Foreign's varieties due to a rise in its environmental tax. Since $\frac{dN'}{dT} > 0$ (see equation (64)), this implies a gain in welfare for Home consumers as there is an expansion in their consumption possibilities. The final term in the r.h.s. is the welfare gain for the Home country through a fall in Foreign's pollution due to an increase in its environmental tax, which in turn leads to a fall in disutility of consumers in the Home country via the positive pollution spillover effect.[11]

We next derive how Home's exports and imports change w.r.t. a change in Foreign's environmental tax, for which we differentiate equations (52) and (53) w.r.t. T to get,

$$\frac{dm_I}{dT} = \frac{[N'n'(Q')^{\theta}(q')^{\theta}(1-\theta)\frac{dQ'}{dT} + (n')^2(q')^{2\theta}\frac{\partial Q'}{\partial T} - n'(q')^{\theta}(Q')^{\theta+1}\frac{dN'}{dT}]}{[n'(q')^{\theta} + N'(Q')^{\theta}]^2} < 0, \tag{83}$$

[11] Again, the amount of welfare gain on this account would depend on the exact functional form used for disutility as well as the value of the pollution spillover parameter (δ), which is not explicitly modeled here.

The sign of the above change follows from $(1-\theta) > 0$, $\frac{\partial Q'}{\partial T} < 0$ and $\frac{\partial N'}{\partial T} > 0$ (see equations (62) and (64)). Also, intuitively, since Foreign's varieties become costlier and lose competitiveness in the world market, their demand falls. Thus, Home's imports fall as the Foreign country raises its environmental tax, which lowers Home's aggregate welfare. Further,

$$\frac{dM_i}{dT} = \frac{n'(q')^{(\theta+1)}(Q')^{(\theta-1)}[Q'\frac{\partial N'}{\partial T} + N'\theta\frac{\partial Q'}{\partial T}]}{[n'(q')^{\theta} + N'(Q')^{\theta}]^2} \lesseqgtr 0. \tag{84}$$

The above change is ambiguous in sign as it depends on whether the negative scale effect or the positive selection effect dominates (see equations (62) and (64) for this). This is because a higher tax in Foreign reduces its production and hence consumers in Foreign demand more Home varieties as they are relatively cheaper leading to a rise in Home's exports. Additionally, selection effect induces new firms to enter the Foreign market thus expanding the choice of consumption possibilities for Foreign consumers, thus decreasing the demand for Home varieties and hence Home's exports.

In sum, the change in Home's aggregate welfare to an increase in Foreign's environmental tax rate comprises the gain in welfare from an enlarged set of consumption varieties and a fall in the Foreign's aggregate pollution, loss in welfare from a reduction in the level of imports and an ambiguous change in the level of exports. The direction of the change in Home's aggregate welfare is thus contingent on which of these effects predominates, and is, in general, indeterminate. Hence,

Proposition 9: *: An exogenous increase in the environmental tax rate by Home (Foreign) leads to ambiguous impacts on its aggregate welfare at both autarky and free trade equilibria. Also, the impact on Home's (Foreign's) aggregate welfare is again ambiguous with respect to a change in in Foreign's (Home's) environmental tax rate.*

6 Conclusion

The paper analyzes the complex interactions between international trade and environmental pollution, specifically in the context of IIT. A bulk of the existing theoretical and empirical literature is devoted to the analysis of linkages between inter-industry trade and the environment. In models of inter-industry, environmental policy affects trade flows through three channels: scale, technique and composition effects. The key premise is that countries trade due to differences in relative factor endowments, as also environmental policy, which is incorporated as an additional factor in determining the trade pattern. But research focusing on intra-industry trade and environment is somewhat scarce. Similar to inter-industry trade, intra-industry trade affects environment through three channels: scale, technique and selection effects, for which scanty literature exists thus far, in spite of the growing importance of IIT between countries.

To bridge the gap in the literature, this paper formulates a model of international trade and environmental pollution arising in production, which is an extension of the Krugman's model of monopolistic competition. We find that at both autarky and free trade equilibria, a unilateral exogenous increase in environmental tax rate by a country leads to a fall in its production, increase in the number of varieties and a fall in aggregate pollution in that country provided that elasticity of substitution between varieties is low. Additional, there are two key results that pertain to the change in the share of IIT due to an exogenous rise in environmental policy stringency. First, when Home is a net exporter and the varieties are substituted inelastically, an increase in environmental stringency by it leads to a rise in its share of IIT with Foreign, while an increase in the Foreign's environmental tax reduces its IIT share as measured by the G-L index. Second, assuming that Home is a net importer, and again the product varieties are inelastically substituted for each other, an increase in environmental stringency by Home leads to a reduction in its share of IIT with Foreign, while an increase in the Foreign's environmental tax raises its share of IIT.

Furthermore, the impacts on aggregate welfare due to a change in environmental policy by one country are also worked out. The main finding is that a rise in the environmental tax by a country leads to ambiguous impacts on its aggregate welfare as well as the welfare of the trading partner. This is due to the counteracting effects on welfare ascribable to a fall in the level of production (scale effect), increase in number of varieties (selection effect) and positive effects of lower aggregate pollution and higher tax revenue generation. Besides these, at the free trade equilibrium, there are two additional effects which are at work via changes in the levels of exports and imports due to an environmental tax change. The resultant change in aggregate welfare is, in general, ambiguous, and depends upon which of these effects dominates.

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Mathematical Appendix I

Maximization of utility of a consumer in Home country yields three FOCs:

$$p' = (\lambda^{-1}\theta)(c_i')^{\theta-1}; \tag{85}$$

$$P' = (\lambda^{-1}\theta)(m_I)^{\theta-1}; \tag{86}$$

$$w' = \sum p'c_i' + \sum P'm_I \tag{87}$$

Also from the two FOCs we can write,

$$\frac{p'}{P'} = (\frac{c_i'}{m_I})^{\theta-1} \tag{88}$$

Similarly, the maximization of utility of a Foreign consumer yields the following FOCs:

$$P' = (\lambda^*)^{-1}\theta(C_I')^{\theta-1}; \tag{89}$$

$$p' = (\lambda^*)^{-1}\theta(M_i)^{\theta-1}, \tag{90}$$

$$W' = \sum P'C_I' + \sum p'M_i \tag{91}$$

And using the two FOCs we can write,

$$\frac{P'}{p'} = (\frac{C_I'}{M_i})^{\theta-1} \tag{92}$$

Using the first order equations, we have,

$$p'c_i' = \lambda^{-1}\theta(c_i')^{\theta}; \tag{93}$$

and

$$P'm_I = \lambda^{-1}\theta(m_I)^{\theta}; \tag{94}$$

Substituting these in Home country's budget constraint (or the third FOC), we get,

$$w' = \sum \lambda^{-1}\theta(c_i')^{\theta} + \sum \lambda^{-1}\theta m_I^{\theta} \tag{95}$$

Since consumption is same across all varieties, we can write,

$$\lambda^{-1}\theta = \frac{w'}{[n'(c_i')^{\theta} + N'm_I^{\theta}]} \tag{96}$$

Substituting this value in the FOCs for Home and using equation(79), we get the demand functions of Home and Foreign varieties by a Home consumer (equations (37) and (38)).
Similarly, solving for the Foreign country, we get,

$$(\lambda^{\star})^{-1}\theta = \frac{W'}{[N'(C_I')^{\theta} + n'M_i^{\theta}]} \tag{97}$$

Substituting this value in the FOCs of Foreign country consumer and using equation (83), we can get the demand functions of Home and Foreign varieties by a Foreign consumer (equations (39) and (40)).

Mathematical Appendix II

Using the FOCs for Home and Foreign country we can write,

$$c_i' = [p'\lambda\theta^{-1}]^{\frac{1}{\theta-1}}; \tag{98}$$

$$M_i = [p'\lambda^{\star}\theta^{-1}]^{\frac{1}{\theta-1}}; \tag{99}$$

$$C_I' = [P'\lambda^{\star}\theta^{-1}]^{\frac{1}{\theta-1}}; \tag{100}$$

$$m_I = [P'\lambda\theta^{-1}]^{\frac{1}{\theta-1}}; \tag{101}$$

The product market clearing equation for Home and Foreign are as following:

$$q' = c_i' + M_i, \tag{102}$$

$$Q' = C_I' + m_I. \tag{103}$$

Substituting for c_i and M_i in the product market clearing equation of Home, we get,

$$q' = (p')^{1/(\theta-1)}[(\lambda\theta^{-1})^{1/(\theta-1)} + (\lambda^{*}\theta^{-1})^{1/(\theta-1)}]. \tag{104}$$

Using the budget constraint and FOCs for the Home country, we find that,

$$(\lambda\theta^{-1})^{\frac{1}{\theta-1}} = \frac{w'}{\sum(p')^{\frac{\theta}{\theta-1}} + \sum(P')^{\frac{\theta}{\theta-1}}} \tag{105}$$

Similarly, using budget constraint and FOCs for Foreign country, we get,

$$(\lambda^{\star}\theta^{-1})^{\frac{1}{\theta-1}} = \frac{W'}{\sum(p')^{\frac{\theta}{\theta-1}} + \sum(P')^{\frac{\theta}{\theta-1}}} \tag{106}$$

Substituting these values in product market clearing equation for Home, we get,

$$q' = (p')^{1/(\theta-1)}[w'\psi + W'\psi] \tag{107}$$

where,

$$\psi = \frac{1}{\sum(p')^{\frac{\theta}{\theta-1}} + \sum(P')^{\frac{\theta}{\theta-1}}}$$

Now taking natural logarithm of the above equation on both sides, we get,

$$lnq' = \frac{1}{\theta-1}lnp' + ln[\phi] \tag{108}$$

where,

$$\phi = [w'\psi + W'\psi]$$

is a constant.

Thus, we can write the price elasticity of demand as:

$$-\frac{\partial lnq'}{\partial lnp'} = \frac{1}{1-\theta} \tag{109}$$

which is the same as the case of autarky.